ECT* workshop: From first-principles QCD to experiments

Non-perturbative characteristics of (QCD) spectral functions at finite temperature

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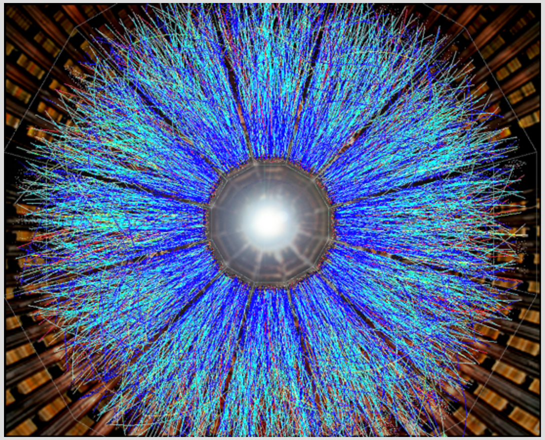
CRC-TR 211 Strong-interaction matter under extreme conditions

Talk outline

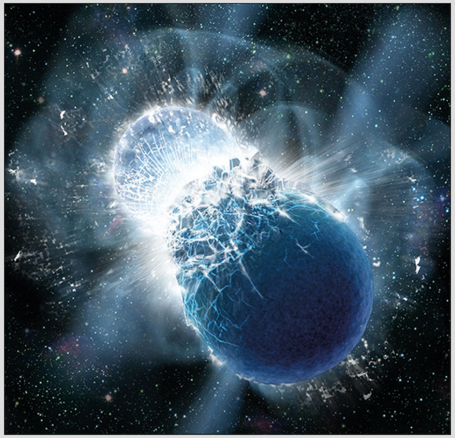
1. QFT beyond the vacuum
2. The importance of spectral functions
3. Causality constraints
4. Spectral properties from Euclidean data
5. Revisiting $T > 0$ perturbation theory

1. QFT beyond the vacuum

- To describe physical phenomena in "extreme environments" one must understand of how QFT applies to systems that are hot, dense, or both



[Brookhaven National Lab]



[Skyworks Digital Inc.]

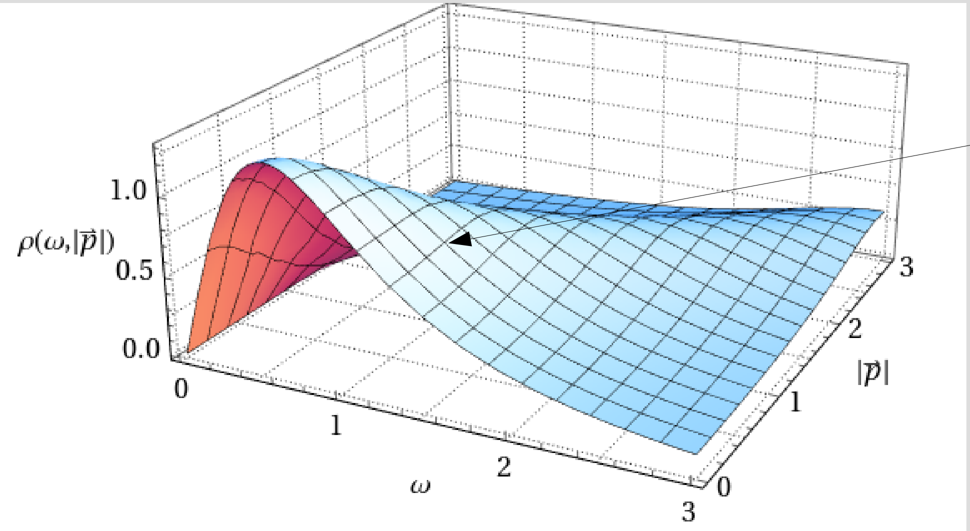
- Correlation functions are the building blocks of *any* QFT $\rightarrow$ they encode the dynamical properties that arise due to changes in temperature $T=1/\beta$ or density. In this talk we will restrict to vanishing density.

$$\langle \Omega_\beta | \phi(x_1) \cdots \phi(x_n) | \Omega_\beta \rangle$$

2. The importance of spectral functions

- At finite T spectral functions $\rho(\omega,\boldsymbol{p})$ play a particularly important role

$$\rho(\omega,\vec{p}) = \int d^4x \; e^{i(\omega x_0 - \vec{p}\cdot\vec{x})} \langle \Omega_\beta | \left[\phi(x), \phi(0)\right] | \Omega_\beta \rangle$$



Peak locations and their dispersion are related to the dynamics of the medium and the underlying degrees of freedom of the theory

- Spectral functions also enter into the calculation of numerous important observables (transport coefficients, particle production rates, etc.)

<u>Important question</u>: *can general spectral function characteristics be disentangled from model-dependent effects?*

3. Causality constraints

- Finite temperature QFT is *significantly* less well-understood than in vacuum, but important progress has been made for spectral functions
- For scalar fields, the fields being **local**, i.e. $[\Phi(x),\Phi(y)]=0$ for $(x\text{-}y)^2 < 0$, implies the following general representation*

$$\rho(\omega,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(\omega)\, \delta\big(\omega^2 - (\vec{p}-\vec{u})^2 - s\big)\, \widetilde{D}_\beta(\vec{u},s)$$

"Thermal spectral density"

- This is the $T>0$ generalisation of the well-known *Källén-Lehmann* spectral representation

$$\rho(\omega,\vec{p}) \xrightarrow{\beta\to\infty} 2\pi\epsilon(\omega)\int_0^\infty ds\; \delta(p^2 - s)\, \rho(s)$$

e.g. $\rho(s)=\delta(s\text{-}m^2)$ for a massive free theory

- Determining the properties of $\widetilde{D}_\beta(\boldsymbol{u},s)$ is clearly key to understanding how in-medium effects manifest themselves in $\rho(\omega,\boldsymbol{p})$

* See: *J. Bros and D Buchholz, Z. Phys. C* 55 (1992), *Ann. Inst. H.Poincare Phys.Theor.* 64 (1996)

3. Causality constraints

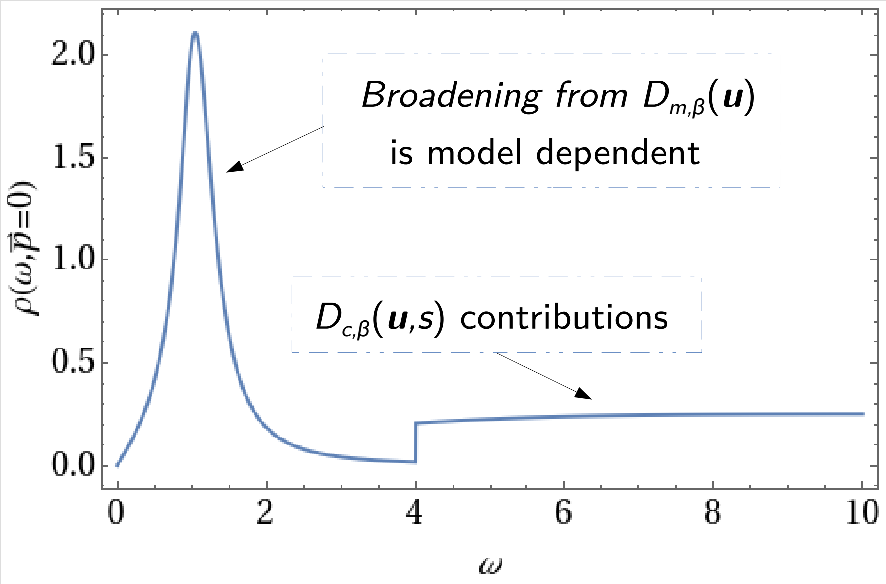
- <u>Proposition</u>: the medium contains "Thermoparticles": particle-like constituents which differ from collective quasi-particle excitations, and show up as ***discrete*** contributions [Bros, Buchholz, *NPB* 627 (2002)]

$$\widetilde{D}_{\beta}(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\, \delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$$

→ Thermoparticle components reduce to those of a vacuum particle state with mass m in the limit $T \rightarrow 0$

→ Non-trivial "Damping factor" $\widetilde{D}_{\beta}(\boldsymbol{u})$ results in thermally-broadened peaks in the spectral function, i.e. parametrises the effects of collisional broadening

→ Component $\widetilde{D}_{c,\beta}(\boldsymbol{u},s)$ contains all other types of excitations, including those that are *continuous* in s



4. Spectral properties from Euclidean data

- In many instances *Euclidean* data is used to calculate $T > 0$ observables, e.g. spectral functions $\rho_\Gamma(\omega, \boldsymbol{p})$ from $C_\Gamma(\tau, \vec{x}) = \langle O_\Gamma(\tau, \vec{x})\, O_\Gamma(0, \vec{0}) \rangle_T$ where O_Γ is some particle-creating operator

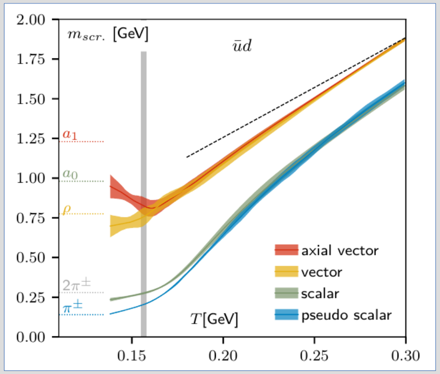
$$\widetilde{C}_\Gamma(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho_\Gamma(\omega, \vec{p})$$

$\rightarrow$ Determine $\rho_\Gamma(\omega, \boldsymbol{p})$ given $\widetilde{C}_\Gamma(\tau, \boldsymbol{p})$: *problem is ill-conditioned, need more information!*

- Another quantity of interest in lattice studies is the *spatial* correlator

$$C_\Gamma(x_3) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, C_\Gamma(\tau, \vec{x}) = \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} e^{ip_3 x_3} \int_0^\infty \frac{d\omega}{\pi\omega}\, \rho_\Gamma(\omega, p_1 = p_2 = 0, p_3)$$

- Large-x_3 behaviour $C_\Gamma(x_3) \sim \exp(-m_{scr}|x_3|)$ used to extract "screening masses" $m_{scr}(T)$



[HotQCD collaboration, *PRD* 100 (2019)]

4. Spectral properties from Euclidean data

<u>Goal</u>: Use the additional constraints imposed by causality to better understand how spectral features manifest themselves in Euclidean data

- Causality implies a general connection between the spatial correlator and thermal spectral density [P.L., *PRD* 106 (2022); P.L., O. Philipsen, *JHEP* 10, 161 (2022)]

$$C(x_3) = \frac{1}{2}\int_0^{\infty} ds \int_{|x_3|}^{\infty} dR \; e^{-R\sqrt{s}} D_{\beta}(R,s)$$

Thermal spectral density in position space

→ Thermoparticle states give rise to $C(x_3)$ contributions that are particularly significant in the large-x_3 region

$$C(x_3) \approx \frac{1}{2}\sum_{i=1}^{n} \int_{|x_3|}^{\infty} dR \; e^{-m_i R} D_{m_i,\beta}(R)$$

- Once the damping factors of these states are know one can use the $T > 0$ spectral representation to compute their analytic contribution to $\rho(\omega,\boldsymbol{p})$

4. Spectral properties from Euclidean data

- Can now apply these relations to QCD lattice data [P.L., O. Philipsen, 2022]

  → Use data [Rohrhofer et al. *PRD* 100 (2019)] for spatial correlator $C_{\mathrm{PS}}(x_3)$ of light-quark pseudo-scalar meson operator $\mathcal{O}^a_{\mathrm{PS}} = \overline{\psi}\gamma_5 \frac{\tau^a}{2}\psi$

Step 1: Perform fits to $C_{\mathrm{PS}}(x_3)$ data to obtain the functional dependence at different temperatures (T = 220-960 MeV) → $c_1 \exp(-m_\pi x_3) + c_2 \exp(-m_{\pi^*} x_3)$ describes data well

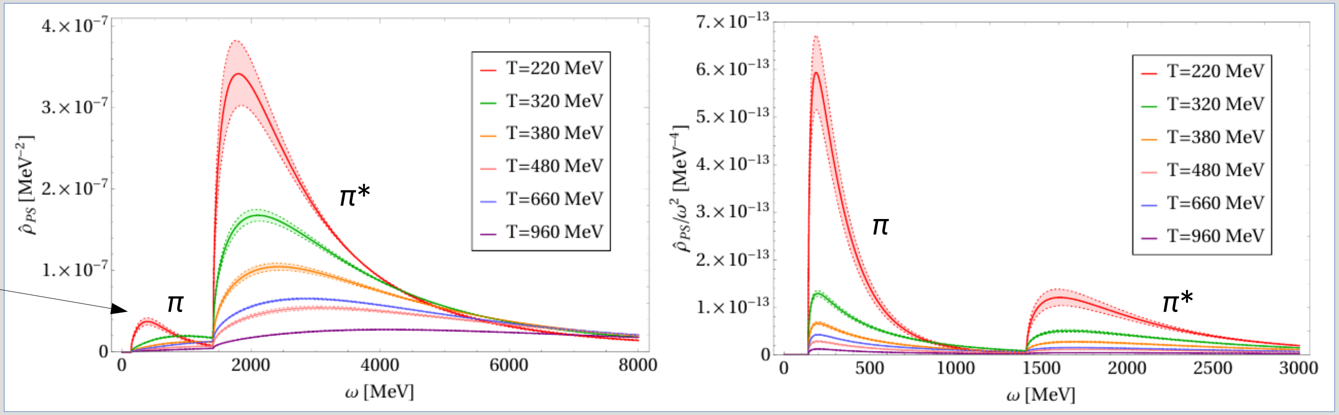
Contribution of 2 lowest-lying states, π and π^*

Step 2: If π and π^* are thermoparticle-type states for $T > 0$, then:

→ Fit ansatz implies $D_{m_i,\beta}(\vec{x}) = \alpha_i\, e^{-\gamma_i|\vec{x}|}$ with screening masses $m_i(T) = m_i(T=0) + \gamma_i(T), \quad i = \pi, \pi^*$

Step 3: Using $D_{m,\beta}(\boldsymbol{x})$ and spectral representation one can compute $\rho_{\mathrm{PS}}(\omega,\boldsymbol{p})$ contributions:

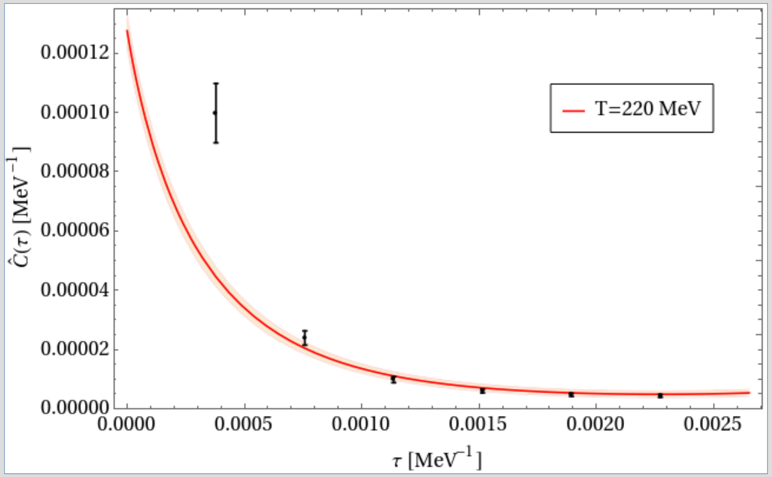
π peak at $T \sim 1.2\ T_{pc}$



4. Spectral properties from Euclidean data

Non-trivial test: Since procedure gives the full analytic structure of $\rho_{\rm PS}(\omega,\boldsymbol{p})$ due to thermoparticle contributions, one can use this to ***predict*** the form of the corresponding temporal correlator $\widetilde{C}_{\rm PS}(\tau,\boldsymbol{p})$

- The spatial and temporal correlators have very different $\rho_{\rm PS}(\omega,\boldsymbol{p})$ dependencies $\longrightarrow$ a *highly non-trivial check!*
- Using the $T = 220$ MeV $\boldsymbol{p} = 0$ temporal data from [Rohrhofer et al. *PLB* 802 (2020)] one obtains:

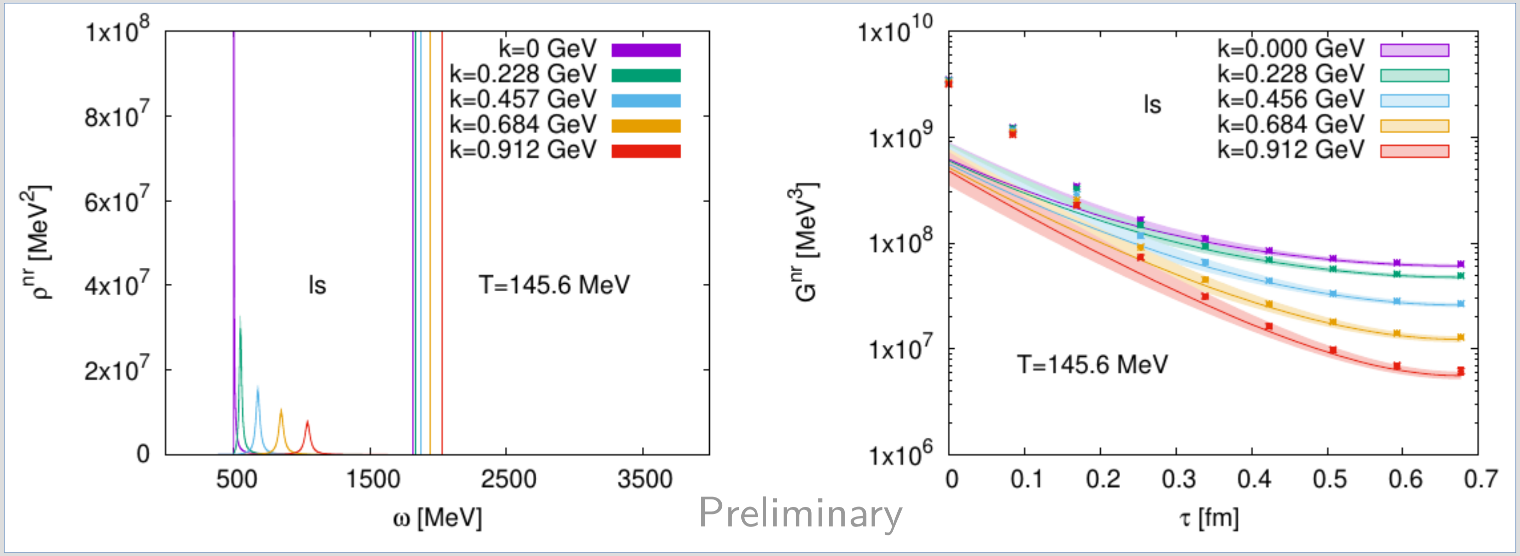


Prediction matches the data well for large τ, and then begins to undershoot
$\longrightarrow$ Missing contributions from higher excited states

- No matter the procedure, comparing temporal and spatial correlator predictions is an important test for *any* extracted spectral function

4. Spectral properties from Euclidean data

- Work is ongoing [D. Bala, O. Kaczmarek, P. Lowdon, O. Philipsen, and T. Ueding] to apply this approach to pseudo-scalar mesons involving heavier quarks (light-strange and strange-strange)



- The temporal correlator predictions are now *also* compared for $\boldsymbol{p} > 0$

  $\rightarrow$ Consistent predictions are obtained in both light-strange and strange-strange channels!

- This approach is straight-forwardly generalisable to higher spin states (work in progress...)

5. Revisiting $T > 0$ perturbation theory

- It has long been understood that finite-temperature perturbation theory has complications: non-analytic contributions, IR divergences, ...

- In fact, more specifically, Weldon [*PRD* 65 (2002)] showed that the perturbative procedure in Φ^4 theory *fails* at 2-loop order because the self-energy $\Pi(k)$ has a branch point on the perturbative mass shell $k_0 = E(k)$

  → This is a generic feature of perturbative computations that use free thermal propagators, or in fact any propagators that have a real dispersion relation $p_0 = E(p)$

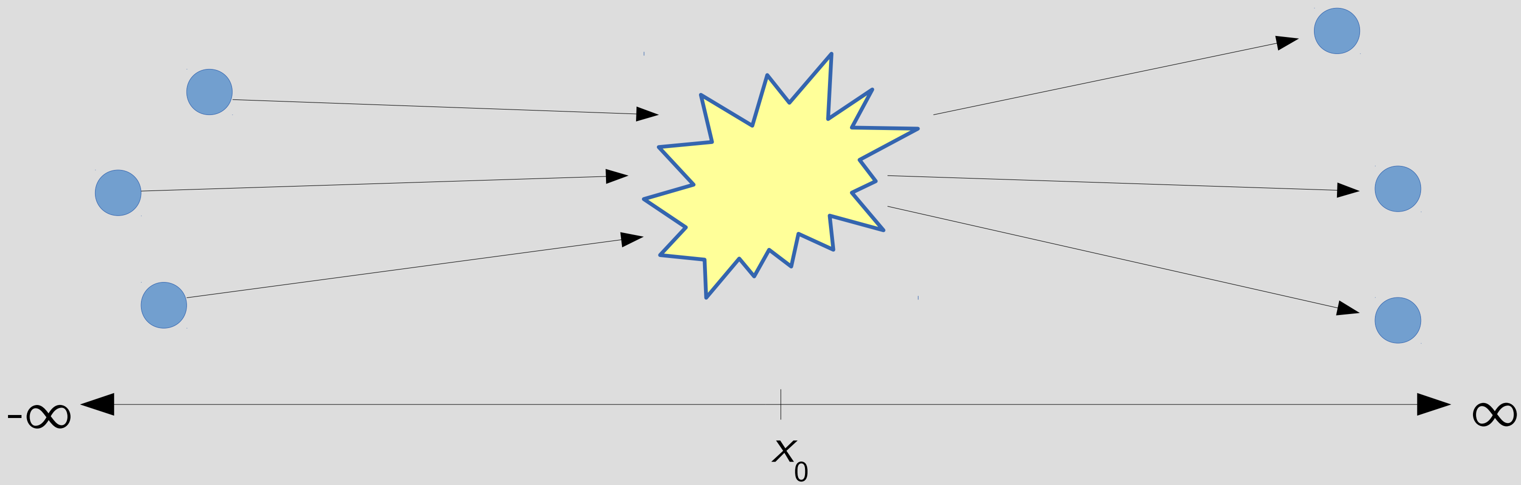
$$D_R(P) = \frac{1}{(p_0 + i\epsilon)^2 - E^2(p)}$$

- Physically, this arises due to the incompatibility of the KMS condition with on-shell states and non-zero interactions [Landsman, *Ann. Phys.* 186, 141 (1988)] (*Narnhofer-Requardt-Thirring Theorem* [Commun. *Math. Phys.* 92, 247 (1983)])

5. Revisiting $T > 0$ perturbation theory

Idea: Start with propagators that are off shell [Weldon, 2002]

- The logic is that interactions with the thermal medium persist, even for large times x_0 → *need to take into account in the definition of scattering states*



→ But how does one decide what form these propagators should take?

- With decomposition $\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$ one can prove that the thermoparticle component *dominates* the two-point function $\langle \Omega_\beta | \phi(x)\phi(0) | \Omega_\beta \rangle$ at large x_0 [Bros, Buchholz, *NPB* 627 (2002)]

→ *Thermoparticles are a natural asymptotic thermal state candidate!*

5. Revisiting $T > 0$ perturbation theory

Idea: thermal scattering states are defined by imposing an asymptotic field condition [Bros, Buchholz, *NPB* 627 (2002)]:

Asymptotic fields Φ_0 are assumed to satisfy dynamical equations, but only at large x_0

In Φ^4 theory

$$(\partial^2 + m^2)\phi_0(x) + \frac{\lambda}{3!}\phi_0^3(x) \xrightarrow{|x_0|\to\infty} 0$$

"Asymptotic mass"

- The thermoparticle damping factor $\widetilde{D}_{m,\beta}(\boldsymbol{u})$ is **uniquely fixed** by the asymptotic field equation

  $\rightarrow$ This means that the non-perturbative effects experienced by thermoparticle states are controlled by the asymptotic dynamics

- Given $D_{m,\beta}(\boldsymbol{u})$ one can simply combine this together with the spectral representation to compute the explicit form of the thermoparticle propagator or spectral function

- Can then start to perform perturbative calculations with *this* propagator instead of a free field propagator $\rightarrow$ suggested that this could give rise to an IR-regularised perturbative expansion for $T > 0$ [Bros, Buchholz hep-th/9511022]

Example: Φ^4 theory [PL, O. Philipsen, *in preparation*]

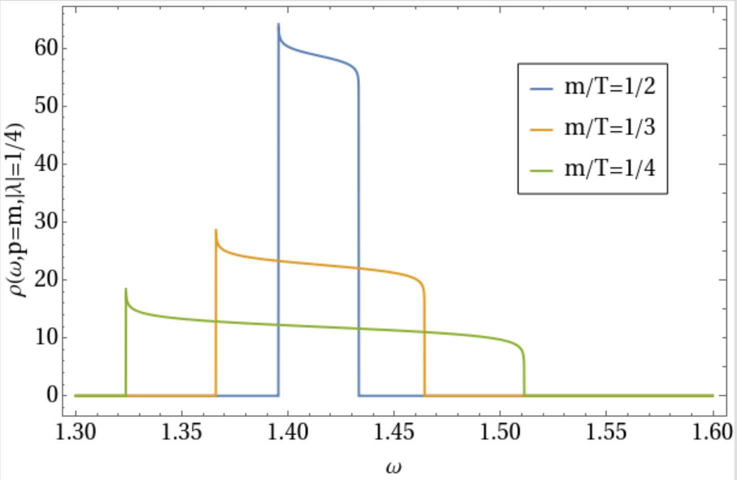
$\rightarrow$ Thermoparticle propagator:

(*Width parameter* $\kappa \sim \sqrt{|\lambda|}T$)

$$\widetilde{G}_{\beta}^{(-)}(k_0,\vec{p}) = \frac{1}{4|\vec{p}|\kappa} \ln\left[\frac{-k_0^2 + m^2 + (|\vec{p}| + \kappa)^2}{-k_0^2 + m^2 + (|\vec{p}| - \kappa)^2}\right]$$

$\rightarrow$ Spectral function already has a width at 1-loop order (and is renormalisable)

$\rightarrow$ At 2-loop the thermoparticle peak plays a dominant role at low energies



Summary & outlook

- Causality imposes **non-perturbative** constraints for $T > 0$ which have significant implications

  → Spectral properties of thermal correlation functions

  → Connection between real-time observables and Euclidean correlators

  → New insights into the characteristics of perturbation theory

- So far, only real scalar fields Φ with $T > 0$ have been considered, but this approach *can* be extended

  → Other hadronic states (baryons, exotic states, ...)

  → Higher spin fields/states (fermions, vectors, ...)

  Work in progress!

  → Non-vanishing density, $|\mu|>0$

- Ultimately, these constraints and methods can help in gaining a better understanding of the phase structure of QCD

Backup: *Euclidean spectral relations*

- One can use the assumptions of local QFT at finite T to put constraints on the the structure of Euclidean correlation functions

→ From the KMS condition and locality:

$$\mathcal{W}_E(\tau,\vec{x}) = \frac{1}{\beta}\sum_{N=-\infty}^{\infty} w_N(\vec{x})\, e^{\frac{2\pi i N}{\beta}\tau}$$

- The Fourier coefficients of the Euclidean two-point function are then related to the thermal spectral density as follows [P.L., *PRD* 106 (2022)]:

$$w_N(\vec{x}) = \frac{1}{4\pi|\vec{x}|}\int_0^{\infty} ds\; e^{-|\vec{x}|\sqrt{s+\omega_N^2}}\, D_\beta(\vec{x},s)$$

- With the Bros-Buchholz decomposition this becomes

$$w_N(\vec{x}) = \frac{1}{4\pi|\vec{x}|}\left[D_m(\vec{x})e^{-|\vec{x}|\sqrt{m^2+\omega_N^2}} + \int_0^{\infty} ds\, e^{-|\vec{x}|\sqrt{s+\omega_N^2}}\, D_c(\vec{x},s)\right]$$

→ *The continuous component $D_c(\mathbf{x},s)$ is increasingly suppressed for large $|\mathbf{x}|$*

Backup: *Damping factors from asymptotic dynamics*

- Applying the asymptotic field condition for Φ^4 theory, the resulting damping factors have the form [Bros, Buchholz, 2002]:

→ **For $\boldsymbol{\lambda} < 0$:** $$D_{m,\beta}(\vec{x}) = \frac{\sin(\kappa|\vec{x}|)}{\kappa|\vec{x}|}$$ → **For $\boldsymbol{\lambda} > 0$:** $$D_{m,\beta}(\vec{x}) = \frac{e^{-\kappa|\vec{x}|}}{\kappa_0|\vec{x}|}$$

where κ is defined with $r = m/T$: $$\kappa = T\sqrt{|\lambda|}K(r), \qquad K(r) = \sqrt{\int \frac{d^3\hat{\vec{q}}}{(2\pi)^3 2\sqrt{|\hat{\vec{q}}|^2 + r^2}} \frac{1}{e^{\sqrt{|\hat{\vec{q}}|^2+r^2}} - 1}}$$

→ *The parameter κ has the interpretation of a thermal width: $\kappa \to 0$ for $T \to 0$*, or equivalently κ^{-1} *is mean-free path*

- Now that one has the exact dependence of $D_{m,\beta}(\boldsymbol{x})$ on the external physical parameters, in this case T, m and λ, one can use this to calculate observables ***analytically***

Backup: *Analytic shear viscosity computation*

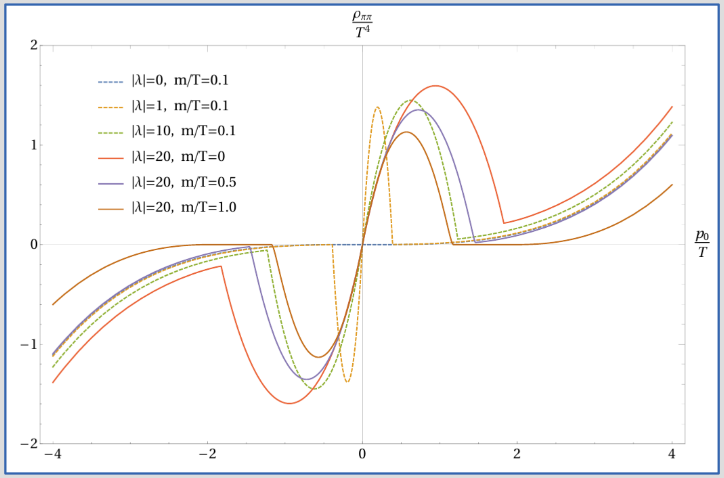
- Of particular interest is the *shear viscosity* η, which measures the resistance of a medium to sheared flow

  → This quantity can be determined from the spectral function of the spatial traceless energy-momentum tensor

$$\rho_{\pi\pi}(p_0) = \lim_{\vec{p}\to 0} \mathcal{F}\left[\langle\Omega_\beta|\left[\pi^{ij}(x), \pi_{ij}(y)\right]|\Omega_\beta\rangle\right](p)$$

  ... and η is recovered via the *Kubo relation* $$\eta = \frac{1}{20}\lim_{p_0\to 0}\frac{d\rho_{\pi\pi}}{dp_0}$$

- Using $D_{m,\beta}(\boldsymbol{x})$ for $\lambda < 0$, the EMT spectral function $\rho_{\pi\pi}$ has the form:



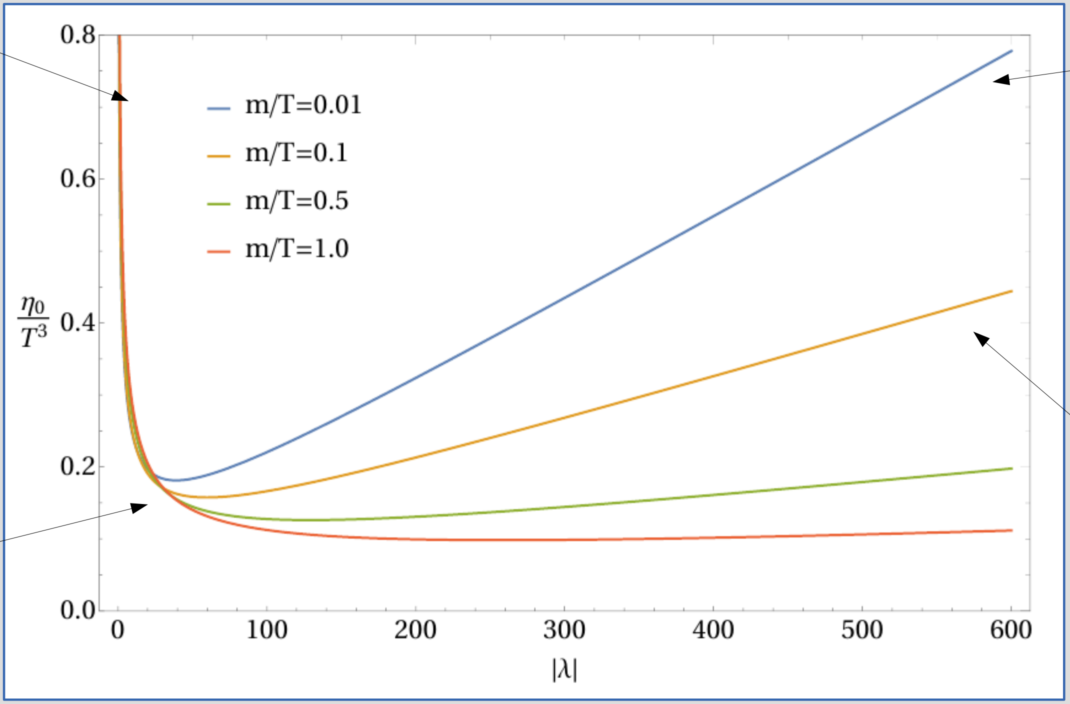
- The presence of interactions causes resonant peaks to appear → peaked when $p_0 \sim \kappa = 1/\ell$
- For $\lambda \to 0$ the free-field result is recovered, as expected
- The dimensionless ratio m/T controls the magnitude of the peaks

Backup: *Analytic shear viscosity computation*

- Applying Kubo's relation, the shear viscosity η_0 arising from the asymptotic states can be written [P.L., R.-A. Tripolt, J. M. Pawlowski, D. H. Rischke, *PRD* 104 (2021)]

$$\eta_0 = \frac{T^3}{15\pi}\left[\frac{\mathcal{K}_3\left(\frac{m}{T},0,\infty\right)}{\sqrt{|\lambda|}} + \sqrt{|\lambda|}\,\mathcal{K}_1\left(\frac{m}{T},0,\infty\right) + \frac{\mathcal{K}_4\left(\frac{m}{T},\sqrt{|\lambda|}K\left(\frac{m}{T}\right),\sqrt{|\lambda|}K\left(\frac{m}{T}\right)\right)}{4|\lambda|}\right]$$

Dominant component for small $|\lambda|$



For large $|\boldsymbol{\lambda}|$, $\boldsymbol{\eta_0} \sim |\boldsymbol{\lambda}|$

Global minima

Magnitude of large $|\boldsymbol{\lambda}|$ growth controlled by m/T

$\rightarrow$ For fixed coupling, η_0/T^3 is entirely controlled by functions of m/T

Backup: *Shear viscosity from FRG data*

- Locality constraints imply that particle damping factors $D_{m,\beta}(\boldsymbol{x})$ can also be calculated from Euclidean momentum space data [P.L., *PRD* 106 (2022)]

$$D_{m,\beta}(\vec{x}) \sim e^{|\vec{x}|m} \int_0^\infty \frac{d|\vec{p}|}{2\pi} \; 4|\vec{p}| \sin(|\vec{p}||\vec{x}|) \; \widetilde{G}_\beta(0,|\vec{p}|).$$

p-space Euclidean propagator

Holds for large separation $|\boldsymbol{x}|$

- In [P.L., R.-A. Tripolt, *PRD* 106 (2022)] pion propagator data from the quark-meson model (FRG calculation) was used to compute the damping factor at different values of T via the analytic relation above
- Fits to the resulting data were consistent with the form: $D_{m_\pi,\beta}(\vec{x}) = \alpha_\pi \, e^{-\gamma_\pi |\vec{x}|}$
- $D_{m,\beta}(\boldsymbol{x})$ can then be used as input for calculations, e.g. shear viscosity

Similar qualitative features to results from chiral perturbation theory