

Trento 2013



Trento, 2023

Mesons effects on the QCD phase diagram at physical and unphysical quark masses

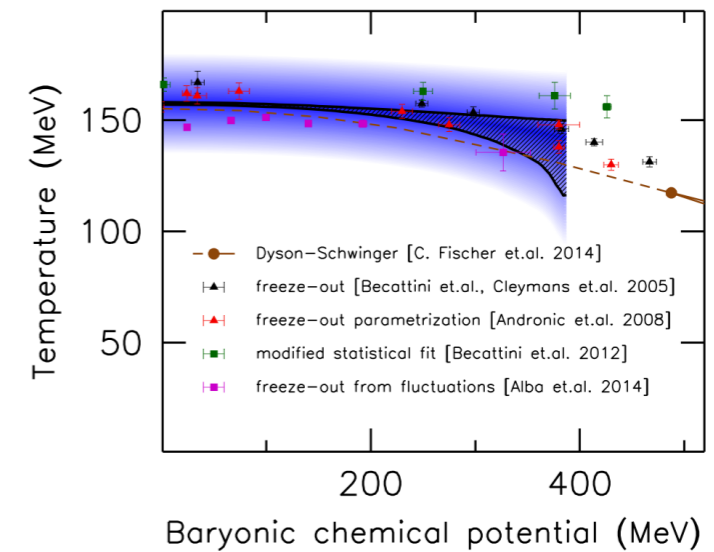
Gunkel, CF, Isserstedt, EPJA 55 (2019) no 9, 169
Gunkel, CF, EPJA 57 (2021) no 4, 147
Gunkel, CF, PRD 104 (2021) 5, 054022
Bernhardt, CF, arXiv: 2305.01434
Bernhardt, CF, in preparation

1. Introduction

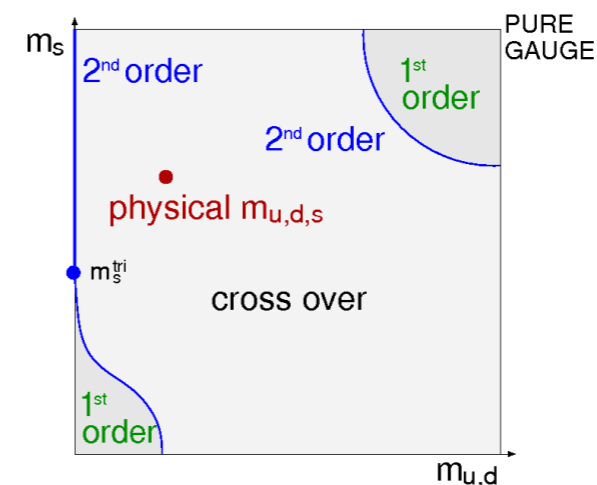
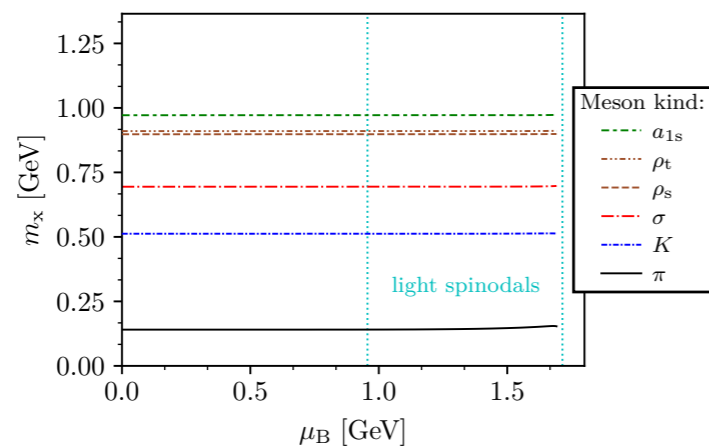


2. Mesons in vacuum

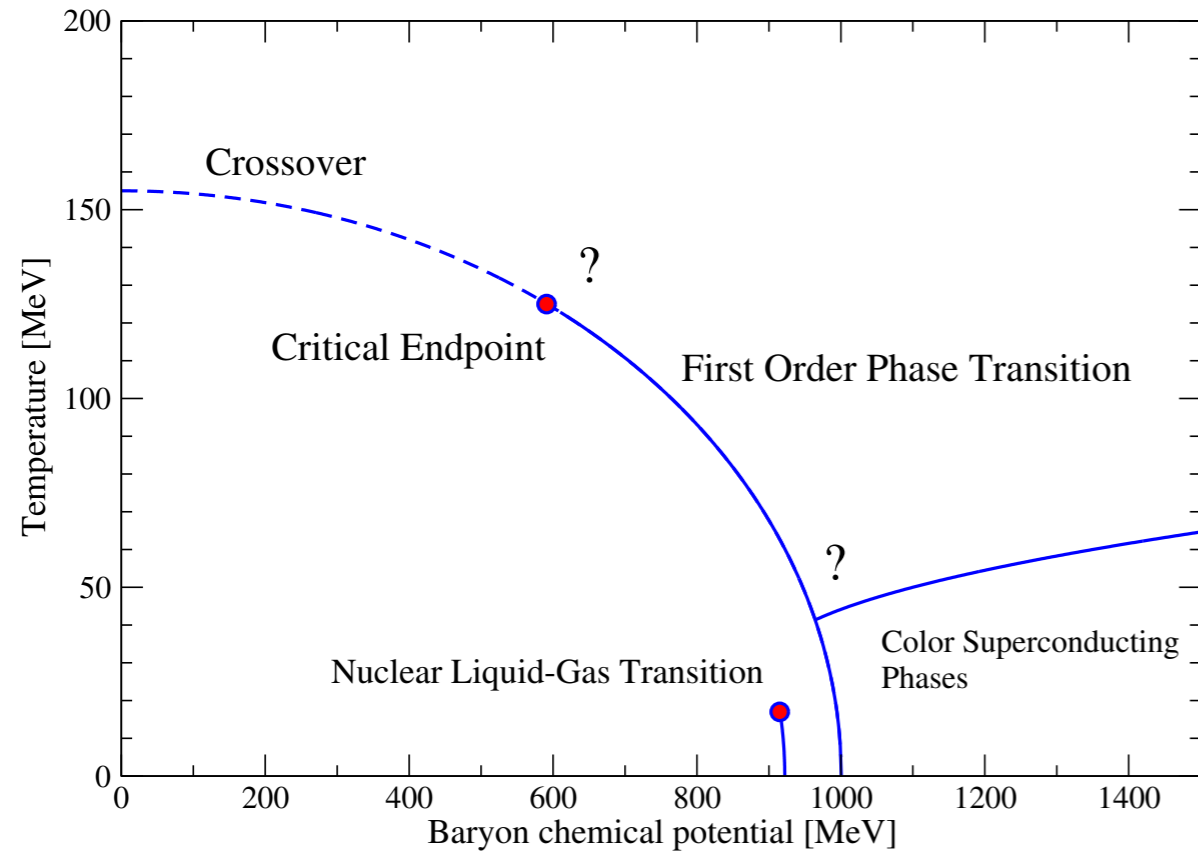
3. QCD phase diagram and the CEP



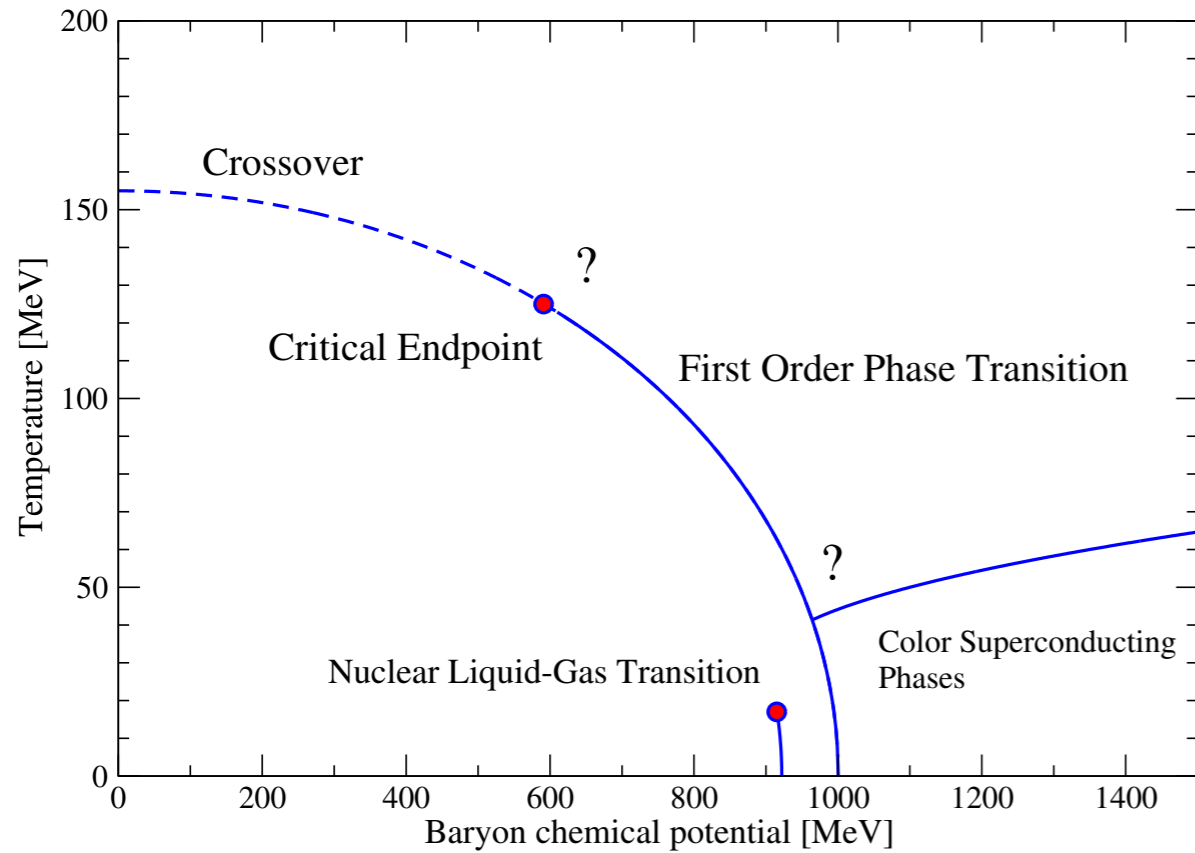
4. Mesons at finite μ , at the CEP and in the chiral limit



QCD phase transitions: fun fact

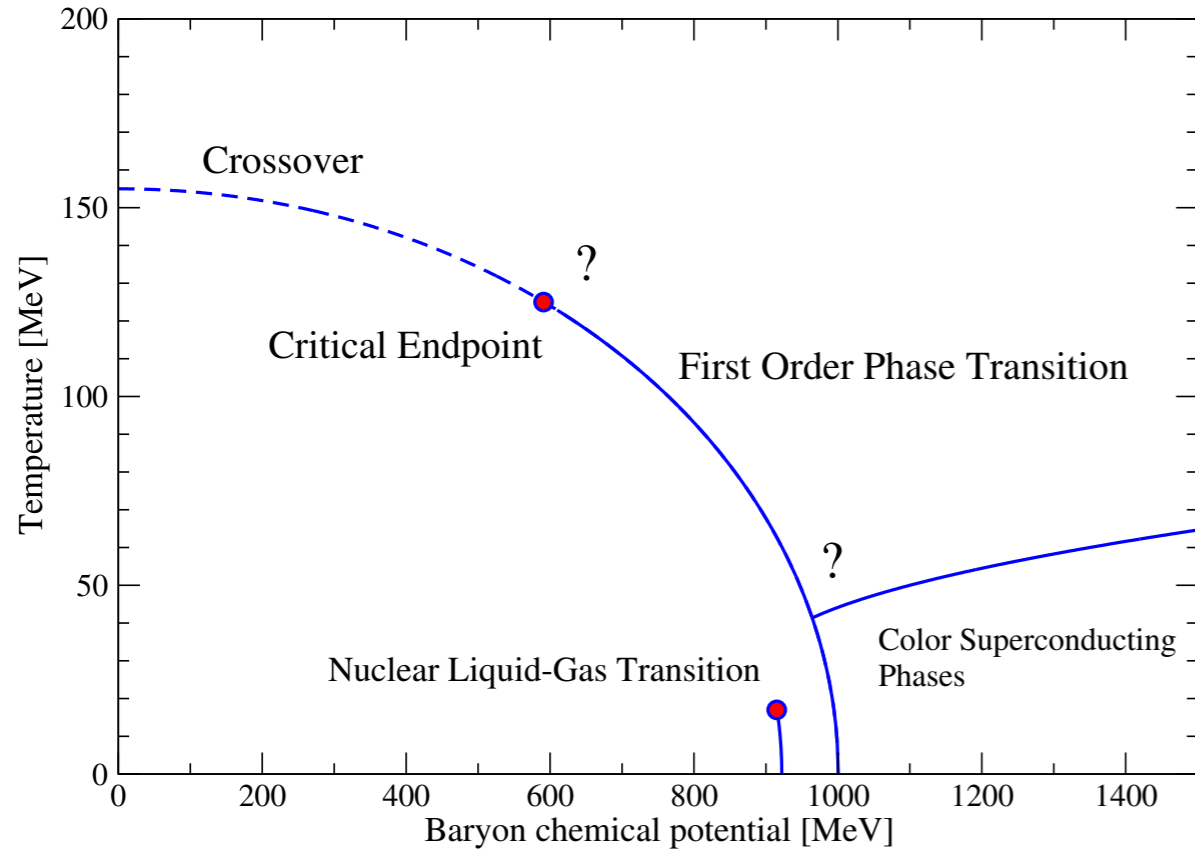


QCD phase transitions: fun fact



$$Z\left(\frac{\mu_B}{T}\right) = Z\left(-\frac{\mu_B}{T}\right)$$

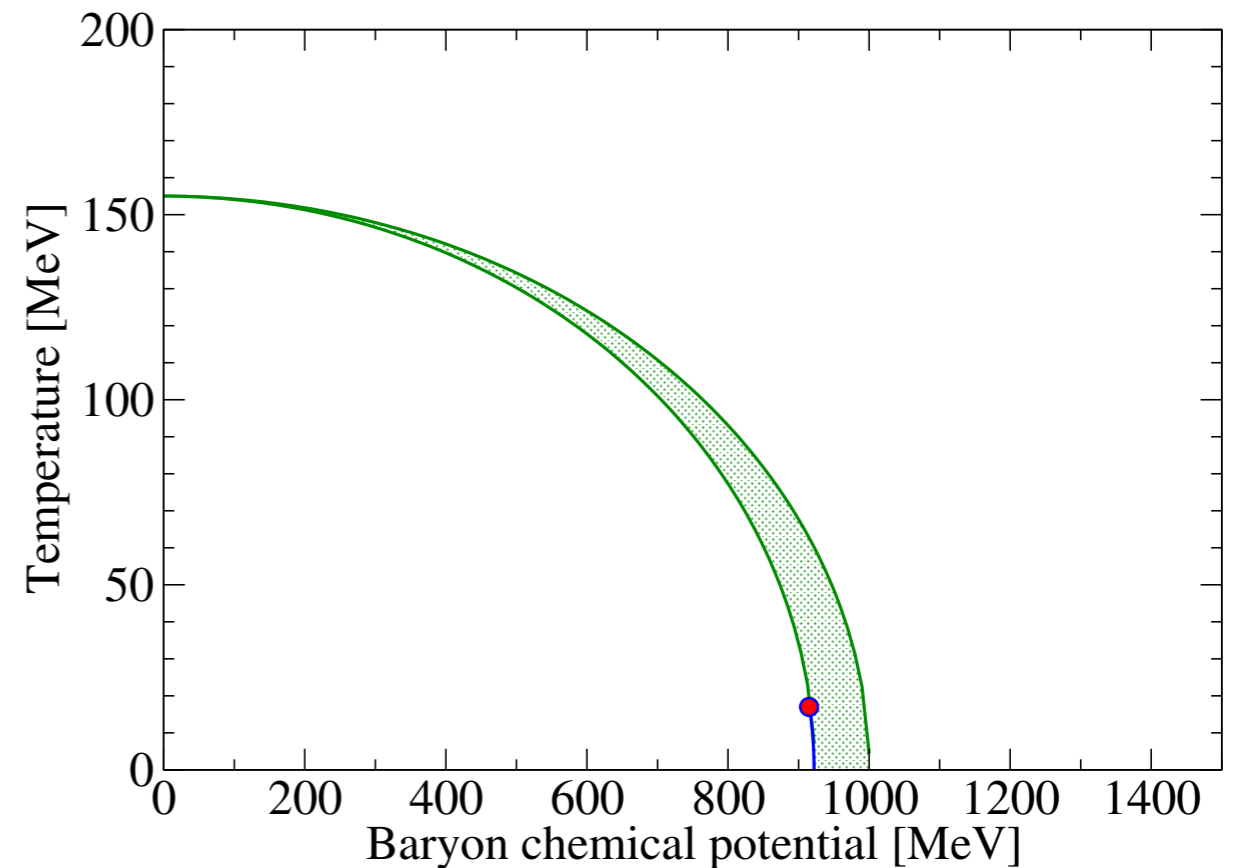
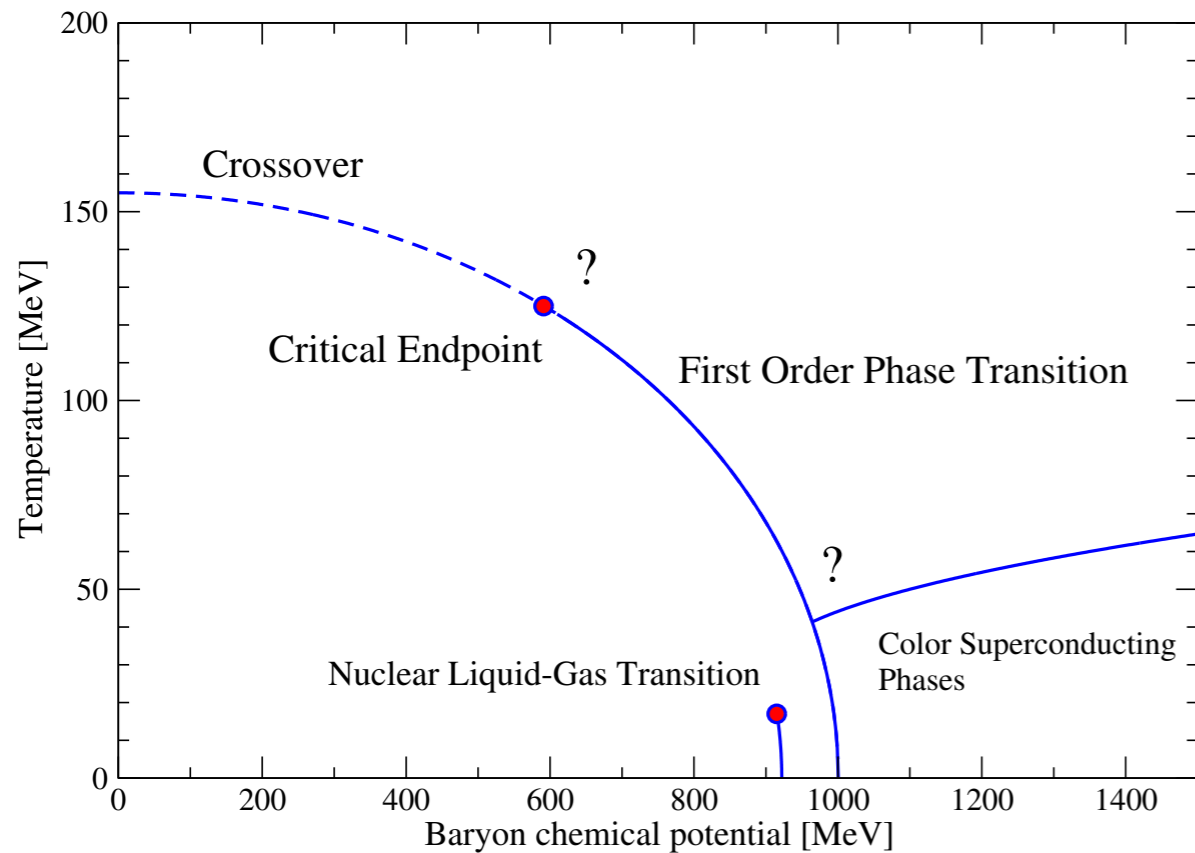
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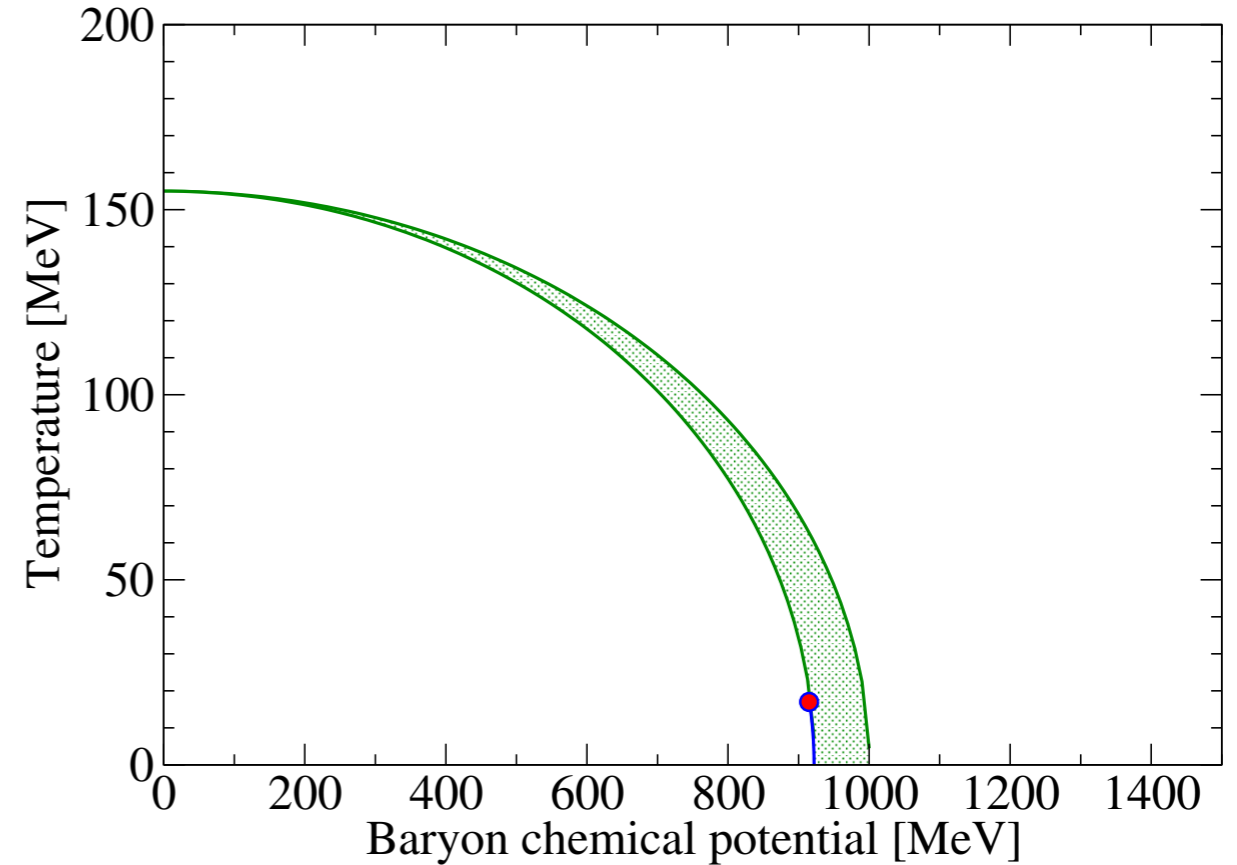
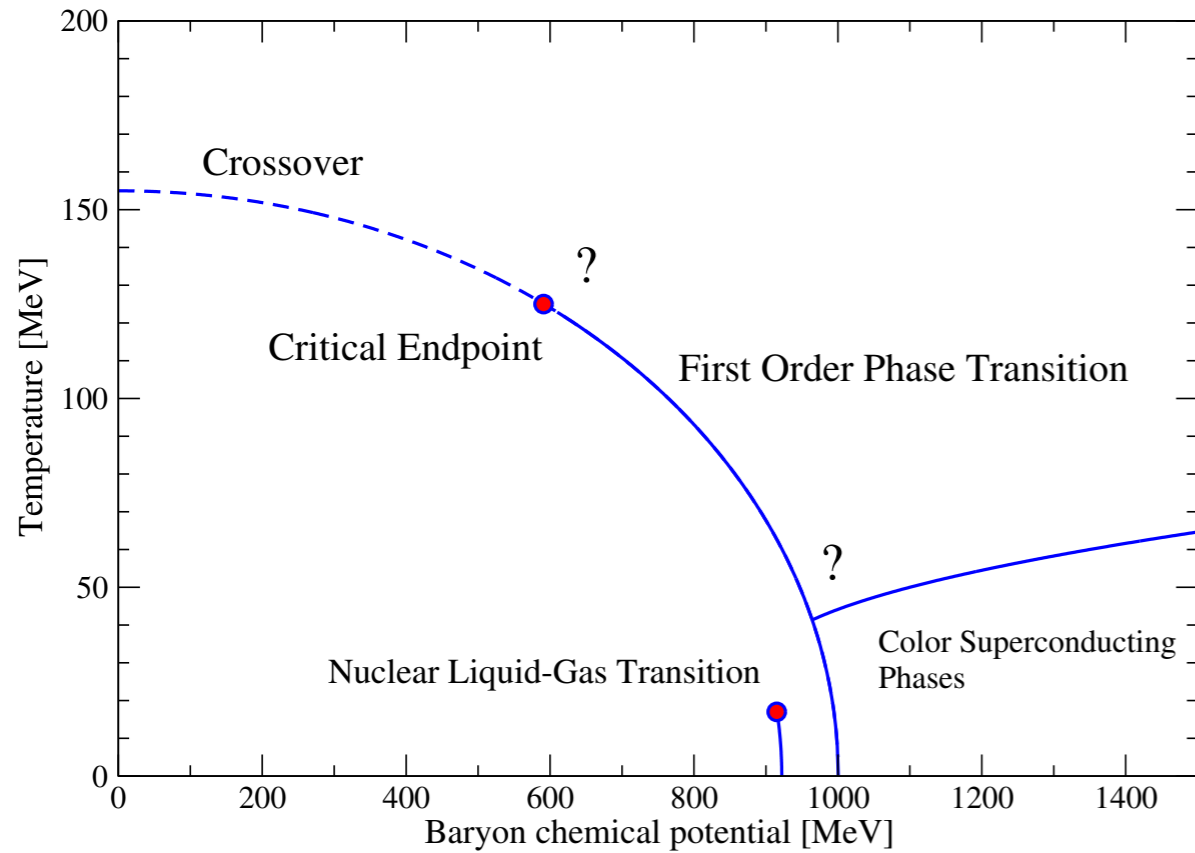


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$$\mu_B^{lg} \approx 922 \text{ MeV} \rightarrow \kappa \leq 0.0141$$

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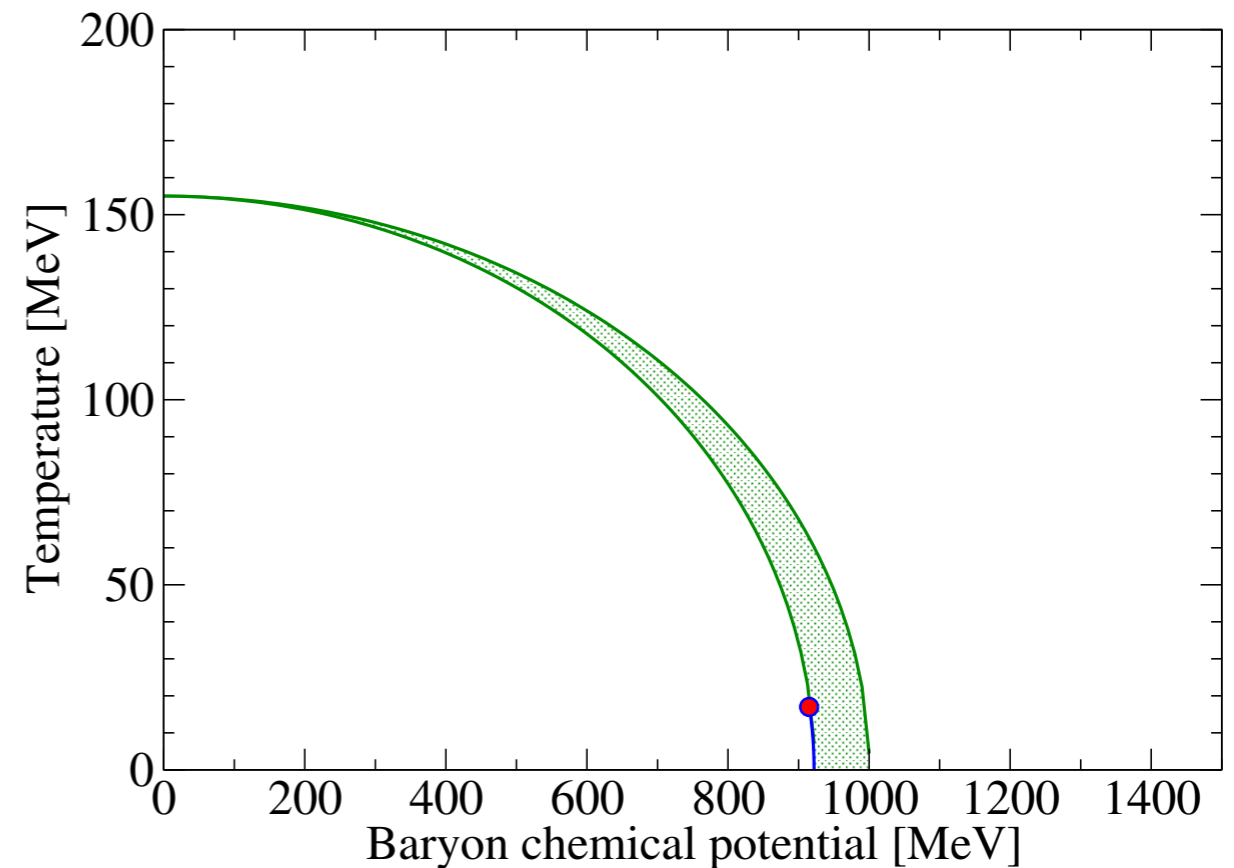
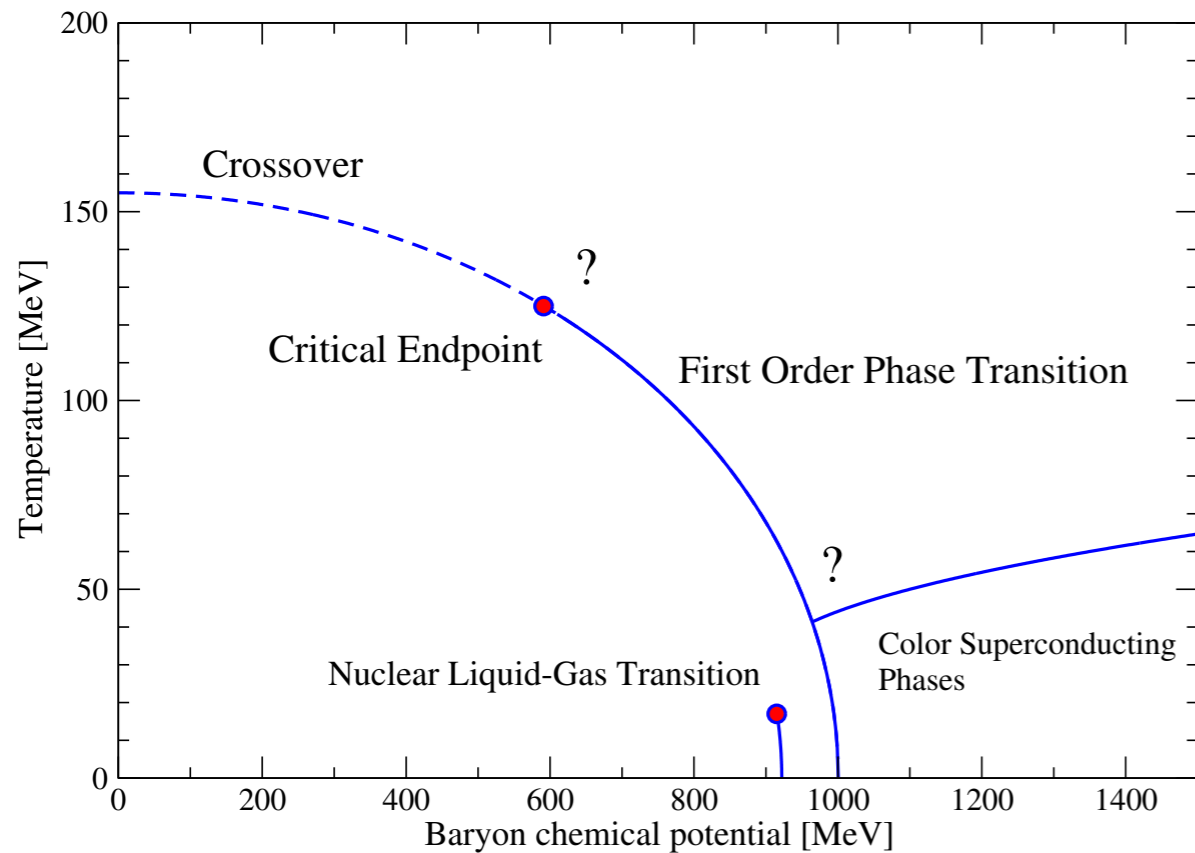
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Lattice QCD:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 - \lambda\left(\frac{\mu_B}{T_c}\right)^4 \dots,$$

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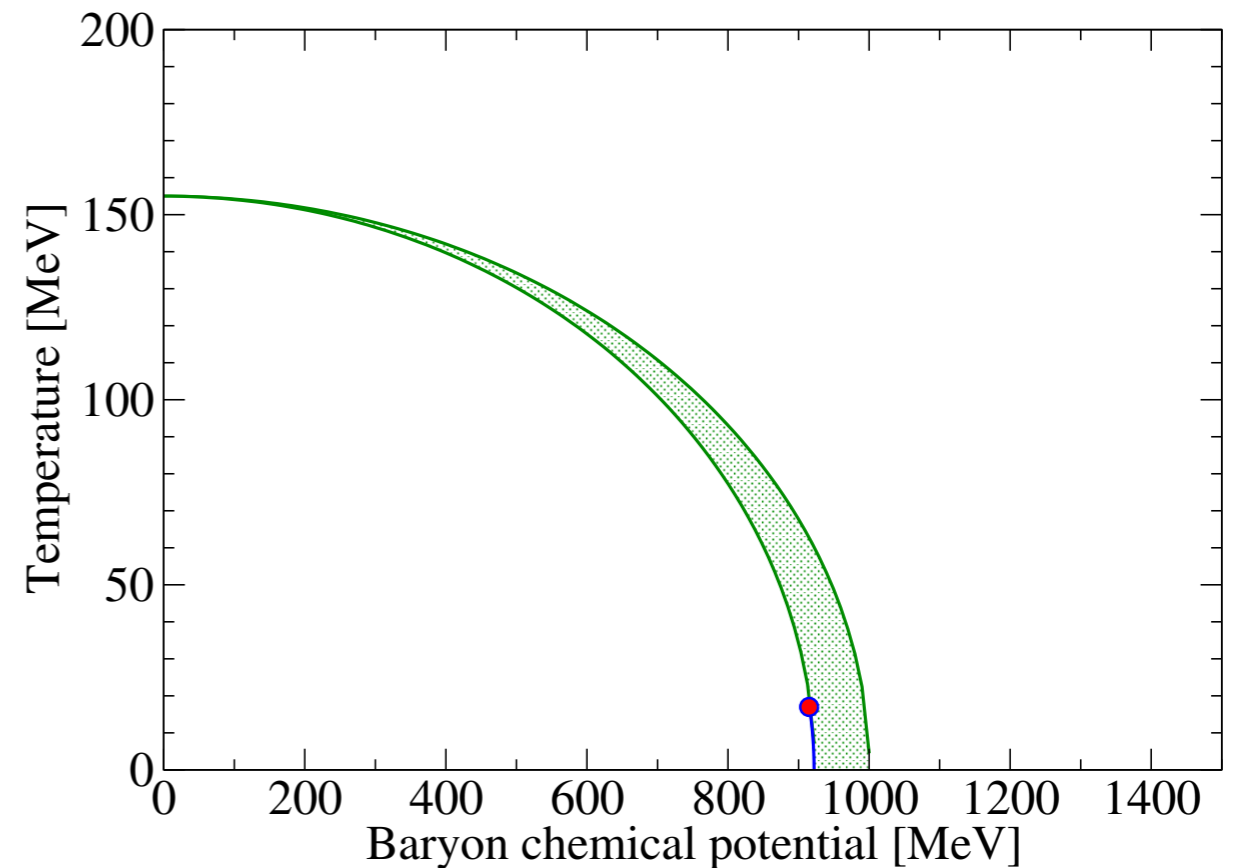
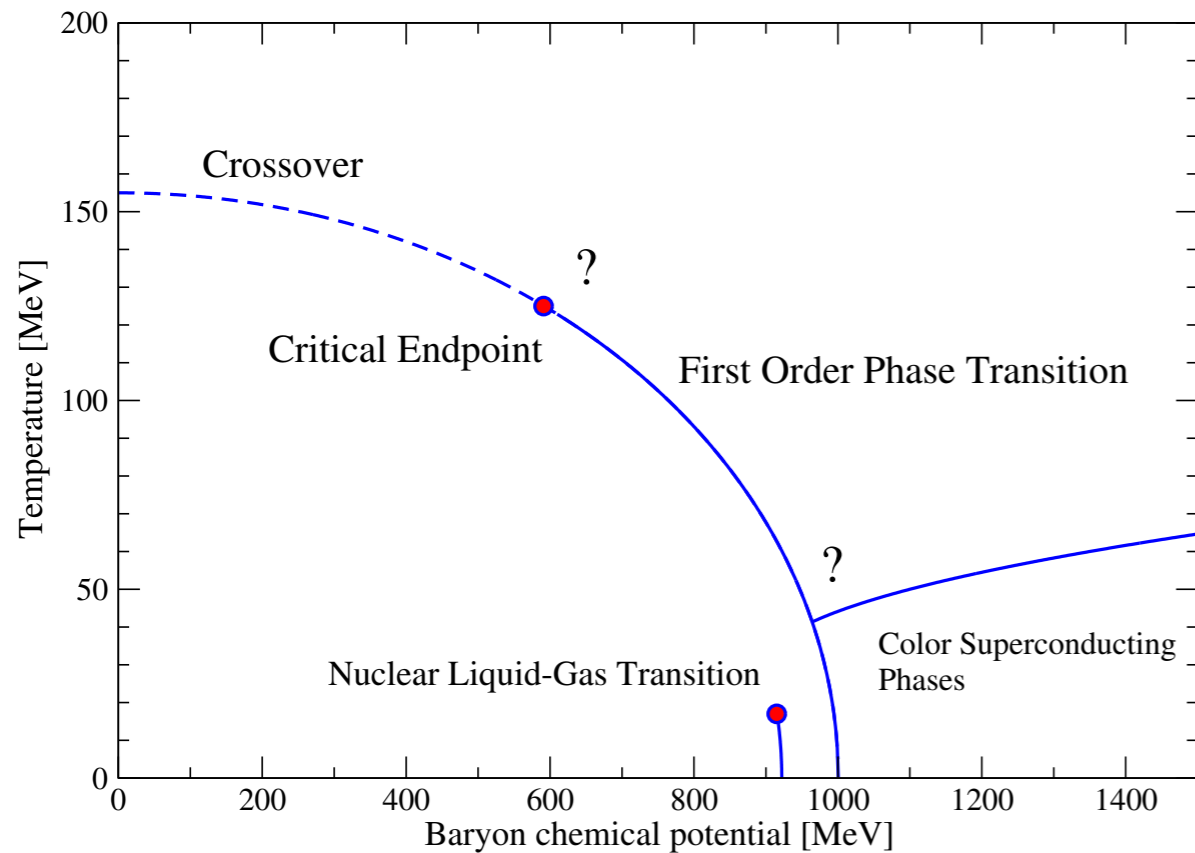
$$\rightarrow \lambda = \frac{\kappa^2}{2} \leq 0.0001$$

CF, PPNP 105 (2019) [1810.12938]

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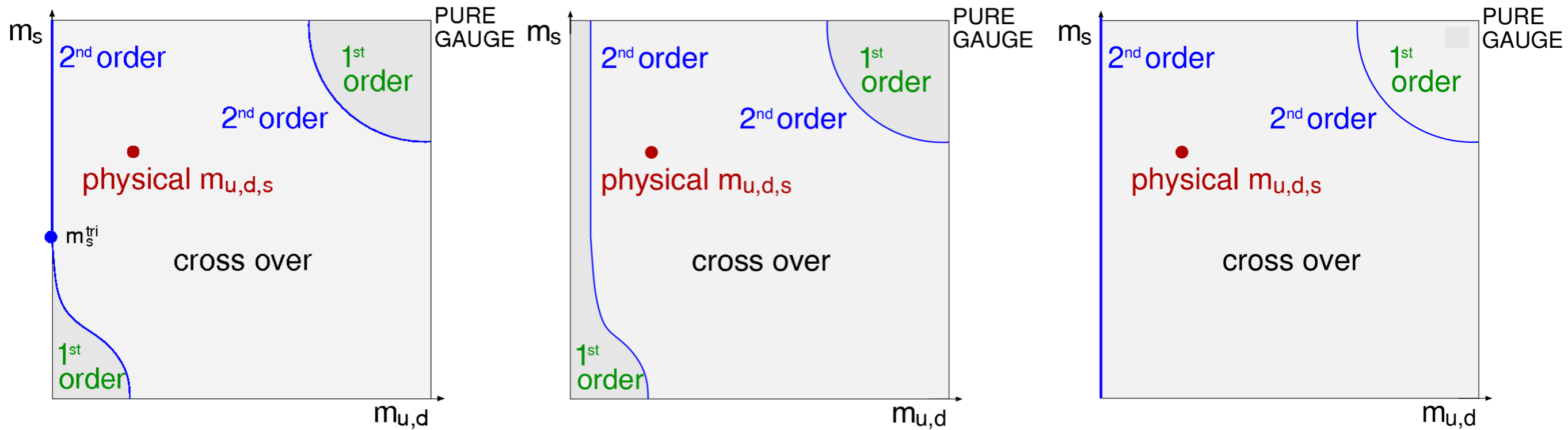
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$$\kappa = \begin{cases} 0.0145(25) & \text{Bonati et al., PRD 98 (2018)} \\ 0.0120(40) & \text{Bazavov et al., PLB 795 (2018)} \\ 0.0153(18) & \text{Borsanyi et al., PRL 125 (2020)} \end{cases}$$

$$\lambda = \begin{cases} 0.000(4) & \text{Bazavov et al., PLB 795 (2018)} \\ 0.00032(67) & \text{Borsanyi et al., PRL 125 (2020)} \end{cases}$$

QCD phase transitions



$U_A(1)$ broken at T_c

$U_A(1)$ restored at T_c

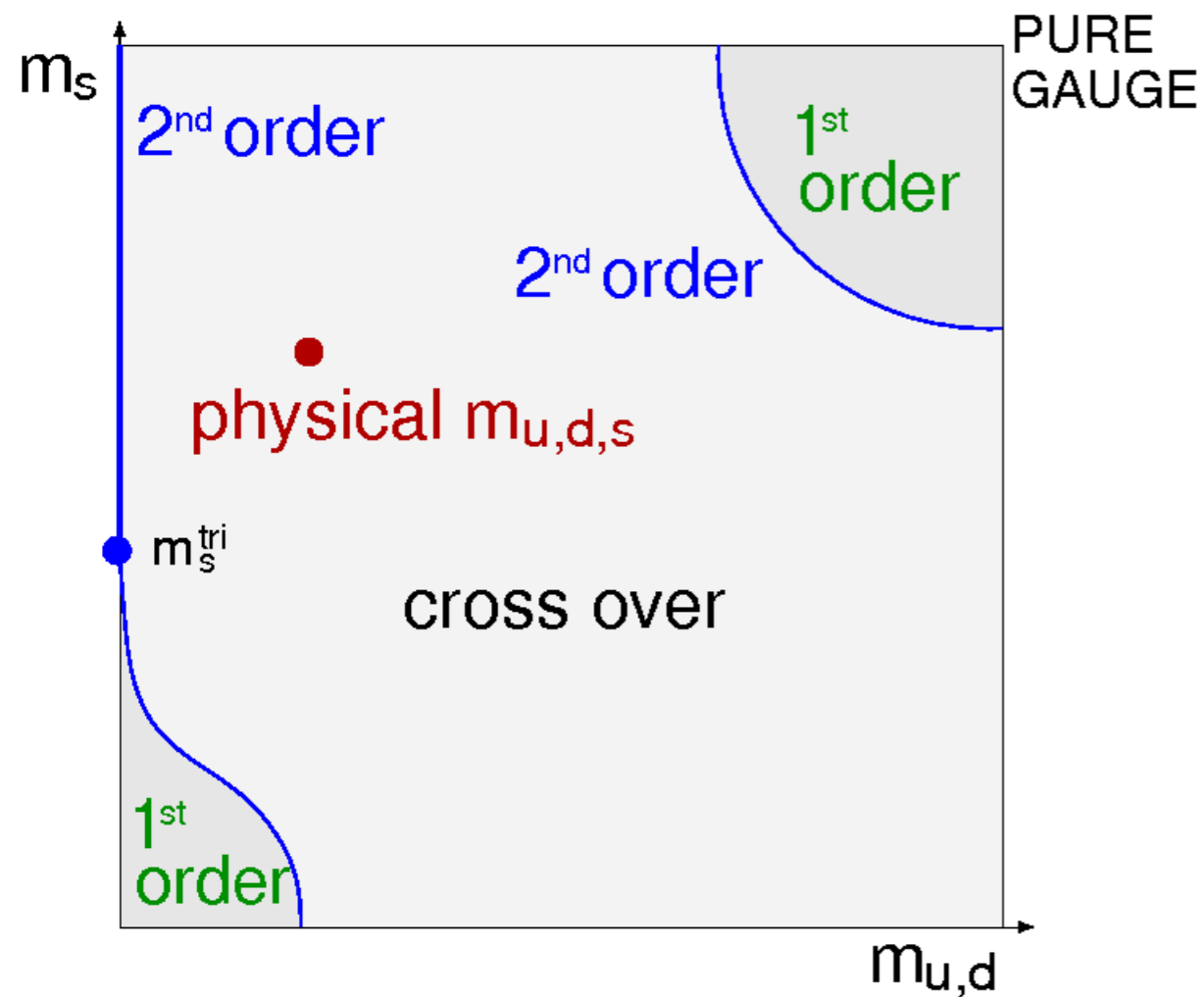
Is there chiral 1st order at all?

Pisarski and Wilczek, PRD 29 (1984), 338-341
Resch, Rennecke and Schaefer, PRD 99 (2019)

Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141
Dini, et al, PRD 105 (2022) no.3, 034510
Fejos, PRD 105 (2022) no.7, L071506

and many more...

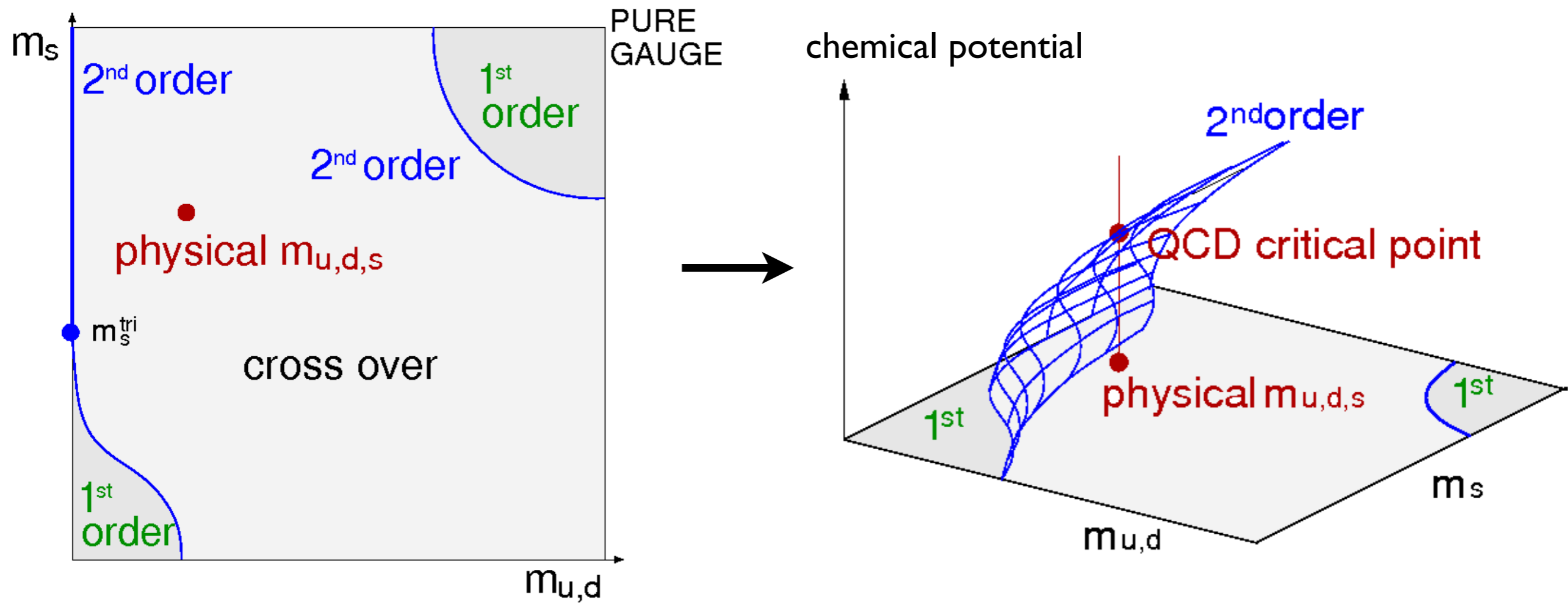
QCD phase transitions



Is this happening ??
Maybe yes, maybe not..

de Forcrand, Philipsen, JHEP 0811 (2008) 012;
NPB 642 (2002) 290

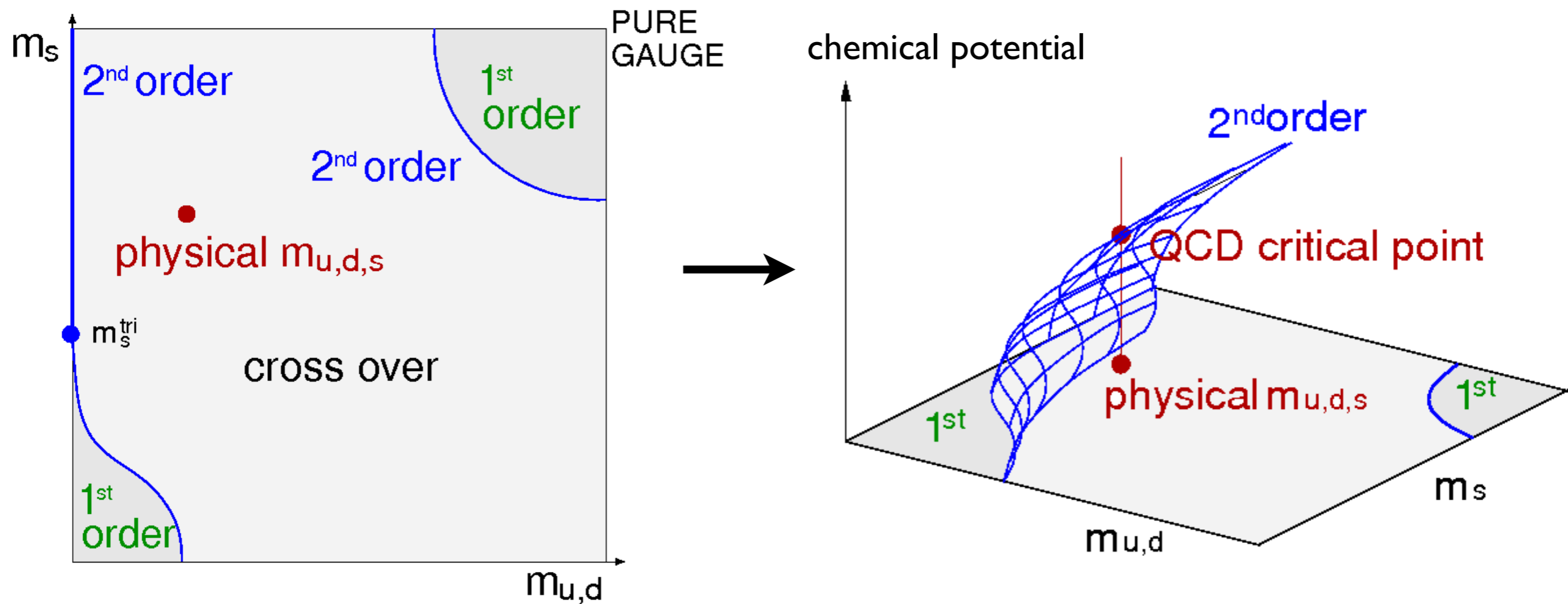
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QCD phase transitions



- Lattice-QCD

- present: extrapolation
- future: exact methods?

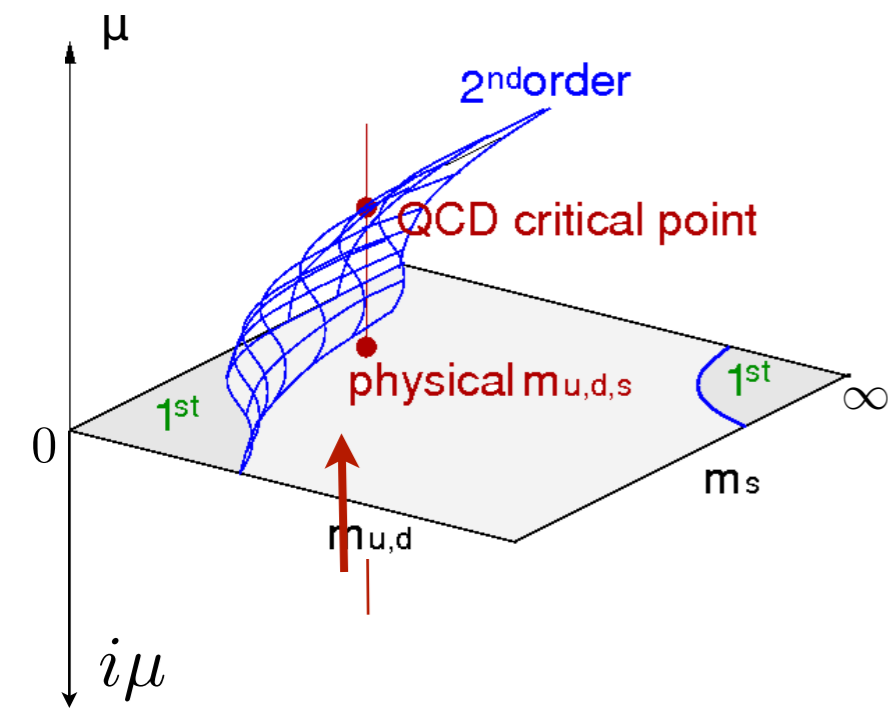
- DSE/FRG

- can do! but typical errors 5-30%

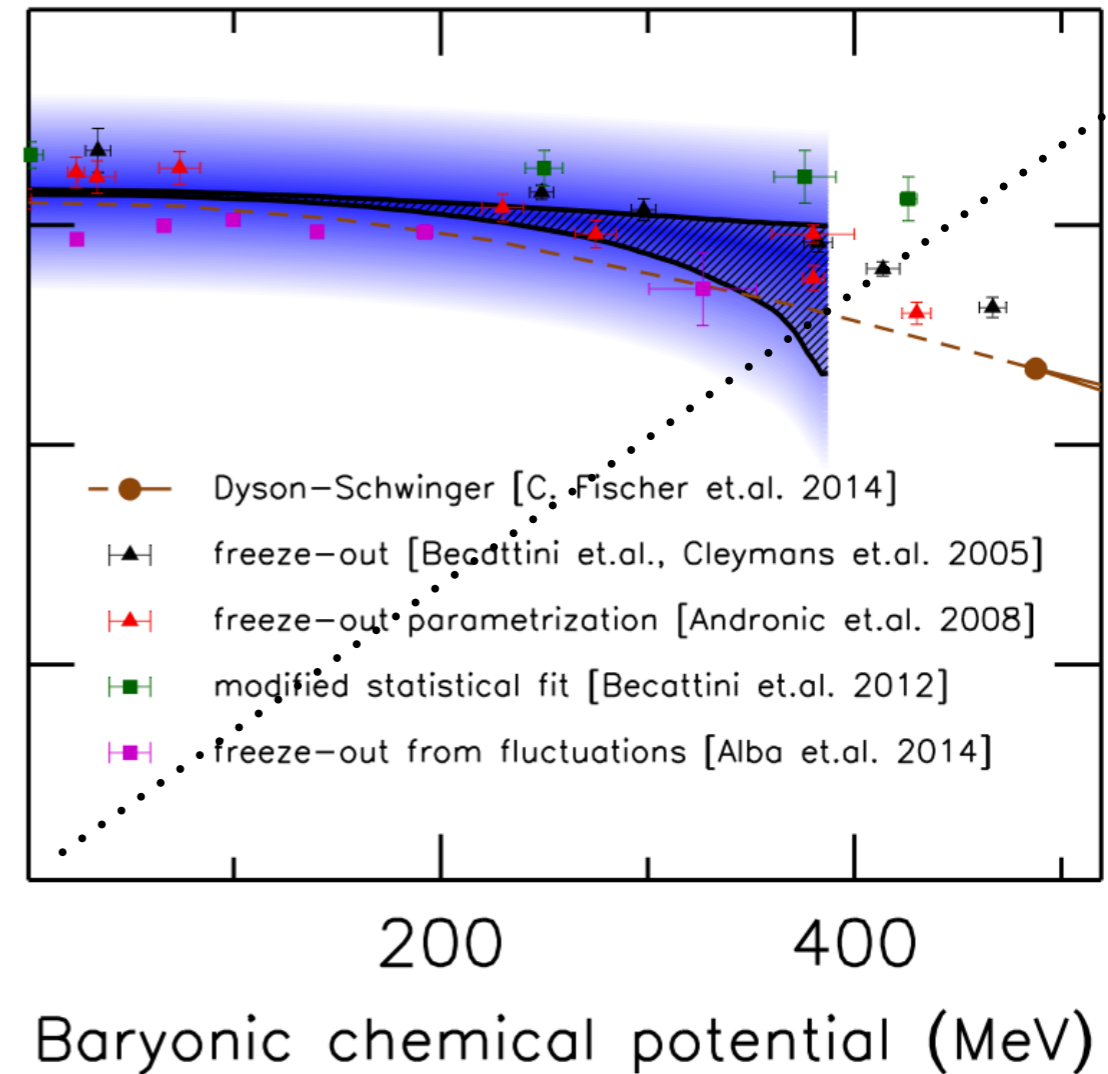
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NPB 642 (2002) 290

Chiral transition line from analytic continuation



Temperature (MeV)



Bellwied, Borsanyi, Fodor, Günther,
Katz, Ratti and Szabo, PLB 751 (2015) 559

HOT-QCD: similar results

Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

Main result:

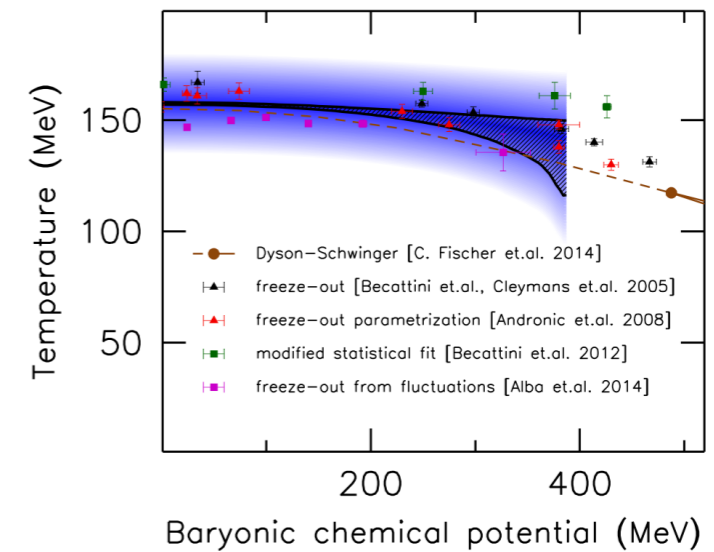
- See no transition for $\mu_B/T < 2-3$

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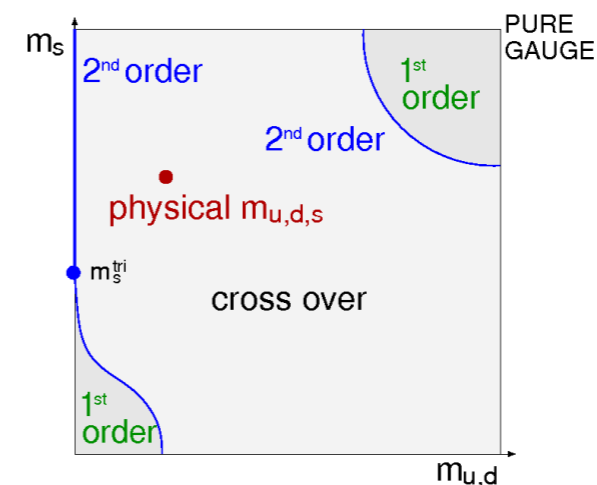
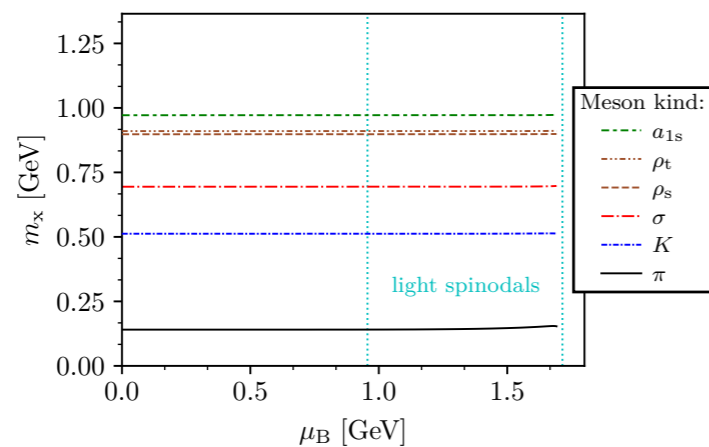


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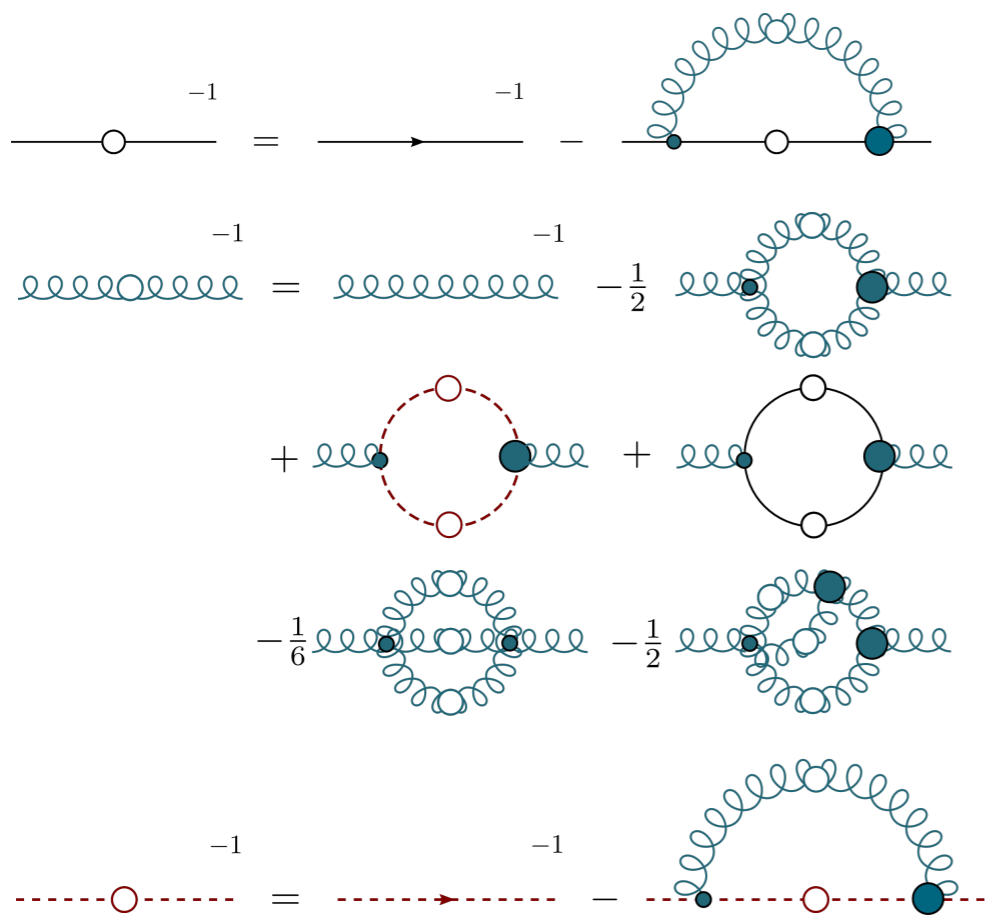


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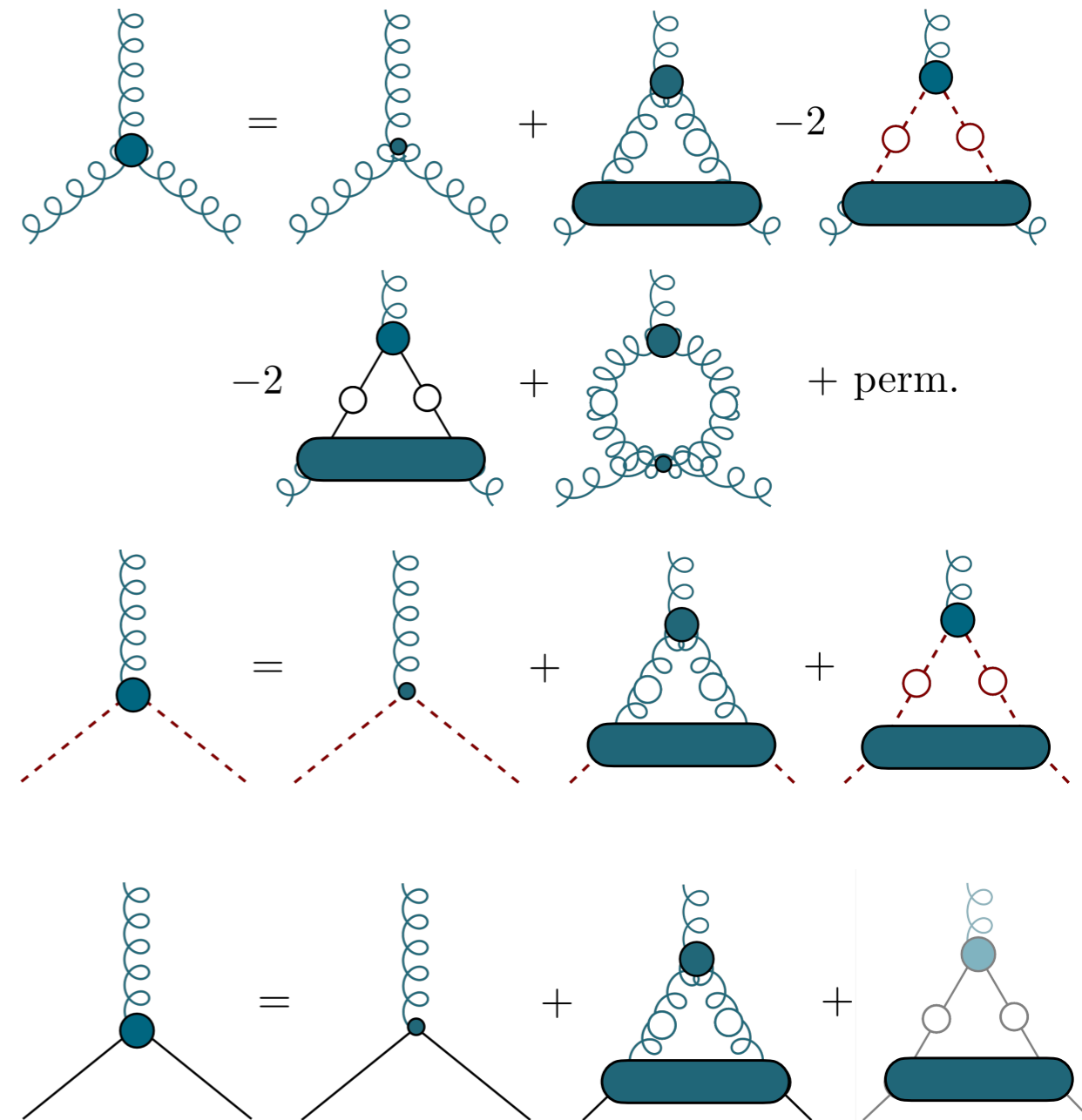
3PI-truncation ($T=0, \mu=0$)

propagators



for different BRL approaches see work of
 Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister,
 Gao, Huber, Maas, Mitter, Papavassiliou, Pawłowski, Roberts, Smekal,
 Strodthoff, Vujanovic, Watson, Williams...

vertices

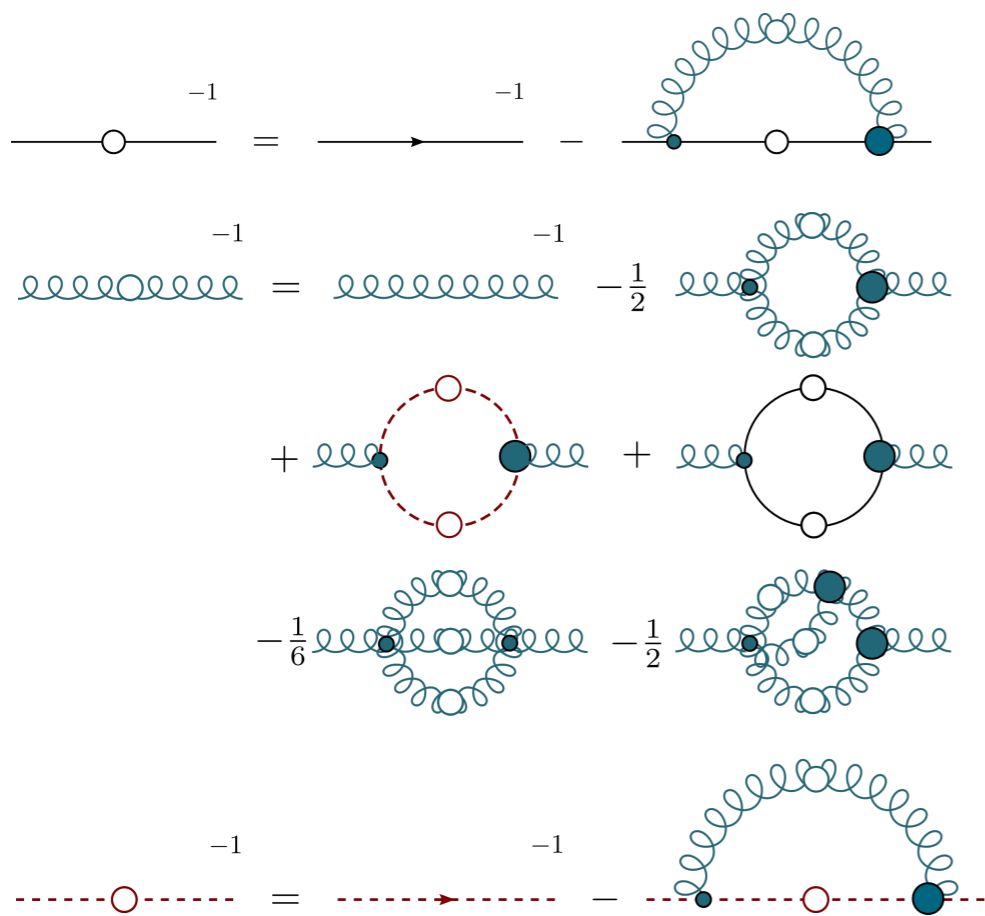


CF, Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

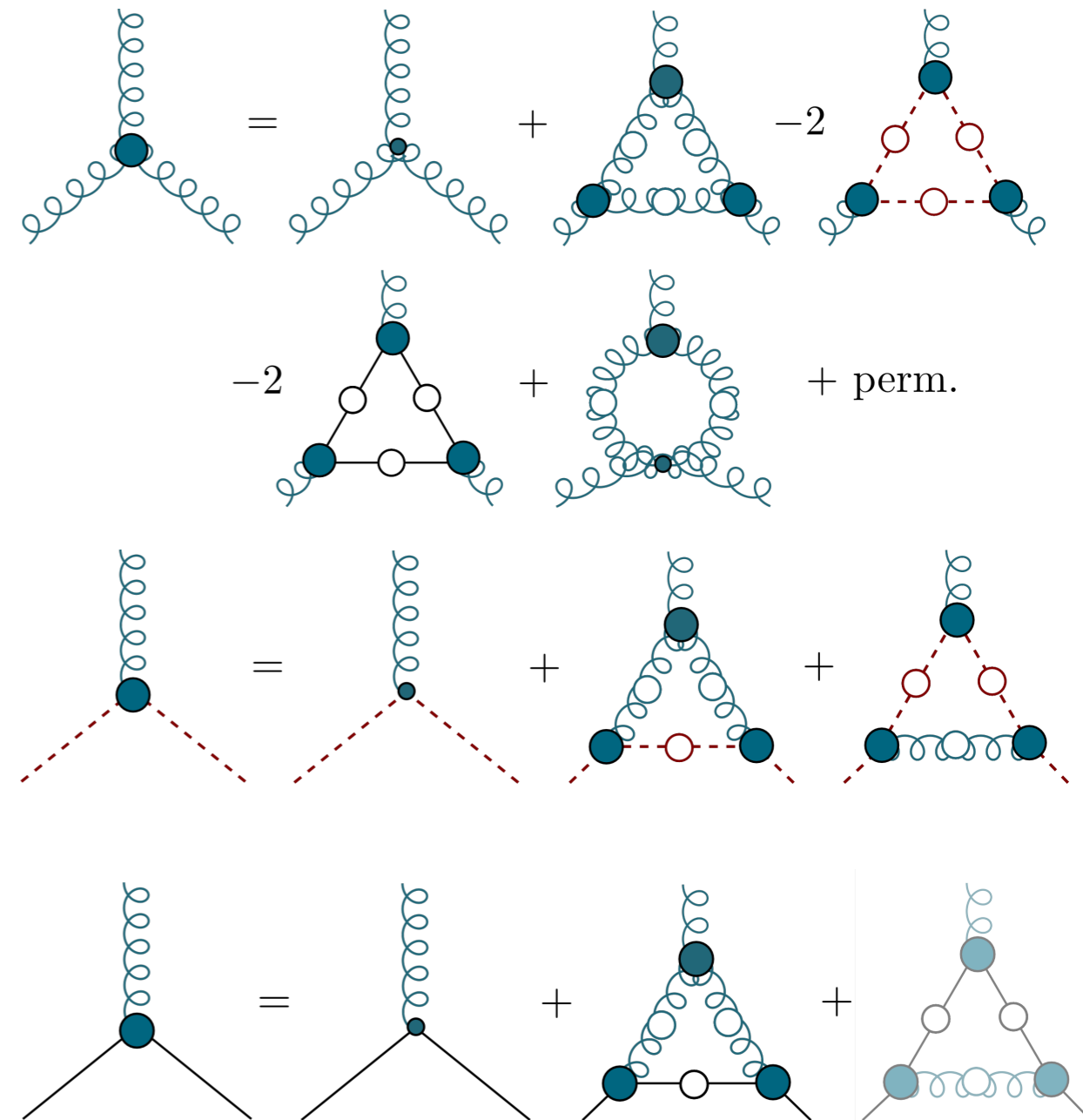
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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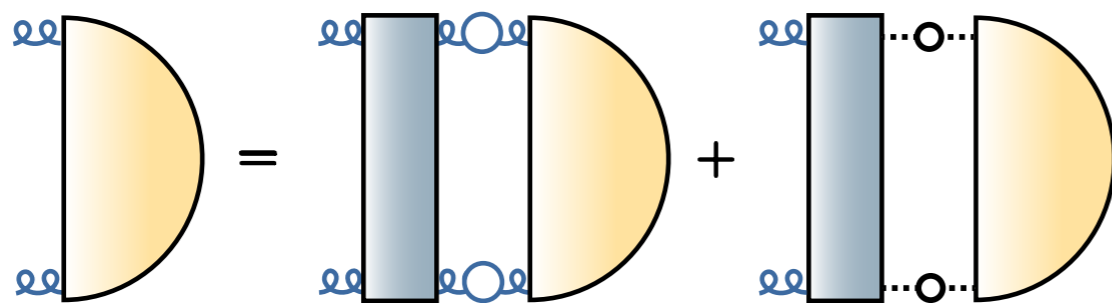
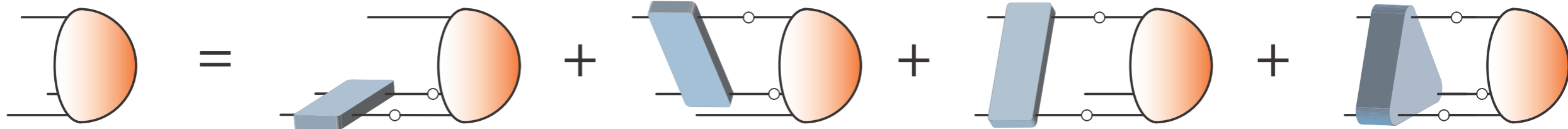
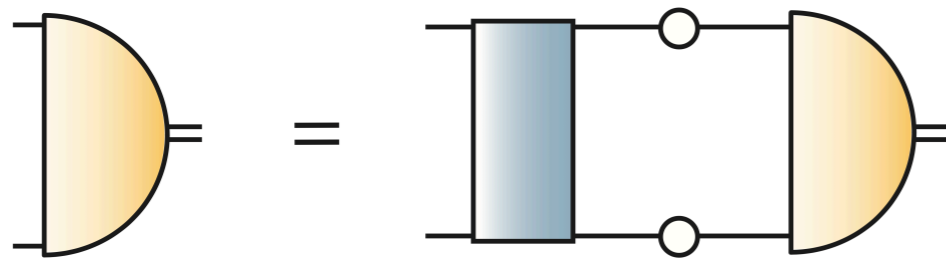
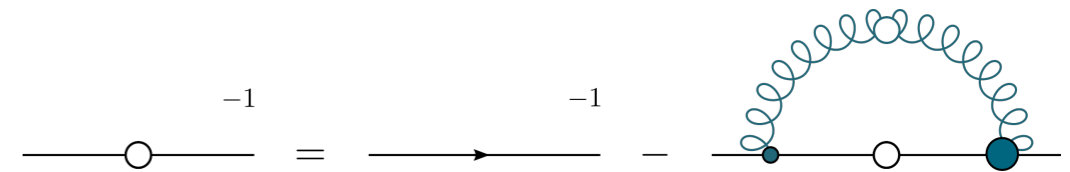
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Bound states and Bethe-Salpeter equations

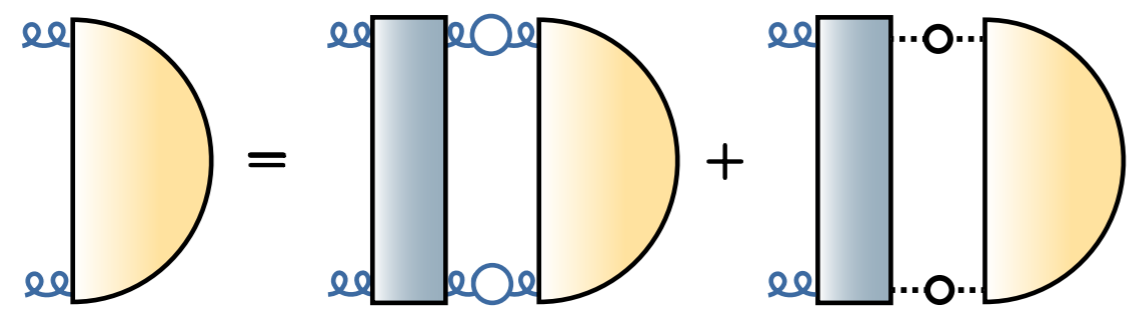
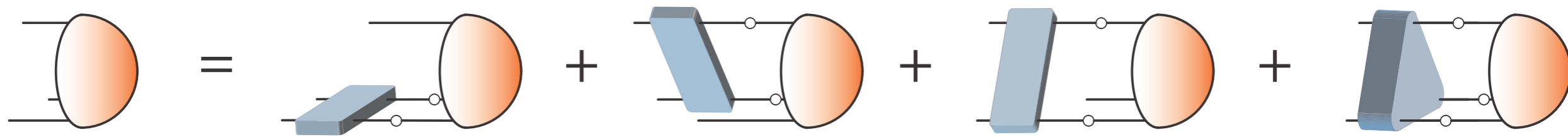
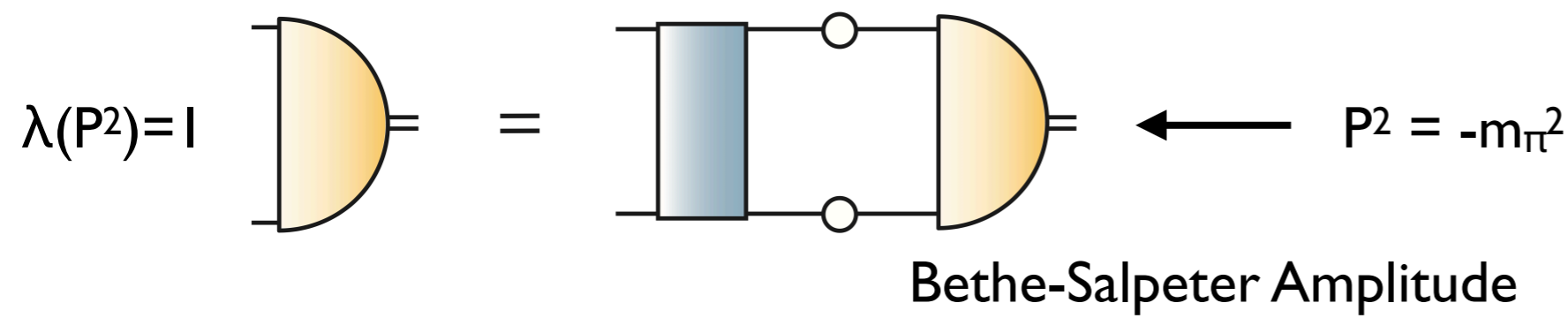
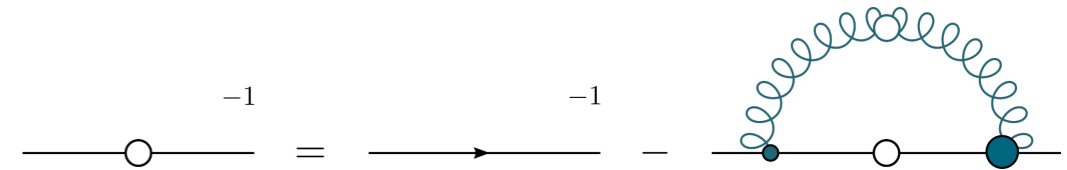
BSEs:



Eigenvalue equations: masses and wave functions

Bound states and Bethe-Salpeter equations

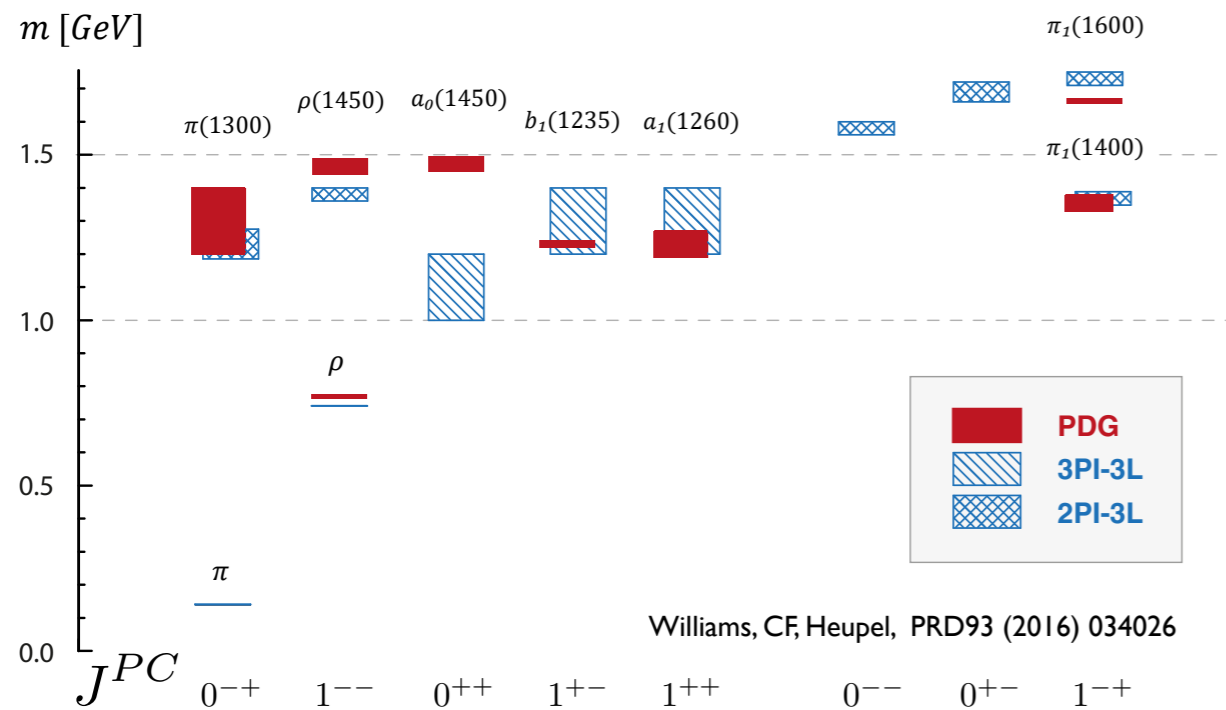
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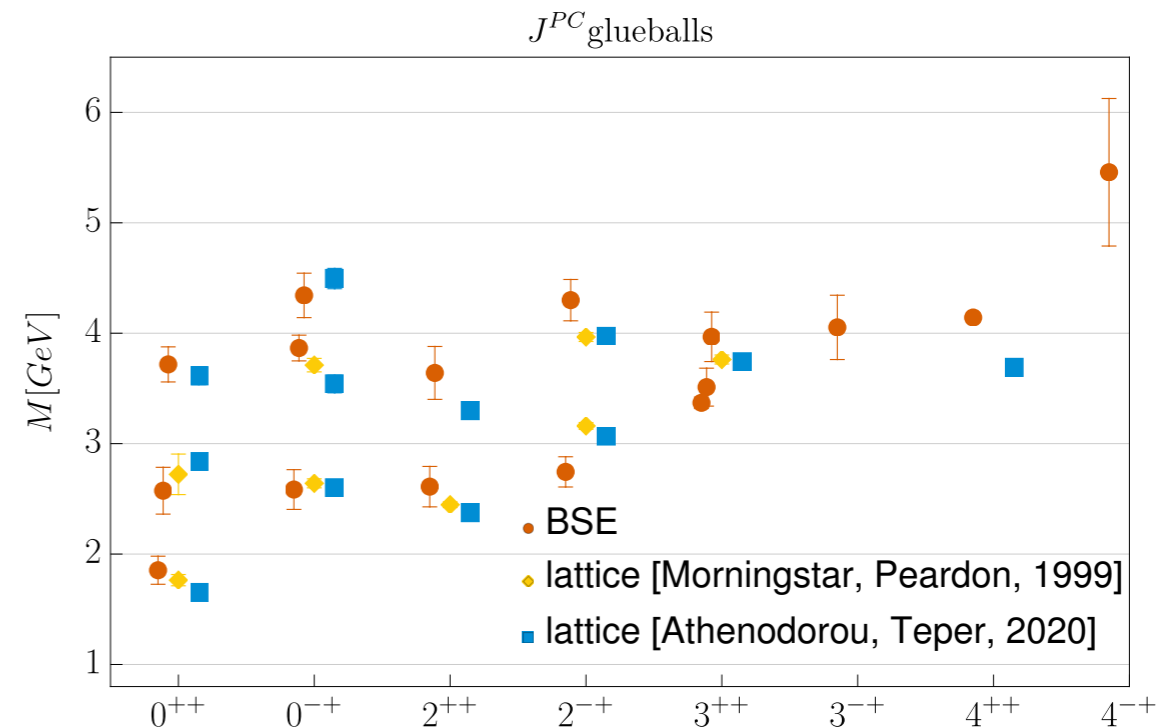
Eigenvalue equations: masses and wave functions

Hadron spectra: mesons, baryons, glueballs

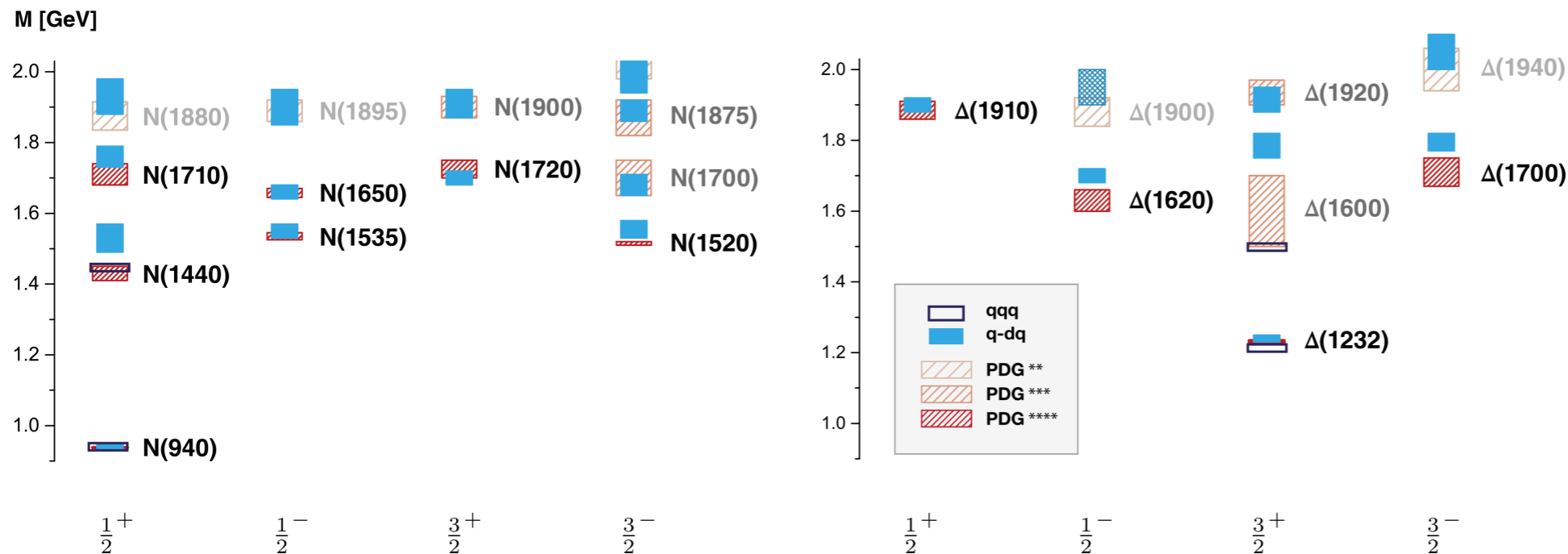
Mesons:



Glueballs:

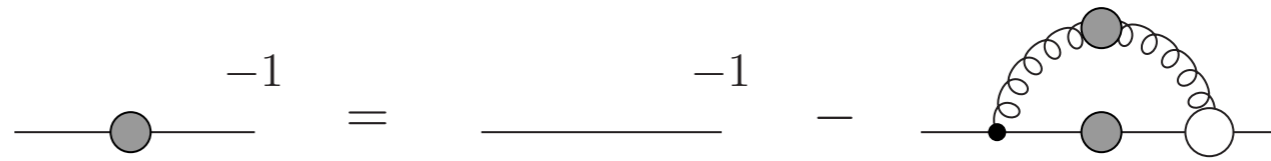


Baryons:



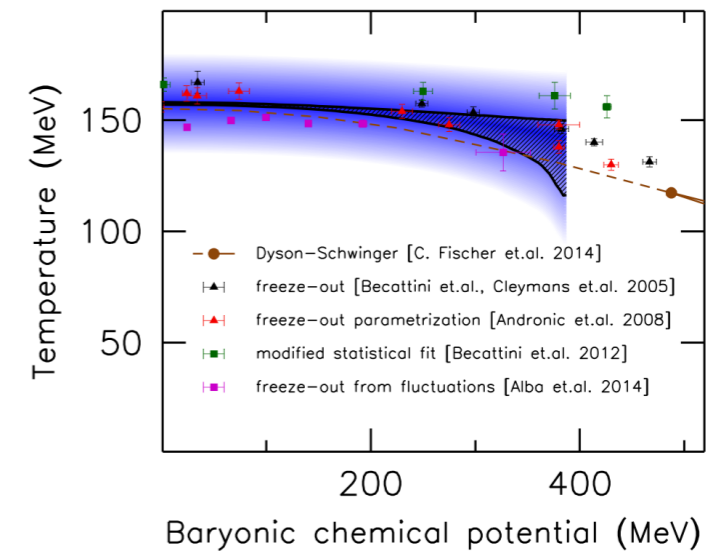
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

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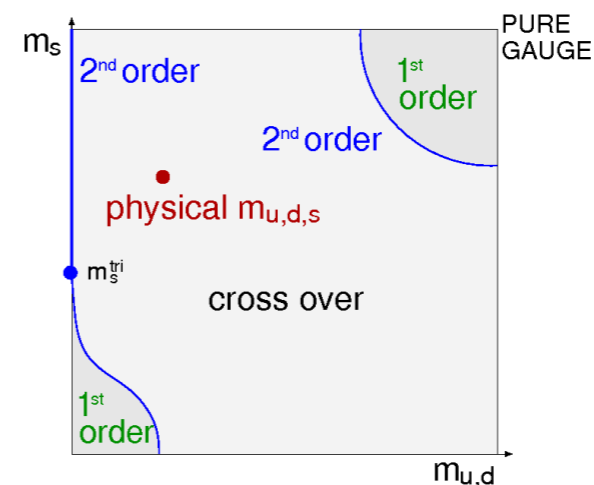
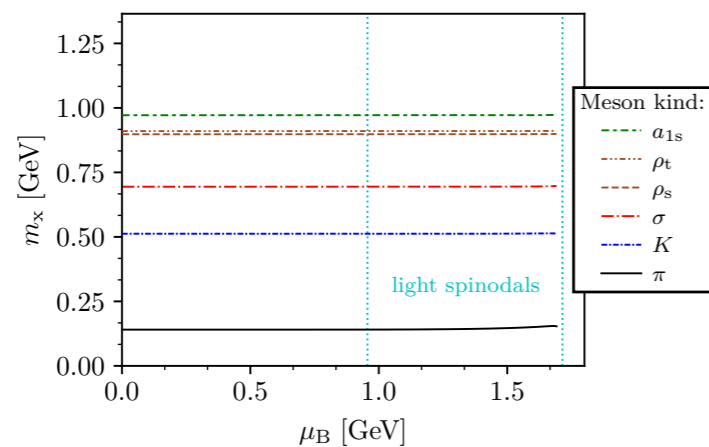


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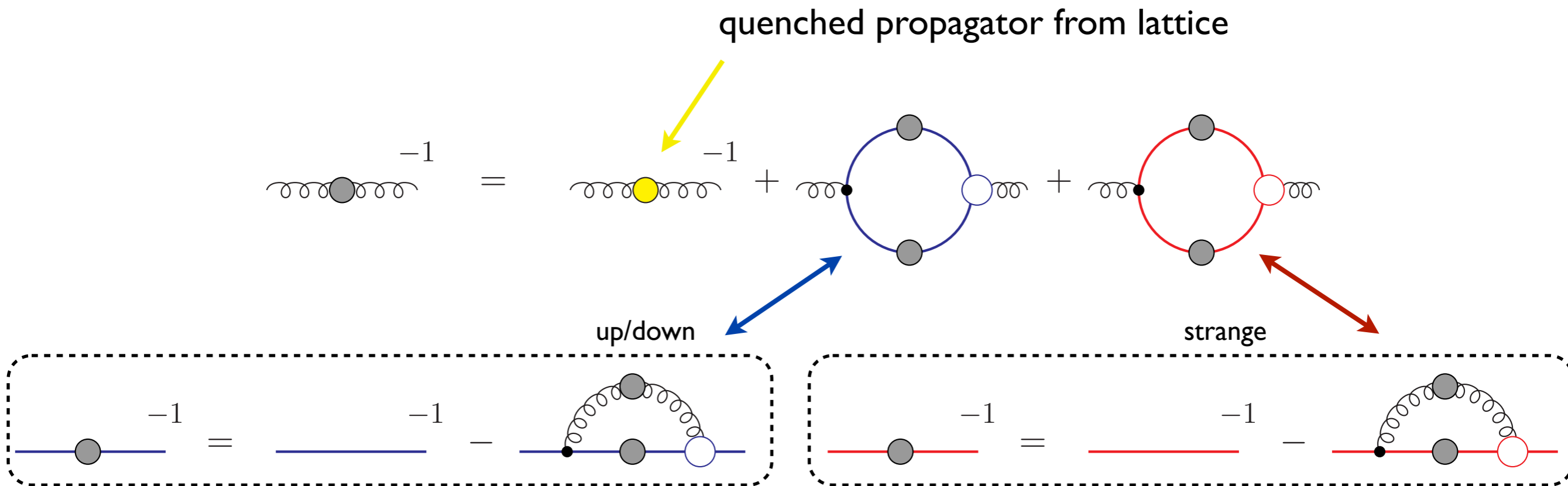
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$N_f=2+1$ -QCD with DSEs

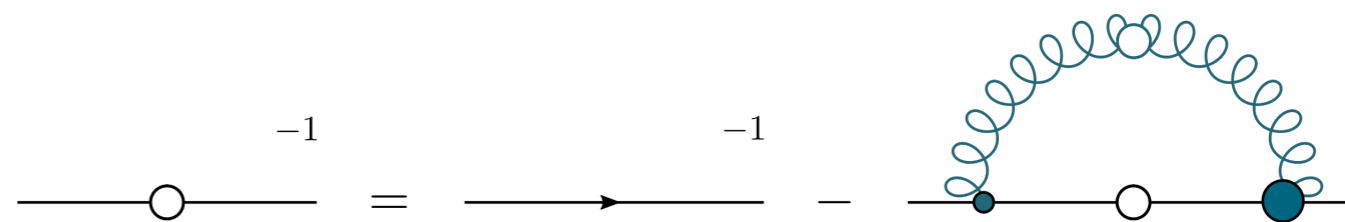


$$S^{-1}(\omega_p, \vec{p}) = i\vec{p} A(\omega_p, \vec{p}) + i\gamma_4 \omega_p C(\omega_p, \vec{p}) + B(\omega_p, \vec{p})$$

- quenched: without quark-loop
- $N_f=2$: isospin symmetry $m_{u/d}$ fixed by m_π
- $N_f=2+1$: coupled system of 2+3+3 equations
- Vertex: ansatz built along STI and known UV/IR behavior
→ T, μ, m -dependent

Chiral order parameter:

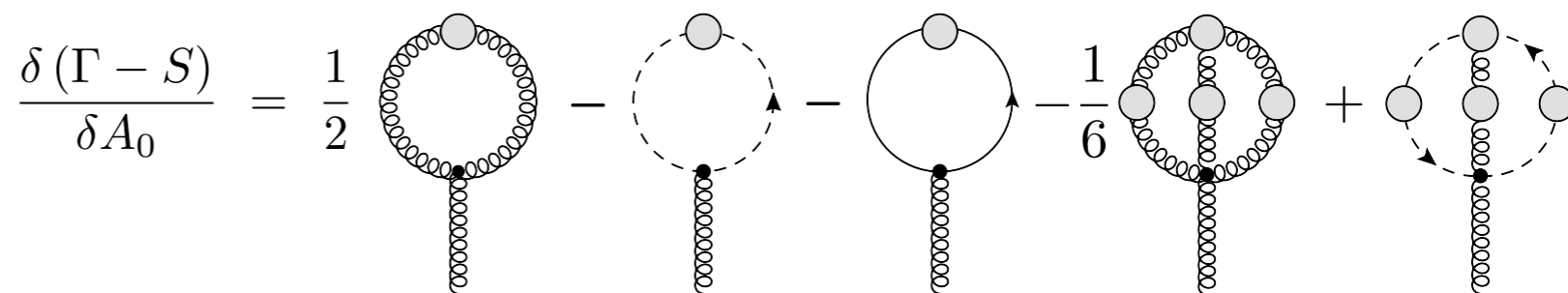
$$\langle \bar{\Psi} \Psi \rangle = Z_2 N_c \text{Tr}_D \frac{1}{T} \sum_{\omega} \int \frac{d^3 p}{(2\pi)^3} S(\vec{p}, \omega)$$



Deconfinement:

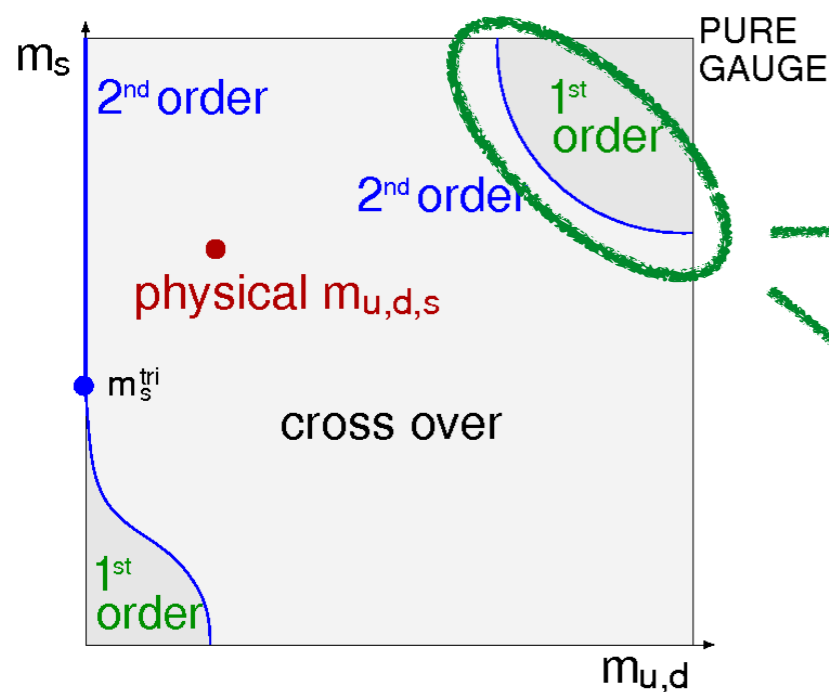
- Polyakov loop potential

$$L = \frac{1}{N_c} \text{Tr} e^{ig\beta A_0}$$

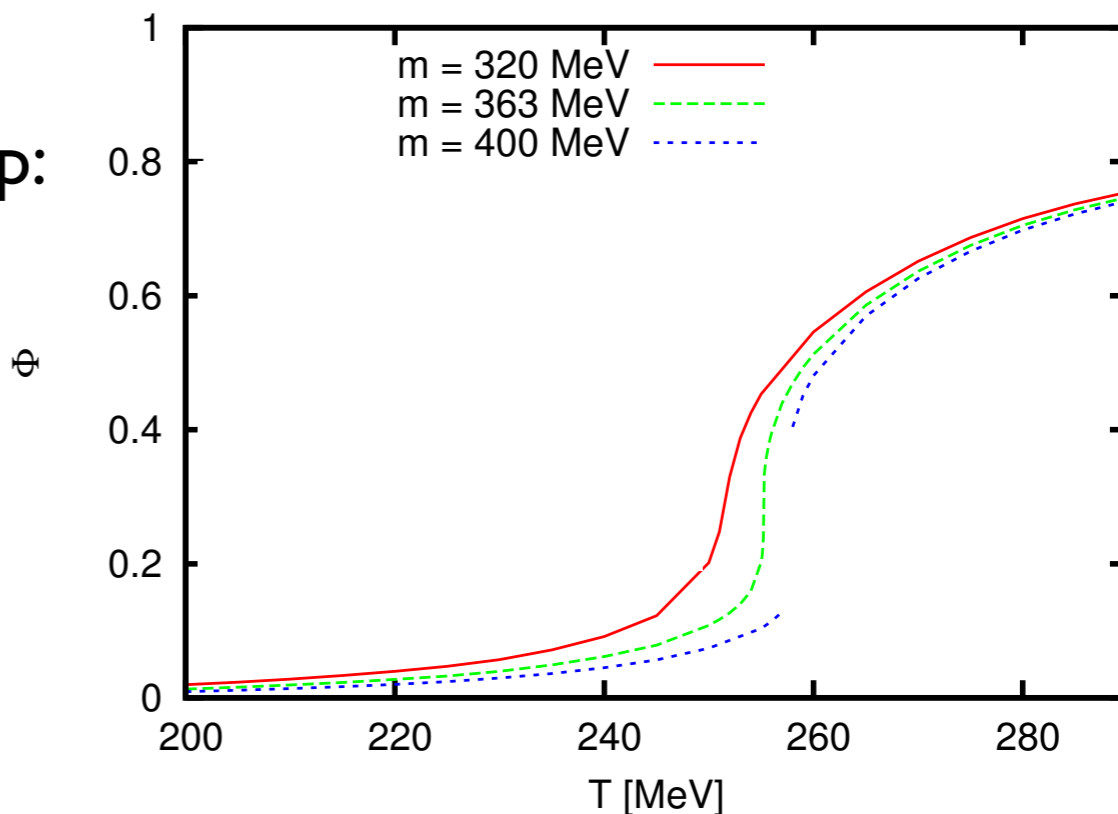


Braun, Gies, Pawłowski, PLB 684, 262 (2010)
 Braun, Haas, Marhauser, Pawłowski, PRL 106 (2011)
 Fister, Pawłowski, PRD 88 045010 (2013)
 CF, Fister, Luecker, Pawłowski, PLB 732 (2013)

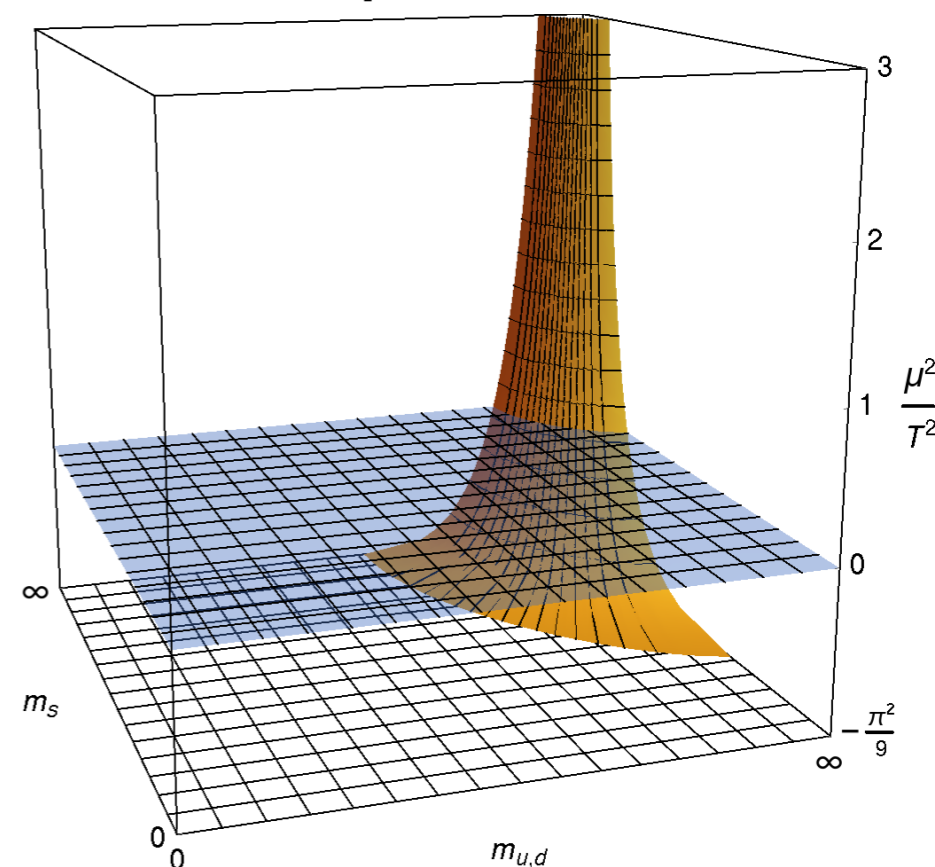
Critical line/surface for heavy quarks



Polyakov Loop:



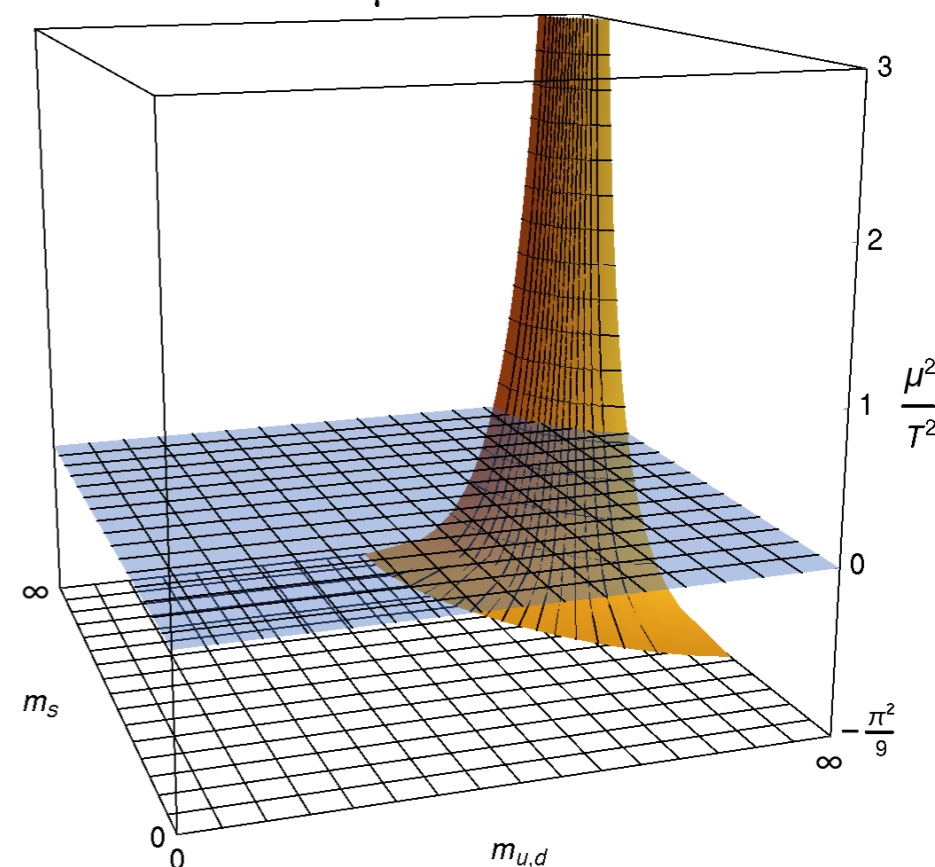
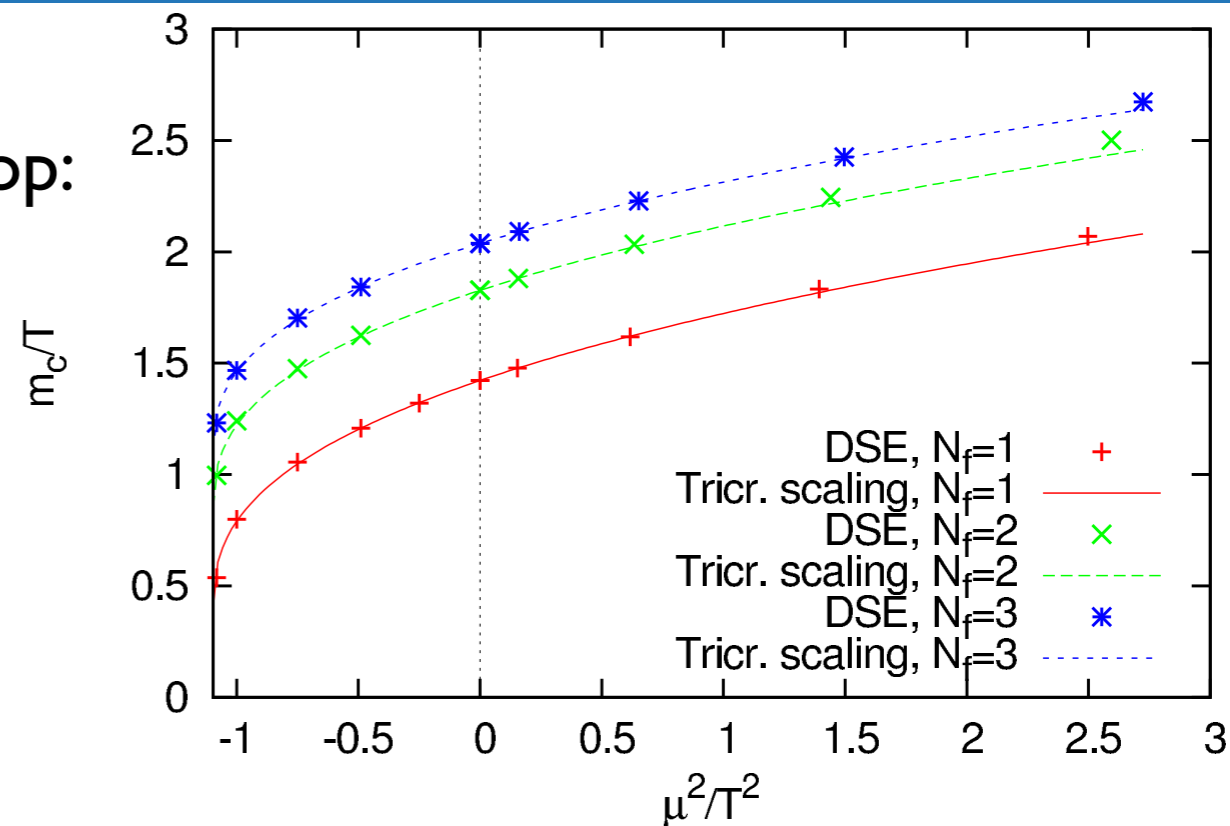
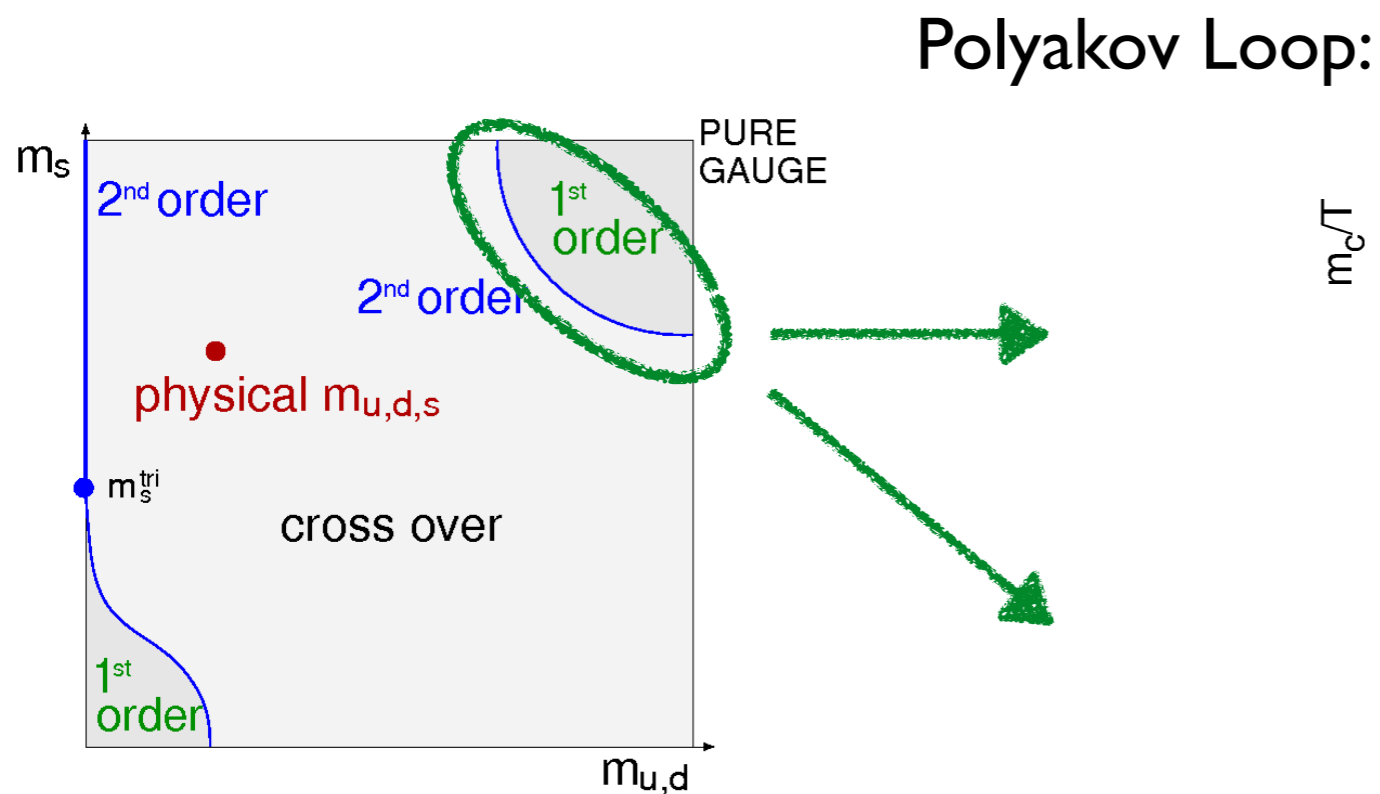
- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen



CF, Luecker, Pawlowski, PRD 91 (2015) 1

Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

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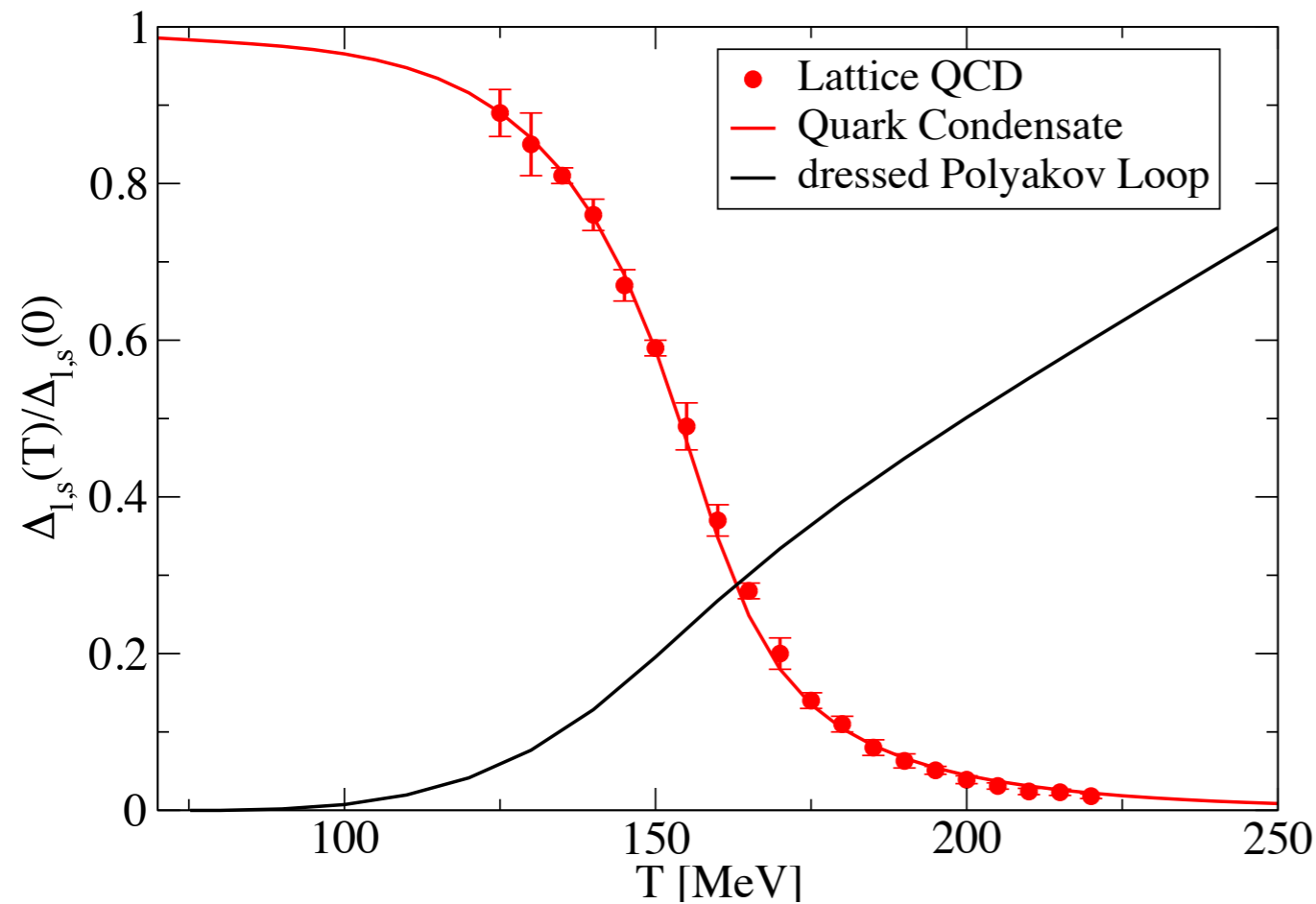
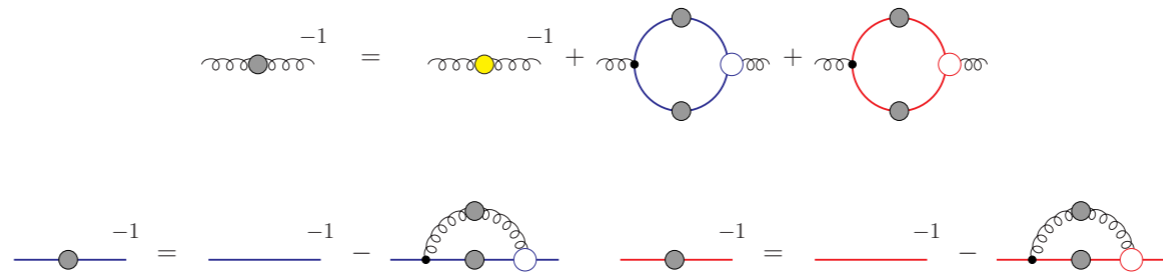
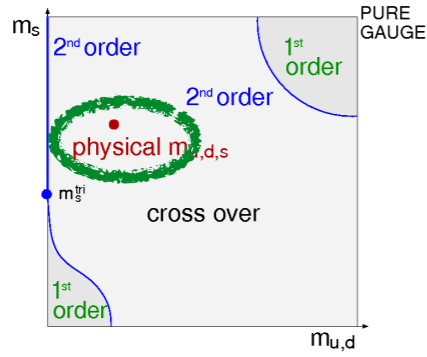


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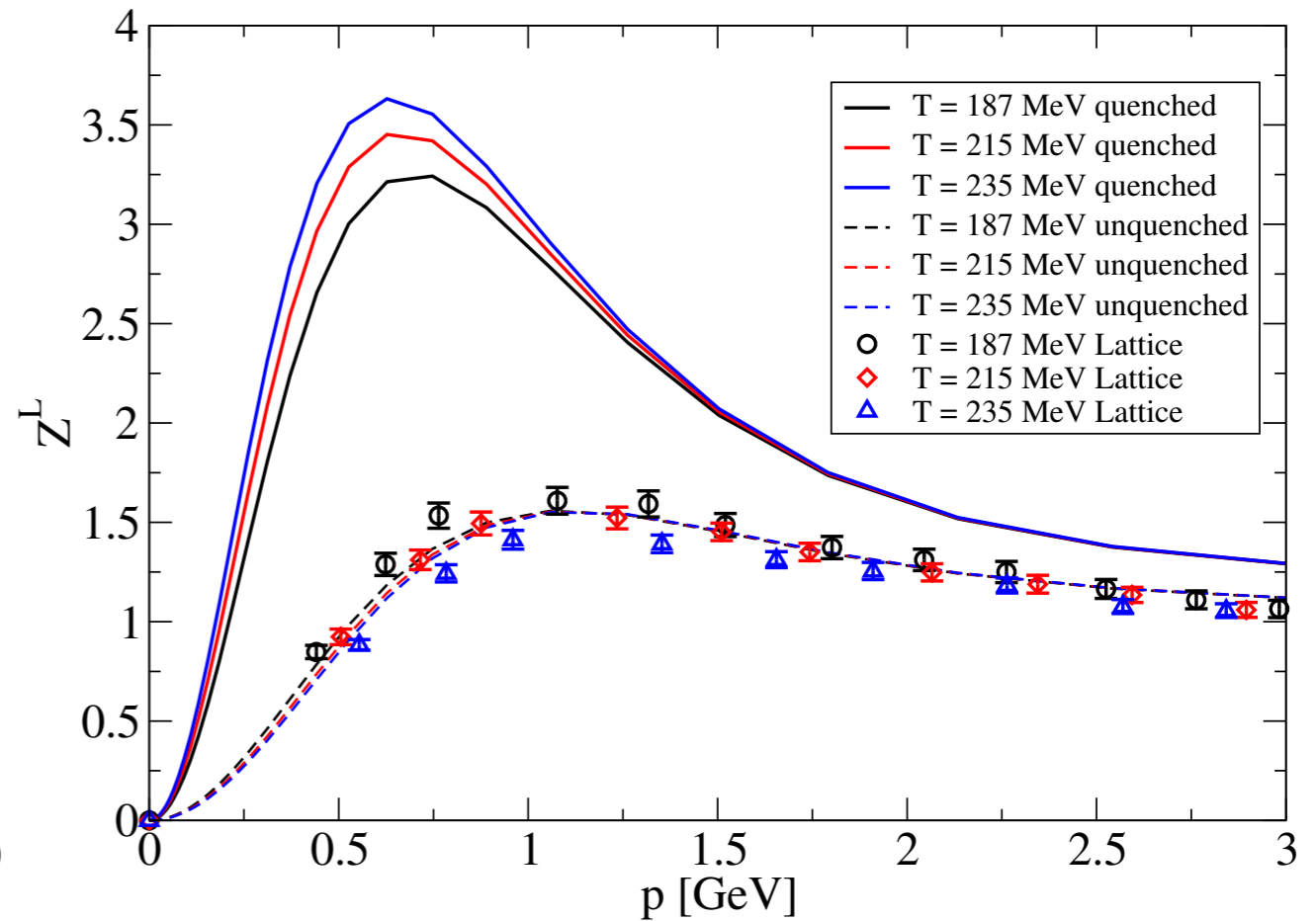
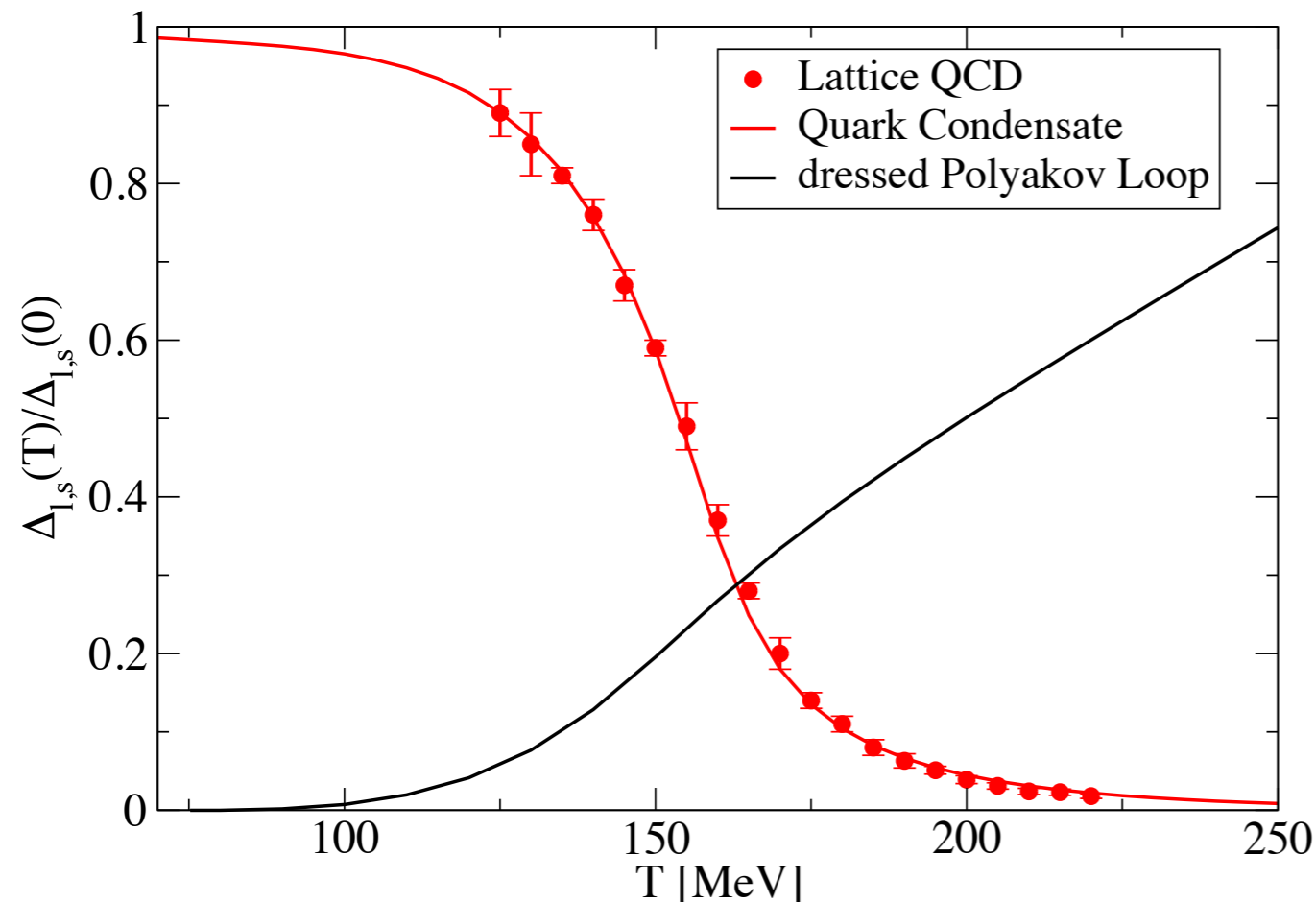
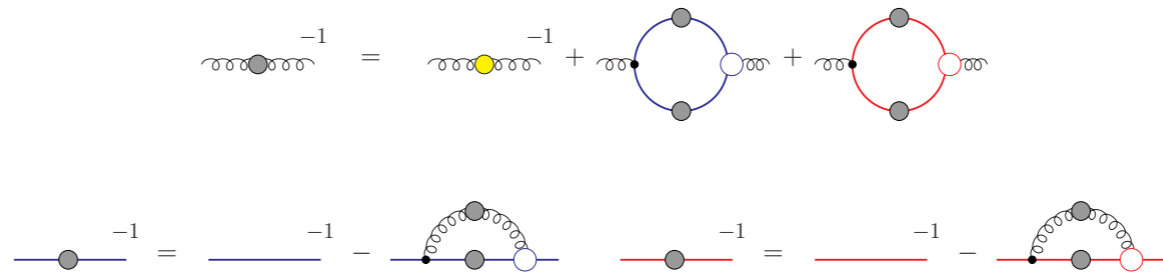
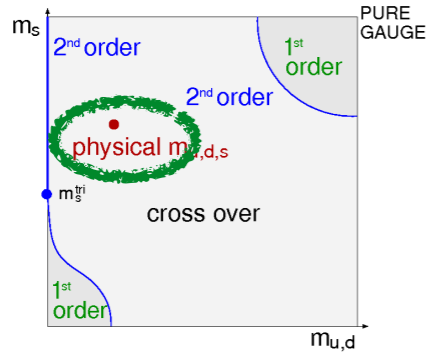
$N_f=2+1, \mu=0$, physical point



Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073

DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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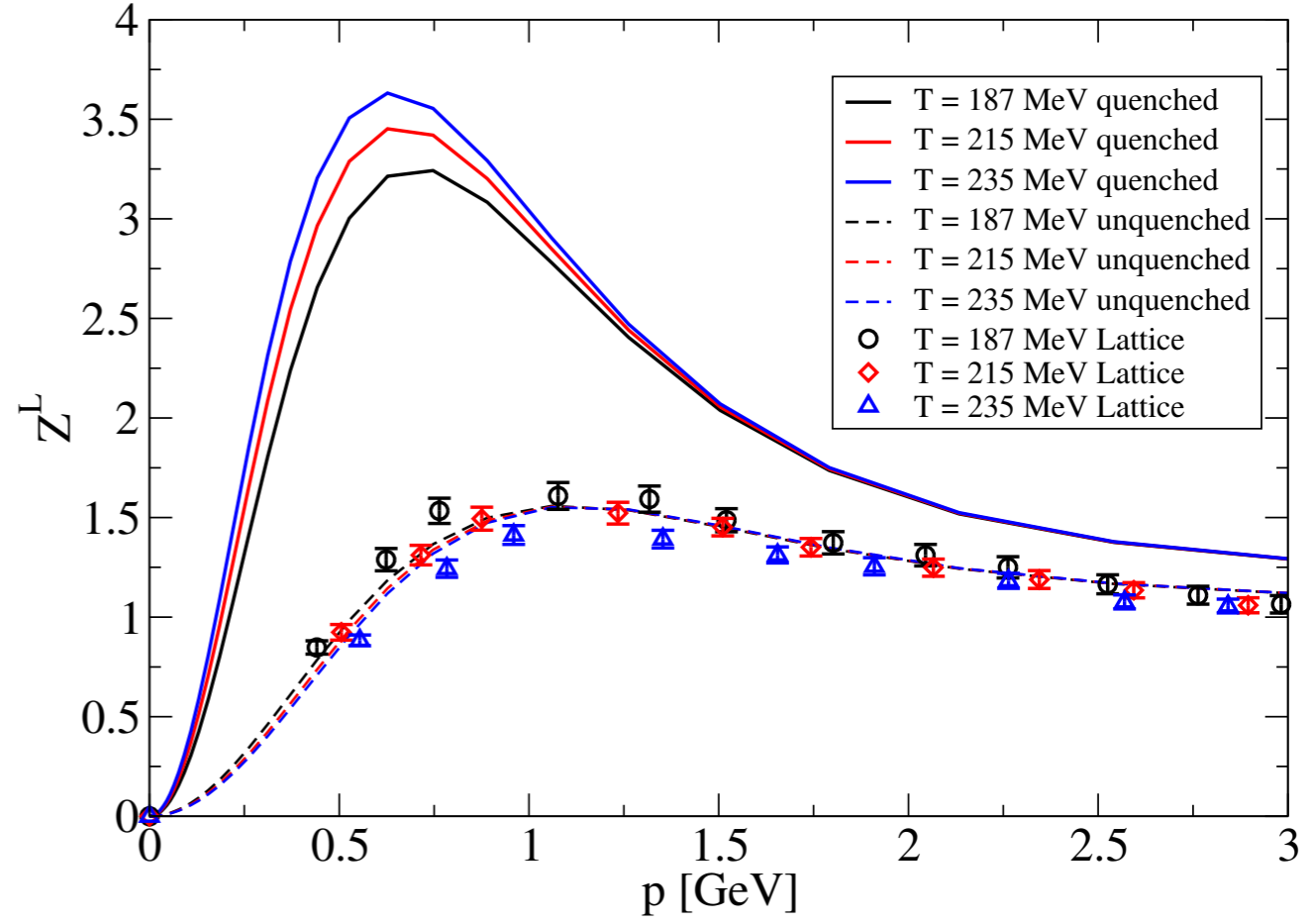
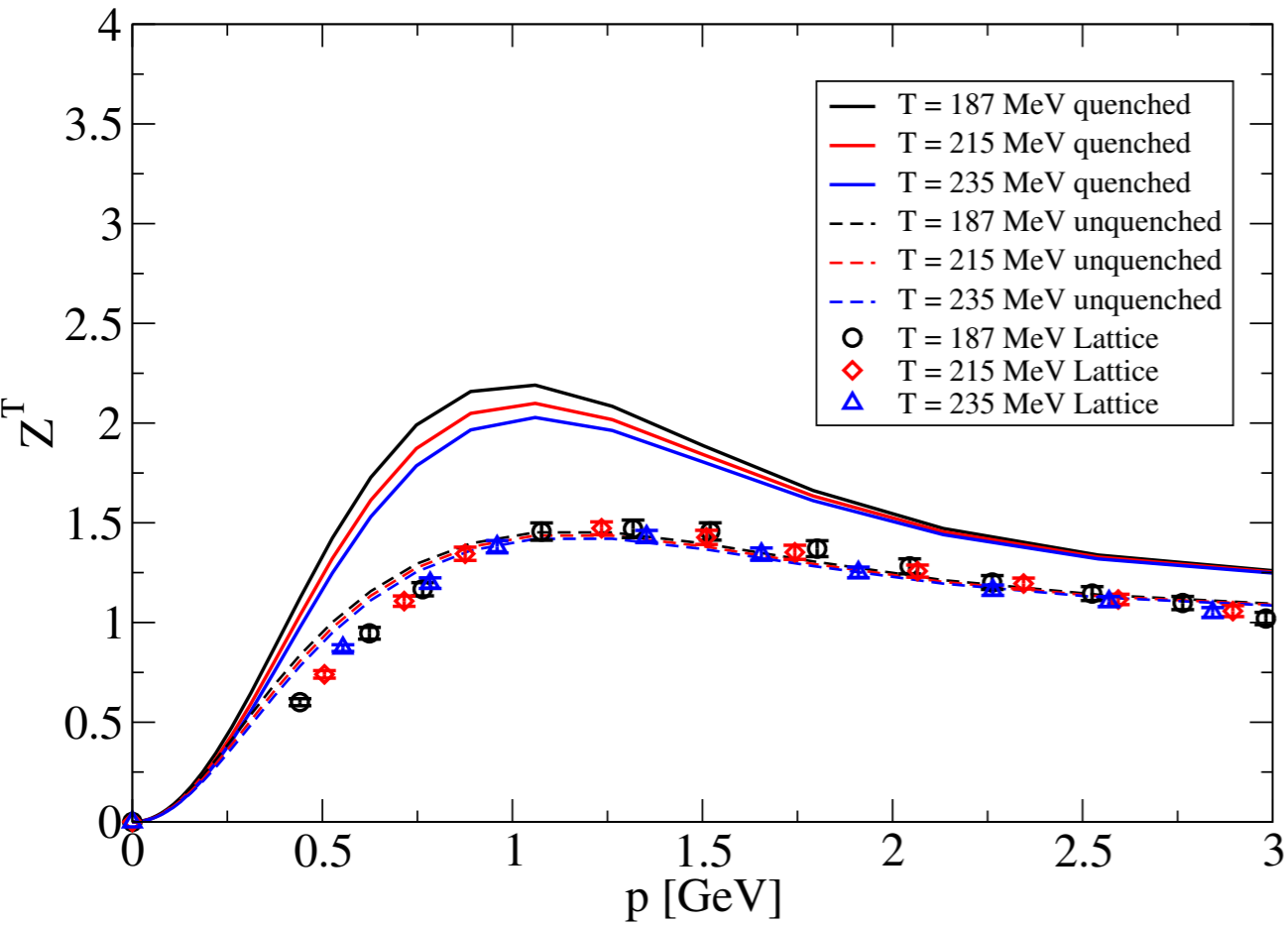
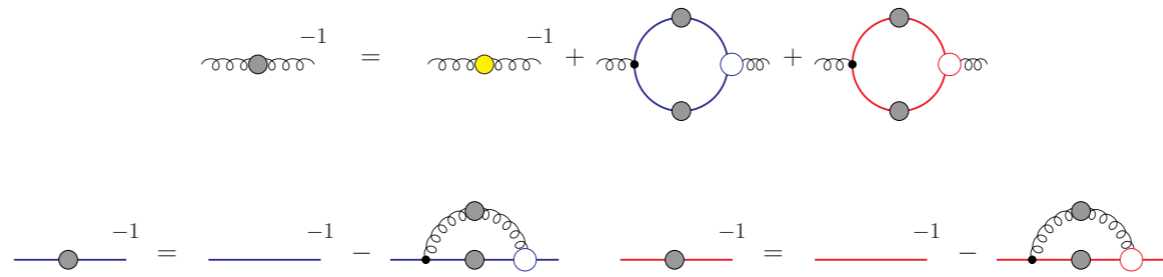
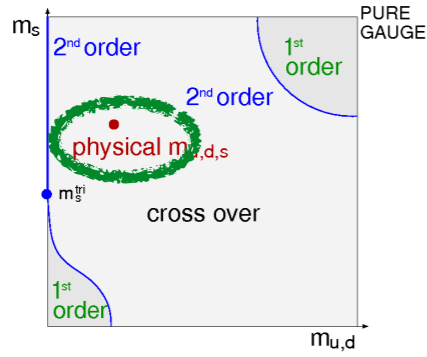


Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
 DSE: CF, Luecker, PLB 718 (2013) 1036,
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Lattice: Aouane, *et al.* PRD D87 (2013), [arXiv:1212.1102]
 DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

● quantitative agreement: DSE prediction verified by lattice

$N_f=2+1, \mu=0$, physical point



Lattice: Aouane, et al. PRD D87 (2013), [arXiv:1212.1102]
 DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
 CF, Luecker, Welzbacher, PRD 90 (2014) 034022

● quantitative agreement: DSE prediction verified by lattice

Location of CEP in freeze-out landscape

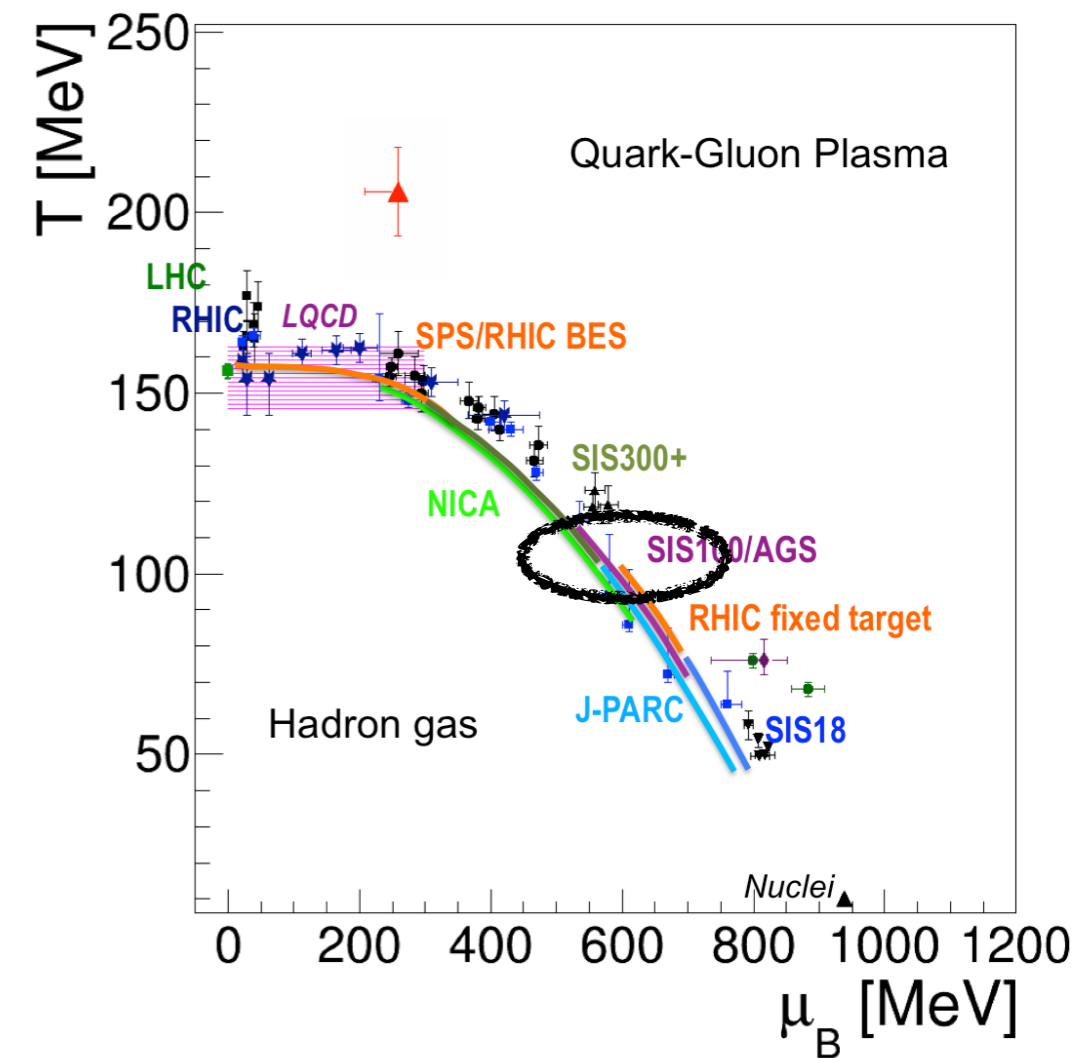
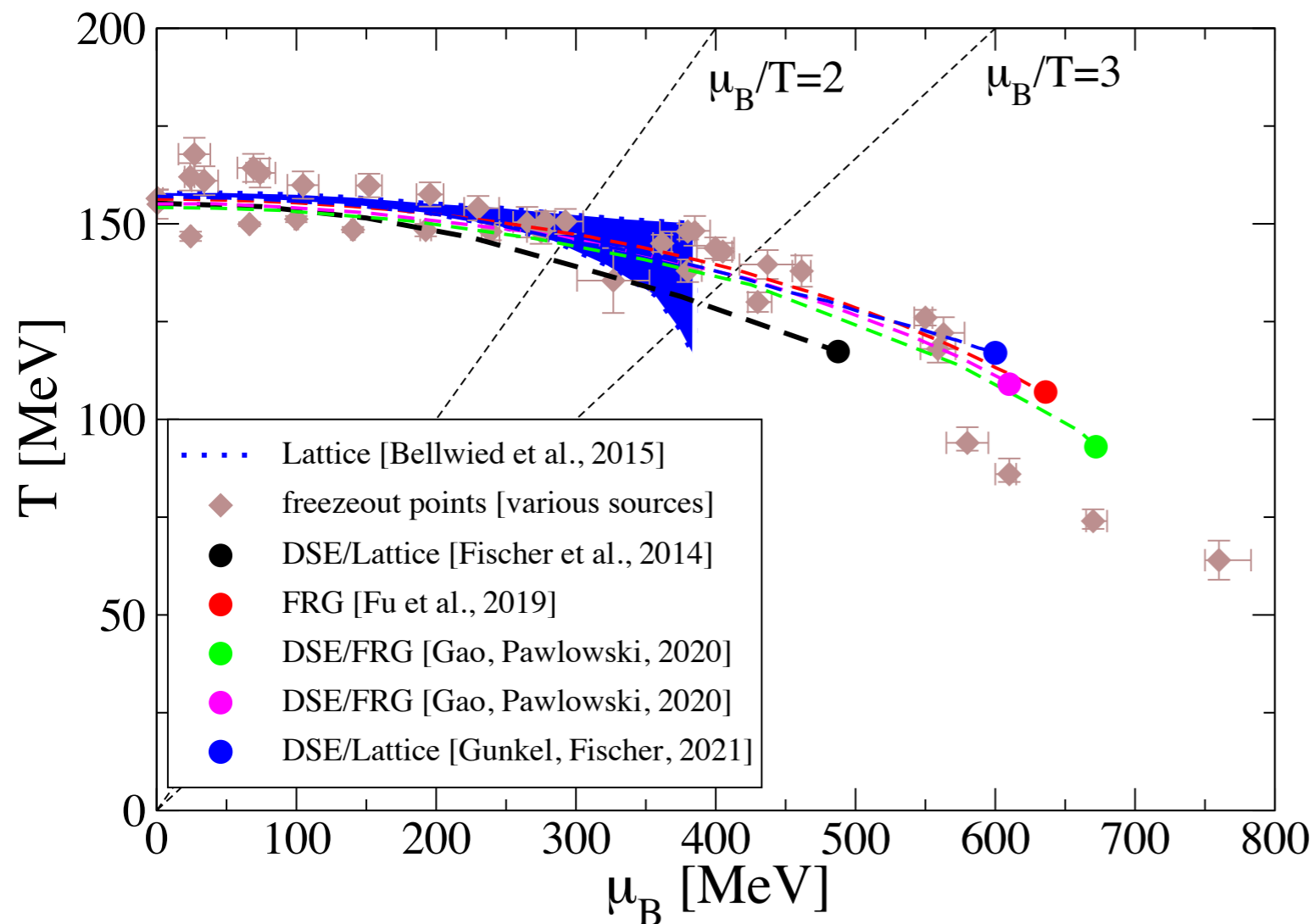


Figure adapted from talk of T. Galatyuk, Erice 2016

● what about truncation error ? how stable is this result ??

✱ $N_f=2+1+1$

✱ baryon and meson effects ?

✱ crosscheck with FRG

Location of CEP in freeze-out landscape

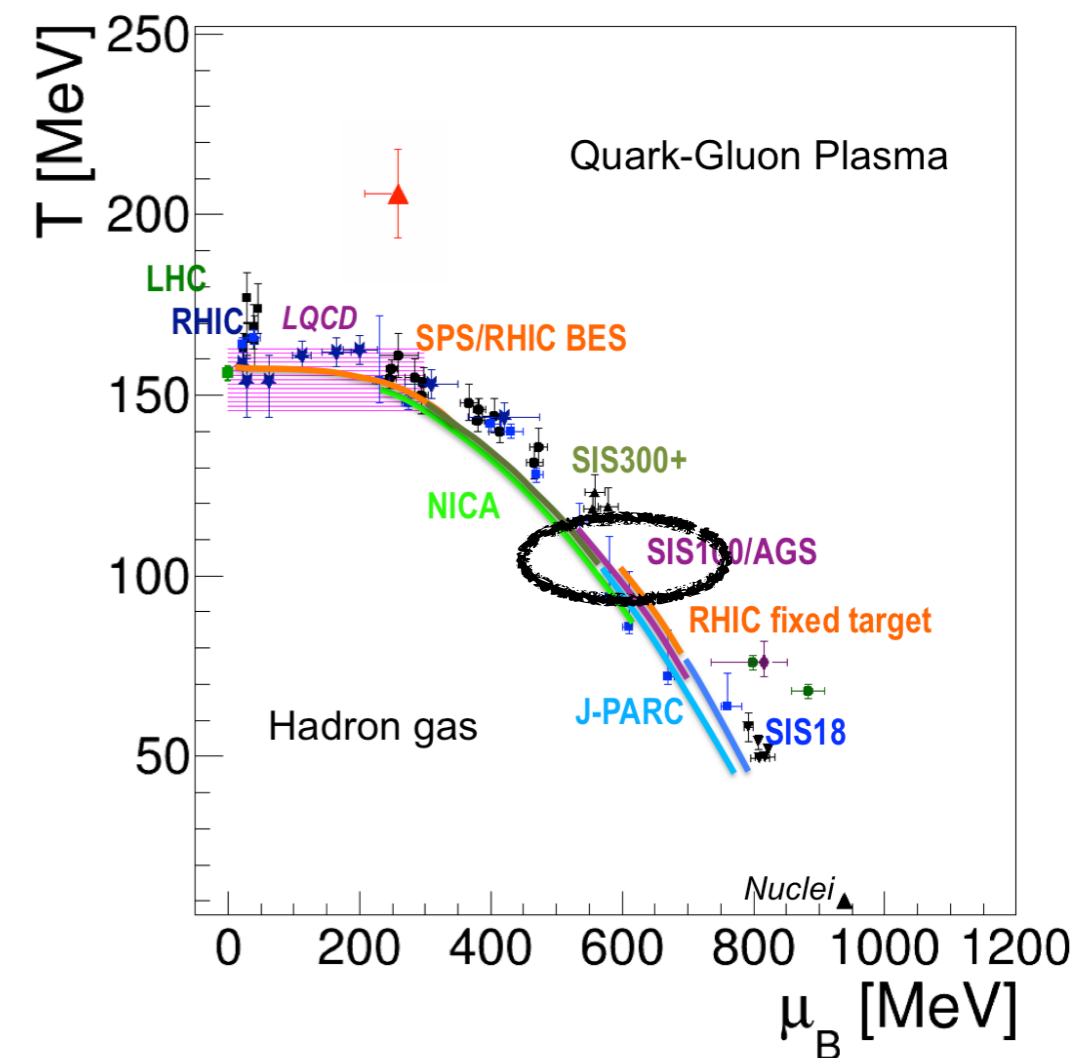
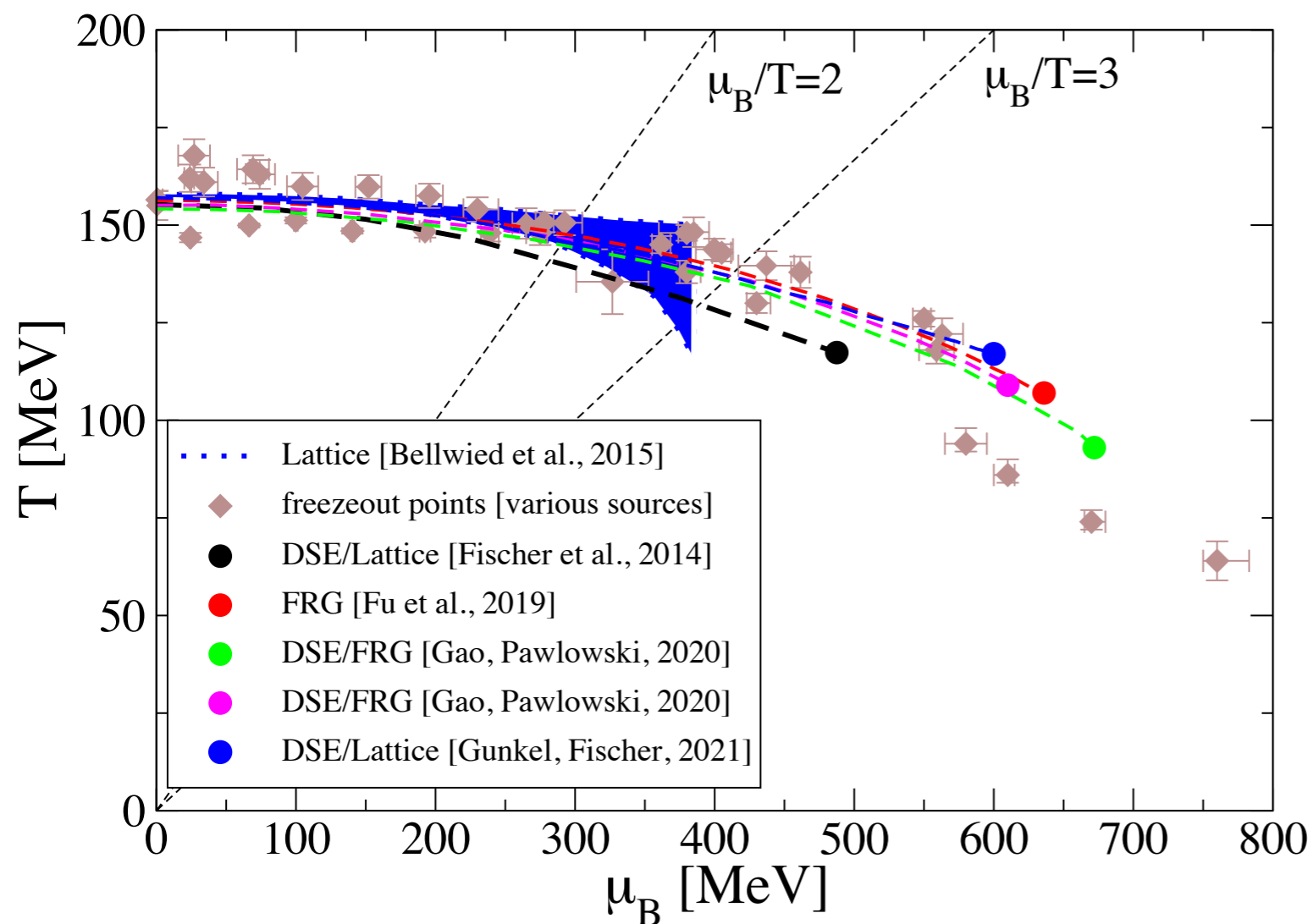
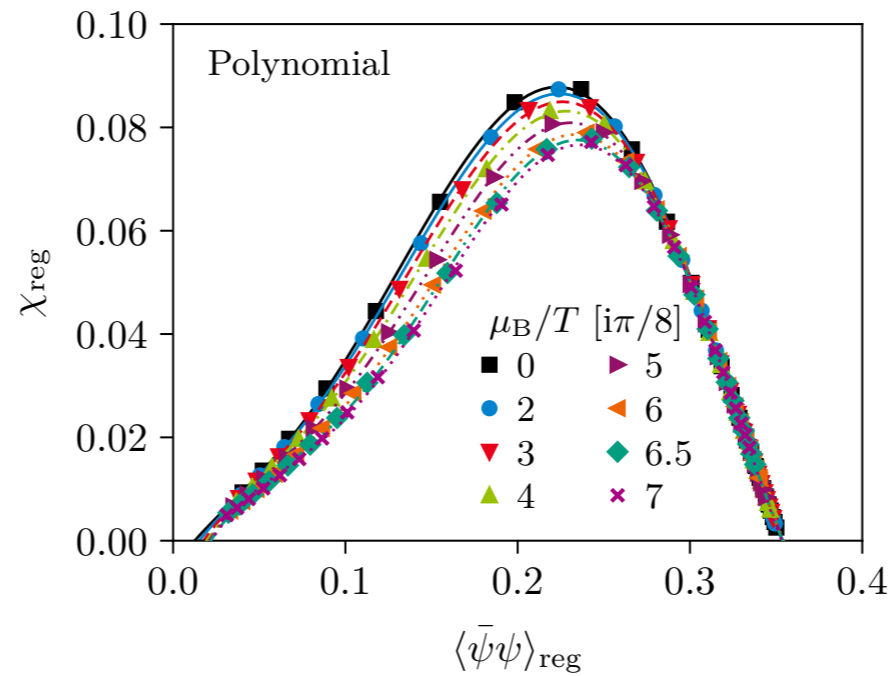
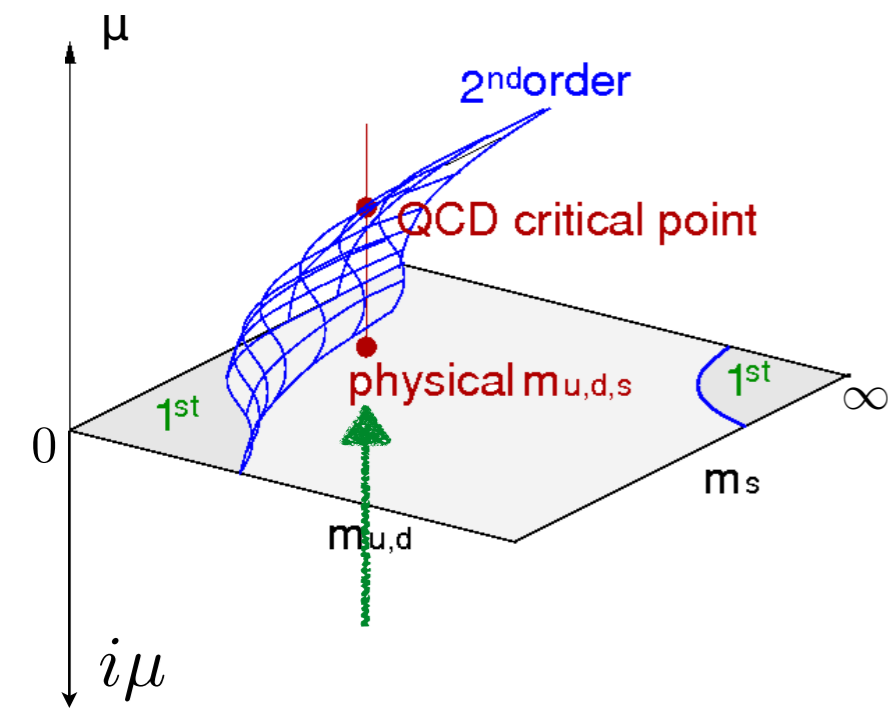


Figure adapted from talk of T. Galatyuk, Erice 2016

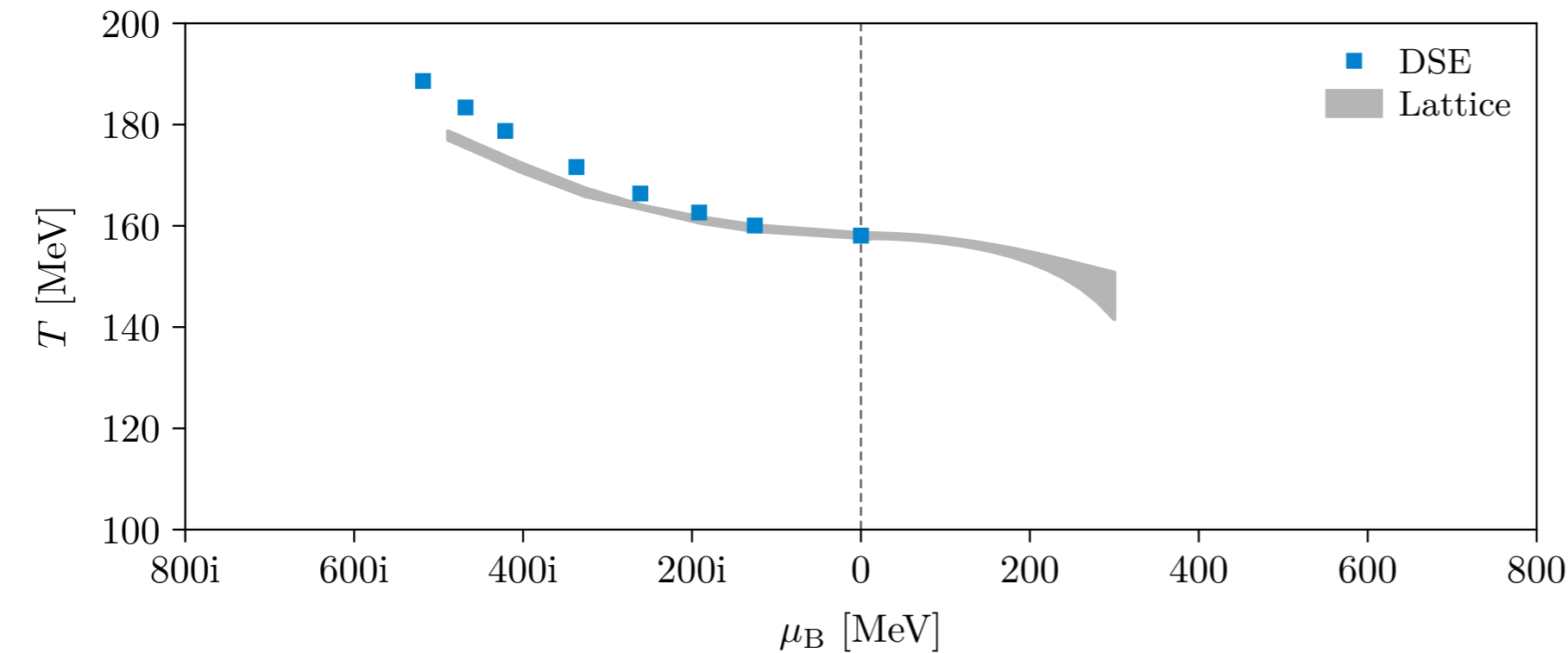
- what about truncation error ? how stable is this result ??
- ✱ $N_f=2+1+1$
- ✱ baryon and meson effects ?
- ✱ crosscheck with FRG
- inhomogeneous phases ?

Buballa and Carignano, PPNP 81 (2015) 39

Extrapolation from imaginary chemical potential



$$\chi(T) = \frac{\partial \langle \bar{\psi} \psi \rangle(T)}{\partial m_u}$$



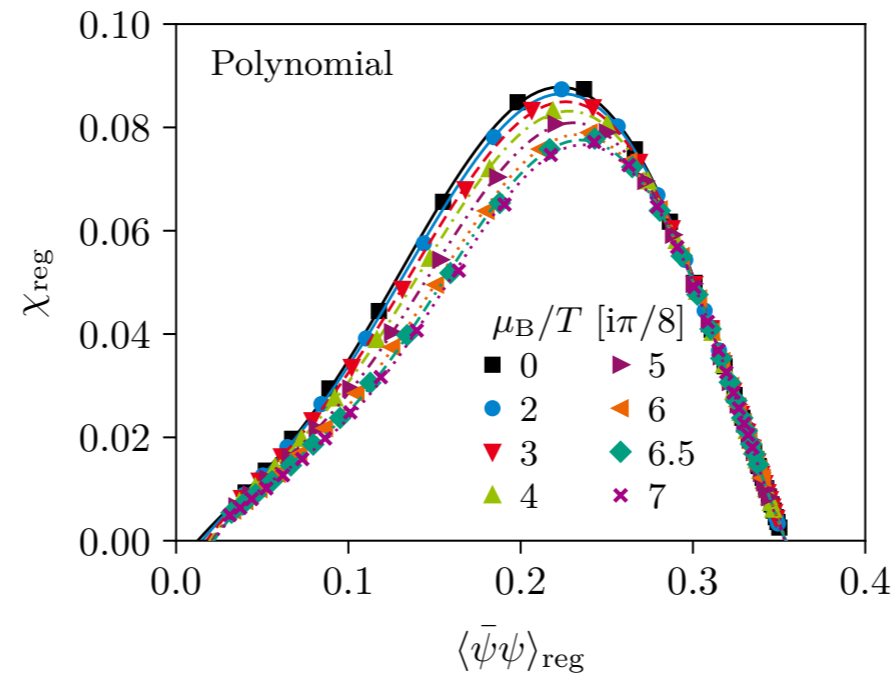
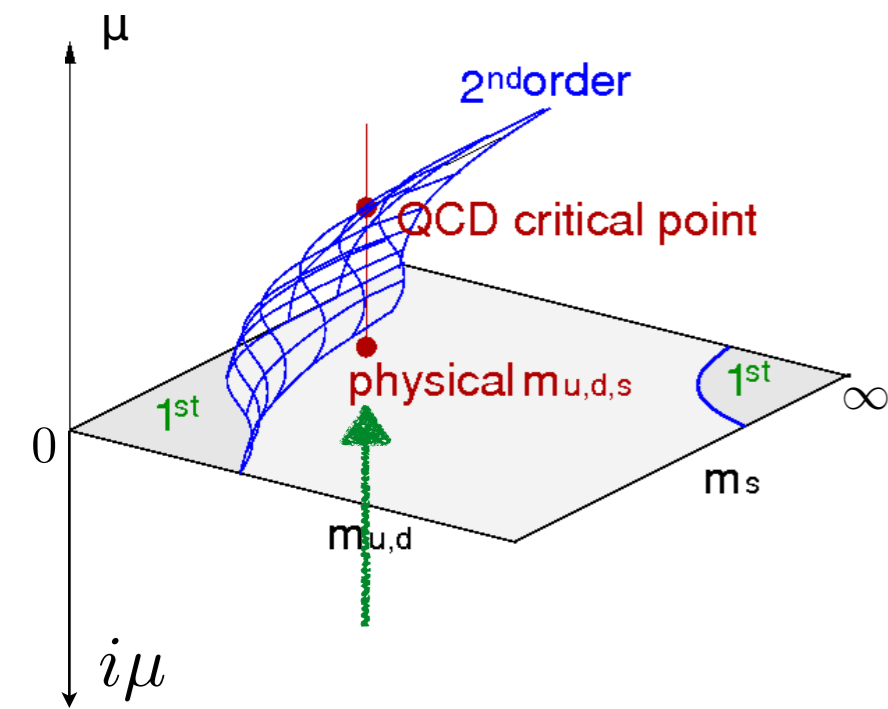
Lattice: Borsanyi et al. PRL 125 052001 (2020)

DSE: Bernhardt, CF, arXiv: 2305.01434

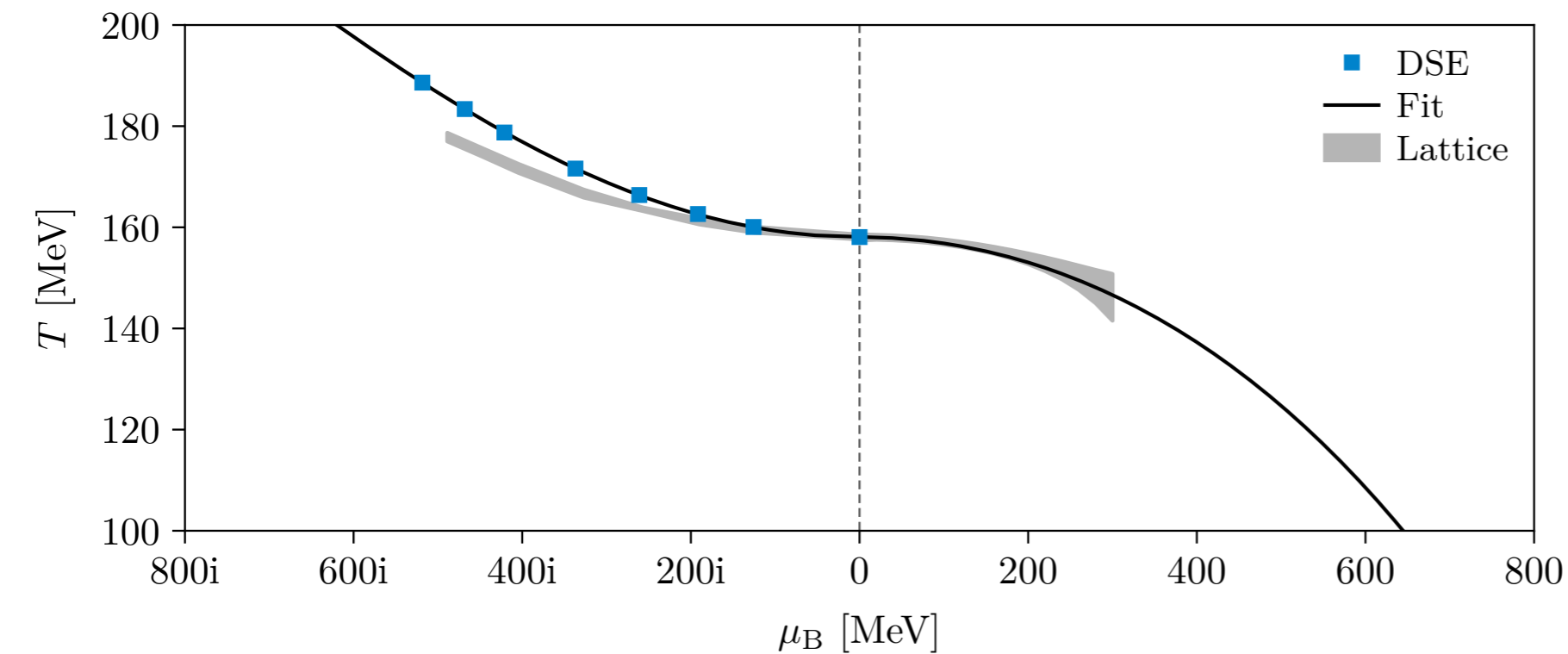
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c} \right)^4$$

$$\kappa_2^{\text{poly}} = 0.0196, \quad \kappa_4^{\text{poly}} = 0.00015,$$

Extrapolation from imaginary chemical potential



$$\chi(T) = \frac{\partial \langle \bar{\psi} \psi \rangle(T)}{\partial m_u}$$



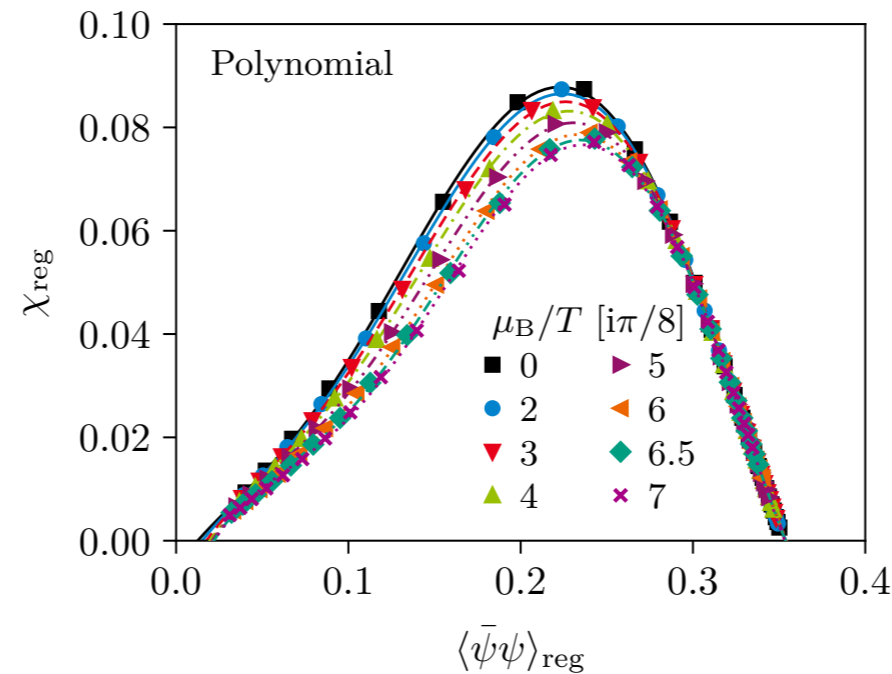
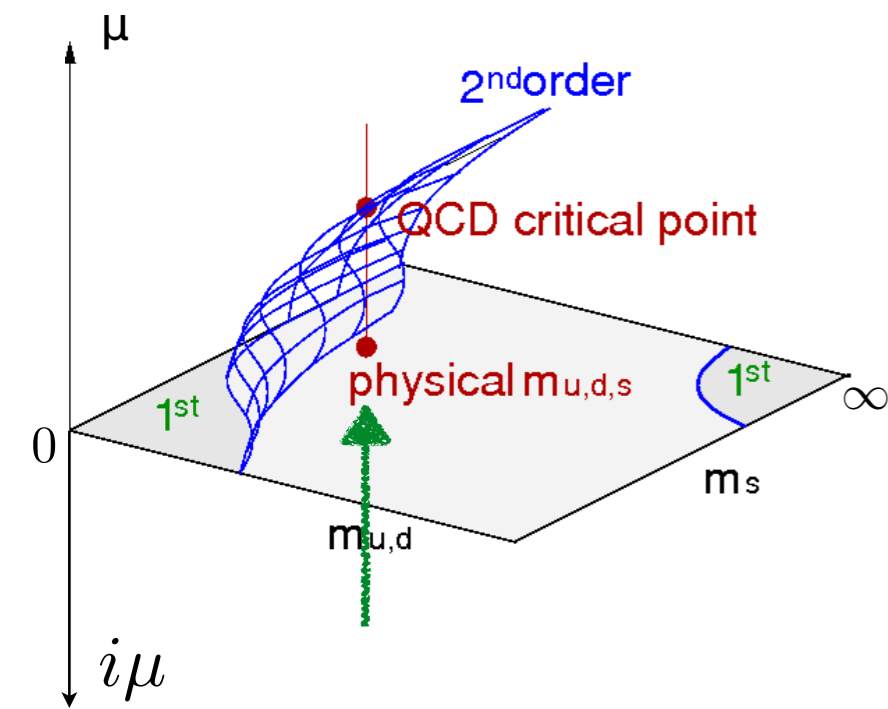
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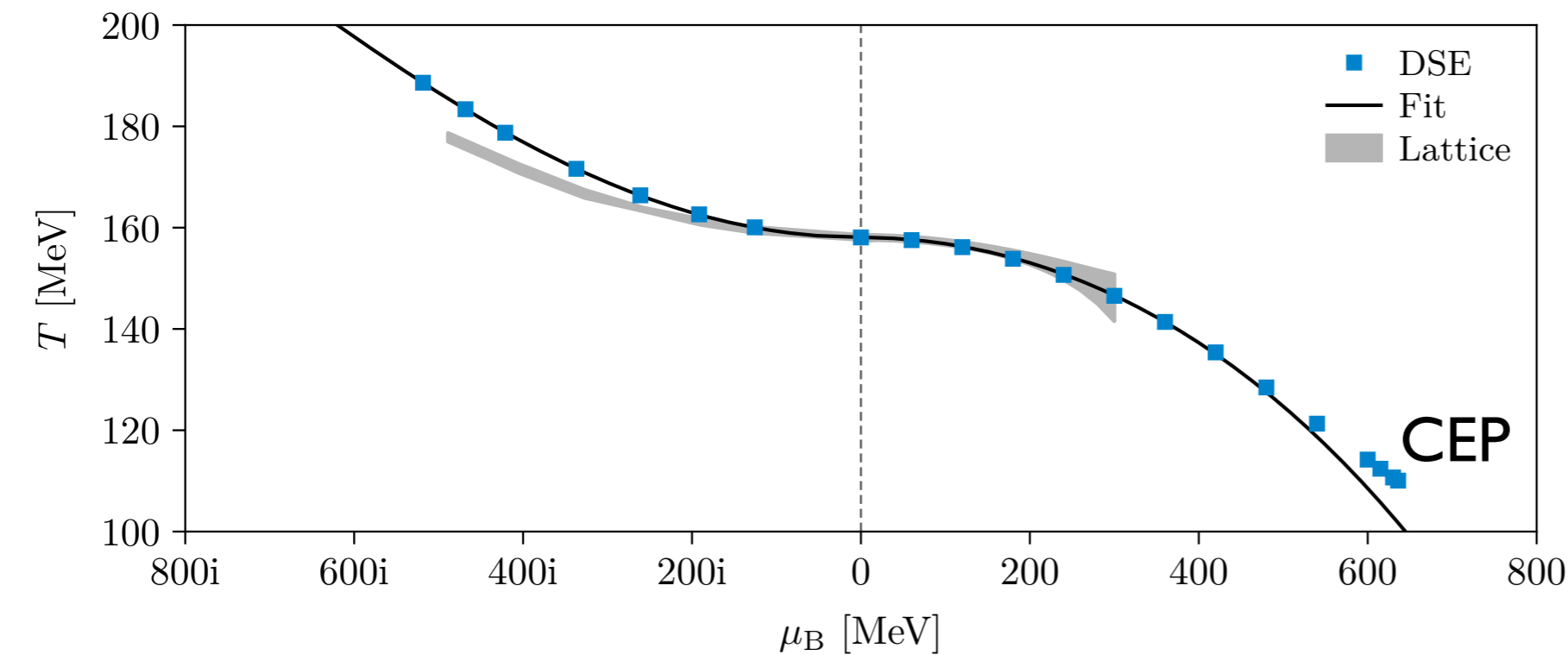
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Extrapolation from imaginary chemical potential



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Lattice: Borsanyi et al. PRL 125 052001 (2020)

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● Extrapolation works very well!

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c} \right)^4$$

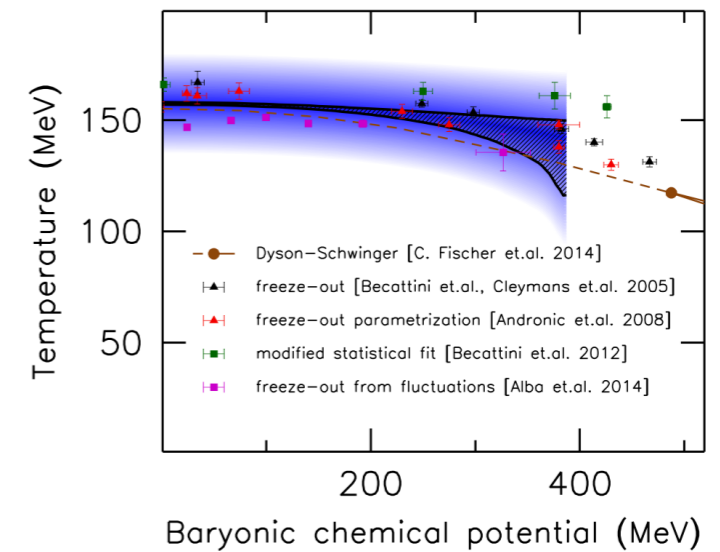
$$\kappa_2^{\text{poly}} = 0.0196, \quad \kappa_4^{\text{poly}} = 0.00015,$$

1. Introduction

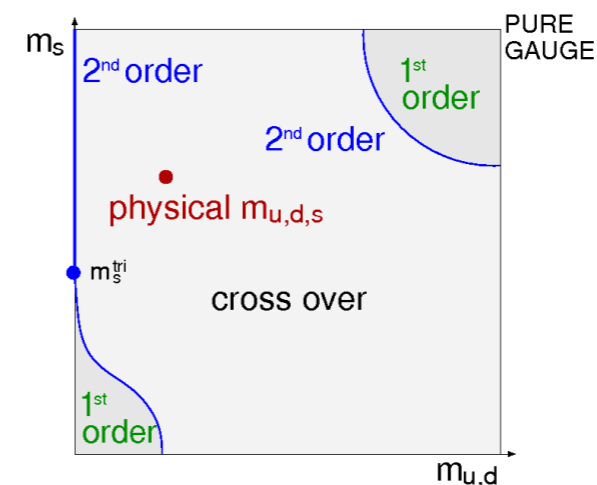
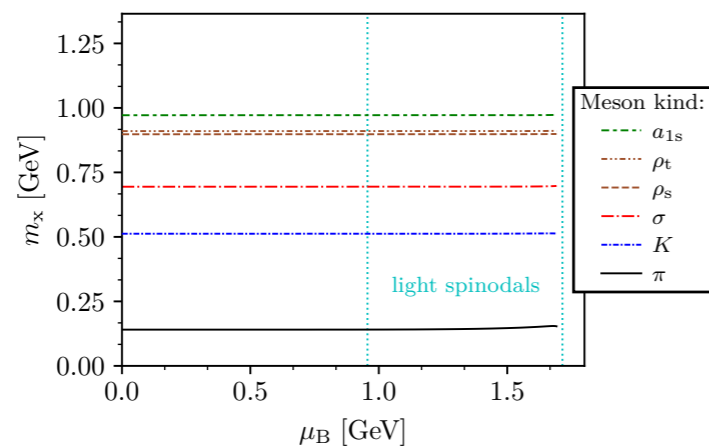


2. Mesons in vacuum

3. QCD phase diagram and the CEP

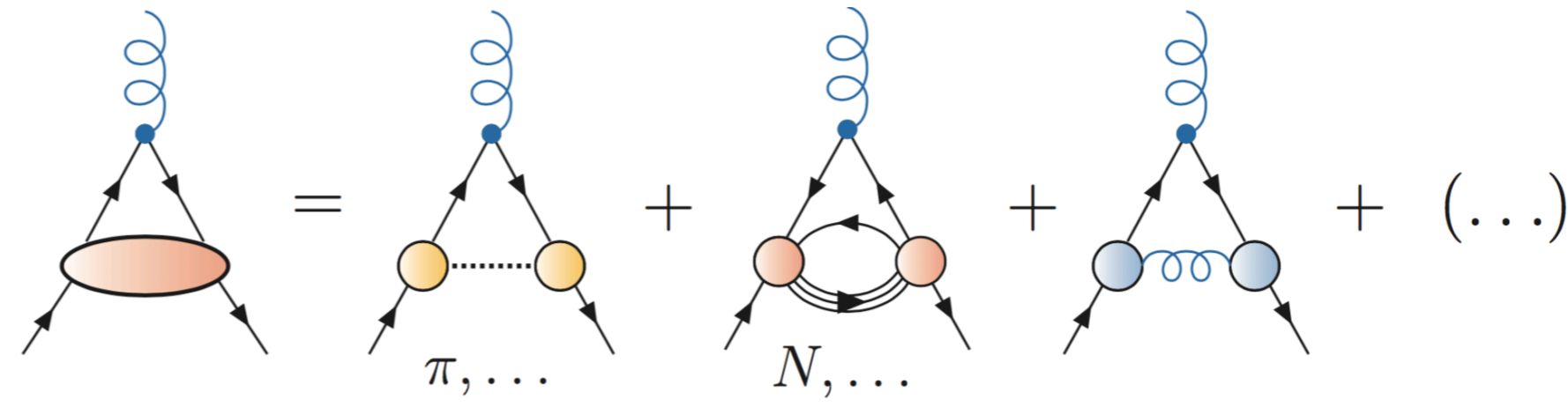
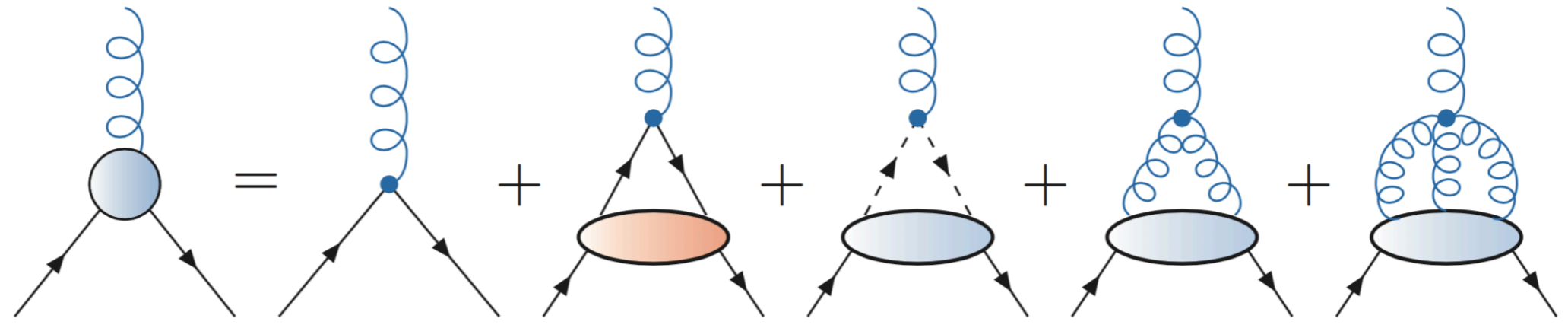


4. Mesons at finite μ , at the CEP and in the chiral limit

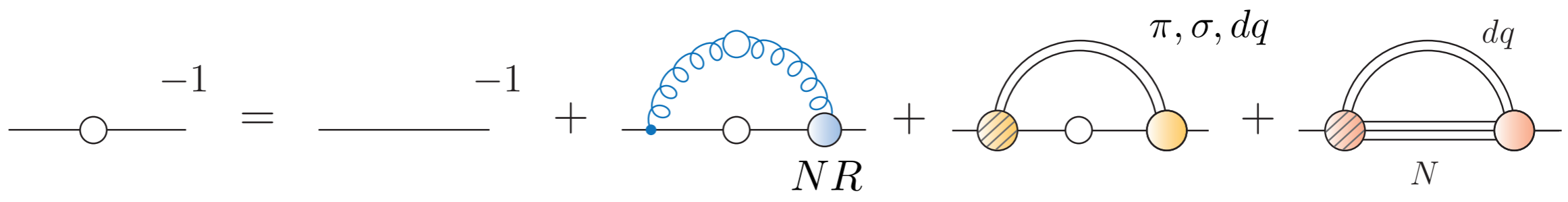


Hadron effects in quark-gluon interaction

quark-gluon vertex:



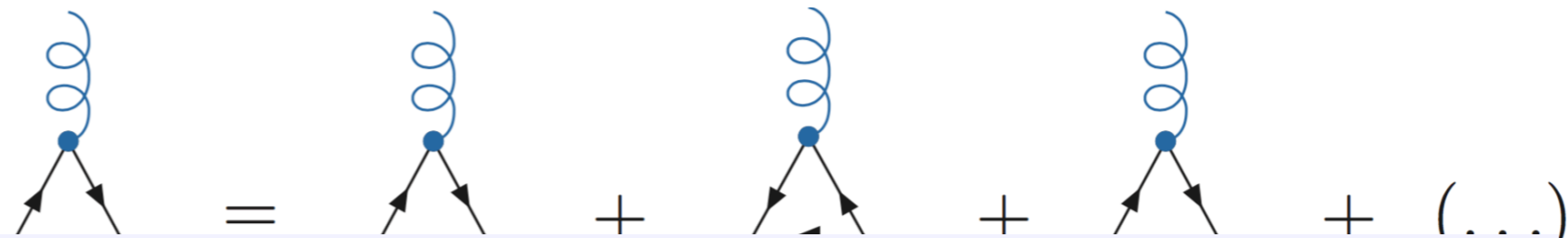
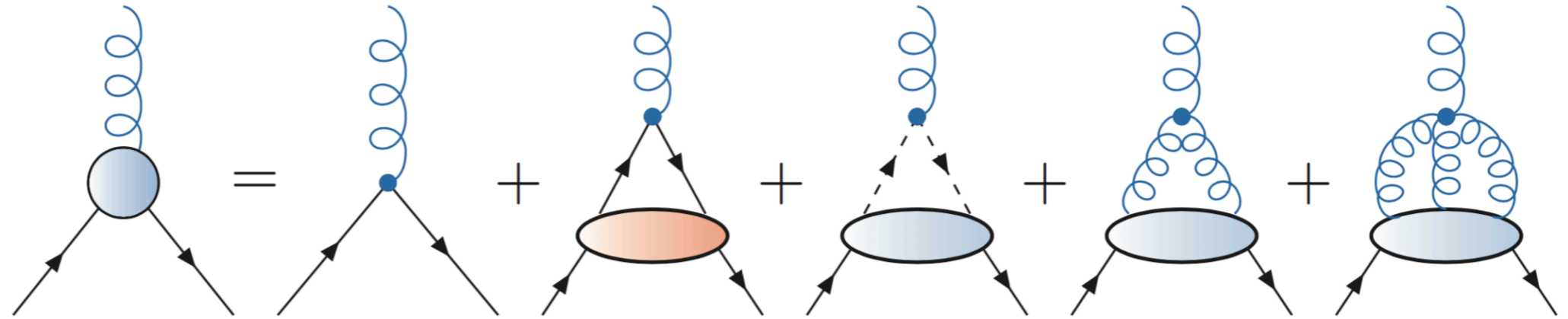
quark:



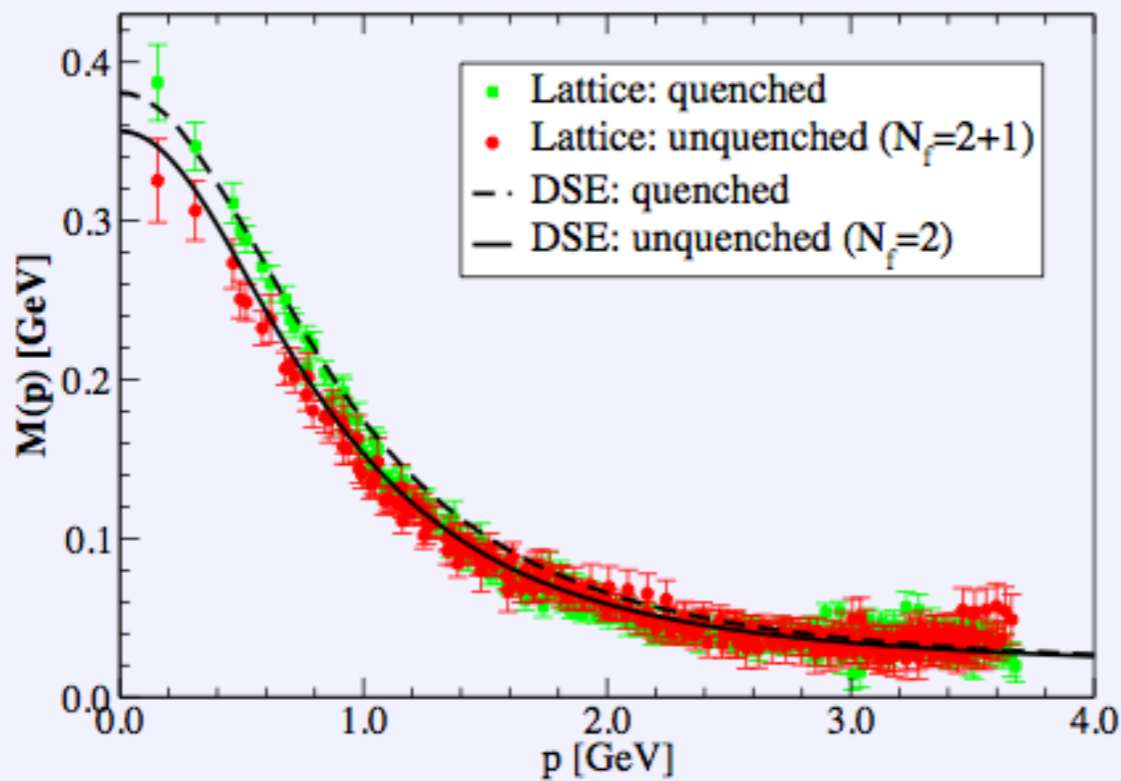
Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

Hadron effects in quark-gluon interaction

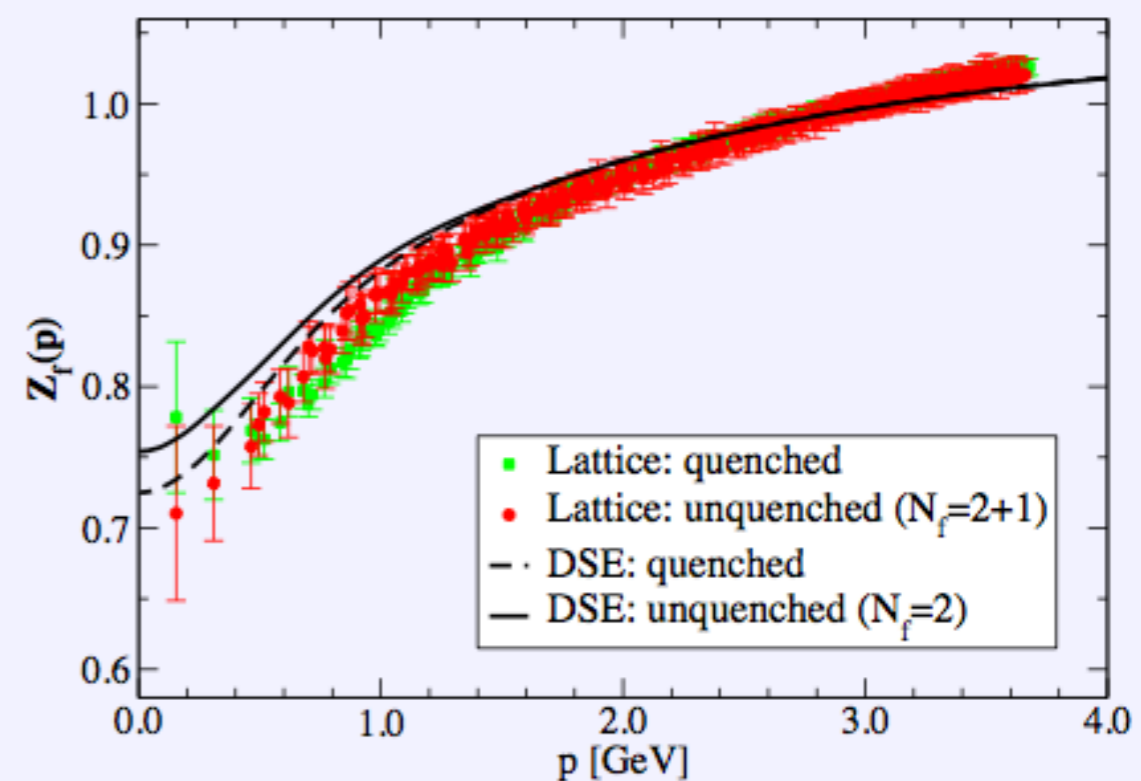
quark-gluon vertex:



quark:

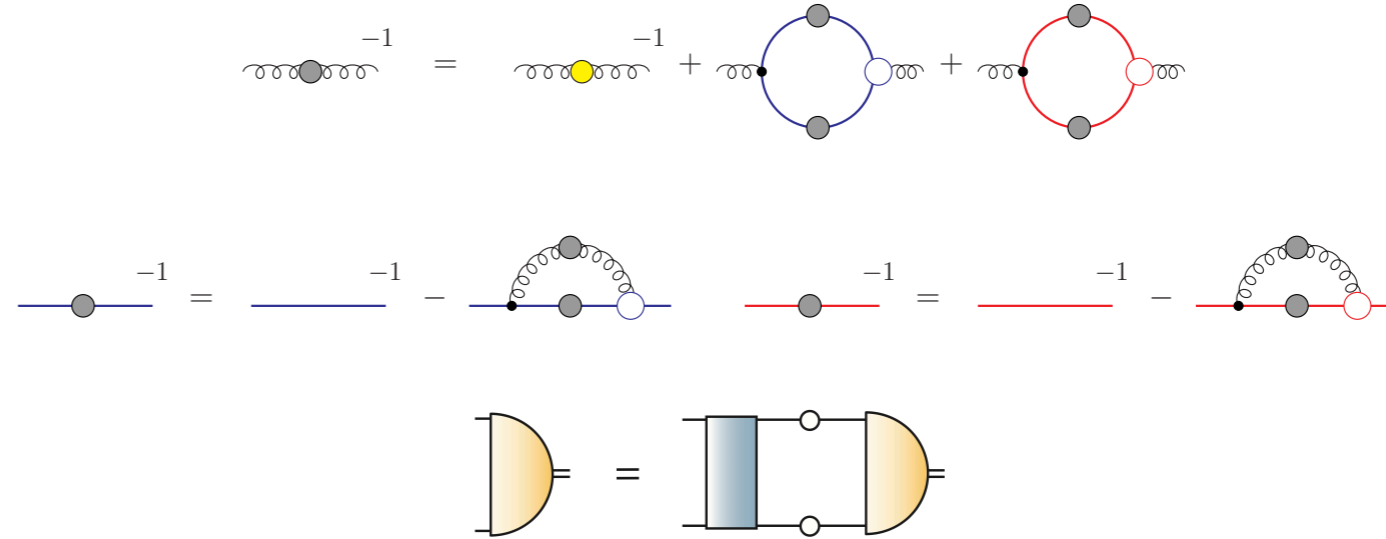
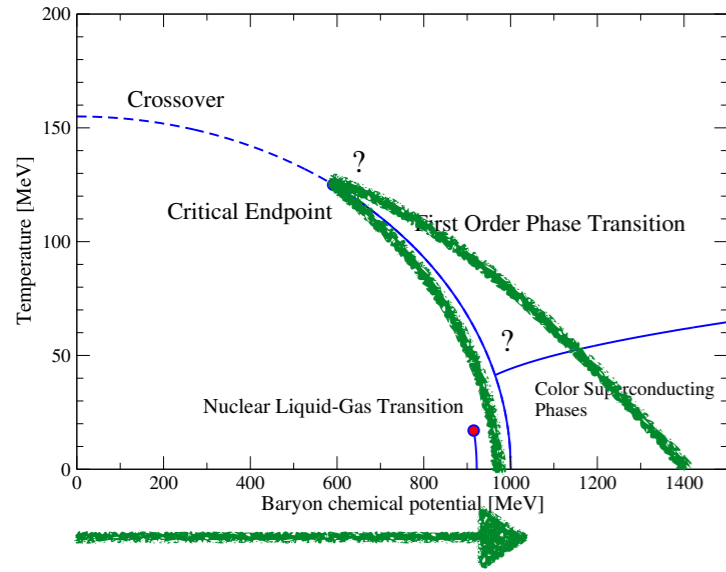


CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)



Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

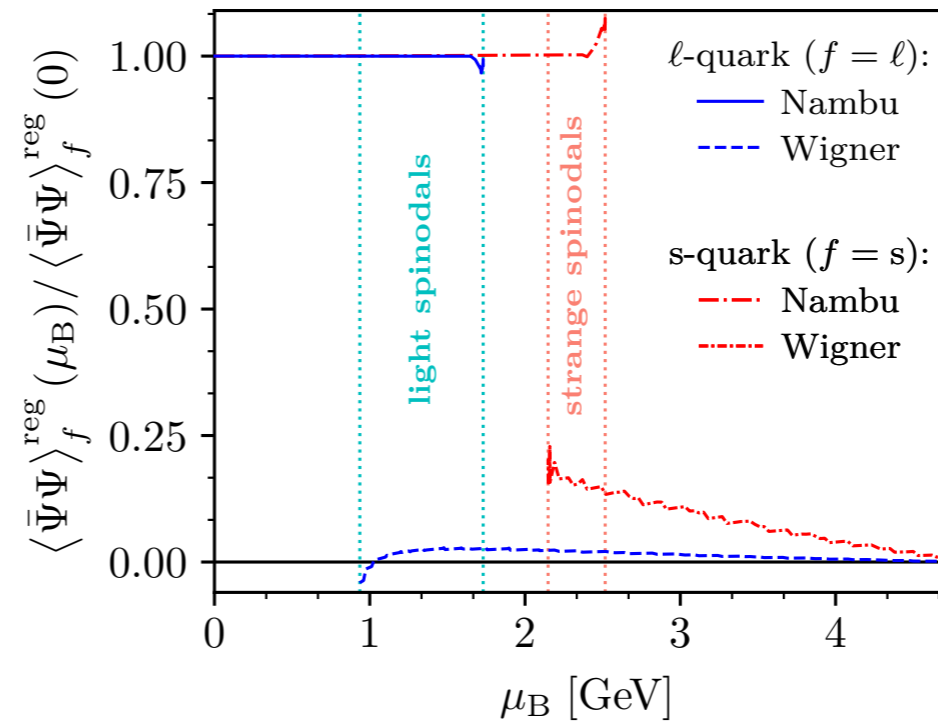
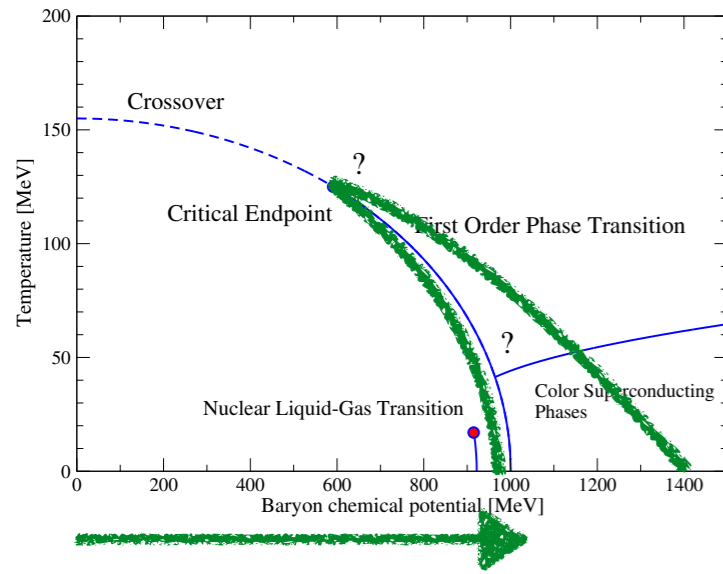
Meson properties at finite chemical potential



● Quarks/meson wave functions do change !

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
 Gunkel, CF, EPJ A 57 (2021) no. 4, 147

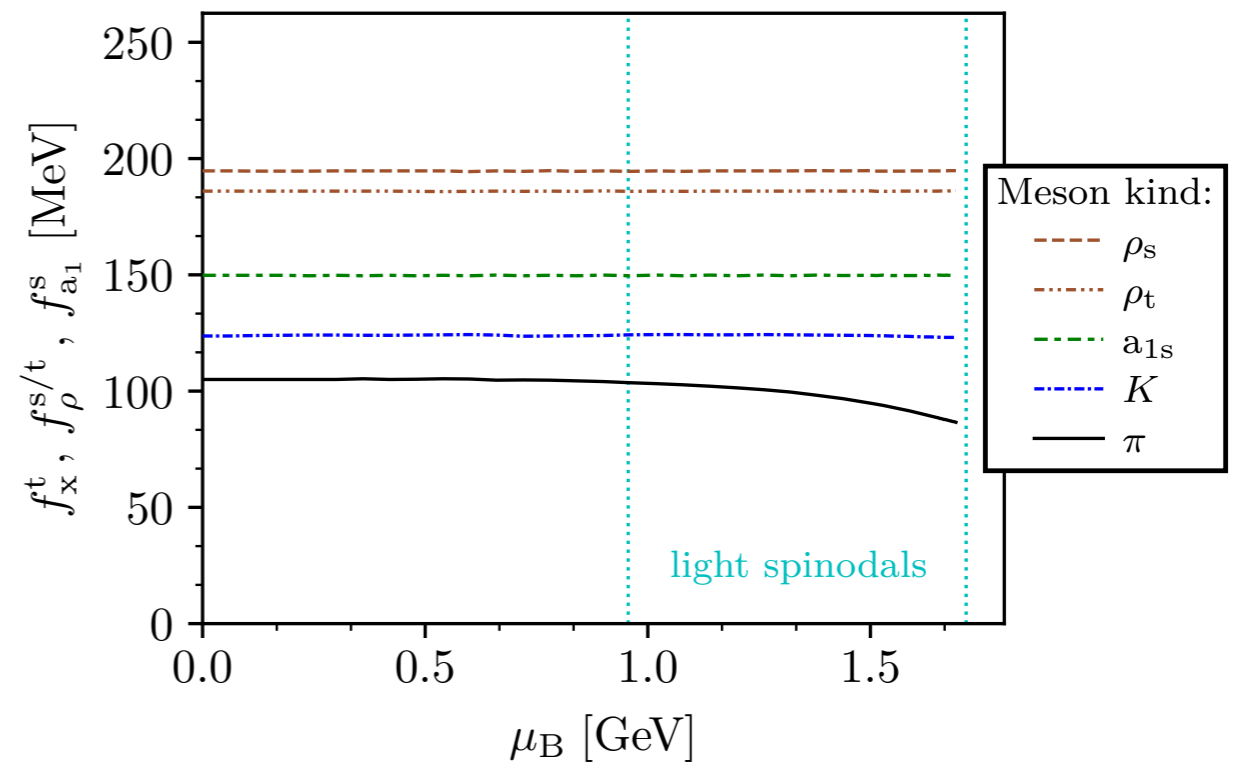
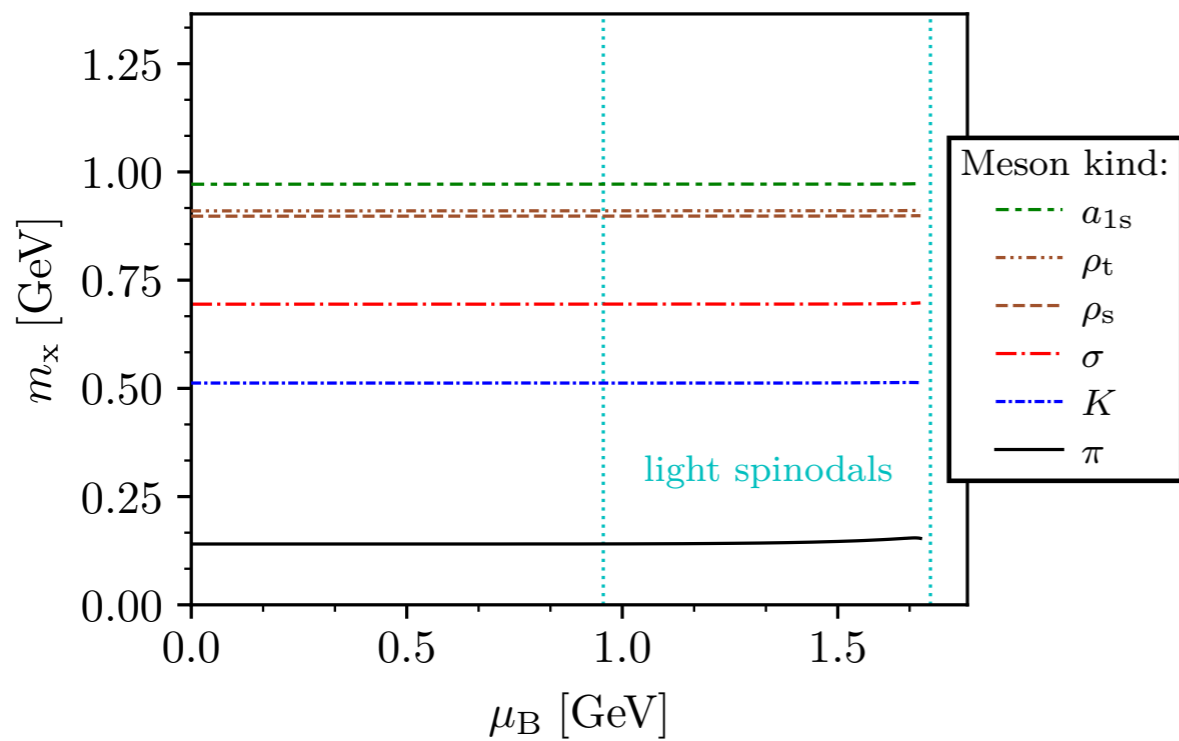
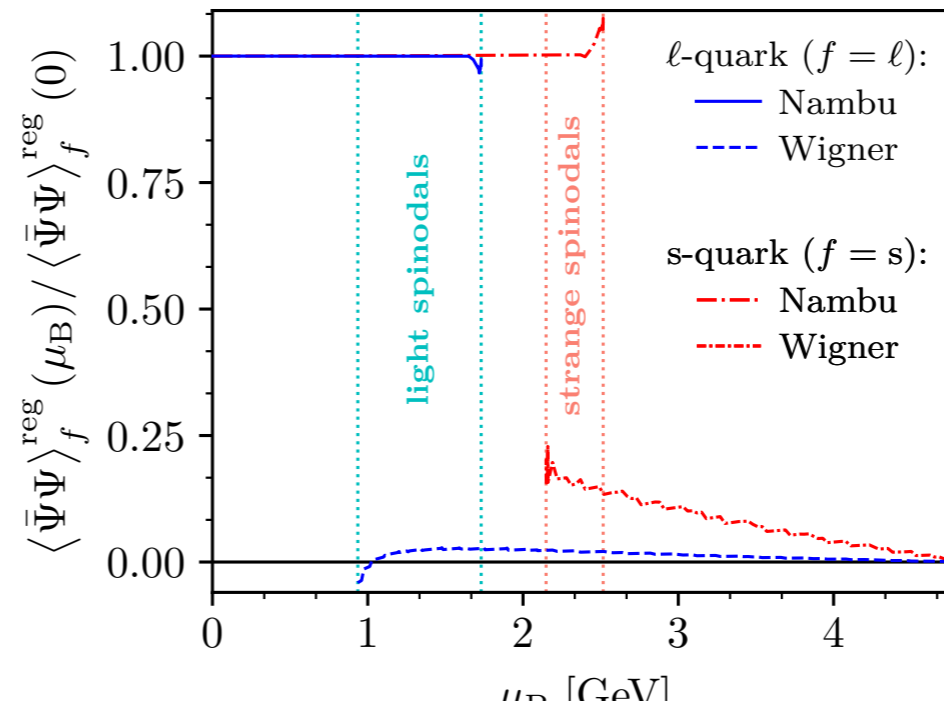
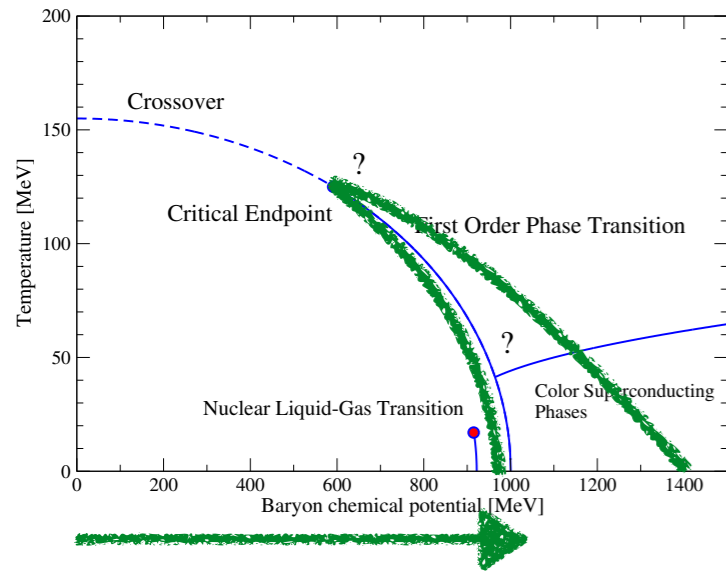
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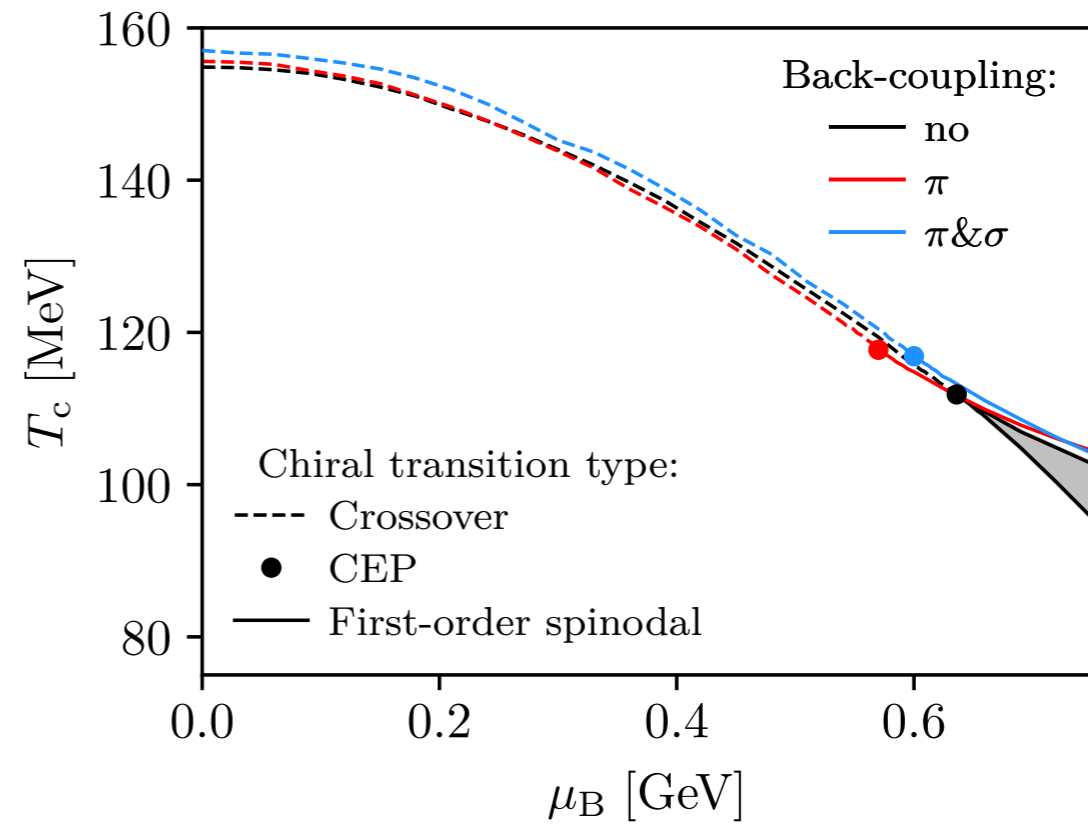
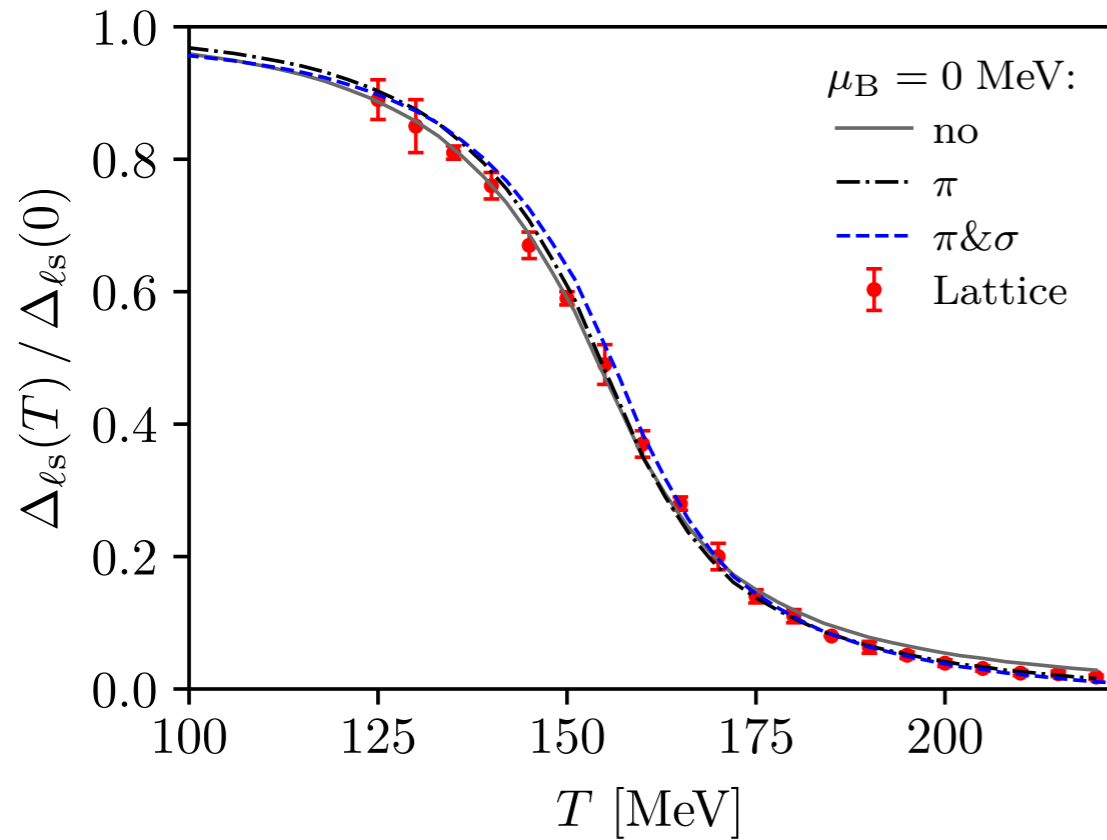
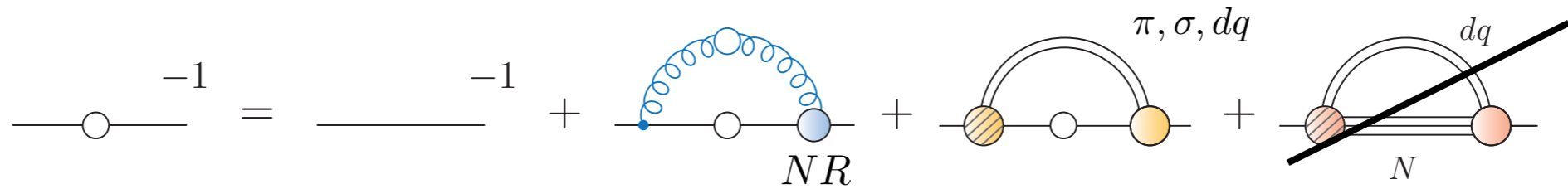


- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

T. D. Cohen, PRL 91 , 222001 (2003)

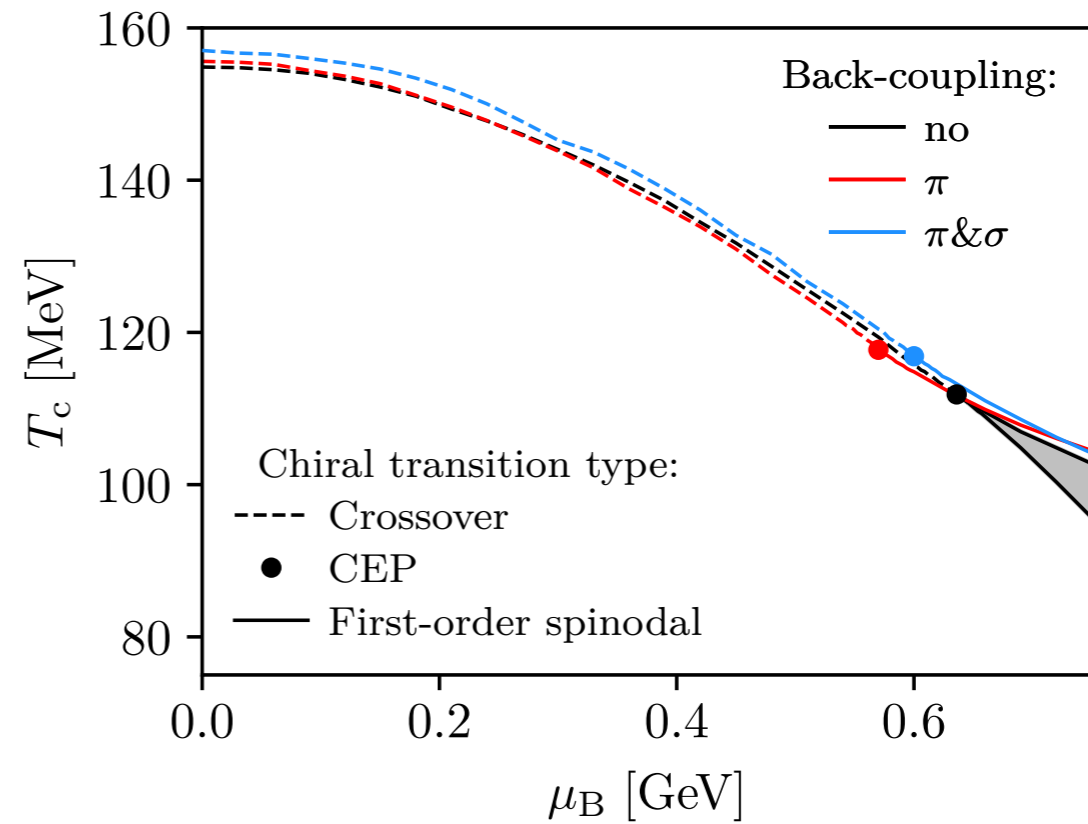
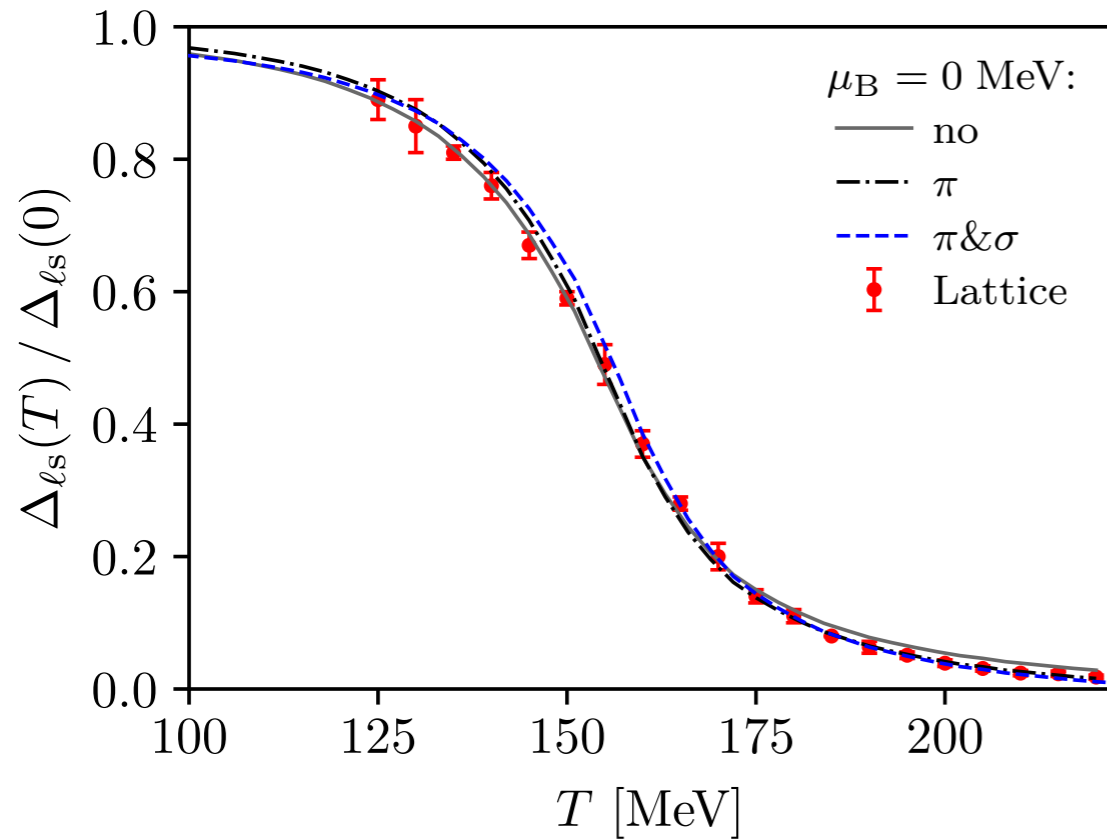
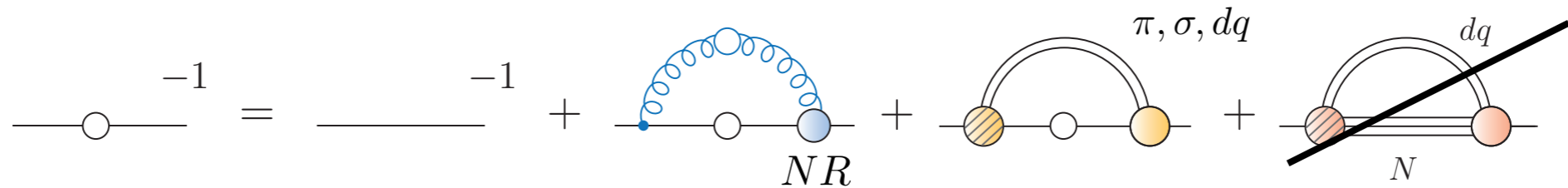
Meson effects on the CEP - results ($N_f=2+1$)



Gunkel, CF, PRD 104 (2021) [2106.08356]

- Vanishing chemical potential: no effect

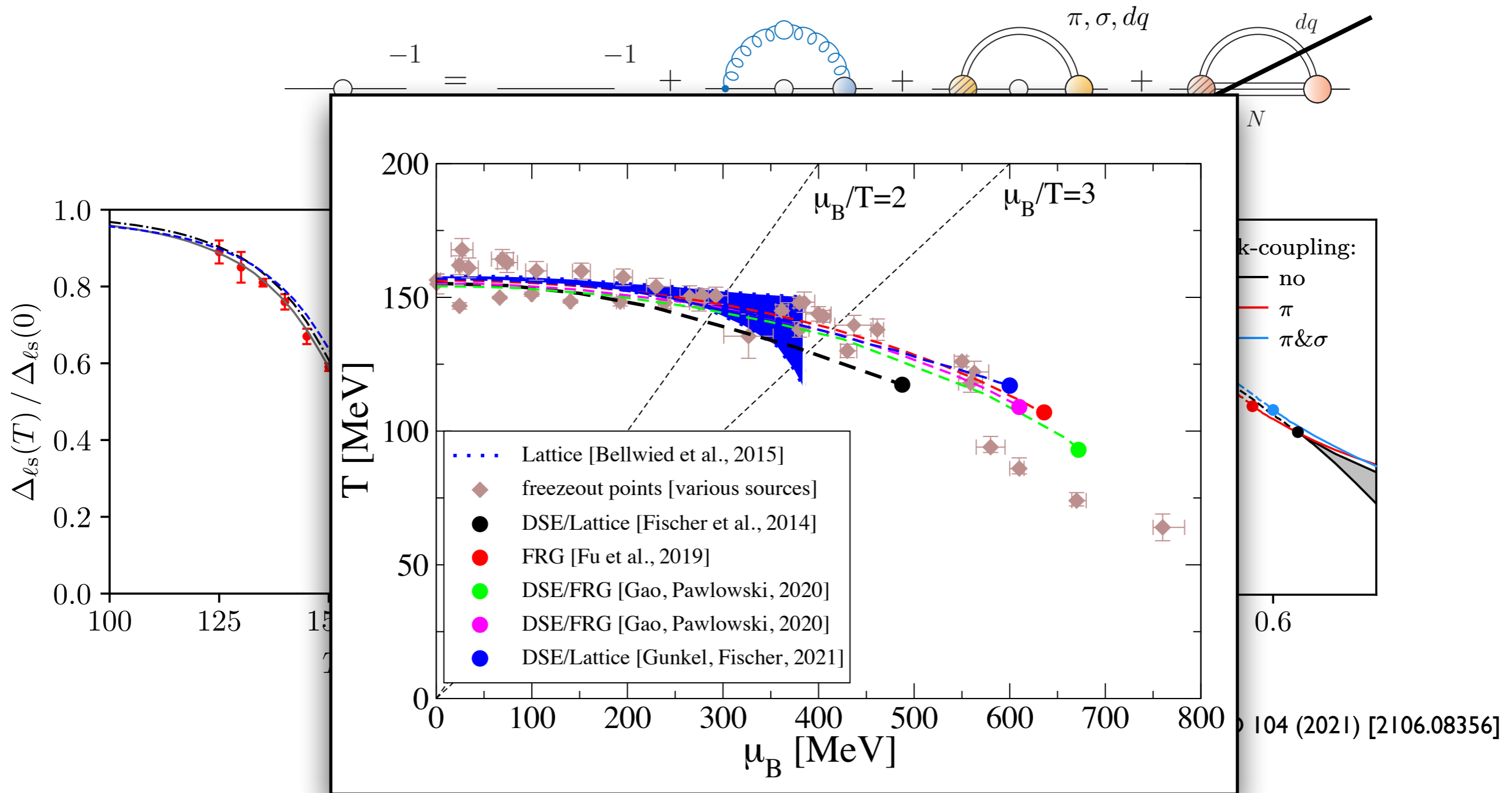
Meson effects on the CEP - results ($N_f=2+1$)



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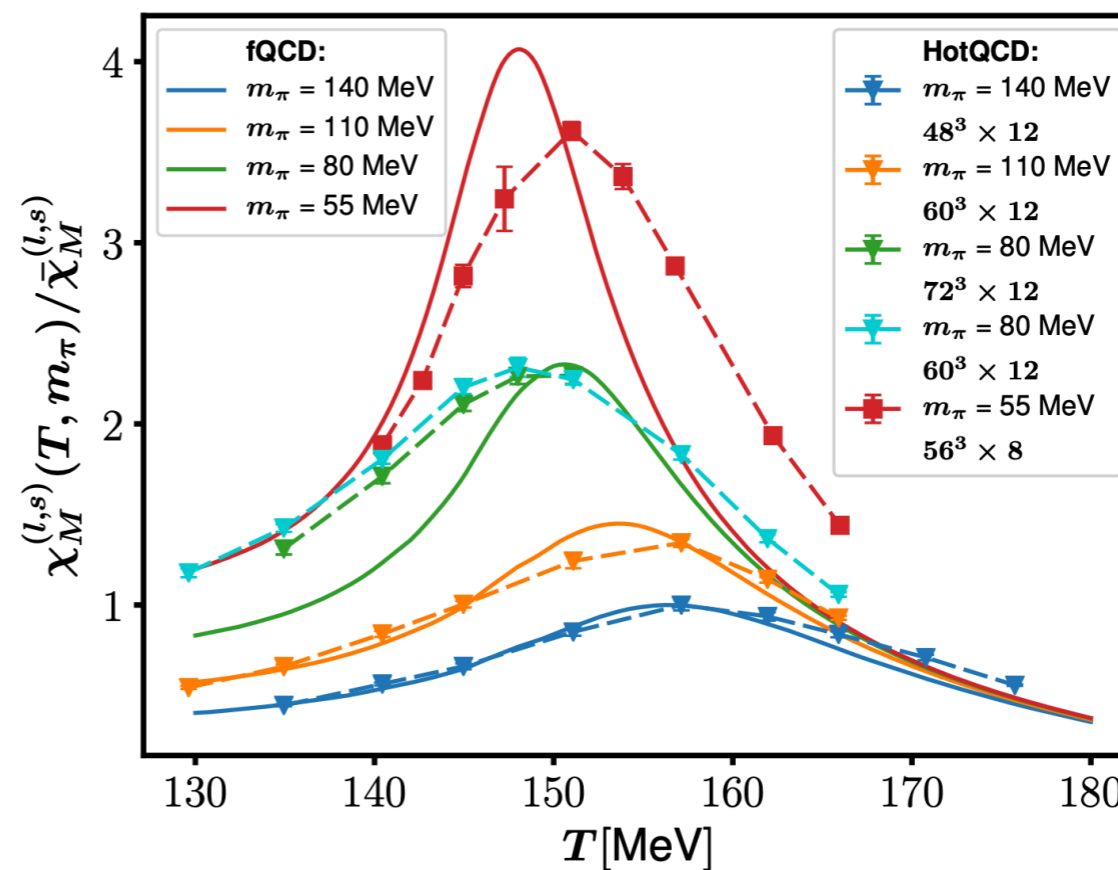
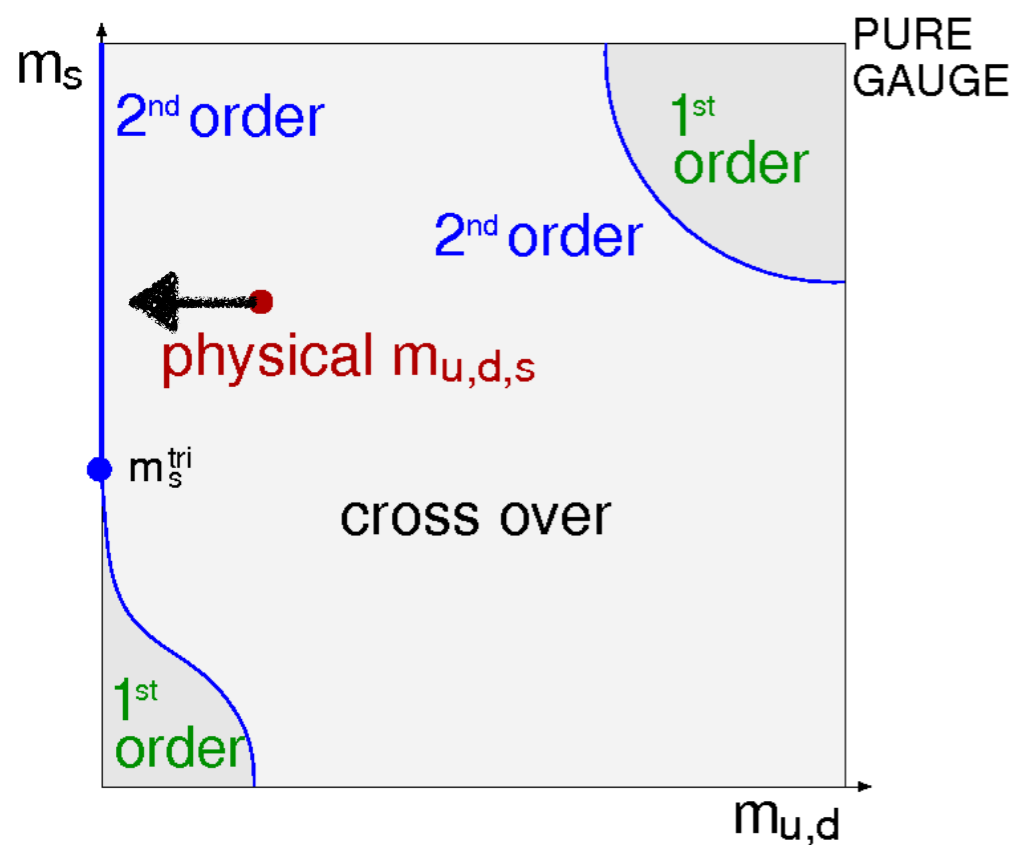
- Vanishing chemical potential: no effect
- small effects on location of CEP
- μ -dependence of meson wave function taken into account

Meson effects on the CEP - results ($N_f=2+1$)



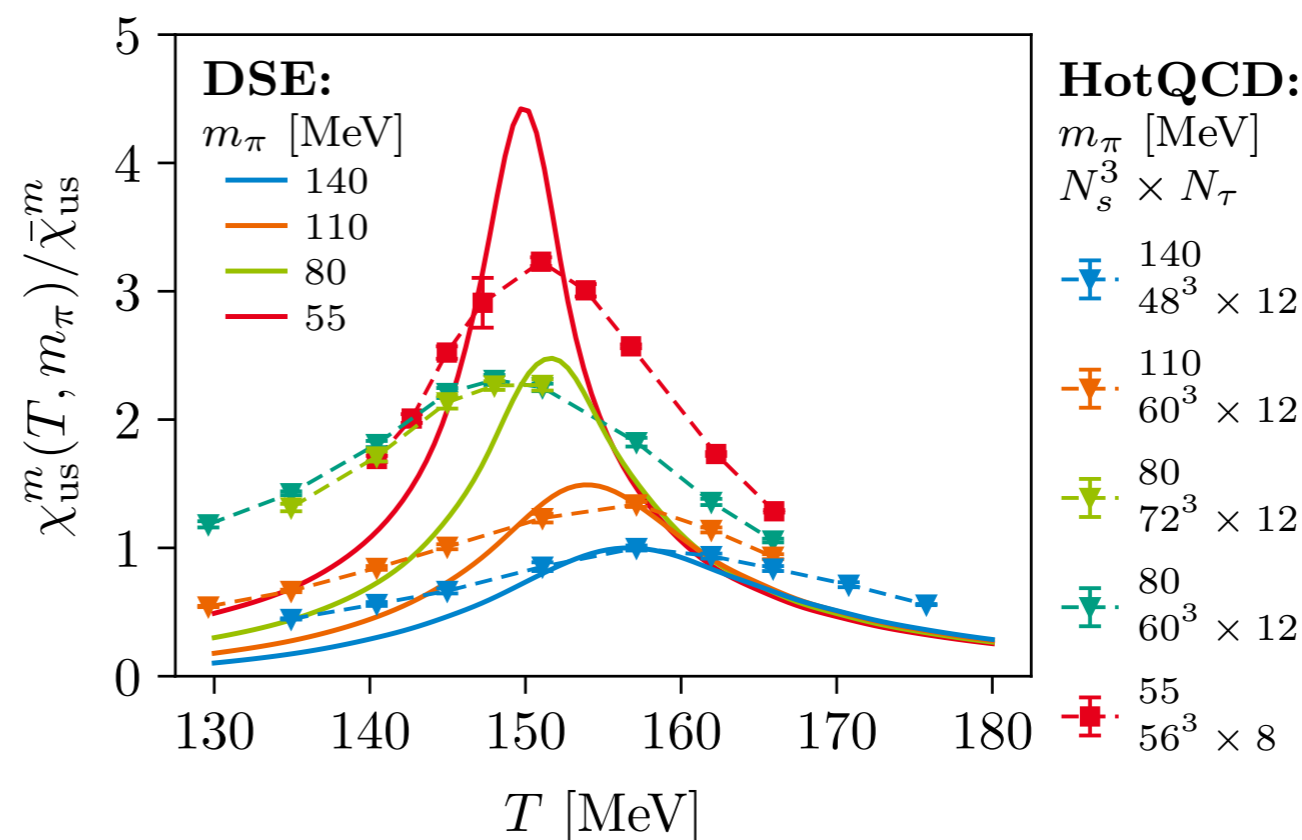
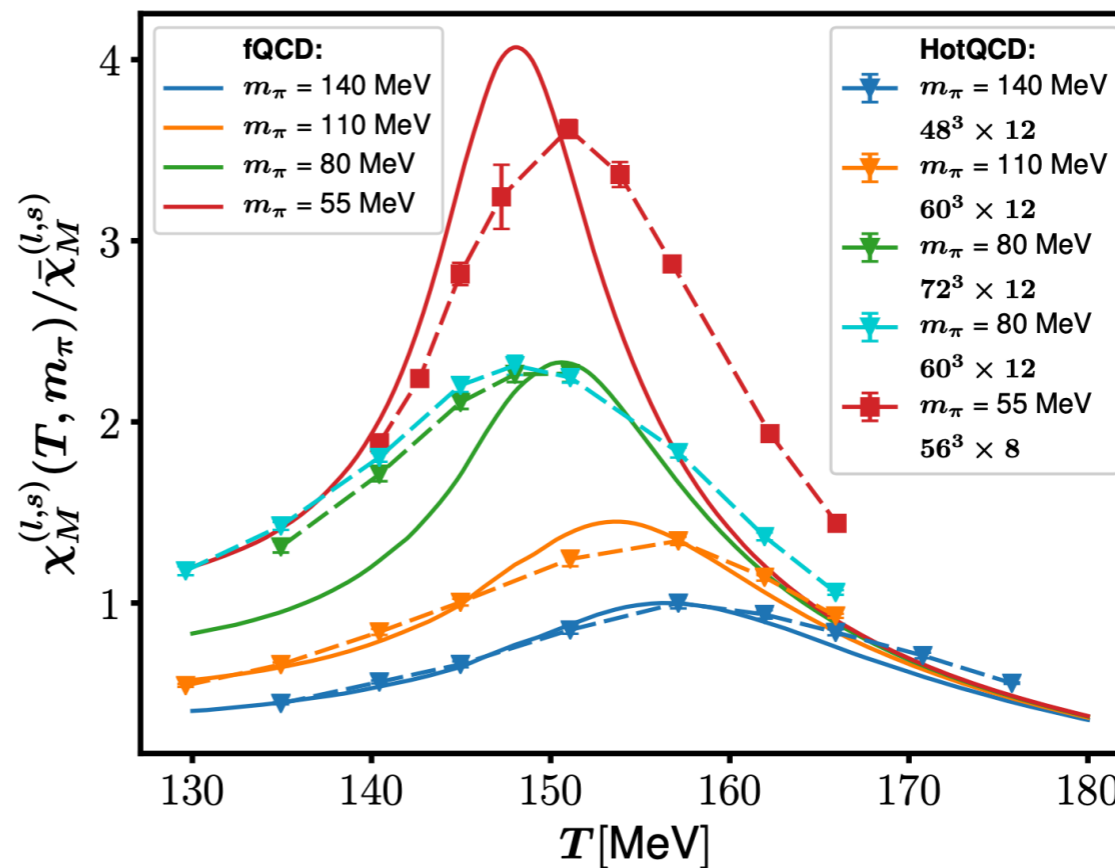
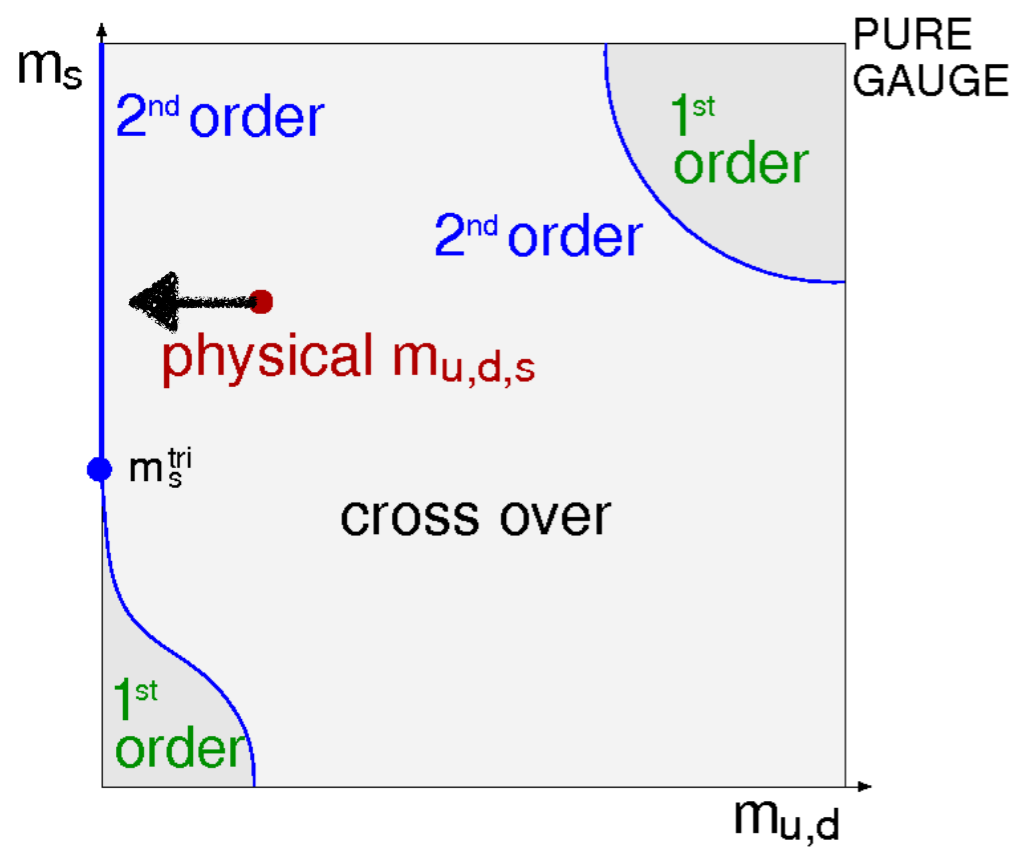
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Towards the chiral limit...



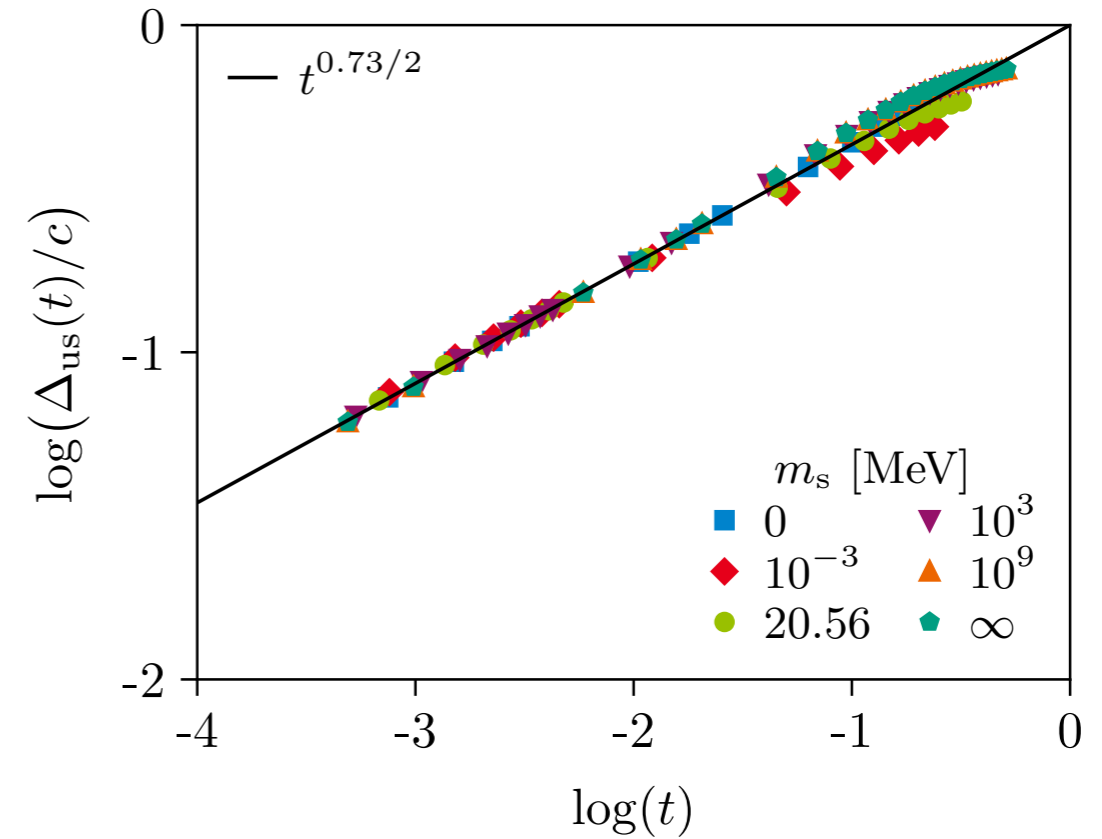
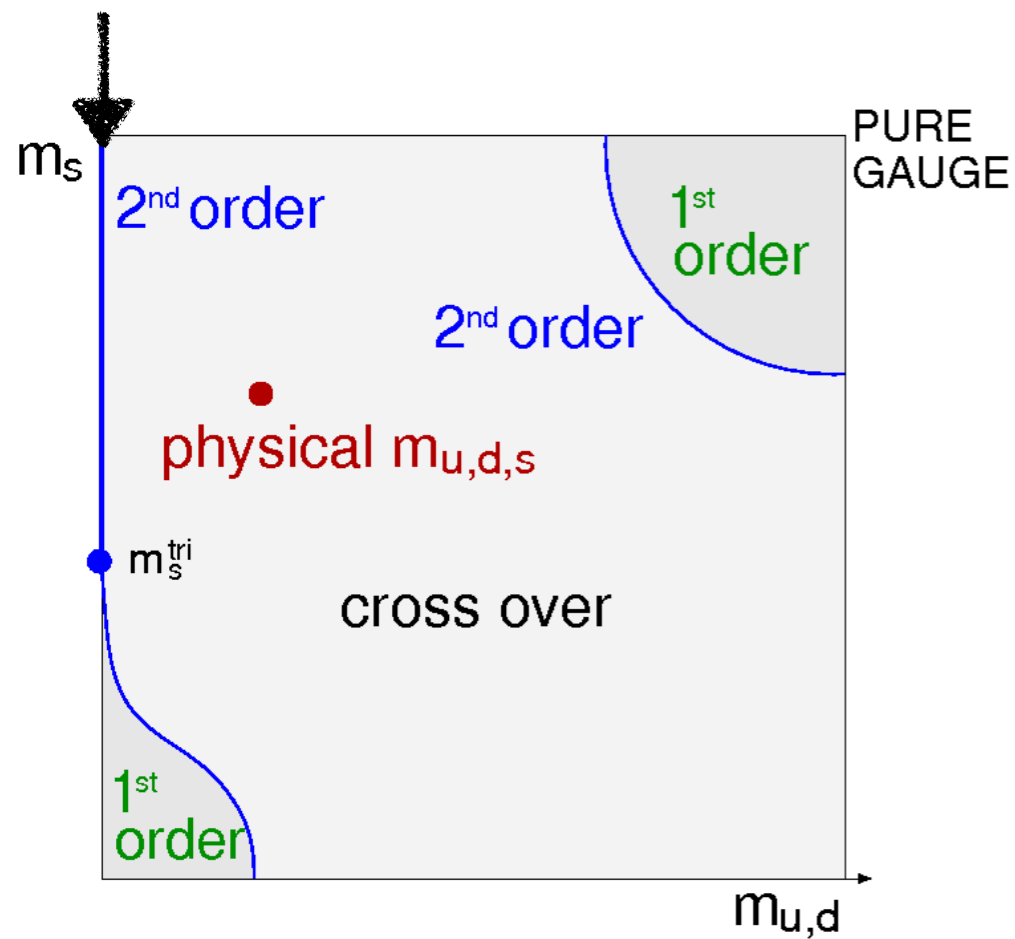
HotQCD: Ding et al. PRL 123, 062002 (2019)
 FRG: Braun et al, PRD 102 (2020) 5, 056010
 DSE: Bernhardt and CF, in preparation

Towards the chiral limit...



HotQCD: Ding et al. PRL 123, 062002 (2019)
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At the chiral limit...



reproduce CF and Mueller, PRD 84 (2011) 054013

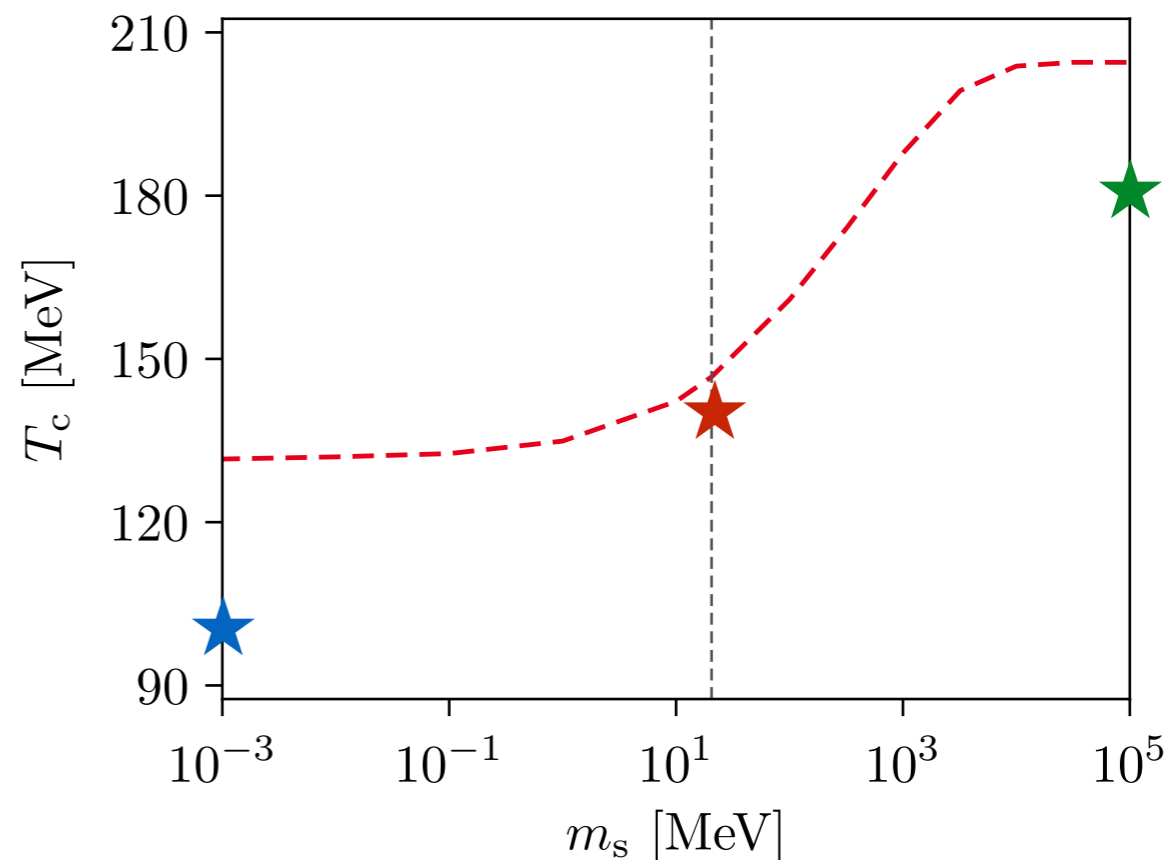
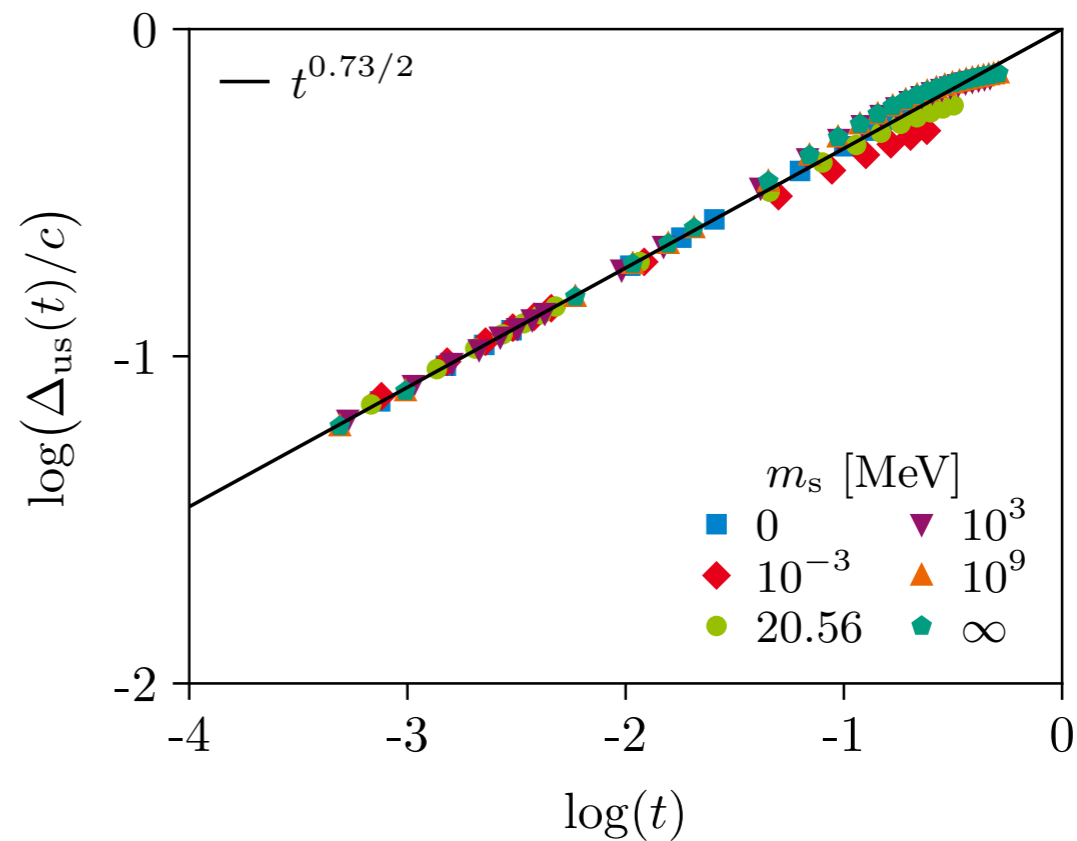
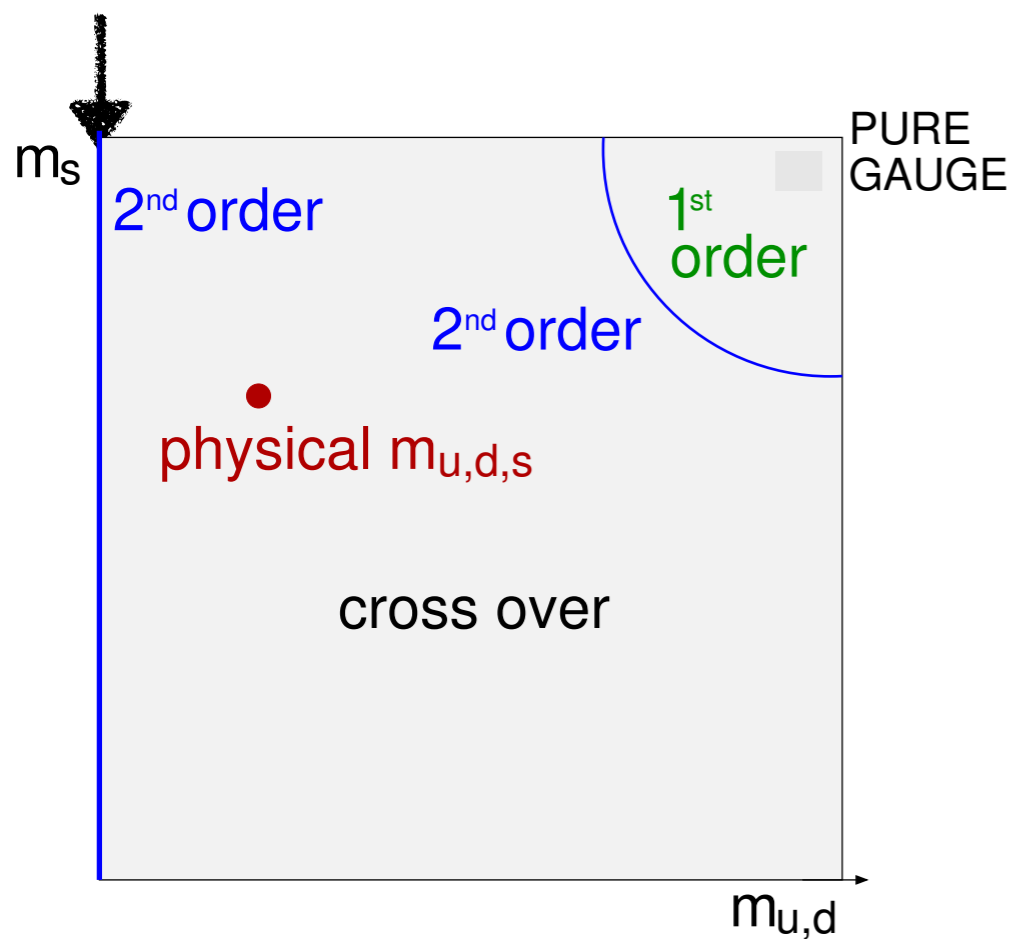
DSE: Bernhardt and CF, in preparation

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510

Ding et al. PRL 123, 062002 (2019)

Bornyakov et al. PRD 82, 014504 (2010)

At the chiral limit...



reproduce CF and Mueller, PRD 84 (2011) 054013

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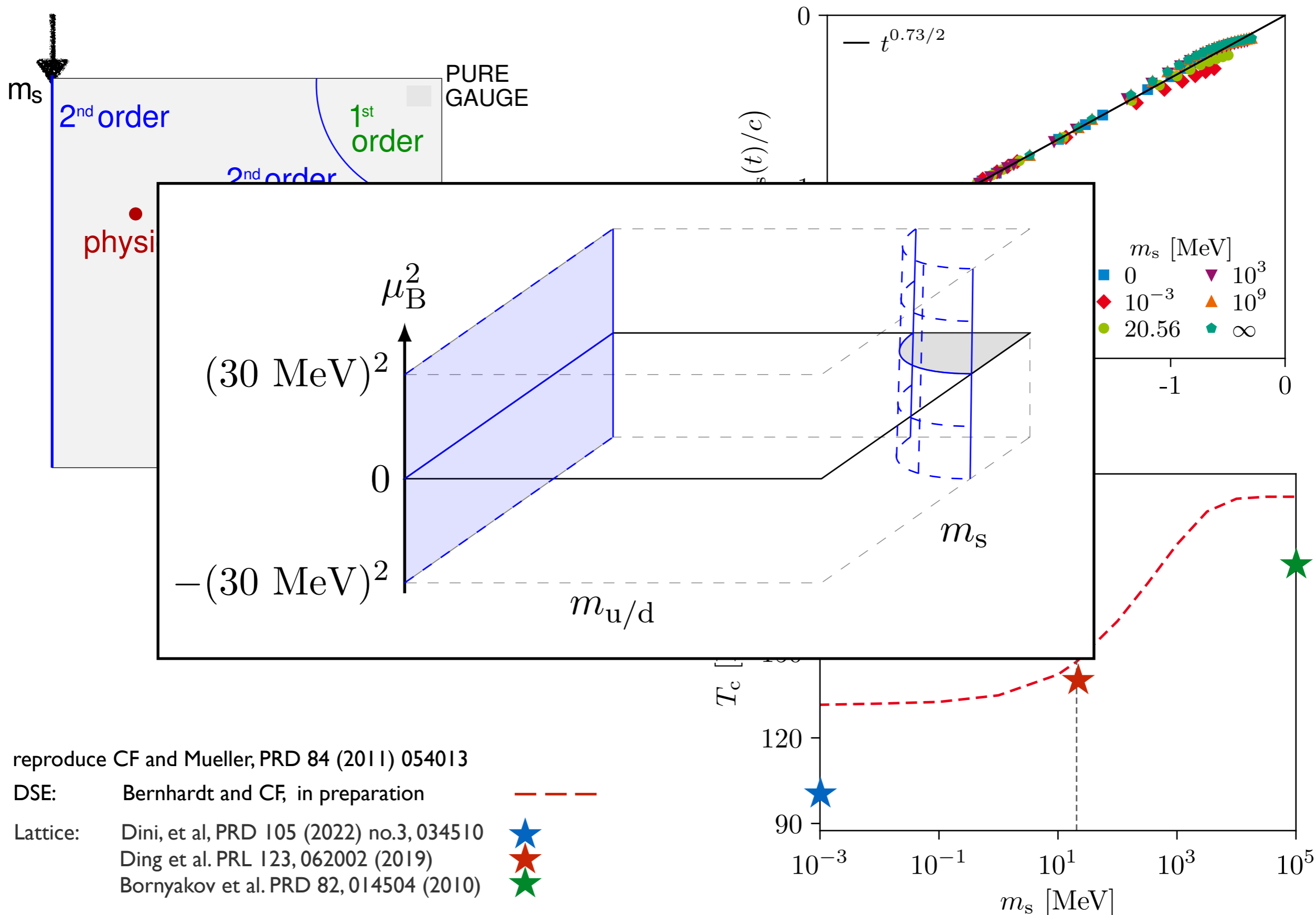
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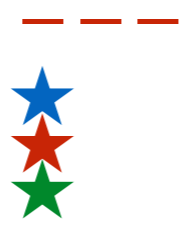
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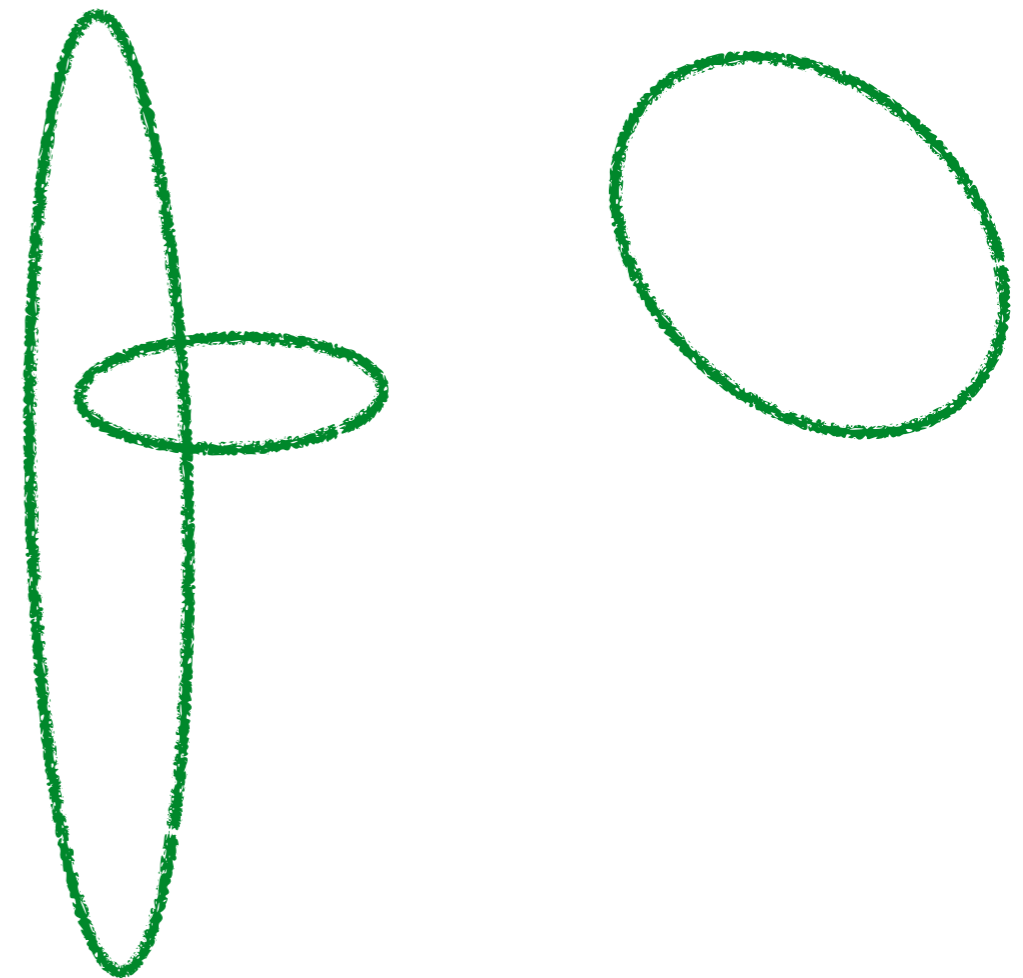
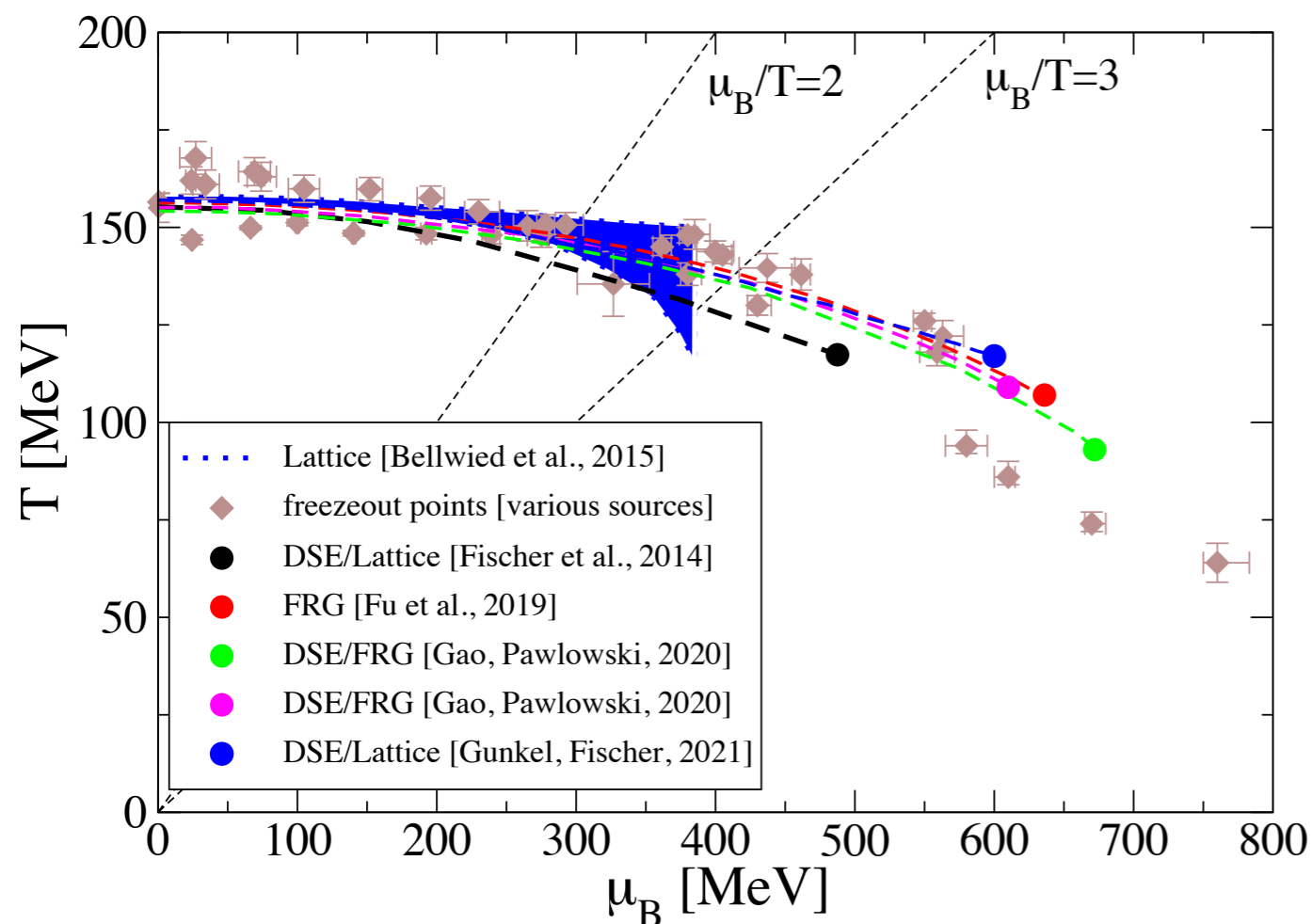


Summary: QCD with functional methods

Main goals:

- **one** framework for all areas of hadron physics: mesons, baryons, 'exotic states', form factors, hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

Main results:

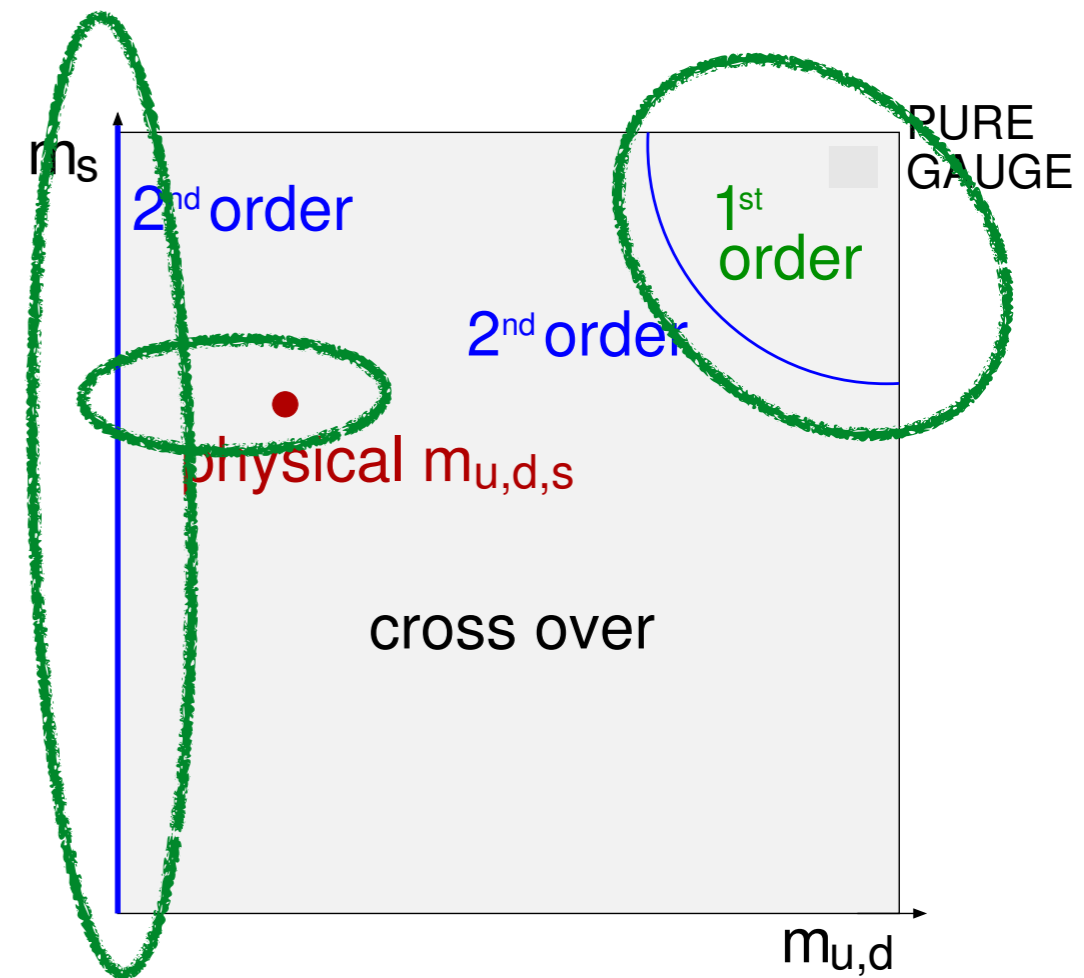
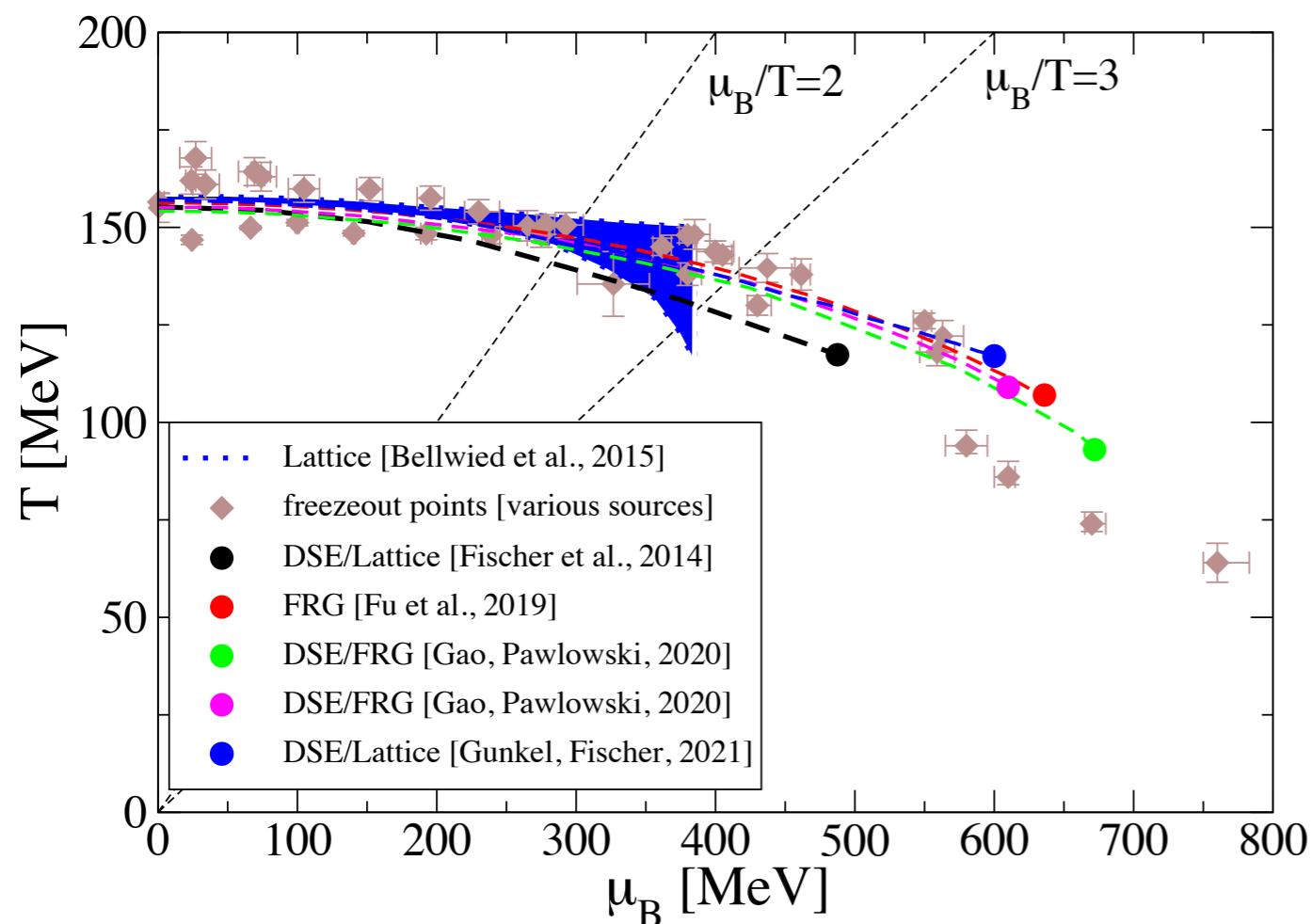


Summary: QCD with functional methods

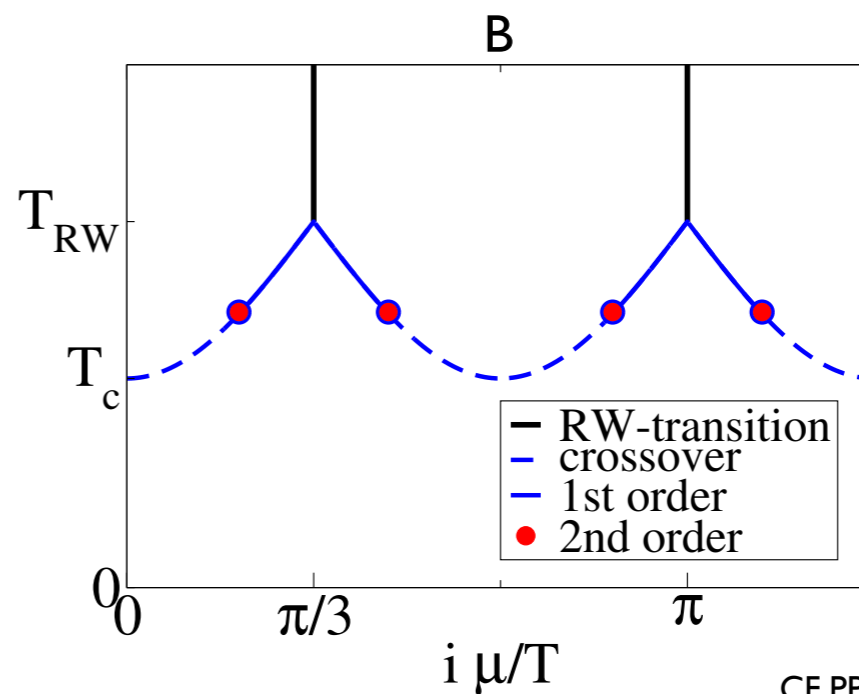
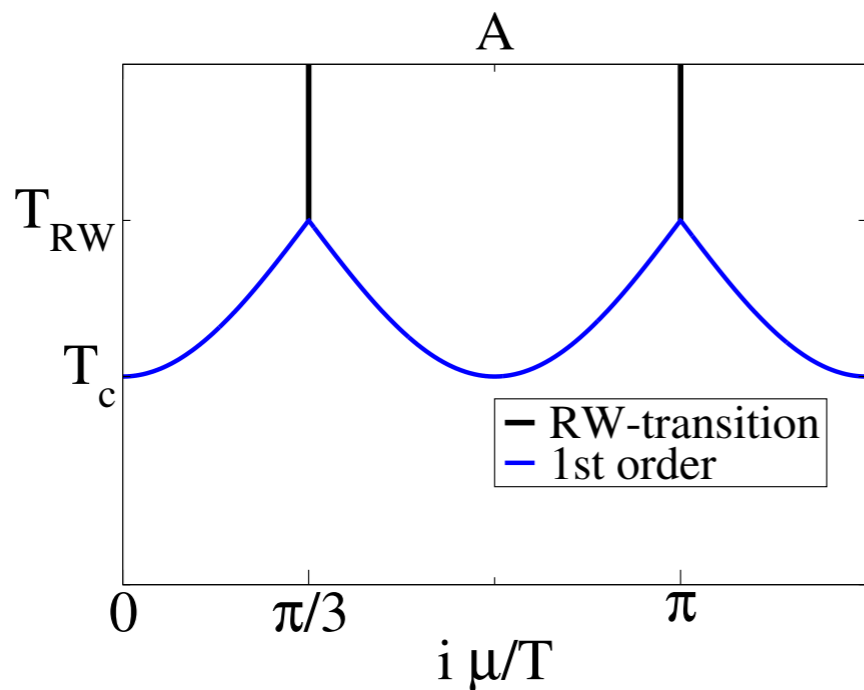
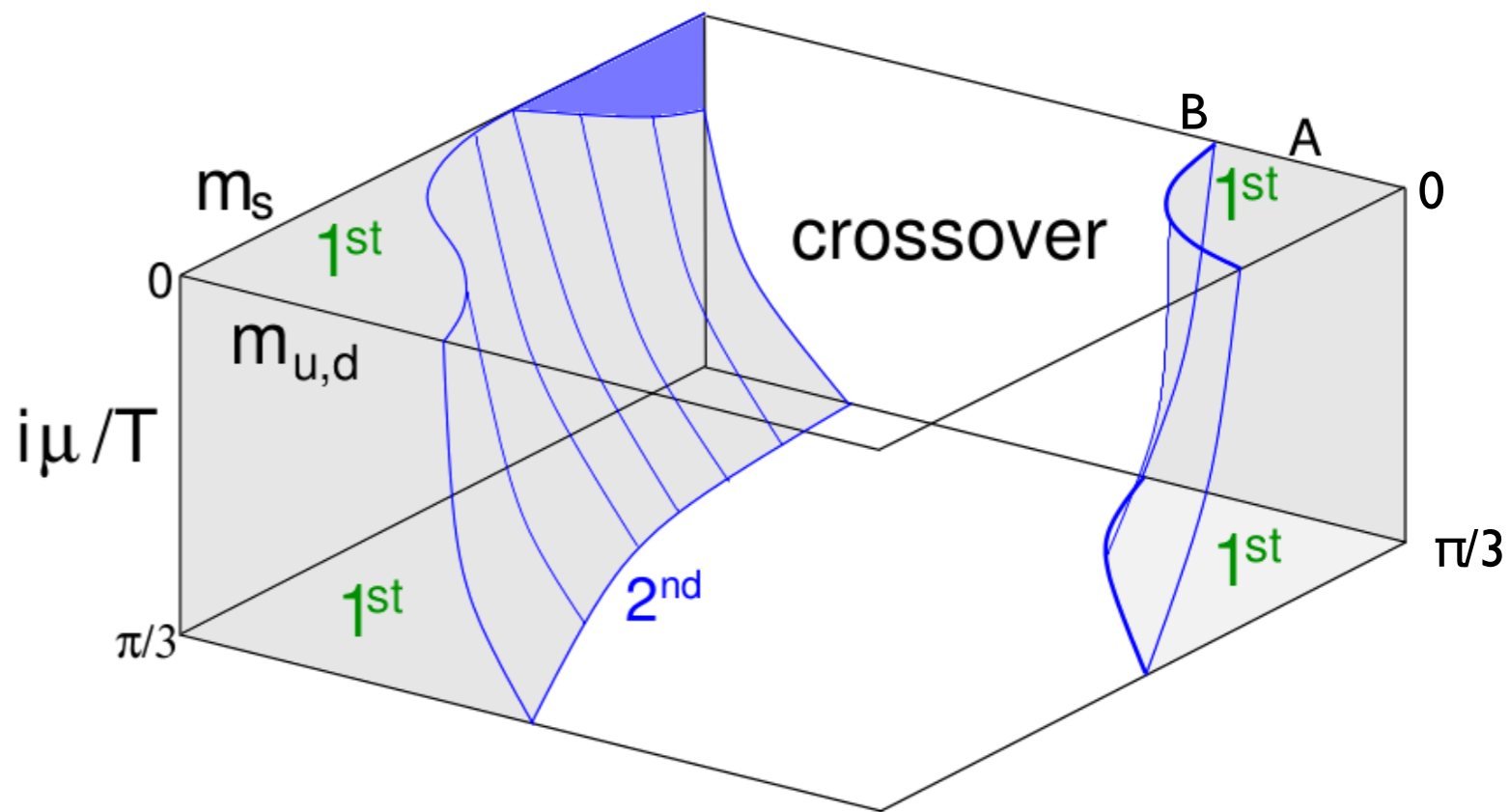
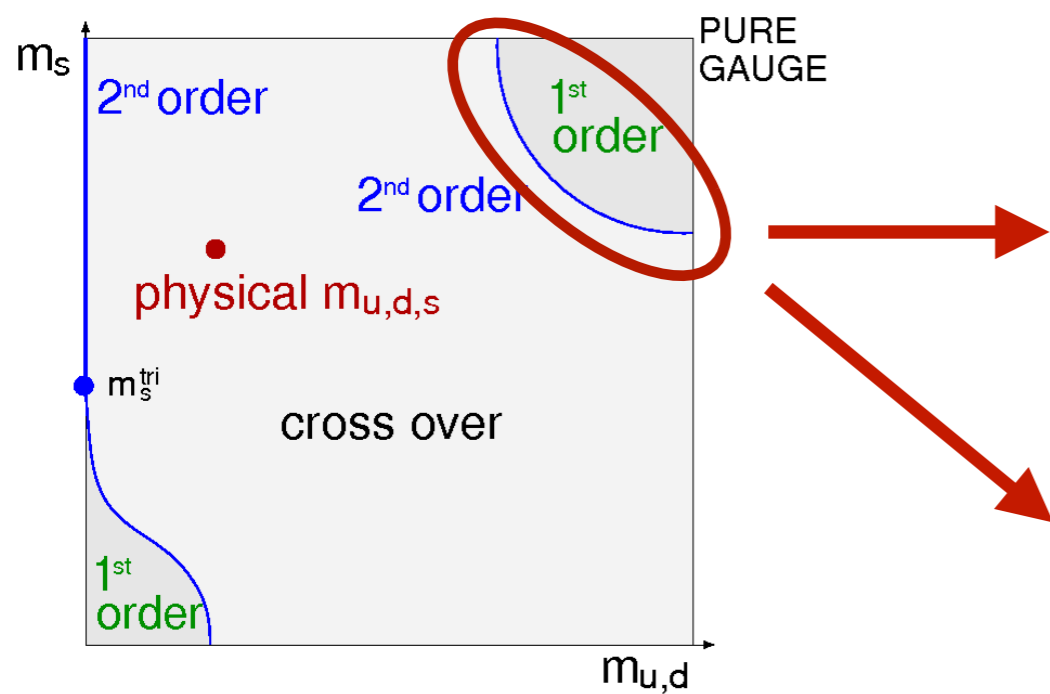
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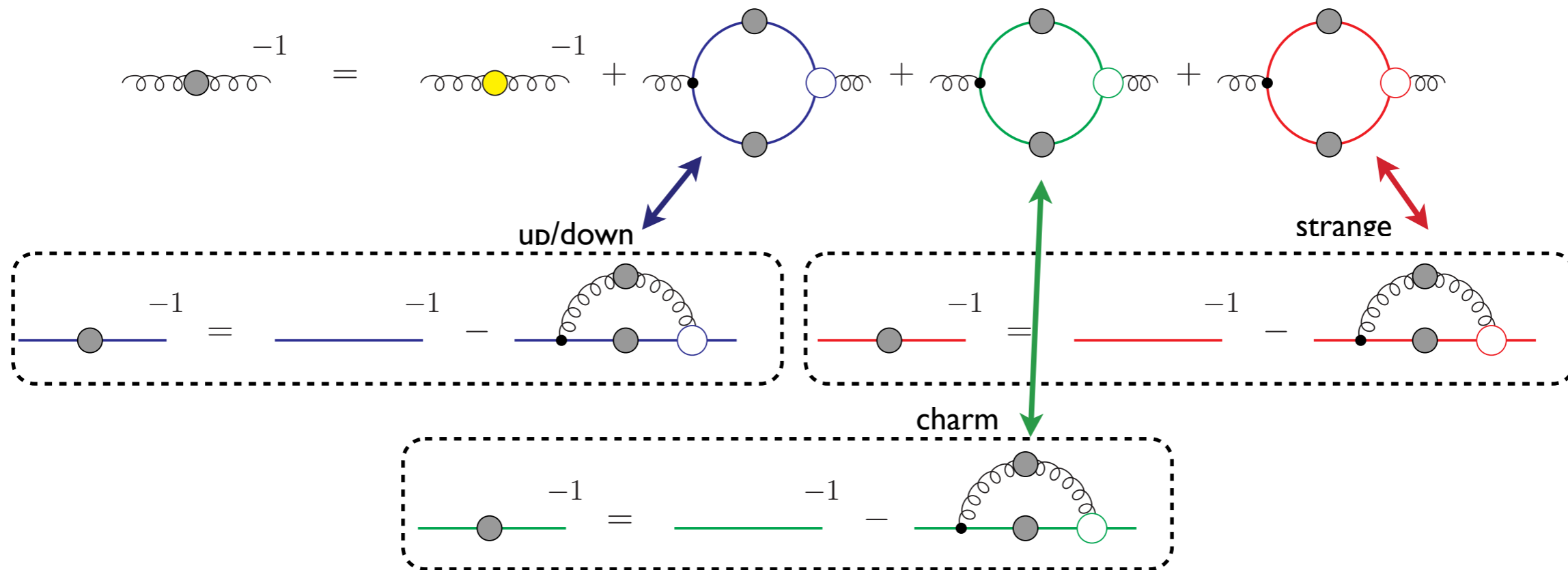


Critical line/surface for heavy quarks



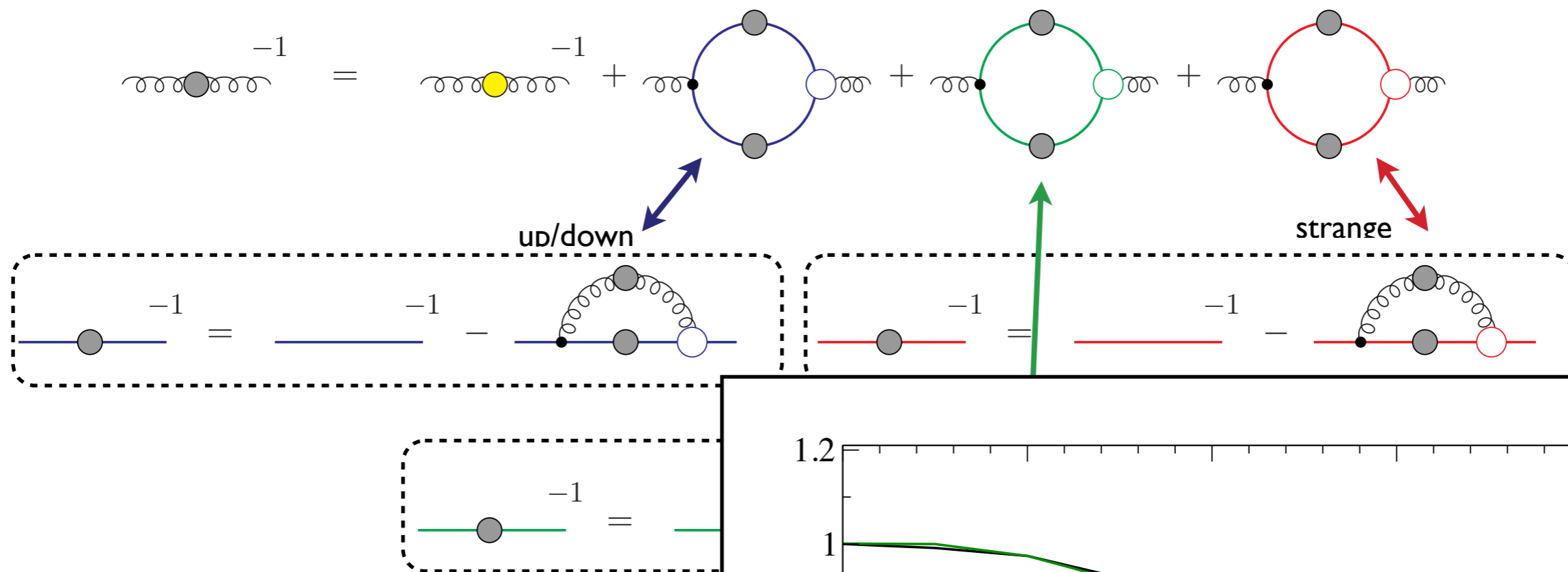
CF, PPNP 105 (2019) [1810.12938]

$N_f=2+1+1$: effects of charm

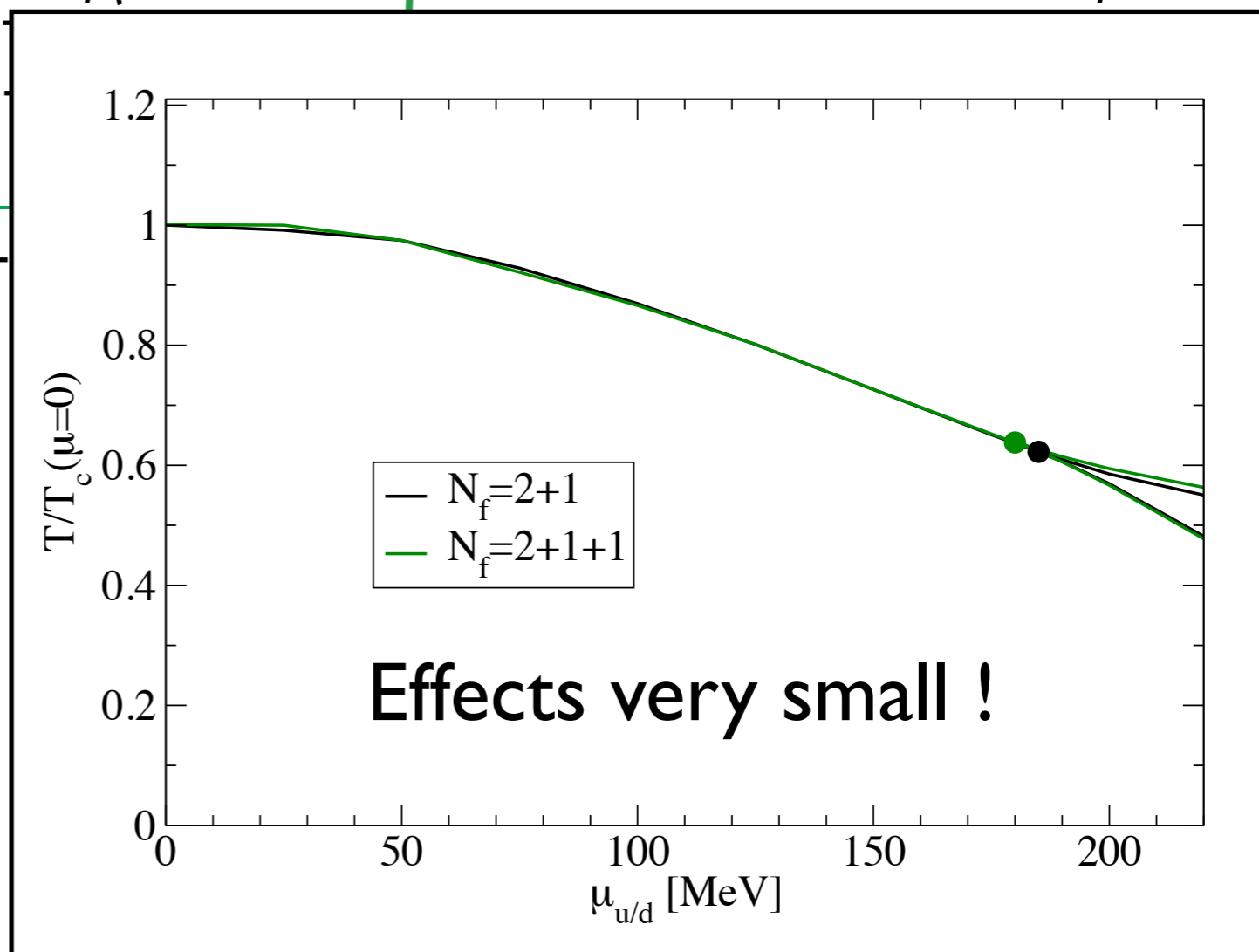


- Physical up/down, strange and **charm quark masses**
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*

$N_f=2+1+1$: effects of charm



- Physical up/down, strange and **charm quark masses**
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- *no lattice or model results available yet*



CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Quark chemical potentials related to those of conserved charges:

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$

$$\mu_d = \mu_B/3 - \mu_Q/3$$

$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

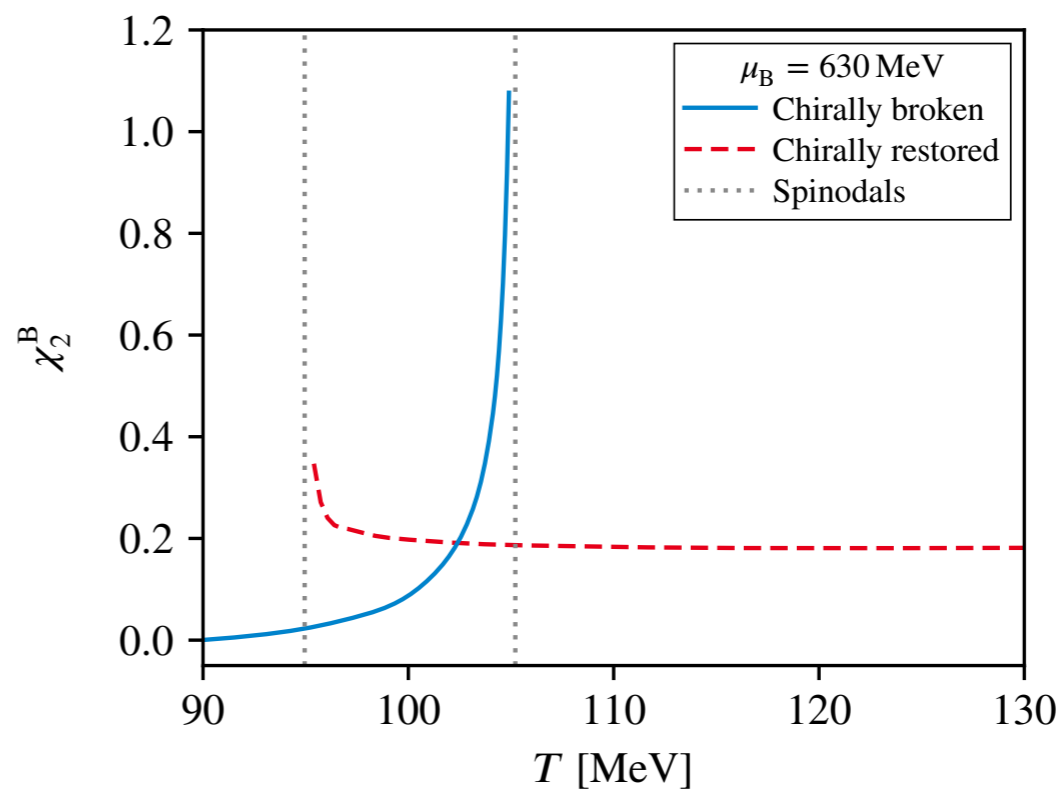
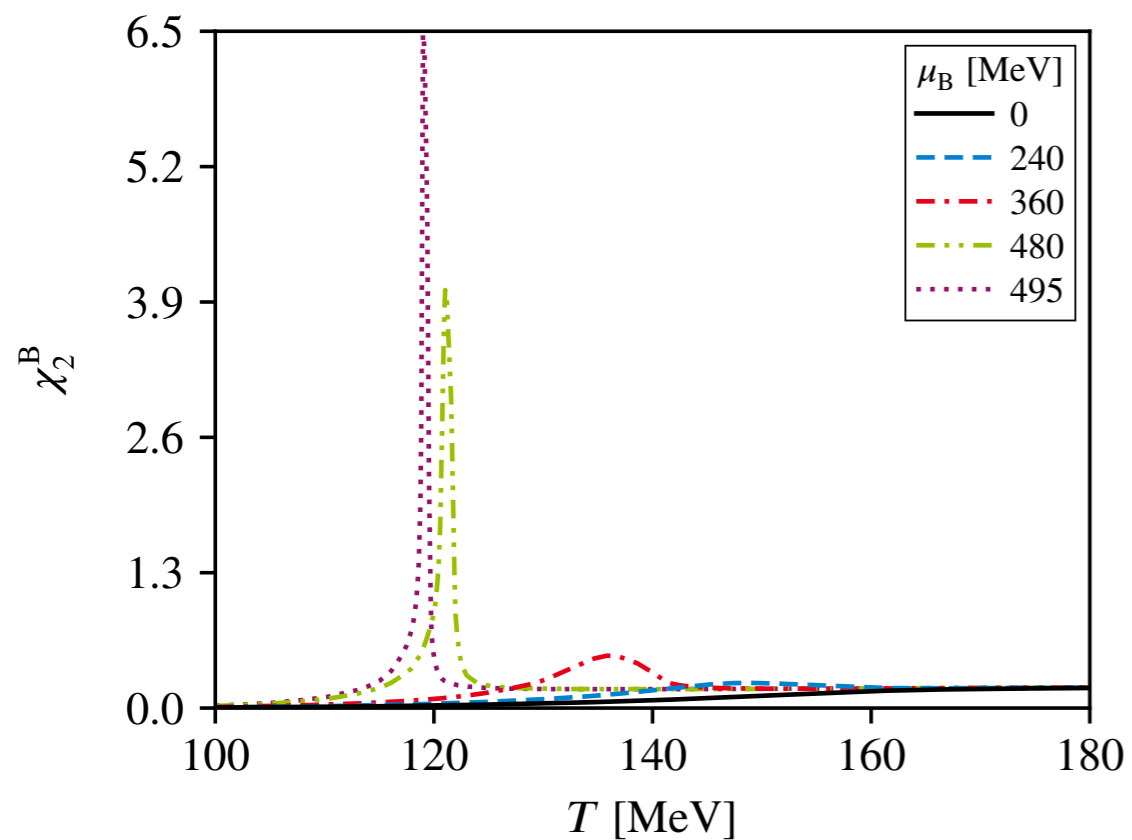
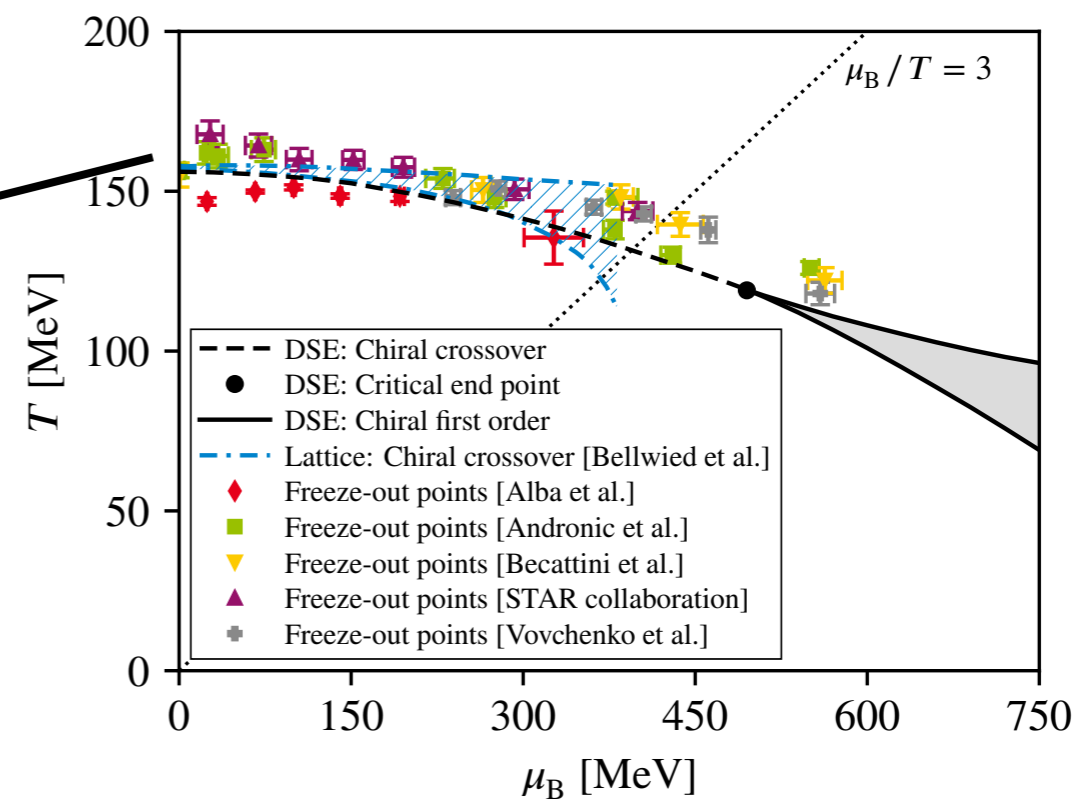
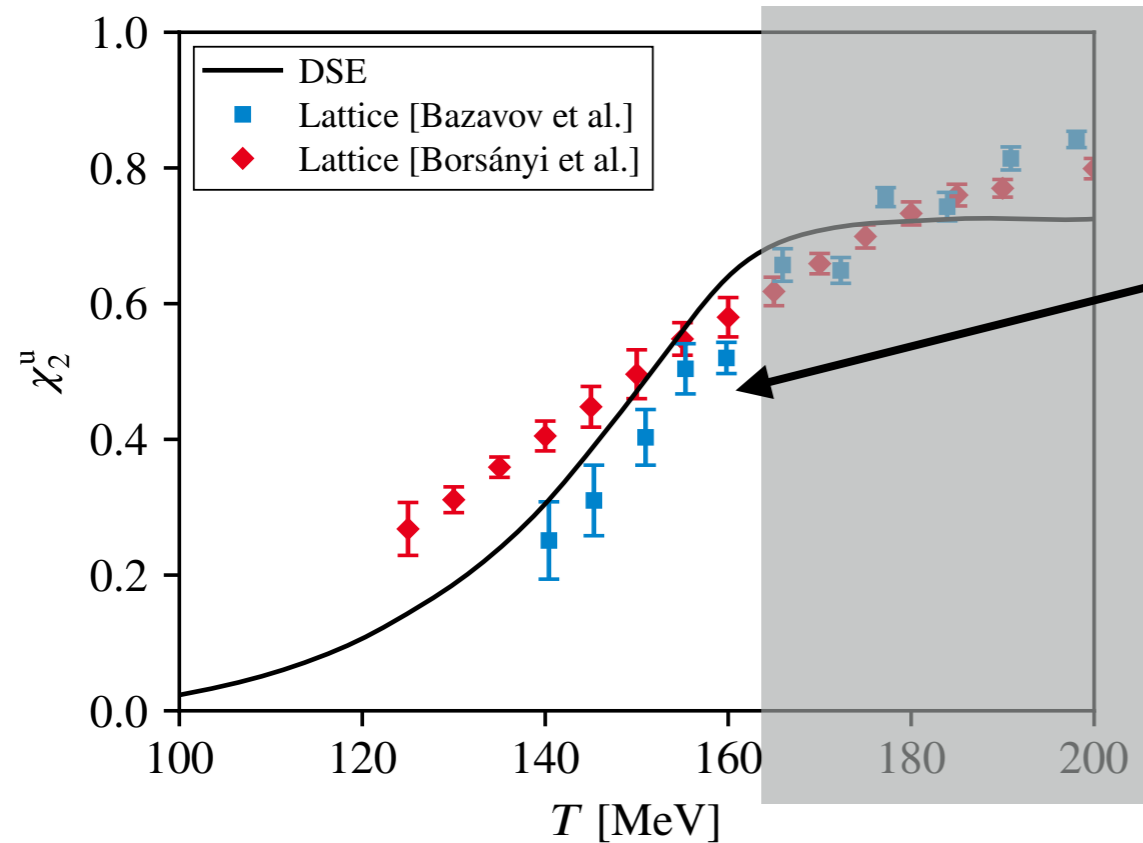
Serve to calculate susceptibilities:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} (p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

Related to cumulants, which can be extracted from experiment:

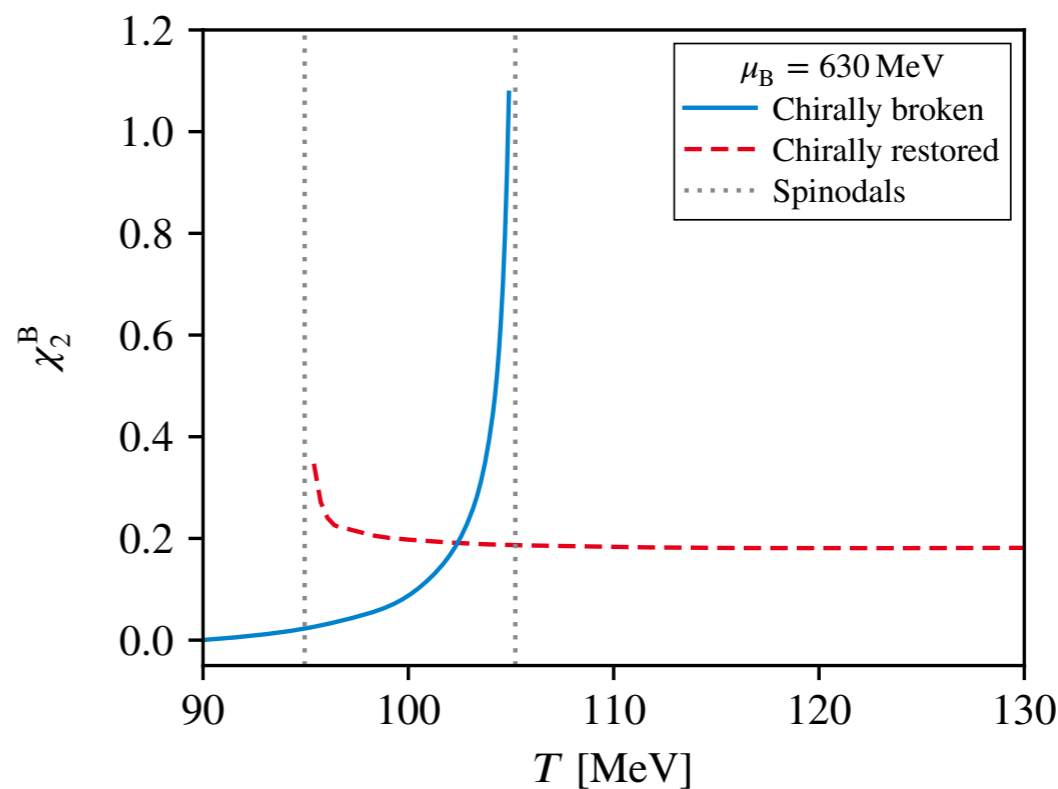
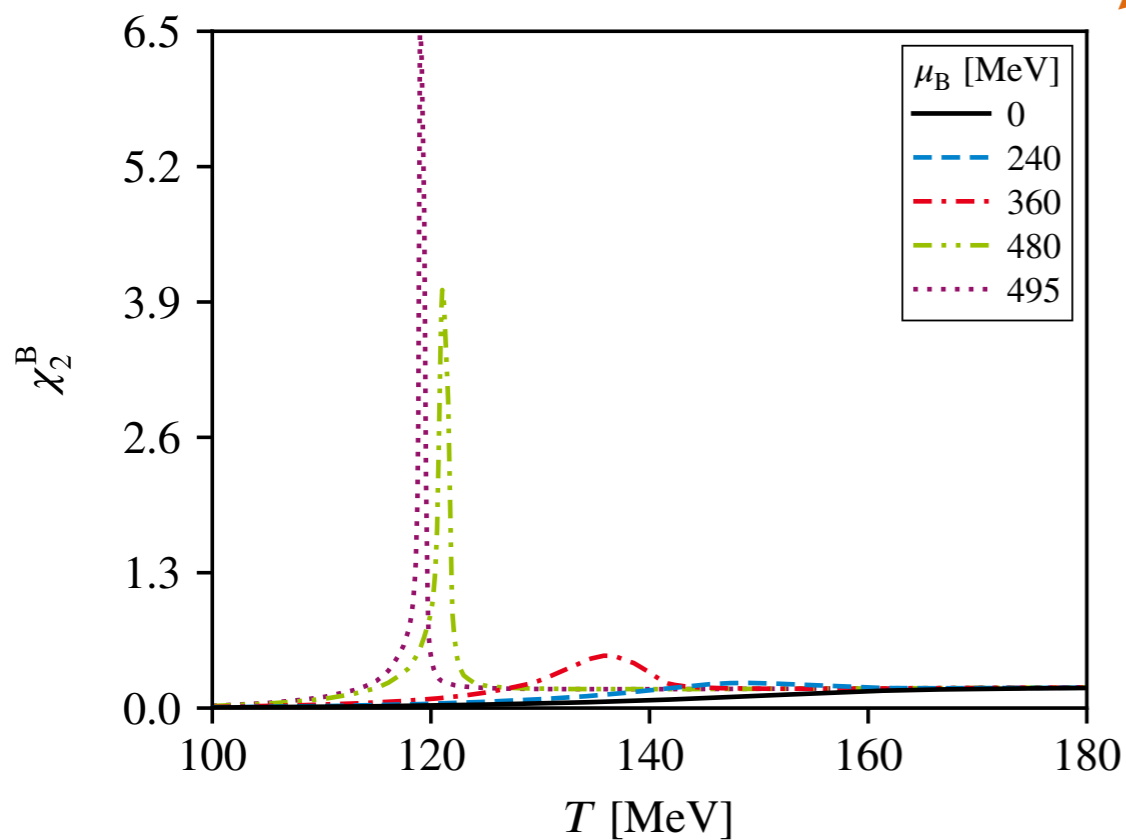
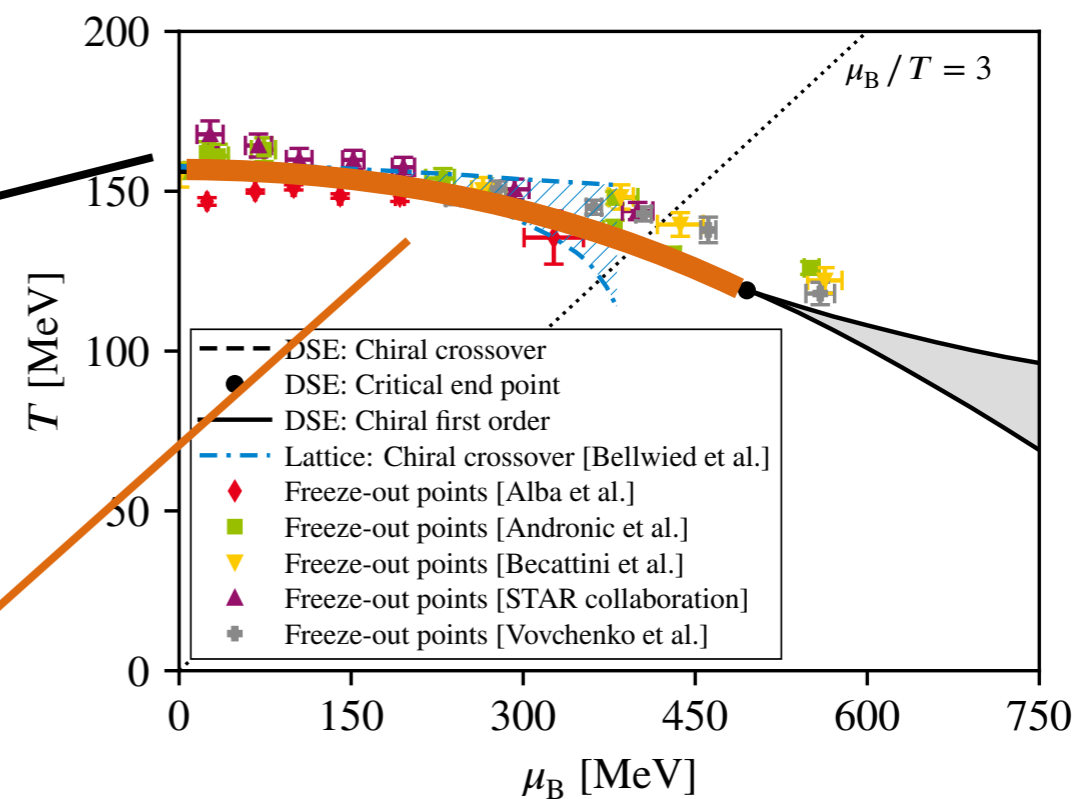
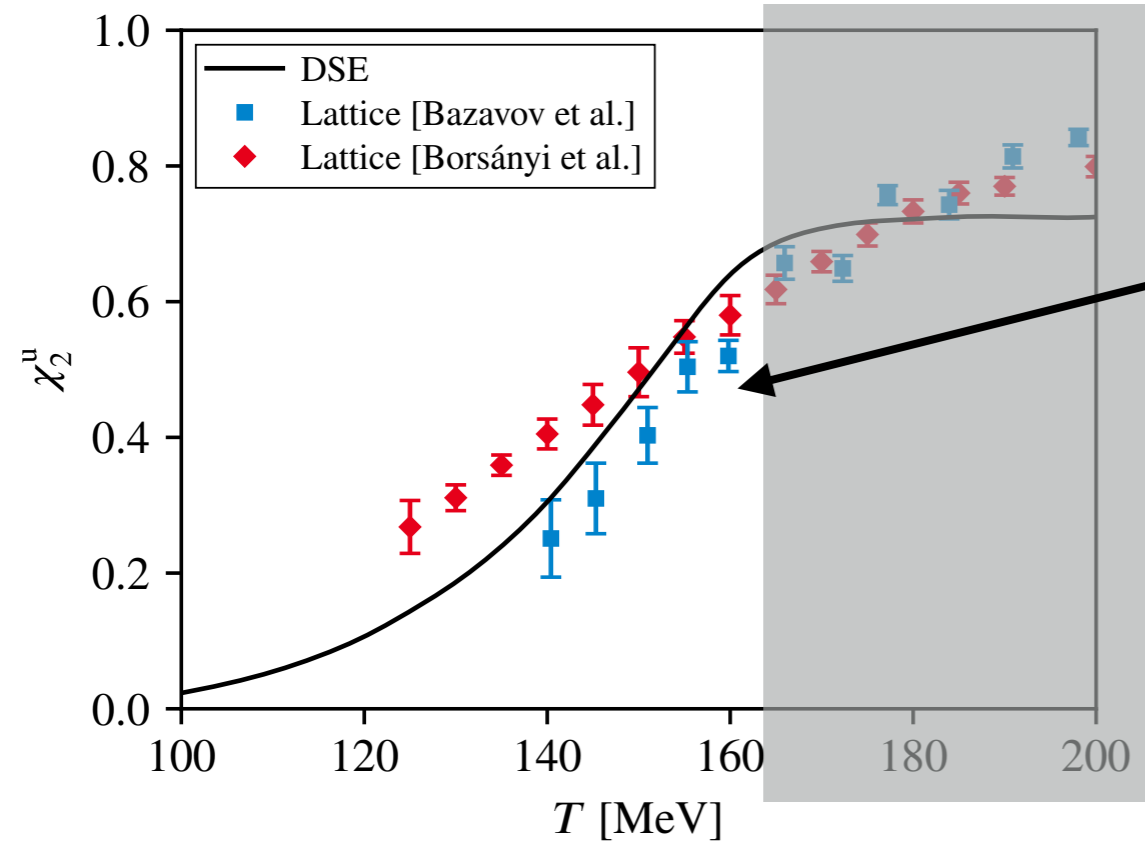
$$C_{lmn}^{BSQ} = VT^3 \chi_{lmn}^{BSQ}$$

Fluctuations at the CEP



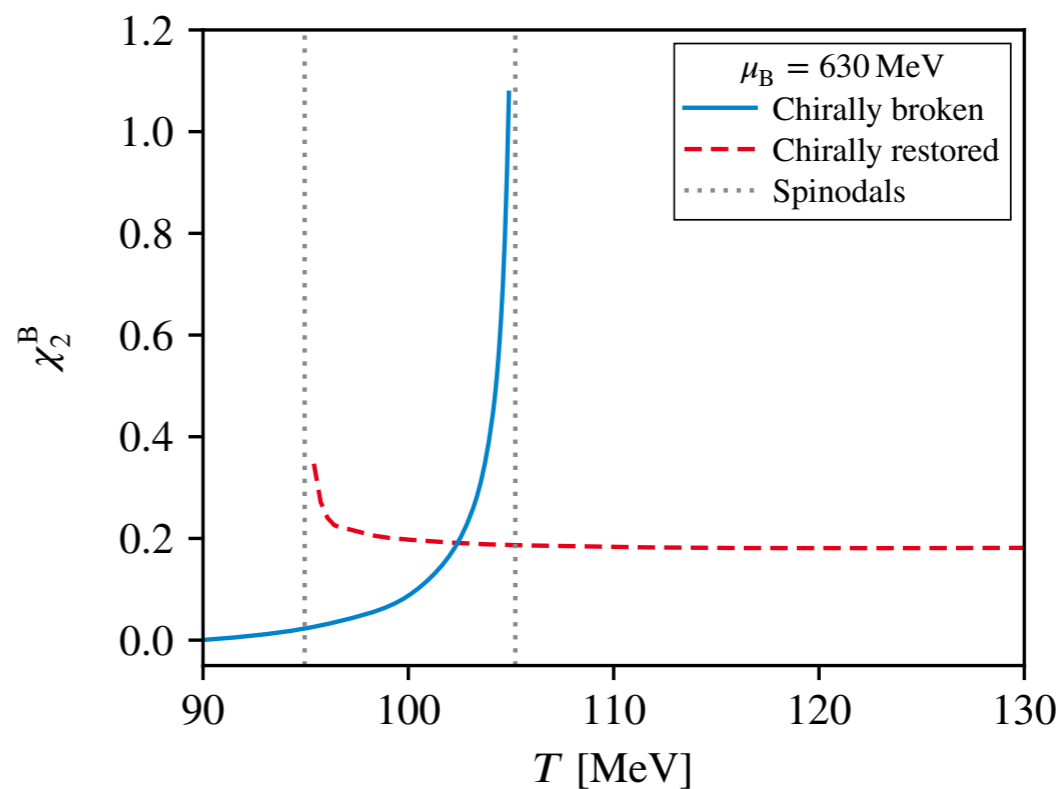
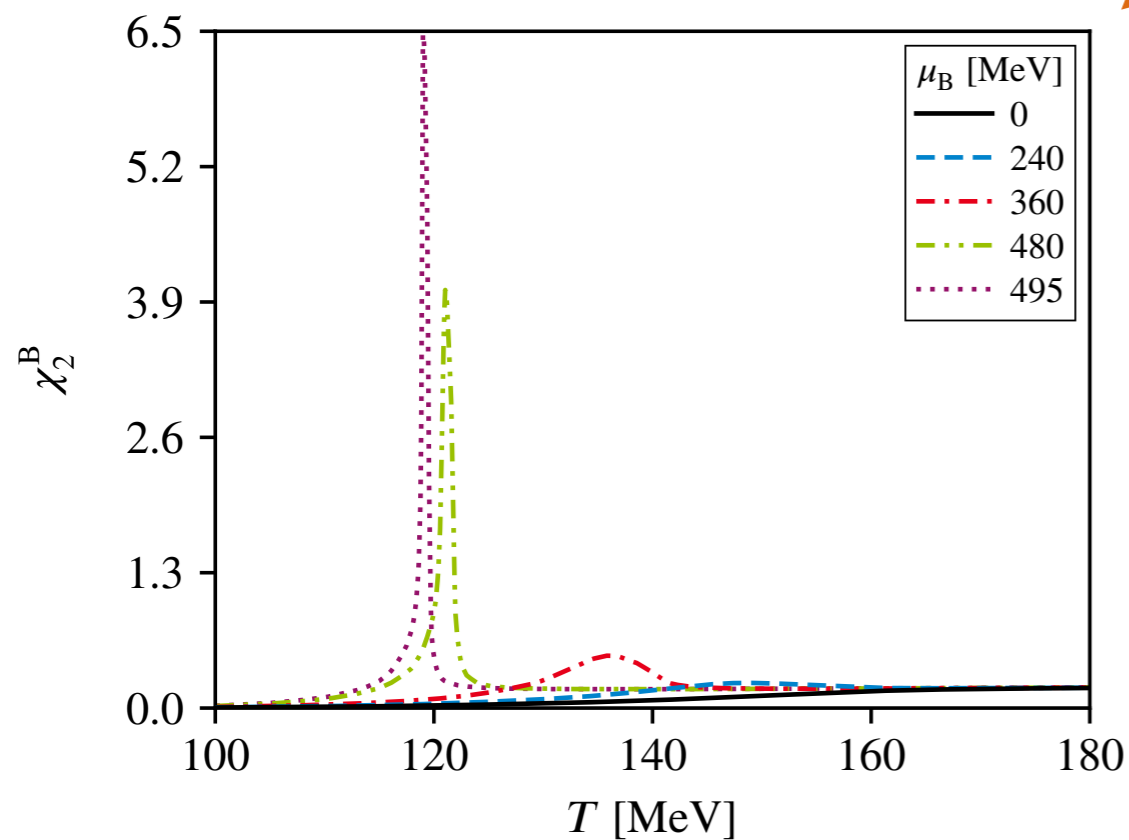
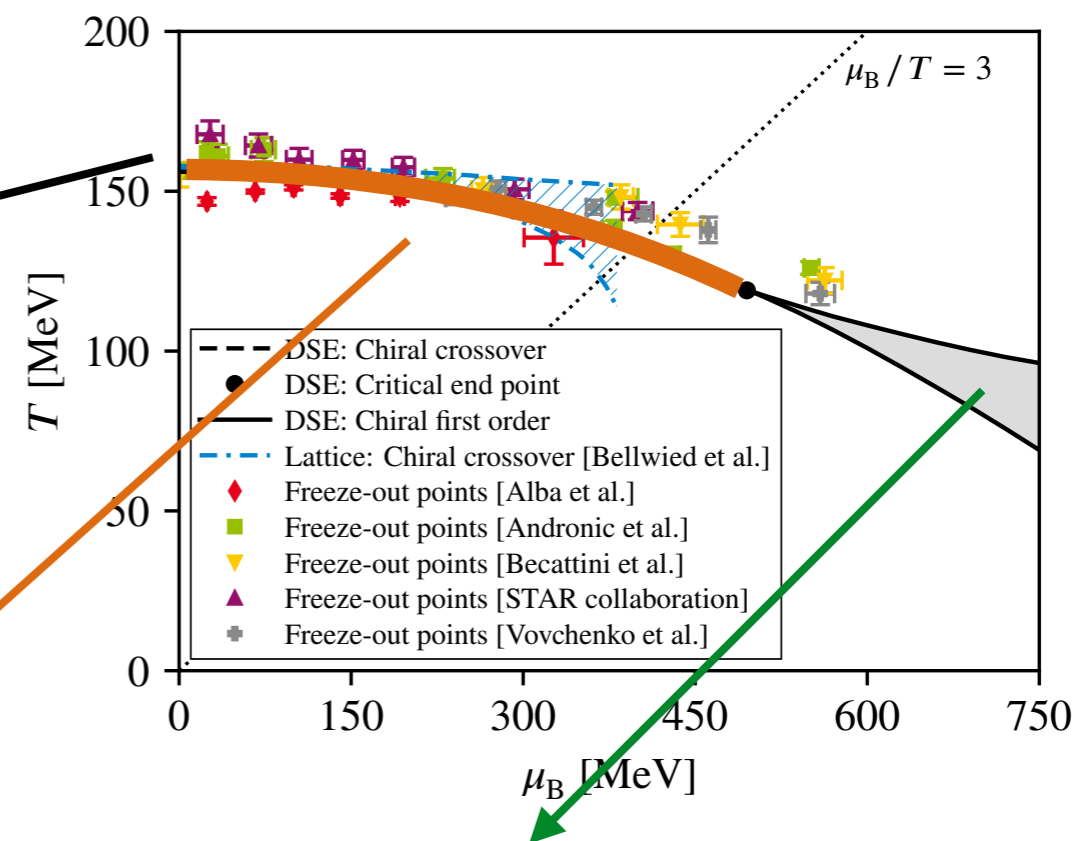
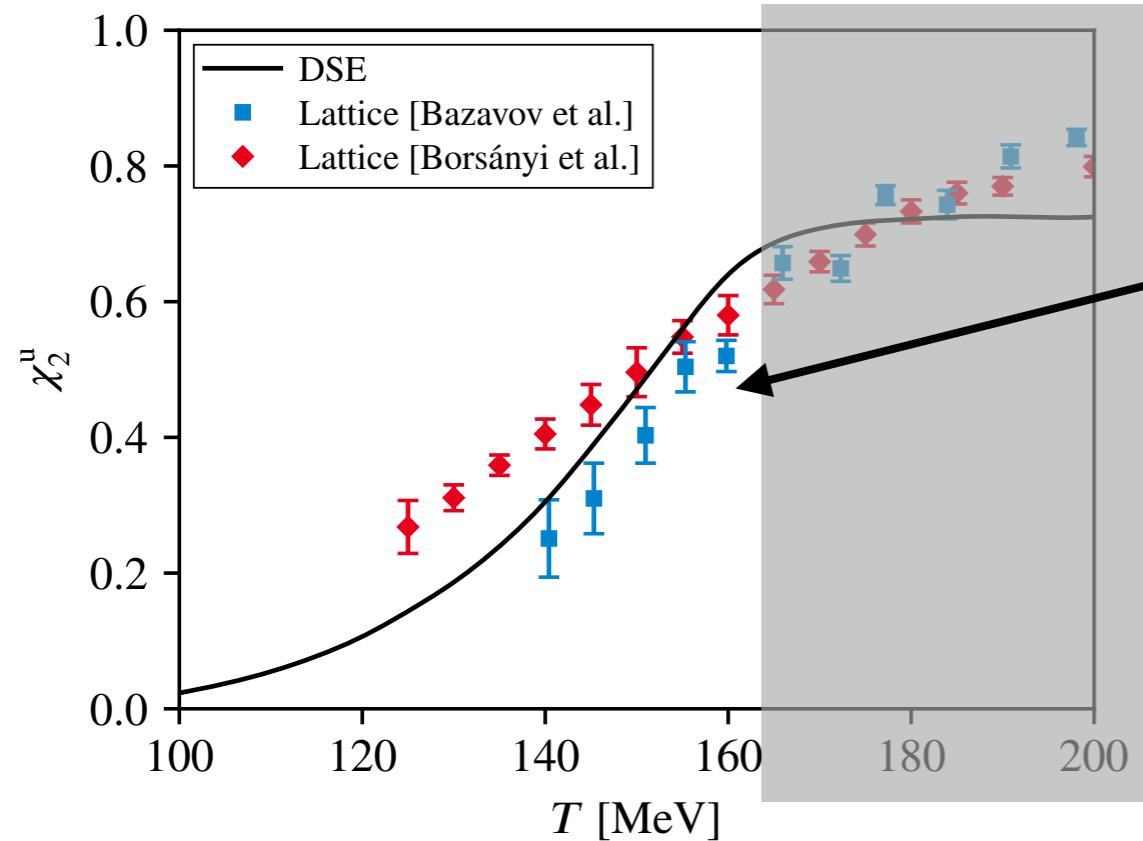
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Fluctuations at the CEP



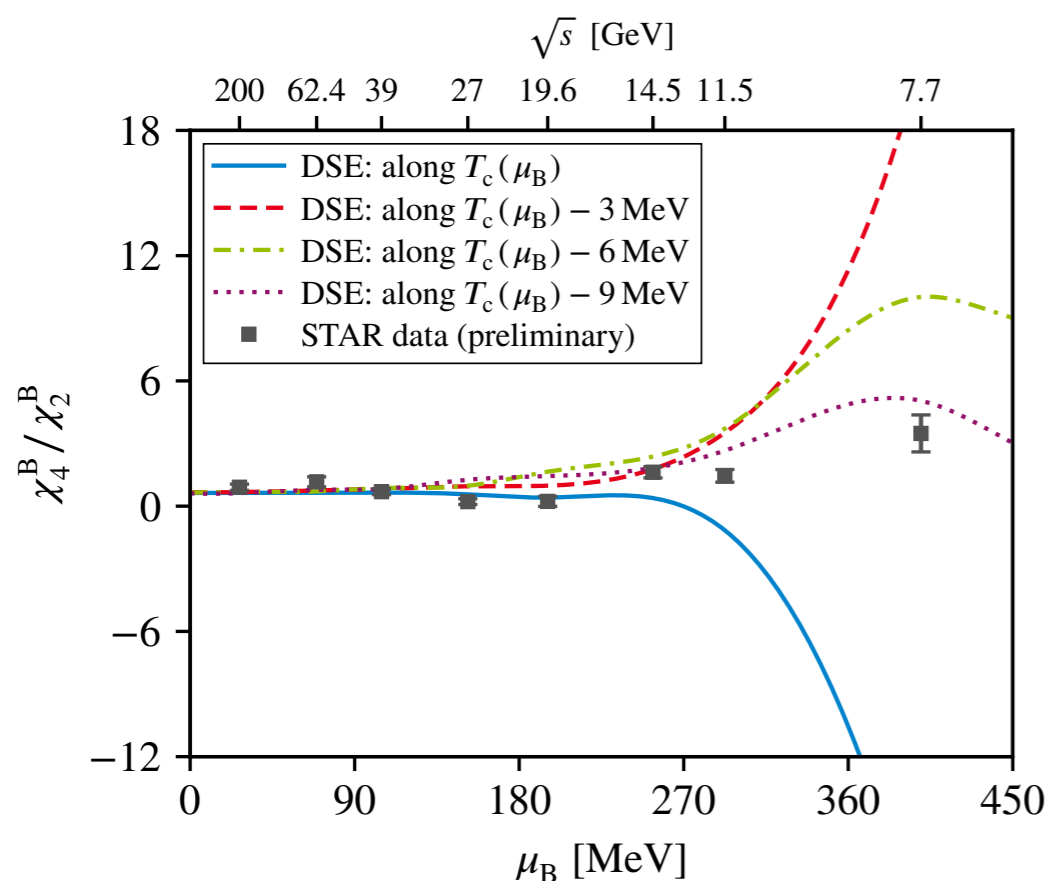
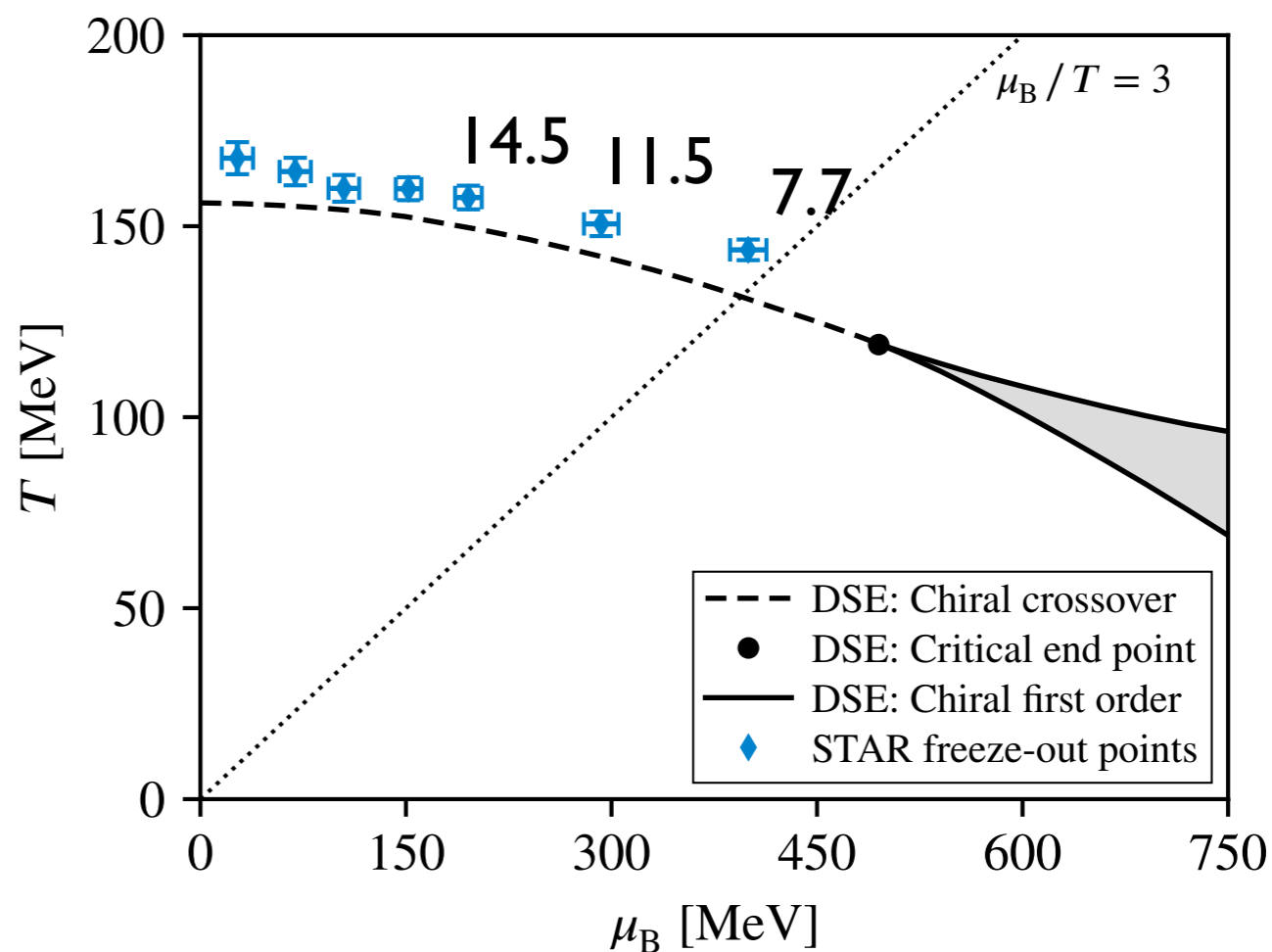
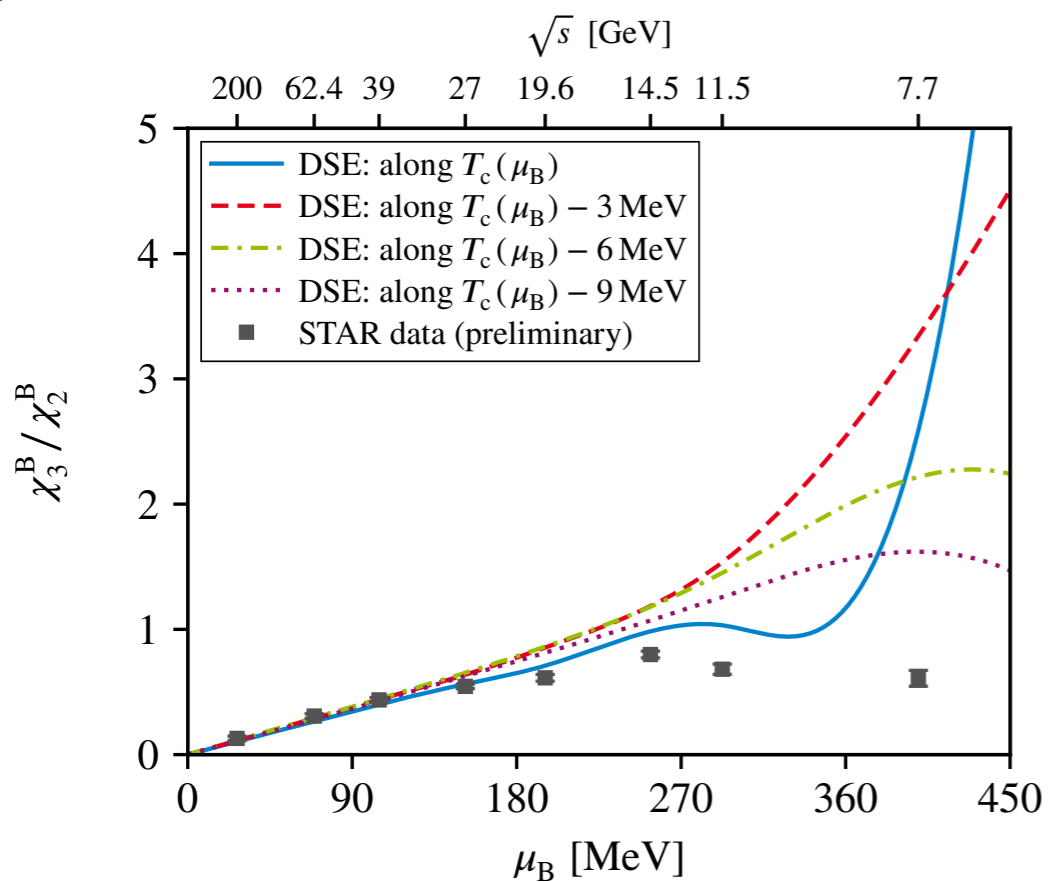
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Fluctuations at the CEP



Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Ratios: skewness and kurtosis



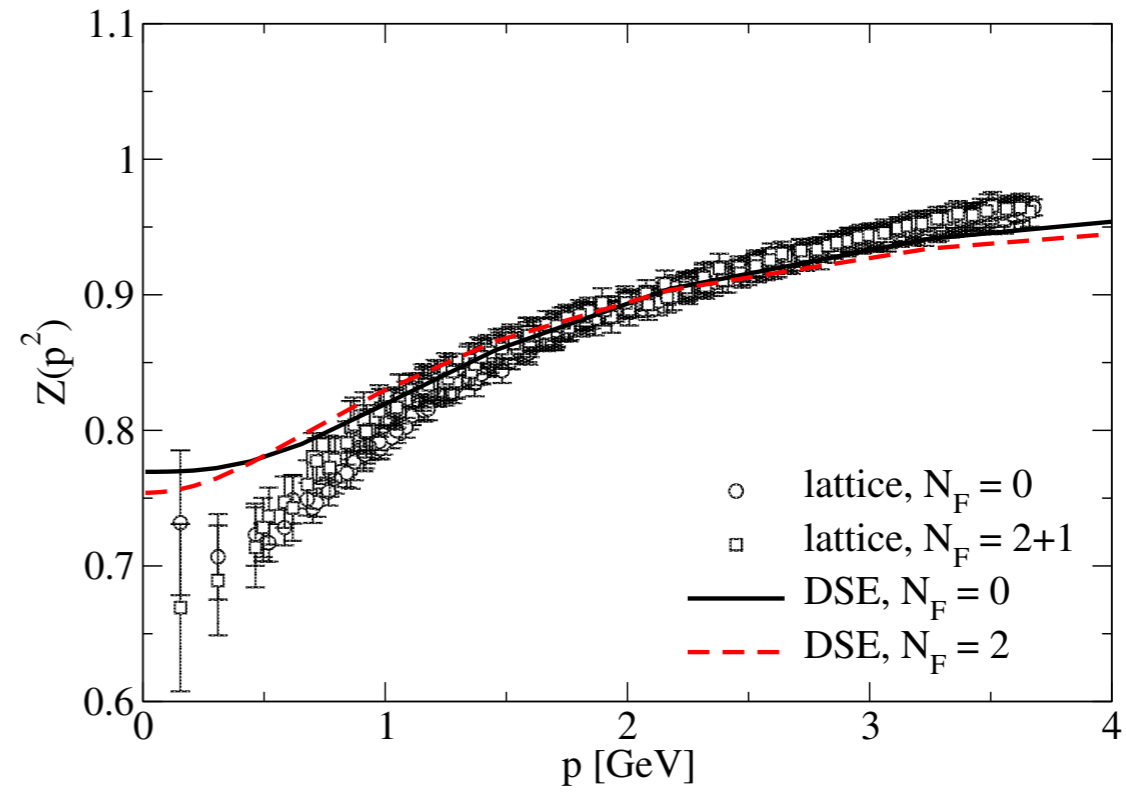
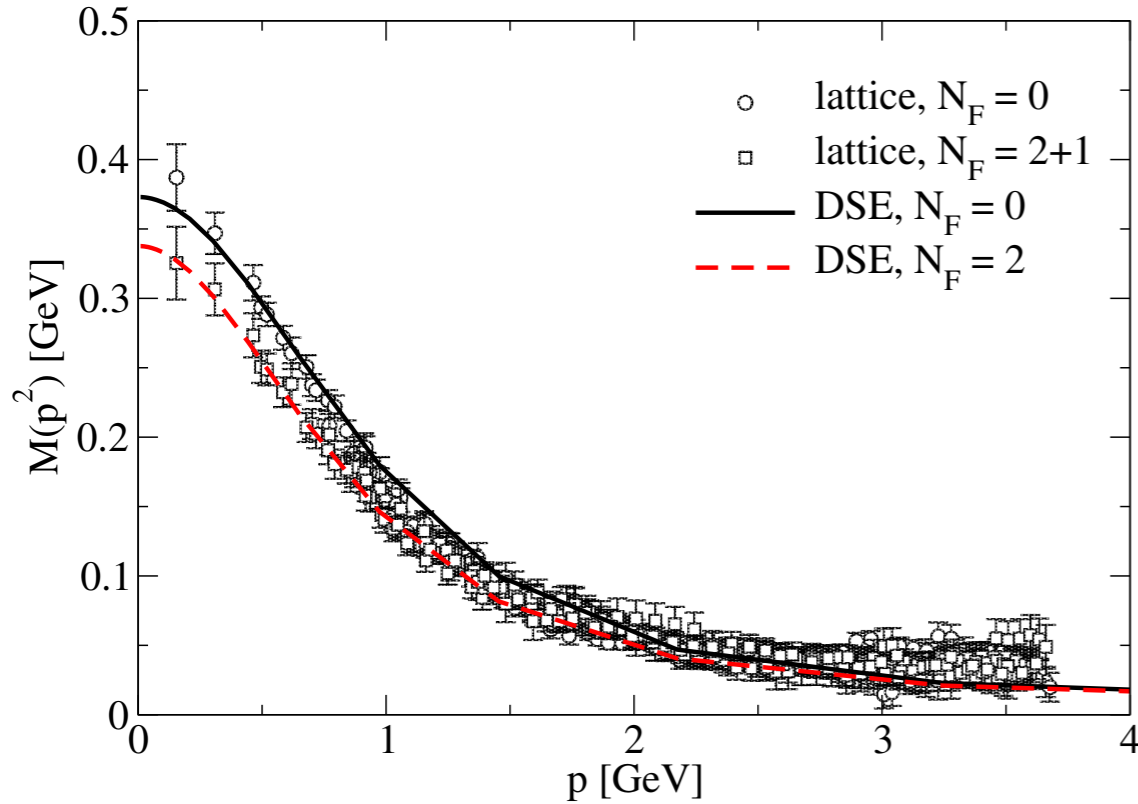
$\sqrt{s} \geq 14.5$: good agreement
 $\sqrt{s} = 11.5$: trend ok!
 $\sqrt{s} \leq 7.7$: freezeout line \neq transition line ?!

Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

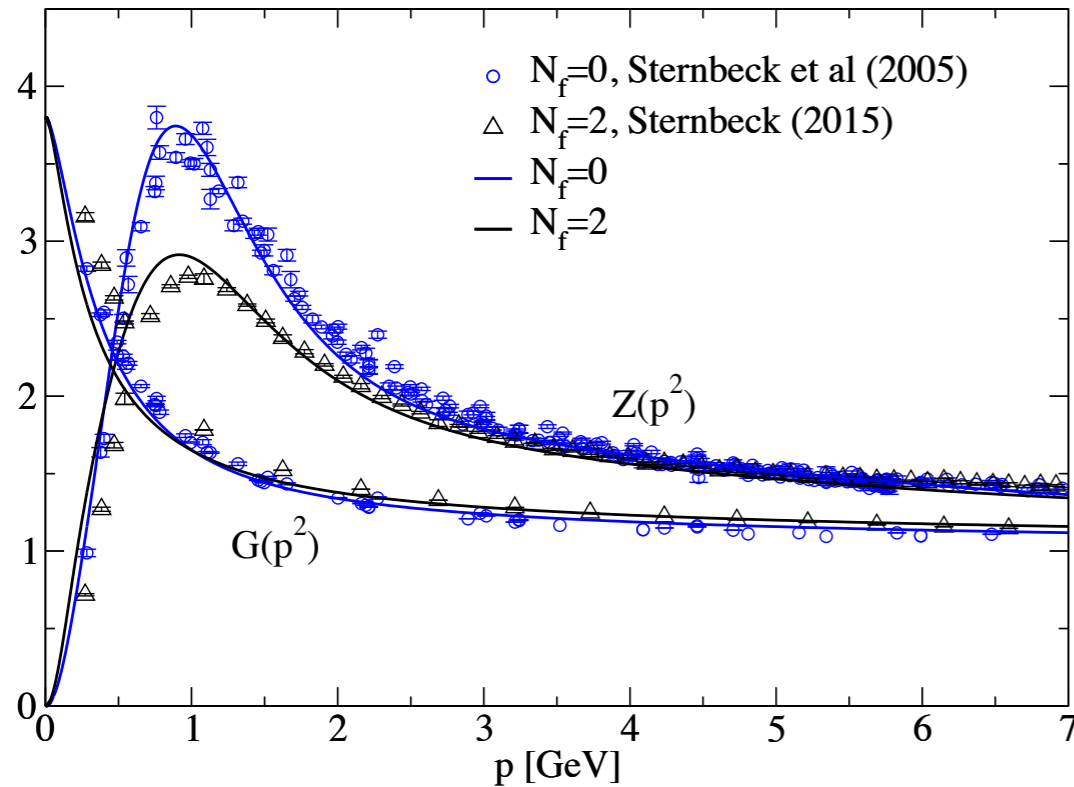
Selected results for Green's functions

Williams, CF, Heupel, PRD 93 (2016) 034026

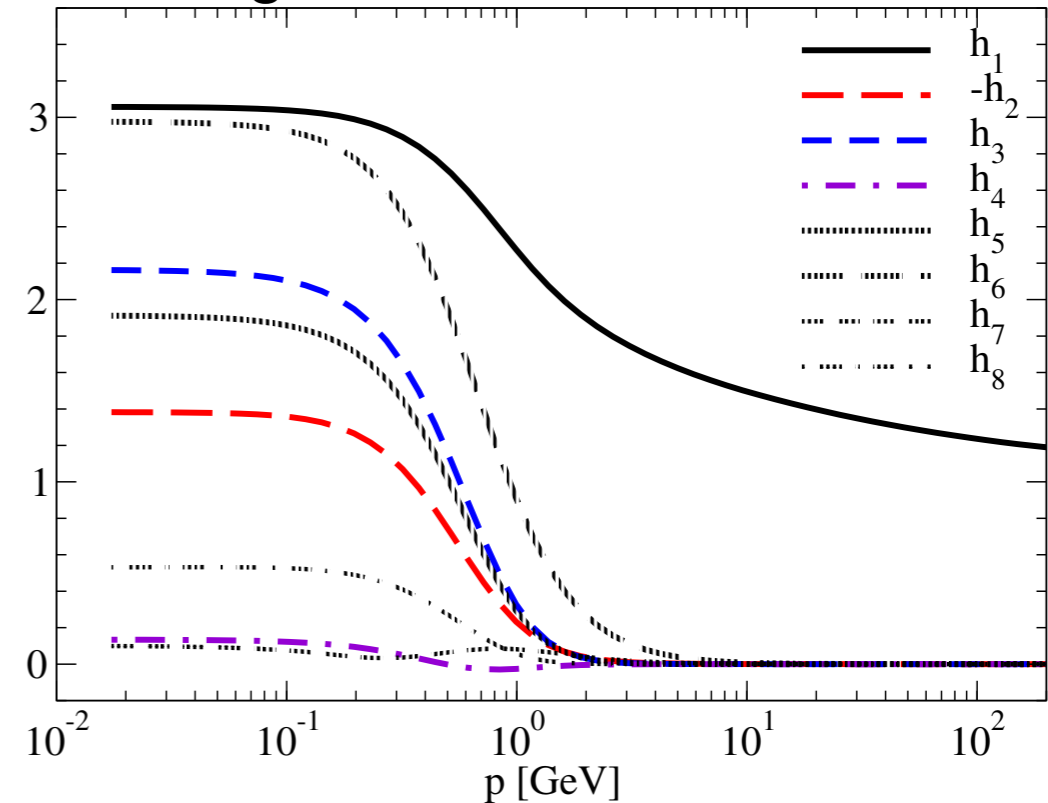
Quark



Gluon



Quark-gluon-vertex



Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet available...
- Diagrammatics: vertex-DSE (see later...)

explicit solutions at T=0: Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

- Slavnov-Taylor identity: T, μ, m-dependent vertex

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left(\delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times$$
$$\times \left(\frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2 / \Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

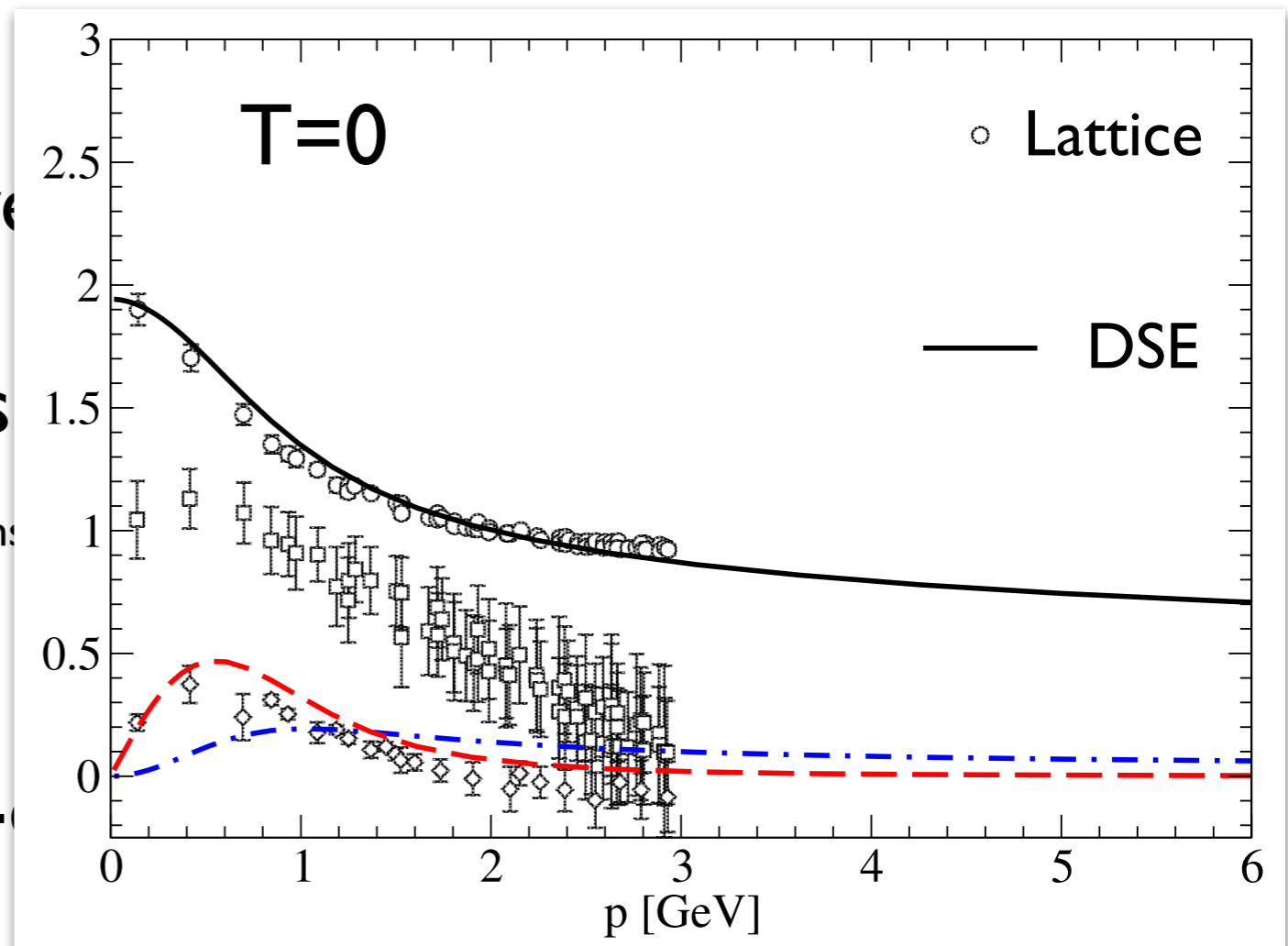
STI

PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet
- Diagrammatics: vertex-DSE (solving for Γ)
explicit solutions
- Slavnov-Taylor identity: T, μ, m -dependence



$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left(\delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times$$

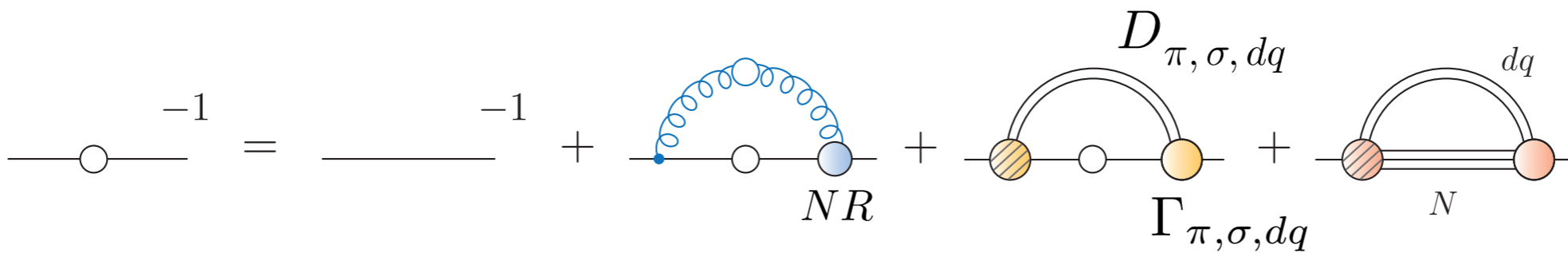
$$\times \left(\frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2 / \Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

STI

PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

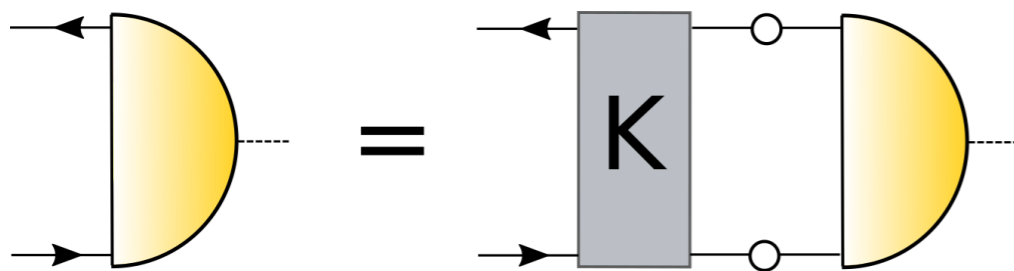
Meson effects at finite T and μ



$$D_\pi(p) = \frac{1}{p_4^2 + u^2(\vec{p}^2 + m_\pi(T, \mu)^2)}$$

$$u = \frac{f_s}{f_t}$$

Son, Stephanov, PRD 66 (2002) 7



$$\Gamma_\pi(P, q) = \gamma_5 E(P, q, T, \mu) + \dots$$

chiral limit: $\Gamma_\pi = \gamma_5 \frac{B}{f_t}$