Multigluon Correlation Functions from lattice QCD

The four gluon vertex

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From first-principles QCD to experiments, Trento, Italy, May 2023

Motivation

QCD Dynamics at a fundamental level

importance of higher order corrections (two loop contributions to gluon DSE, etc.)

Test our understanding of QCD Green functions (ghost dominance at IR)

Effective charge $$\frac{g^2}{4\,\pi}\;\Gamma^{(4g)}(p^2)\;p^2\,D(p^2)$$

Landau gauge **(pure gauge) Lattice point of view**

Lattice Approach

Importance Sampling **allows to access** the **gluon field** $A^a_\mu(x)$

$$\mathcal{G}^{(n)}(x_1, \ldots, x_n) = \langle 0 | \, T \, \left(A^{a_1}_{\mu_1}(x_1) \cdots A^{a_n}_{\mu_n}(x_n) \right) \, | 0 \rangle$$

Larger n: → **Noisy** **Disconnected parts**

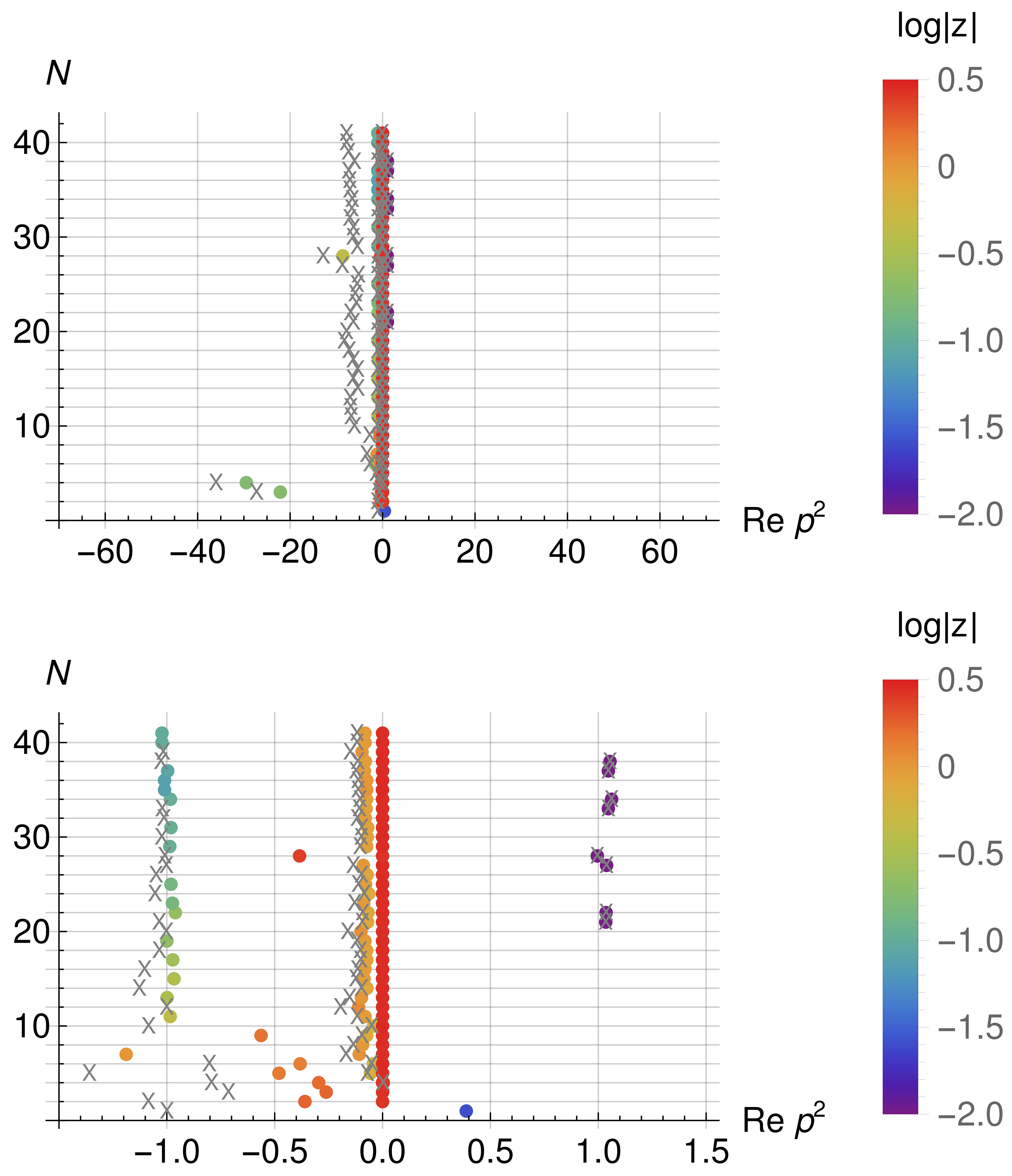
Larger sets of configurations

kinematical configurations
or
Combinations of components

Massless Ghost

$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$

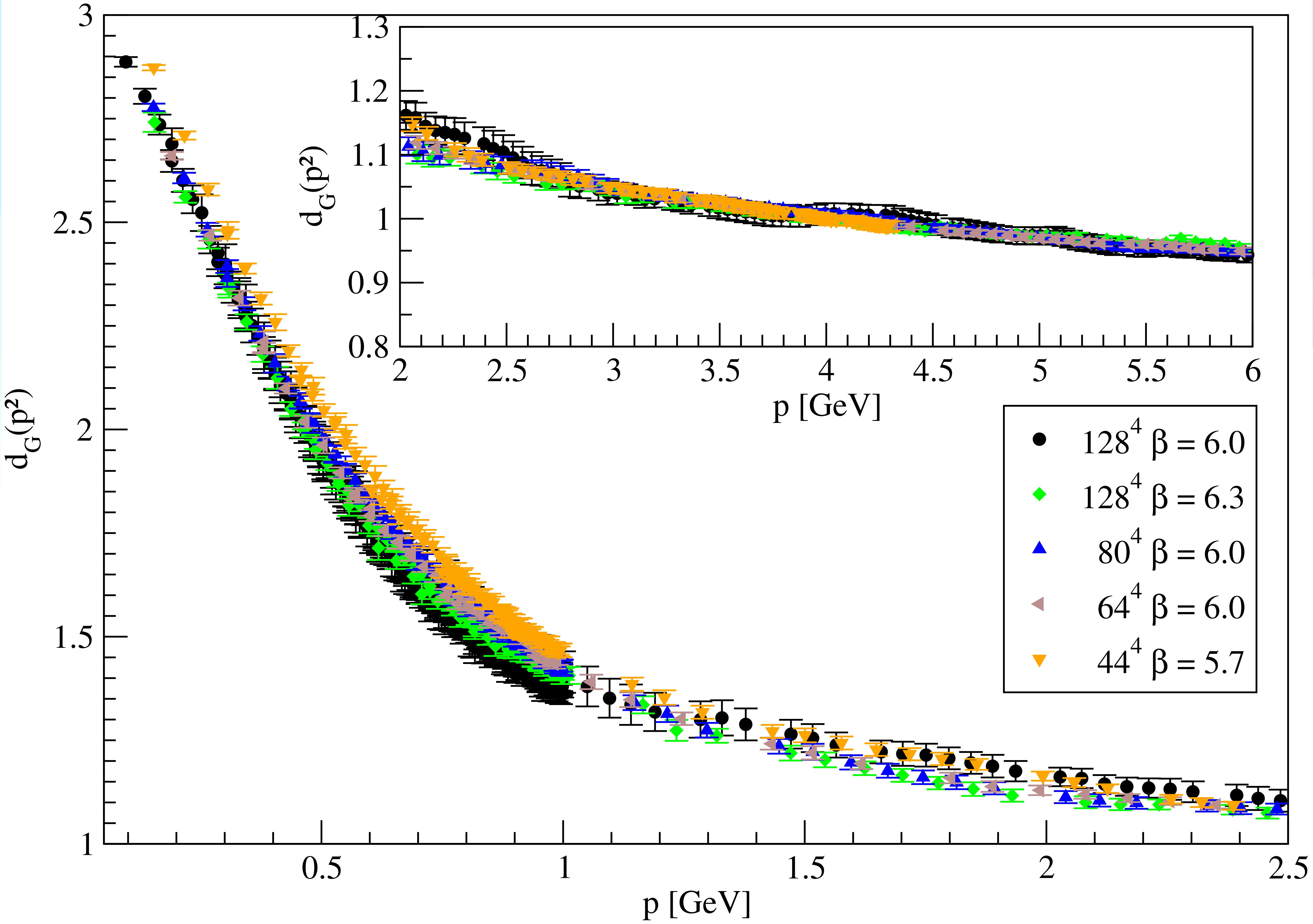
lattice studies support a massless ghost



A F Falcão et al, *Phys. Rev. D* **102** (2020) 11, 114518

IR Ghost Dominance / Log Divergence

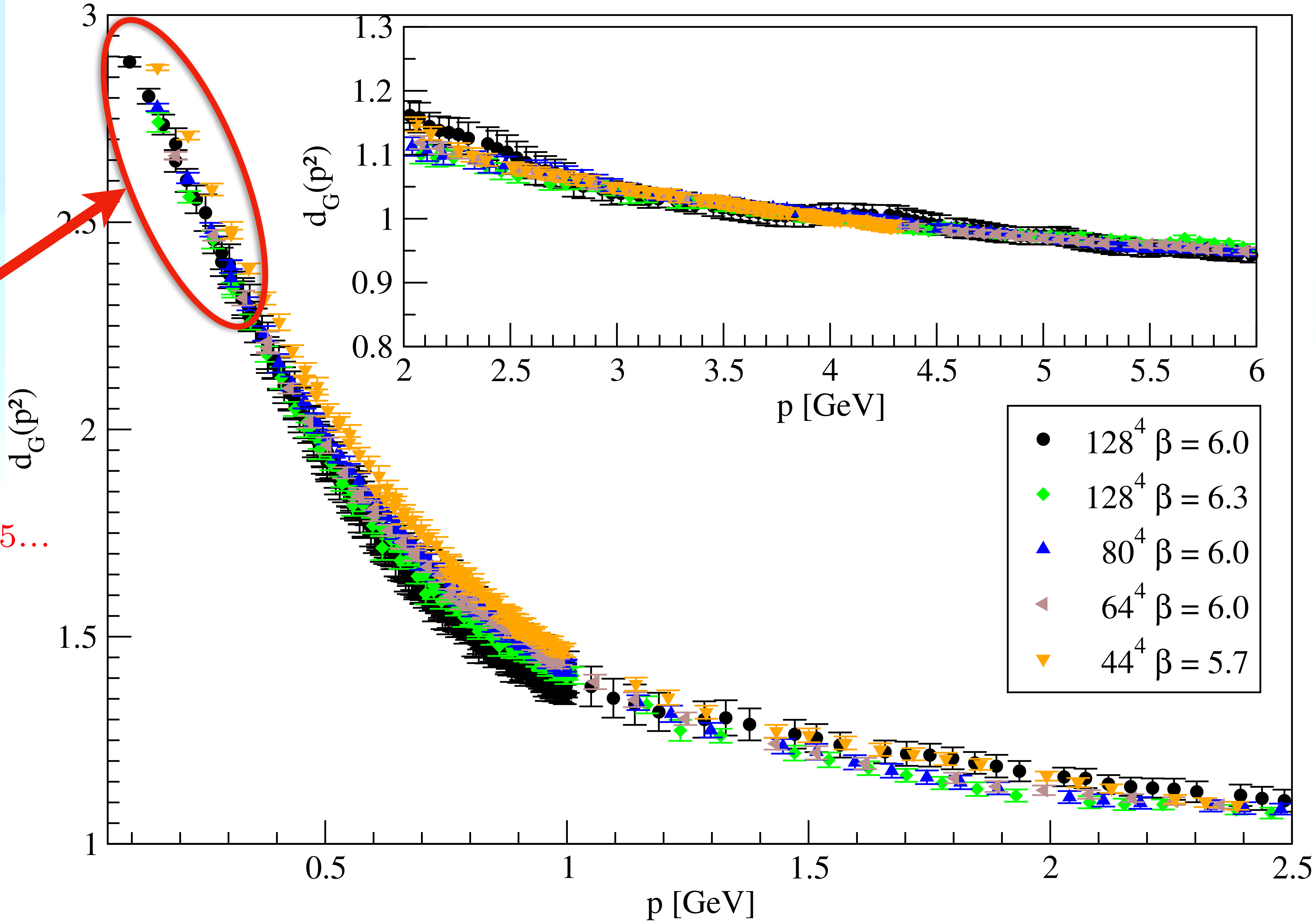
$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$



A G Duarte et al, *Phys. Rev. D* **94** (2016) 1, 014502

IR Ghost Dominance / Log Divergence

$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$



$$d_G(p^2) \propto \left(\ln \frac{p^2 + (1.7\text{ GeV})^2}{(0.4\text{ GeV})^2}\right)^{-0.2045\ldots}$$

A G Duarte et al, *Phys. Rev. D* **94** (2016) 1, 014502

IR Ghost Dominance / Log Divergence

Ward identities/Slavnov-Taylor identities

See Ferreira talk

Ward identity displacement in QCD

$$q^\alpha \mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

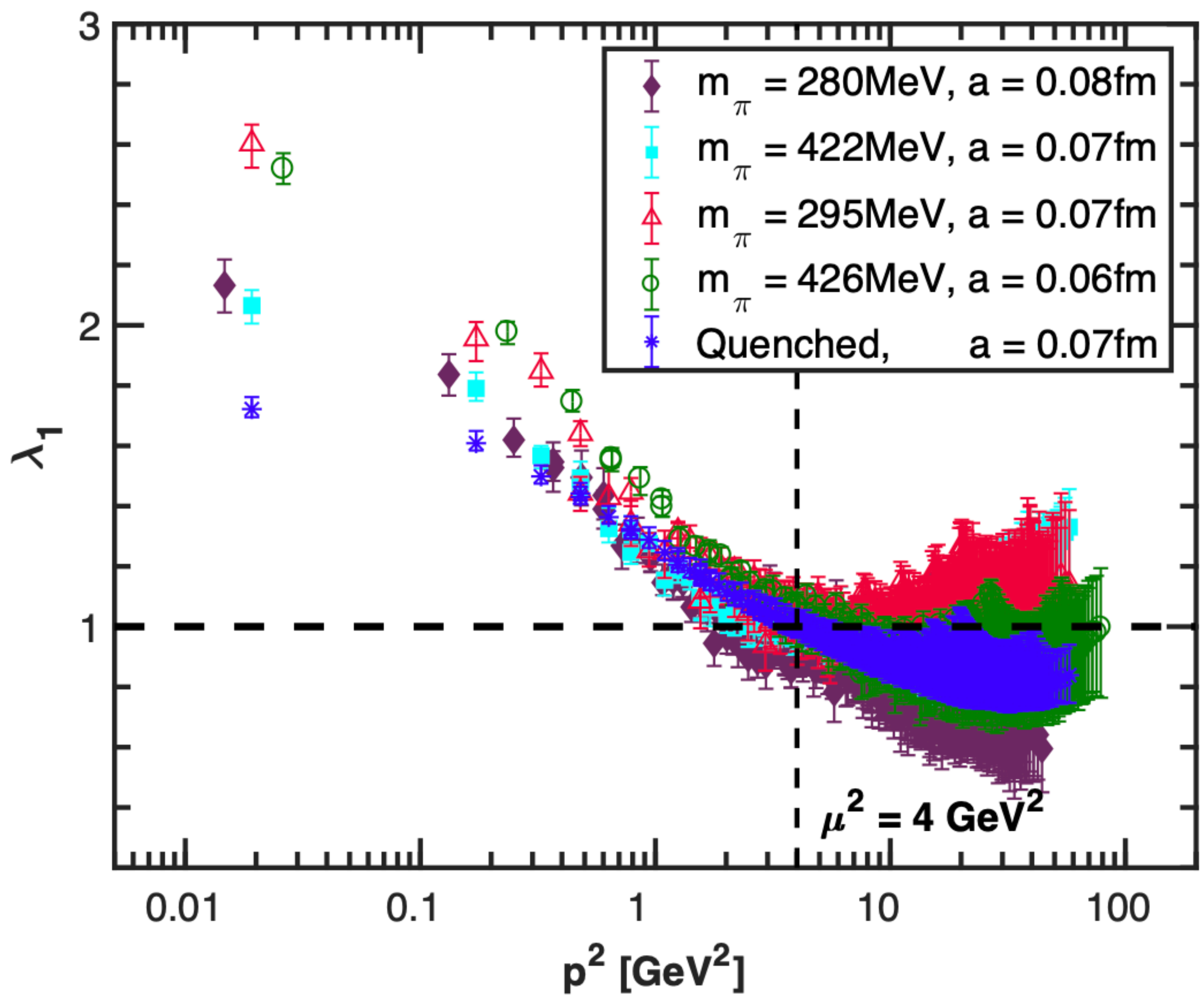
IR Ghost Dominance / Log Divergence

Full QCD

$$\lambda_1(p_1, p_2, p_3) = \frac{F(p_3^2)}{2}\Big\{ A(p_1^2)\Big[X_0 + \big(p_1^2 - (p_1 p_2)\big)\ X_3\Big] + A(p_2^2)\Big[\overline{X}_0 + \big(p_2^2 - (p_1 p_2)\big)\ \overline{X}_3\Big] + B(p_1^2)\Big[X_1 + X_2\Big] + B(p_2^2)\Big[\overline{X}_1 + \overline{X}_2\Big]\Big\}$$

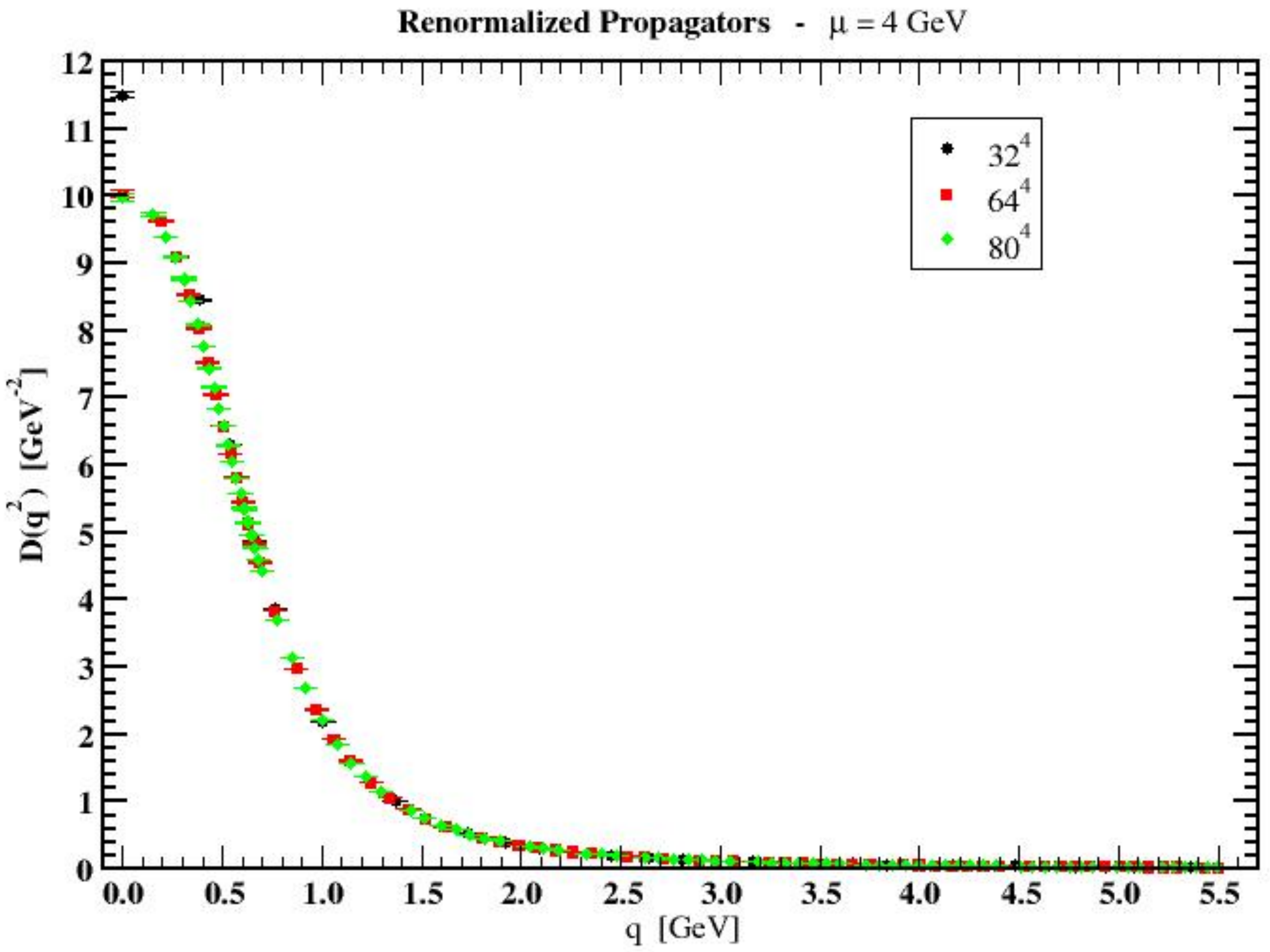
IR Ghost Dominance / Log Divergence

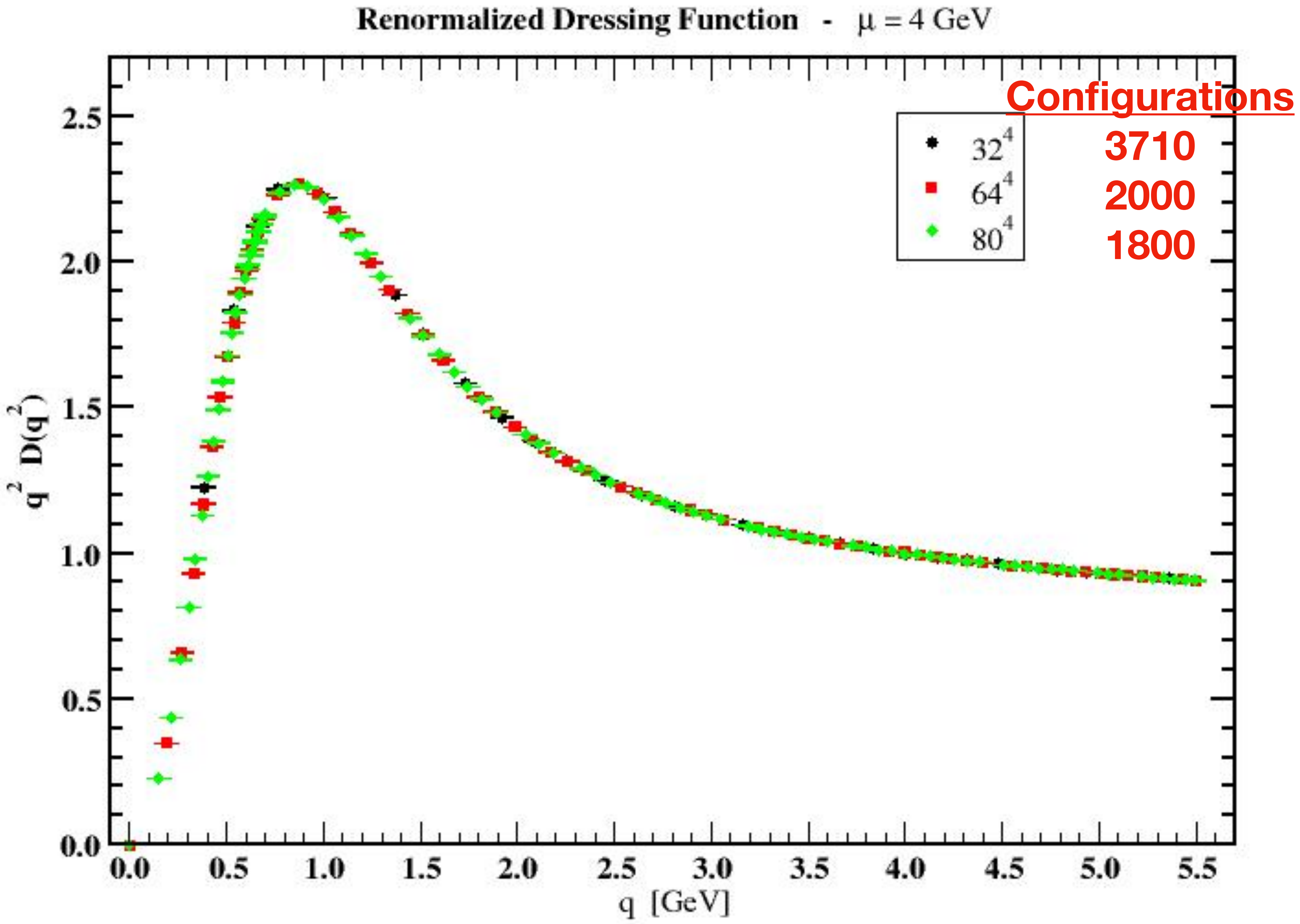
Full QCD $\lambda_1(p_1, p_2, p_3) =$



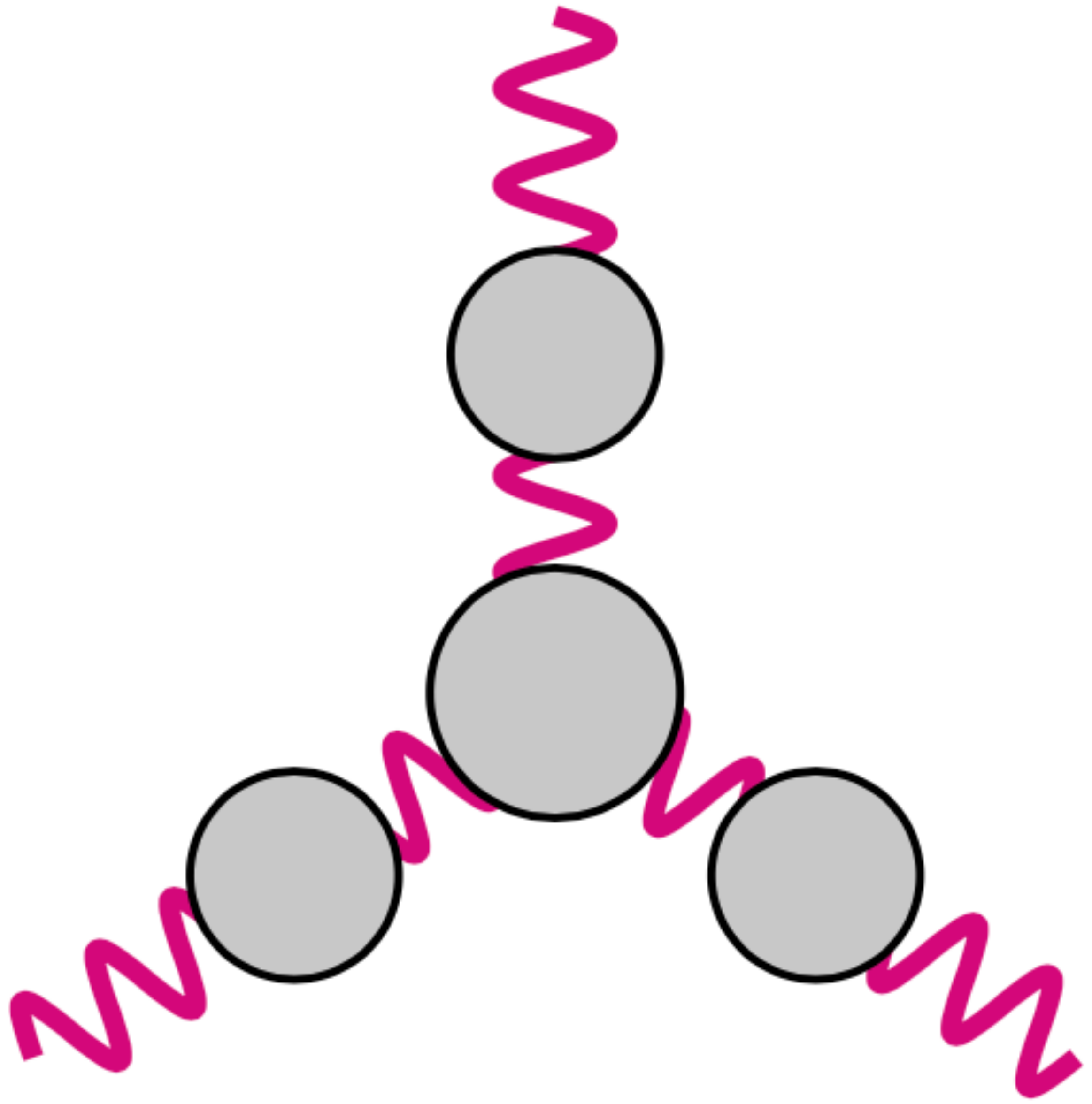
A. Kizilersu et al, Phys.Rev.D **103** (2021) 11, 114515

Gluon Propagator





3-point function



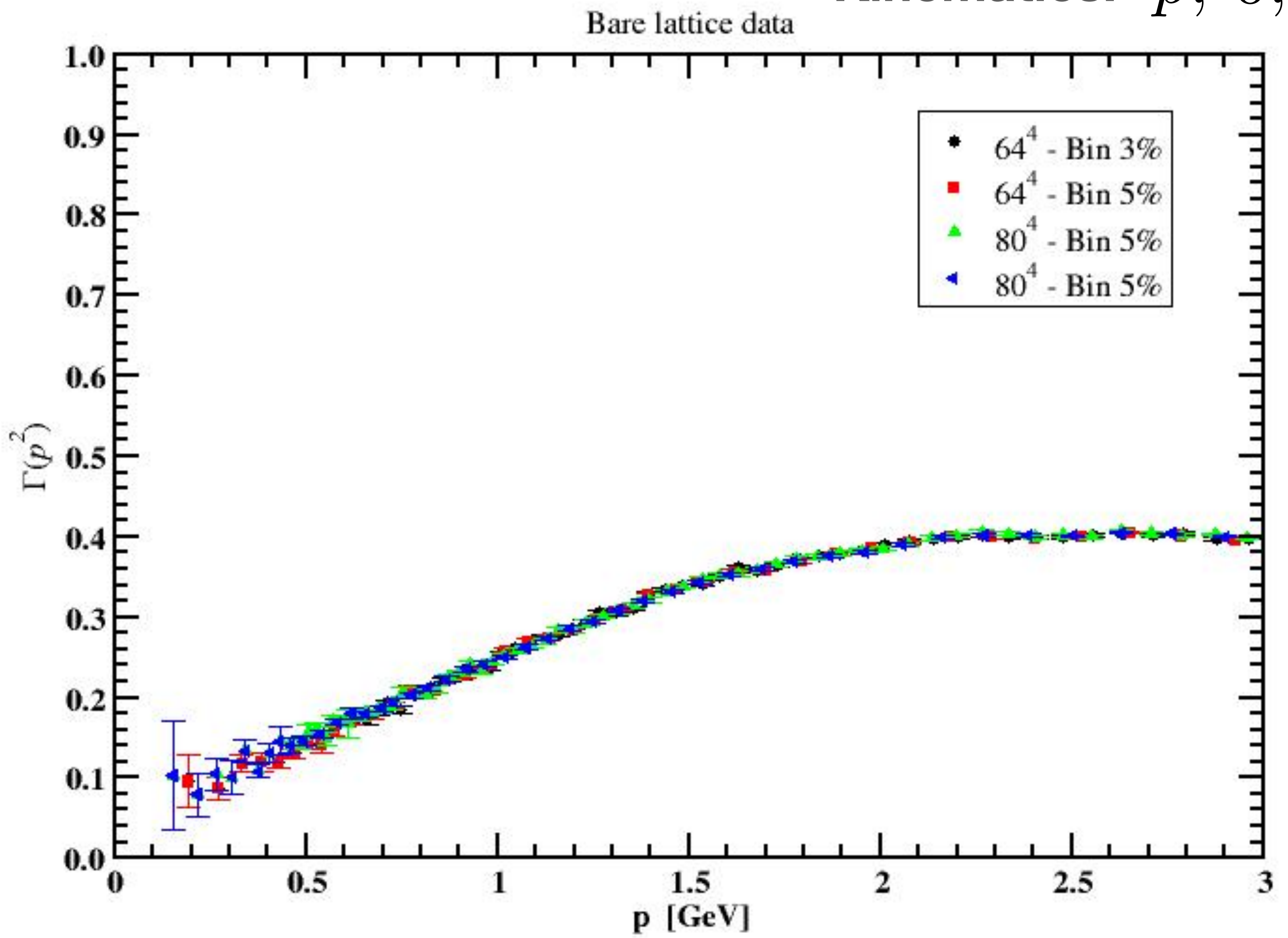
$$\mathcal{G}^{(3)\,abc}_{\alpha\beta\zeta}(p_1,p_2,p_3) = D^{aa'}_{\alpha\alpha'}(p_1)\,D^{bb'}_{\beta\beta'}(p_2)\,D^{cc'}_{\zeta\zeta'}(p_3)\,\underbrace{\Gamma^{a'b'c'}_{\alpha'\beta'\zeta'}(p_1,p_2,p_3)}_{f_{a'b'c'}\,\Gamma_{\alpha'\beta'\zeta'}(p_1,p_2,p_3)}$$

Six form factors

J S Ball, T.-W. Chiu, PRD22, 2550 (1980)

Landau gauge: only the transverse form factors

Kinematics: $\vec{p},\ \vec{0},\ -\vec{p}$

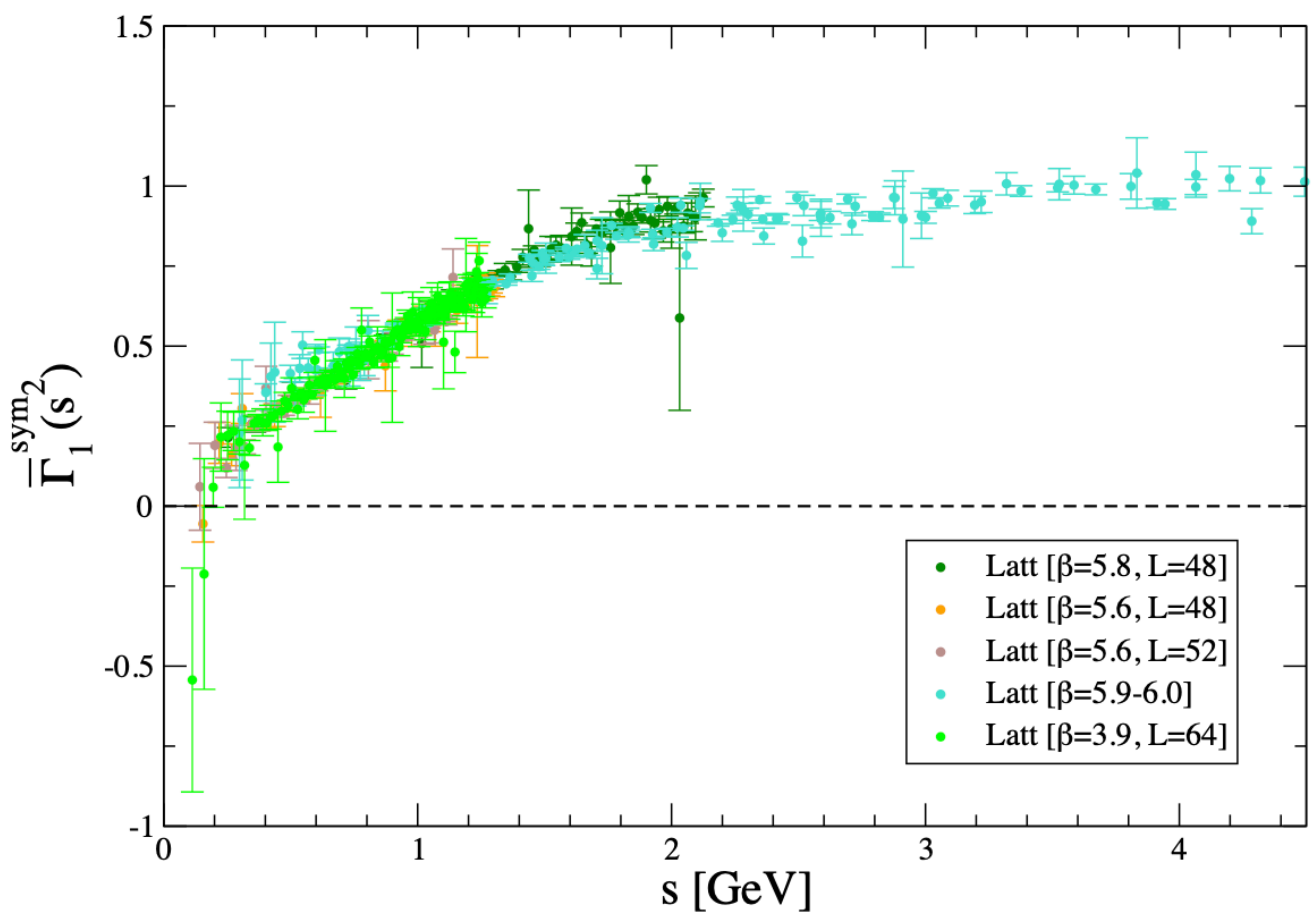


G. T. R. Catumba, O. Oliveira, P,. J. Silva, EPJ Web Conf. **258** (2022) 02008
A G Duarte, O. Oliveira, Rhys Rev D**94** (2016) 074502

A. Cucchieri, A. Maas, T. Mendes, Phys Rev D**77** (2008) 0945001
and many other works by several groups

See all talks by:
Feliciano de Soto

Typically more than ~1000 configurations



A. C. Aguilar et al, Phys. Lett. B **818** (2021) 136352

4-gluon correlation function

$\vec{p}_i \propto \vec{p}$

Single momentum scale

In Landau gauge simplifies the tensor analysis

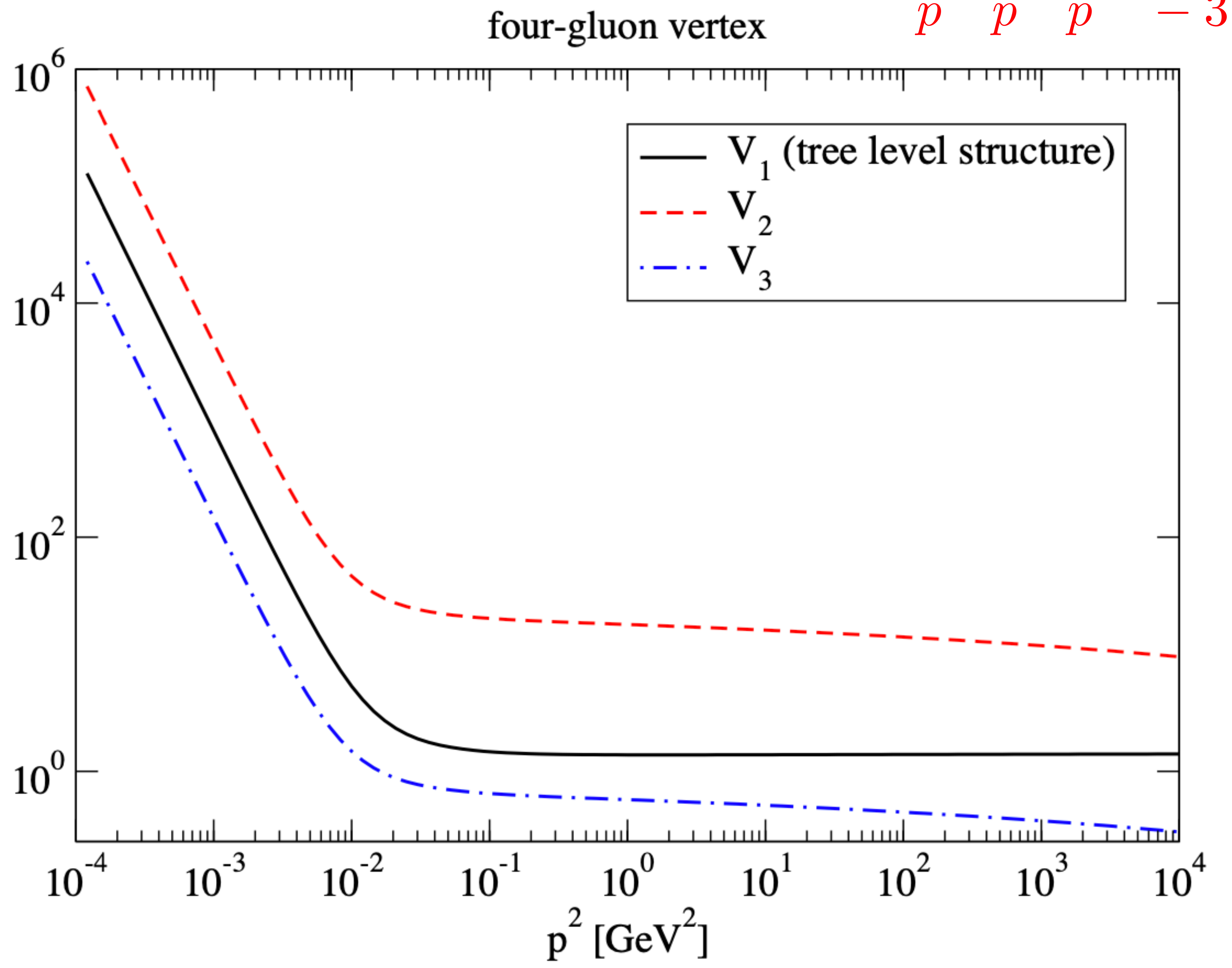
Contributions only from the tensors proportional to $\delta_{\mu\nu}$

Recent Continuum Calculations

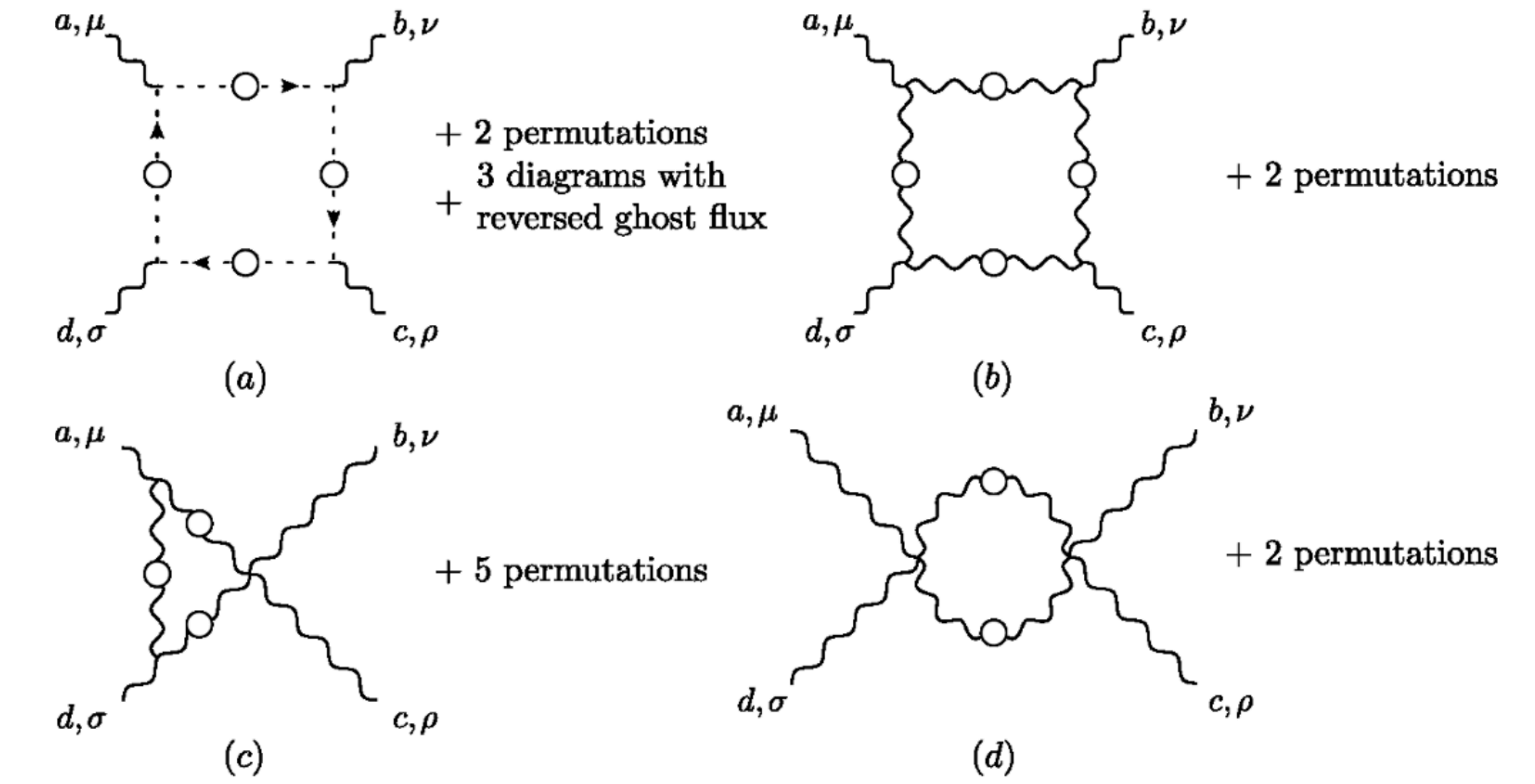
C Kellermann, C S Fischer, *Phys. Rev. D* ***78*** *(2008) 025015*
D Binosi, D Ibañez, J Papavassiliou, *JHEP* ***1409*** *(2014) 059*
A K Carol, M Q Huber, L von Smekal, *Eur Phys J C* ***75*** *(2015) 102*

C Kellermann, C S Fischer, *Phys. Rev. D* ***78*** *(2008) 025015*

$\vec{p} \quad \vec{p} \quad \vec{p} \quad -3\,\vec{p}$

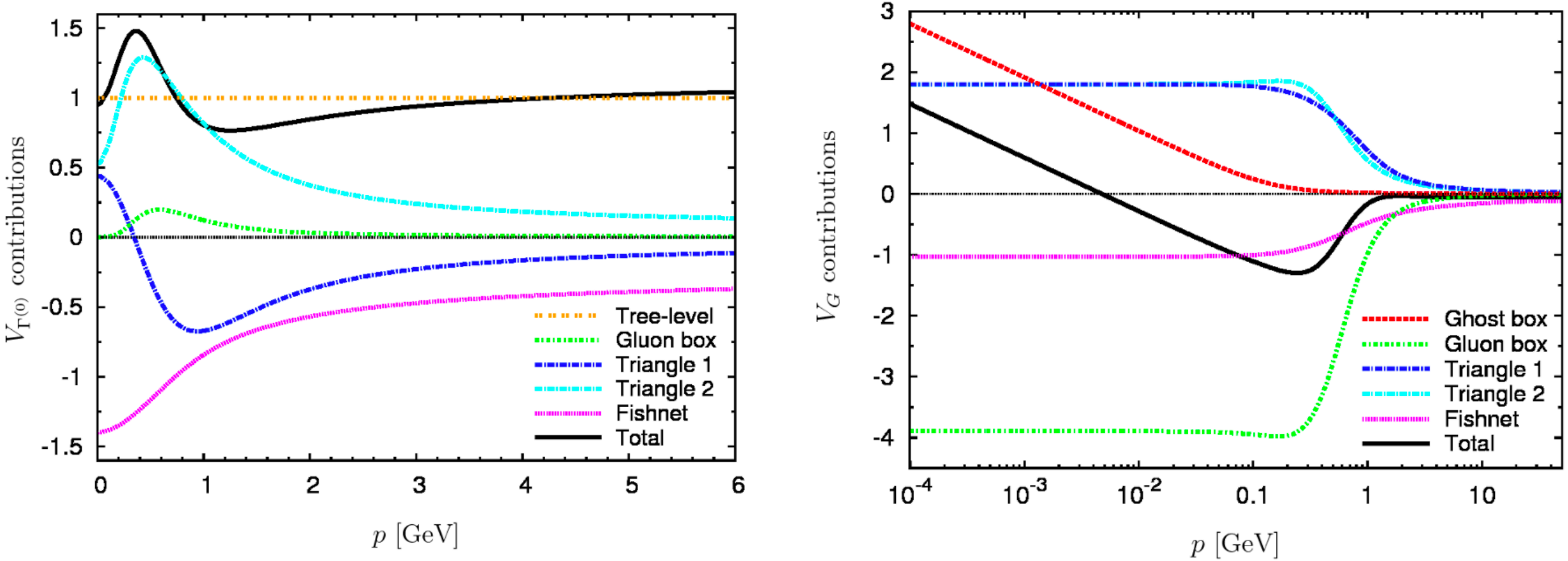


D Binosi, D Ibañez, J Papavassiliou, *JHEP* **1409** *(2014) 059*



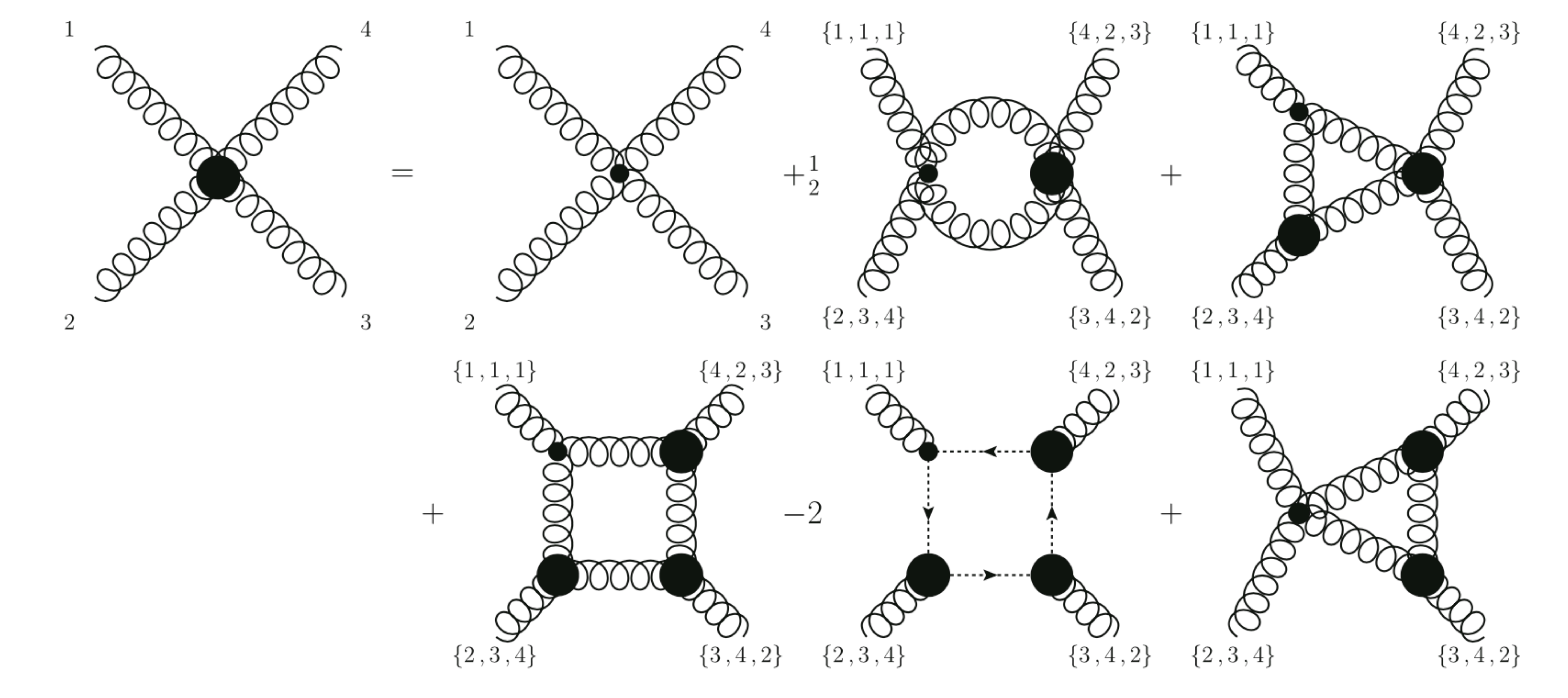
$$\Gamma^{abcd}_{\mu\nu\rho\sigma}(p,p,p,-3p)\Big|_{gg} = V_{\Gamma^{(0)}}(p^2)\Gamma^{abcd(0)}_{\mu\nu\rho\sigma} + V_G(p^2)G^{abcd}_{\mu\nu\rho\sigma},$$

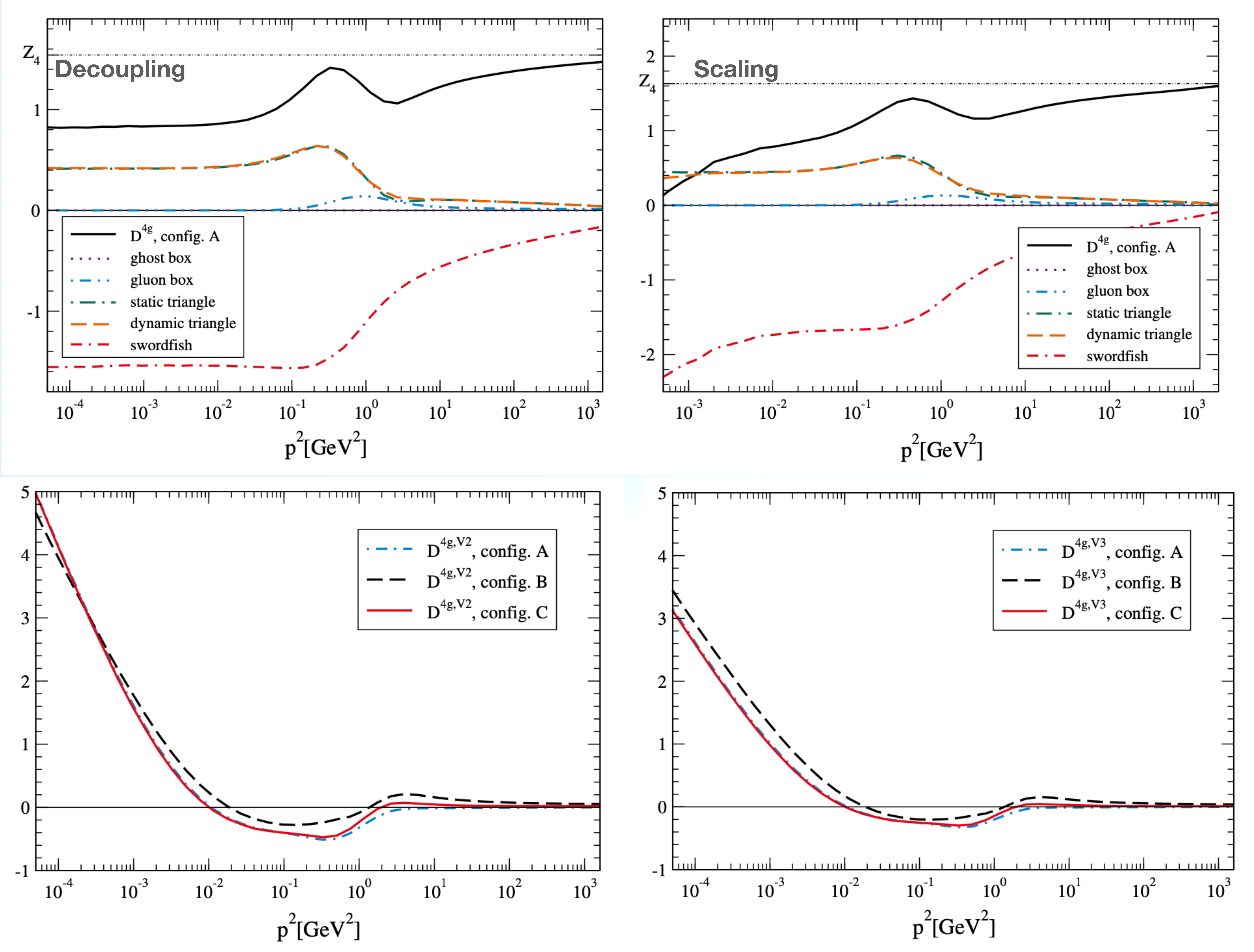
$$G^{abcd}_{\mu\nu\rho\sigma} = (\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\underbrace{(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho})}_{R_{\mu\nu\rho\sigma}}$$



$V_{\Gamma^{(0)}}$ contributions
Tree-level
Gluon box
Triangle 1
Triangle 2
Fishnet
Total
p [GeV]
V_G contributions
Ghost box
Gluon box
Triangle 1
Triangle 2
Fishnet
Total
p [GeV]

A K Cyrol, M Q Huber, L von Smekal, *Eur Phys J C* **75** *(2015) 102*





Decoupling
Z4
D4g, config. A
ghost box
gluon box
static triangle
dynamic triangle
swordfish
p2[GeV2]
Scaling
Z4
D4g, config. A
ghost box
gluon box
static triangle
dynamic triangle
swordfish
p2[GeV2]
D4g,V2, config. A
D4g,V2, config. B
D4g,V2, config. C
p2[GeV2]
D4g,V3, config. A
D4g,V3, config. B
D4g,V3, config. C
p2[GeV2]

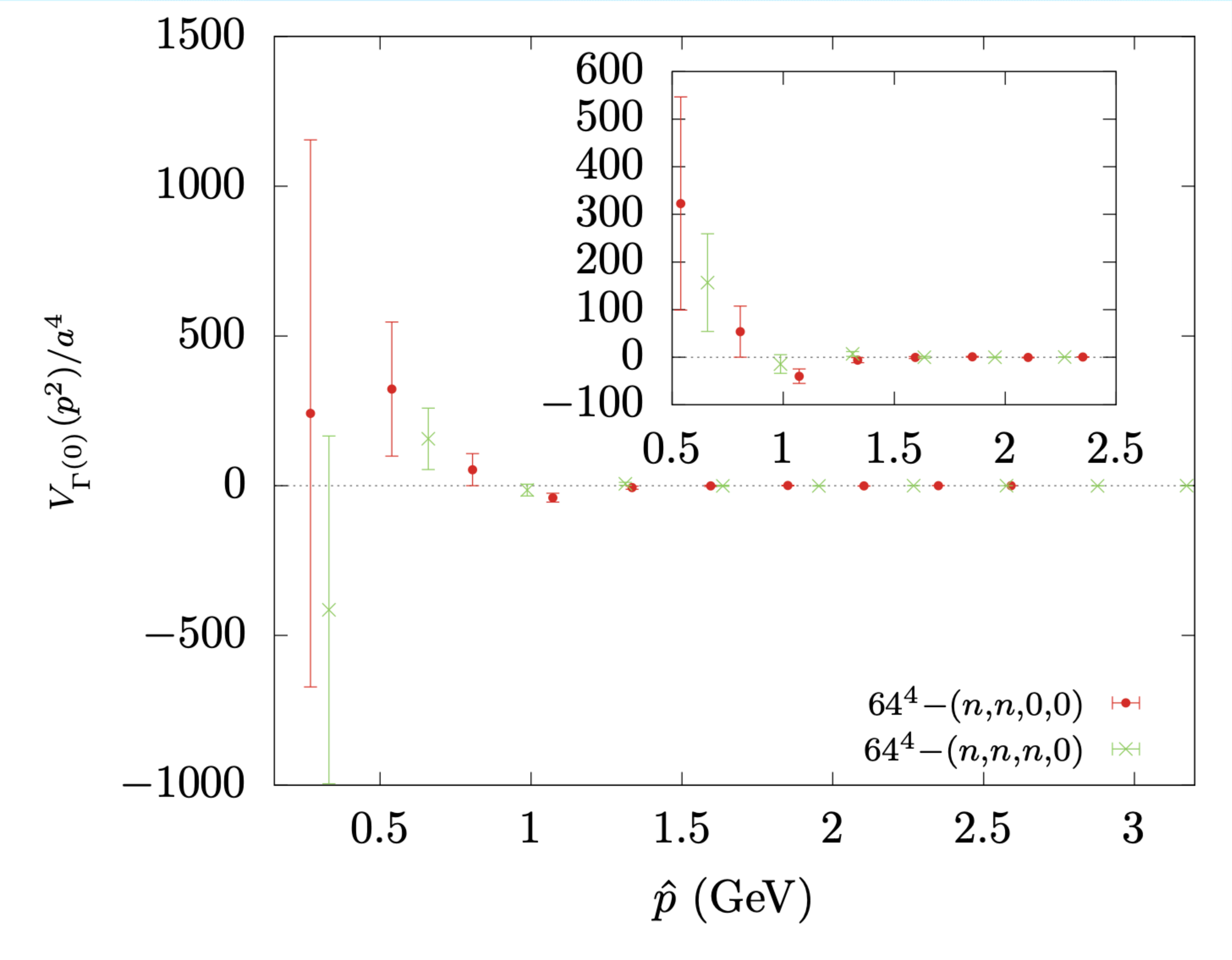
First tentative of a lattice calculation can be found in
G T R Catumba Master Thesis ArXiv: 2101.06074

D. Binosi, D. Ibañez, J. Papavassiliou JHEP 9, 059 (2014) arXiv:1407.3677

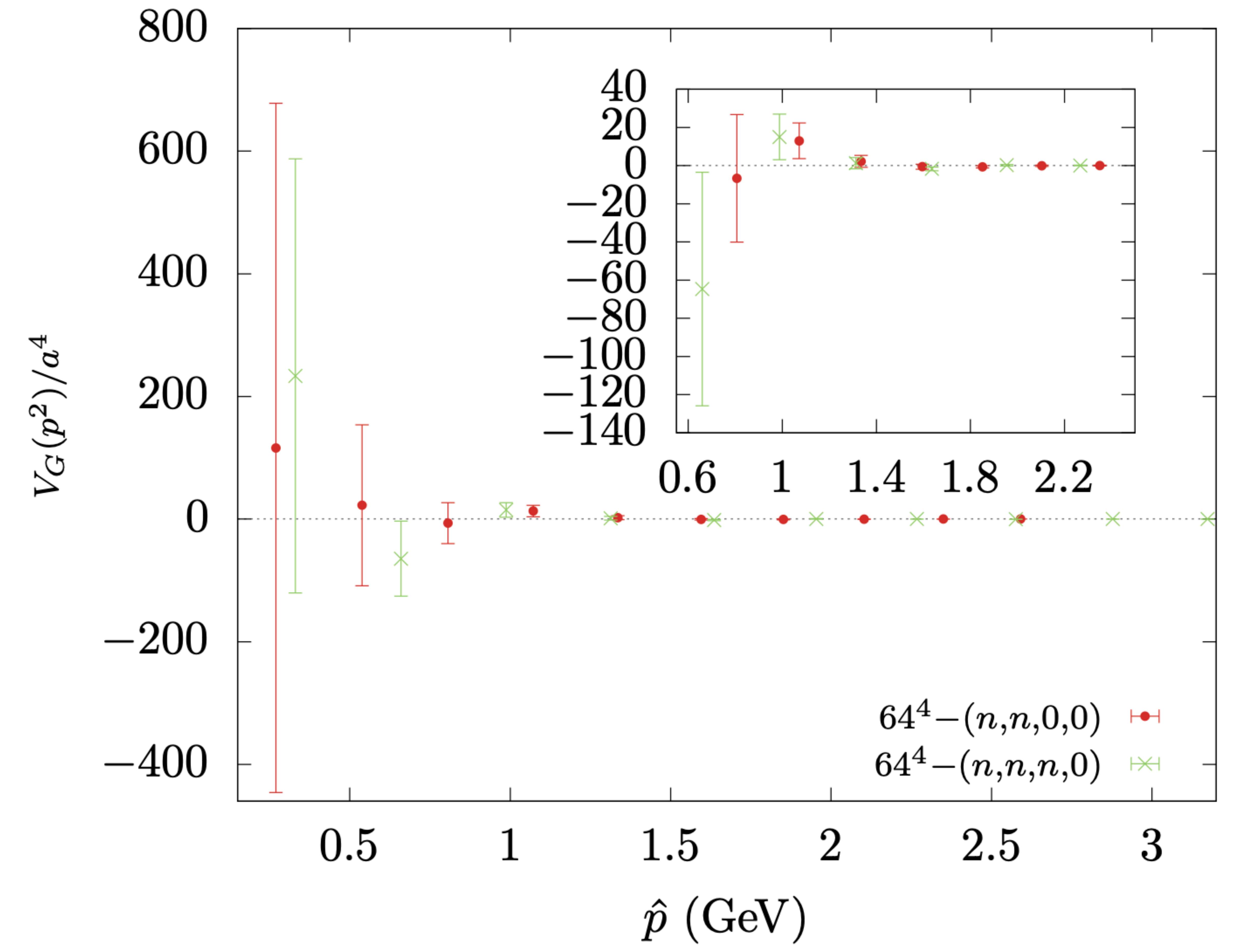
p, p, p, -3p Two tensor structures

$$\widetilde{\Gamma}^{(0)\,abcd}_{\mu\nu\eta\zeta} = f_{abr} f_{cdr} \left(\delta_{\mu\eta}\delta_{\nu\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{acr} f_{bdr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{adr} f_{bcr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\eta}\delta_{\nu\zeta}\right)$$

$$G^{abcd}_{\mu\nu\eta\zeta} = \left(\delta^{ab}\,\delta^{cd} + \delta^{ac}\,\delta^{bd} + \delta^{ad}\,\delta^{bc}\right)\left(\delta_{\mu\nu}\,\delta_{\eta\zeta} + \delta_{\mu\eta}\,\delta_{\nu\zeta} + \delta_{\mu\zeta}\,\delta_{\nu\eta}\right)$$



$V_{\Gamma(0)}(p^2)/a^4$
$\hat{p}$ (GeV)
$64^4-(n,n,0,0)$
$64^4-(n,n,n,0)$



$V_G(p^2)/a^4$
$\hat{p}$ (GeV)
$64^4-(n,n,0,0)$
$64^4-(n,n,n,0)$

For **tensor analysis** see:

J A Gracey, Phys Rev D90, 025011 (2014) arXiv: 1406.1618
G Eichmann, C S Fischer, W Heupel, Phys Rev D92, 056006 (2015) arXiv: 1505.06336

$$\widetilde{\Gamma}^{(0)\,abcd}_{\;\mu\nu\eta\zeta} = f_{abr}f_{cdr}\left(\delta_{\mu\eta}\delta_{\nu\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{acr}f_{bdr}\left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{adr}f_{bcr}\left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\eta}\delta_{\nu\zeta}\right)$$

$$\widetilde{\Gamma}^{(1)\,abcd}_{\;\mu\nu\eta\zeta} = d_{abr}d_{cdr}\left(g_{\mu\eta}g_{\nu\zeta} + g_{\mu\zeta}g_{\nu\eta}\right) + d_{acr}d_{bdr}\left(g_{\mu\zeta}g_{\nu\eta} + g_{\mu\nu}g_{\eta\zeta}\right) + d_{adr}d_{bcr}\left(g_{\mu\nu}g_{\eta\zeta} + g_{\mu\eta}g_{\nu\zeta}\right)$$

$$\widetilde{\Gamma} = F(p^2)\,\widetilde{\Gamma}^{(0)} + G(p^2)\,\widetilde{\Gamma}^{(1)} + H(p^2)\,\widetilde{\Gamma}^{(2)}$$

Not an orthogonal basis

$$\mathcal{G}^{(4)} = \widetilde{\Gamma} \left(P^{\perp}(p) D(p^2) \right)^3 \left(P^{\perp}(3p) D(9p^2) \right)$$

p, p, p, -3p	**Kinematics**	0, p, p, -2p

Measure the two form factors

$\beta = 6.0 \qquad a = 0.102 \text{ fm} \qquad a^{-1} = 1.943 \text{ GeV}$

Lattice	Configs	
32^4	3710	381 MeV
64^4	2000	191 MeV
80^4	1801	153 MeV

Averaged over equivalent momenta, including the negative momenta !

(1 1 1 1)

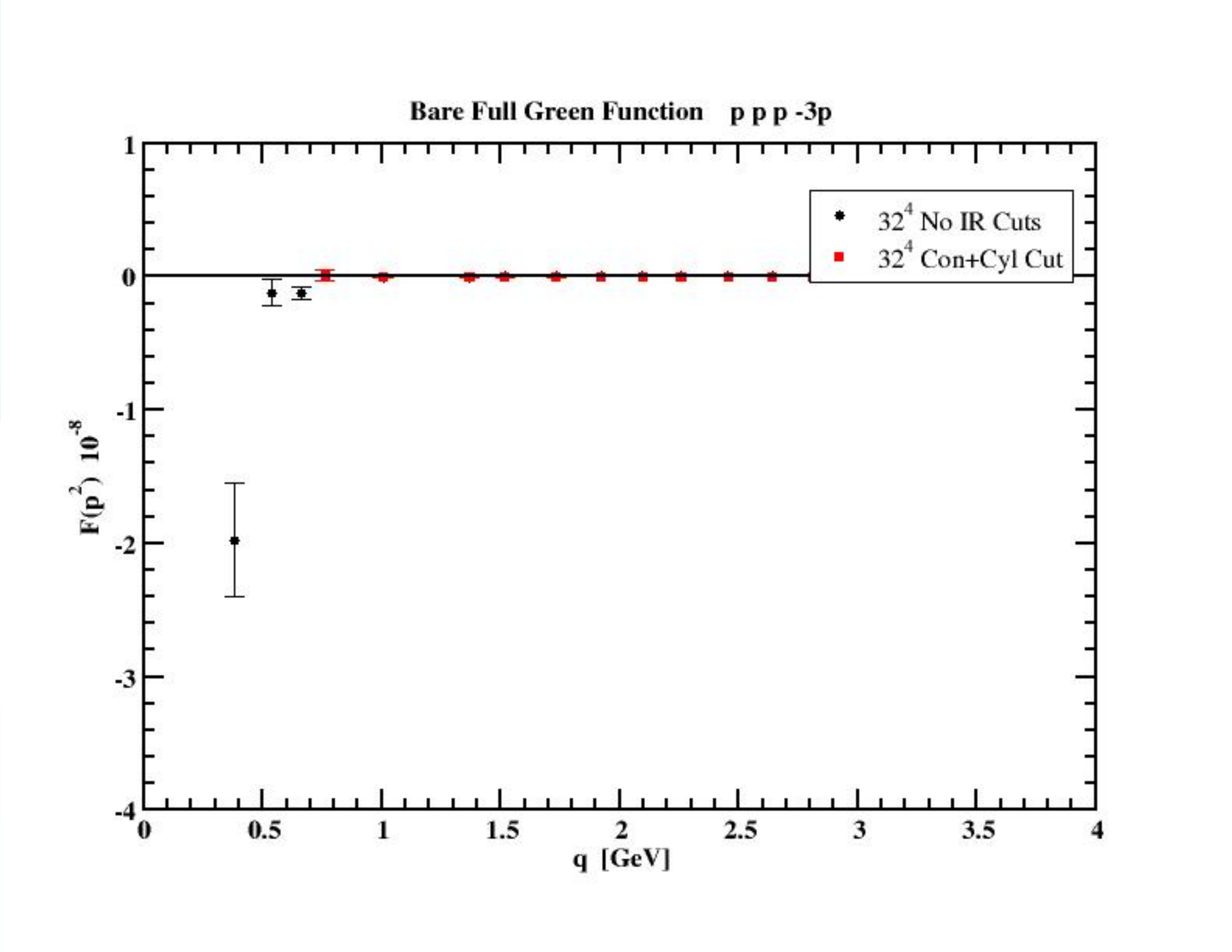
(-1 1 1 1) (1 -1 1 1) (1 1 -1 1) (1 1 1 -1)

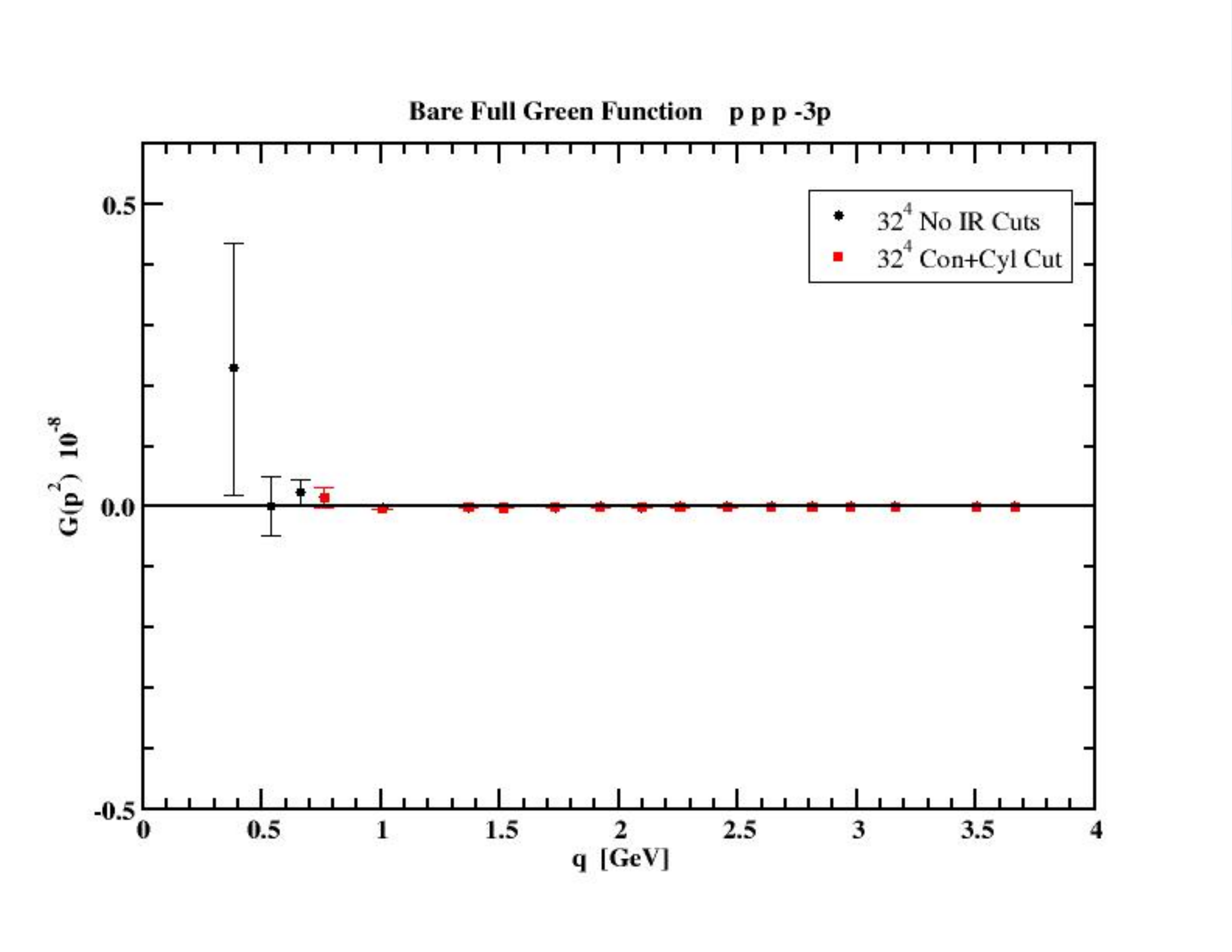
(-1 -1 1 1) (-1 1 -1 1) (-1 1 1 -1) (1 -1 -1 1) (1 -1 1 -1) (1 1 -1 -1)

(1 -1 -1 -1) (-1 1 -1 -1) (-1 -1 1 -1) (-1 -1 -1 1)

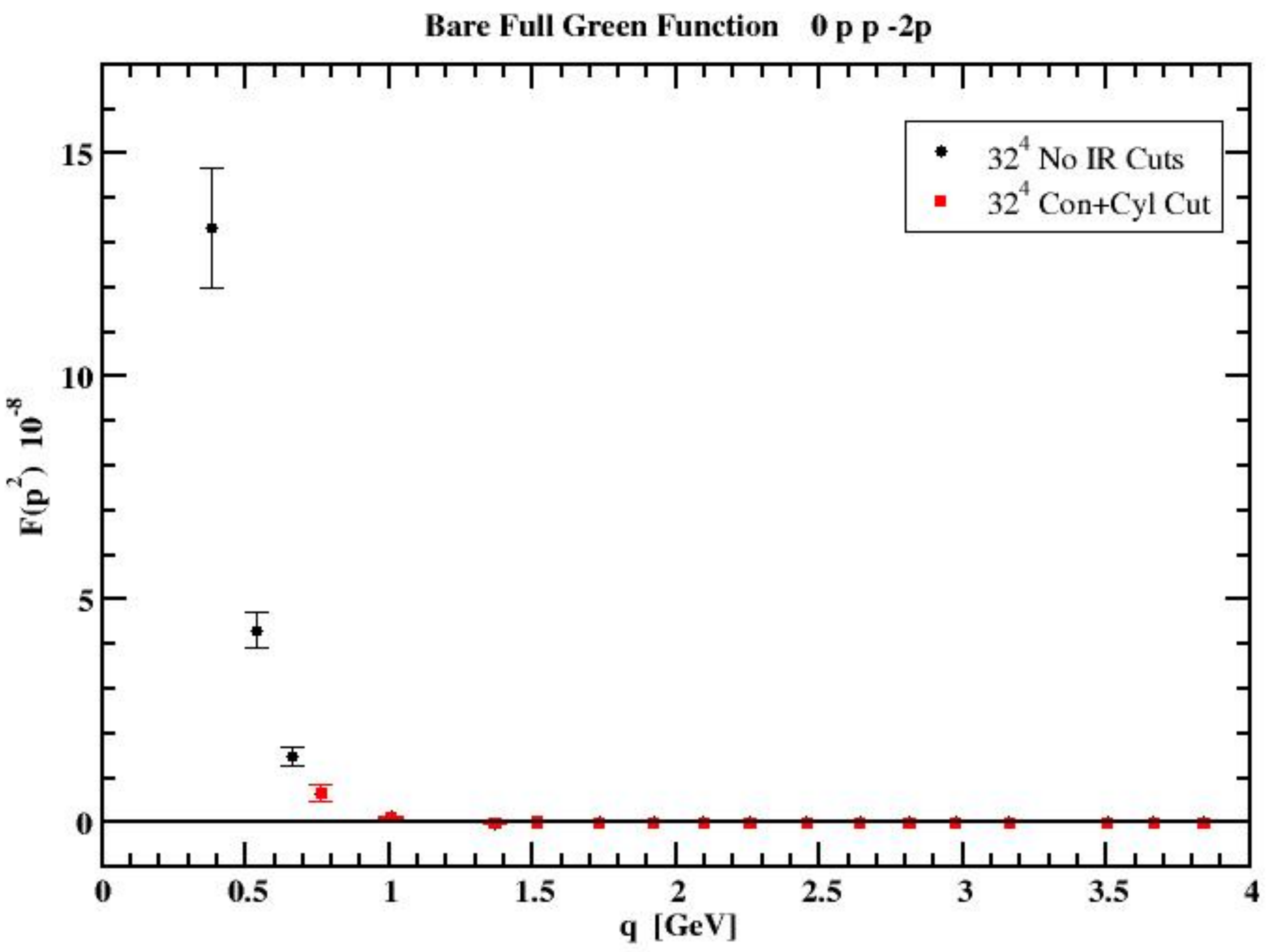
(-1 -1 -1 -1)

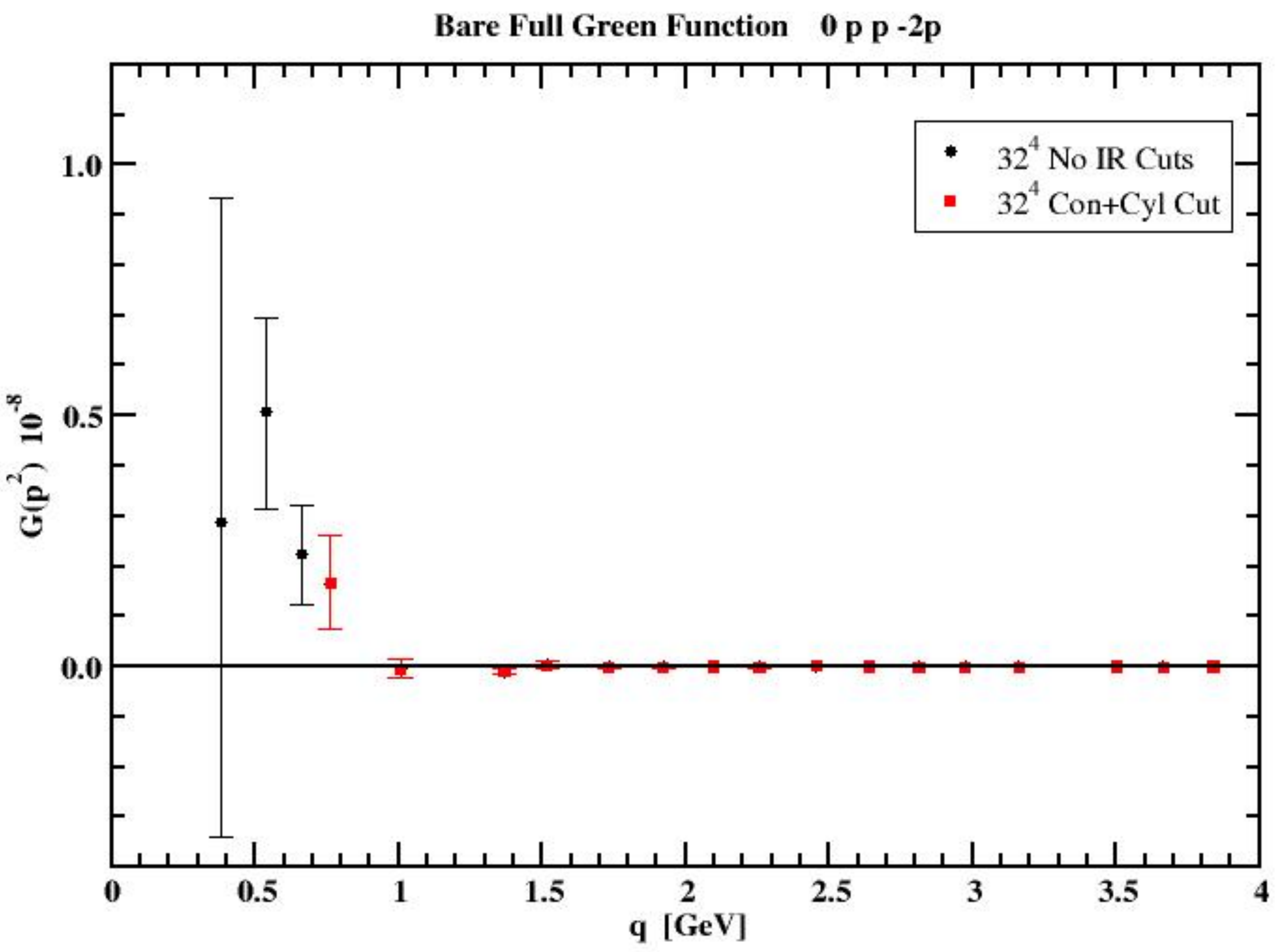
Bare Green Function for kinematics: p p p -3p



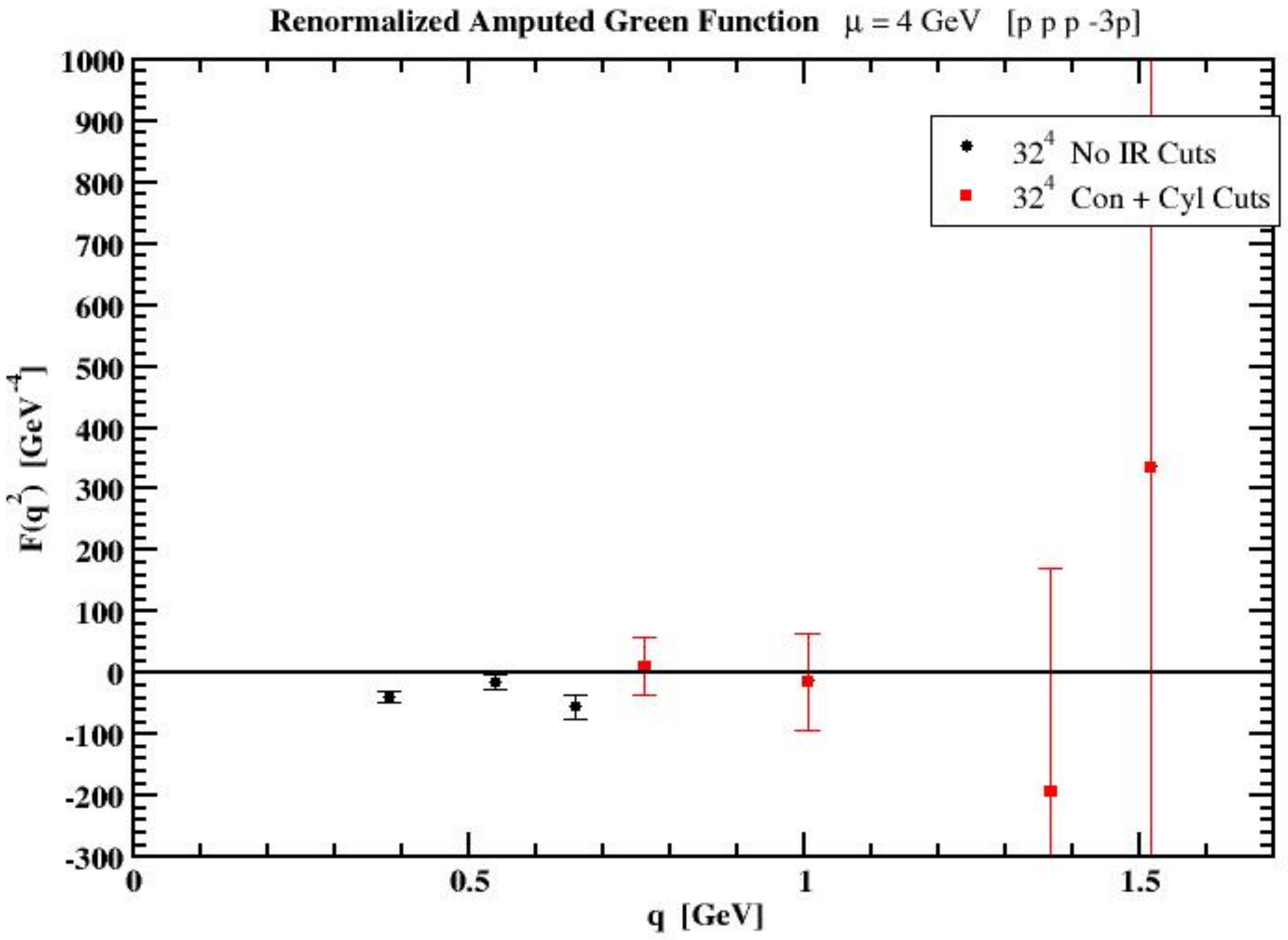


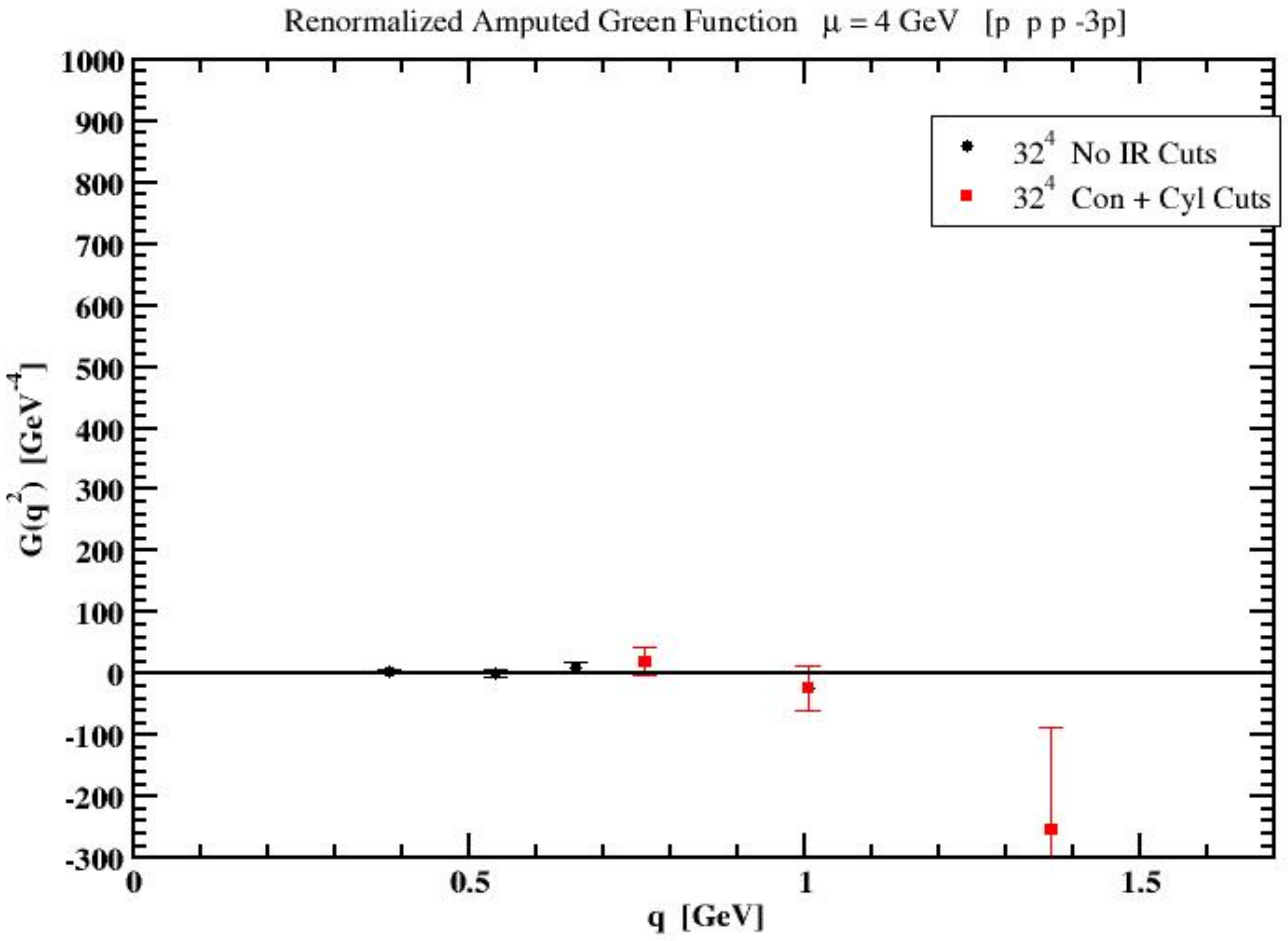
Bare Green Function for kinematics: 0 p p -2p





Vertex Form Factors Renormalized @ μ = 4 GeV





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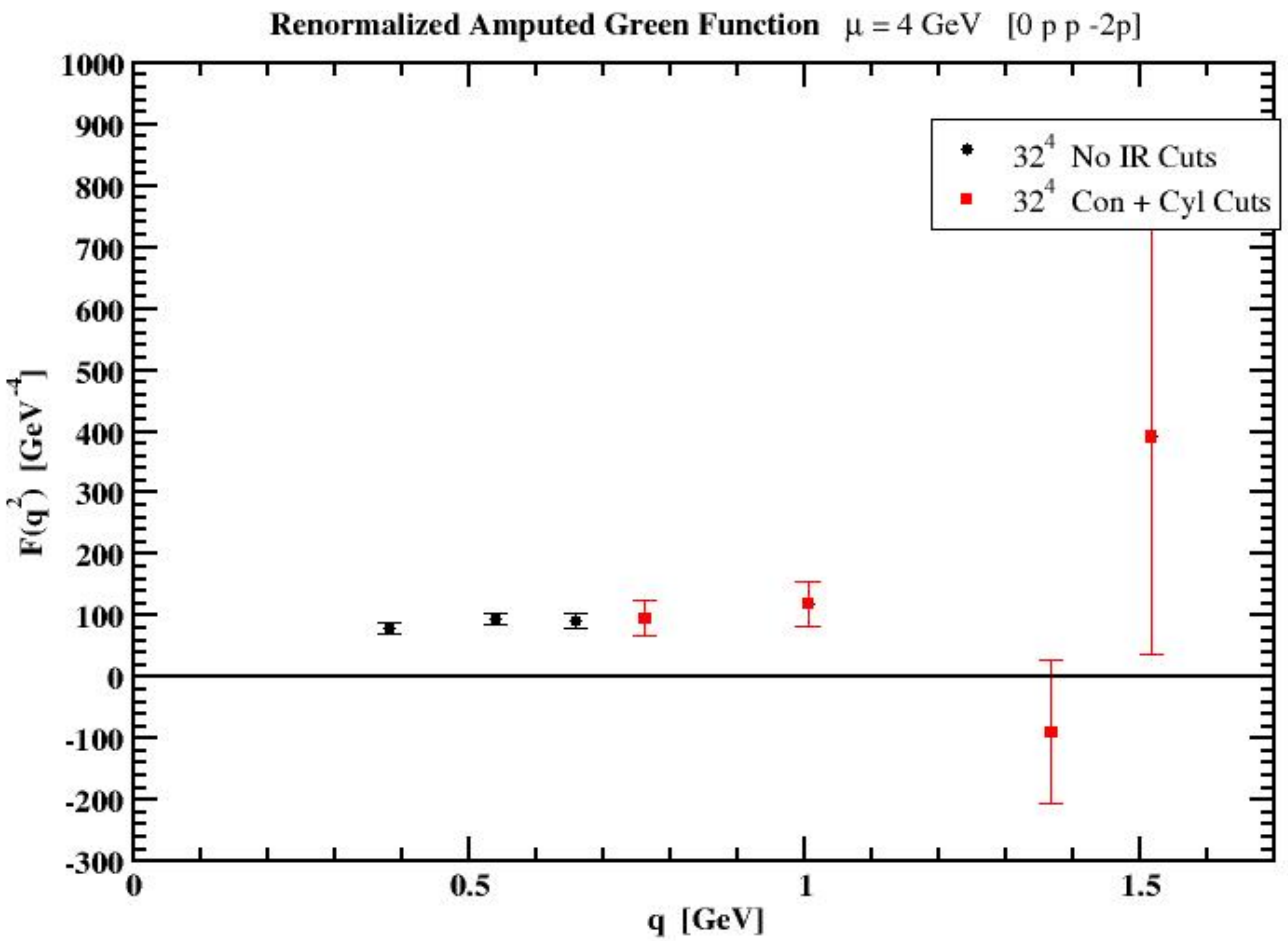
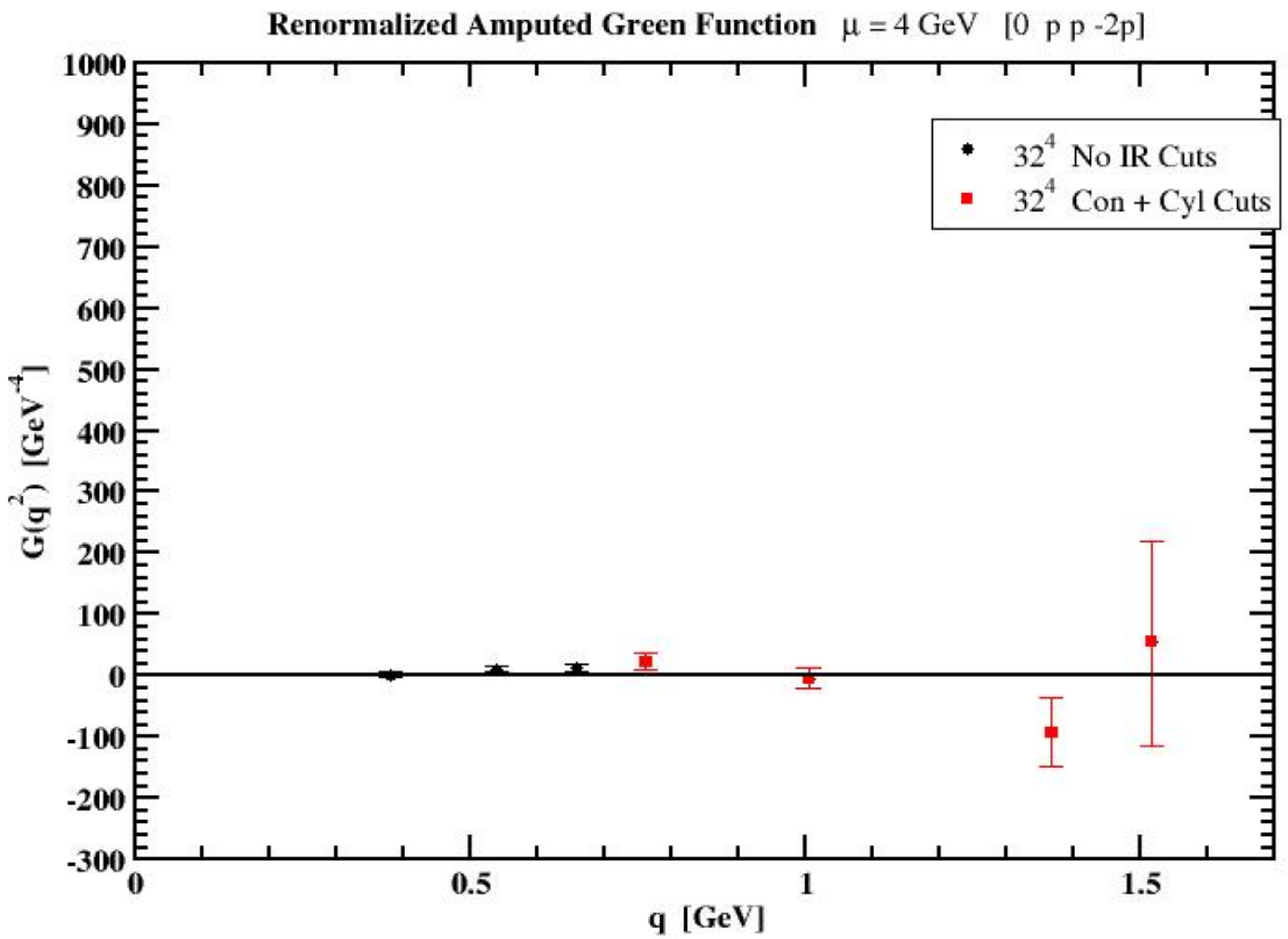
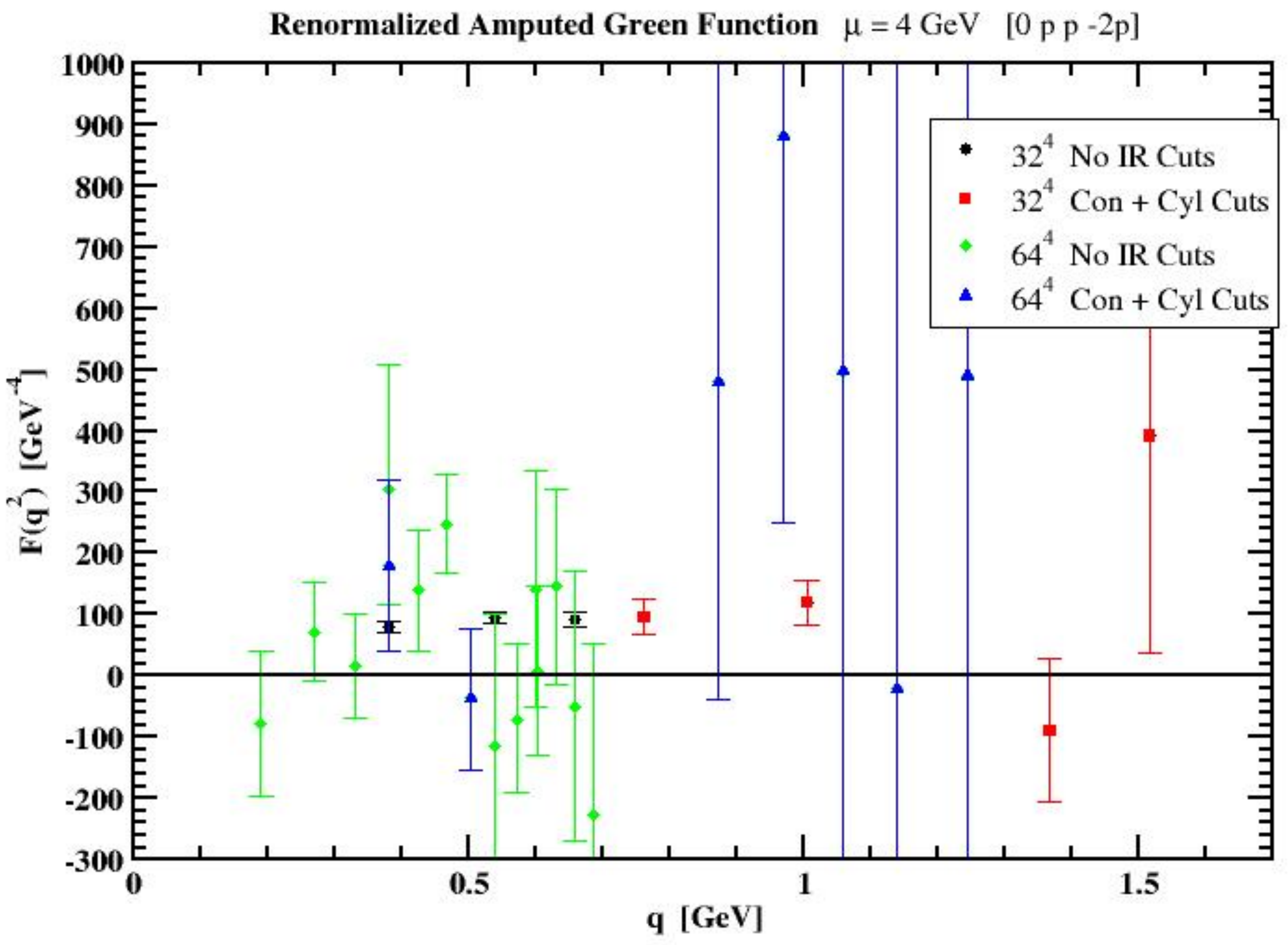
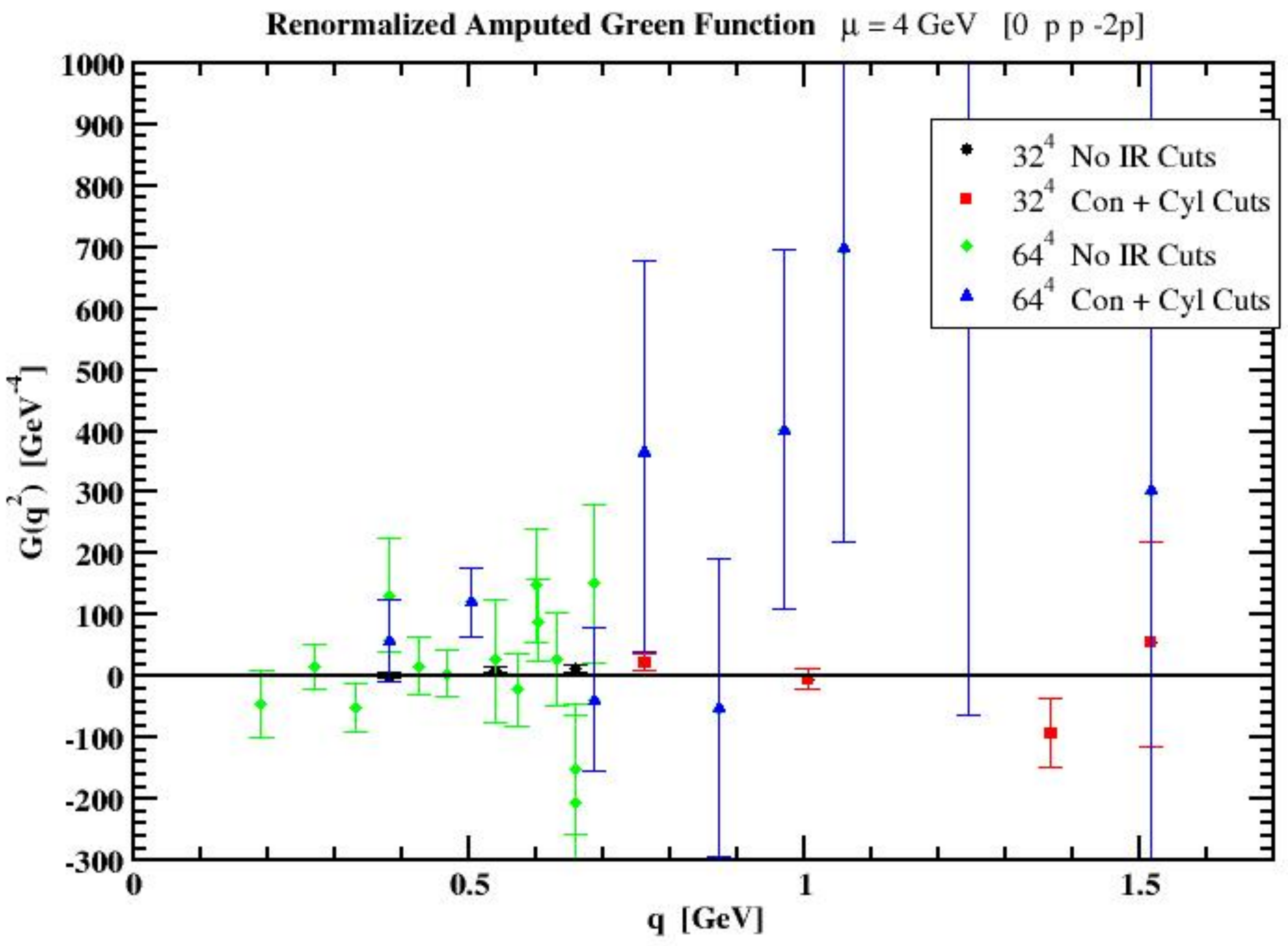




Vertex Form Factors Renormalized @ μ = 4 GeV





Summary and Conclusions

- gluon correlation functions up to 4-external legs are possible with standard lattice methods
- need to increase the statistics and improve the tensor analysis
- It would be great to explore further kinematics (need to be studied)
- How low can we go in momentum?

Work supported by national funds from FCT -- Fundação para a Ciência e a Tecnologia, I.P., Portugal, within projects UIDB/04564/2020, UIDP/04564/2020 and CERN/FIS-PAR/0023/2021. Simulations performed in supercomputers Navigator, managed by LCA -- University of Coimbra [url: www.uc.pt/lca], Lindgren, Sisu (through PRACE projects COIMBRALATT [DECI-9] and COIMBRALATT2 [DECI-12]) and Bob through FCT project CPCA/A2/6816/2020.