On time-like electromagnetic pion form factors

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From first-principles QCD to Experiments,
ECT*, Trento, May 23, 2023

Outline

1. Analytic Structure of *n*-point Functions
2. Effect of resonances on hadron structure
3. Pion form factor & Vector Meson Dominance
4. Microscopic interaction model
5. Justification of Vector Meson Dominance
6. Conclusions
7. Outlook: $\gamma\ \pi\ \pi\ \pi$ form factor

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Analytic Structure of *n*-point Functions

- Strong Interactions in Theory: QCD
  - The model quantum gauge field theory:
    Locality, Unitarity, Asymptotic Freedom
  - Non-perturbative phenomena:
    Dimensional Transmutation, Chiral Anomaly, DχSB, Confinement
- Strong Interactions in Experiment: Hadrons
  - Hadron spectroscopy:
    many "unexpected" resonances, many "missing" resonances
  - Hadron structure: surprising results
- Quark-hadron duality:
  - Orthogonality of quark-glue d.o.f. vs. hadronic states
  - Physical *S*-matrix elements:
    Singularities (i.e., poles and cuts) **only** from intermediate physical (hadronic, leptonic, ...) states

Analytic Structure of *n*-point Functions

Lesson to be learned for <u>any</u> calculation in a gauge theory:

Singularities of gauge-dependent *n*-point functions

[need to, resp., should]

cancel in every $N(= N_i + N_f)-$point function / amplitude of N physical (composite!) states describing the scattering of N_i to N_f particles.

(NB: cf. kinematical aspect of confinement!)

Analytic Structure of *n*-point Functions

Domain of holomorphy of *n*-point Functions:

(see, e.g., beginning of Chapter 2 of RA & L. von Smekal, Phys. Rept. 353 (2001) 281)

- construction based on axioms of local quantum field theory in Minkowski spacetime[1]
- time-ordered *n*-point Green functions:
  boundary values of analytic functions
- several steps to arrive at the permuted extended tubes as envelope of holomorphy domain
- the latter contains non-coincident Euclidean region:
  Justifies the incorporation of time-like vectors as complex four-vectors in an analytically continued Euclidean formulation!

[1] for curved spacetimes see E. Witten, "Why Does Quantum Field Theory In Curved Spacetime Make Sense?", arXiv:2112.11614, and references therein

Analytic Structure of *n*-point Functions

NB:
No room for essential singularities, complex conjugated poles, etc.!

Nevertheless successful phenomenology ...

(see, however, S. Ahlig et al., PRD 64 (2001) 014004)

In practice:
First-principles investigations of the analytical structure, there are

- many for propagators / two-point functions (P. Maris 1991, ...),
- a few for three-point functions,[2]
- none (?) for higher *n*-point functions.

[2] see, e.g., M. Q. Huber, W. J. Kern, R.A., "Analytic structure of three-point functions from contour deformations," Phys. Rev. D **107** (2023) 074026

Effect of resonances on hadron structure

Only physical thresholds in S-matrix elements:
Hadron spectroscopy and hadron structure interrelated:
Microscopic understanding of effect of
resonances on form factors, structure functions, etc.?

Test case: **Pion form factor**[3]
Method: Functional method, in particular combination of
Dyson-Schwinger / Bethe-Salpeter eqs.

Important for the time-like pion form factor:

(i) Pion as pseudo Goldstone Boson
(ii) Mixing of ρ-meson with virtual photon
(iii) ρ-meson decay

[3] A topic for me since the eighties [K. Langfeld *et al.*, Z. Phys. **C42 (1989)** 159]

Effect of resonances on hadron structure

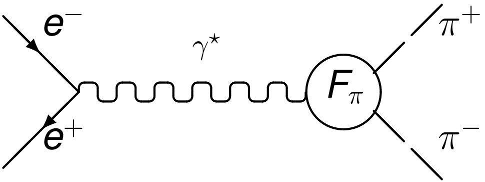
Important for the time-like pion form factor:

(i) Pion as $\bar{q}q$ bound state & as pseudo Goldstone Boson:
composite, highly collective state in QCD,
reflecting χSB patterns

(ii) Mixing of ρ-meson with virtual photon:
ρ as $\bar{q}q$ bound state in quark-photon vertex (QCD & QED),
theoretical explanation of Vector Meson Dominance

(iii) ρ-meson decay $\rho \to \pi\pi$:
two-pion cut and ρ-meson pole (on 2nd Riemann sheet)
in quark-photon vertex and
thus in e.m. timelike pion form factor.

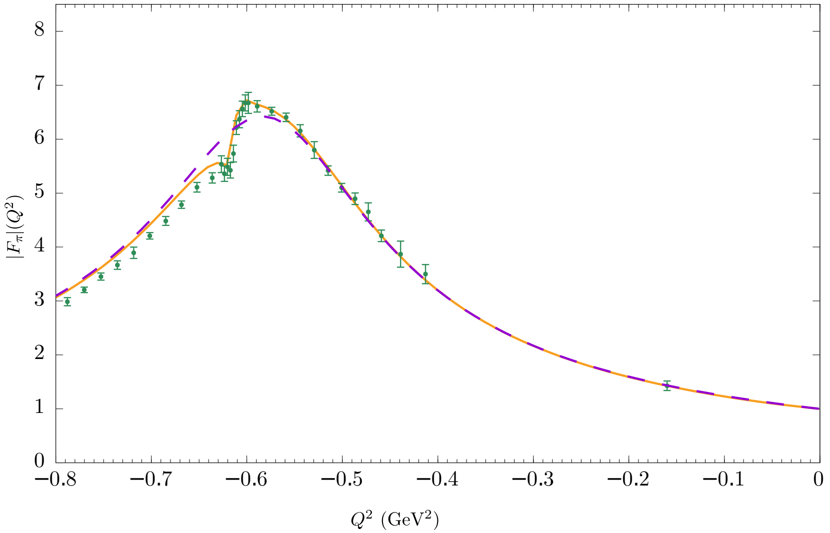
Time-like pion form factor & Vector Meson Dominance



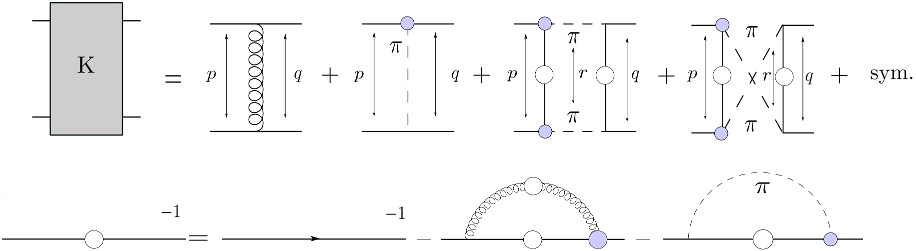
Experimentally, *e.g.*, from e^+e^- annihilation to $\pi\ \pi$

Convention: Negative Q^2 relates to timelike photon virtuality.

Time-like pion form factor & Vector Meson Dominance



Experimental data vs. VMD fit with / without ρ-ω mixing



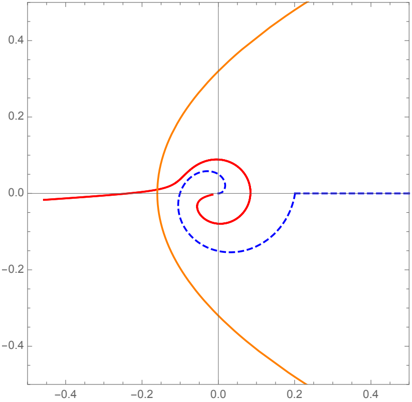
Interactions in this <u>exploratory calculation</u>:

- gluon exchange (Maris-Tandy model)
- pion exchange
- *s*- and *u*-channel pion decay contributions

Dyson-Schwinger/Bethe-Salpeter approach to time-like pion form factor

Disclaimer: To keep this calculation feasible a number of technically motivated approximations have been made, see arXiv:2102.12541 for details.

Major technical challenge: Find integration contour in presence of cuts generated by quark propagator poles, pion propagator pole as well as 2-pion cuts and ρ pole in quark-photon vertex!



Results

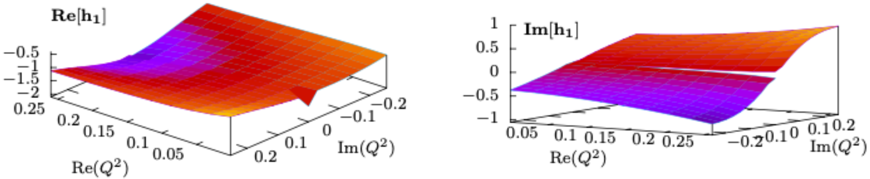
For two different parameters η of the Maris-Tandy model:

	m_π	f_π	m_ρ	m_ω	M_ρ	Γ_ρ
$\eta = 1.5$	0.139	0.138	0.768	0.778	0.750	0.100
$\eta = 1.6$	0.126	0.138	0.774	0.784	0.759	0.105

m_ρ and m_ω: Masses (in GeV) without two-pion decay kernel

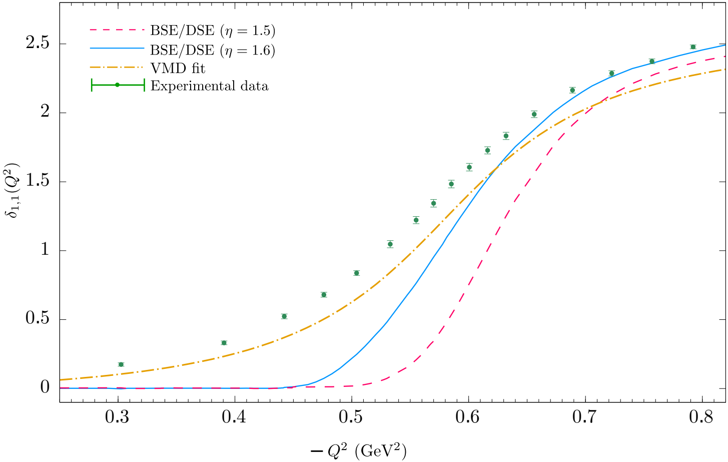
M_ρ and Γ_ρ (in GeV) determined from ρ-meson pole position defined as $M_{pole}^2 = M_\rho^2 - iM_\rho\Gamma_\rho$ with two-pion decay kernel taken into account

Results



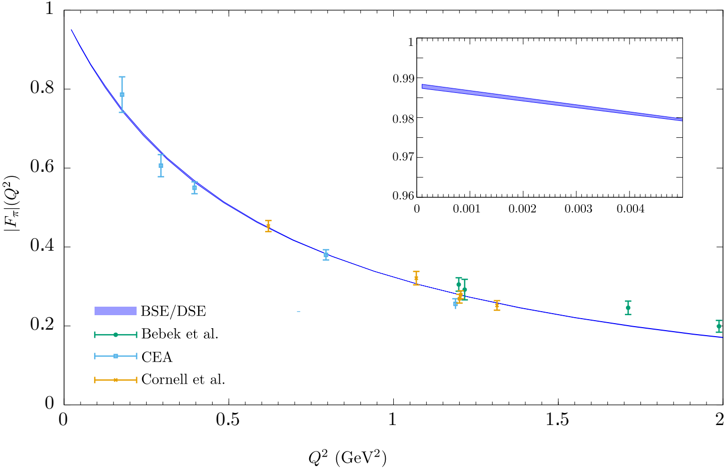
Real and imaginary part of the leading (transversely projected) amplitude of the quark-photon vertex for $p \cdot Q = 0$.
The two-pion branch cut starts at $Q^2 = -4m_\pi^2$.
[A. S. Miramontes,H. Sanchis-Alepuz, EPJA **55** (2019) 170 [arXiv:1906.06227].]

Results



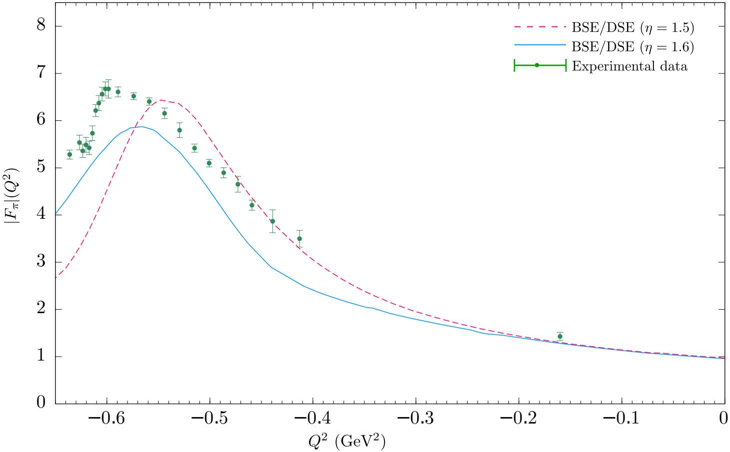
Phase of the pion form factor in the time-like $Q^2 < 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$ compared to experimental data on pion-pion phase shift.

Results



Pion form factor in the space-like $Q^2 > 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$ compared to experimental data.
(The inset illustrates the impact of one of the technically motivated approximations.)

Results



Absolute value of the pion form factor in the time-like $Q^2 < 0$ domain for the model parameters $\eta = 1.5$ and $\eta = 1.6$.

Results

Predicted by VMD (without ρ-ω mixing):

$$Re\,F_\pi(Q^2) - 1 = -\frac{a_1 Q^2 + a_2 (Q^2)^2}{b_0 + b_1 Q^2 + b_2 (Q^2)^2}$$

$$Im\,F_\pi(Q^2) = \frac{c_1 Q^2 + c_2 (Q^2)^2}{d_0 + d_1 Q^2 + d_2 (Q^2)^2} .$$

and verified by our "microscopic model" calculation

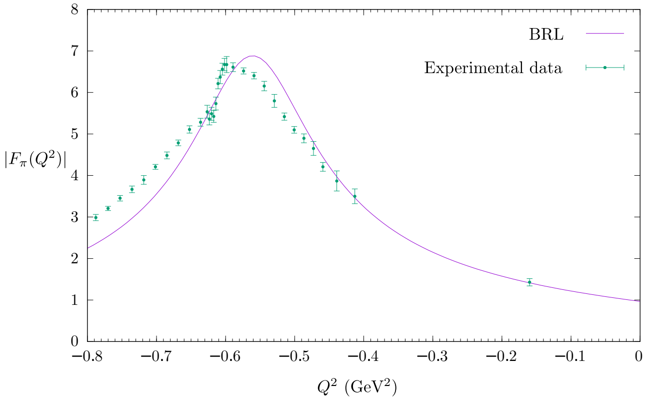
	η =1.5	η =1.6	VMD
a_1	0.5587	0.4149	0.72
a_2	0.8828	0.6827	1.2
b_0	0.3600	0.3600	0.36
b_1	1.2307	1.2517	1.2
b_2	1.0722	1.1000	1.0037
c_1	0.0591	0.0997	0
c_2	0.1295	0.2383	0.2308
d_0	0.3600	0.3600	0.36
d_1	1.1924	1.2464	1.2
d_2	0.9973	1.0916	1.0037

Results

- Other terms than VMD-predicted ones are tiny:
  Elaborated **calculation yields** within error margin
  the VMD predicted functional form.
- No significant impact from quark propagator poles!
  (Wanted in view of confinement! But why in this model-based calculation?)
- **The resulting time-like pion form factor in the region $0 > Q^2 > 0.8$GeV^2 is determined by the ρ-meson pole and the two-pion cut!**

Pion time-like form factor: All BSE amplitudes

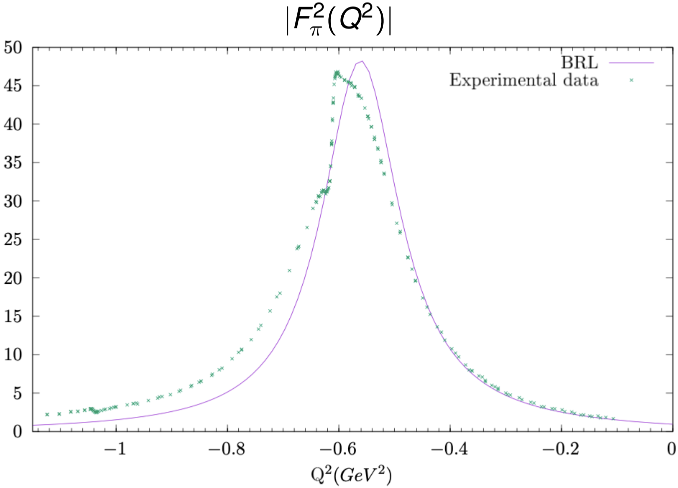
Absolute value of the pion form factor, $Q^2 < 0$, $\eta = 1.5$ with all BS amplitudes in 2-π-exchange kernels taken into account (requires 10 $\times$ CPU time) [A. Miramontes, unpublished]



extracted pole position:
$M_\rho = 755$ MeV, $\Gamma_\rho = 123$ MeV (before: 100 MeV, expt.: 149 MeV).

Pion time-like form factor: Large virtuality

New experimental results from 0.32 to 1.2 GeV
from CMD-3 at VEPP-2000 [arXiv:2302:08834]:



Pion time-like form factor: Large virtuality

At appr. one GeV:

- “strong deviation from the theoretical prediction” (citation from abstract)

  Impact on determination of anomalous magnetic moment of muon!
- Some deviation from predictions based on dispersion relations
- Similar for Vector Meson Dominance
- Large discrepancy of our results to experimental data!!!

. . . further analysis on-going . . .

Conclusions

- ☺ Exploratory DSE/BSE calculation of pion time-like form factor

  (. . . we can do time-like . . .)
- ☺ ρ-meson resonance & 2π cut determine time-like pion form factor: Detailed verification of VMD from microscopic model!
- ☺ Despite modelling and technical limitations: Remarkable agreement with experiment.

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Outlook

$\Longrightarrow$ Isospin breaking:

- Effect of different quark masses vs. electric charges

  cf. A.S. Miramontes et al., Phys. Lett. **B 833** (2022) 137291
- ρ-ω mixing, resp. ρ-ω-ϕ–mixing . . .

Outlook

$\Longrightarrow$ $\gamma\ \pi\ \pi\ \pi$ form factor:

(Experimental data from COMPASS currently analysed.)

- Anomaly determining soft-point value in symmetry-preserving truncation confirmed.
- Spacelike momenta: Results of S. Cotanch and P. Maris (Phys. Rev. **D68** (2003) 036006) verified.
- Effect of hadron resonances (ρ & ω) for timelike *s* . . .

Outlook

$\Longrightarrow$ Long-term wish list: Time-like form factors from
first-principle “functional” calculations.

Outlook

To appear soon:

[F. Llanes-Estrada, A. Salas-Bernadez, RA]

Verification of $^3P_0$ meson production mechanism from
- chromo-electric flux-tube,
- non-linear Breit-Wheeler process and
- χSB tensor structures of the quark-gluon vertex!

☺