# The Internal Structure of Light Baryon Excitations

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Langtian Liu, Chen Chen, Ya Lu, Craig Roberts and Jorge Segovia, Phys.Rev.,D.105,114047

Langtian Liu, Chen Chen, Craig Roberts, Phys.Rev., D107,014002

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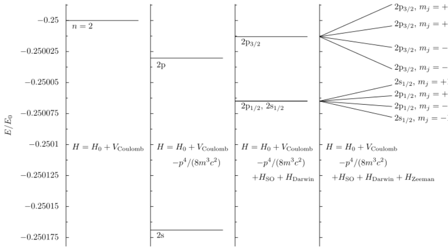
# Introduction

① The baryons and their excitations are essential for us to understand the emergent hadron mass feature of QCD.
Many interaction details are encoded in the baryon excited states.



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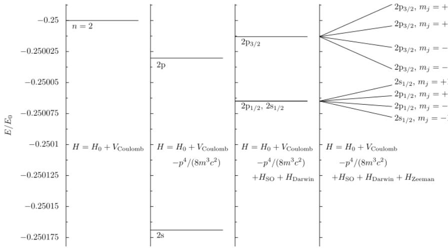


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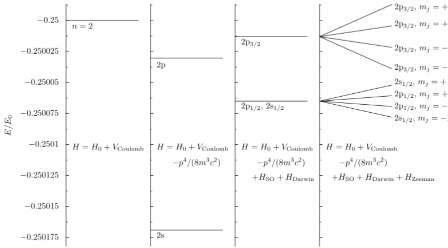

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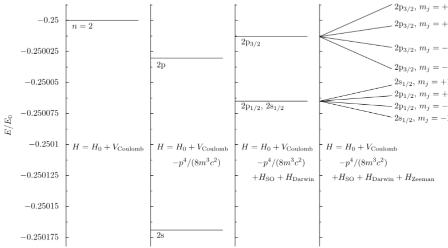


② The Poincaré covariance is important when we explore QCD.

③ We investigated such baryons in a Poincaré-covariant quark diquark Faddeev equation approach. This continuum Schwinger function method connects the QCD tightly.

④ The similarities and differences about the structure of light baryon excitations in constituent quark model and quark-diqaurk Faddeev equation approach.

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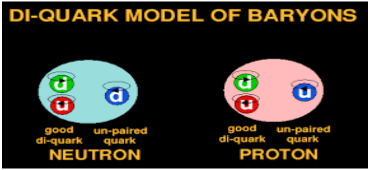
# Quark-diquark Faddeev equation



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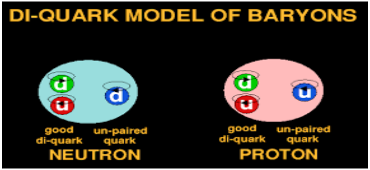


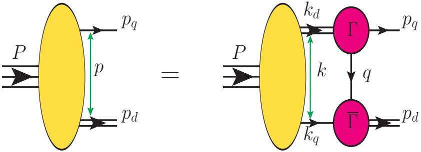

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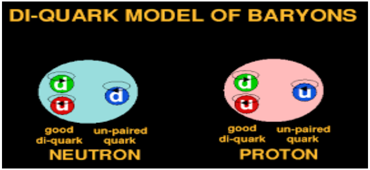
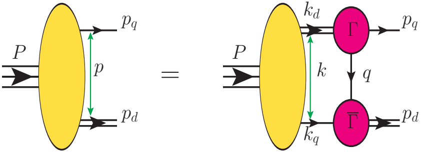
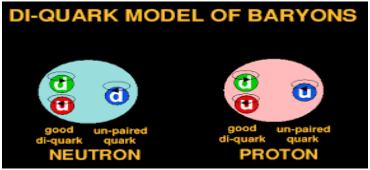
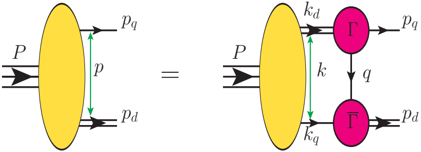



$$\psi_i(p,P) = \sum_a \left[\Gamma_i^a(l,p_d) D_i^a(p_d)\right]\left[\Phi_i^a(p,P)u(P)\right] , \quad i = 1, 2, 3 ,$$

# Quark-diquark Faddeev equation

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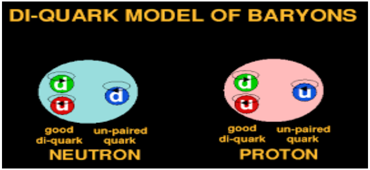
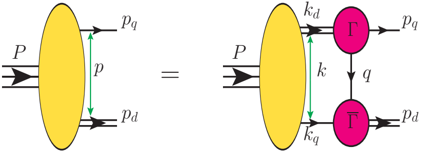
Faddeev equation: $\Phi_i^a = \sum_b \sum_{j\neq i}^{3} \int \frac{d^4k}{(2\pi)^4} K_{ij}^{ab}(k,p,P) G_j^b(k,P) \Phi_j^b(k,P) .$

# Quark-diquark Faddeev equation

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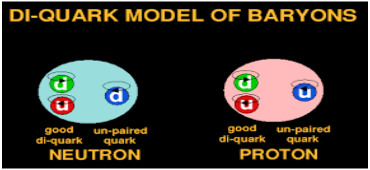
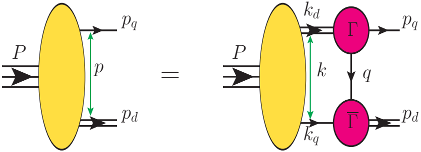
Wave function: $\Psi_i^a(p,P) = S_i(p_q) D_i^a(p_d) \Phi_i^a(p,P) .$

# Quark-diquark Faddeev equation




$$\psi_i(p,P) = \sum_a \left[\Gamma_i^a(l,p_d) D_i^a(p_d)\right]\left[\Phi_i^a(p,P)u(P)\right] \,, \quad i = 1,2,3\,,$$

Faddeev equation: $\Phi_i^a = \sum_b \sum_{j\neq i}^{3} \int \frac{d^4k}{(2\pi)^4} K_{ij}^{ab}(k,p,P) G_j^b(k,P) \Phi_j^b(k,P)\,.$

Wave function: $\Psi_i^a(p,P) = S_i(p_q) D_i^a(p_d) \Phi_i^a(p,P)\,.$

In our calculations, we used 5 diquarks: isoscalar-scalar, isovector-axial vector, isoscalar-pseudoscalar, isoscalar-vector, isovector-vector diquarks. Gernot Eichmann, Few Body Systems, 57, 965 (2016)

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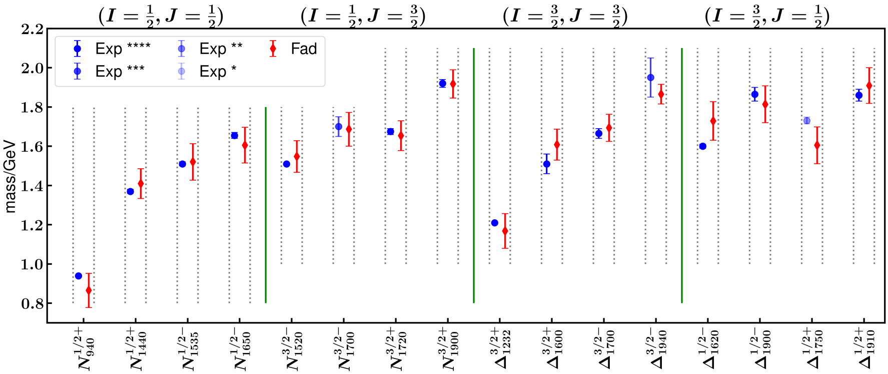
# Mass spectrum from quark-diquark Faddeev equation

Isospin: I; Angular momentum: J. Christian's talk also shows their results. Gernot Eichmann, Few Body Systems, 58, 81 (2017)



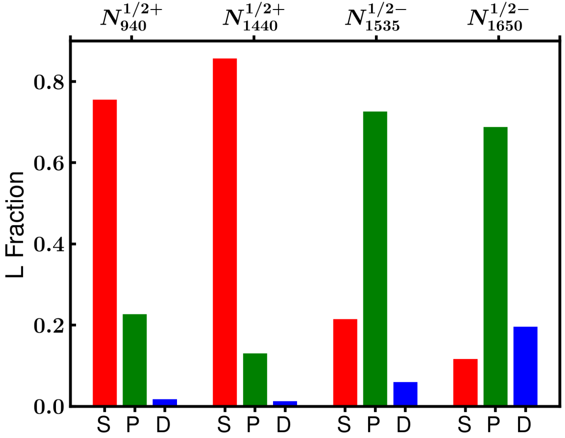

In general, the mass spectrum is consistent with the experimental values.

- ♣ Here we have subtract the meson-baryon interactions effect.
- ♣ For negative parity states with $J = \frac{1}{2}$, we reduce the interaction strength of positive parity diquarks.

# Partial wave fraction of I $= \frac{1}{2}$, J $= \frac{1}{2}$ baryons

The partial wave norm fraction:

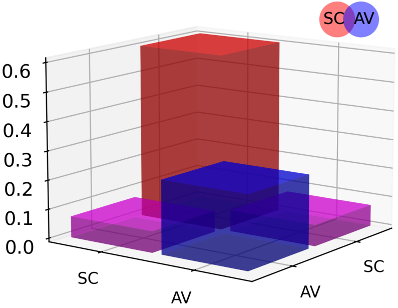
$$\mathcal{W}_i = \int \frac{d^4p}{(2\pi)^4} |w_i(p^2, p\cdot P)|^2 , \quad \mathbb{Q}_L = \frac{1}{\mathbb{T}} \sum_{i\in L} \mathcal{W}_i , \qquad \mathbb{T} = \sum_L \sum_{i\in L} \mathcal{W}_i .$$




- For positive parity states, S-wave dominated.
- For negative parity states, P-wave dominated.
- Consistent with quark model predictions.

# Orbital angular momentum decomposition of charge of $I = \frac{1}{2}$, $J = \frac{1}{2}$ baryons

A. $N(940)\frac{1}{2}^+$ Phys.Rev.,D105,114047




♣ isovector-axial vector diquark contribute to the nucleons.

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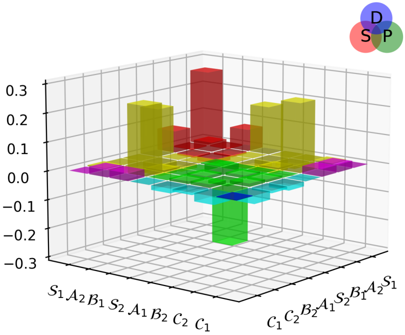


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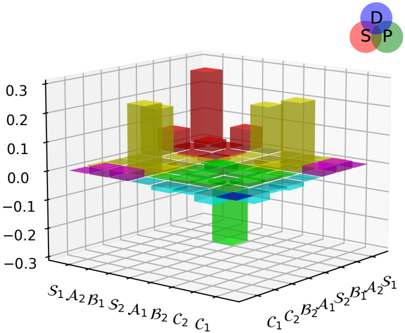
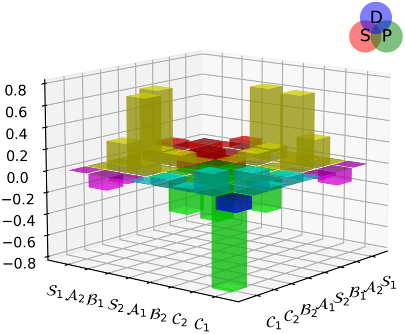



B. $N(1440)\frac{1}{2}^+$




- ♣ isovector-axial vector diquark contribute to the nucleons.

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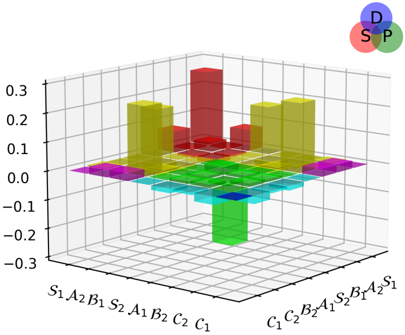


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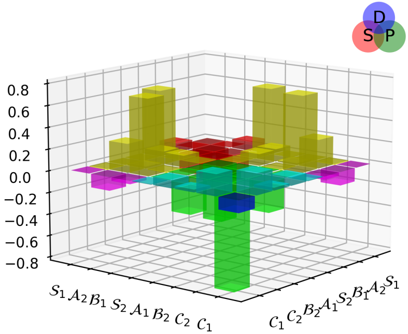
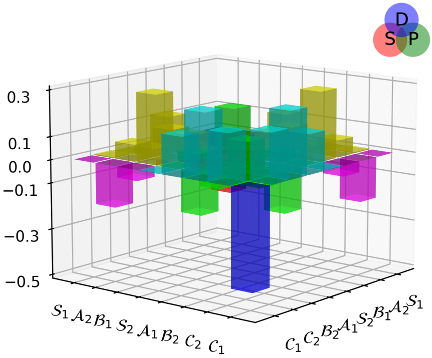


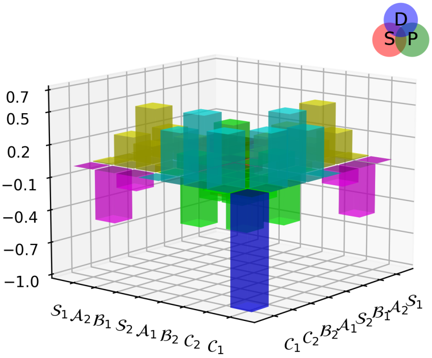

- ♣ isovector-axial vector diquark contribute to the nucleons.
- ♣ P-wave and S-P waves interactions contribute to nucleons.

# Orbital angular momentum decomposition of charge of I = $\frac{1}{2}$, J = $\frac{1}{2}$ baryons

C. N(1535)$\frac{1}{2}^-$

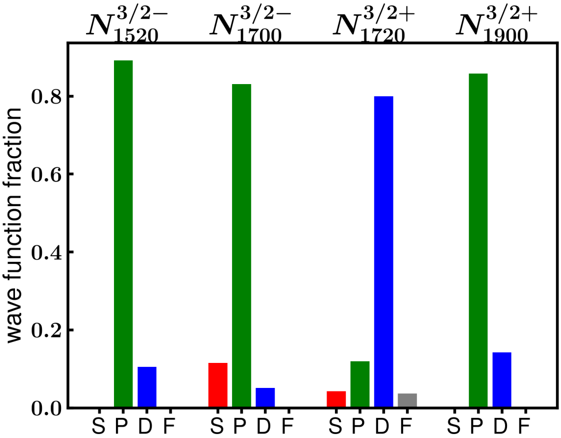



D. N(1650)$\frac{1}{2}^-$




♣ Here the P- and D- waves are important for negative parity states.

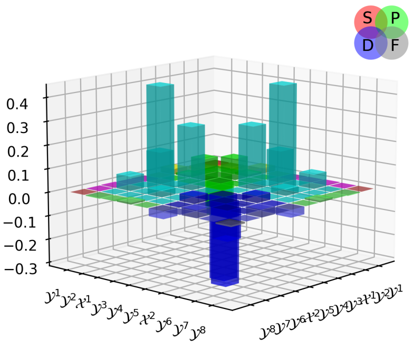
# Partial wave fraction of $I = \frac{1}{2}$, $J = \frac{3}{2}$ baryons



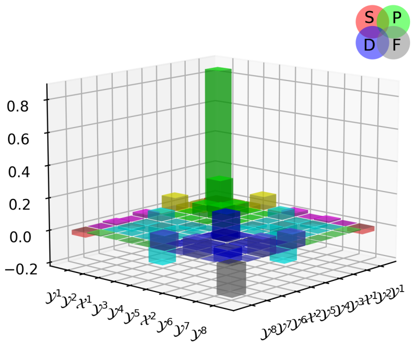

- ♣ For negative parity states, P-wave dominated.
- ♣ For positive parity states, N(1720)$\frac{3}{2}^+$ D-wave dominated while N(1900)$\frac{3}{2}^+$ P-wave dominated.

# Orbital angular momentum decomposition of charge of $I = \frac{1}{2}$, $J = \frac{3}{2}$ baryons

A. $N(1520)\frac{3}{2}^-$ Phys.Rev.,D107,014002



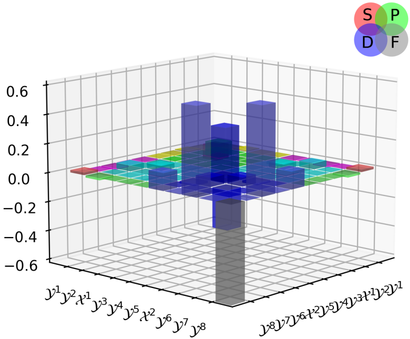

B. $N(1700)\frac{3}{2}^-$



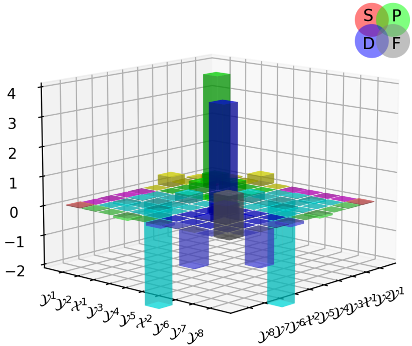

- ♣ For $N(1520)\frac{3}{2}^-$, the main contributions are P-D wave interactions and D waves.
- ♣ For $N(1700)\frac{3}{2}^-$, less complex, mainly P-waves contribute.

# Orbital angular momentum decomposition of charge of $I = \frac{1}{2}$, $J = \frac{3}{2}$ baryons

C. $N(1720)\frac{3}{2}^+$

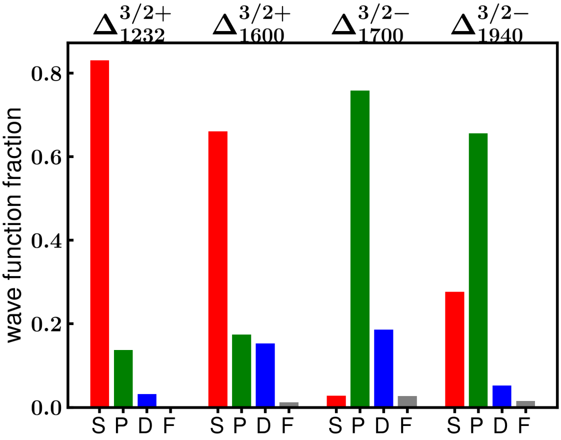



D. $N(1900)\frac{3}{2}^+$




- ♣ For $N(1720)\frac{3}{2}^+$, D-, F- waves mainly contribute.
- ♣ For $N(1900)\frac{3}{2}^+$, besides D-waves, the P-waves contribute.

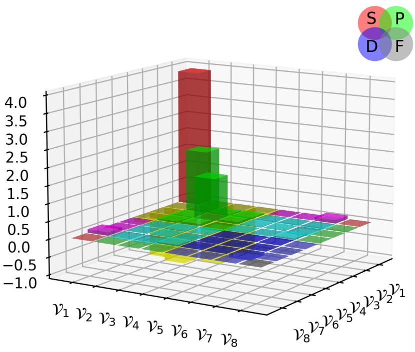
# Partial wave fraction of $I = \frac{3}{2}$, $J = \frac{3}{2}$ baryons



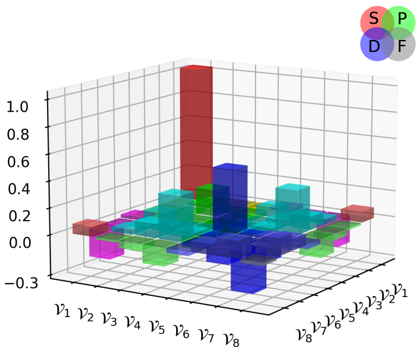

- For positive parity states, S-wave dominated.
- For negative parity states, P-wave dominated.
- Consistent with quark model predictions.

# Orbital angular momentum decomposition of charge of $I = \frac{3}{2}$, $J = \frac{3}{2}$ baryons

A. $\Delta(1232)\frac{3}{2}^+$ Phys.Rev.,D105,114047



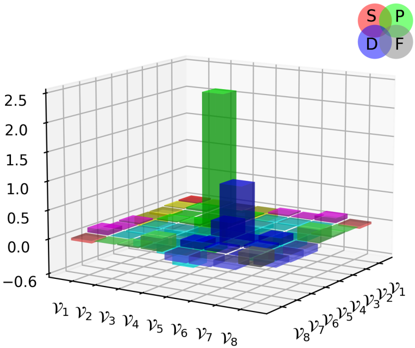

B. $\Delta(1600)\frac{3}{2}^+$



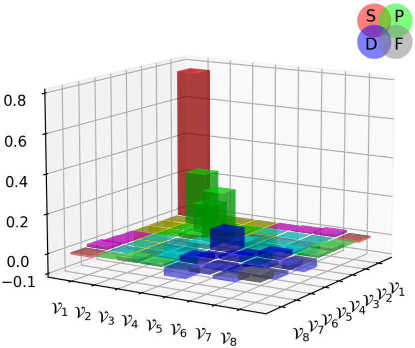

- ♣ They are both S-wave mainly contribute.
- ♣ For $\Delta(1600)\frac{3}{2}^+$, besides S-wave, the D-waves contribution rises.

# Orbital angular momentum decomposition of charge of $I = \frac{3}{2}$, $J = \frac{3}{2}$ baryons

C. $\Delta(1700)\frac{3}{2}^-$

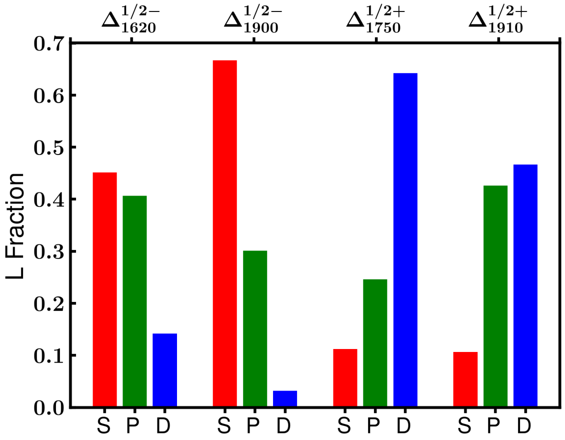



D. $\Delta(1940)\frac{3}{2}^-$




- ♣ For $\Delta(1700)\frac{3}{2}^-$, P-waves mainly contribute.
- ♣ For $\Delta(1940)\frac{3}{2}^-$, besides P-waves, S-wave contribution becomes main part.

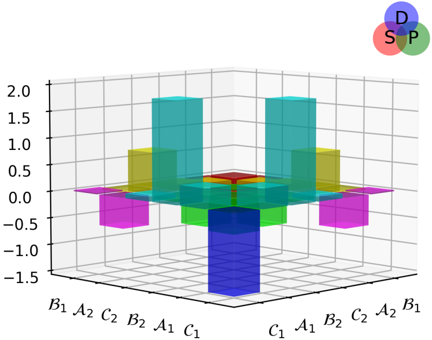
# Partial wave fraction of $I = \frac{3}{2}$, $J = \frac{1}{2}$ baryons



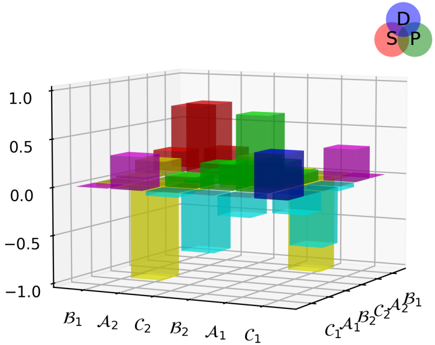

- ♣ For negative parity states, both S- and P-waves are important.
- ♣ For positive parity states, P- and D-waves are important.

# Orbital angular momentum decomposition of charge of $\mathrm{I} = \frac{3}{2}$, $\mathrm{J} = \frac{1}{2}$ baryons

A. $\Delta(1620)\frac{1}{2}^-$



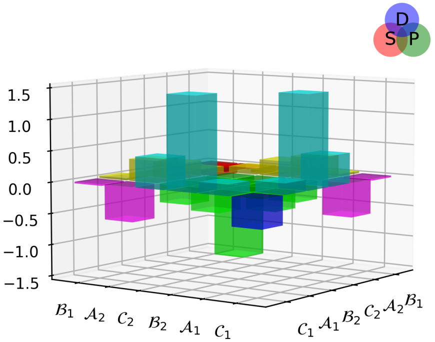

B. $\Delta(1900)\frac{1}{2}^-$



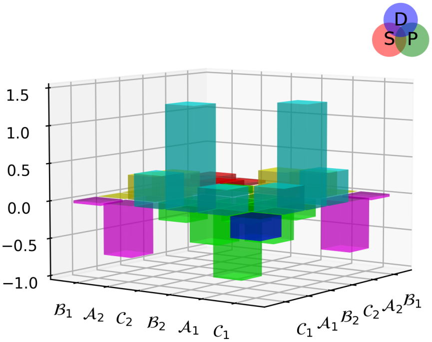

- ♣ For $\Delta(1620)\frac{1}{2}^-$, P- and D-waves are important.
- ♣ For $\Delta(1900)\frac{1}{2}^-$, S- and P-waves are important.

# Orbital angular momentum decomposition of charge of $I = \frac{3}{2}$, $J = \frac{1}{2}$ baryons

C. $\Delta(1750)\frac{1}{2}^+$




D. $\Delta(1910)\frac{1}{2}^+$




♣ Both $\Delta(1750)\frac{1}{2}^+$ and $\Delta(1940)\frac{1}{2}^+$, P- and D-waves are important.

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# Conclusion

- ♣ We use the Poincaré-covariant quark diquark Faddeev equation approach to investigate the light baryons with $\mathsf{I} = \frac{1}{2}, \frac{3}{2}$ and $\mathsf{J} = \frac{1}{2}, \frac{3}{2}$, including positive parity and negative parity states.
- ♣ In most case, the main partial wave from quark-diquark Faddeev equation method is consistent with the quark model prediction.
- ♣ Our analysis shows much more complex wave function structure of these light baryons.
- ♣ In some cases, the main partial wave from our analysis is different from quark model predictions. These predictions need further tests in the future experiments, e.g., the transition form factors at large $Q^2$ region.
- ☆ In the future, we will try to calculate the electromagnetic form factors of some of these light baryon excitations.

# THANK YOU!

# Quark and diquark propagators

The quark propagator has the form

$$S(p) = -i\gamma \cdot p\sigma_v(p^2) + \sigma_s(p^2)\,, \tag{1}$$

$$\sigma_v(x) = \frac{1}{x+\bar{m}^2}\left[1-\mathcal{F}\left(2(x+\bar{m}^2)\right)\right]\,, \quad \mathcal{F}(x) = \frac{1-\exp[-x]}{x}\,, \tag{2}$$

$$\sigma_s(x) = 2\bar{m}\mathcal{F}\left(2(x+\bar{m}^2)\right) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left[b_0 + b_2\mathcal{F}(\epsilon x)\right]\,, \tag{3}$$

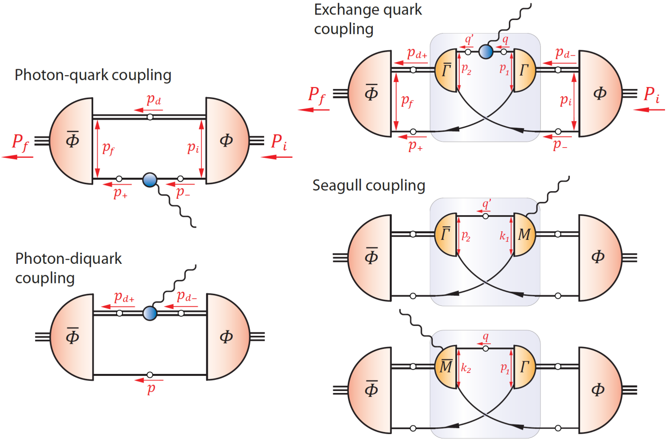
| $\lambda$ | $\bar{m}$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $\varepsilon$ |
|---|---|---|---|---|---|---|
| 0.566 | 0.00897 | 0.131 | 2.90 | 0.603 | 0.185 | 0.0001 |

The diquark propagators:

$$\triangle_{0^+}(k) = \frac{1}{M_{0^+}^2}\mathcal{F}(k^2/\omega_{0^+}^2)\,, \tag{4}$$

$$\triangle_{1^+}^{\mu\nu}(k) = \left(g^{\mu\nu} + \frac{k^\mu k^\nu}{M_{1^+}^2}\right)\frac{1}{M_{1^+}^2}\mathcal{F}(k^2/\omega_{1^+}^2)\,. \tag{5}$$

# Electromagnetic current in quark-diquark Faddeev equation




taken from Gernot Eichmann's thesis.