Towards a Stability Analysis of Inhomogeneous Phases in QCD

Theo F. Motta (JLU Gießen & TU Darmstadt)

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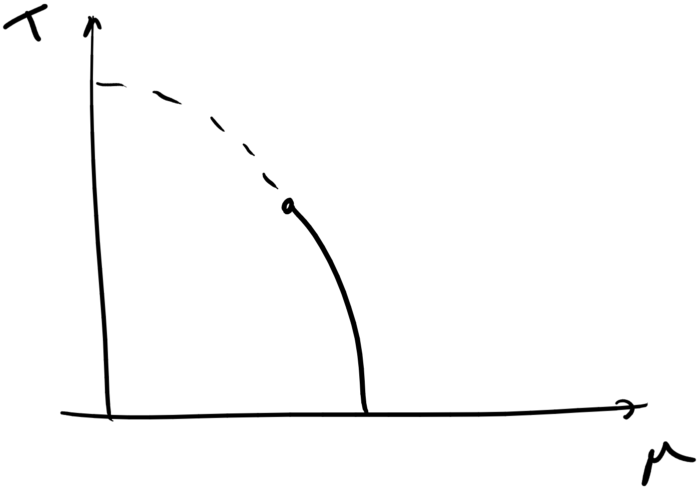
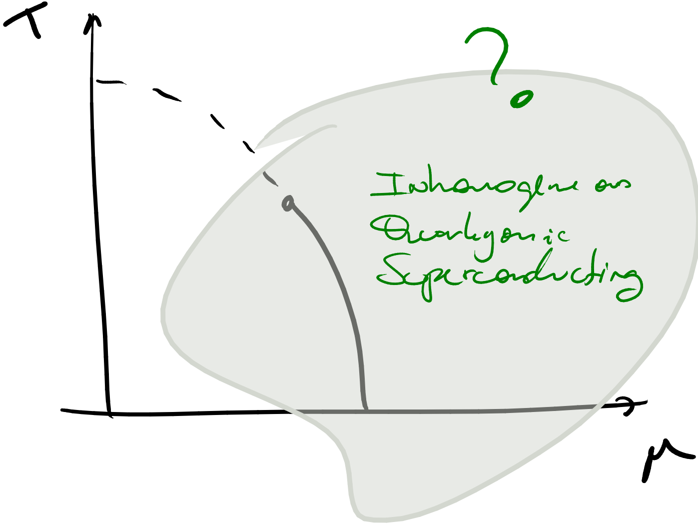
in collaboration with C.S. Fischer, M. Buballa & J. Bernhardt
ECT* workshop 2023

Overview of Inhomogeneous Phases

Inhomogeneous Phases



Inhomogeneous Phases



How to Study *Inhomogeneous* Phases?

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- Usually, with Models of QCD:
  - Gross-Neveu
  - NJL
  - QM

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- By Direct Ansatz
- By Stability Analysis

NJL

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left\{(\bar{\psi}\psi)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right\}$$

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$$\phi_S(\mathsf{x}) = \langle\bar{\psi}(\mathsf{x})\psi(\mathsf{x})\rangle, \quad \phi_P(\mathsf{x}) = \left\langle\bar{\psi}(\mathsf{x})i\gamma_5\tau^3\psi(\mathsf{x})\right\rangle$$

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$$\Omega_{\mathrm{MF}} = -\frac{T}{V}\,\mathrm{Tr}\log\left(\frac{S^{-1}}{T}\right) + G\frac{1}{V}\int d^3x\left(\phi_S^2(\mathsf{x}) + \phi_P^2(\mathsf{x})\right)$$

Ansatz

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- Chiral Density Wave:

$$\phi_S(\vec{x}) = -\frac{\Delta}{2G_S}\cos(\vec{q}\cdot\vec{x}), \qquad \phi_P(\vec{x}) = -\frac{\Delta}{2G_P}\sin(\vec{q}\cdot\vec{x})$$

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- Real-Kink-Crystal:

$$M(x) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta x|\nu)$$

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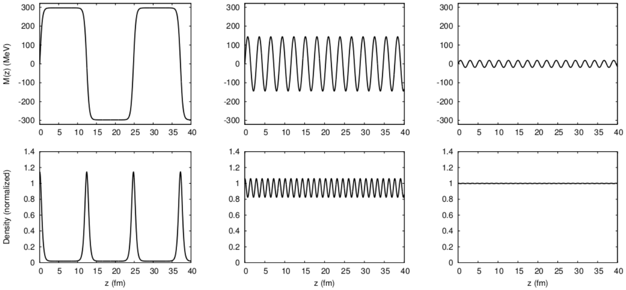

Ansatz

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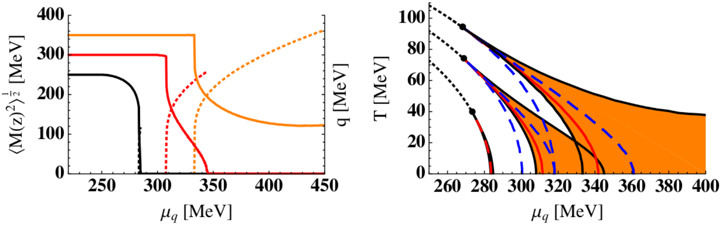
Inhomogeneous phases in the Nambu–Jona-Lasinio and quark-meson model

Dominik Nickel

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 10 July 2009; published 22 October 2009)



Stability Analysis

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$$\phi_S(\mathsf{x}) = \bar{\phi}_S + \delta\phi_S(\mathsf{x}), \quad \phi_P(\mathsf{x}) \equiv \delta\phi_P(\mathsf{x})$$

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$$\Omega^{(1)} \equiv 0$$

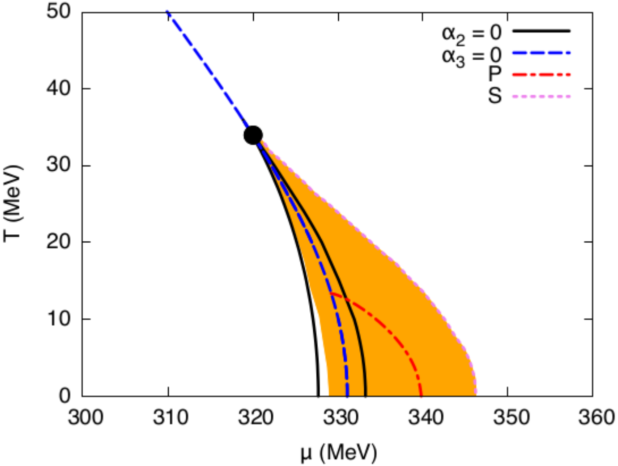
$$\Omega^{(2)} = 2G^2 \sum_{\mathsf{q}_k} \left\{ \left|\delta\phi_{S,\mathsf{q}_k}\right|^2 \Gamma_S^{-1}\left(\mathsf{q}_k^2\right) + \left|\delta\phi_{P,\mathsf{q}_k}\right|^2 \Gamma_P^{-1}\left(\mathsf{q}_k^2\right) \right\}$$

Inhomogeneous chiral phases away from the chiral limit

Michael Buballa[1] and Stefano Carignano[2]

[1] *Theoriezentrum, Institut für Kernphysik, Technische Universität Darmstadt, Schlossgartenstr. 2, D-64289 Darmstadt, Germany*

[2] *Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Catalonia, Spain.*



Quantum Chromodynamics

Stability Analysis

Stability Analysis

- We start from a 2PI effective action

$$\Gamma[S] = \mathrm{Tr}\log\left[S^{-1}\right] - \mathrm{Tr}\left[\mathbf{1} - S_0^{-1}S\right] + \Phi_{2PI}[S]$$

- We start from a 2PI effective action

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and we perturb the propagator (test-function)

$$S(x,y) = \bar{S}(x,y) + \delta S(x,y)$$

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- Fundamentally, we want a Taylor expansion

$$\begin{aligned} F[\varphi(u)] =& F[\varphi_0(u)] + \mathrm{Tr}\left[\left.\frac{\delta F[\varphi(u)]}{\delta\varphi(u')}\right|_{\varphi=\varphi_0} \times (\varphi(u) - \varphi_0(u'))\right] \\ &+ \frac{1}{2!}\mathrm{Tr}\left[\left.\frac{\delta^2 F[\varphi(u)]}{\delta\varphi(u')\delta\varphi(u'')}\right|_{\varphi=\varphi_0} \times (\varphi(u) - \varphi_0(u')) \times (\varphi(u) - \varphi_0(u''))\right] + \cdots \end{aligned}$$

- So zero-th order is

$$\Gamma^{(0)} = -\mathrm{Tr}\log[\bar{S}] - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}\bar{S}\right] + \Phi_{2\mathrm{PI}}[\bar{S}] = \Gamma[\bar{S}]$$

Stability Analysis

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- First order is zero, as it should

$$\Gamma^{(1)} = \mathrm{Tr}\left[\overline{\frac{\delta\Gamma}{\delta S}}\delta S\right] = \mathrm{Tr}\left[\left(\bar{S}^{-1} - S_0^{-1} - \overline{\frac{\delta\Phi_{2PI}}{\delta S}}\right)\delta S\right]$$

Stability Analysis

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- Second order is the leading order

$$\Gamma^{(2)} = \frac{1}{2!}\mathrm{Tr}\left[\overline{\frac{\delta^2\Gamma}{\delta S\delta S}}\delta S\delta S\right] = \frac{1}{2}\mathrm{Tr}[(\bar{S}^{-1}\delta S)^2] + \frac{1}{2}\mathrm{Tr}\left[\overline{\frac{\delta^2\Phi_{2PI}}{\delta S_{12}\delta S_{34}}}\delta S_{12}\delta S_{34}\right]$$

Proofs of Principle

- Can this formalism reproduce the NJL stability analysis?

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Yes.

Proofs of Principle

- Can this formalism reproduce the *homogeneous* chiral phase transition?

A Test Case: The Chiral Phase Transition

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$$\Gamma_\nu(k, q; l) = Z_{1F}\, \gamma_\nu \, .$$

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- Not dynamical symmetric gluon

$$D^{ab}_{\mu\nu}(l) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{l_\mu l_\nu}{l^2} \right) D(l)$$

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- Watson Model

$$D(l) = \frac{(Z_2)^2}{g^2\,(Z_{1F})^2} \frac{8\pi^2}{\omega^4} D e^{-l^2/\omega^2}$$

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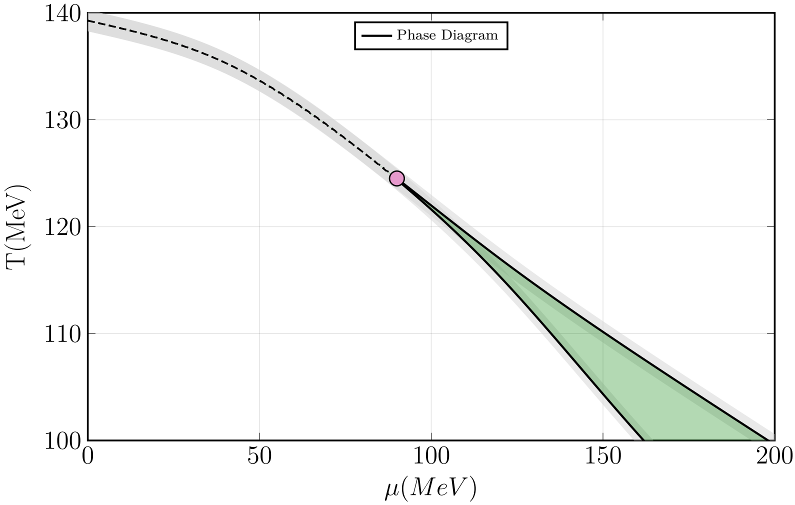

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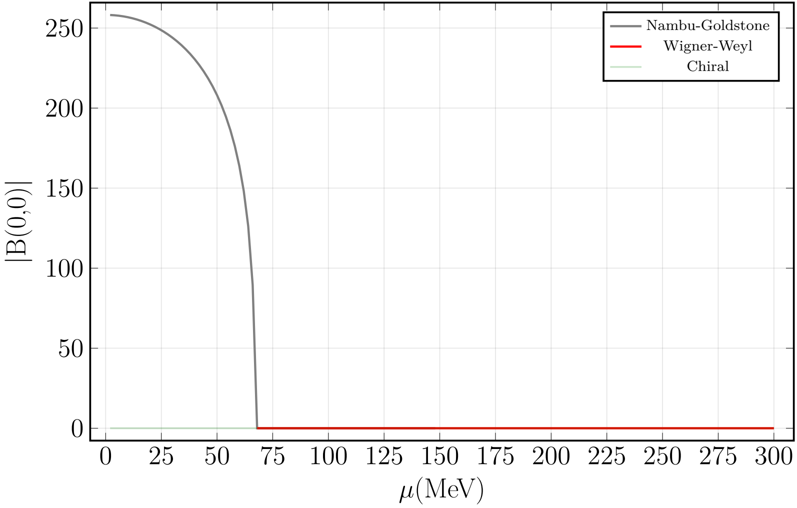

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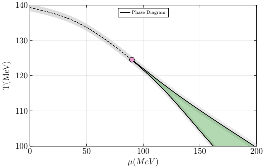

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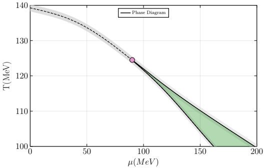

- Chiral

$$S(k) = \frac{-i\vec{k}A_k - i(\omega + i\mu)\gamma_4 C_k}{\vec{k}^2 A_k^2 + (\omega + i\mu)^2 C_k^2}$$

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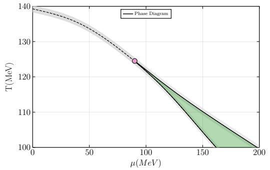

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- Chiral Broken

$$S = S_{\text{chiral}} + \delta S_{\text{breaks}}$$

$$\delta S_{\text{breaks}} = \frac{\delta m(k)}{\vec{k}^2 A_k^2 + (\omega + i\mu)^2 C_k^2}$$

Conditions on the test-function

- Let's look at my stability condition ($\Omega \propto -\Gamma$)

$$\Omega^{(2)}_{\mu}[\delta m] = \sum\!\!\!\!\!\!\!\int_k \left(4\frac{\delta m(k)^2}{d(k)} - 12 C_F Z_2^2 \sum\!\!\!\!\!\!\!\int_q \frac{\delta m(k)}{d(k)} \frac{\delta m(k-q)}{d(k-q)} \mathcal{G}(q) \right)$$

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So the $\sum_\omega$ is real. Otherwise $\Omega^{(2)}$ could turn out complex.

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  So the $\sum_{\omega}$ is real. Otherwise $\Omega^{(2)}$ could turn out complex.
- Also the imaginary part of the test-function has to be fixed

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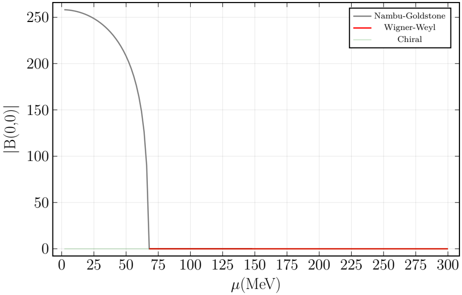

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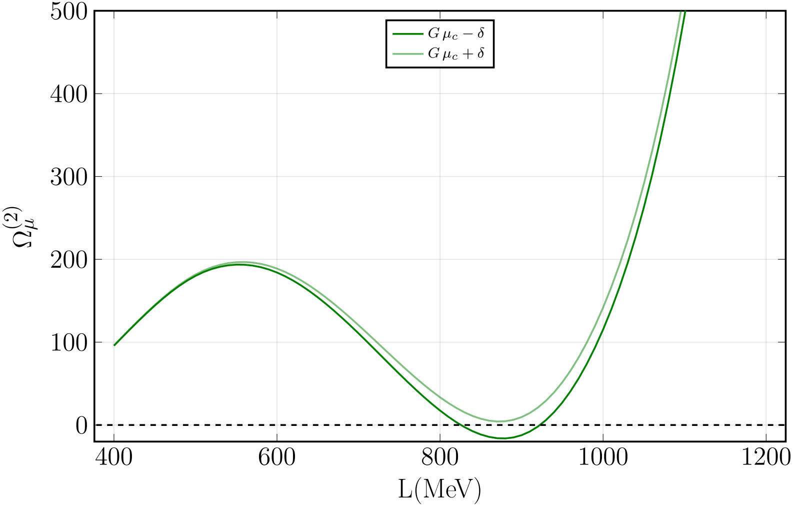
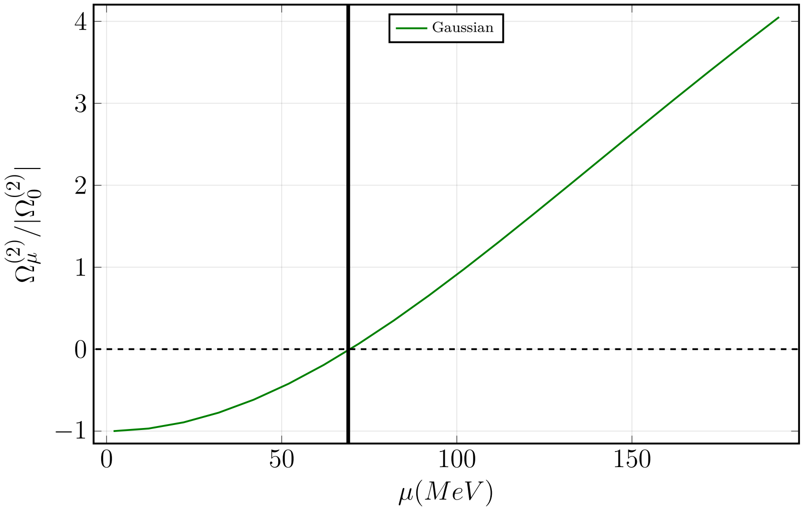


$G\,\mu_c - \delta$
$G\,\mu_c + \delta$
500
400
300
200
100
0
$\Omega_\mu^{(2)}$
400
600
800
1000
1200
L(MeV)

The Test



The Test

- What if we didn't know what the ”real answer” was?

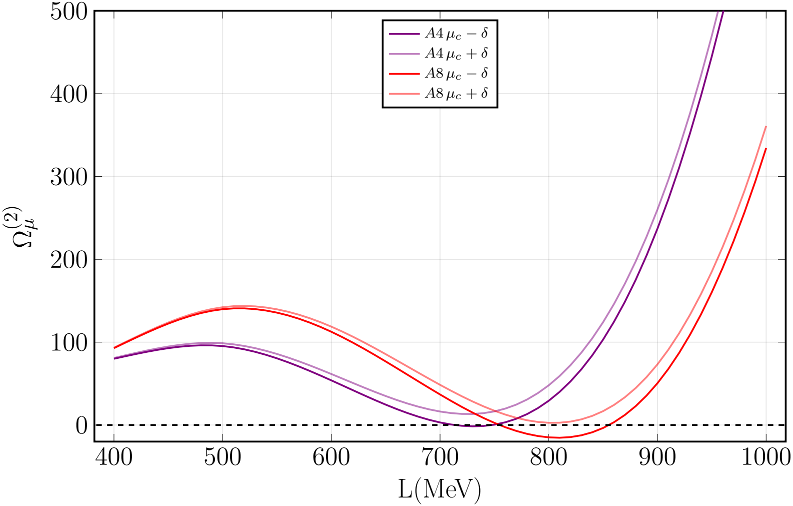
The Test

- What if we didn't know what the "real answer" was?
- Take an "Algebraic decaying function"

$$\delta m(k) = \lambda \left(1 + \frac{k^2}{L^2}\right)^{-N}$$

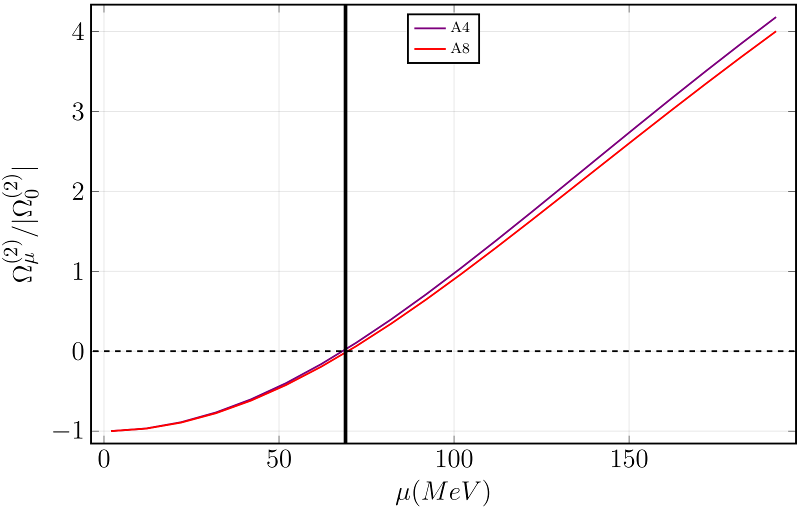
with $N = 2, 3, 4, \ldots$

The Test



$A4\,\mu_c - \delta$
$A4\,\mu_c + \delta$
$A8\,\mu_c - \delta$
$A8\,\mu_c + \delta$
$\Omega_\mu^{(2)}$
500
400
300
200
100
0
400
500
600
700
800
900
1000
L(MeV)

The Test



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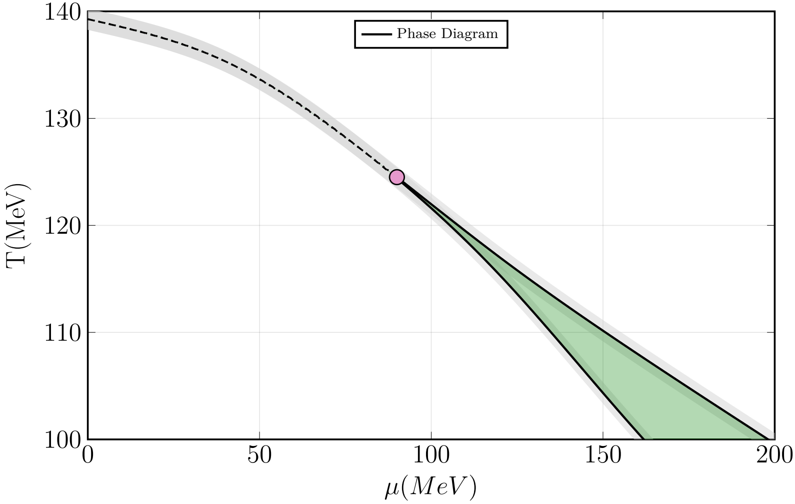

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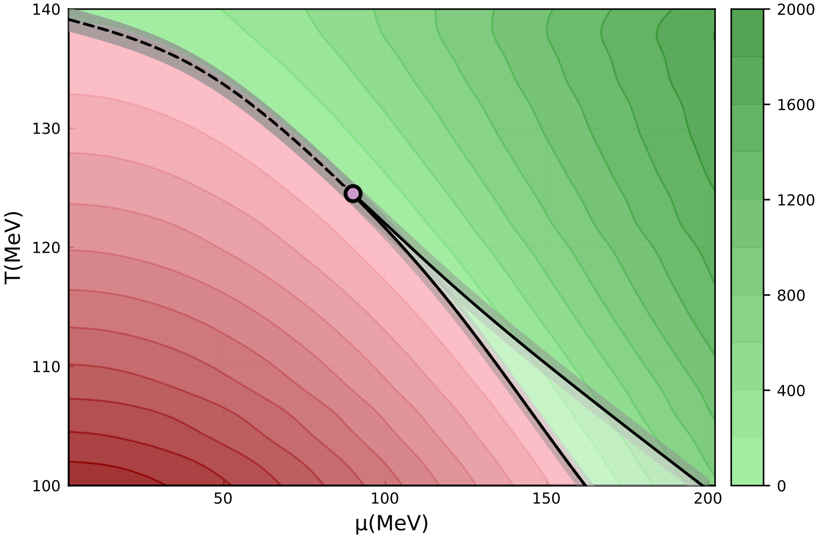
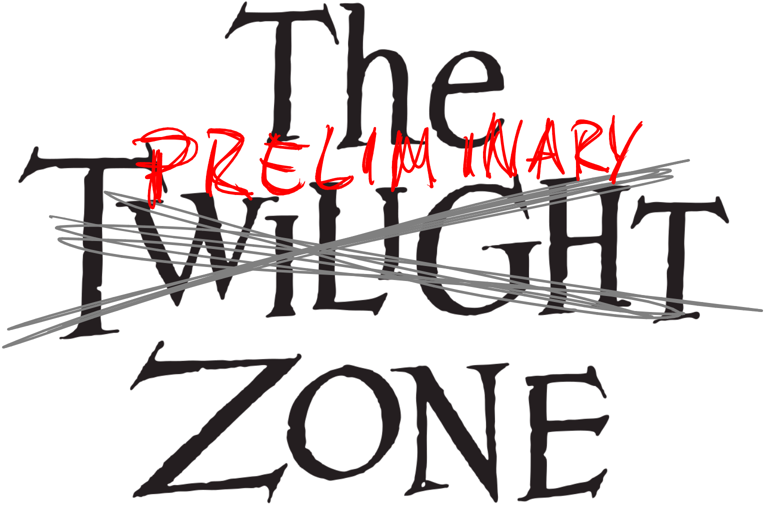


140
130
120
110
100
T(MeV)
50
100
150
200
μ(MeV)
2000
1600
1200
800
400
0

Inhomogeneous Tests

Inhomogeneous Tests



Inhomogeneous Tests

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$$\delta S(k_1, k_2) = H(k_1, k_2)F(k_1 - k_2)$$

  where H is symmetric. This way I can be completely agnostic with respect to F.

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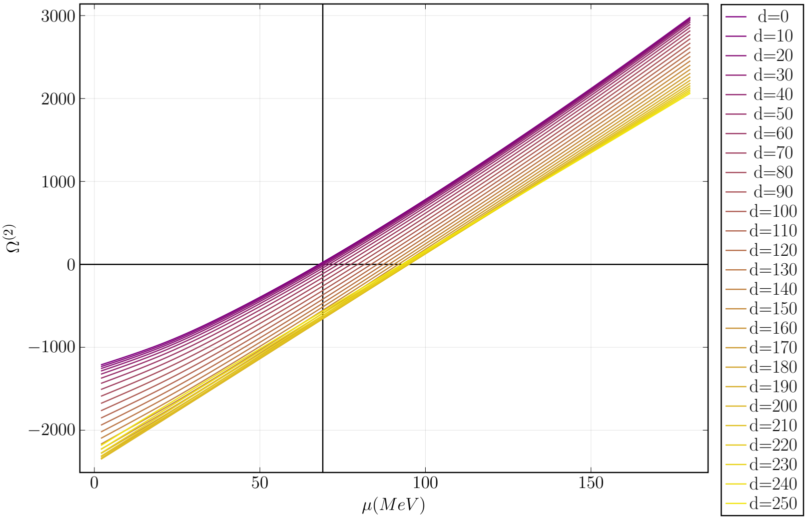
  where H is symmetric. This way I can be completely agnostic with respect to F.
- It’s nice if when $d = k_1 - k_2 = 0$, I recover my previous test-function...

$$H(k, k) = \frac{\delta m(k)}{\vec{k}^2 A_k^2 + (\omega + i\mu)^2 C_k^2}$$

Inhomogeneous Tests



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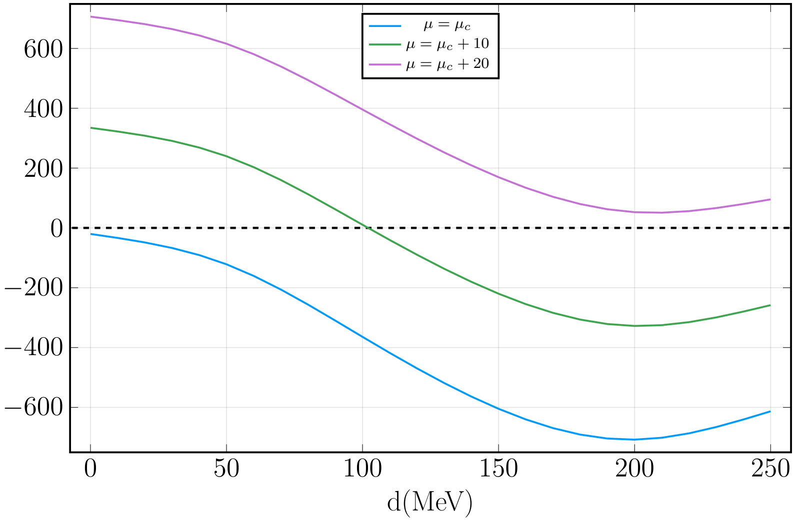


Outlook: Towards Inhom. Phases

- The goal now is to apply this to inhomogeneous phases.
- Some preliminar results: Too simple, too bad.
    - No local fluctuations
    - Beyond local, Watson model is too simplistic
    - Gluons have to be dynamic
    - Etc...
- Results within Watson and with an improved truncations to be released soon.

Thanks!

Backups

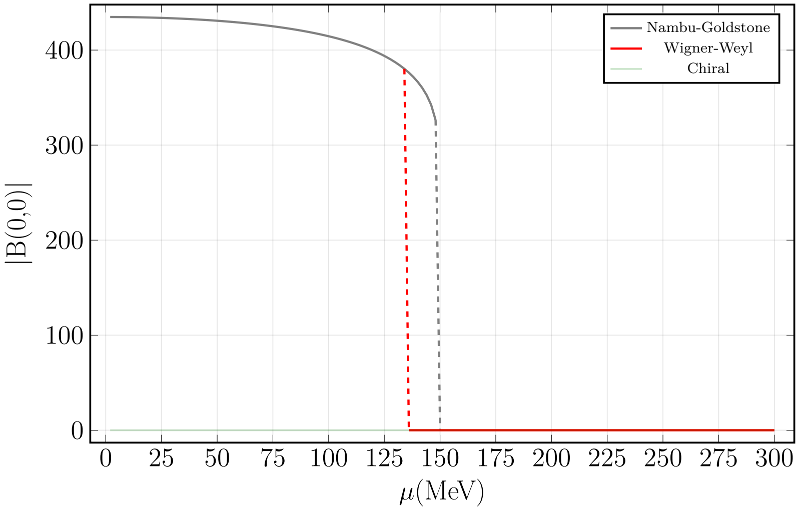
Inhomogeneous Tests

- Test Functions?
  $\delta S(\omega_1, \vec{k}_1, \omega_2, \vec{k}_2)^\dagger = \gamma_4 \delta S(-\omega_2, \vec{k}_2, -\omega_1, \vec{k}_1)\gamma_4$

$$\text{test function 1:} \quad \delta S(k_1, k_2) = \left(\frac{\delta m(k_1)}{\mathrm{d}(k_1)} + \frac{\delta m(k_2)}{\mathrm{d}(k_2)}\right) F(k_1 - k_2)$$

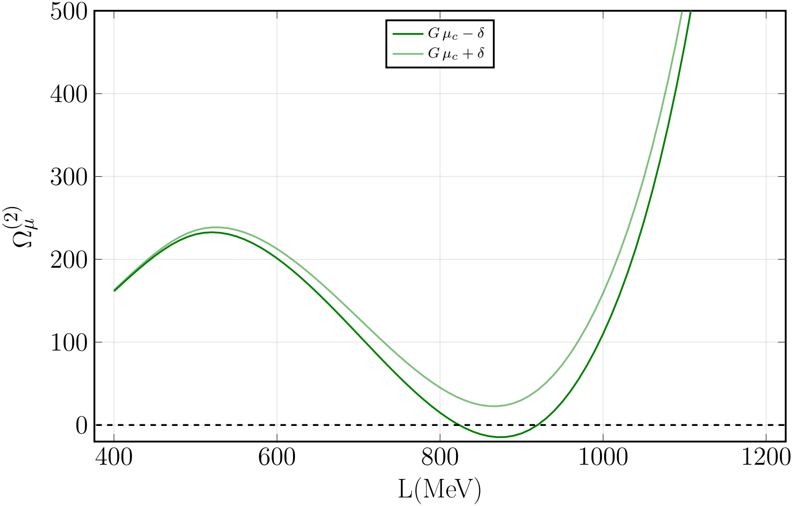
$$\text{test function 2:} \quad \delta S(k_1, k_2) = \left(\frac{\delta m(k_1 + k_2)}{\mathrm{d}(k_1 + k_2)}\right) F(k_1 - k_2)$$

Lower T



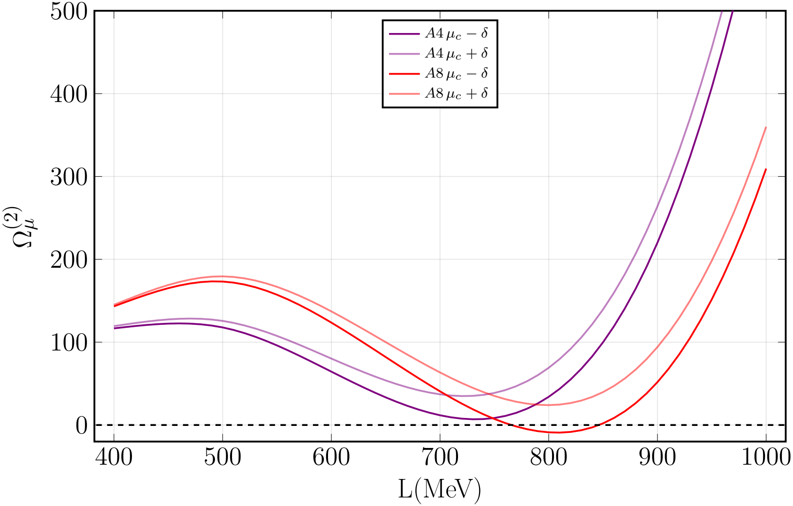
Nambu-Goldstone
Wigner-Weyl
Chiral
400
300
200
100
0
|B(0,0)|
0
25
50
75
100
125
150
175
200
225
250
275
300
μ(MeV)

Lower T



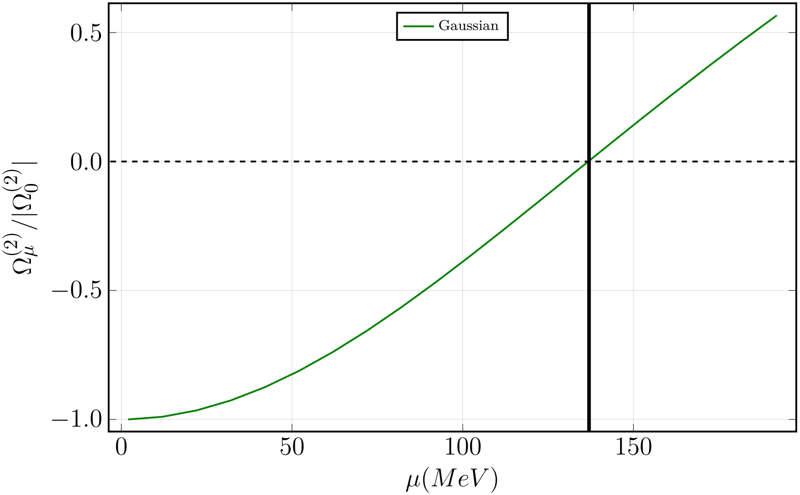
$G\,\mu_c - \delta$
$G\,\mu_c + \delta$
500
400
300
200
100
0
$\Omega_\mu^{(2)}$
400
600
800
1000
1200
L(MeV)

Lower T

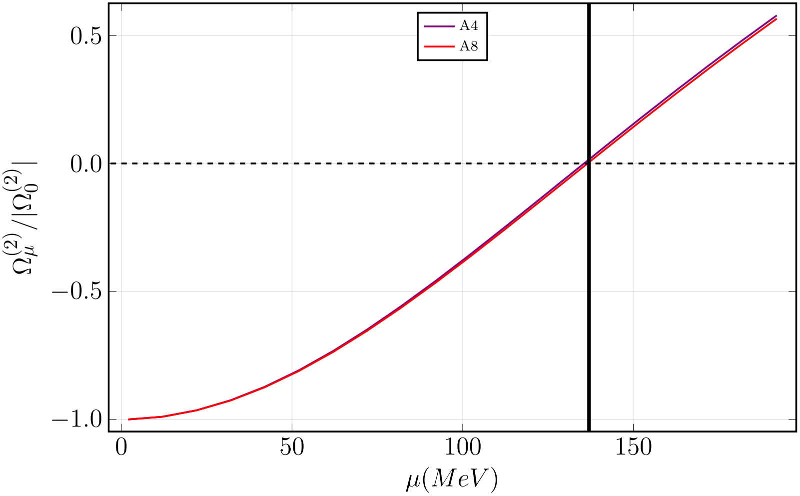


$A4\,\mu_c - \delta$
$A4\,\mu_c + \delta$
$A8\,\mu_c - \delta$
$A8\,\mu_c + \delta$
500
400
300
200
100
0
$\Omega_\mu^{(2)}$
400
500
600
700
800
900
1000
L(MeV)

Lower T



Lower T

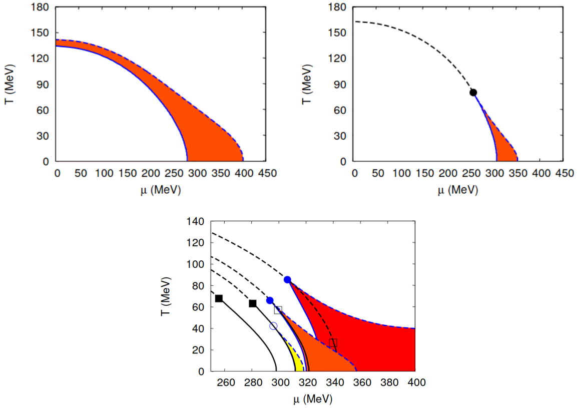


Fluctuations and m_σ in QM

Inhomogeneous phases in the quark-meson model with vacuum fluctuations

Stefano Carignano,[1] Michael Buballa,[2] and Bernd-Jochen Schaefer[3]

[1] *Department of Physics, The University of Texas at El Paso, USA*
[2] *Theoriezentrum, Institut für Kernphysik, Technische Universität Darmstadt, Germany*
[3] *Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, Germany*



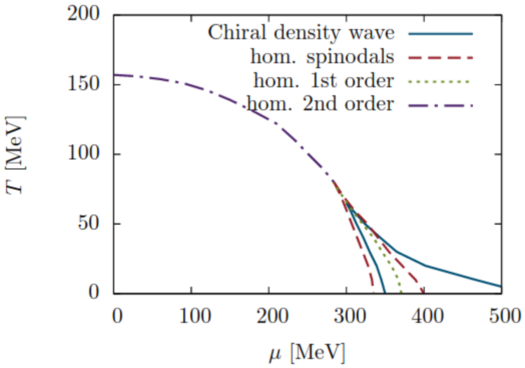
How about QCD?

How about QCD?

Dyson-Schwinger study of chiral density waves in QCD

D. Müller[a], M. Buballa[a], J. Wambach[a,b]

[a] Institut für Kernphysik (Theoriezentrum), Technische Universität Darmstadt, Germany
[b] GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany



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- You need an ansatz for the propagator that supports a self-consistent solution of the Dyson-Schwinger Equations

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- You need an ansatz for the propagator that supports a self-consistent solution of the Dyson-Schwinger Equations

$$
\begin{aligned}
S^{-1}\left(p, p'\right) = \Big[&- i\left(\omega_n + i\mu\right)\gamma_4 C(p) - i p_3 \gamma_3 E(p) - i\vec{p}_\perp A(p) \\
&- i\left(\omega_n + i\mu\right)\gamma_5\gamma_4 C_5(p) - i p_3 \gamma_5\gamma_3 E_5(p) - i\gamma_5\vec{p}_\perp A_5(p)\Big]\delta\left(p - p'\right) \\
&+ \left(B\left(p,p'\right) - i\gamma_4\gamma_3 F\left(p,p'\right) - i\gamma_4\frac{\vec{p}_\perp}{|\vec{p}_\perp|}G\left(p,p'\right) - i\gamma_3\frac{\vec{p}_\perp}{|\vec{p}_\perp|}H\left(p,p'\right)\right)\frac{(\mathrm{I}-\gamma_5)}{2}\delta\left(p - p' + Q\right) \\
&+ \left(B\left(p,p'\right) + i\gamma_4\gamma_3 F\left(p,p'\right) + i\gamma_4\frac{\vec{p}_\perp}{|\vec{p}_\perp|}G\left(p,p'\right) + i\gamma_3\frac{\vec{p}_\perp}{|\vec{p}_\perp|}H\left(p,p'\right)\right)\frac{(\mathrm{I}+\gamma_5)}{2}\delta\left(p - p' - Q\right).
\end{aligned}
$$

How about QCD?

- You need an ansatz for the propagator that supports a self-consistent solution of the Dyson-Schwinger Equations

$$
\begin{aligned}
S^{-1}\left(p, p'\right) = \Big[& -i\left(\omega_n + i\mu\right)\gamma_4 C(p) - i p_3 \gamma_3 E(p) - i\vec{p}_\perp A(p) \\
& - i\left(\omega_n + i\mu\right)\gamma_5\gamma_4 C_5(p) - i p_3 \gamma_5 \gamma_3 E_5(p) - i\gamma_5 \vec{p}_\perp A_5(p)\Big]\,\delta\left(p - p'\right) \\
& + \left(B\left(p, p'\right) - i\gamma_4\gamma_3 F\left(p, p'\right) - i\gamma_4 \frac{\vec{p}_\perp}{|\vec{p}_\perp|} G\left(p, p'\right) - i\gamma_3 \frac{\vec{p}_\perp}{|\vec{p}_\perp|} H\left(p, p'\right)\right) \frac{(\mathrm{I} - \gamma_5)}{2}\,\delta\left(p - p' + Q\right) \\
& + \left(B\left(p, p'\right) + i\gamma_4\gamma_3 F\left(p, p'\right) + i\gamma_4 \frac{\vec{p}_\perp}{|\vec{p}_\perp|} G\left(p, p'\right) + i\gamma_3 \frac{\vec{p}_\perp}{|\vec{p}_\perp|} H\left(p, p'\right)\right) \frac{(\mathrm{I} + \gamma_5)}{2}\,\delta\left(p - p' - Q\right).
\end{aligned}
$$

- Then you solve the DSE **and**, in theory, you must calculate whether or not this solution is favoured!

A plot twist? Gross-Neveu Model!

Revised Phase Diagram of the Gross-Neveu Model

Michael Thies and Konrad Urlichs
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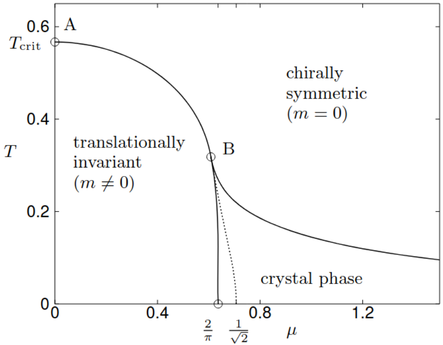
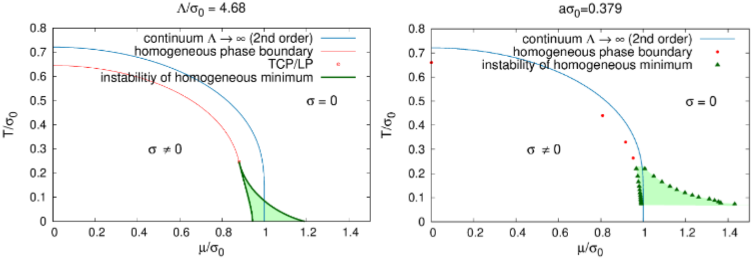
(Dated: October 25, 2018)



A plot twist? Gross-Neveu Model!

Regulator dependence of inhomogeneous phases in the 2+1-dimensional Gross-Neveu model

Michael Buballa[a,c], Lennart Kurth[a], Marc Wagner[b,c], Marc Winstel[b]



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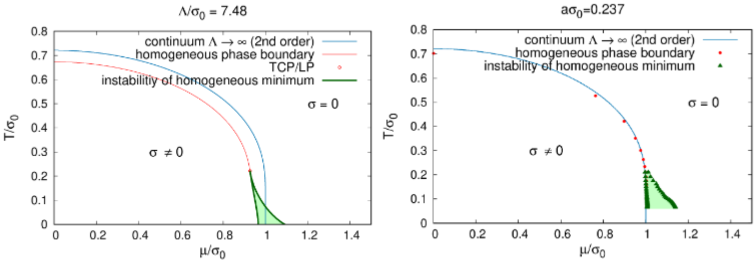

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