Quark Pressure inside hadrons at finite density

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Outline

- Pressure and shear forces inside hadrons can be estimated through electron scattering experiments

  Burkert, Elouadrhiri, Girod, Nature 2018 and 2104.02031

  Polyakov and Schweitzer, Int.J.Mod.Phys.A 33 (2018) 26, 1830025

- Much easier to compute from the energy momentum tensor of quarks!
- A chiral model with vector mesons

  Drago and Mantovani Sarti, Phys.Rev.C 86 (2012) 015211

- How to go to finite density: Wigner-Seitz approach
  - Can we get “saturation”?
- Pressure and shear forces at finite density

What are the EMT form factors?

From Polyakov and Schweitzer 2018

$$\langle p',s'|\hat{T}^a_{\mu\nu}(x)|p,s\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{m} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)g_{\mu\nu}\right]u\;e^{i(p'-p)x}.$$

$$P = \tfrac{1}{2}(p'+p),\ \Delta = p'-p,\ t = \Delta^2$$

In the Breit frame

$$\langle p',s'|\hat{T}^{00}_a(0)|p,s\rangle = 2\,m\,E\left[A^a(t) - \frac{t}{4m^2}\left[A^a(t) - 2J^a(t) + D^a(t)\right] + \bar{c}^a(t)\right]\delta_{ss'}$$

$$\langle p',s'|\hat{T}^{ik}_a(0)|p,s\rangle = 2\,m\,E\left[D^a(t)\,\frac{\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2}{4m^2} - \bar{c}^a(t)\,\delta^{ik}\right]\delta_{ss'}$$

$$\langle p',s'|\hat{T}^{0k}_a(0)|p,s\rangle = 2\,m\,E\left[J^a(t)\,\frac{(-i\,\boldsymbol{\Delta}\times\boldsymbol{\sigma}_{s's})^k}{2\,m}\right]$$

The shear tensor can be decomposed into shear force s(r) and pressure p(r)

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\,\delta^{ij}\right) s(r) + \delta^{ij}\, p(r)$$

Conservation of the energy momentum tensor implies

$$\frac{2}{3}\, s'(r) + \frac{2}{r}\, s(r) + p'(r) = 0$$

And the equilibrium of internal forces for an isolated nucleon implies

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

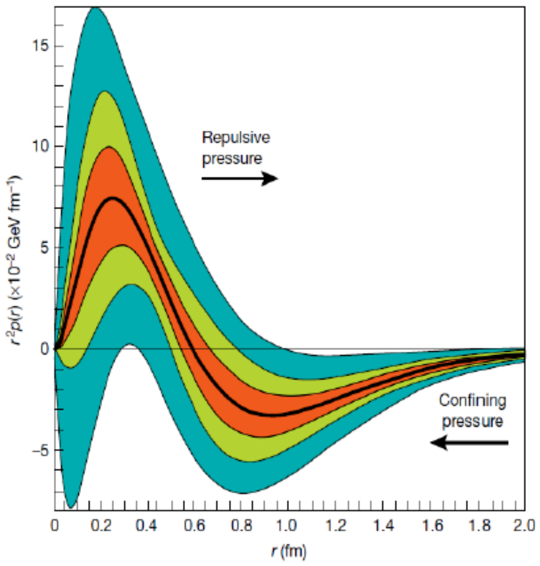
From Polyakov and Schweitzer 2018

The most natural way to probe EMT form factors, scattering off gravitons in Fig. 1a, is also the least practical one. A practical opportunity to access EMT form factors emerged with the advent of GPDs [7–21], which describe hard-exclusive reactions, such as deeply virtual Compton scattering (DVCS) $eN \to e'N'\gamma$ sketched in Fig. 1b or hard exclusive meson production $eN \to e'N'M$. In the case of the nucleon, the second Mellin moments of unpolarized GPDs yield the EMT form factors

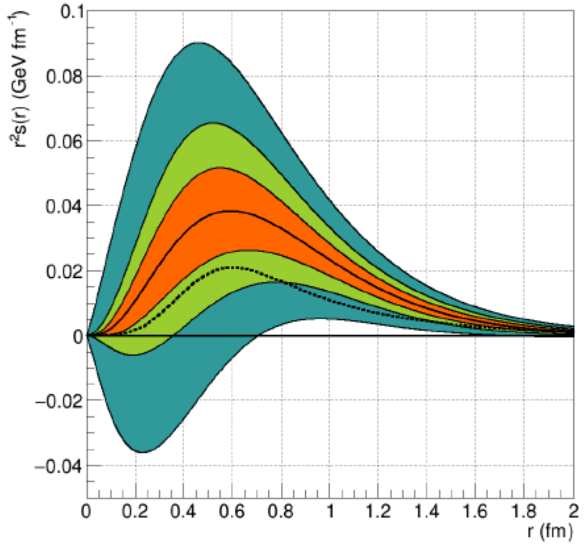
$$\int_{-1}^{1} \mathrm{d}x \, x \, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t) \, , \qquad \int_{-1}^{1} \mathrm{d}x \, x \, E^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t) \, . \tag{15}$$

The GPDs can be viewed as "amplitudes" for removing from the nucleon a parton carrying the fraction $x - \xi$ of the average momentum P and putting back in the nucleon a parton carrying the fraction $x + \xi$, while the nucleon receives the momentum transfer Δ. For $\xi = 0$ the momentum transfer is purely transverse and the Fourier transform $H^a(x, b_\perp) = \int d^2\Delta_\perp/(2\pi)^2 \exp(-i\boldsymbol{\Delta}_\perp \boldsymbol{b}_\perp) \, H^a(x, 0, -\boldsymbol{\Delta}_\perp^2)$ describes the probability to find a parton carrying the momentum fraction x of the hadron and located at the distance $b_\perp$ from the hadrons (transverse) center-of-mass on the lightcone. This allows one to do femtoscale tomography of the nucleon [75–77].

Pressure and Shear forces



(a) Pressure[1]



(b) Shear Forces[1]

Figure: The thick black line, in (a), (or black continous line in (b)) is the fit results. The blue area is the range of uncertainties before the CLAS data were included. The light green area are based on the CLAS data. The red area represents the expected errors when the results from the planned experiments will be included in the fits. The dashed black curve, present only in (b), is a model prediction.

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[1] from Burket, Elouadhriri and Girod, 2018 (a), 2021 (b)

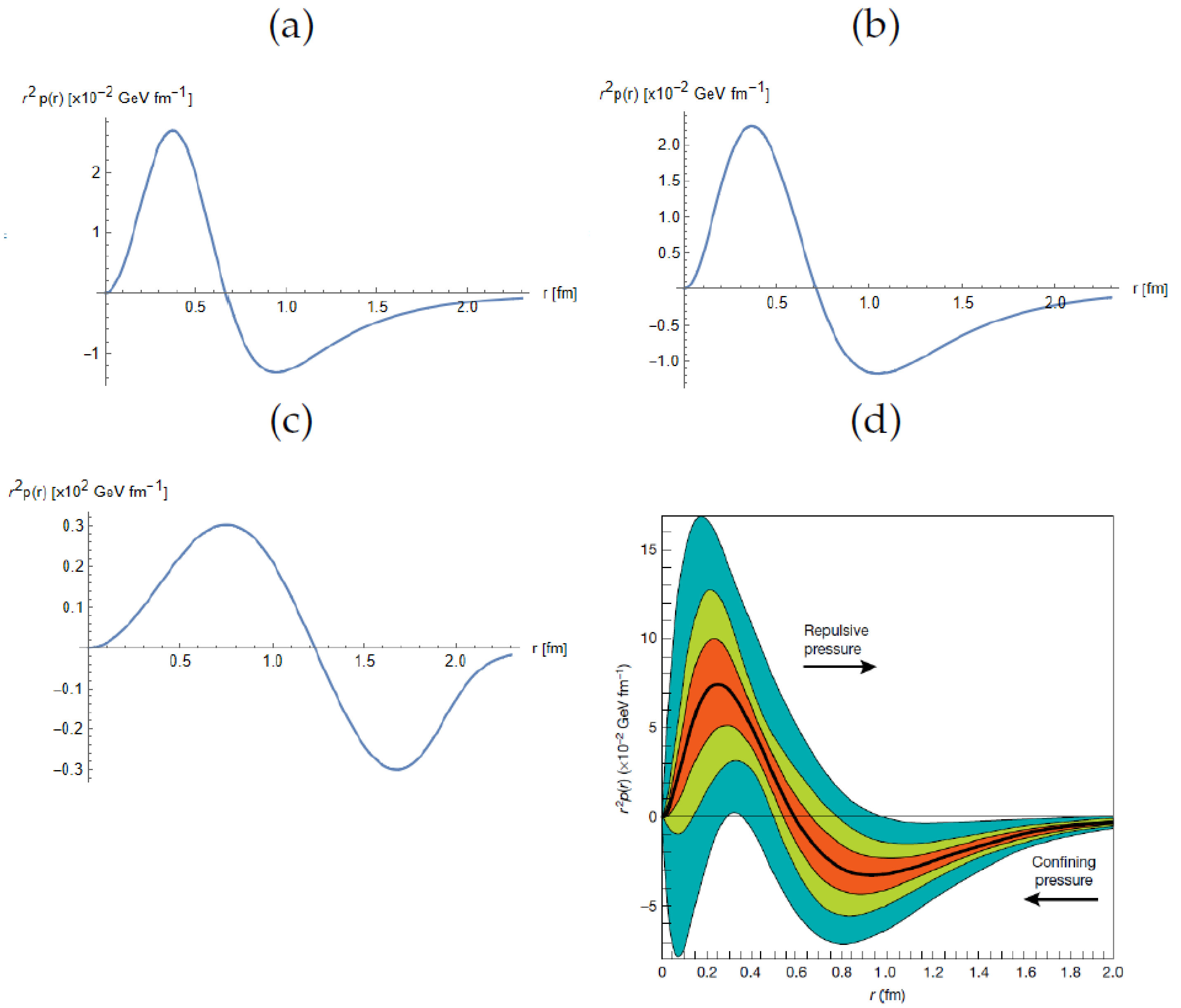


Figure 3.1 *Pressures: (a) σ-model; (b) Chiral dilaton model; (c) Dielectric model; (d) From experiments*

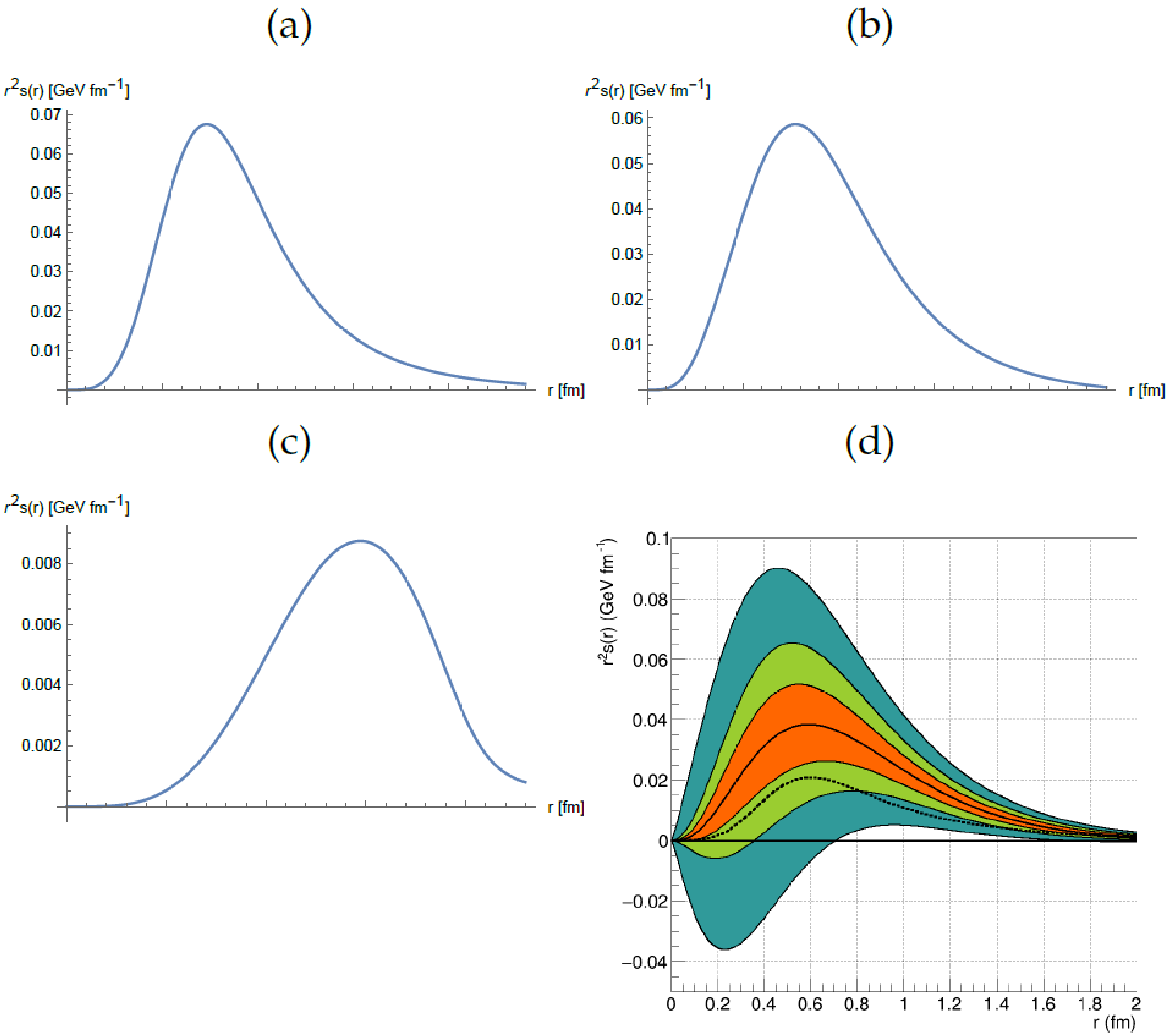
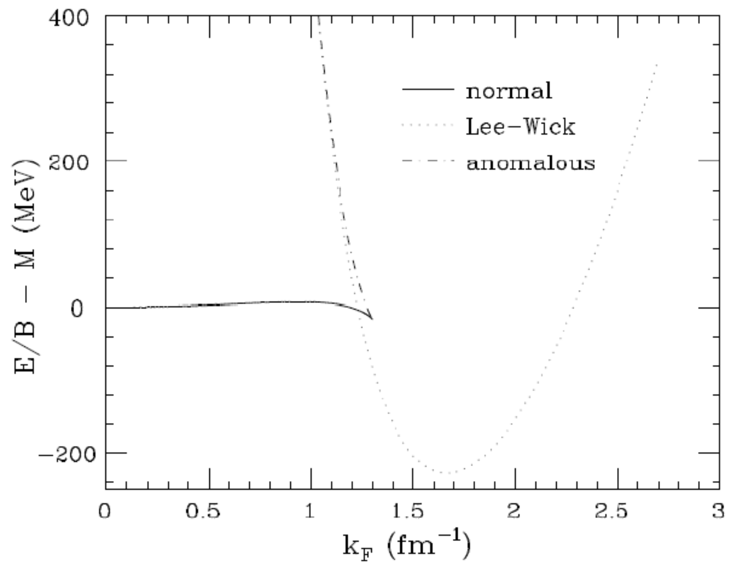


Figure 3.2 *Shear Forces: (a) σ-model; (b) Chiral dilaton model; (c) Dielectric model; (d) From experiments*

Chiral models at finite density

Failure of Linear-σ model at finite density

- The ground state at high densities is not the normal solution, but the Lee-Wick one, having effective nucleon mass $M^* = 0$
- restauration of chiral symmetry already at $\rho \approx \rho_0$

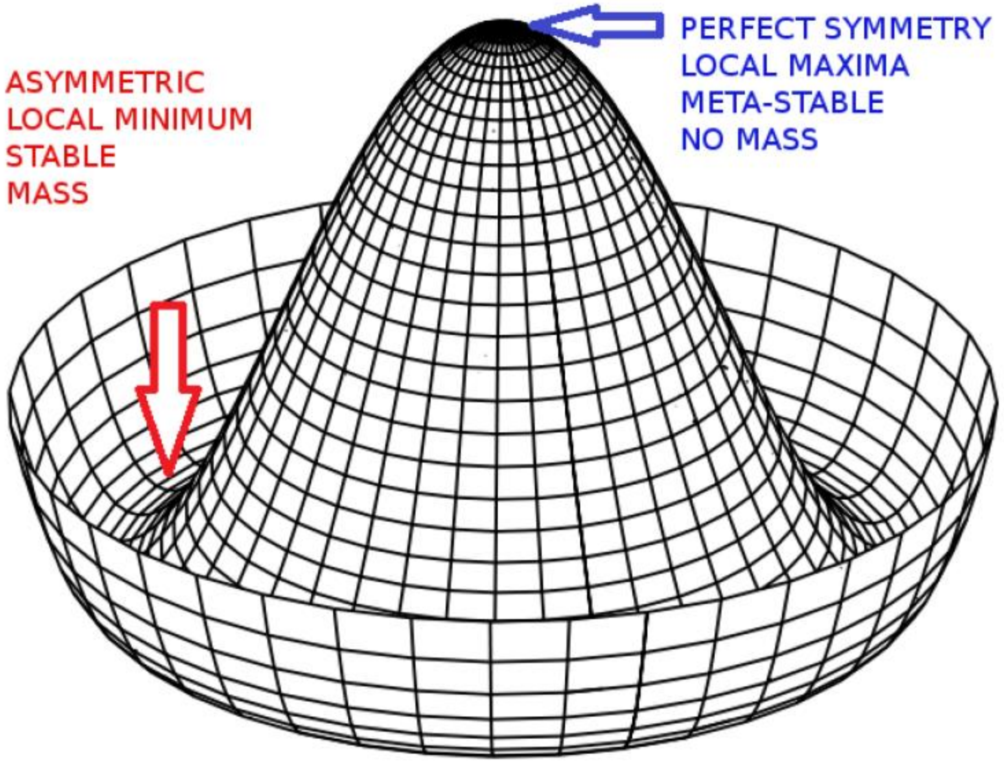


(R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))

HOW CAN WE REACH HIGHER DENSITIES AND STILL INCLUDE CHIRAL SYMMETRY?

Linear realization of chiral symmetry with scale invariance

PROBLEM: the linear sigma model fails to yield saturation. It provides chiral symmetry restoration ($m_N = 0$) already at low density due to the form of the meson self-interaction



Breaking of Scale Invariance in QCD

- In QCD, scale symmetry is broken by trace anomaly. This mechanism is responsible for the existence of Λ_{QCD} parameter, which sets the scale of hadron masses and radii
- Formally the non conservation of the dilatation current is strictly connected to a non vanishing gluon condensate

$$\langle \partial_\mu j^\mu_{QCD} \rangle = \frac{\beta(g)}{2g} \langle F^a_{\mu\nu}(x) F^{a\mu\nu}(x) \rangle$$

- In an effective model, the dynamics of the gluon condensate at mean-field level, is obtained by introducing a scalar field ϕ, the dilaton field ϕ (Schechter (1980), Migdal, Shifman (1982)), so that the potential is determined by:

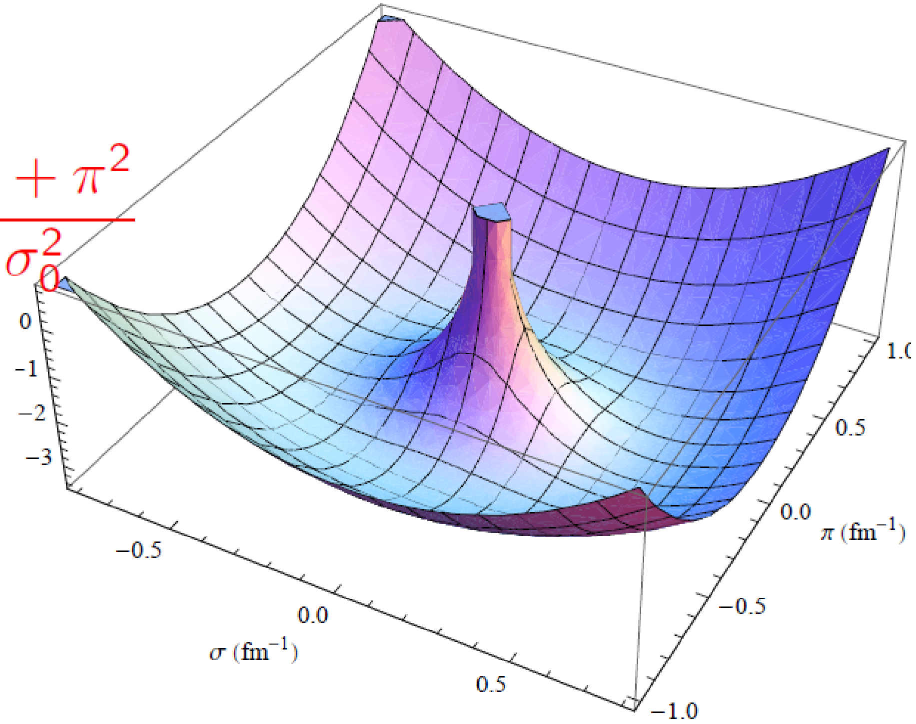
$$\Theta^\mu_\mu = 4V(\phi) - \phi \frac{\partial V}{\partial \phi} = 4\epsilon_{vac} \left(\frac{\phi}{\phi_0} \right)^4$$

The dilatonic potential

The dilaton field potential:

$$V(\phi, \sigma, \pi) =$$

$$B\phi^4\left(ln\frac{\phi}{\phi_0} - \frac{1}{4}\right) - \frac{1}{2}B\delta\phi^4 ln\frac{\sigma^2+\pi^2}{\sigma_0^2}$$



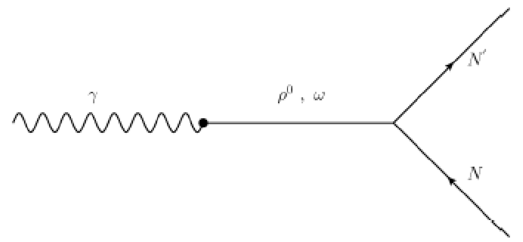
$$+\frac{1}{2}B\delta\zeta^2\phi^2\left(\sigma^2+\pi^2-\frac{1}{2}\frac{\phi^2}{\zeta^2}\right)$$

$$-\frac{3}{4}\epsilon_1 - \frac{1}{4}\epsilon_1\left(\frac{\phi}{\phi_0}\right)^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) - \left(\frac{\phi}{\phi_0}\right)^2\right]$$

E. K. Heide, S. Rudaz, and P. J. Ellis, Phys. Lett. B293, 259 (1992)

The introduction of vector mesons

- Vector mesons dominance → better description of nucleon properties



- $N - N$ interaction → provide the necessary repulsion at short distances (OBE model)

How to introduce the vector mesons in an effective Lagrangian

- VM as massive Yang-Mills fields of $SU(2)_L \otimes SU(2)_R$ symmetry group

- principle of universality → ρ meson couples to isospin current and ω meson couples to the baryonic current:

$$g_{\rho NN} = g_{\rho qq} = g_{\rho\pi\pi} = g_{\rho\rho\rho} \quad , g_{\omega qq} = \frac{1}{3} g_{\omega NN}(q^2 = 0)$$

The Lagrangian of the Chiral Dilaton Model

- in the hadronic sector $\rightarrow$ fermionic fields are *nucleons*;
- chiral fields $(\sigma, \boldsymbol{\pi})$ $\rightarrow$ nuclear physics at low densities (Heide, Rudaz, Ellis, Nucl.Phys.A571, 713 (1994)), restoration of chiral symmetry at quite high densities (Drago, Bonanno, Phys.Rev.C79:045801,2009);

MAIN IDEA: use the same nucleon Lagrangian, but now introducing quarks degrees of freedom $\rightarrow$ fermionic fields are quarks

The Lagrangian density becomes:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_\pi(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5) + g_\rho\gamma^\mu\frac{\boldsymbol{\tau}}{2}\cdot(\boldsymbol{\rho}_\mu + \gamma_5\mathbf{A}_\mu) - \frac{g_\omega}{3}\gamma^\mu\omega_\mu\right)\psi \\
& + \frac{\beta}{2}(D_\mu\sigma D^\mu\sigma + D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi}) - \frac{1}{4}(\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \mathbf{A}_{\mu\nu}\cdot\mathbf{A}^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}) \\
& + \frac{1}{2}m_\rho^2(\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \mathbf{A}_\mu\cdot\mathbf{A}^\mu) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - V(\phi,\sigma,\pi)
\end{aligned}
$$

Soliton in vacuum: projecting the Hedgehog

- the Hedgehog is not an eigenstate of spin and isospin $\rightarrow$ projection on physical states with good quantum numbers J and I(Ruiz-Arriola et al.NPA591, Birse PRD33)
- it also breaks translational invariance $\rightarrow$ projection on linear momentum
  - easier approach to estimate the center-of-mass corrections(Dethier et al. PRD27,(1983)): $M_J = (E_J - \mathbf{P}^2)^{1/2}$

Set of parameters

- model without VM: $m_\sigma = 550$ MeV, $g = 5$
- model with VM:
  1. better fit to nucleon properties: $g = 3.6$, $g_\omega = 13$, $g_\rho = 4$ and $m_\sigma = 1200$ (SET I)

Soliton in vacuum

Contributions to the total soliton energy at MFL:

Quantity	Log. Model	Linear σ-Model
Quark eigenvalue	114.5	112.9
Quark kinetic energy	1075.8	1080.6
E_σ (mass+kin.)	213.8	212.2
E_π (mass+kin.)	393.2	397.3
Potential energy $\sigma - \pi$	81.2	80.4
E_ω (mass+kin.)	-194.4	-196.5
E_ρ (mass+kin.)	162.6	165.4
E_A (mass+kin.)	329.5	334.1
Total energy	1329.5	1331.7

- chiral and vector mesons contributions are comparable $\rightarrow$ VMs play a fundamental role in building up the soliton
- results with the logarithmic model and the linear-σ model are very similar

Soliton in vacuum

Model without VM :

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1075	1002	
$M_N\,(MeV)$	960	894	938
$E_{3/2}\,(MeV)$	1140	1075	
$M_\Delta\,(MeV)$	1032	975	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.55	0.61	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.02	-0.02	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.7	0.72	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.72	0.75	0.77
$\mu_p\,(\mu_N)$	2.25	2.27	2.79
$\mu_n\,(\mu_N)$	-1.97	-1.92	-1.91
g_a	1.52	1.10	1.26

Model with VM, SET I:

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1020	1008	
$M_N\,(MeV)$	926	912	938
$E_{3/2}\,(MeV)$	1148	1147	
$M_\Delta\,(MeV)$	1066	1063	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.67	0.66	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.05	-0.05	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.77	0.76	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.78	0.77	0.77
$\mu_p\,(\mu_N)$	2.63	2.64	2.79
$\mu_n\,(\mu_N)$	-2.37	-2.38	-1.91
g_a	1.58	1.46	1.26

Going to finite density: the Wigner-Seitz approximation to nuclear matter

- Approximating nuclear matter by a lattice of solitons $\rightarrow$ we consider the meson fields configuration centered at each lattice point, generating a periodic potential in which the quarks move
- Wigner-Seitz approximation: replace the cubic lattice by a spherical symmetric one $\rightarrow$ each soliton sits on a spherical cell of radius R with specific boundary conditions on the surface of the sphere

The Hamiltonian for a periodic system must obey Bloch's theorem, so the quark spinor must be of the form:

$$\psi_{\mathbf{k}}(r) = e^{i\mathbf{k}\cdot\mathbf{r}}\Phi_{\mathbf{k}}(r), \quad (\mathbf{k} = 0 \text{ for the ground state})$$

The bottom of the band is defined as the state satisfying the following periodic boundary conditions, dictated by symmetry arguments (parity):

$$v(R) = h(R) = \rho(R) = 0,$$
$$u'(R) = \sigma_h'(R) = \omega'(R) = A_S'(R) = A_T'(R) = 0.$$

Going to finite density: how to define the band width

In our work we use two different methods to estimate the band width:

- A (rather crude) approximation to the width of a band can be obtained by using (Glendenning, Banerjee PRC 34(1986)):

$$\Delta = \sqrt{\epsilon_0^2 + \left(\frac{\pi}{2R}\right)^2} - |\epsilon_0|,$$
$$\epsilon_{top} = \epsilon_0 + \Delta.$$

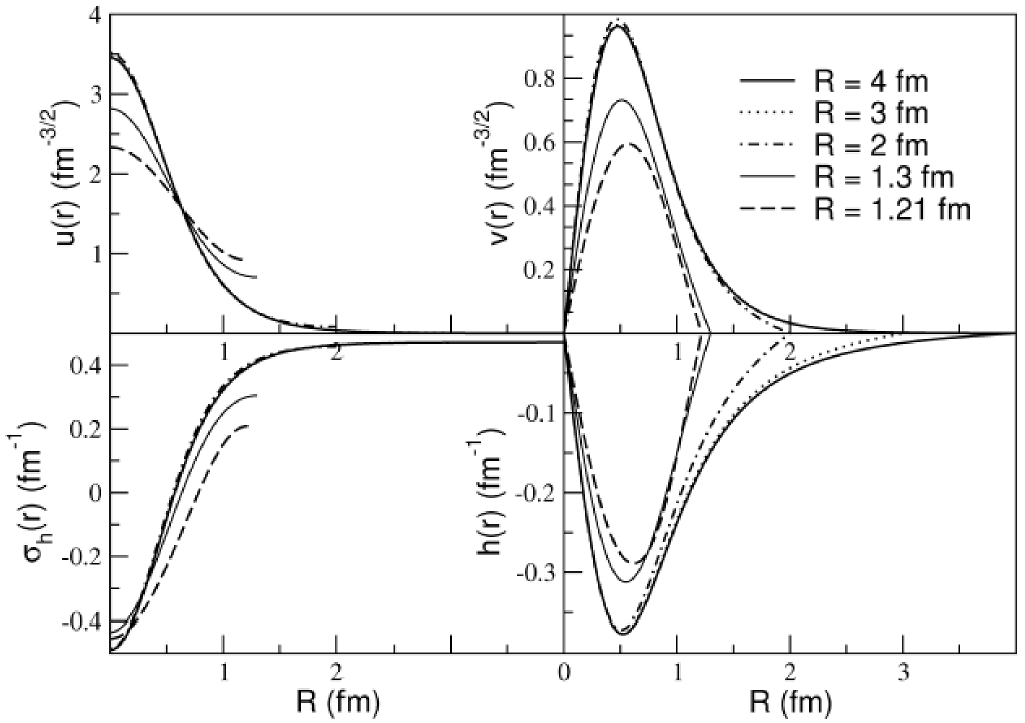
- An alternative approximation is obtained by imposing that the upper Dirac component vanishes at the boundary(Birse,Rehr,Wilets PRC38 (1988)):

$$u(R) = 0$$

- the eigenvalue obtained imposing this boundary condition represents an upper limit to the top and the true top would be about half way between this upper limit and the bottom of the band

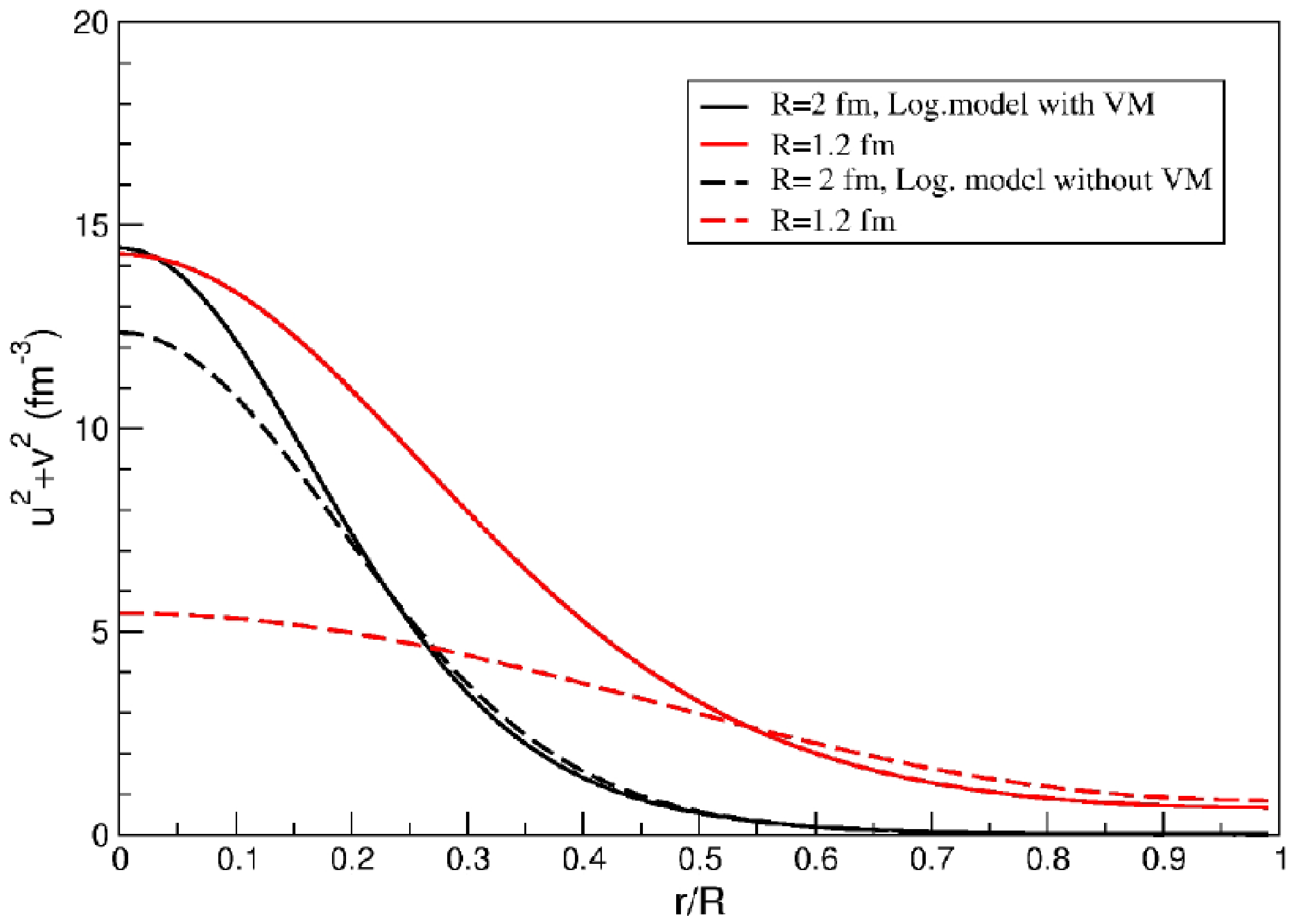
- uniform filling of the band $\rightarrow$ lower band has $G = 0$, color is the only degeneracy left $\rightarrow$ 3 quarks per soliton completely fill the band

Fields at finite density: model without VM

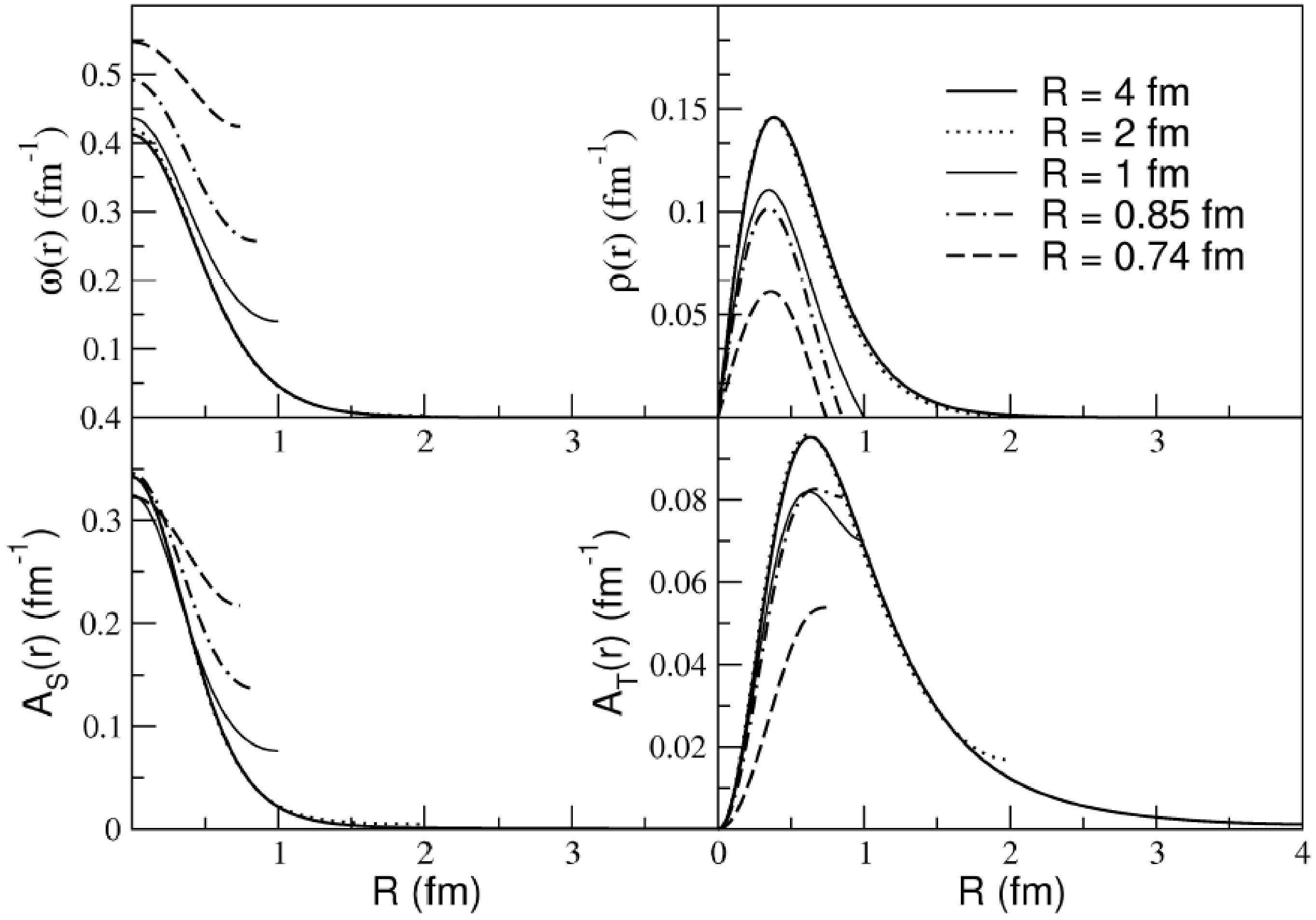


- down to $R \approx 2$ fm the fields do not change significantly, at lower values the finite density effects deeply modify the behaviour of fields.

Flattening of the w.f. at saturation density without and with vector mesons



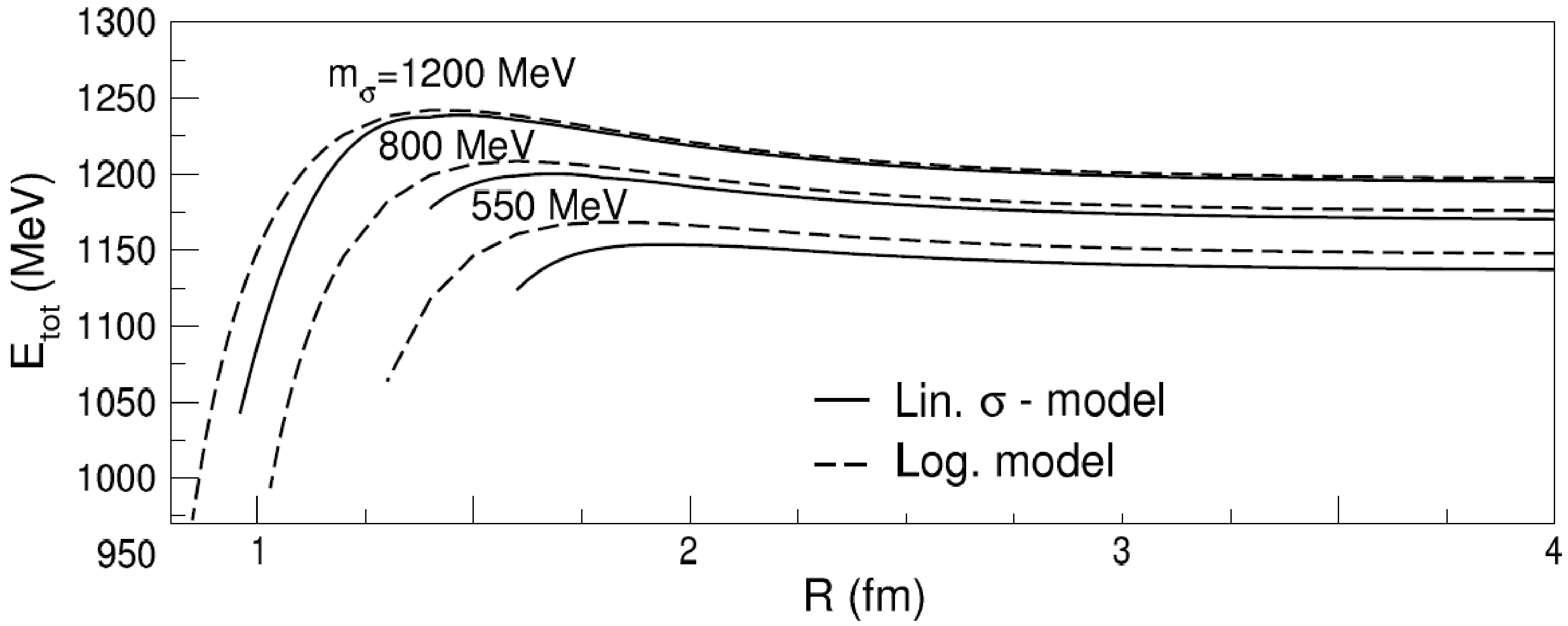
Fields at finite density: model with VM



- more stable solutions $\rightarrow$ fields start to get deformed at $R \approx 1$

Results at finite density: the effect of the dilaton potential

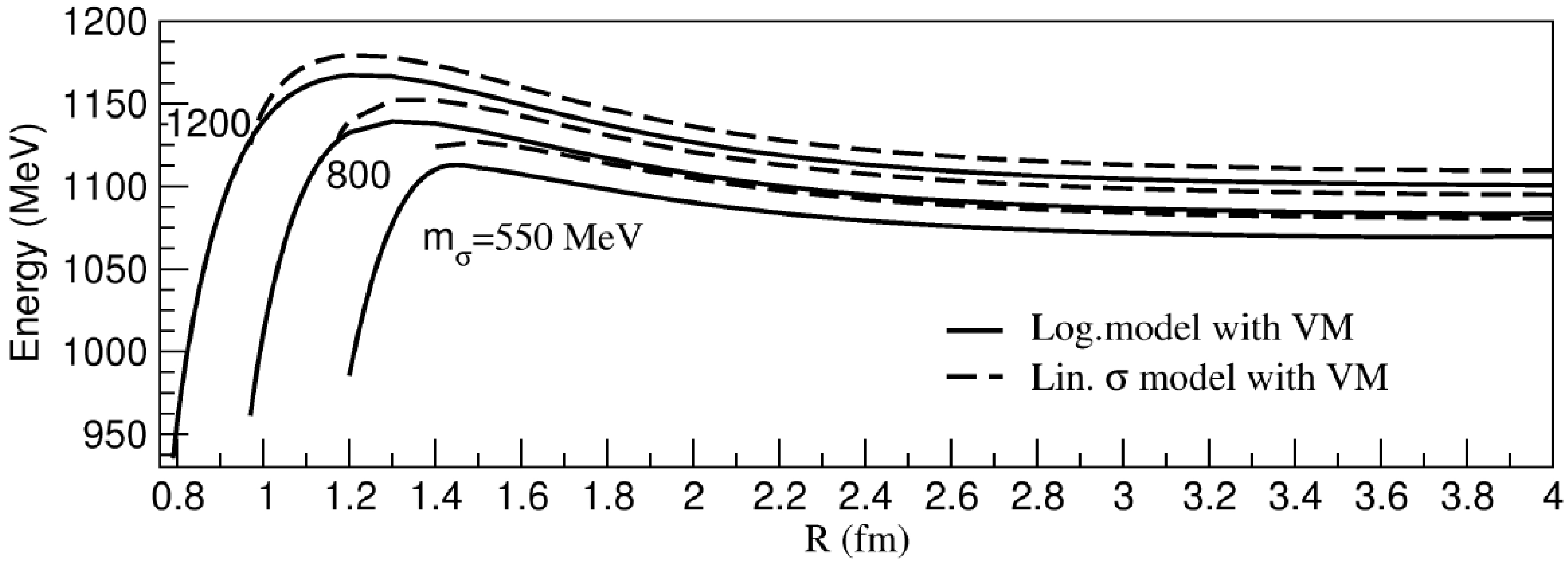
For the model without VM:



- For a fixed value of m_σ, the CDM allows the system to reach higher densities
- as m_σ raises, the system remains stable to lower R

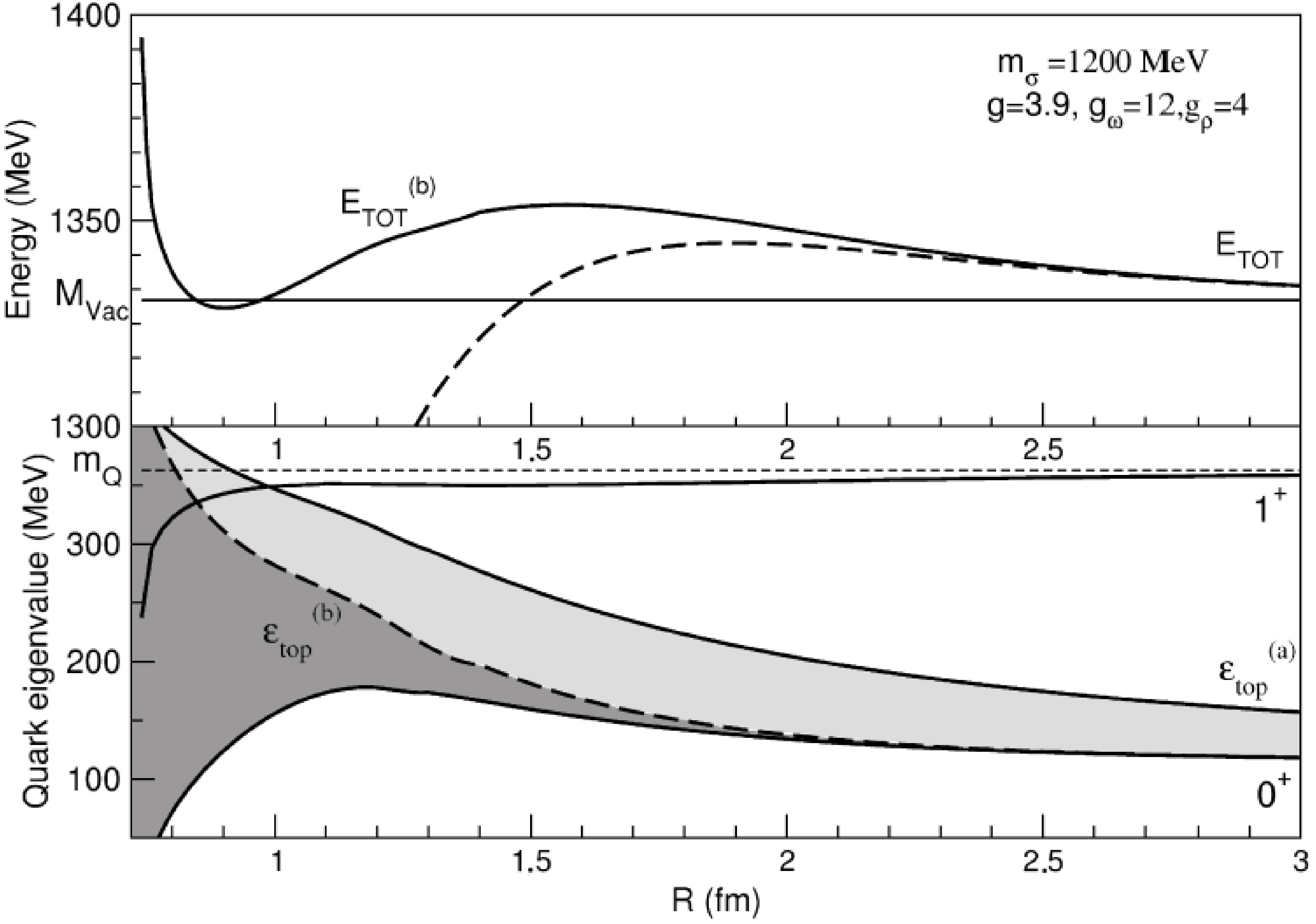
Results at finite density: the effect of vector mesons (I)

For the model with VM:



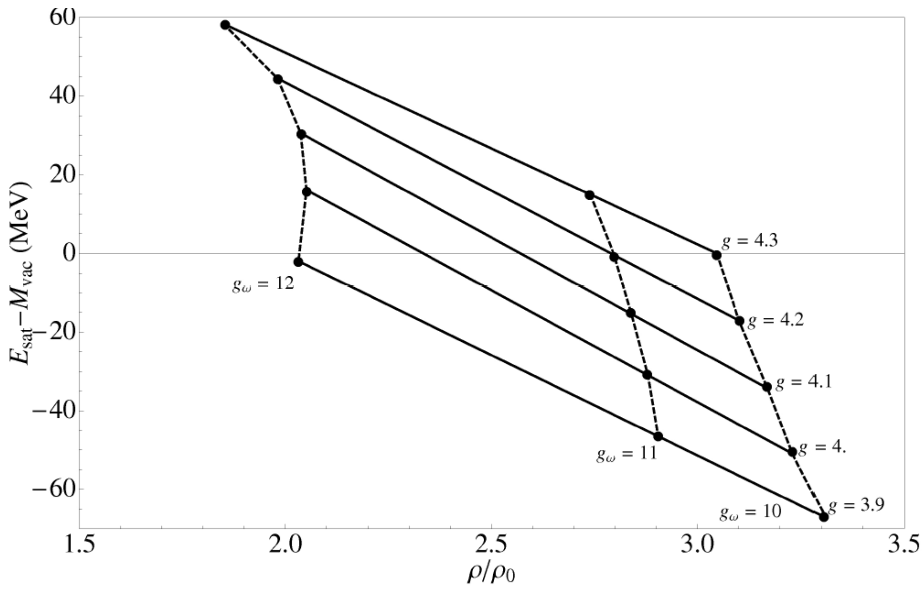
- the introduction of VM stabilizes the solution at high densities → reach even higher densities in comparison to the model with only chiral fields

Stabilizing effect of the band at large densities



Going to finite density: how do we obtain saturation?

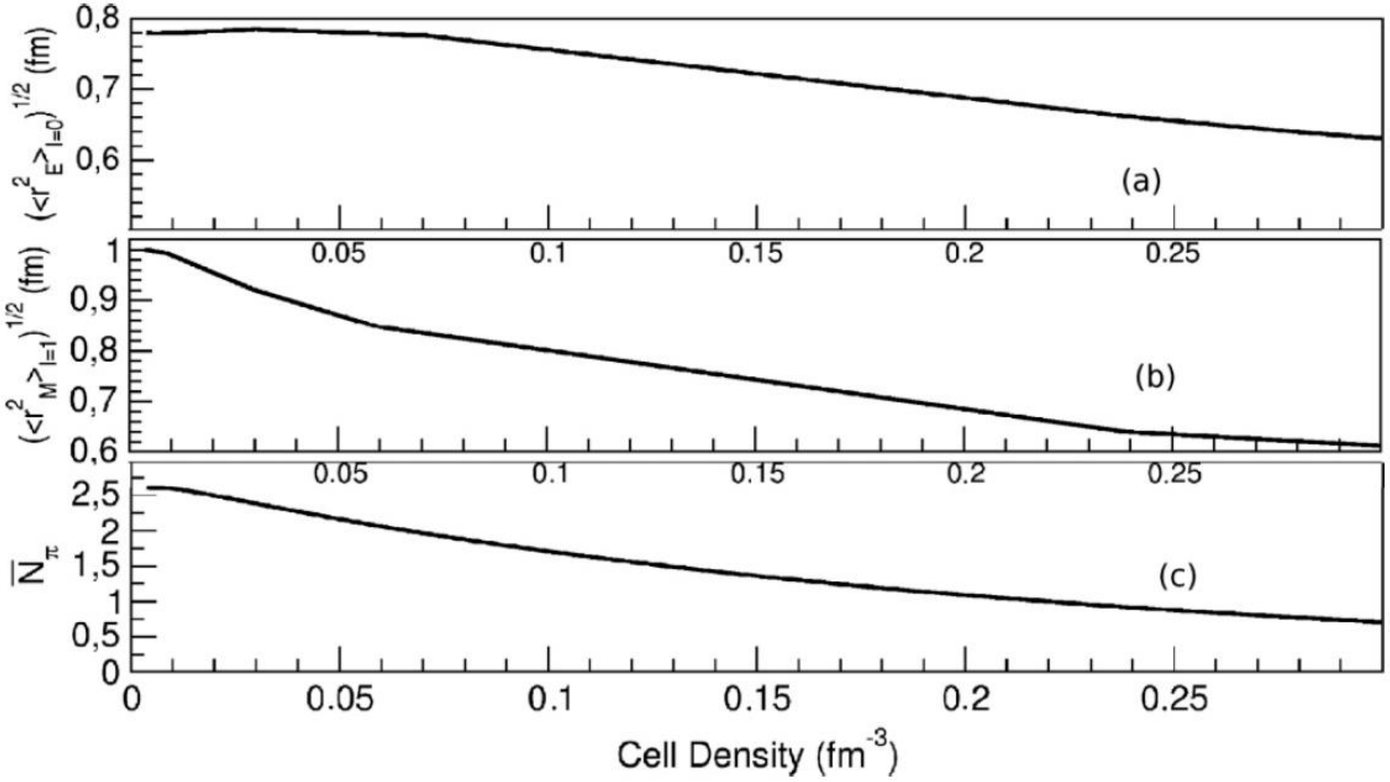
- interplay between attraction from chiral fields and repulsion from vector mesons, dominant up to ρ_0
- the logarithmic potential is fundamental to keep the soliton stable at densities large enough that the vector mesons start to provide repulsion
- at densities $\rho > \rho_0$ the band effect provides the necessary repulsion to obtain saturation



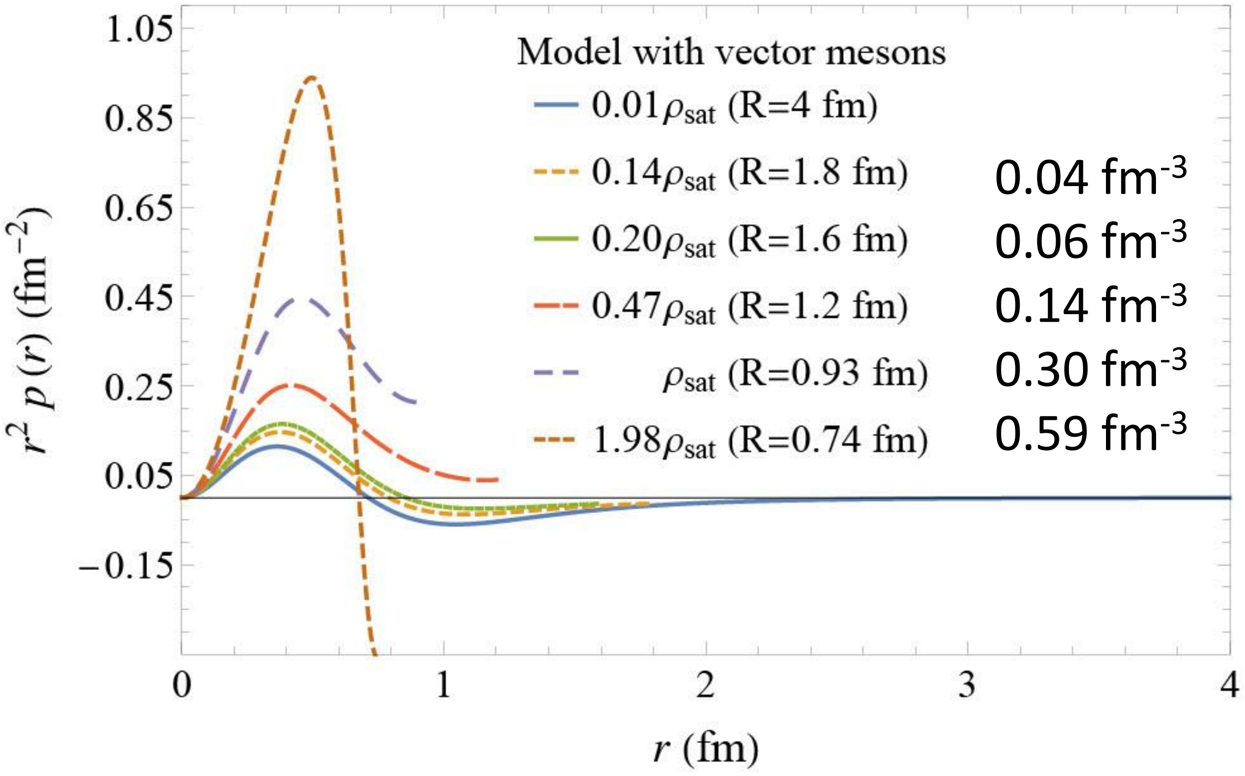
- the model admits "saturation" for different sets of parameters → partial overlap with parameters for the single nucleon (Broniowski, Banerjee PRD34 (1986))

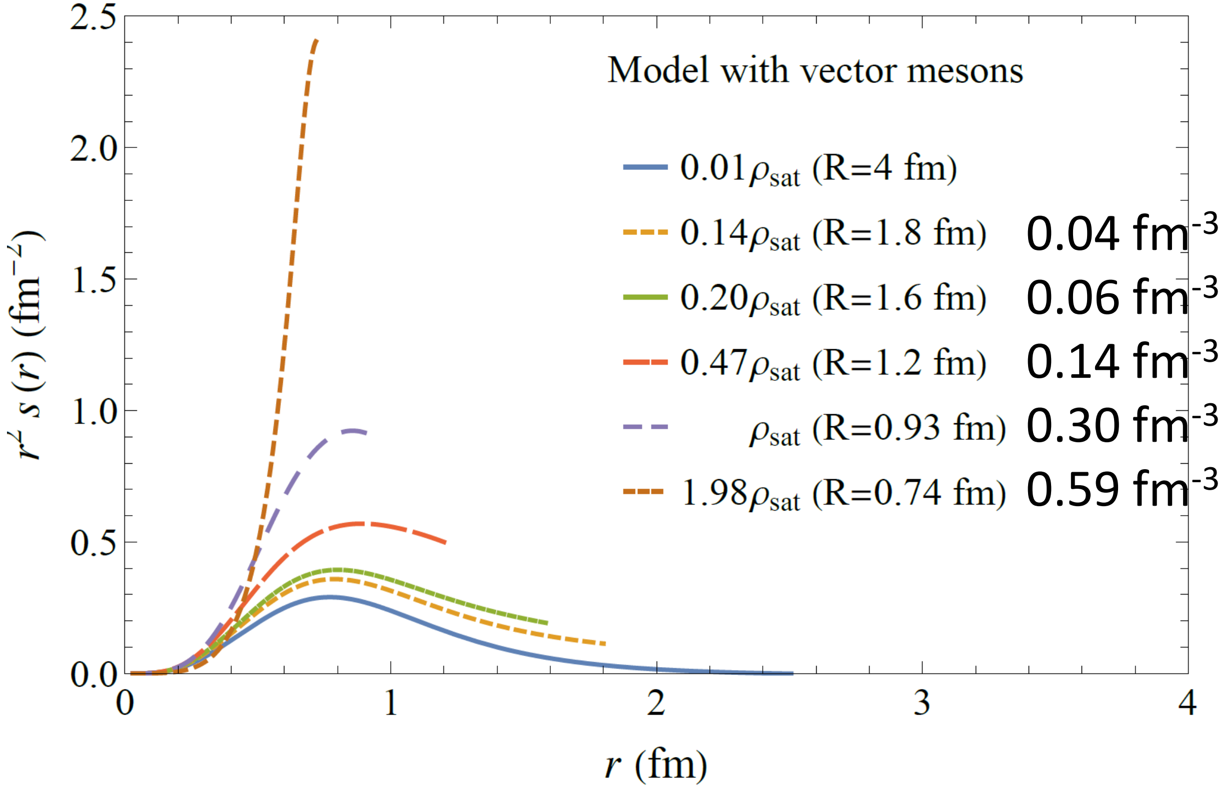
Dependence of the radius on the density

Electric isoscalar and magnetic isovector do not appear at the same order in N_c

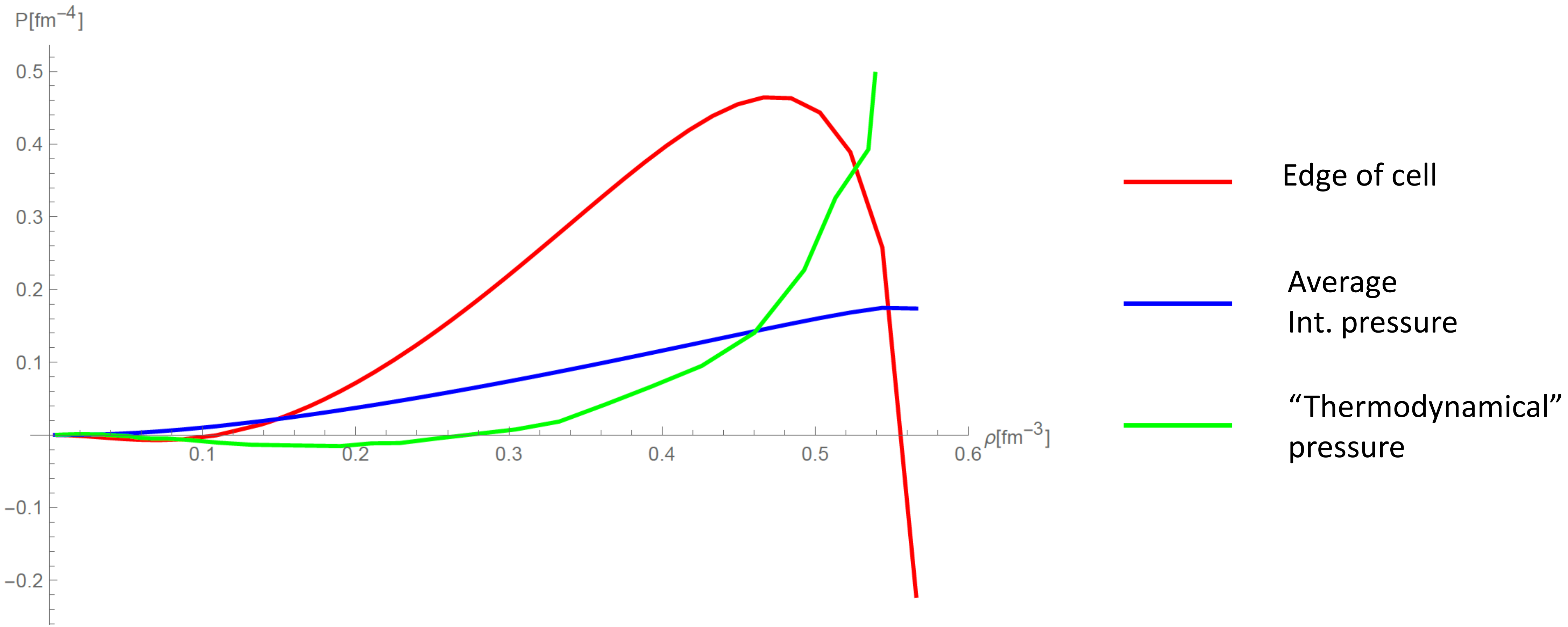


Pressure and shear forces





Evolution of the pressures: what happens at deconfinement?



Conclusions and open questions

- A Wigner-Seitz lattice of chiral solitons with vector mesons can be built and it provides a model for nuclear matter, although VERY schematic
- Within that model we can obtain an «equation of state»
- The modifications of integrated quantitites as e.g. the electric isoscalar radius are small till saturation
- We can observe deconfinement, at the density at which quarks start populating the conduction band
- Strong changes in the internal forces, even at moderate densities
- Still open problems: which is the relation between the internal pressure and the thermodynamical pressure? Which is the equivalent of Von Laue condition for a soliton in a Wigner-Seitz crystal?