Schwinger mechanism in QCD

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VNIVERSITAT DE VALÈNCIA

From first-principles QCD to experiments
Trento, Italy, May 23, 2023

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Dynamical mass generation in QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{c}^a\partial^\mu D^{ab}_\mu c^b + \mathcal{L}_{\rm quarks}$$

- All fundamental **fields are massless** at the level of the QCD Lagrangian
- Perturbation theory cannot generate mass at any finite order

And yet we are brimming with masses!

- 98% of the mass of the hadrons **generated by the nonperturbative QCD dynamics**.
- To study **dynamical mass generation**, we look at the behavior of the nonperturbative QCD Green's function (propagators and vertices).

Mass generation leaves **distinctive signals in the infrared** momentum region of several Green's functions.

Gluon mass generation

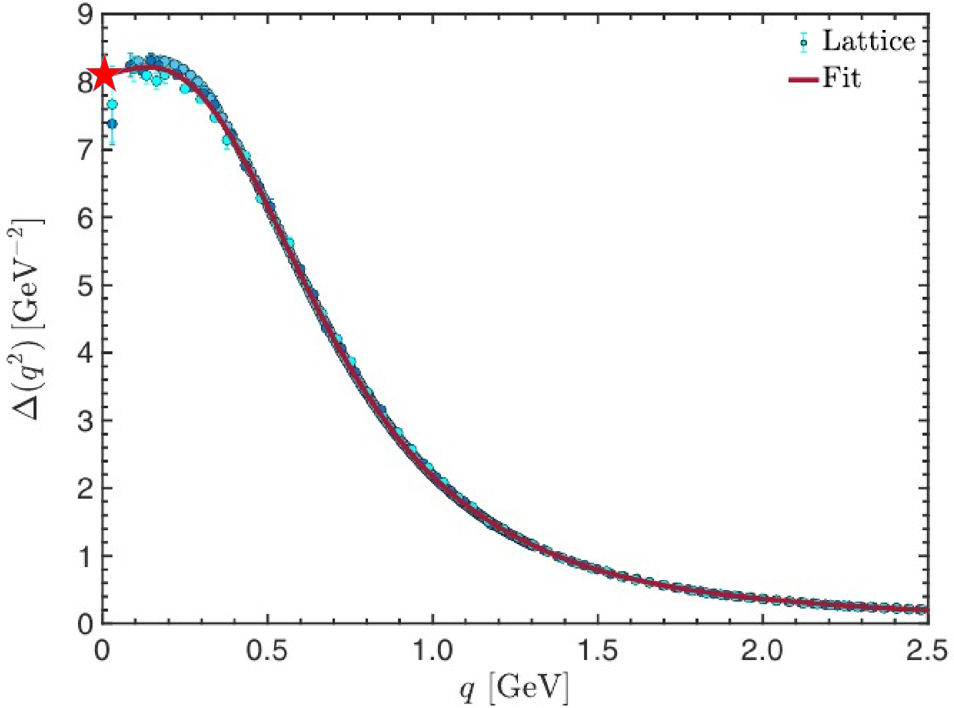
Gluon self-interactions can generate a dynamical mass
J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

- Lattice QCD: **The gluon propagator saturates in the deep infrared**
  I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
  A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
  P. Bicudo, et al, Phys. Rev. D **92**, no.11, 114514 (2015).
  A. C. Aguilar, et al, Eur. Phys. J. C **80**, no.2, 154 (2020).

- Unequivocal signal of gluon mass generation

- All symmetries must be explicitly preserved

- A mass term $m^2 A$ in the Lagrangian is forbidden by gauge invariance.



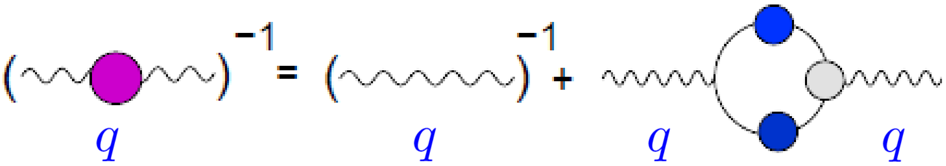
How can the gluon acquire a mass gap?

Schwinger mechanism

A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer.

J. S. Schwinger, Phys. Rev. 125, 397 (1962); Phys. Rev. 128, 2425 (1962)

Schwinger-Dyson equation for gauge boson propagator



$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

If, for some reason

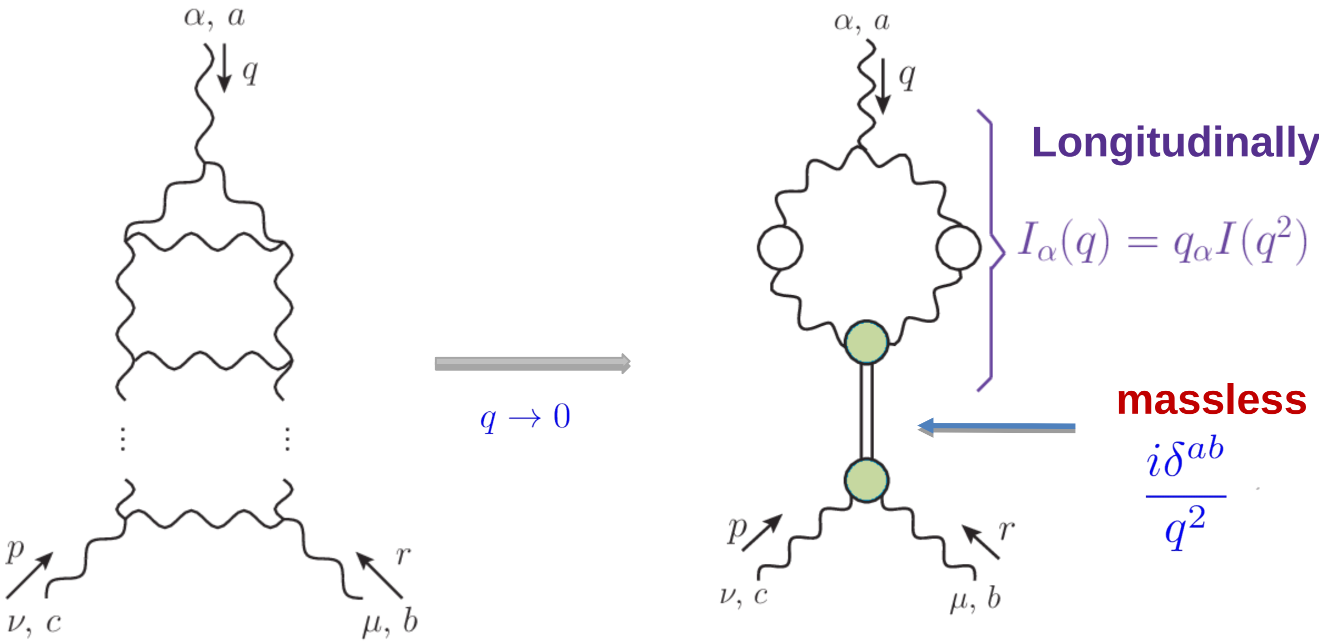
$$\lim_{q \to 0} \Pi(q^2) = \frac{c}{q^2}, \qquad c > 0$$

$$\Delta^{-1}(0) = c > 0$$

But how can the vacuum polarization acquire such a pole?

Massless bound state formalism

If the interaction is sufficiently strong → formation of **massless bound states**



massless colored excitation

$$\frac{i\delta^{ab}}{q^2}$$

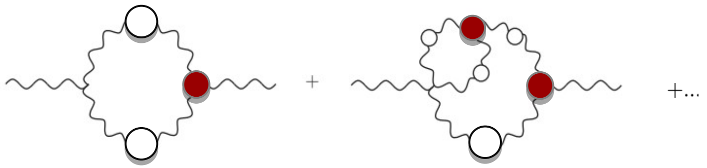
$$\mathbb{\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p) = \Gamma^{abc}_{\alpha\mu\nu}(q,r,p) + g f^{abc}\frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \ldots$$

E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254-3279 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D 104, no.11, 114016 (2021).

Massless bound state formalism

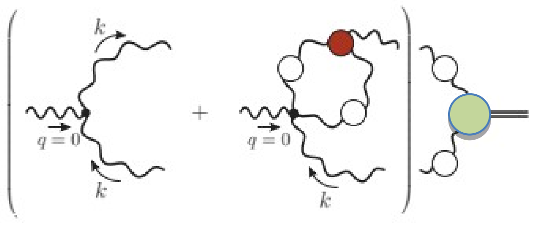
Massless poles in the three-gluon vertex lead to pole in the gluon vacuum polarization:

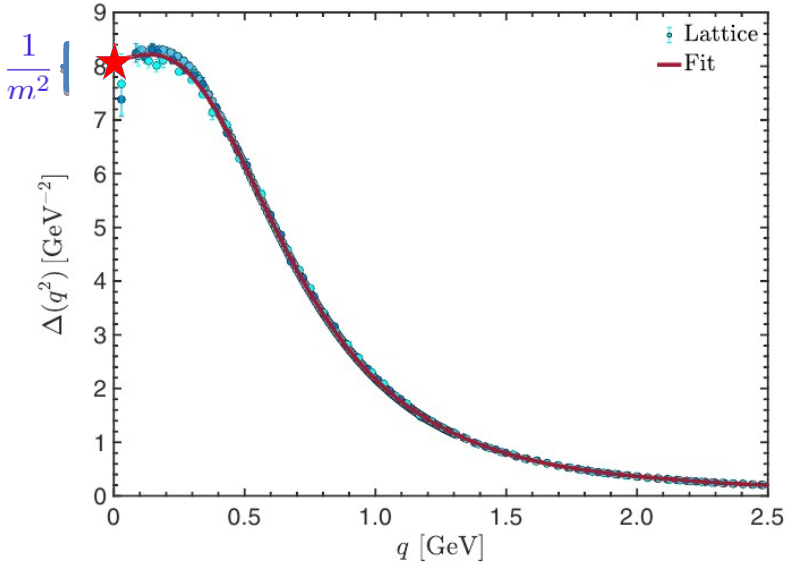
$$\Delta^{-1}(q^2) = q^2 \quad +$$



$q \to 0$

$$m^2 =$$





A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi, D. Ibanez and J. Papavassiliou, Phys. Rev. D 86, 085033 (2012).

Bethe-Salpeter equation

The formation of massless bound state is **dynamical and governed by a Bethe-Salpeter equation.**

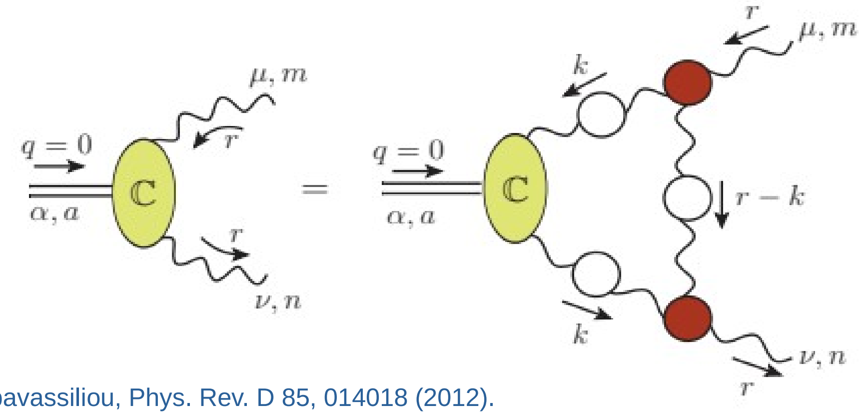
Recalling:

$$\mathrm{I\!\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p) = \Gamma^{abc}_{\alpha\mu\nu}(q,r,p) + g f^{abc}\frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \dots$$

Bose symmetry $\Longrightarrow$ $C_1(0,r,-r) = 0$ $\Longrightarrow$

$$\lim_{q\to 0} C_1(q,r,p) = 2(q\cdot r)\underbrace{\left[\frac{\partial C_1(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)} + \mathcal{O}(q^2)$$

The function $\mathbb{C}(r^2)$ satisfies the equation



BS amplitude

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

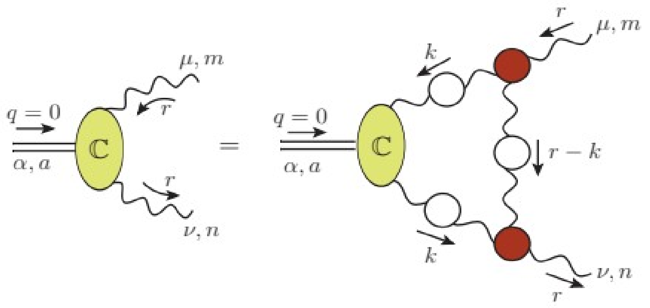
Bethe-Salpeter equation

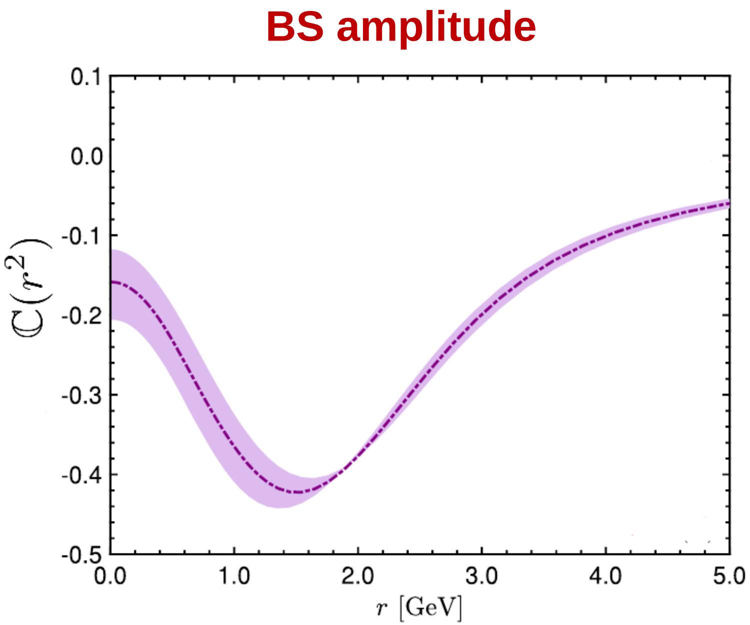
The Bethe-Salpeter equation admits **nontrivial solutions compatible with lattice ingredients** for the:

- Propagator;
- Vertex;
- and, value of the coupling

$$\alpha_s \approx 0.3 \ \ @ \ \ \mu = 4.3 \text{ GeV}$$

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi and J. Papavassiliou, Phys. Rev. D 97, no.5, 054029 (2018).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C 78, no.3, 181 (2018).





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism poles do not show in lattice results

A typical vertex form factor on the lattice is given by:

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p) P_{\alpha'\alpha}(q) P_{\mu'\mu}(r) P_{\nu'\nu}(p) \mathbb{I}\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p) P_{\alpha'\alpha}(q) P_{\mu'\mu}(r) P_{\nu'\nu}(p) \Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu / q^2$

$$\mathbb{I}\Gamma^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\text{poles}}$$

Given that the poles are longitudinally coupled:

$$P_{\alpha\alpha'}(q) P_{\mu\mu'}(r) P_{\nu\nu'}(p) V^{\alpha\mu\nu}(q,r,p) = 0$$

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p) P_{\alpha'\alpha}(q) P_{\mu'\mu}(r) P_{\nu'\nu}(p) \Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p) P_{\alpha'\alpha}(q) P_{\mu'\mu}(r) P_{\nu'\nu}(p) \Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

Lattice extracts the pole-free part of the vertex.

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).
F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).

Testing the Schwinger mechanism with lattice QCD

Question:

Is there a smoking-gun signal of the massless bound state poles, which can be tested with lattice inputs?

Answer:

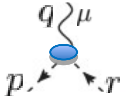
Yes, the **displacement of the Ward identities** satisfied by the vertices.

- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- If there is a massless bound state pole, the **propagators and pole-free parts of the vertices must change in shape to accommodate the pole contribution to the Ward identities**.

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

A toy example: scalar QED

Schwinger mechanism **off**



Takahashi identity

$$\underbrace{q^\mu \Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$, $p \to -r$ — Taylor expansion

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Tensorial decomposition

$$\Gamma_\mu(0,r,-r) = L(r^2) r_\mu$$

$$L(r^2) = 2\frac{\partial D^{-1}(r^2)}{\partial r^2}$$

Schwinger mechanism **on**

$$\mathbb{\Gamma}_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Takahashi identity does not change

$$q^\mu \mathbb{\Gamma}_\mu(q,r,p) = q^\mu \Gamma_\mu(q,r,p) + C(q,r,p)$$
$$= D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$ — Taylor expansion

Ward identity

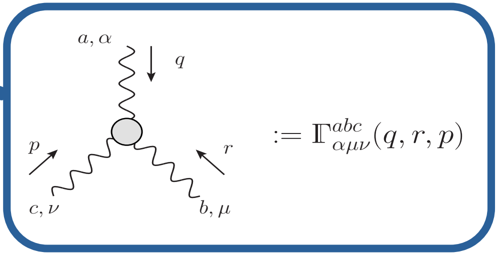
$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

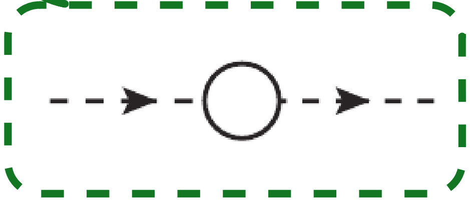
$$\Rightarrow \quad L(r^2) = 2\frac{\partial D^{-1}(r^2)}{\partial r^2} - \underbrace{2\,\mathbb{C}(r^2)}_{\text{Displacement = BS amplitude}}$$

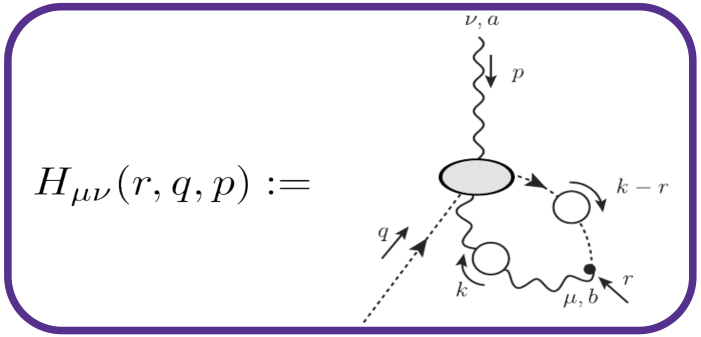
Ward identity displacement in QCD

The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**:

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P^\sigma_\nu(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P^\sigma_\mu(r)H_{\sigma\nu}(r,q,p)]$$







Then, assume the three-gluon vertex has a massless bound state pole:

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \dots$$

And expand around $q = 0$

Ward identity displacement in QCD

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Ward identity

$$L_{\mathrm{sg}}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

- Ingredients can (mostly) be computed with lattice simulations.
- Combine ingredients and determine if there is a nontrivial displacement.

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

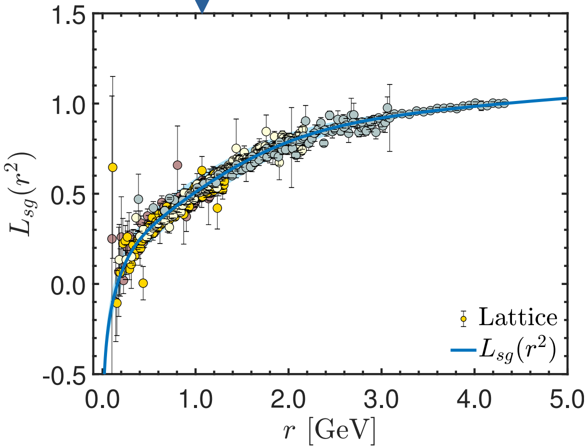
Ward identity displacement in QCD

$$q^\alpha \mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$



Soft-gluon form factor of the three-gluon vertex

$$P_\mu^{\mu'}(r)P_\nu^{\nu'}(r)\mathrm{I\!\Gamma}_{\alpha\mu'\nu'}(0,r,-r) = 2L_{\rm sg}(r^2)r_\alpha P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$$

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

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$q \to 0$

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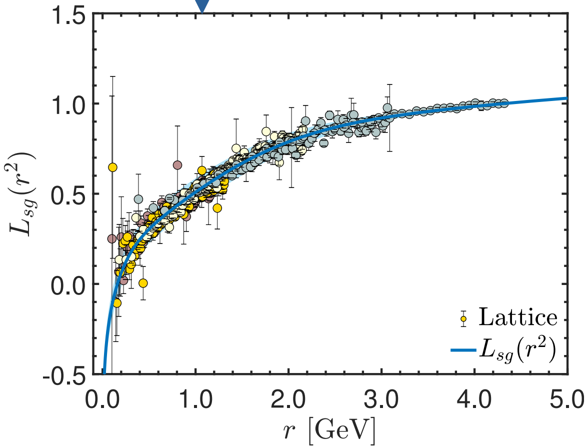

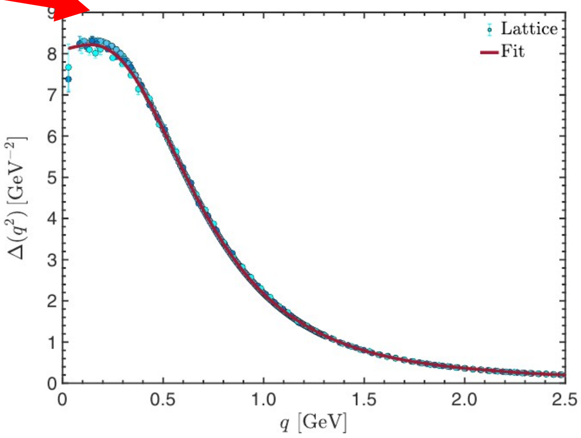

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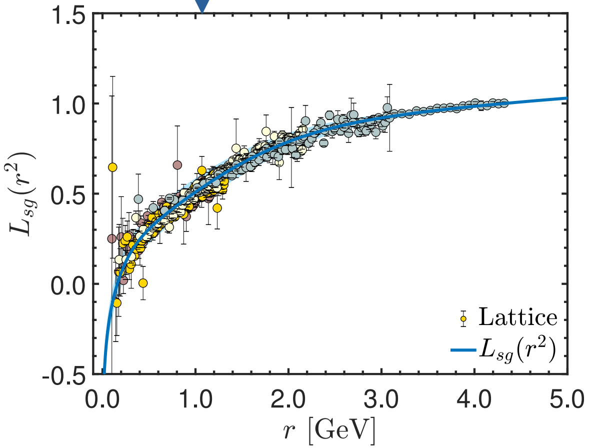
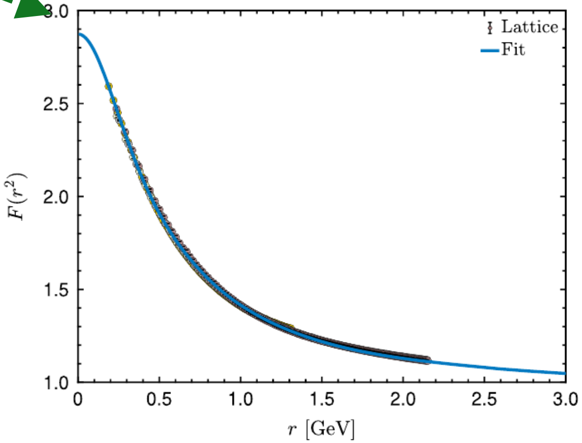
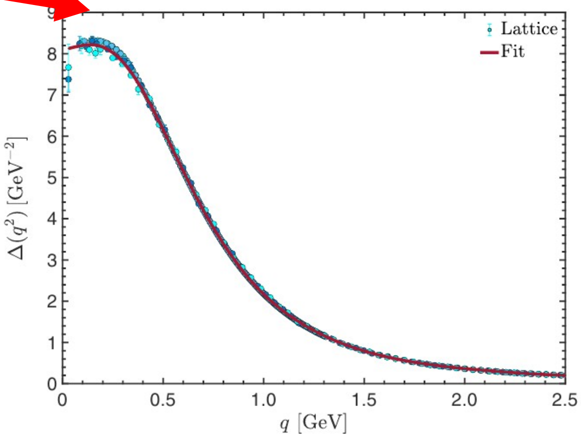
$q \to 0$

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$







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Ward identity displacement in QCD

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$q \to 0$ ⇩

Ward identity

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★ **Only ingredient not yet determined directly by lattice simulations.**

Ward identity displacement in QCD

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

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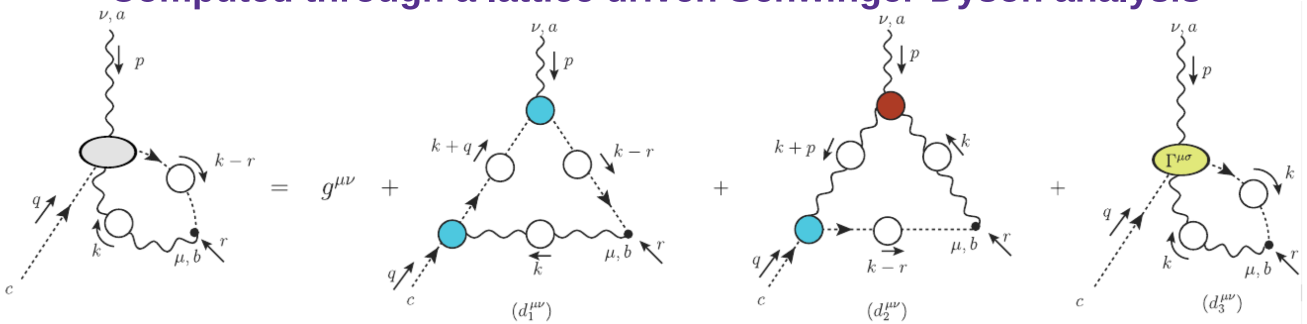
Partial derivative of the ghost-gluon kernel
Computed through a lattice driven Schwinger-Dyson analysis



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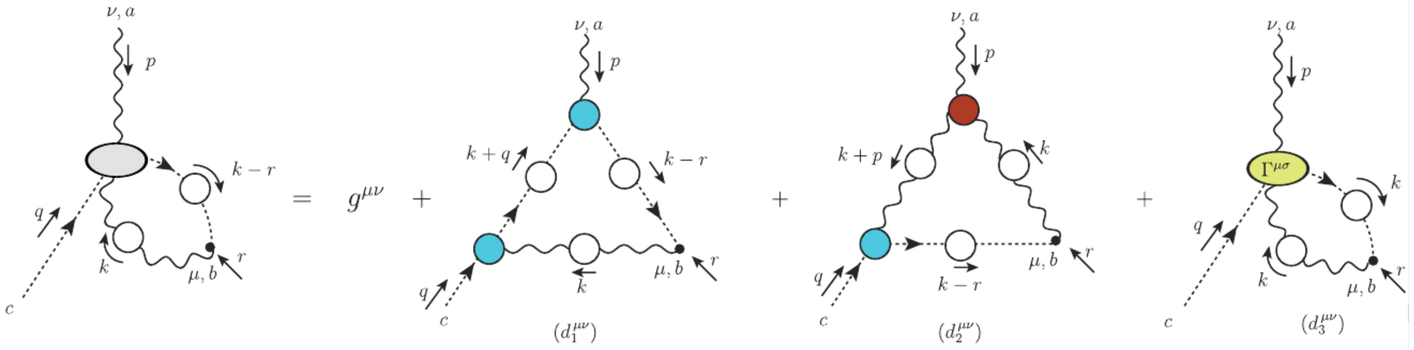
Lattice driven Schwinger-Dyson calculations

The $\mathcal{W}(r^2)$ can be obtained from the **Schwinger-Dyson** equation for the **ghost-gluon scattering kernel**



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Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;

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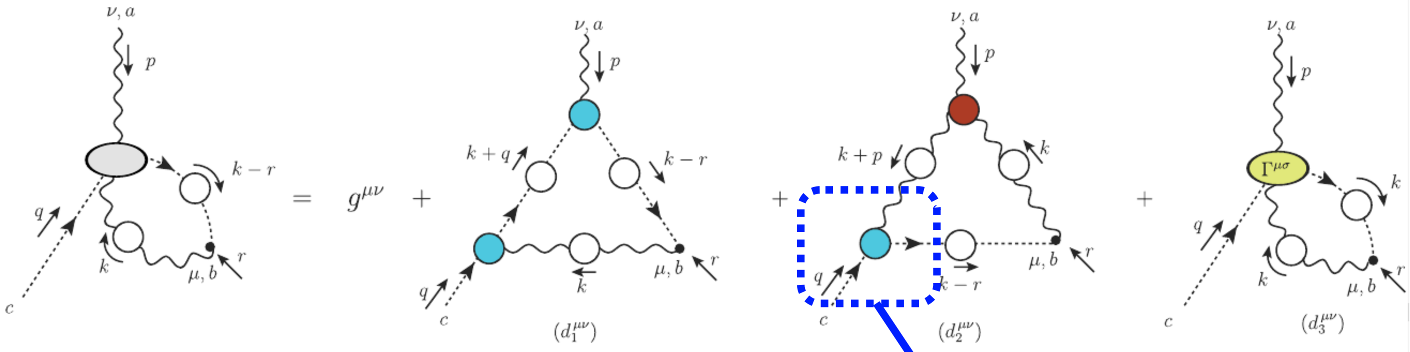

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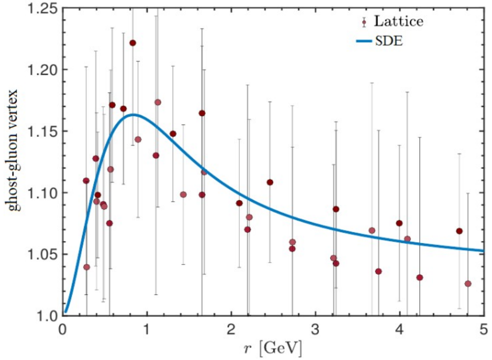
Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;
3) Ghost-gluon vertex;



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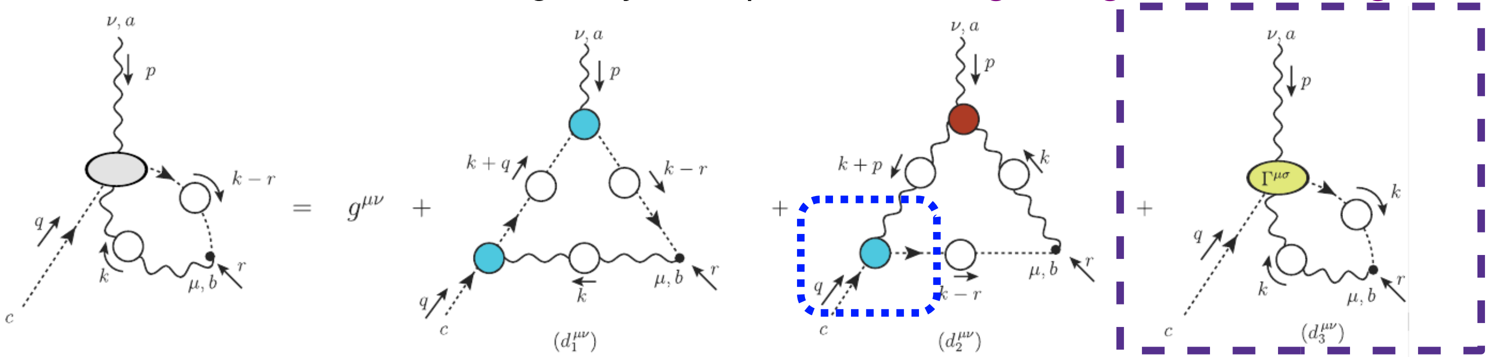
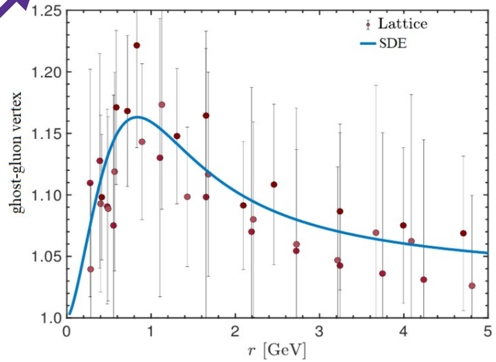


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

Depends on 5 ingredients, all of which are now well constrained:

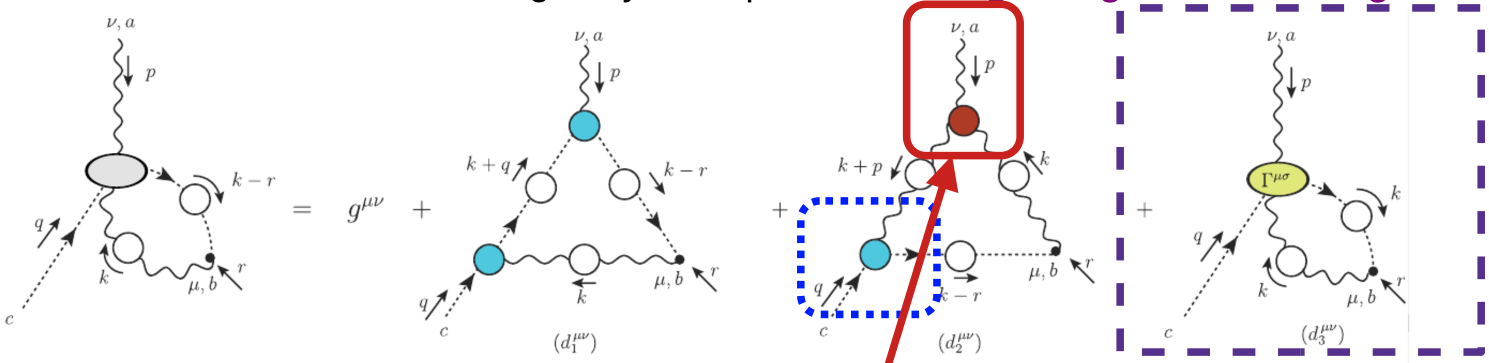
1) Ghost propagator;
2) Gluon propagator;
3) Ghost-gluon vertex;
4) The four-point function is expected to be subleading.



E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).
M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
N. Barrios, M. Pelaez, U. Reinosa, et al. Phys. Rev. D102, 11401 (2020).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

Lattice driven Schwinger-Dyson calculations

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



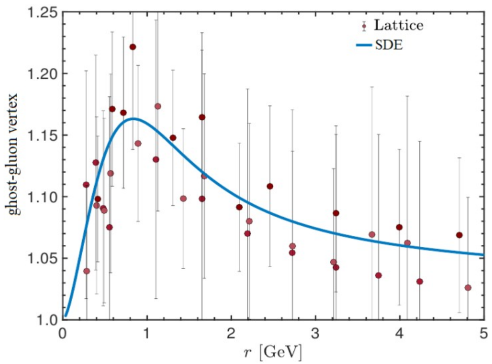
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;
3) Ghost-gluon vertex;
4) The four-point function is expected to be subleading

5) The main uncertainty is in the three-gluon vertex



E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).
M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
N. Barrios, M. Pelaez, U. Reinosa, et al. Phys. Rev. D102, 11401 (2020).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

Three-gluon vertex content

For $\mathcal{W}(r^2)$, only a particular projection contributes:

$$\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2) := \frac{1}{2}(q-r)^{\nu}\overline{\Pi}^{\mu}_{\mu\nu}(q,r,p)$$

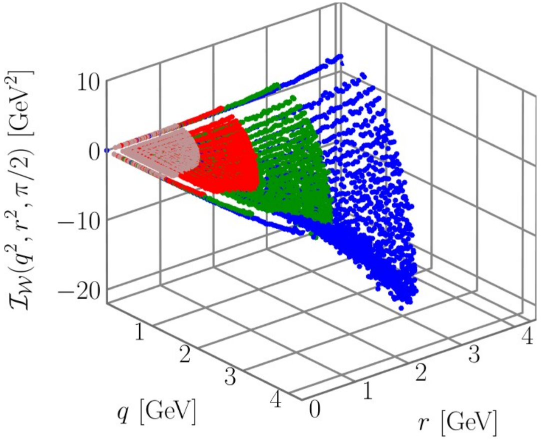
$$\overline{\Pi}_{\alpha\mu\nu}(q,r,p) := P^{\alpha'}_{\alpha}(q)P^{\mu'}_{\mu}(r)P^{\nu'}_{\nu}(p)\Pi_{\alpha'\mu'\nu'}(q,r,p)$$

- **Transverse projection of the vertex.**
- **Pole-free**
- **Accessible to lattice simulations.**

The projection $\mathcal{I}_{\mathcal{W}}$ can be computed by two different methods that rely on lattice results:

A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

M1) Direct simulation



- No approximation.
- Restricted to momenta < 5 GeV

M2) Planar degeneracy (see F. De Soto's talk)

Lattice and continuum studies have shown the accuracy of the approximation:

$$\overline{\Pi}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)L_{\rm sg}(s^2) \qquad s^2 := (q^2+r^2+p^2)/2$$

where $\overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)$ is the tree-level transverse vertex.

- $L_{\rm sg}(s^2)$ is accurately known for entire range of momenta.

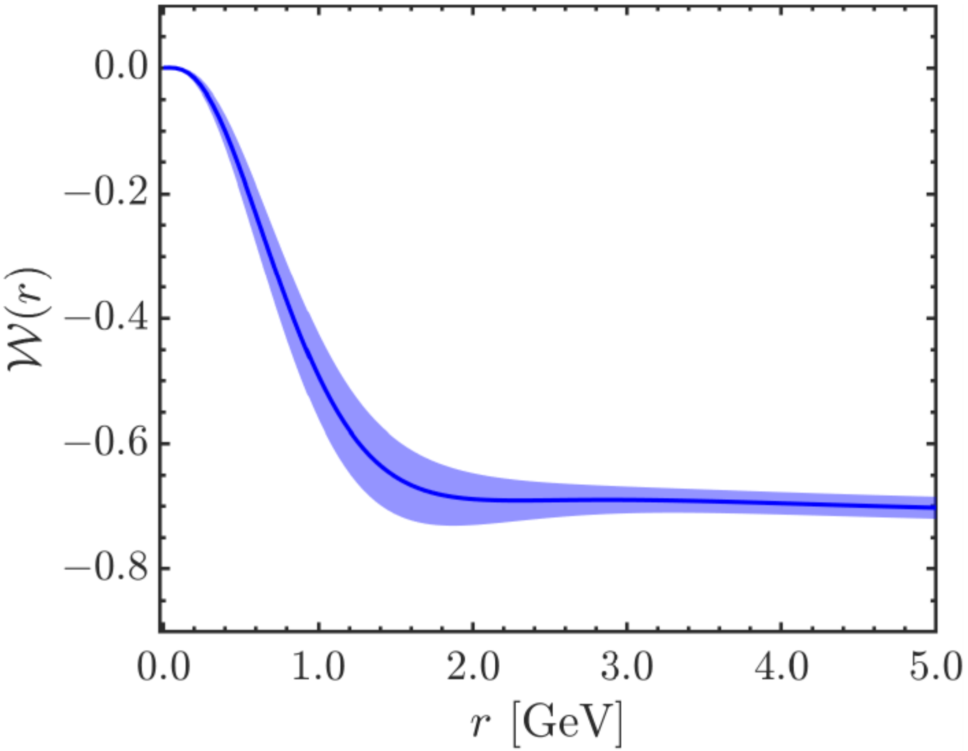
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016).
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016).
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).
F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).
A. C. Aguilar, M. N. Ferreira, J. Papavassiliou and L. R. Santos, arXiv:2305.05704

Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

- **Errors are propagated from known error of the planar degeneracy approximation.**

Impact of three-gluon vertex under control



A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $\mathbb{C}(r^2)$ from the **WI displacement**

$$\mathbb{C}(r^2) = L_{\rm sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

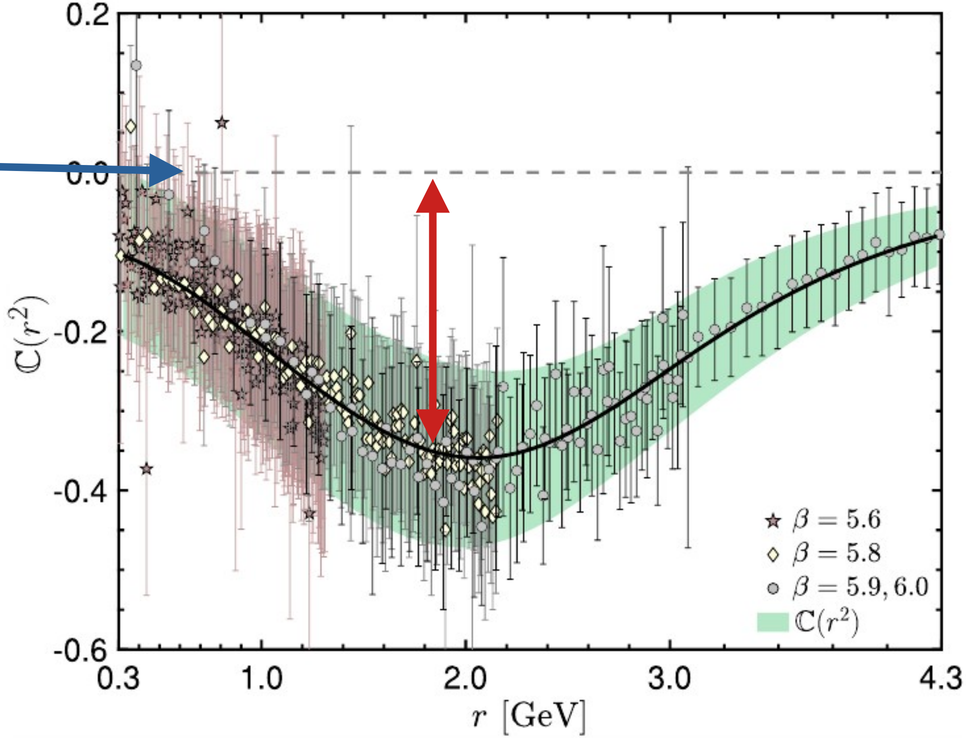
- Clearly nonzero.
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

***p*-value of null hypothesis is tiny:**

$$P_{\mathbb{C}_0} = \int_{\chi^2=2\,630}^{\infty} \chi^2_{\rm PDF}(515, x)dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the 5σ level.



A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Results for $\mathbb{C}(r^2)$

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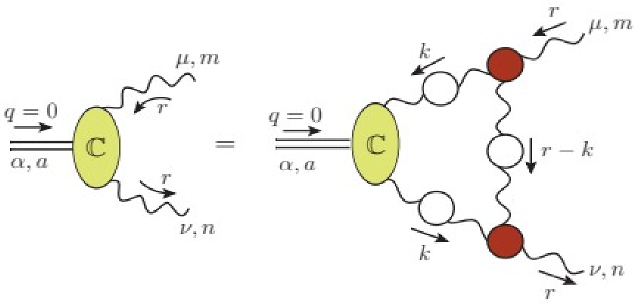
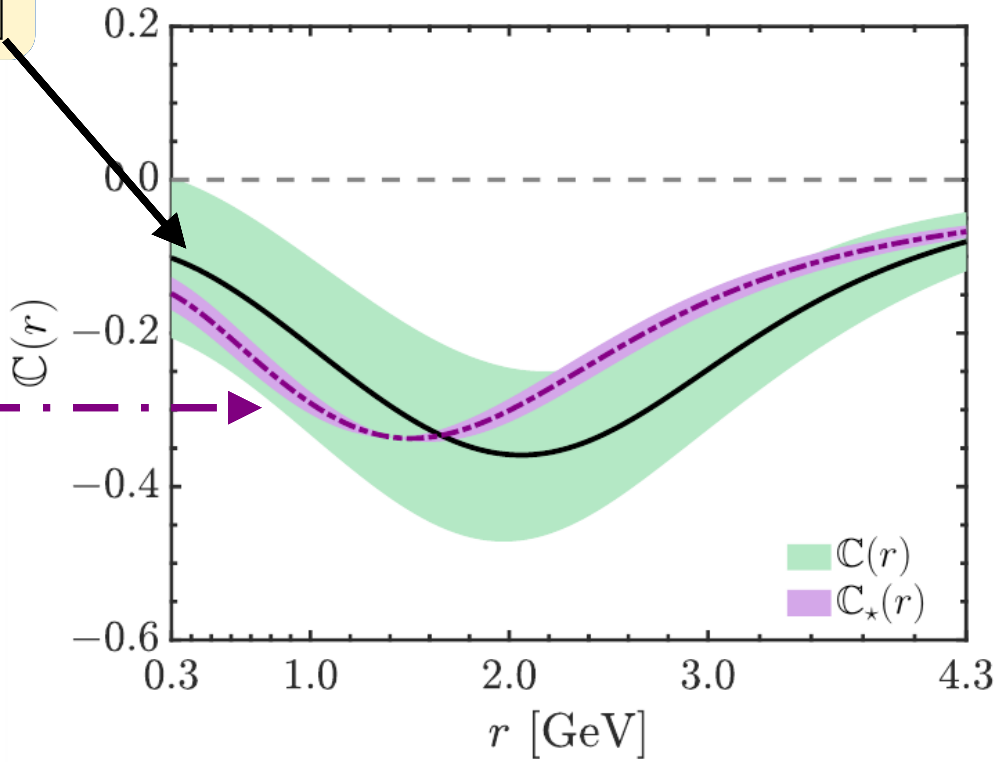
$$\mathbb{C}(r^2) = L_{\rm sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

- Moreover, we find good agreement to the BSE prediction.





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).
M. N. Ferreira and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

Conclusions and outlook

- The **WI displacement** allows us to determine the amplitude $\mathbb{C}(r^2)$ of the **poles that trigger the Schwinger mechanism** in **QCD**.

- Only one ingredient, $\mathcal{W}(r^2)$, not directly accessed by lattice simulations.

- We capitalize on **general kinematics lattice simulations of the three-gluon vertex to stringently control** $\mathcal{W}(r^2)$.

- Our results show **a nonzero** $\mathbb{C}(r^2)$ **with probability near unity**.

In the near future we will explore additional predictions of the Schwinger mechanism:

- Indirect signals: value of the Kugo-Ojima function at the origin (see P. J. Silva's talk).

- Schwinger poles are expected to appear in other vertices, *e.g.*: **quark-gluon** and **four-gluon,** that can also be simulated on the lattice (see O. Oliveira's talk).

- These vertices might exhibit their own WI displacements.

Backup slides

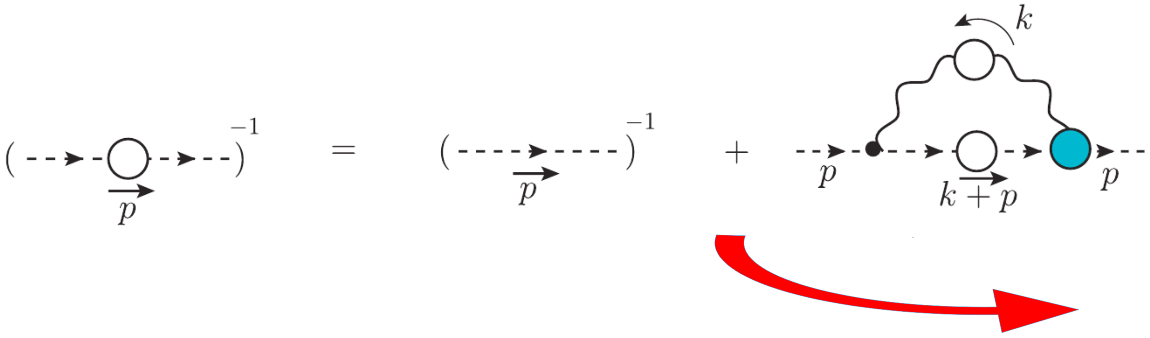
Indirect signals: Finite ghost dressing function

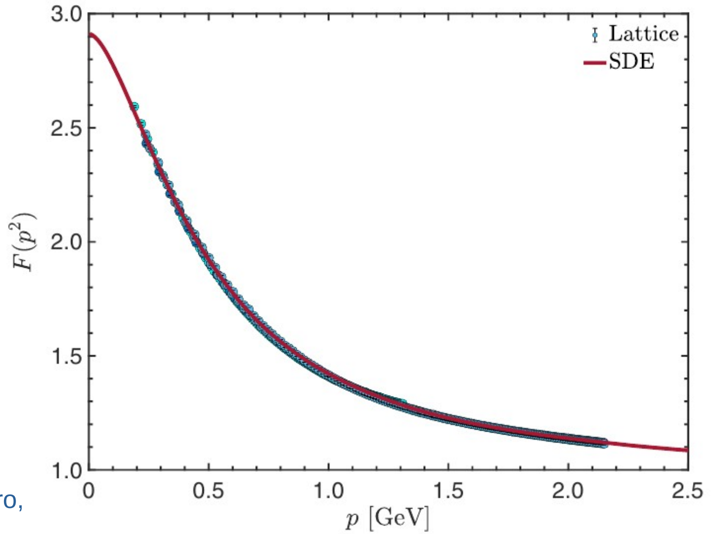
The generation of a gluon mass gap leaves distinctive imprints in other Green's functions.

- The Schwinger mechanism leaves the **ghost propagator,** $D(q^2)$**, massless.**
- But its **dressing function**, $F(q^2)$, given by

$$D(q^2) = \frac{iF(q^2)}{q^2}$$

becomes IR finite.





A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

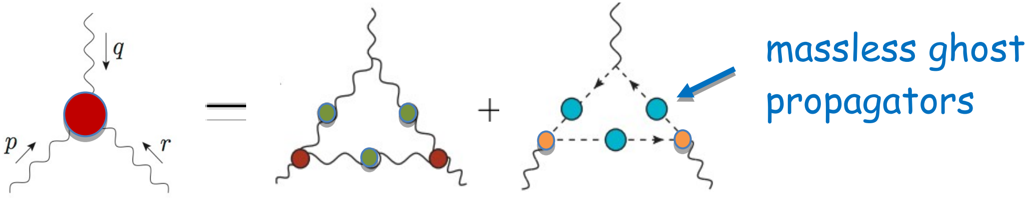
Indirect signals: IR divergence of three-gluon vertex

Three-gluon vertex in the IR exhibits suppression and zero crossing

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014).
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).

A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).

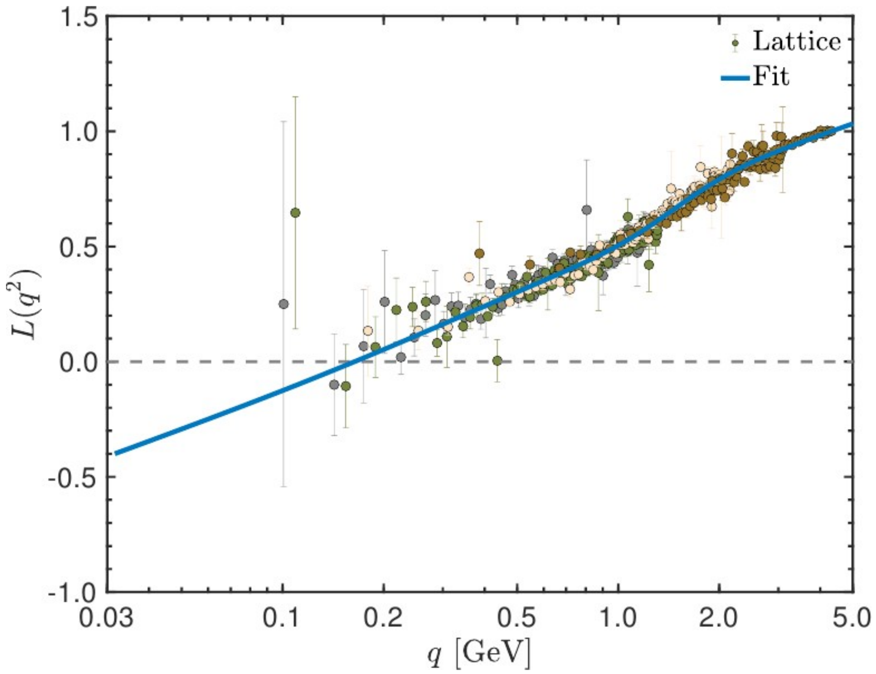
Within the Schwinger mechanism, the infrared behavior of the classical form factor of the three-gluon vertex is characterized by the interplay between two types of logarithms:



$$L_{\rm sg}(r^2) = a + b\ln\left(\frac{r^2+m^2}{\mu^2}\right) + c\ln\left(\frac{r^2}{\mu^2}\right)$$

"protected" "unprotected"

- In the IR, $L_{\rm sg}(r^2) \to -\infty$, logarithmically.
- Explains IR suppression and zero-crossing



A. C. Aguilar, F. De Soto, M.N.F., J. Papavassiliou, J. Rodriguez-Quintero, Phys. Lett. B818 (2021) 136352

Method 2): Planar degeneracy

To quantify the accuracy of the approximation it is convenient to define

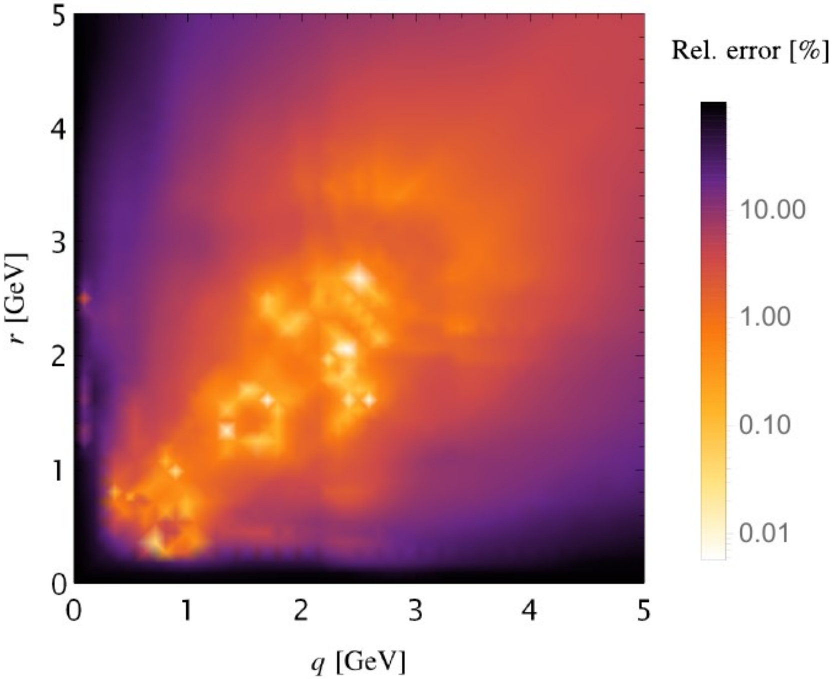
$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^0_{\mathcal{W}}(q^2,r^2,p^2)}$$

Planar degeneracy →

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\rm sg}(s^2)$$

Then we can measure the relative difference between $L_{\rm sg}(s^2)$ and $\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)$

- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

Errors are propagated from known error of the planar degeneracy approximation.

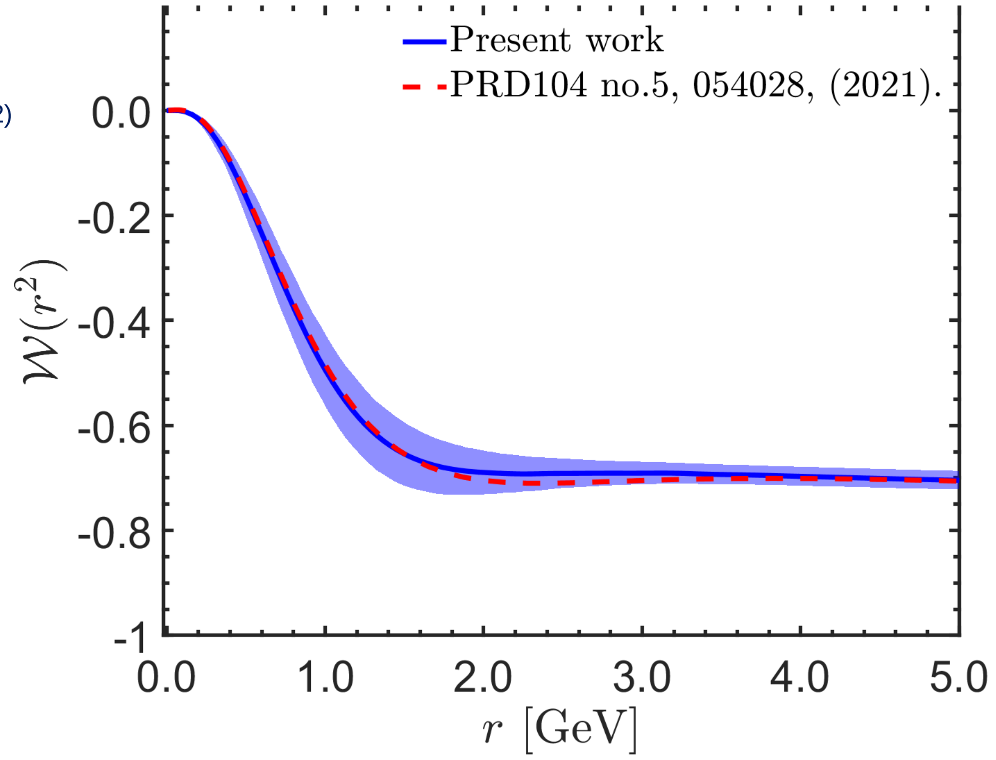
- **Result agrees well with previous calculation**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022)

- Previous result employed a particular *Ansatz* for three-gluon vertex.

- **New result stringently constrained by lattice simulation of the three-gluon vertex.**

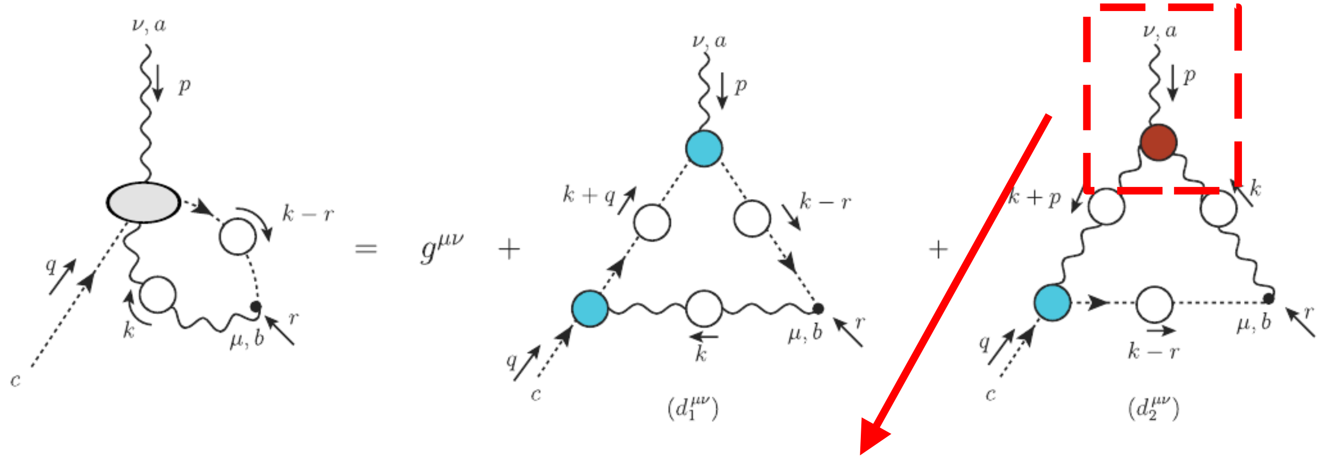
Impact of three-gluon vertex under control

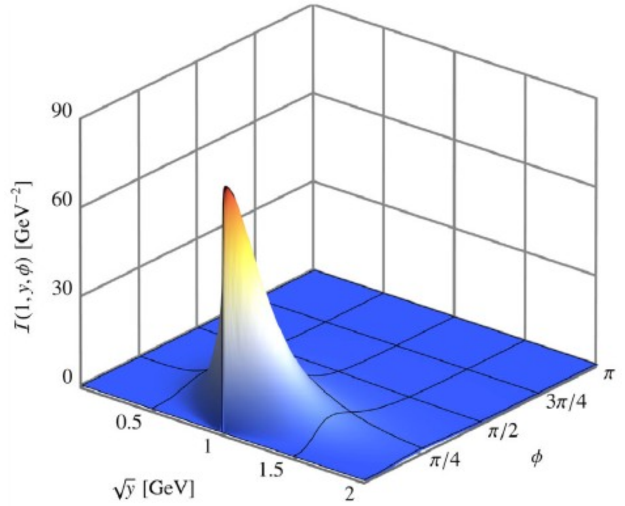


A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Truncation error

The full Schwinger-Dyson equation for $\mathcal{W}(r^2)$ is





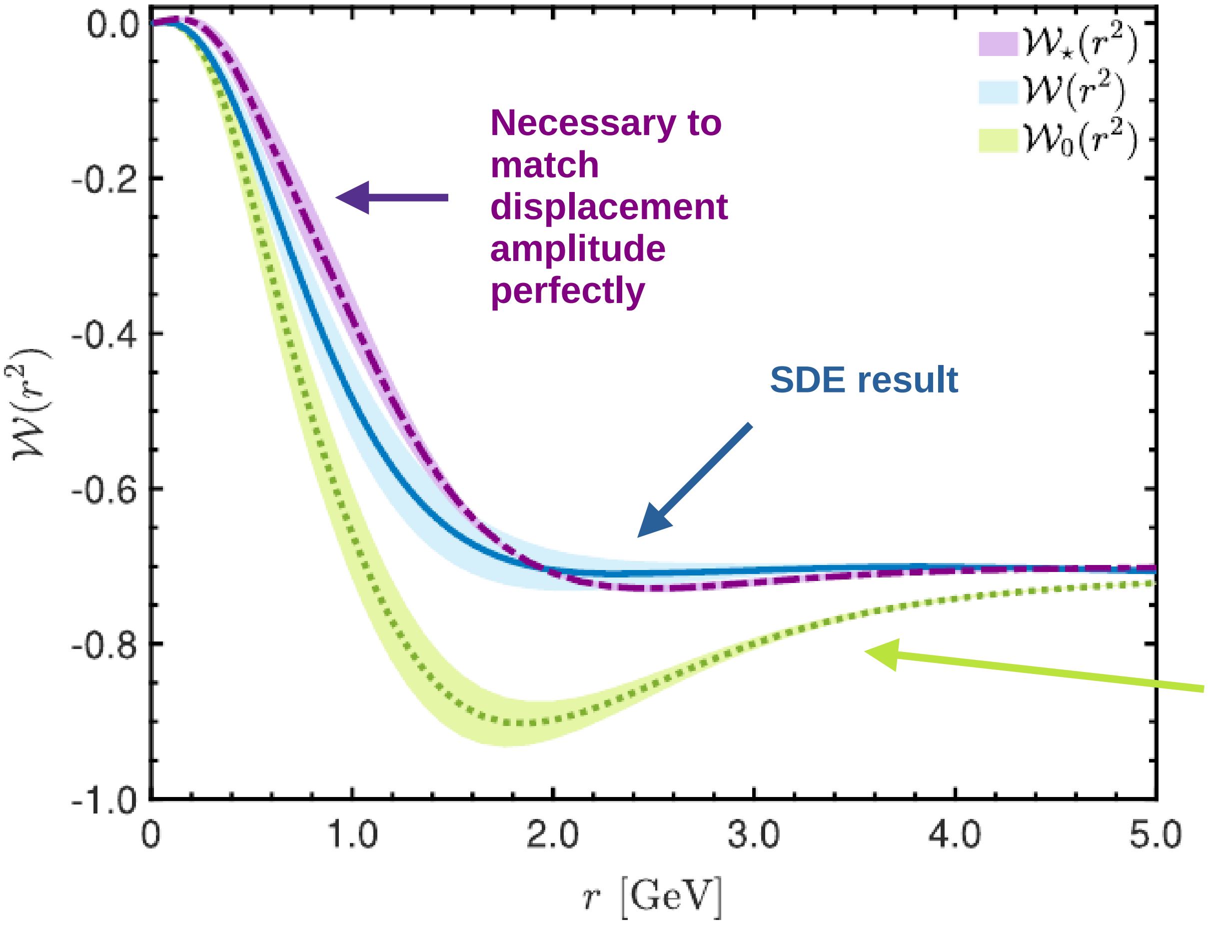
- Three-gluon vertex is a complicated object, with 14 tensor structures.

A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.9, 094010 (2019).
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980). [erratum: Phys. Rev. D **23**, 3085 (1981)].

- But $\mathcal{W}(r^2)$ integrand is sharply peaked, and is sensitive only to the particular projection $L_{\rm sg}(r^2)$ which is well determined by **lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error



Necessary to match displacement amplitude perfectly

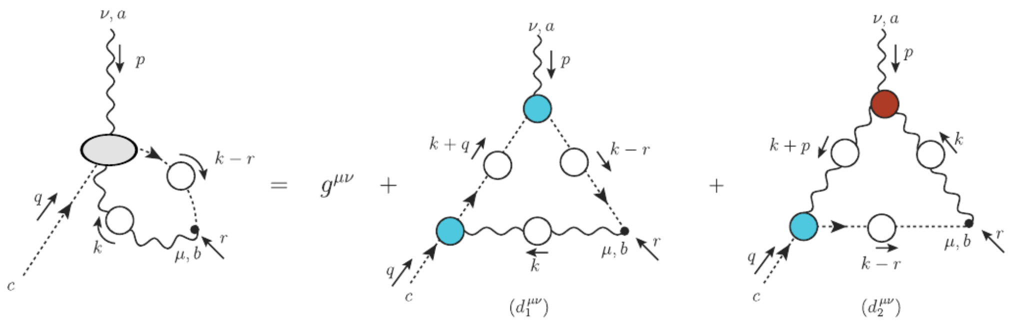
SDE result

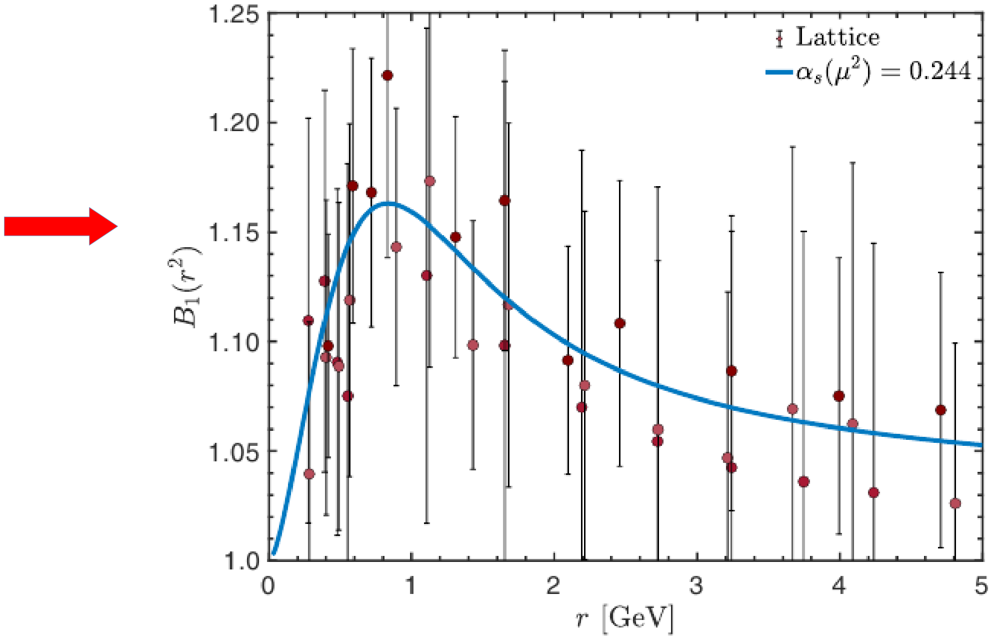
Necessary to match null hypothesis

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error

The same truncation used to determine $\mathcal{W}(r^2)$, reproduces the available lattice data for the ghost-gluon vertex:



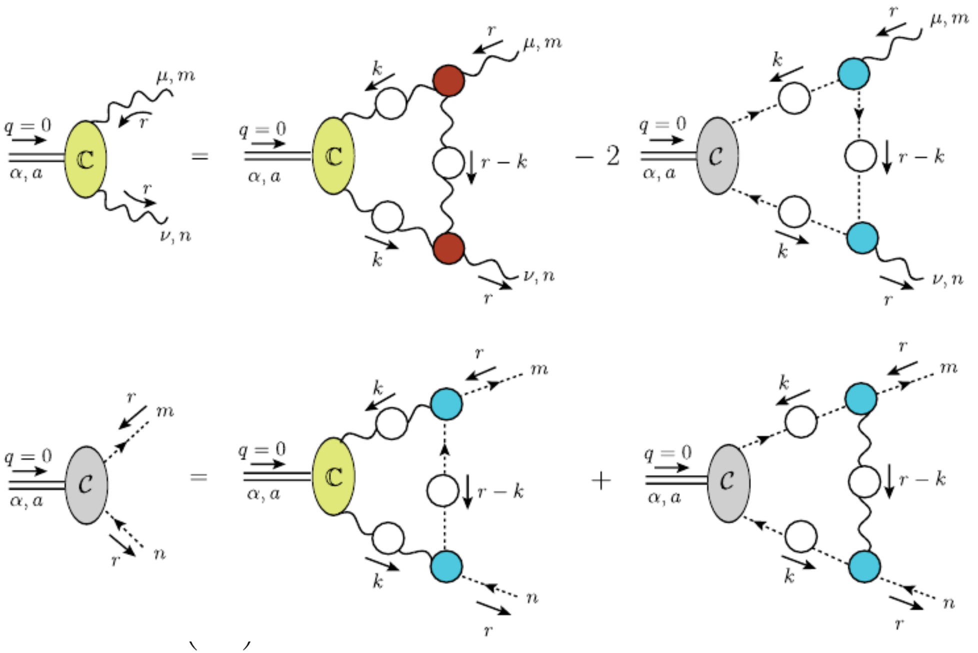


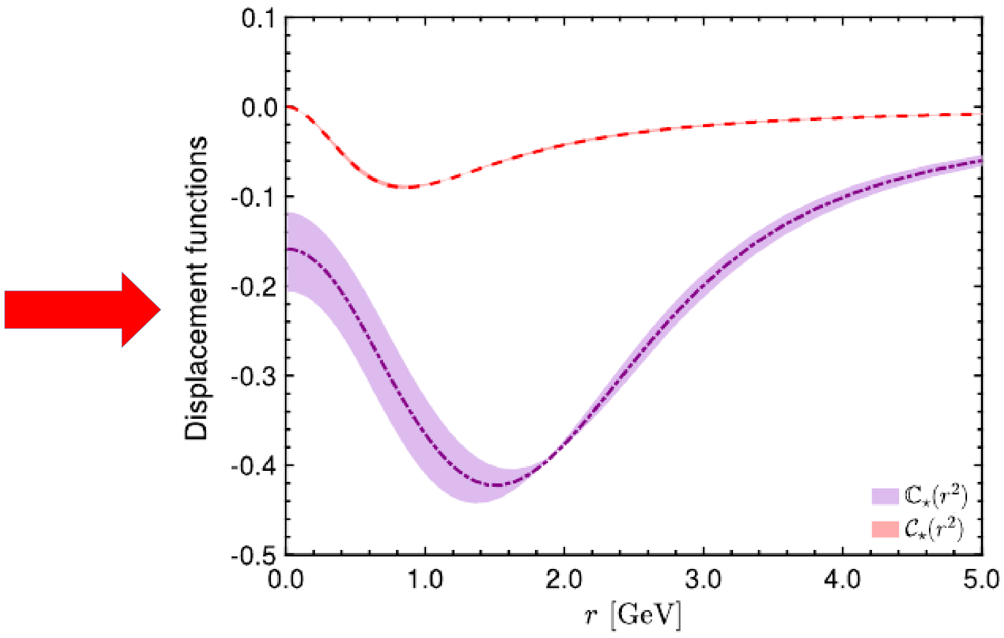
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Lattice data from: A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).

Pole of the ghost-gluon vertex

The Schwinger-Dyson equation for the displacement amplitude $\mathbb{C}(r^2)$ can be coupled to a pole also in **ghost-gluon vertex**



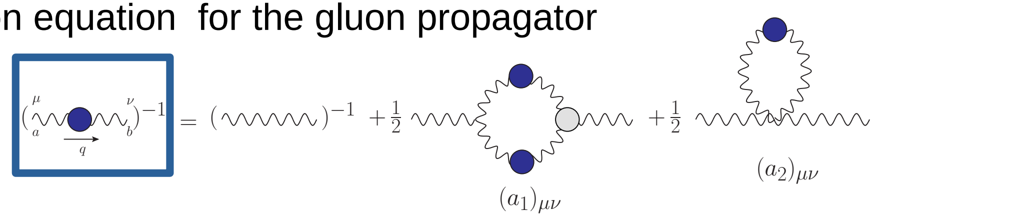


Effect on $\mathbb{C}(r^2)$ is negligible because ghost-gluon pole amplitude, $\mathcal{C}(r^2)$, is subleading.

A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

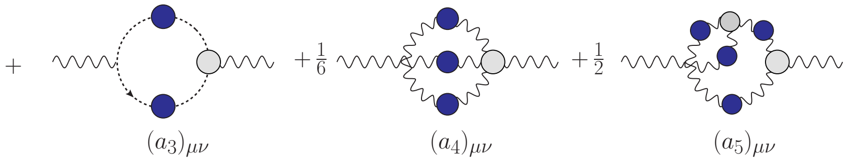
Schwinger mechanism

- The **gluon mass generation must occur without violating gauge symmetry**.
- Recalling the Schwinger-Dyson equation for the gluon propagator



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$



It can be shown that

Gauge symmetry + Regular vertices at *q*² = *0* → $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

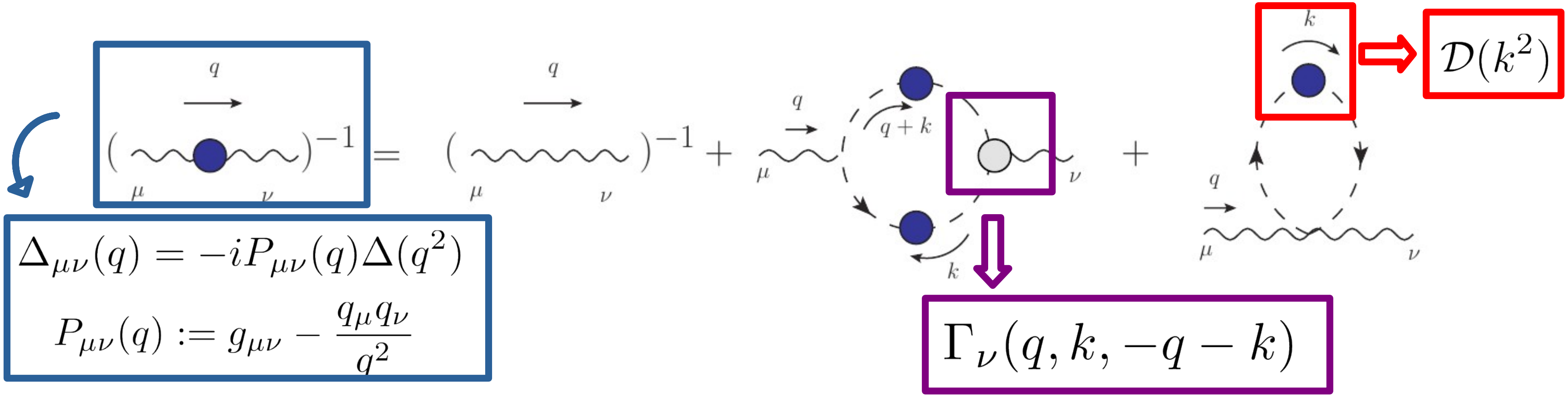
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Seagull cancellation

To understand **how gauge fields can become massive by the Schwinger mechanism**, let us first recall how gauge symmetry ***usually*** implies their masslessness.

A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

To this end, consider the **Schwinger-Dyson equation** for the scalar QED **photon propagator**



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Gamma_\nu(q, k, -q-k)$$

$$\mathcal{D}(k^2)$$

At q = 0, we obtain:

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

Seagull cancellation

Now, **gauge symmetry** implies the **Ward identity**:

$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

q = 0 ⇒

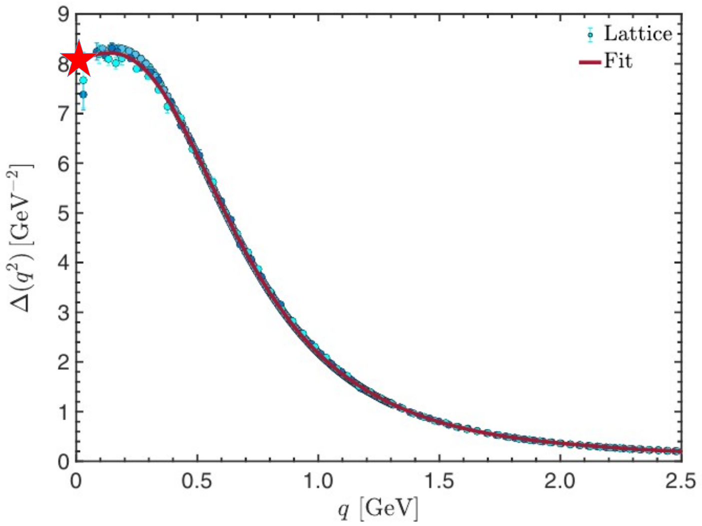
$$\Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^\mu}$$

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2) k^\mu \Gamma_\mu(0,k,-k) - 2ie^2 \int_k \mathcal{D}(k^2)$$

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\left[\int_k k^2 \frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k^2)\right] = 0$$

Seagull identity (integration by parts in *d* dimensions).

Then, how can we have saturation?



A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

Evading the seagull cancellation

Suppose the vertex has a **pole at *q*=0, coupled longitudinally to *q***, i.e.

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

$$\Gamma_\mu(q,r,p) \to \mathrm{I}\Gamma_\mu(q,r,p) = \boxed{\frac{q_\mu}{q^2} C(q,r,p)} + \Gamma_\mu(q,r,p)$$

Does not contribute explicitly to $\Delta(q^2)$ because it is longitudinal.

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2) k^\mu \Gamma_\mu(0,k,-k) - 2ie^2 \int_k \mathcal{D}(k^2)$$

However, now the regular part satisfies a "displaced" Ward identity:

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^\mu} - 2r_\mu \mathcal{C}(r^2)$$

$$\mathcal{C}(r^2) := \left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\int_k k^2 \mathcal{D}^2(k^2) \mathcal{C}(k^2)$$

Ward identity and its displacement in QCD

In the ***q*=0 limit**, the regular part has the tensor decomposition

$$\Gamma_{\alpha\mu\nu}(0,r,-r) = 2L_{\rm sg}(r^2)r_\alpha g_{\mu\nu} + \mathcal{A}_2(r^2)(r_\mu g_{\nu\alpha} + r_\nu g_{\mu\alpha}) + \mathcal{A}_3(r^2) r_\alpha r_\mu r_\nu$$

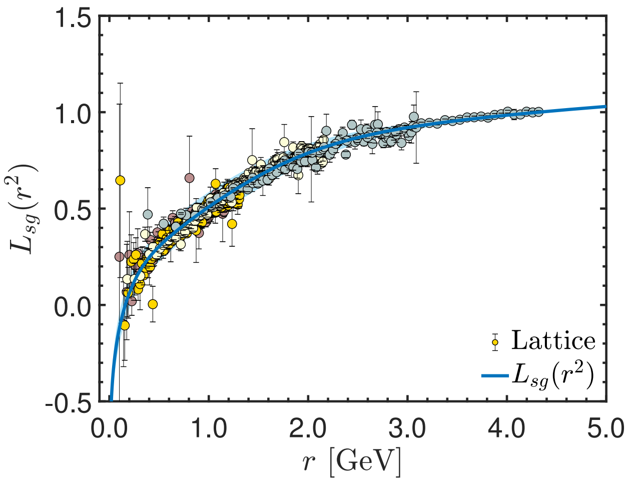
and from the Slavnov-Taylor we obtain A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

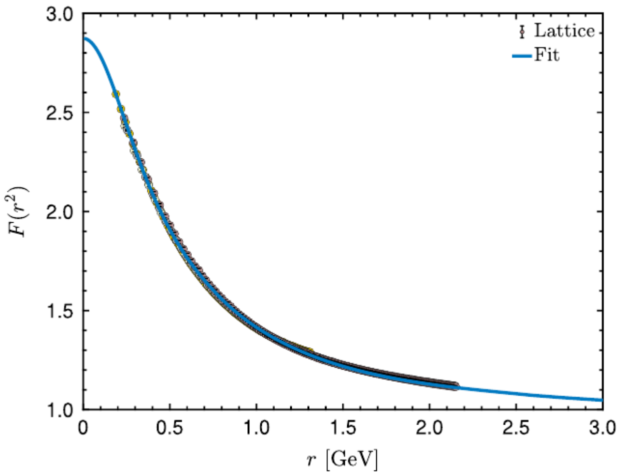
$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

where $$\mathcal{W}(r^2) = -\frac{r^\alpha P^{\mu\nu}(r)}{3}\left[\frac{\partial H_{\mu\nu}(r,q,p)}{\partial q^\alpha}\right]_{q=0}$$ and $$\mathbb{C}(r^2) := \left[\frac{\partial C_1(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude; saturates gluon propagator

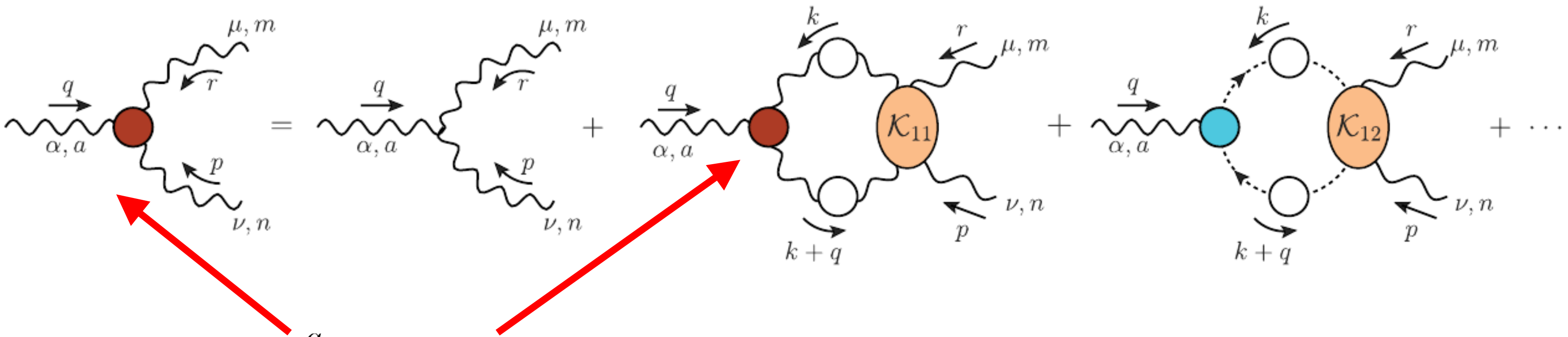
- $L_{\rm sg}(r^2)$, $\Delta(q^2)$ and $F(q^2)$ known from lattice simulations.
A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).
- Allows us to test, whether displacement of WI occurs in QCD.
- We only need to compute $\mathcal{W}(r^2)$.





Derivation of the Schwinger pole Bethe-Salpeter equation

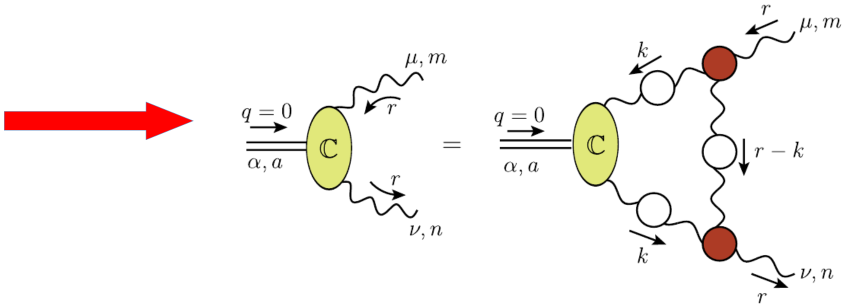
We start with the Schwinger-Dyson (or more generaly nPI) equation for the vertex and assume the presence of a massless pole:



$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \ldots$$

Now multiply by q^2 and take $q = 0$. Only terms containing poles remain:

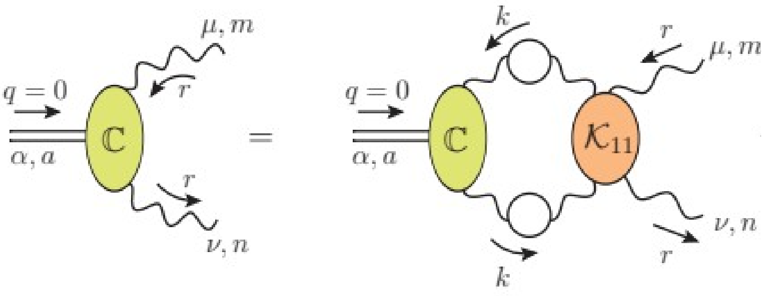
- **Inhomogeneous Schwinger-Dyson** equation becomes a **Homogeneous Bethe-Salpeter** equation.



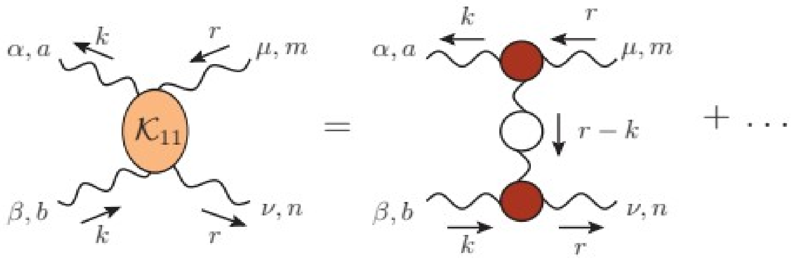
A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

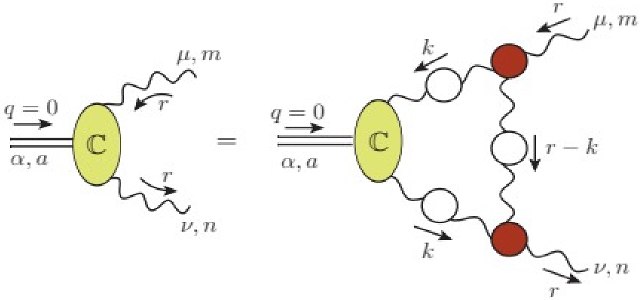
One-gluon exchange approximation

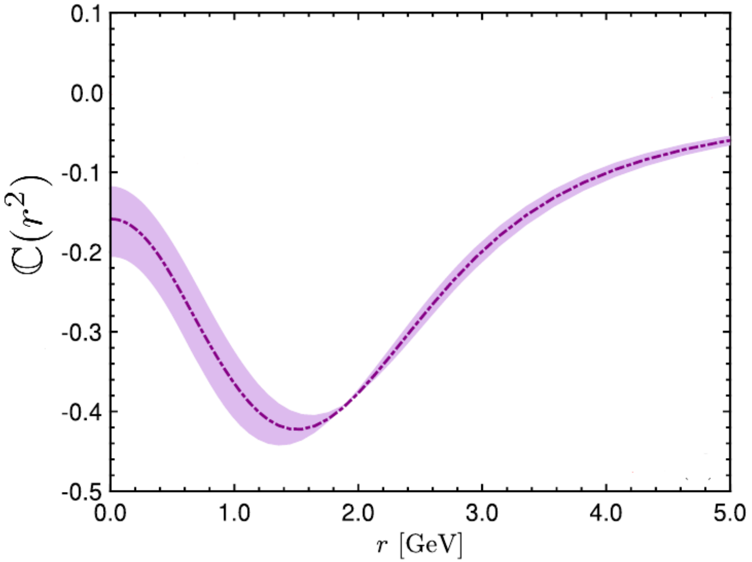
From the Bethe-Salpeter equation, we can



Truncation







A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).