Dynamically generated four-quark interactions in QCD

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Workshop 'From first-principles QCD to experiments'

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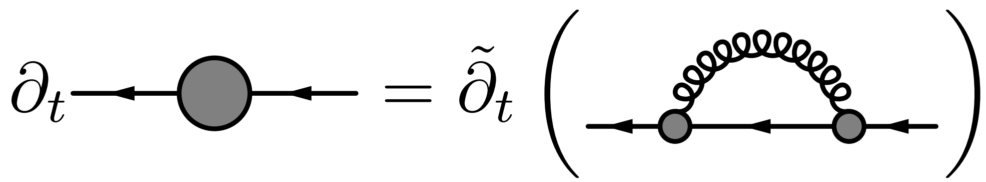
Based on :
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, '*Four-quark scatterings in QCD I*' SciPost Phys. 14 (2023) 069, arXiv:2209.13120;
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, '*Four-quark scatterings in QCD II*' in preparation;
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, Li-jun Zhou, '*Four-quark scatterings in QCD III*' in preparation

Rethinking of mass production in RG

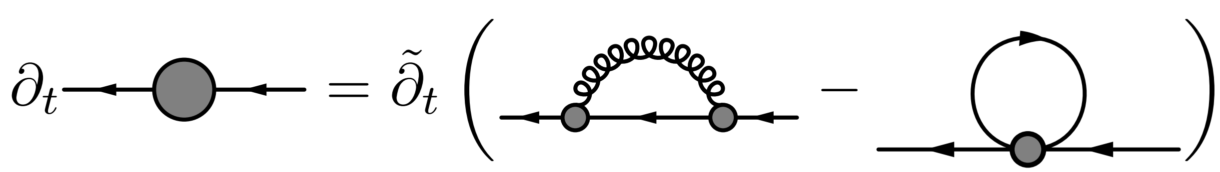
Is it enough to use the flow of quark self-energy as follows to describe the chiral symmetry breaking and quark mass production?



Similar with the quark gap equation in DSE

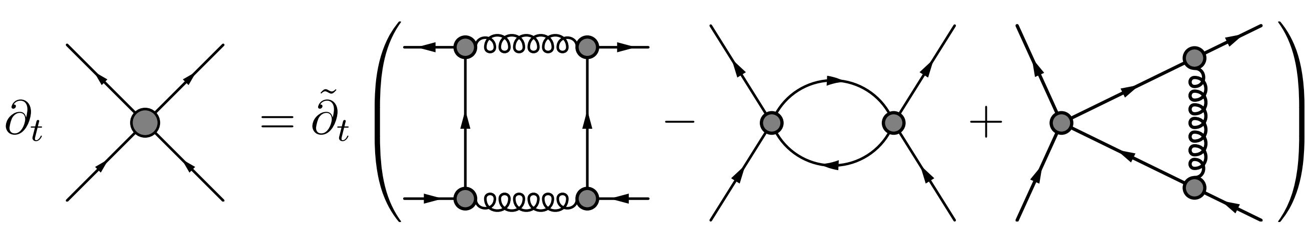
Our findings: Probably not!

How about to include an effective four-quark vertex generated dynamically in the regime of low energy?



fRG analogue of quark gap equation

coupled with



fRG analogue of Bethe-Salpeter equation

Our findings: Seemingly yes!

QCD with dynamical hadronization

Introducing a RG scale dependent composite field:

$$\hat{\phi}_k(\hat{\varphi}), \text{ with } \hat{\varphi} = (\hat{A}, \hat{c}, \hat{\bar{c}}, \hat{q}, \hat{\bar{q}}), \qquad \langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k \, \bar{q} \tau q + \dot{B}_k \, \phi + \dot{C}_k \, \hat{e}_\sigma ,$$

Gies, Wetterich , *PRD* 65 (2002) 065001; 69 (2004) 025001; Pawlowski, *AP* 322 (2007) 2831; Flörchinger, Wetterich, *PLB* 680 (2009) 371

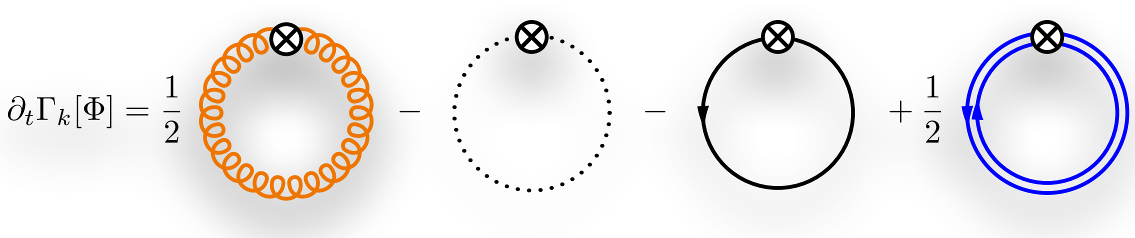
Wetterich equation is modified as

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\big(G_k[\Phi]\, \partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_a}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_k \rangle}{\delta \Phi_a} R_\phi\right)$$

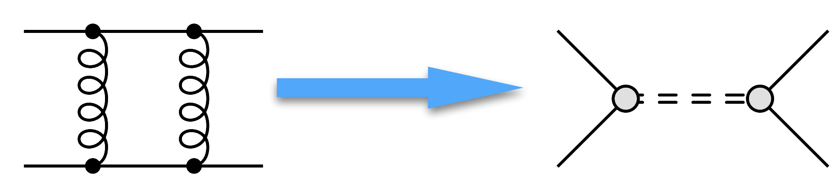
$$- \int \langle \partial_t \hat{\phi}_{k,i} \rangle \left(\frac{\delta \Gamma_k[\Phi]}{\delta \phi_i} + c_\sigma \delta_{i\,\sigma} \right) ,$$

Mitter, Pawlowski, Strodthoff, *PRD* 91 (2015) 054035, arXiv:1411.7978; Braun, Fister, Pawlowski, Rennecke, *PRD* 94 (2016) 034016, arXiv:1412.1045; Cyrol, Mitter, Pawlowski, Strodthoff, *PRD* 97 (2018) 054006, arXiv:1706.06326; WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Flow equation:



four-quark interaction encoded in Yukawa coupling:

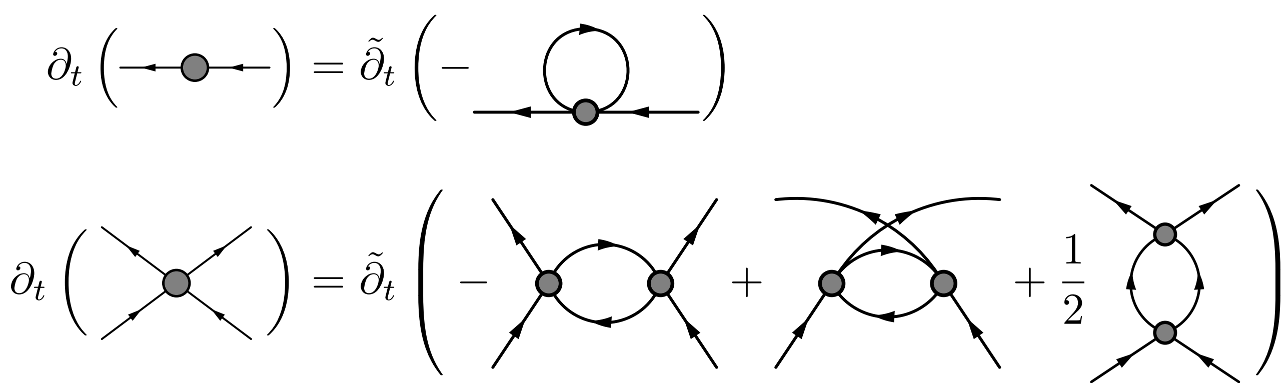


Outline

- Introduction
- LEFT with momentum-independent 4-quark vertex
- Three momentum (s, t, u) channels-dependent 4-quark vertex
- Embedding in unquenched QCD
- Summary and outlook

Momentum-independent 4-quark Vertex

Flow of 2- and 4-quark functions in LEFT:



- momentum-independent approximation:

$$\lambda_\alpha = \lambda_\alpha(p_i = 0)\,, \qquad (i = 1, \cdots, 4)$$
$$M_q = M_q(p = 0)$$

dimensionless variables

$$\bar{\lambda}_\alpha = \lambda_\alpha k^2\,, \qquad \bar{M}_q = \frac{M_q}{k}$$

- single channel approximation:

$$\lambda_{\sigma\text{-}\pi} \equiv \lambda_\pi = \lambda_\sigma\,, \qquad \lambda_{\alpha \notin \{\sigma,\pi\}} = 0$$

using e.g., the flat regulator

$$\partial_t \bar{\lambda}_{\sigma-\pi} = 2\bar{\lambda}_{\sigma-\pi} - \mathscr{C}(\bar{M}_q)\bar{\lambda}^2_{\sigma-\pi}$$
$$\partial_t \bar{M}_q = -\bar{M}_q\left[1 + \bar{\lambda}_{\sigma-\pi} C(\bar{M}_q)\right]$$

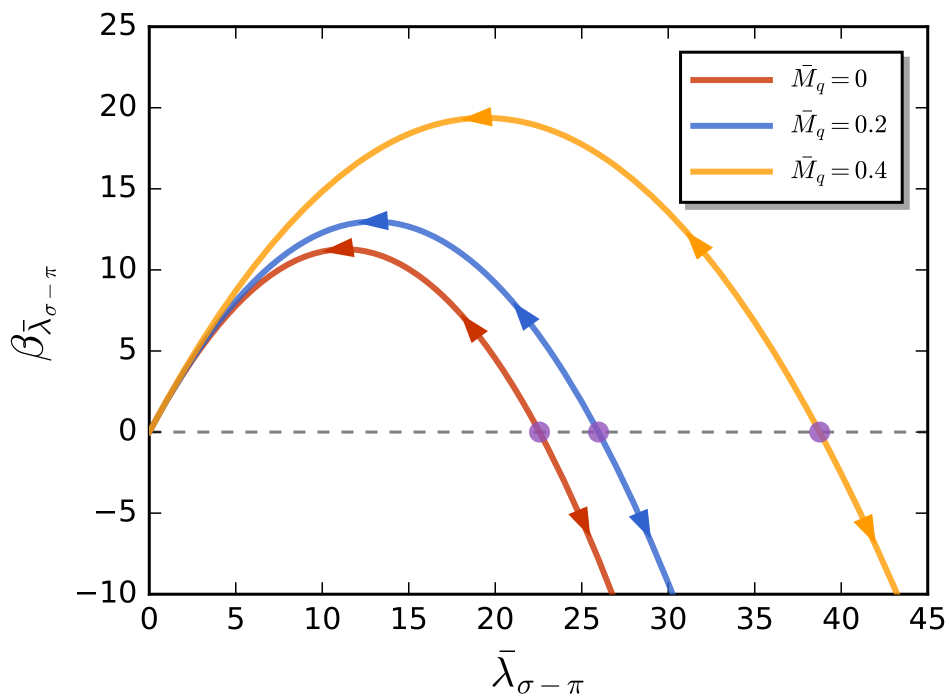
with

$$\mathscr{C}(\bar{M}_q) = \frac{7 - 4\bar{M}_q^2}{8\pi^2\left(1 + \bar{M}_q^2\right)^3}$$

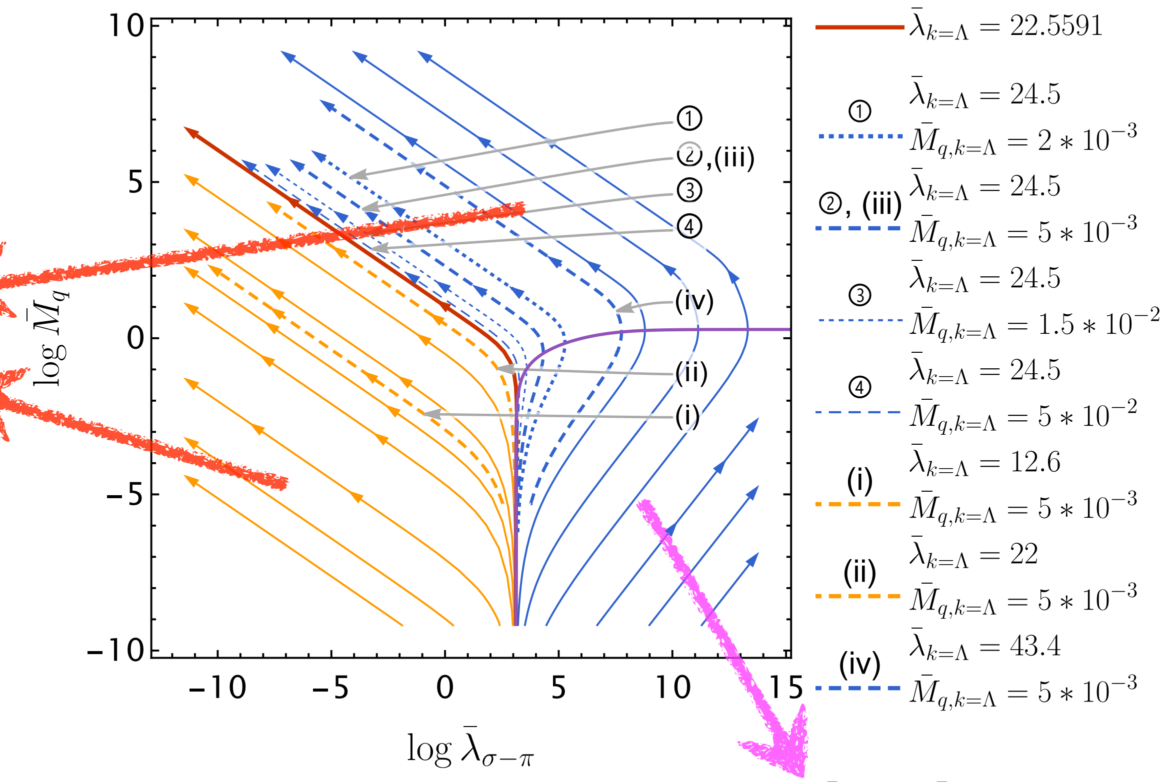
$$C(\bar{M}_q) = \frac{13}{16\pi^2\left(1 + \bar{M}_q^2\right)^2}$$

Flow diagram

β function of 4-quark coupling:



Flow in the plane of the mass and coupling:



Gaussian region

Chiral symmetry breaking

- β function of 4-quark coupling:

$$\beta_{\bar{\lambda}_{\sigma-\pi}} \equiv \partial_t \bar{\lambda}_{\sigma-\pi} = 2\bar{\lambda}_{\sigma-\pi} - \mathscr{C}(\bar{M}_q)\bar{\lambda}^2_{\sigma-\pi}$$

with flat regulator

$$\mathscr{C}(\bar{M}_q) = \frac{7 - 4\bar{M}_q^2}{8\pi^2\left(1 + \bar{M}_q^2\right)^3}$$

fixed point

$$\bar{\lambda}^*_{\sigma-\pi}(\bar{M}_q) = \frac{2}{\mathscr{C}(\bar{M}_q)}$$

When

$$\bar{M}_q \to \bar{M}_q^{\text{Gauss}} = \frac{\sqrt{7}}{2}$$

one has

$$\mathscr{C}(\bar{M}_q) \to 0$$

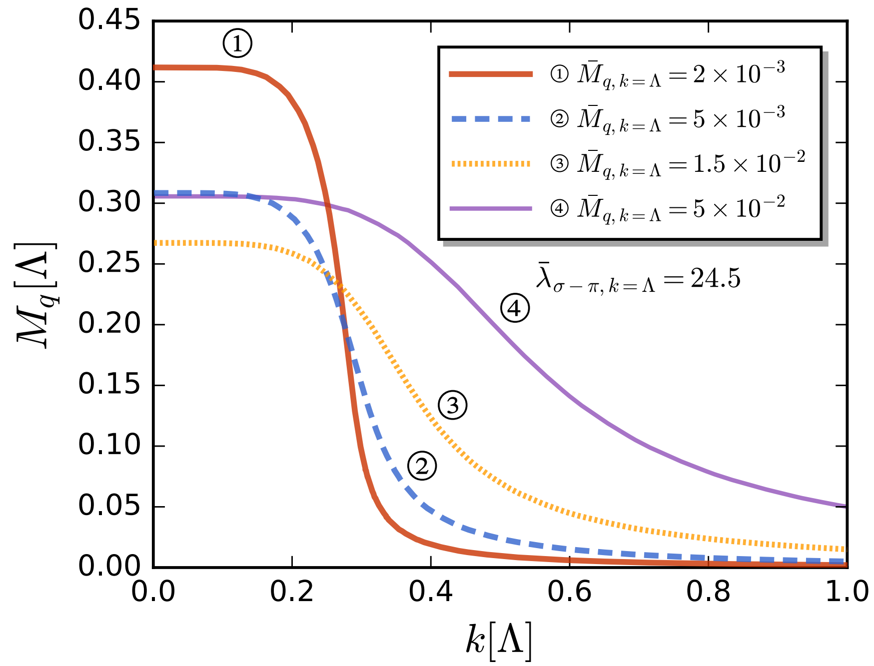
and

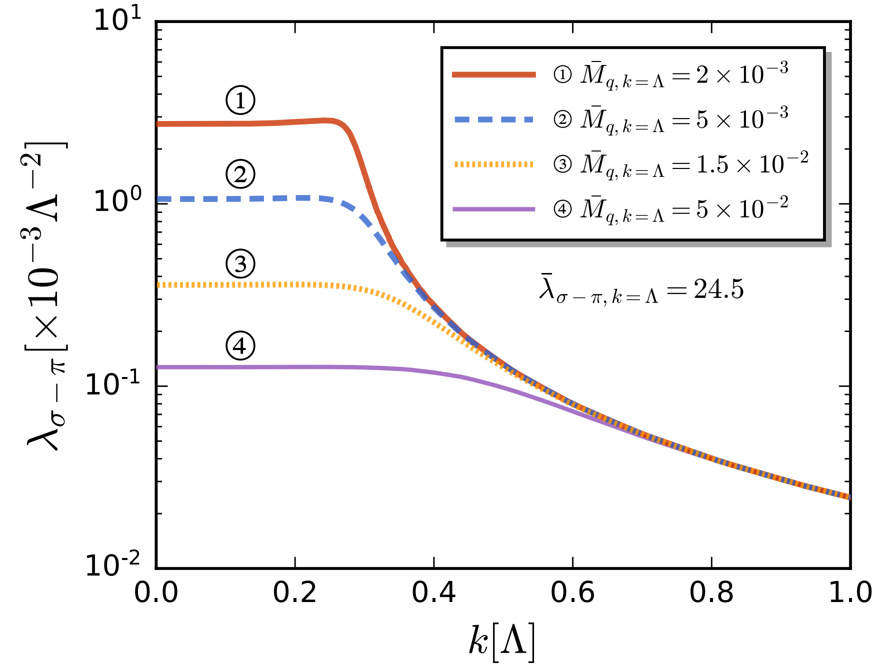
$$\bar{\lambda}^*_{\sigma-\pi}(\bar{M}_q) \to \infty$$

WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

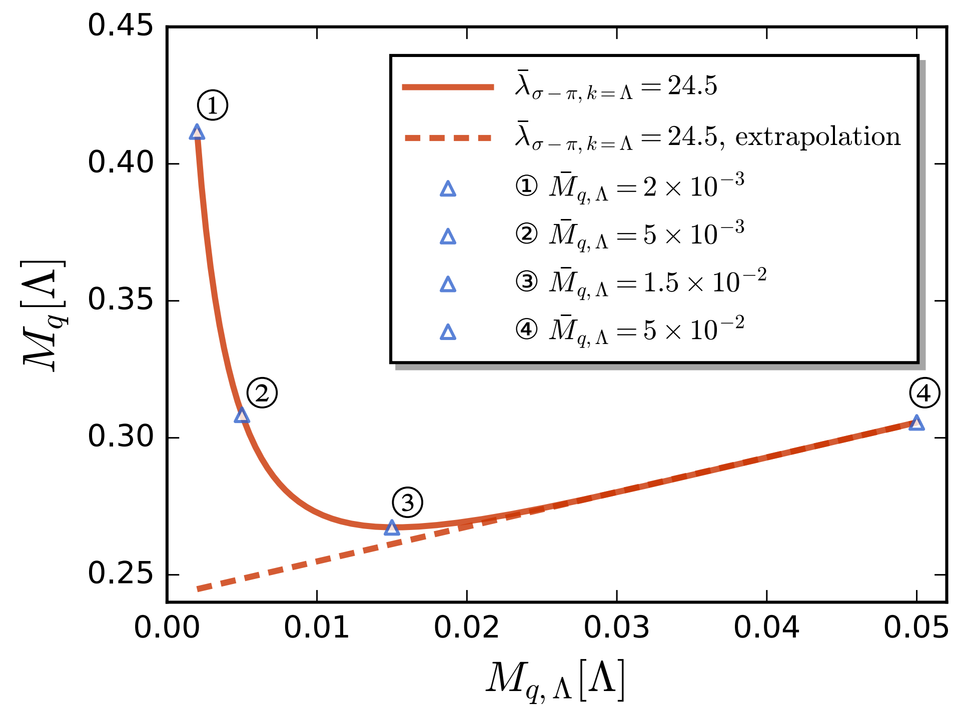
Chiral limit

- quark mass and couplings vs RG scale for different initial quark masses:





- Constituent quark mass vs UV current quark mass:



WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

Chiral limit

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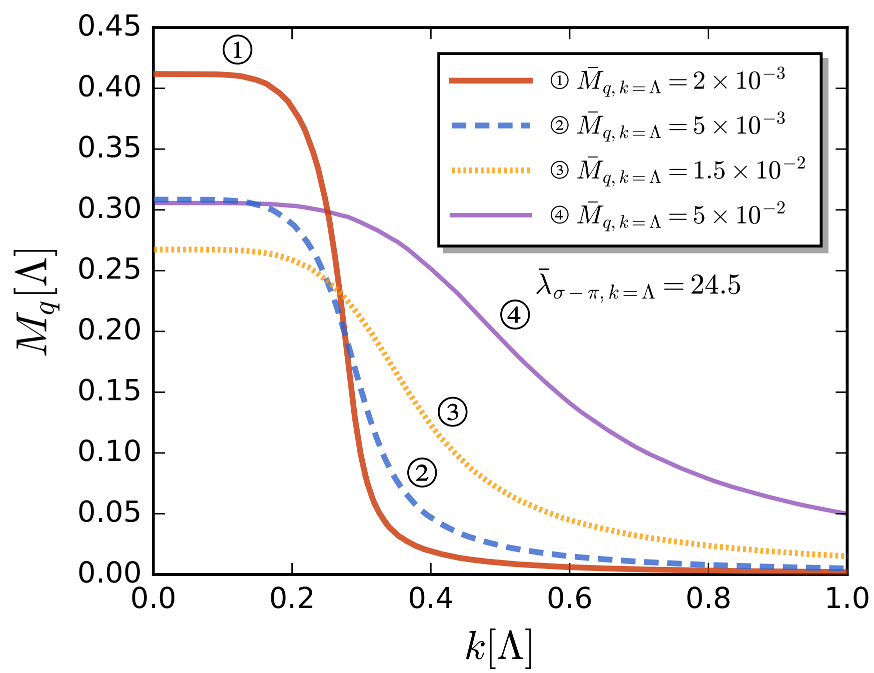
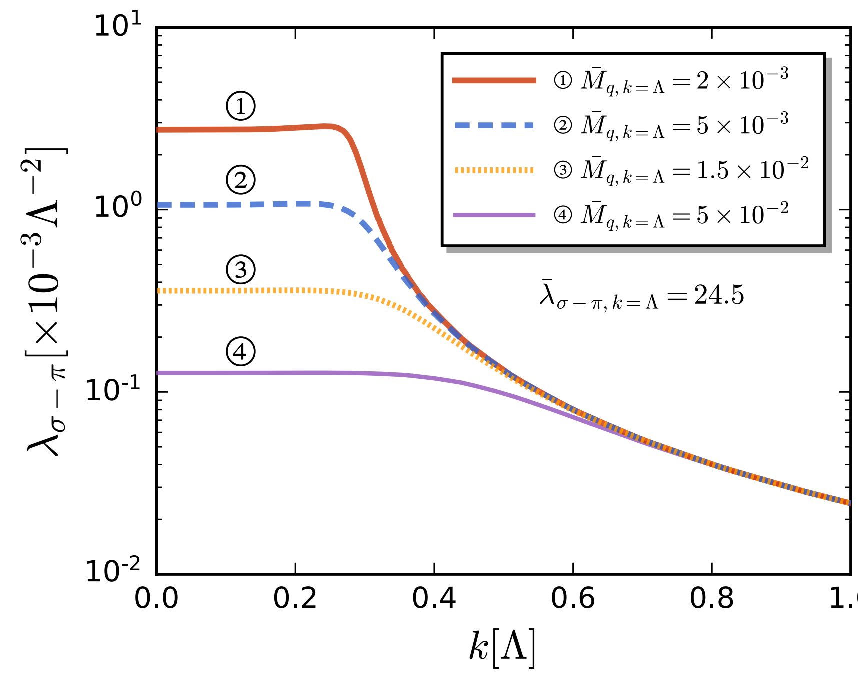
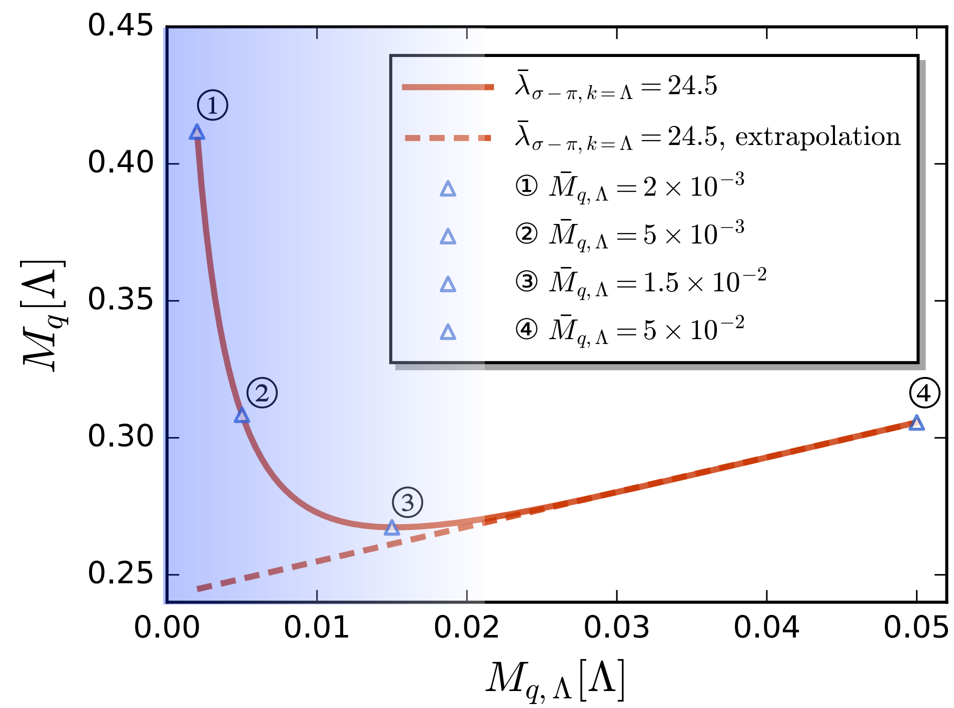




- Constituent quark mass vs UV current quark mass:

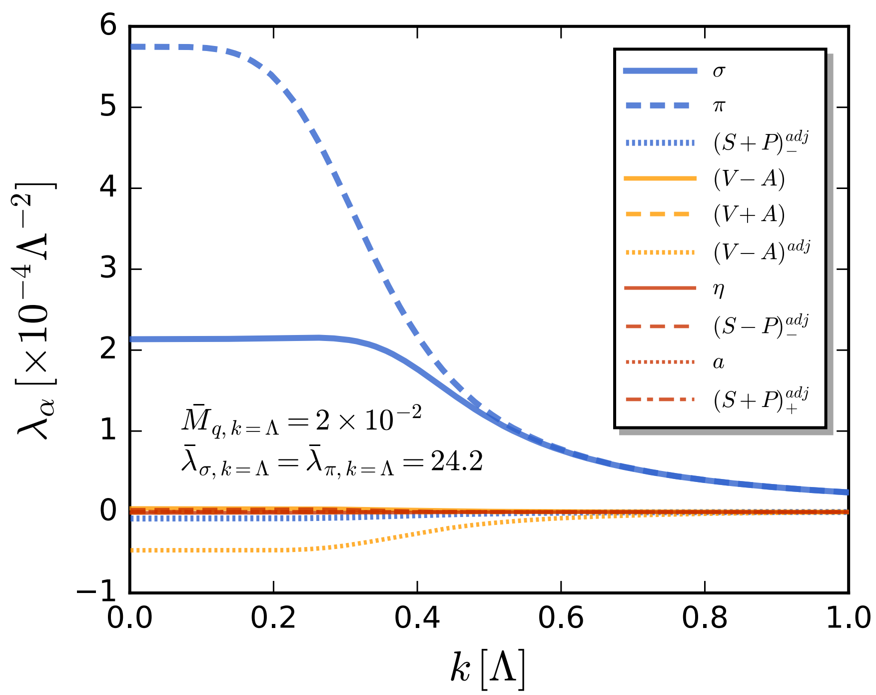
Artifact due to loss of sufficient momentum dependence for the 4-quark vertex

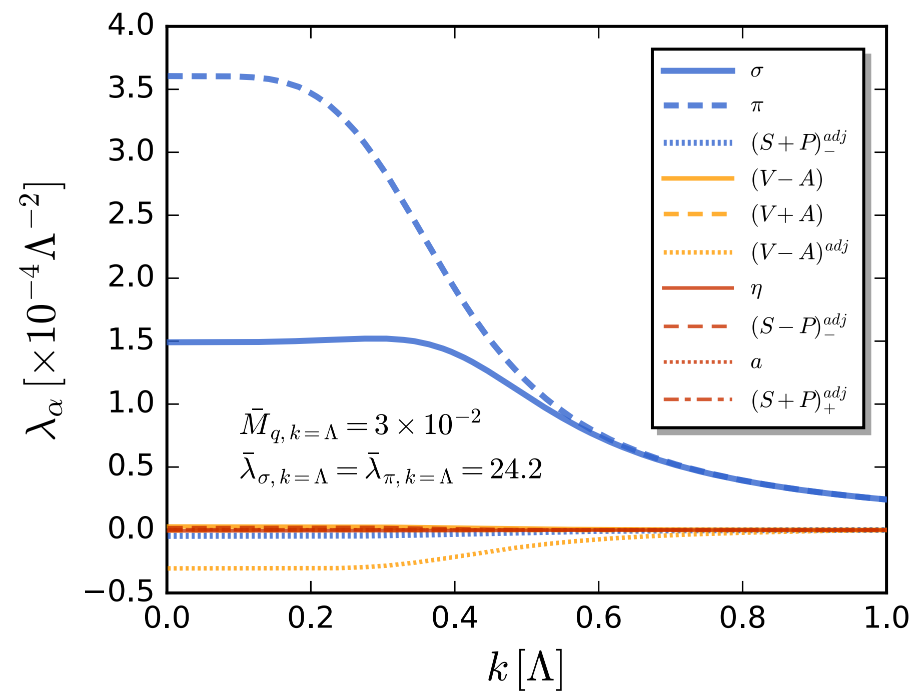


WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

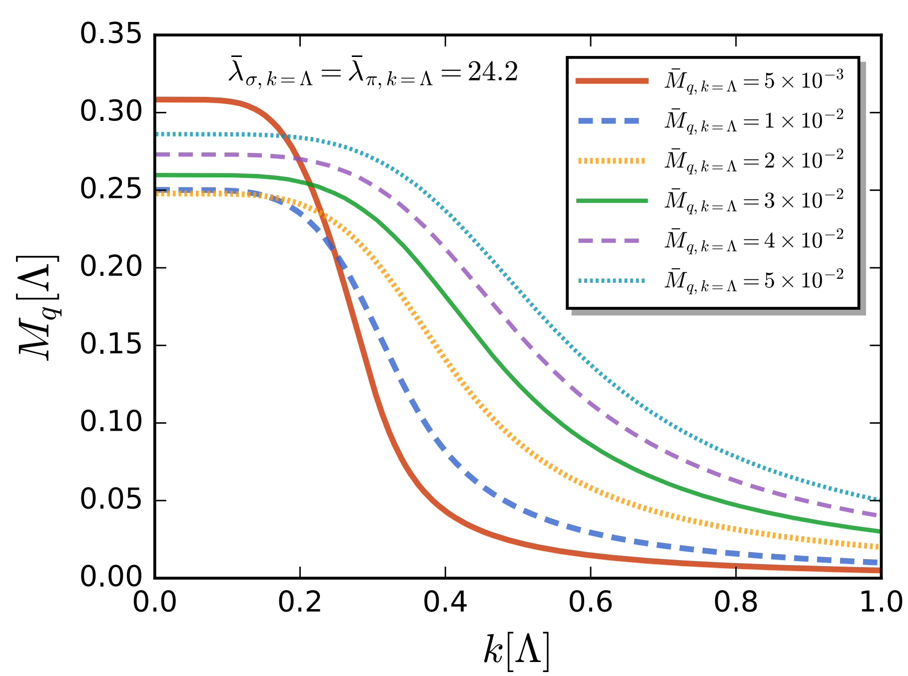
Fierz complete 4-quark vertices

- couplings of different channels:





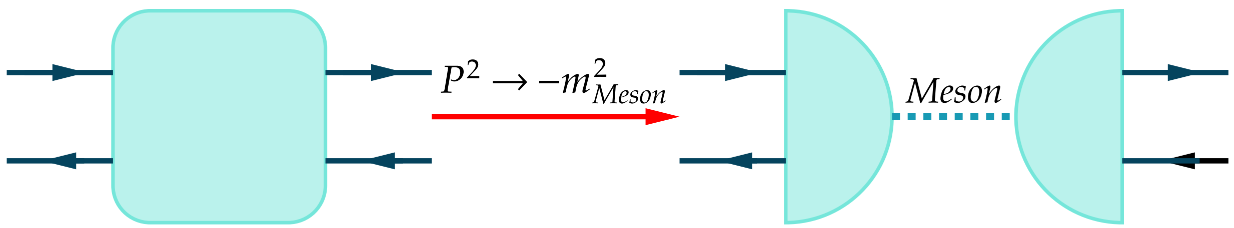
- quark mass:



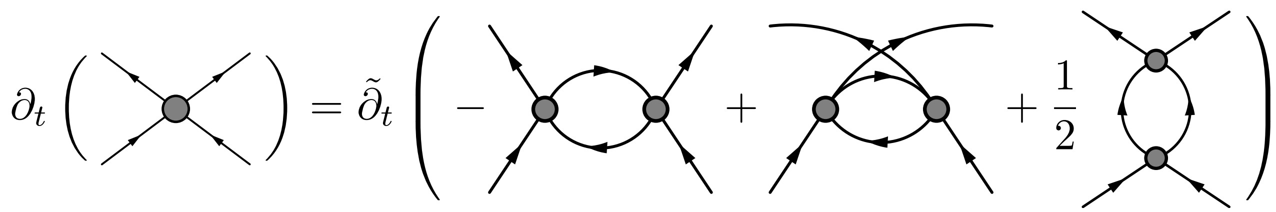
WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

Emergent bound states

- **Bound states encoded in *n*-point correlation functions:**



- **Flow equation of 4-quark interaction:**



Note: playing the same role as the Bethe-Salpeter equation.

single momentum channel (t-channel) approximation:

Using

$$t = P^2 \to -m_\pi^2\,, \qquad s = u \to 0$$

one is led to

$$\partial_t \lambda_{\pi,k}(P^2) = \mathscr{A}_k(P^2) + \mathscr{B}_k(P^2)\,\lambda_{\pi,k}(P^2) + \mathscr{C}_k(P^2)\,\lambda^2_{\pi,k}(P^2)$$

for $\mathscr{A}_k = 0$, one obtain immediately

$$\lambda_{\pi,k=0}(P^2) = \frac{\lambda_{\pi,\Lambda}\mathscr{D}_0(P^2)}{1-\lambda_{\pi,\Lambda}\int_\Lambda^0 \frac{dk}{k}\,\mathscr{D}_k(P^2)\,\mathscr{C}_k(P^2)}$$

with

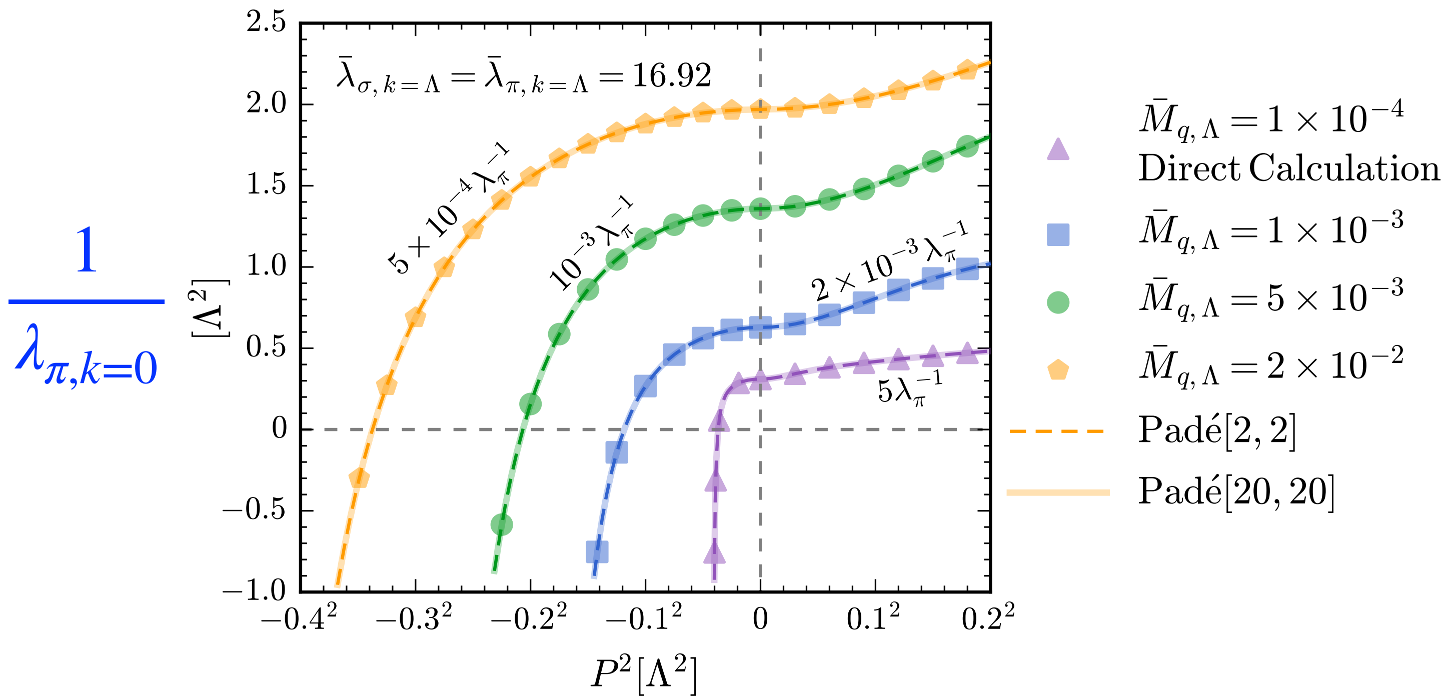
$$\mathscr{D}_k(P^2) \equiv \exp\left\{\int_\Lambda^k \frac{dk'}{k'}\mathscr{B}_{k'}(P^2)\right\}$$

pion mass is determined by the zero of denominator:

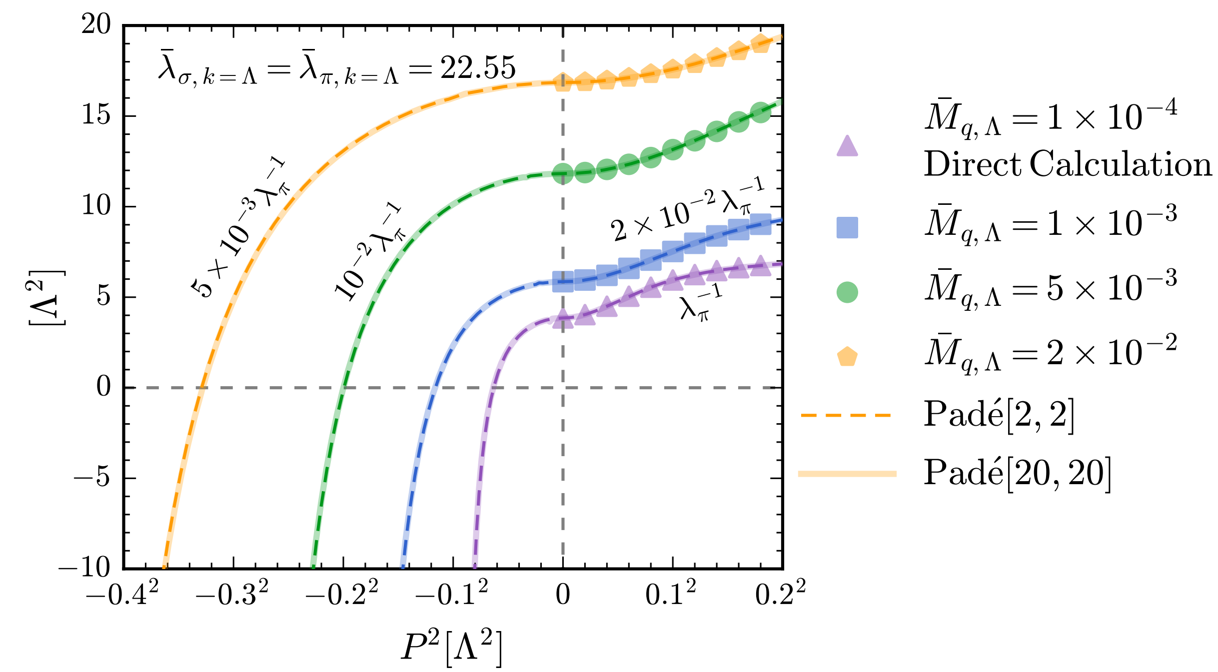
$$1-\lambda_{\pi,\Lambda}\int_\Lambda^0 \frac{dk}{k}\,\mathscr{D}_k(P^2)\,\mathscr{C}_k(P^2) = 0$$

Chiral symmetry and Goldstone theorem

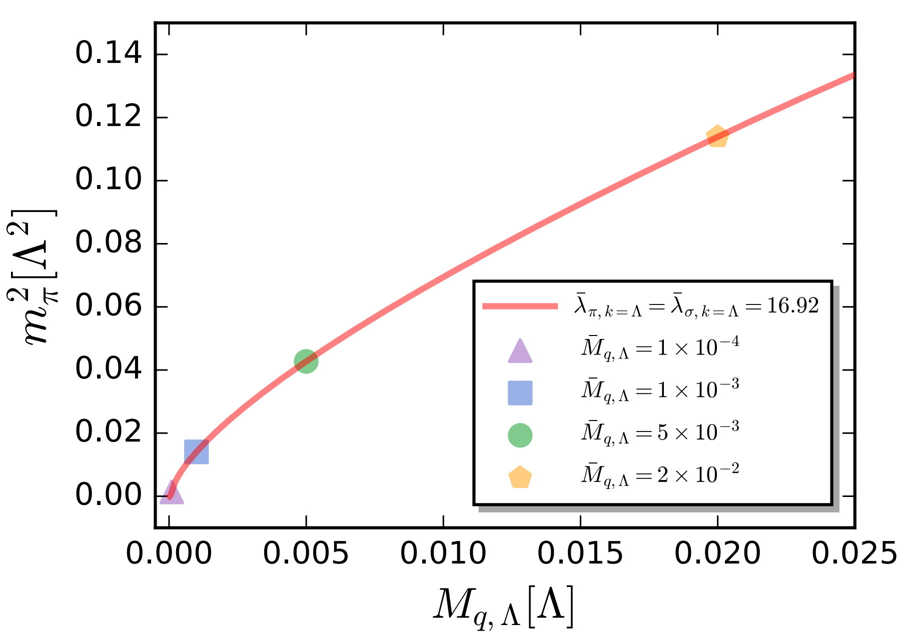
3d regulator:



4d regulator:



Gell-Mann--Oakes--Renner relation:



WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

Note: chiral symmetry and Goldstone theorem are guaranteed automatically in this approach.

Momentum dependence of 4-quark vertices

- 4-quark effective action:

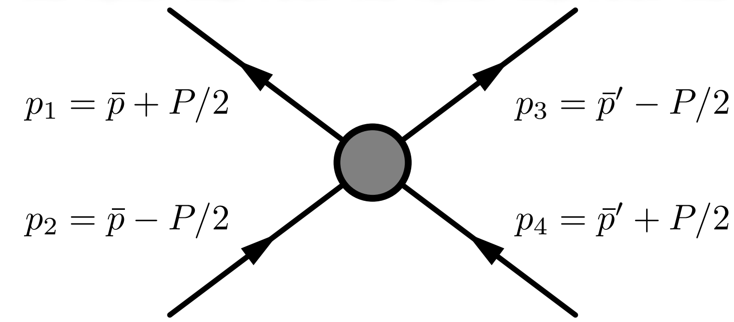
$$\Gamma_{4q,k} = -\sum_{\alpha}\int_{\vec{p}} \lambda_\alpha(\vec{p})\, \mathcal{O}^{(\alpha)}_{ijlm} \bar{q}_i(p_1) q_j(p_2) \bar{q}_l(p_3) q_m(p_4)$$

With $\alpha = 1,\ldots,10$ standing for ten Fierz-complete basis

$$\alpha \in \Big\{ \sigma\,,\pi\,,a\,,\eta\,,(V\pm A)\,,(V-A)^{\mathrm{adj}}\,, \\ (S\pm P)^{\mathrm{adj}}_{-}\,,(S+P)^{\mathrm{adj}}_{+} \Big\}\,,$$

- 4-quark vertex:

$$\Gamma^{(4)}_{\bar{q}_i q_j \bar{q}_l q_m}(p_1,p_2,p_3,p_4) \\ \equiv \frac{\delta}{\delta q_m(p_4)}\frac{\delta}{\delta \bar{q}_l(p_3)}\frac{\delta}{\delta q_j(p_2)}\frac{\delta}{\delta \bar{q}_i(p_1)}\Gamma_k[q,\bar{q}] \\ = \sum_\alpha \Big(\lambda_\alpha(p_1,p_2,p_3,p_4)\mathcal{O}^{(\alpha)}_{ijlm} - \lambda_\alpha(p_3,p_2,p_1,p_4)\mathcal{O}^{(\alpha)}_{ljim}\Big) \\ \times(-2)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4) \\ = \sum_\alpha \Big(\lambda^+_\alpha(p_1,p_2,p_3,p_4)\mathcal{T}^{(\alpha^-)}_{ijlm} + \lambda^-_\alpha(p_1,p_2,p_3,p_4)\mathcal{T}^{(\alpha^+)}_{ijlm}\Big) \\ \times(-4)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4)$$



$$= -\Gamma^{(4)}_{\bar{q}_i q_j \bar{q}_l q_m}(p_1,p_2,p_3,p_4)$$

where we have used 4-quark dressings and tensor structures with definite symmetries, viz.,

$$\lambda^+_\alpha(p_1,p_2,p_3,p_4) \\ \equiv \frac{1}{2}\Big[\lambda_\alpha(p_1,p_2,p_3,p_4) + \lambda_\alpha(p_3,p_2,p_1,p_4)\Big]\,, \\ \lambda^-_\alpha(p_1,p_2,p_3,p_4) \\ \equiv \frac{1}{2}\Big[\lambda_\alpha(p_1,p_2,p_3,p_4) - \lambda_\alpha(p_3,p_2,p_1,p_4)\Big]$$

and

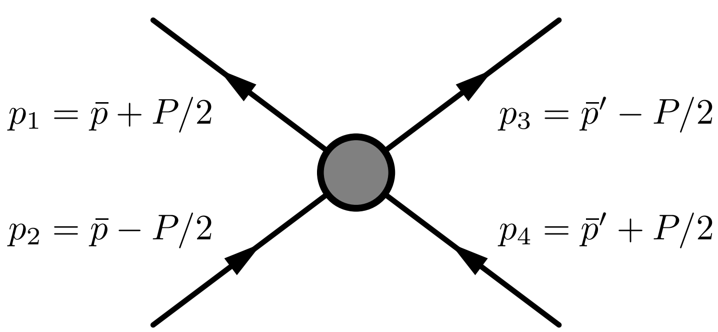
$$\mathcal{T}^{(\alpha^+)}_{ijlm} \equiv \Big(\mathcal{O}^{(\alpha)}_{ijlm} + \mathcal{O}^{(\alpha)}_{ljim}\Big)/2\,, \\ \mathcal{T}^{(\alpha^-)}_{ijlm} \equiv \Big(\mathcal{O}^{(\alpha)}_{ijlm} - \mathcal{O}^{(\alpha)}_{ljim}\Big)/2$$

with the symmetry relations

$$\lambda^+_\alpha(p_1,p_2,p_3,p_4) = \lambda^+_\alpha(p_3,p_2,p_1,p_4) \\ = \lambda^+_\alpha(p_1,p_4,p_3,p_2) = \lambda^+_\alpha(p_3,p_4,p_1,p_2)\,, \\ \lambda^-_\alpha(p_1,p_2,p_3,p_4) = -\lambda^-_\alpha(p_3,p_2,p_1,p_4) \\ = -\lambda^-_\alpha(p_1,p_4,p_3,p_2) = \lambda^-_\alpha(p_3,p_4,p_1,p_2)$$

Three momentum (s, t, u) channel approximation

- parameterization of external momenta of 4-quark vertices:



$$= -\Gamma^{(4)}_{\bar{q}_i q_j \bar{q}_l q_m}(p_1, p_2, p_3, p_4)$$

- three momentum (s, t, u) channel approximation for 4-quark dressings of definite symmetries:

$$\lambda^{\pm}_{\alpha}(p_1, p_2, p_3, p_4) \approx \lambda^{\pm}_{\alpha}(t, u, s)$$

with

$$t = (p_1 - p_2)^2 = P^2\,,$$
$$u = (p_1 - p_4)^2 = (\bar{p} - \bar{p}')^2\,,$$
$$s = (p_1 + p_3)^2 = (\bar{p} + \bar{p}')^2$$

- for the convenience of computation, we choose a subspace of the full momentum of 4-quark vertices as follows

$$P_\mu = \sqrt{P^2}\left(1, 0, 0, 0\right),$$
$$\bar{p}_\mu = \sqrt{\bar{p}^2}\left(1, 0, 0, 0\right),$$
$$\bar{p}'_\mu = \sqrt{\bar{p}^2}\left(\cos\theta,\ \sin\theta, 0, 0\right)$$

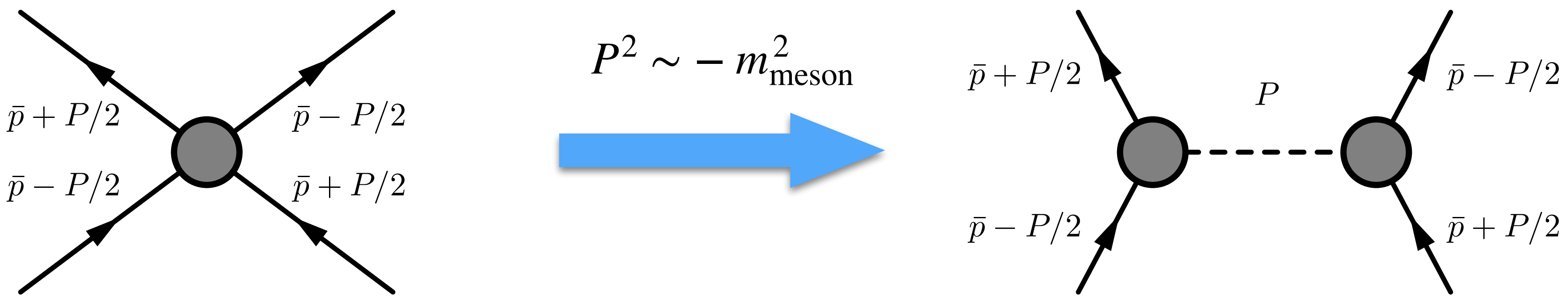
one is led to

$$t = P^2\,, \quad u = 2\bar{p}^2(1 - \cos\theta)\,, \quad s = 2\bar{p}^2(1 + \cos\theta)$$

Here, $\{\sqrt{P^2}, \sqrt{\bar{p}^2}, \cos\theta\}$ is in one-by-one correspondence with respect to $\{t, u, s\}$

Bethe-Salpeter amplitude (quark-meson coupling)

- **Bethe-Salpeter amplitude can be extracted from the 4-quark vertex in the proximity of on-shell momentum of bound states:**



That is

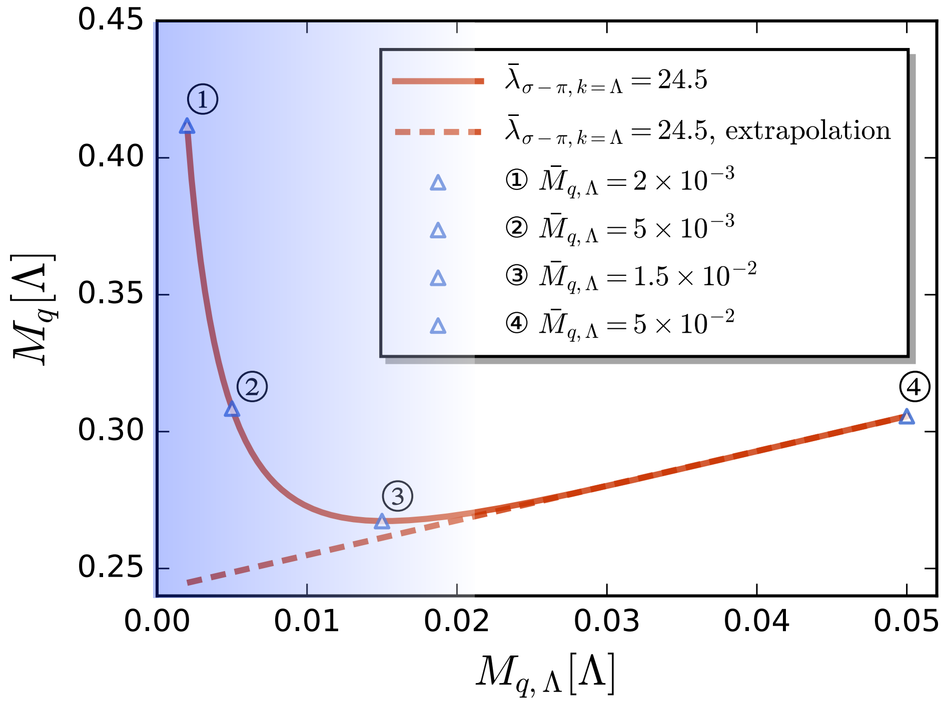
$$\lambda(\bar{p},P) \sim \frac{h^2(\bar{p},P)}{P^2+m^2_{\text{meson}}}$$

where $h(\bar{p},P)$ is the BS amplitude. Note that the angular dependence of $h(\bar{p},P)$ between $\bar{p}$ and P, i.e., θ, can be further recovered by computing the flows on the solutions with

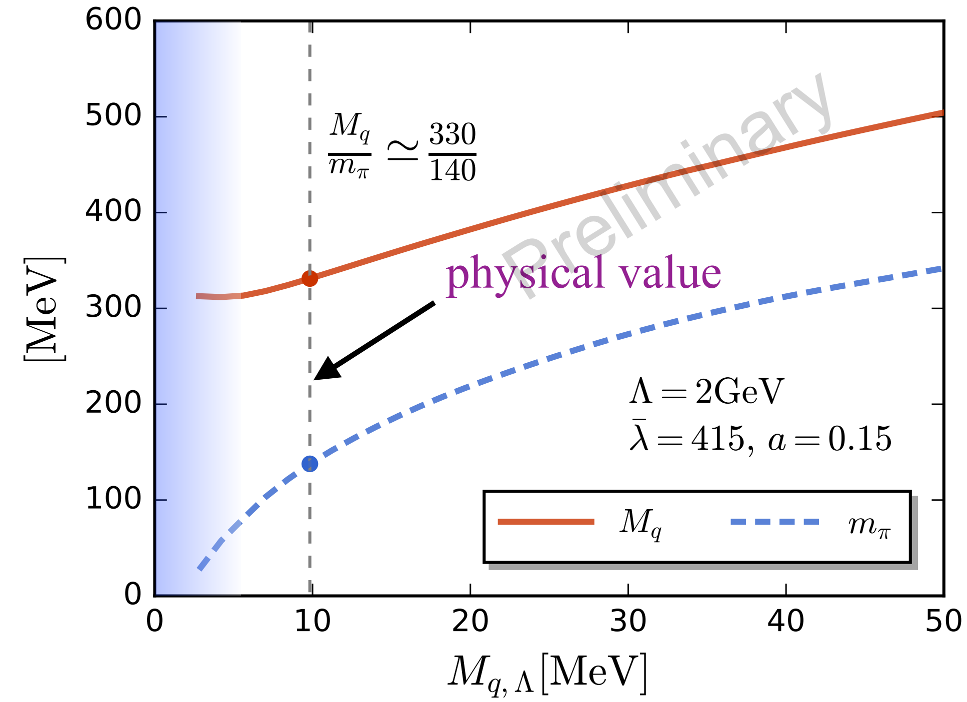
$$\bar{p}_\mu = \bar{p}'_\mu = \sqrt{\bar{p}^2}\left(\cos\theta,\ \sin\theta, 0, 0\right)$$

Chiral limit revisited

momentum-independent 4-quark vertex:



three momentum channel approximation:

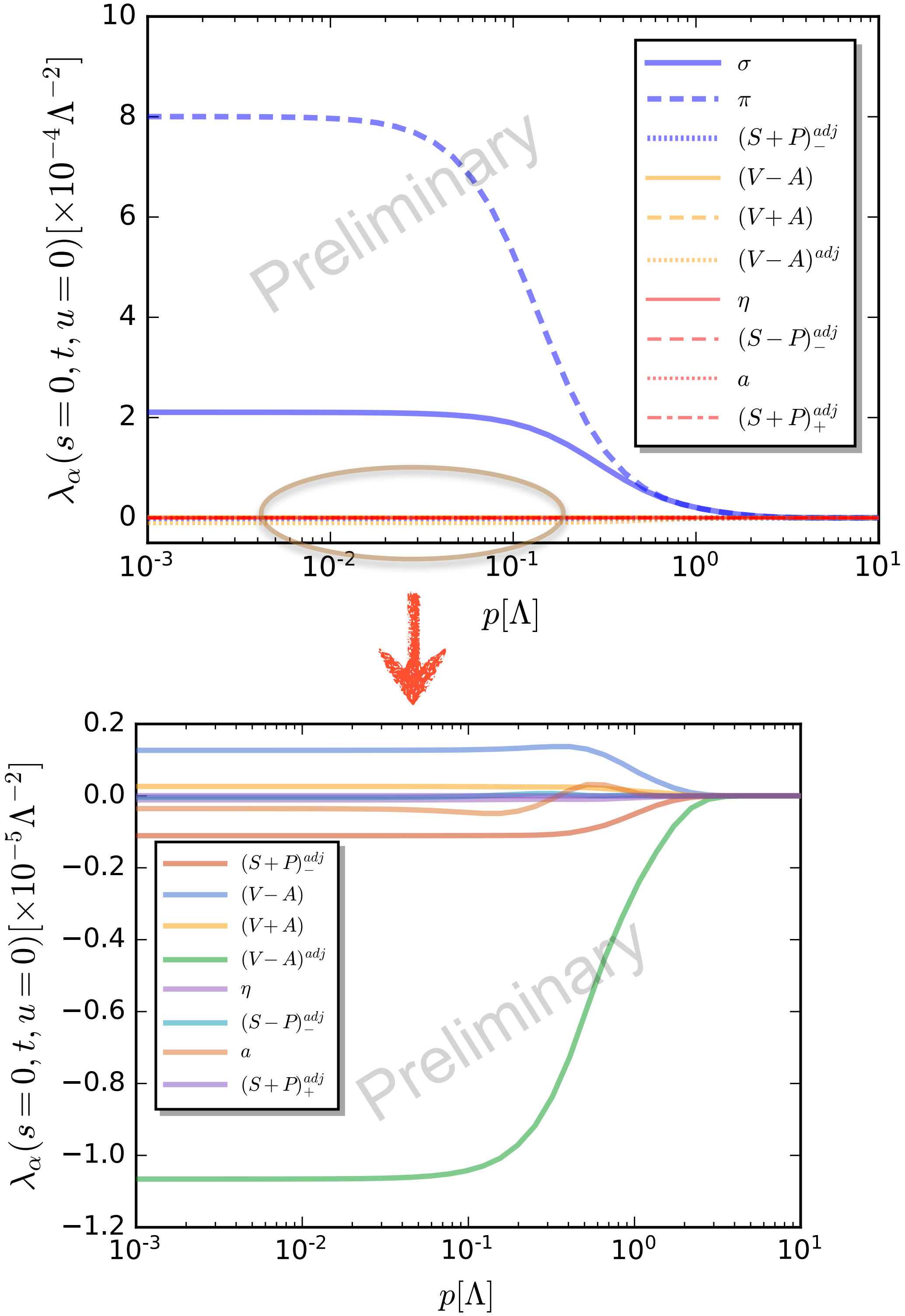


WF, Huang, Pawlowski, Tan, in preparation

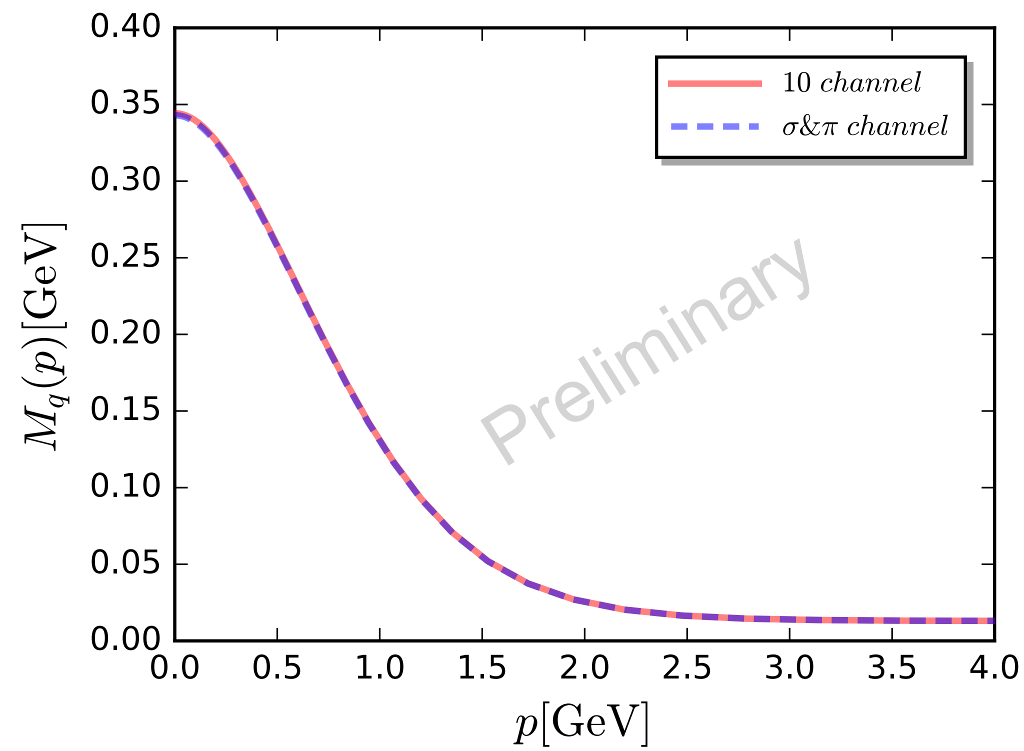
- **Artifact region, blue area, shrinks obviously when more momentum dependence is included**
- **It is expected that this effect would be more prominent in QCD**

σ, π vs other channels

4-quark dressings of different channels:



Quark mass function:

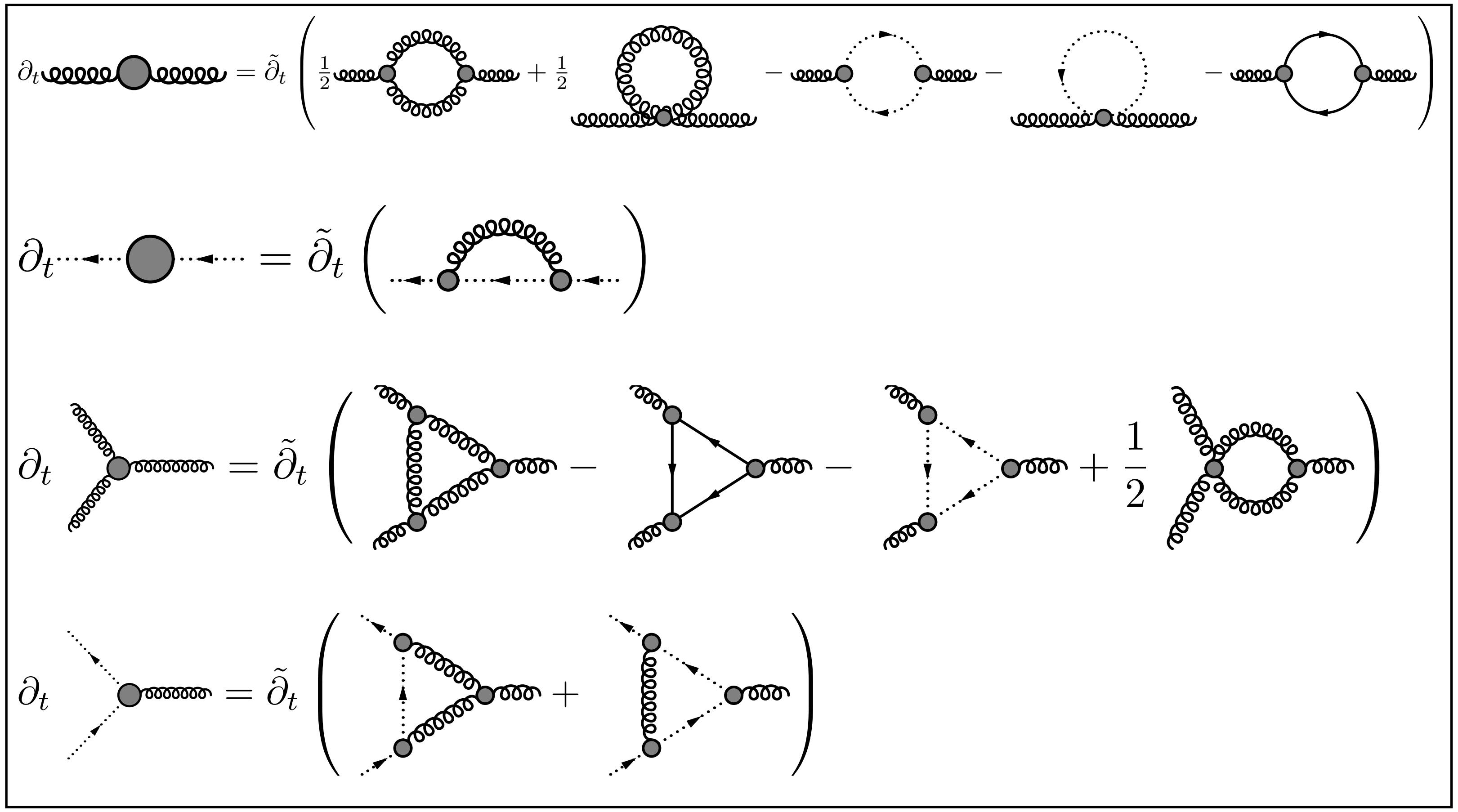


WF, Huang, Pawlowski, Tan, in preparation

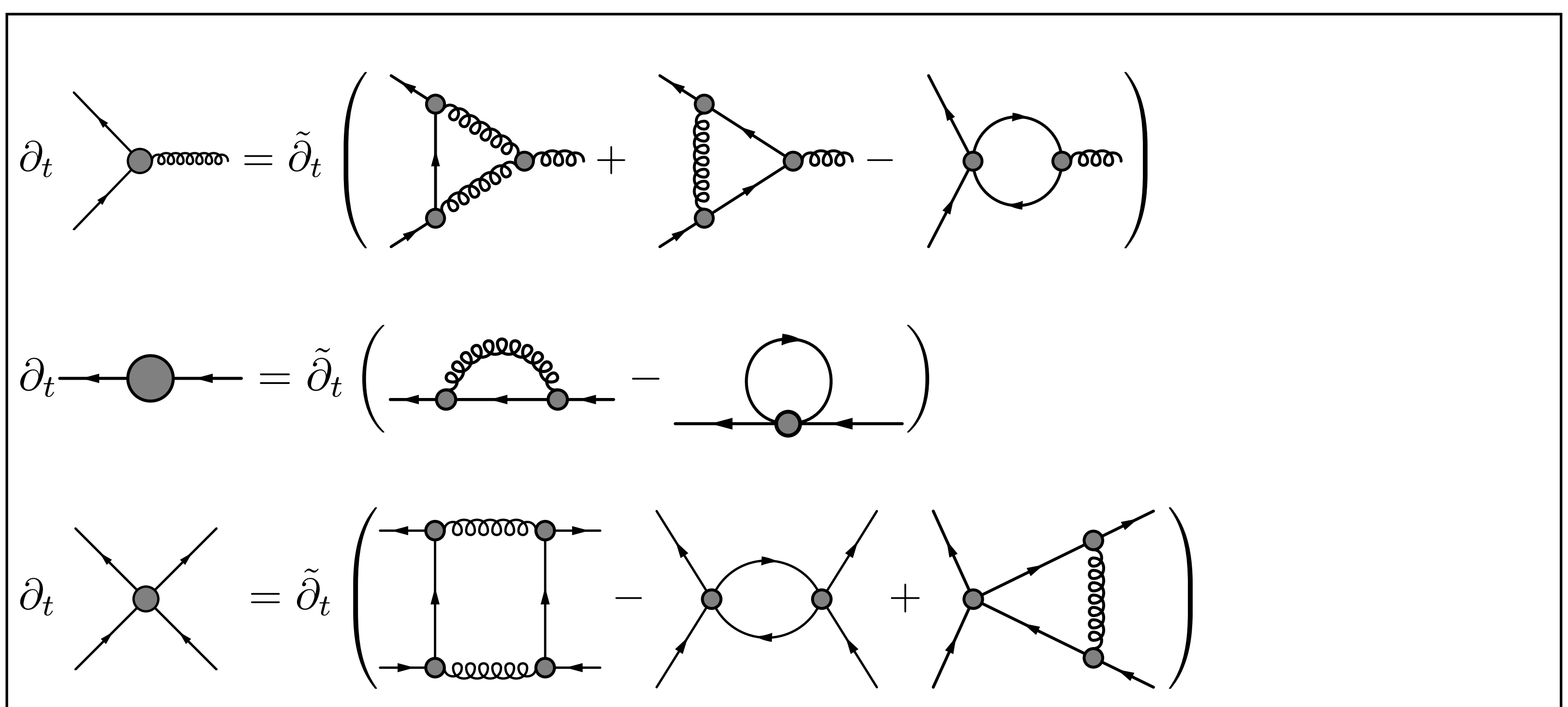
- **Dominance of scalar-pseudoscalar channels over other channels is more pronounced in the momentum-dependent approximations**
- **One can safely include 4-quark vertices of only sigma and pion channels in the calculations**

Embedding in unquenched QCD

Glue sector:



Matter sector:

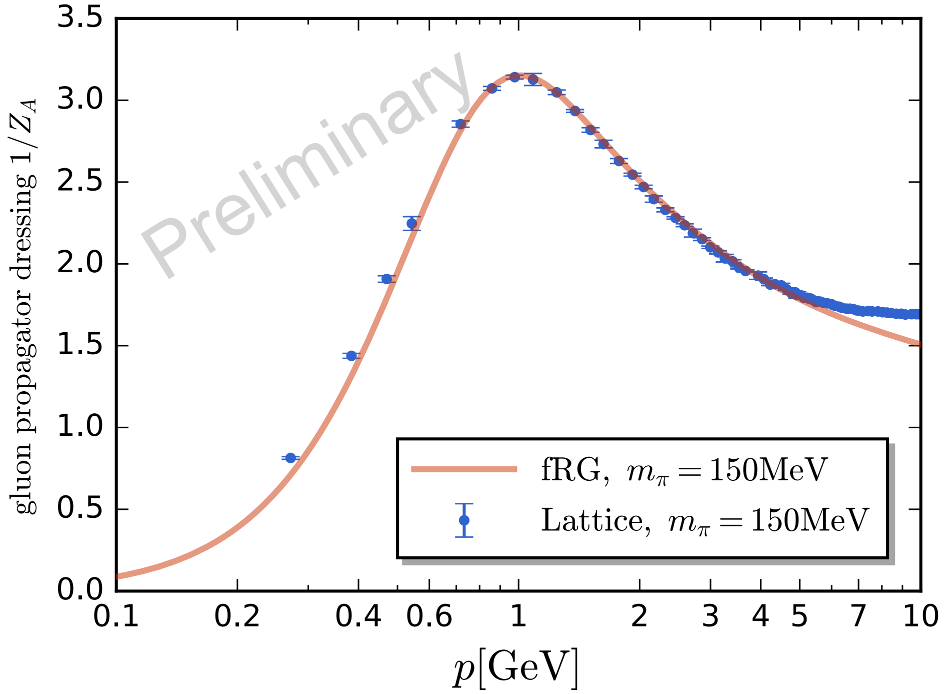


See also e.g., Cyrol, Mitter, Pawlowski, Strodthoff, *PRD* 97 (2018) 054006, unquenched QCD in fRG with dynamical hadronization

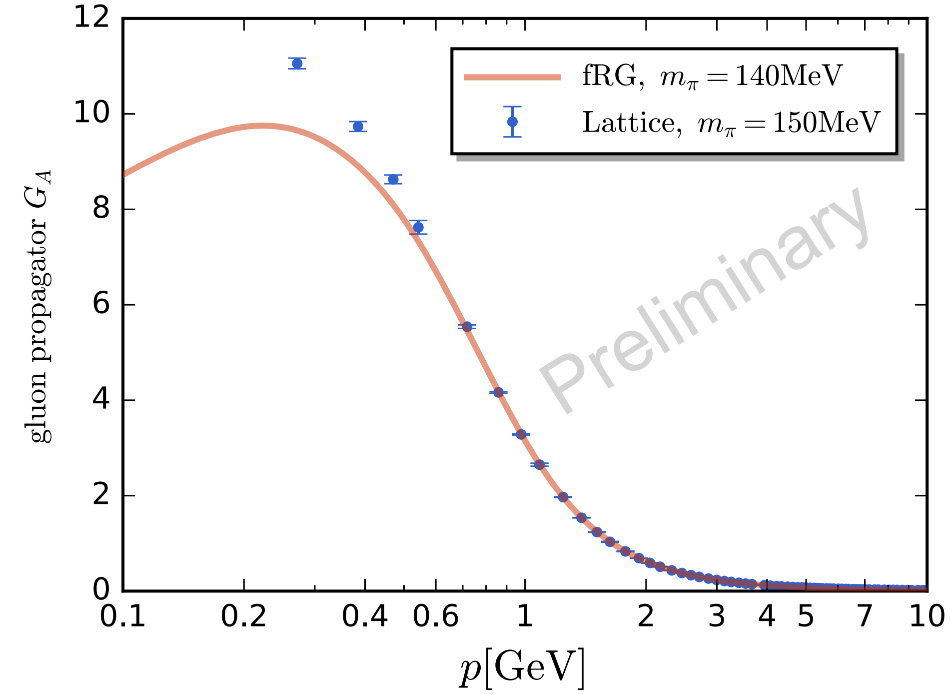
WF, Huang, Pawlowski, Tan, Zhou, in preparation

Gluon, ghost dressings and strong couplings

Gluon dressing:



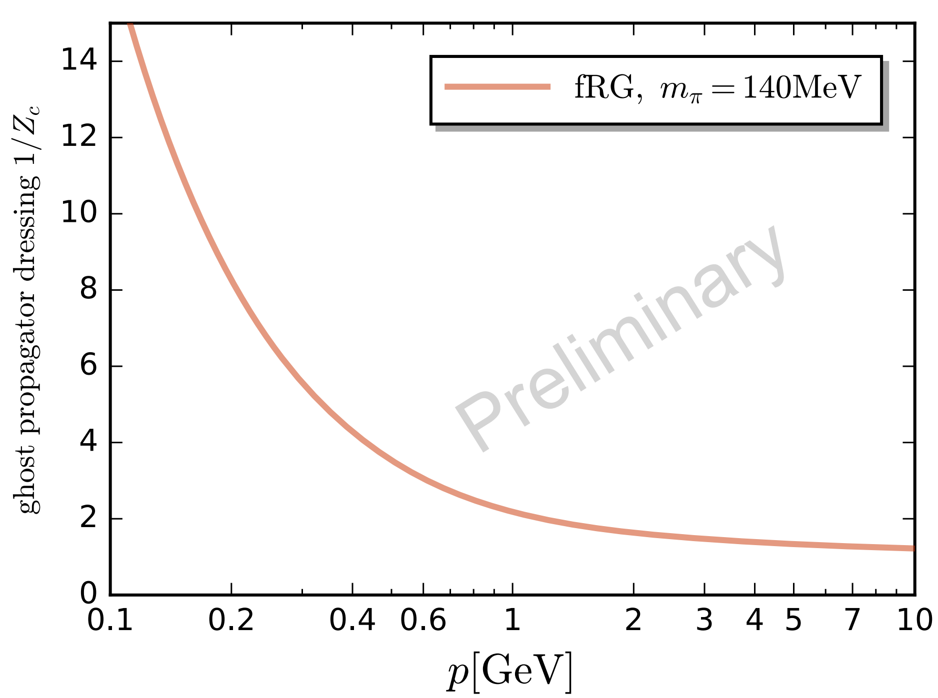
Gluon propagator:



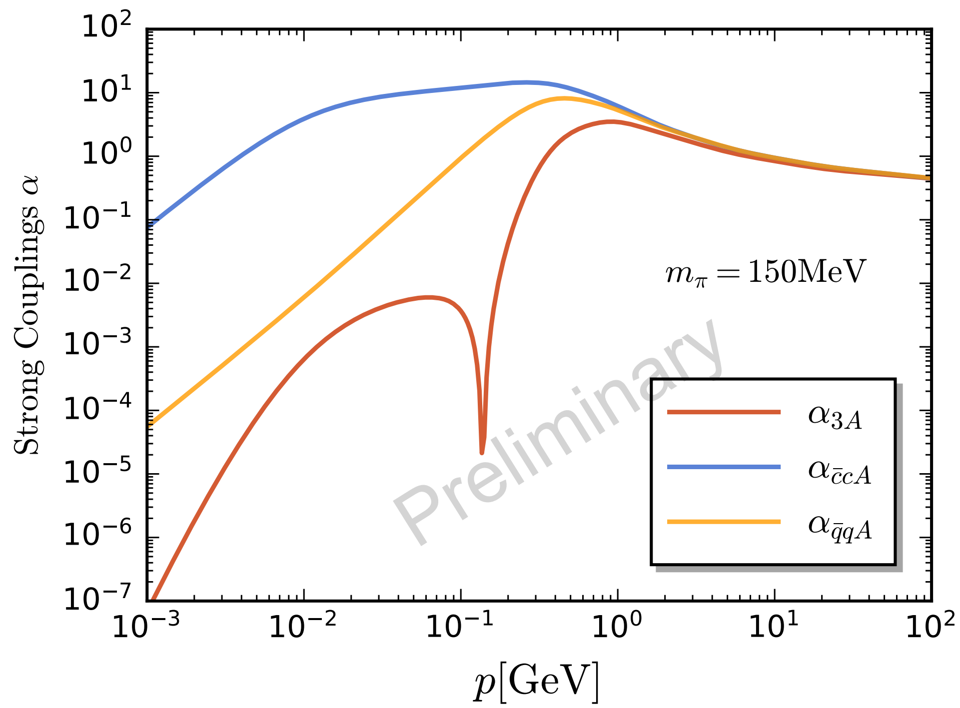
Lattice: Sternbeck *et al.*, PoS LATTICE2012 (2012) 243

fRG: WF, Huang, Pawlowski, Tan, Zhou, in preparation

Ghost dressing:



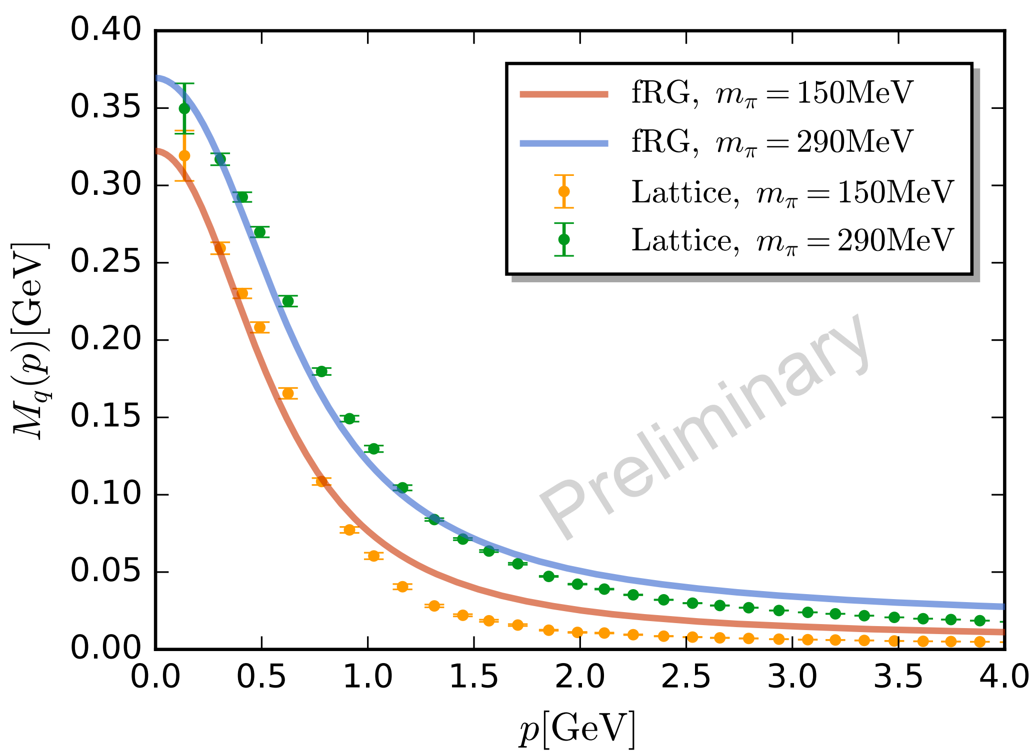
Strong couplings:



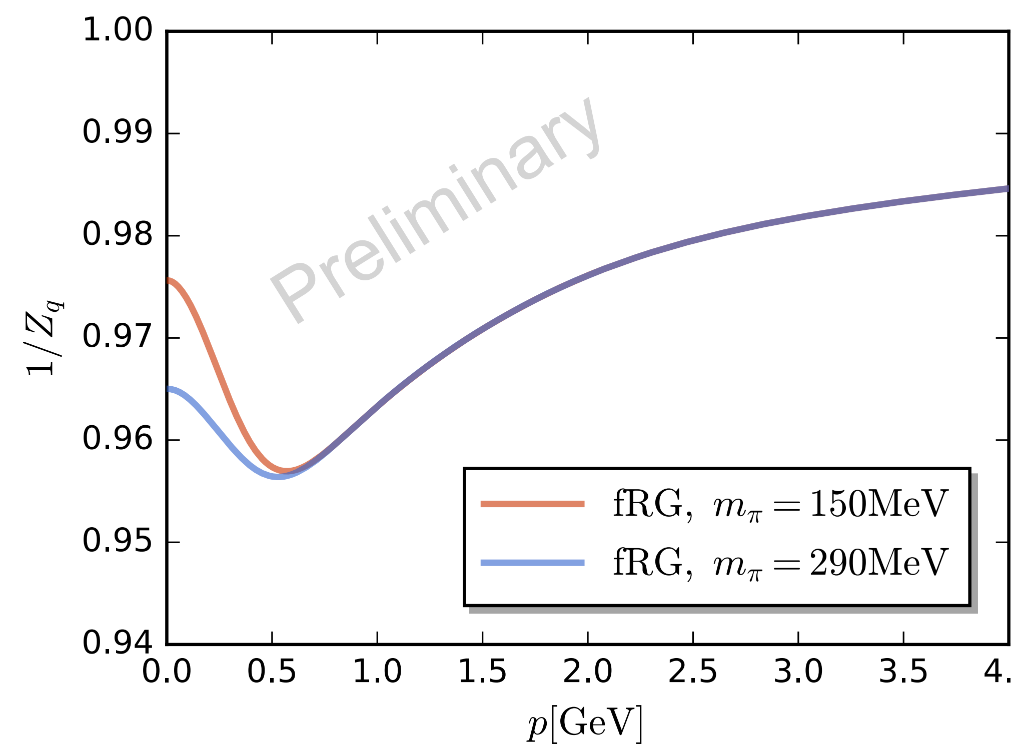
Note: Comparison with the DSE and fRG with DH has not done yet.

Quark propagator and quark-gluon vertex of different channels

Quark mass function:



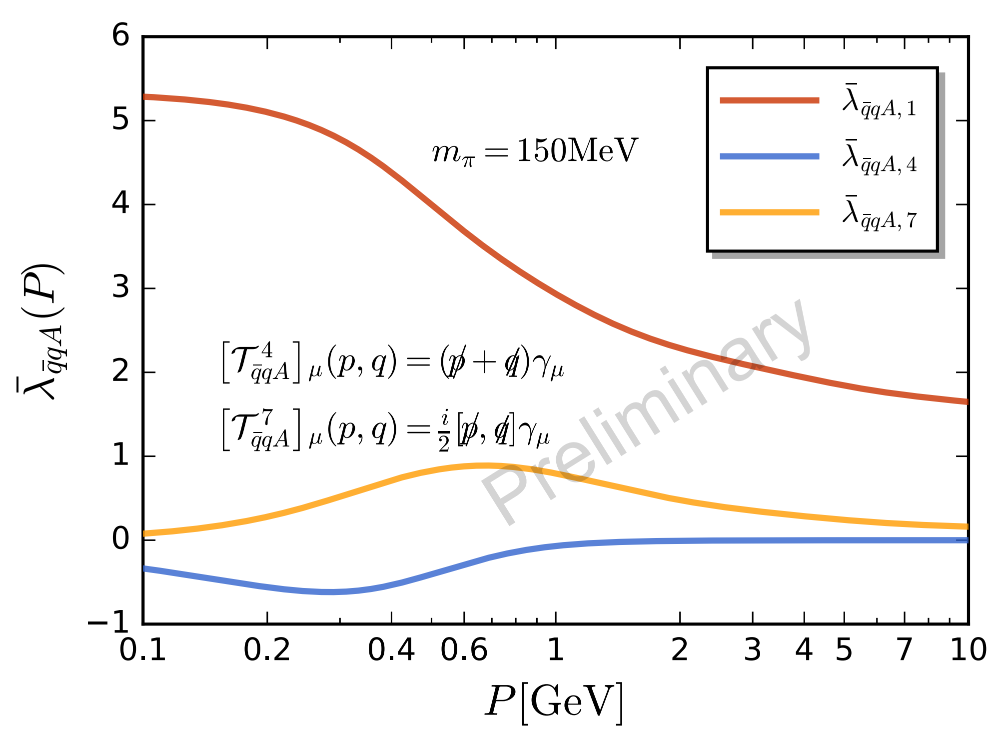
Quark wave function:



Lattice: Oliveira *et al.*, PRD 99 (2019) 094506

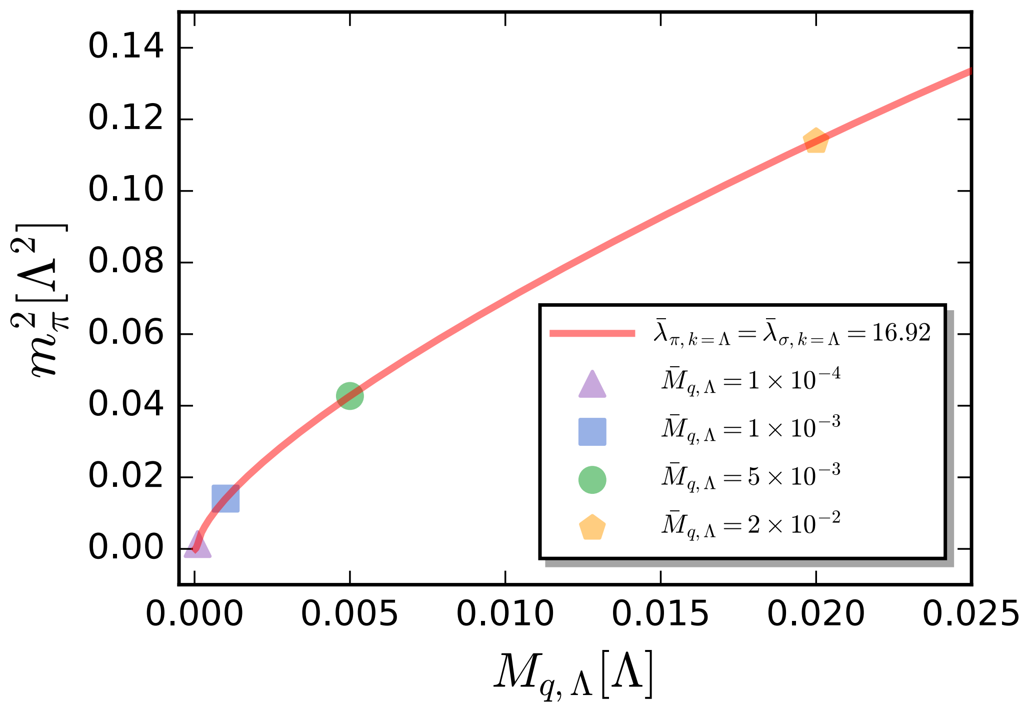
fRG: WF, Huang, Pawlowski, Tan, Zhou, in preparation

Quark-gluon vertex of different channels:



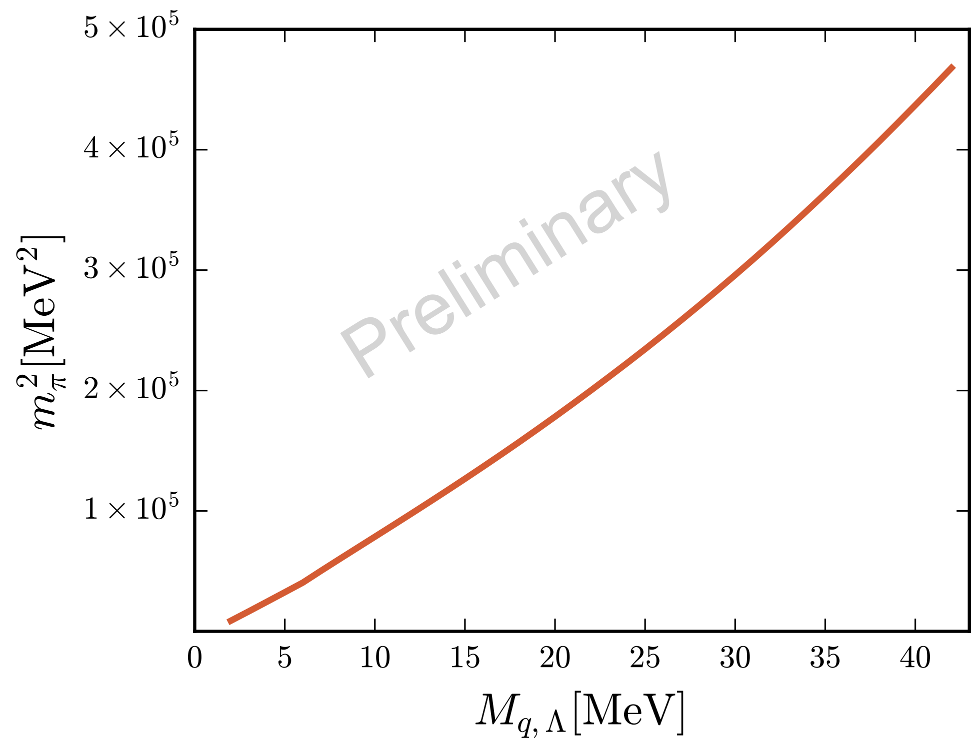
Gell-Mann--Oakes--Renner relation

LEFT with single momentum dependence:



WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

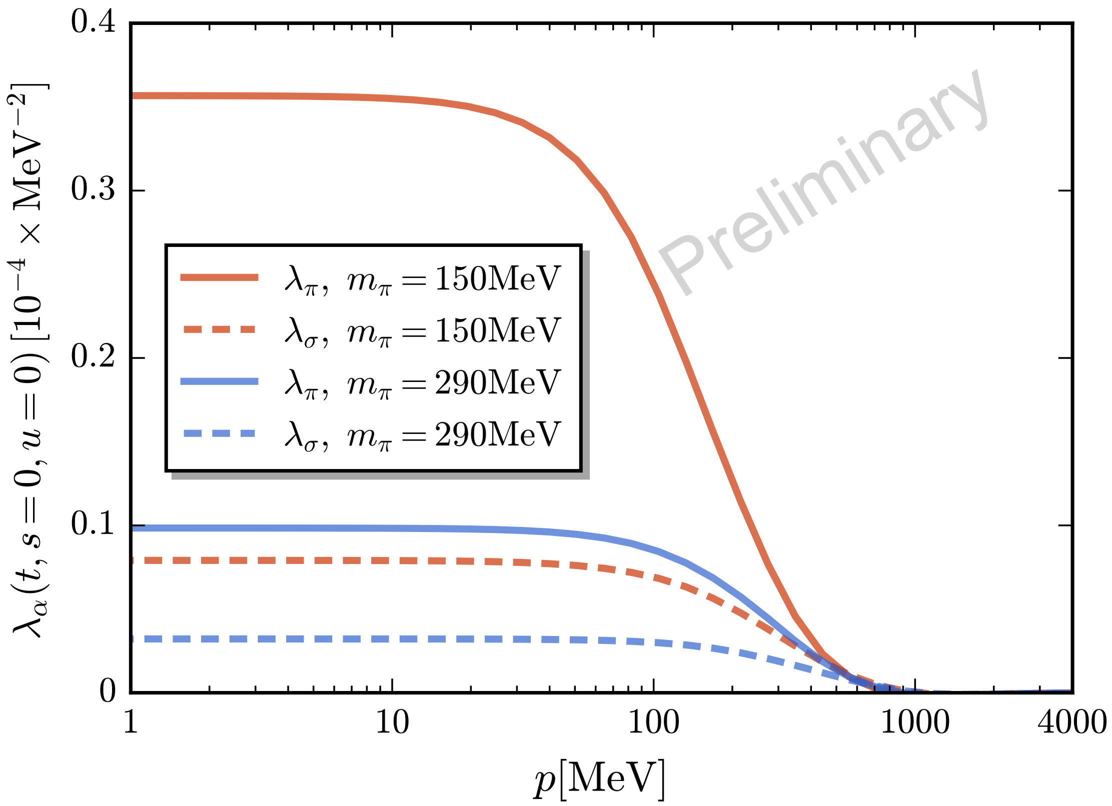
QCD:



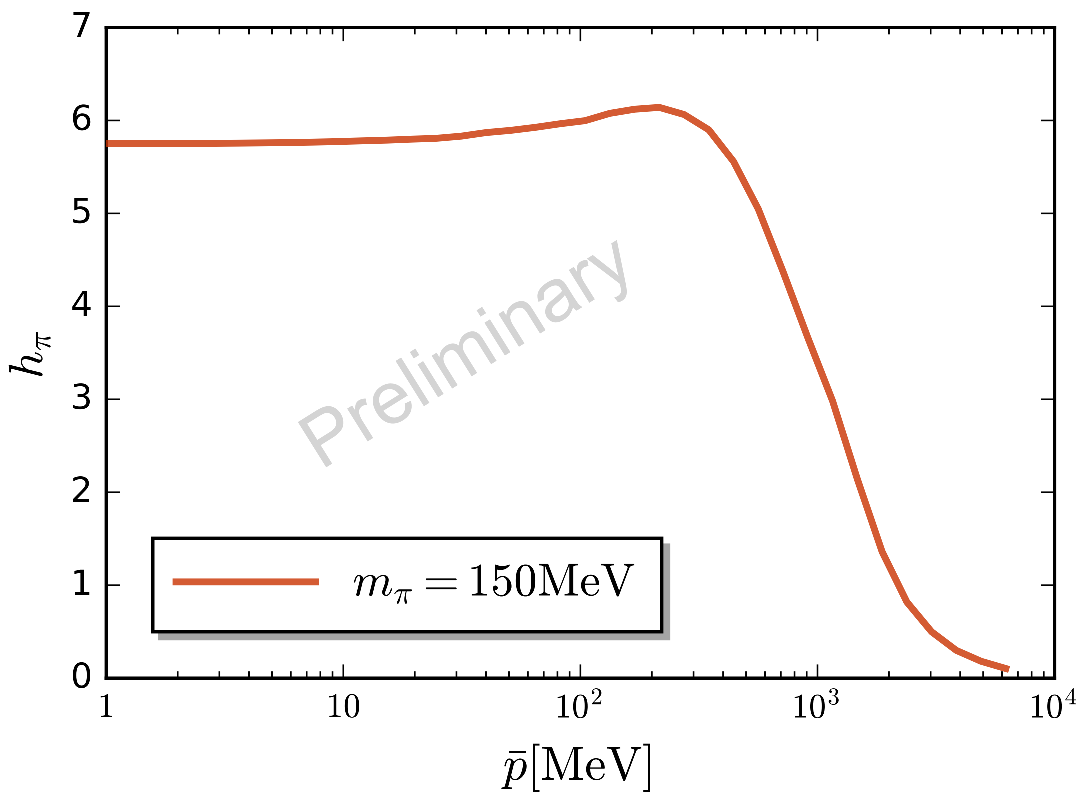
WF, Huang, Pawlowski, Tan, Zhou, in preparation

4-quark dressings and pion BS amplitude

4-quark dressings:



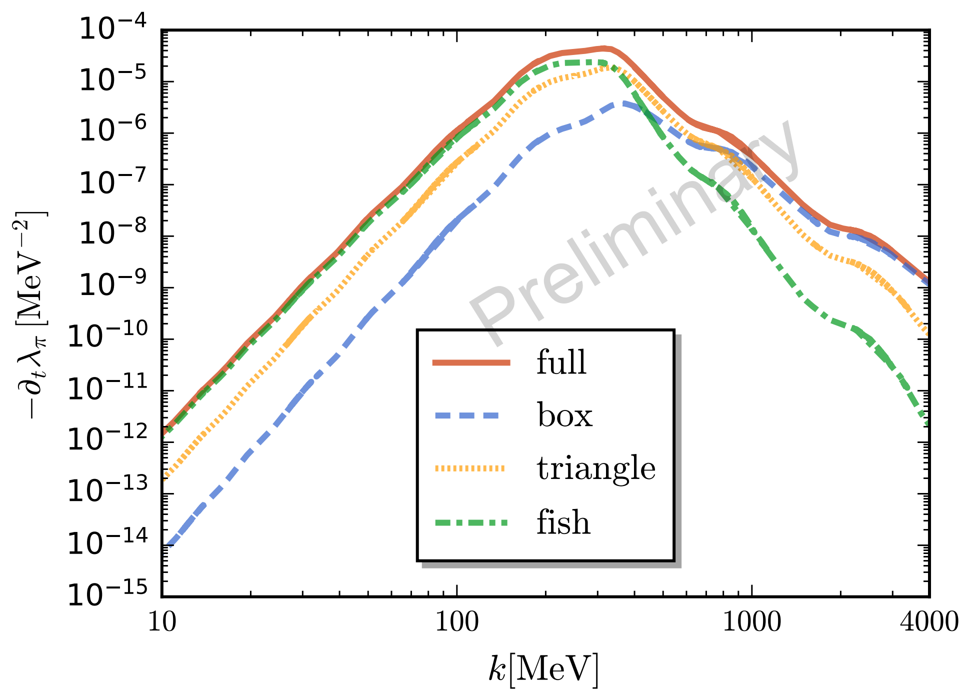
Pion BS amplitude:



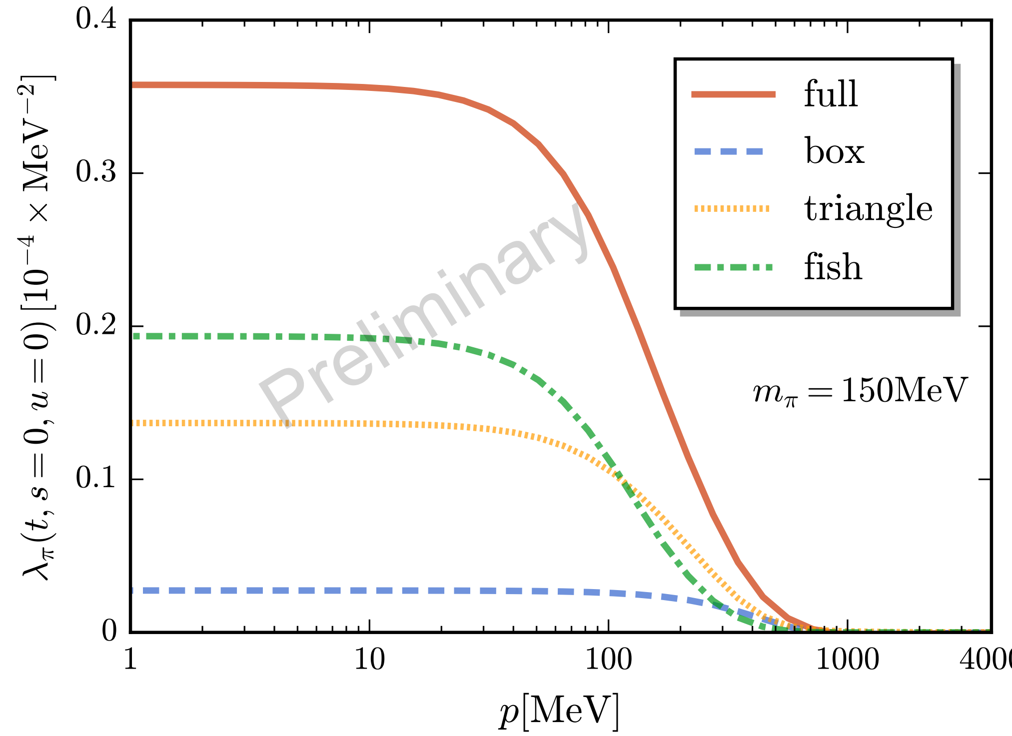
WF, Huang, Pawlowski, Tan, Zhou, in preparation

Contributions to λ_π from different diagrams

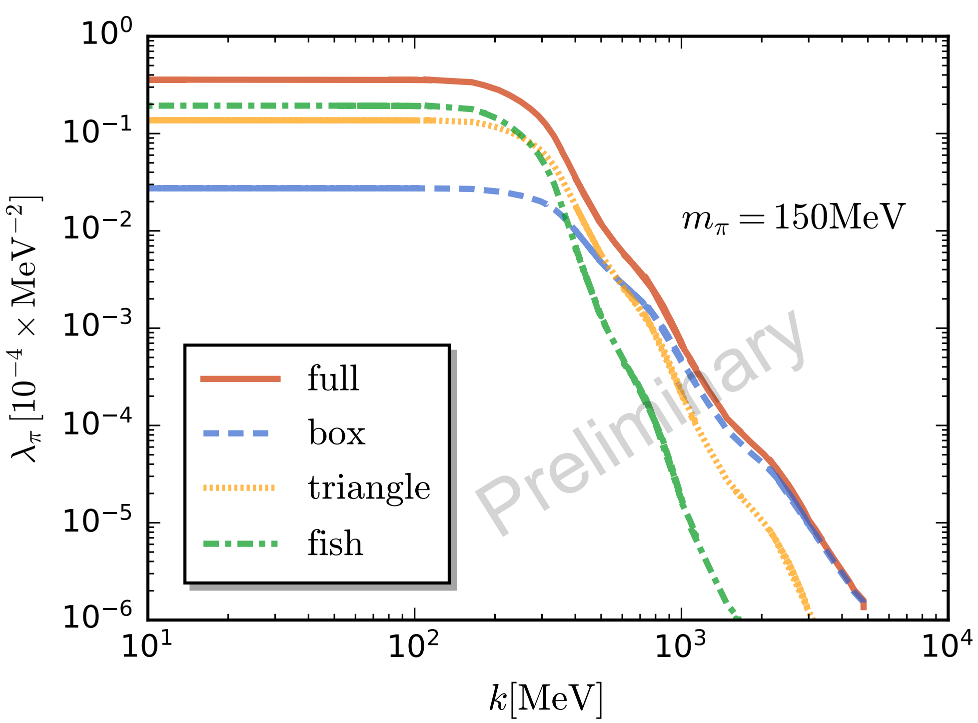
flow of λ_π vs RG scale:



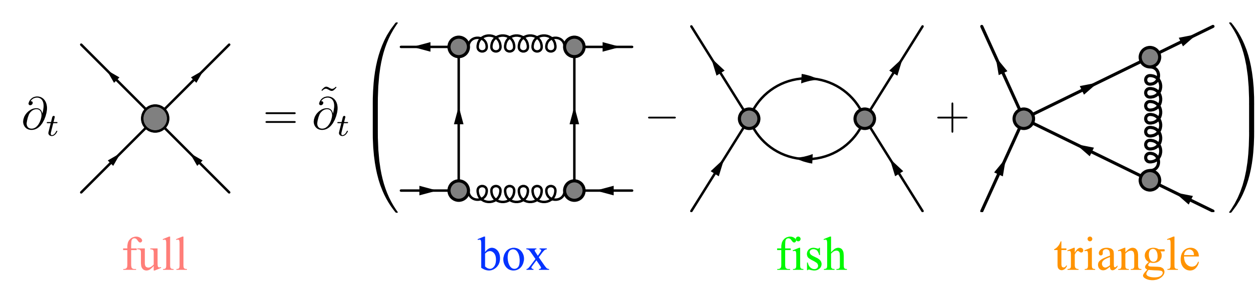
integrated flow of λ_π vs $P = \sqrt{t}$:



integrated flow of λ_π vs RG scale:

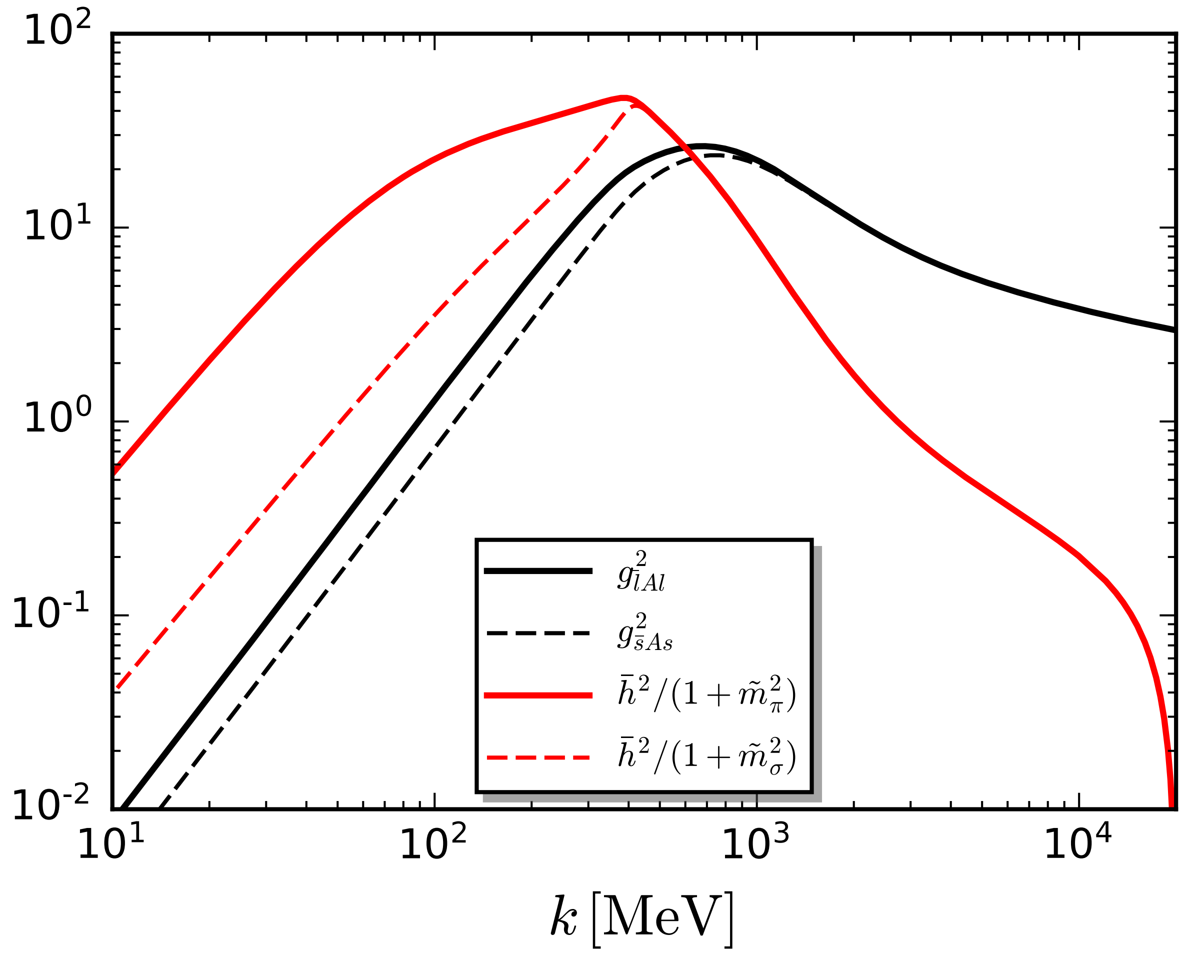


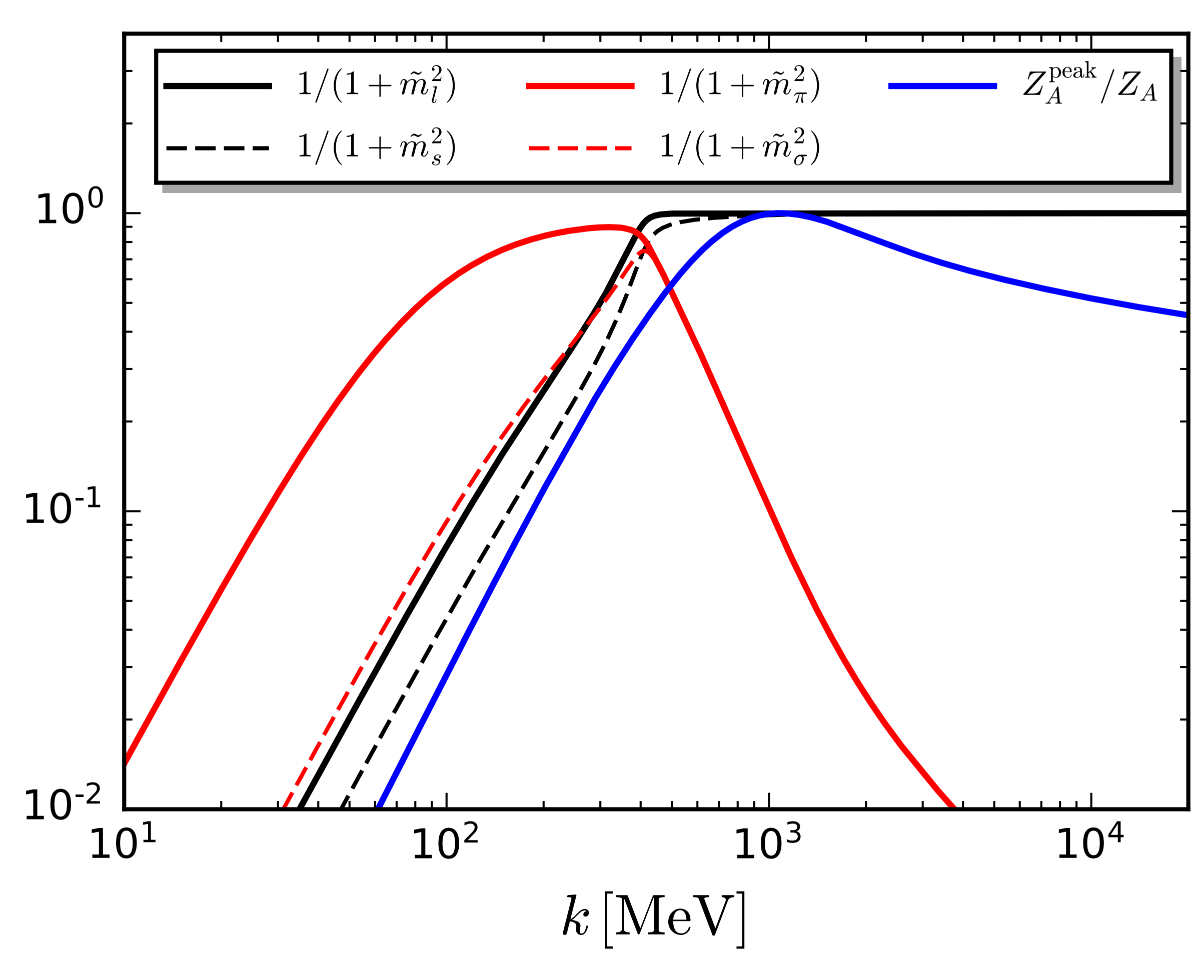
WF, Huang, Pawlowski, Tan, Zhou, in preparation



Comparison to results in DH

- **Exchange couplings**
- **Propagator gapping**



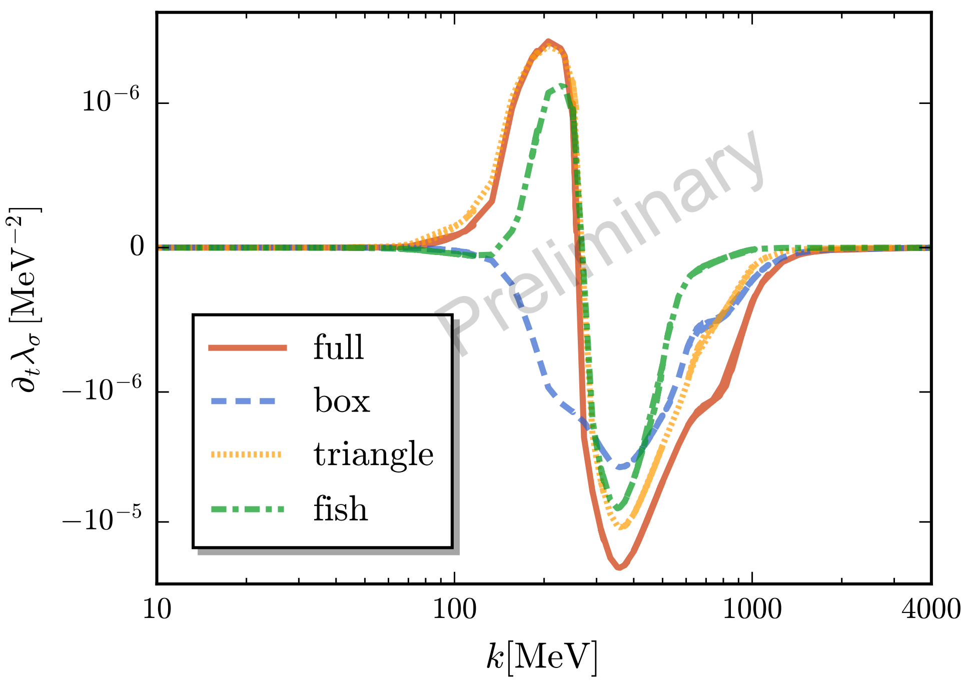


- Composite (mesonic) degrees of freedom take over active dynamics from partonic ones when the RG scale is lowered down $k \lesssim 600 \sim 800$ MeV.
- LEFTs emerge naturally from fundamental theory in the regime of low energy, in agreement with the viewpoint of RG.

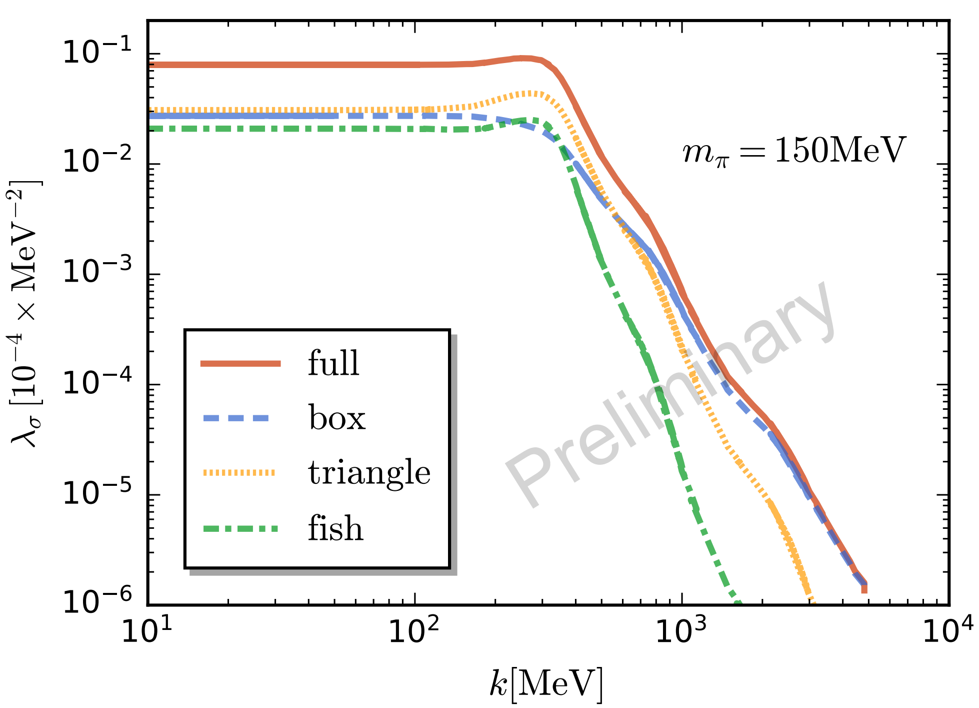
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Contributions to λ_σ from different diagrams

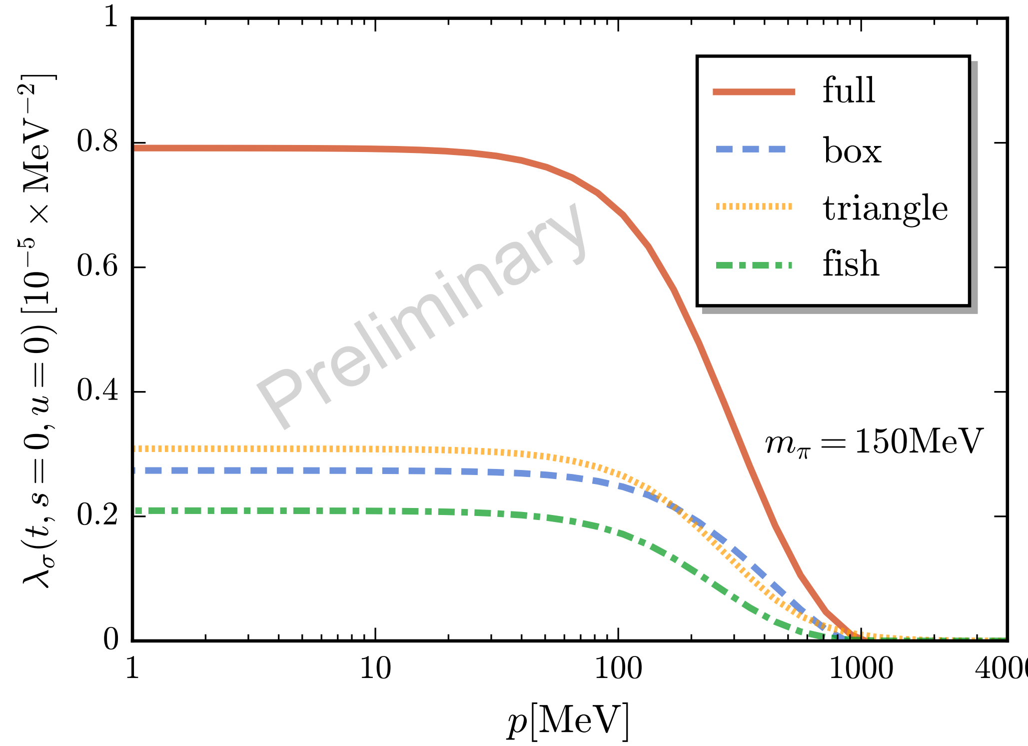
flow of λ_σ vs RG scale:



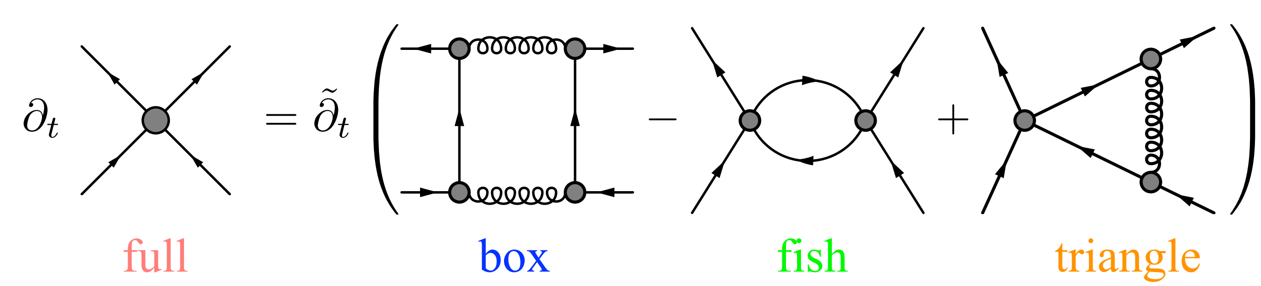
integrated flow of λ_σ vs RG scale:



integrated flow of λ_σ vs $P = \sqrt{t}$:

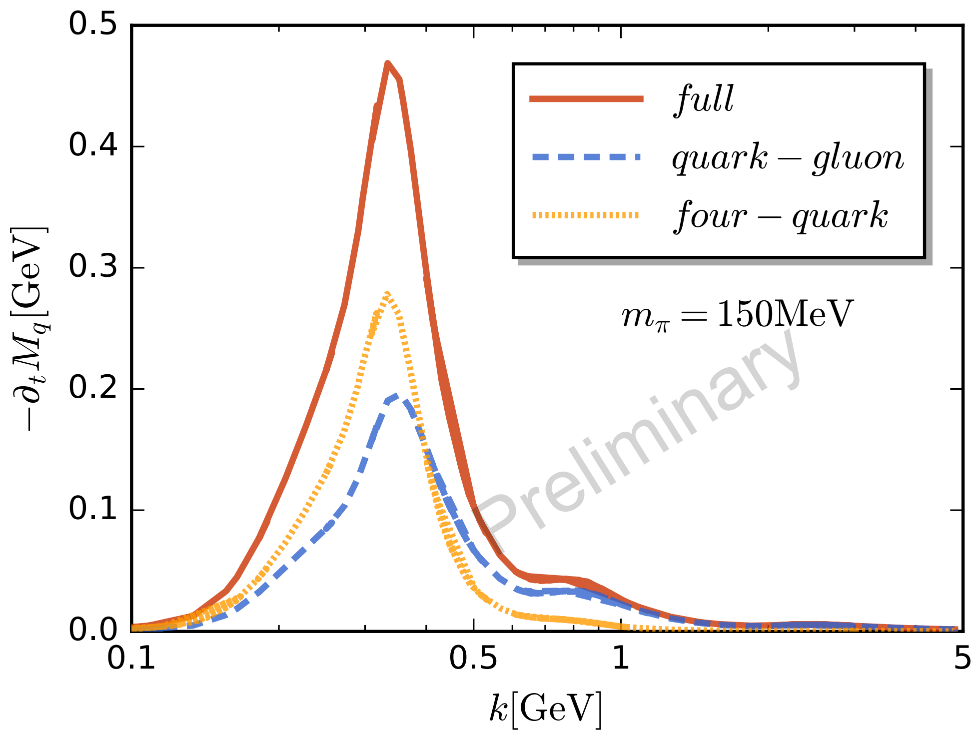


WF, Huang, Pawlowski, Tan, Zhou, in preparation

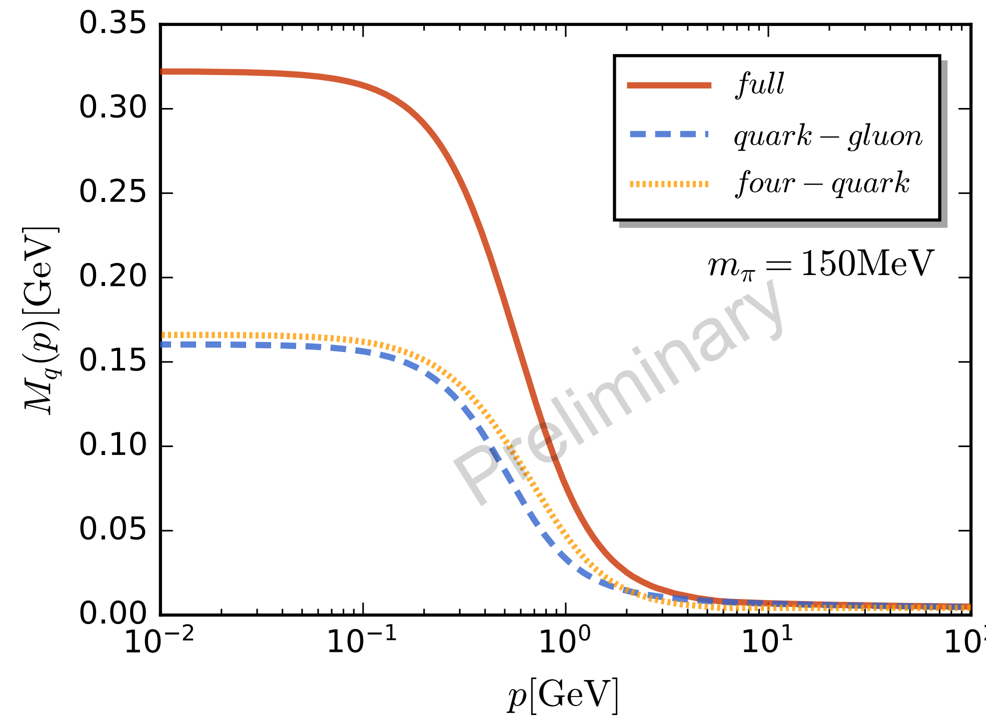


Contributions to quark mass from different diagrams

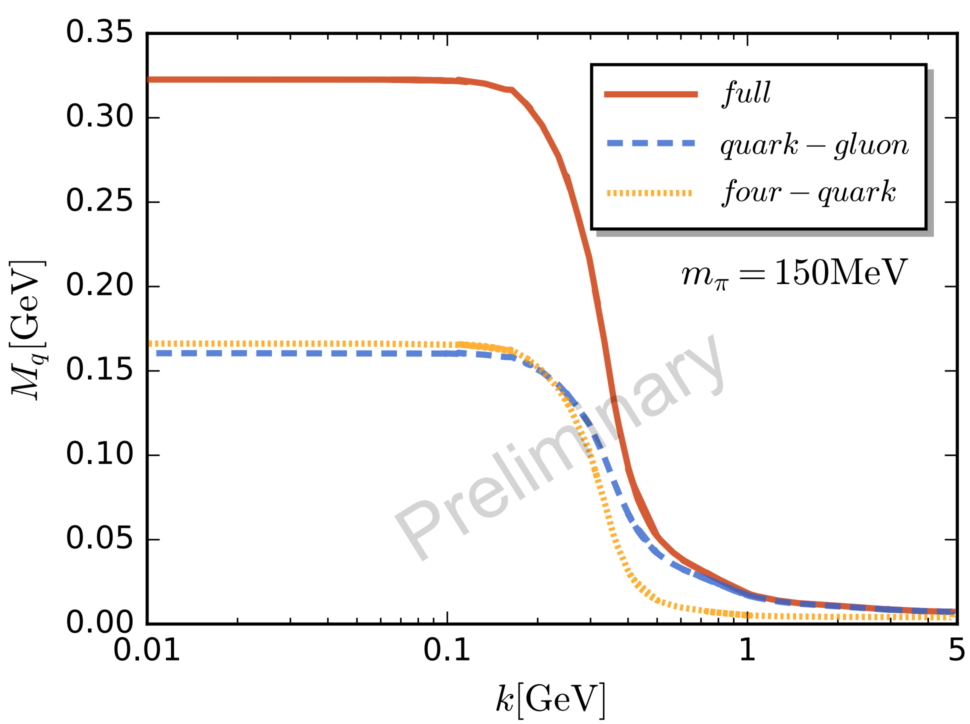
Flow of quark mass:



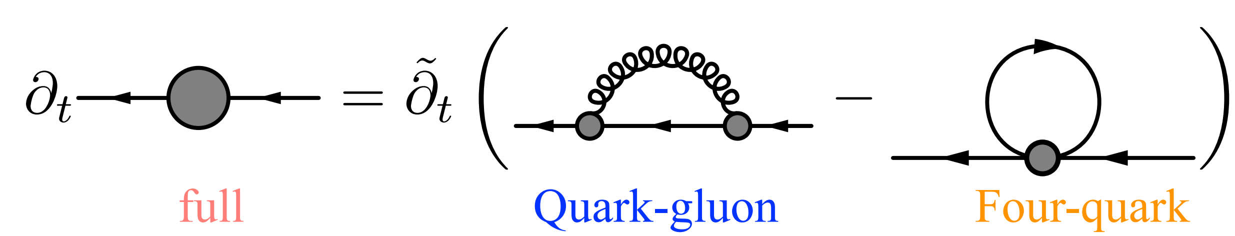
Quark mass function:



Integrated flow of quark mass:



WF, Huang, Pawlowski, Tan, Zhou, in preparation



Pion decay constant

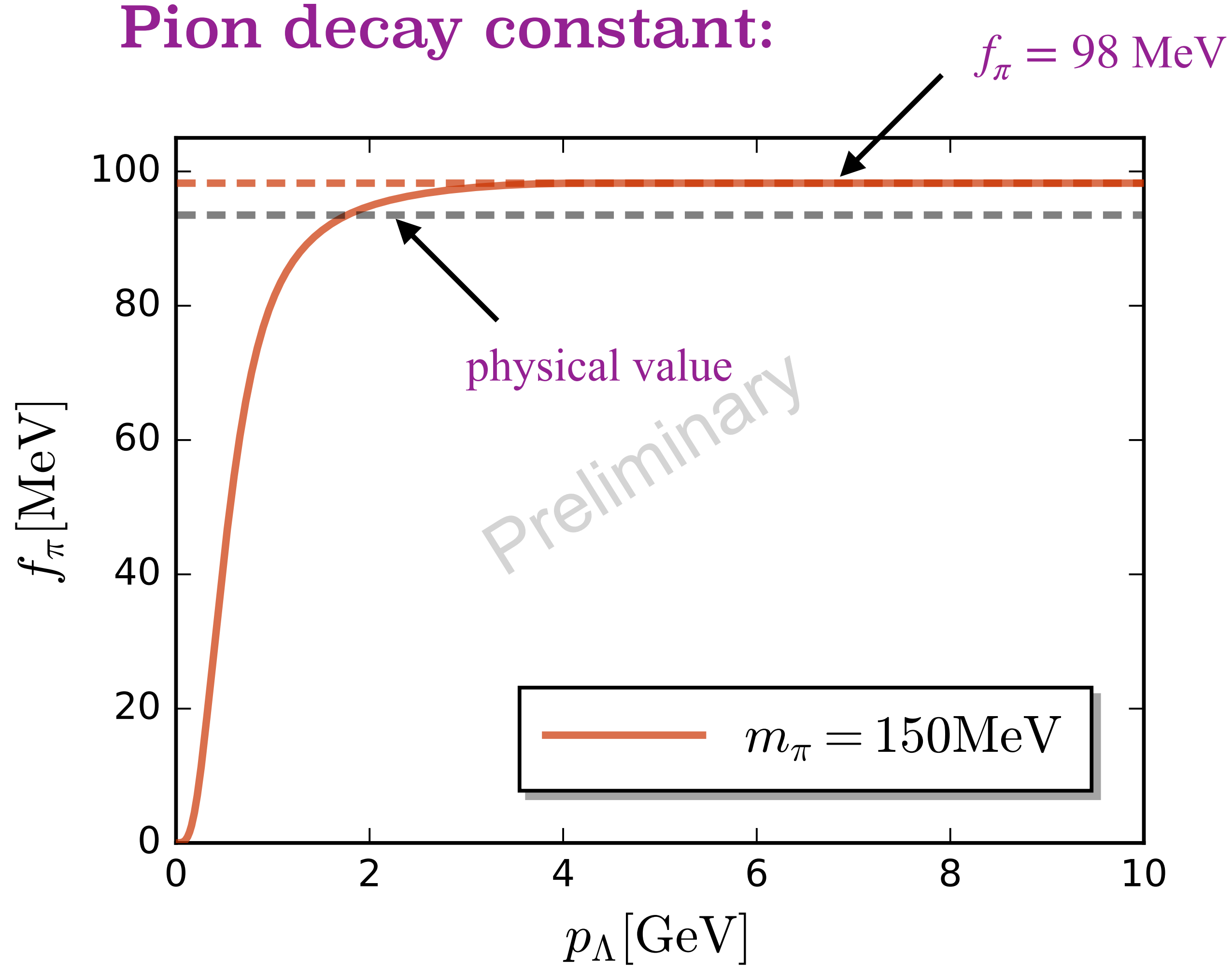
- The pion weak decay constant reads

$$\langle 0 | J^a_{5\mu}(x) | \pi^b \rangle = i P_\mu f_\pi \delta^{ab}$$

The left side is given by

$$\langle 0 | J^a_{5\mu}(x) | \pi^b \rangle$$
$$= \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu \gamma_5 t^a G_q(q+P) h_\pi(q,P) \gamma_5 t^b G_q(q)\right]$$
$$\simeq \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu \gamma_5 t^a G_q(q) h_\pi(q,P) \gamma_5 t^b G_q(q)\right]$$

Pion decay constant:



WF, Huang, Pawlowski, Tan, Zhou, in preparation

Summary and outlook

- ★ Analogues of self-consistent quark gap equation and Bethe-Salpeter equation are developed in terms of RG flows, where effective multi-quark interactions generated dynamically in the regime of low energy are found to play a crucial role.

- ★ This formalism has been embedded in $N_f = 2$ flavor unquenched QCD, a number of interesting observables have been obtained.

- ★ This formalism provides a promising approach to study hadron physics, e.g., PDA of mesons, spectrum, etc., from first-principles QCD, and related work is in progress.

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Thank you very much for your attentions!