

INCOHERENT DIFFRACTIVE DIJET PRODUCTION IN ELECTRON DIS OFF NUCLEI

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“Color Glass Condensate at the EIC”

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B. Rodriguez-Aguilar, DT, S.Y. Wei, 2302.01106, PRD

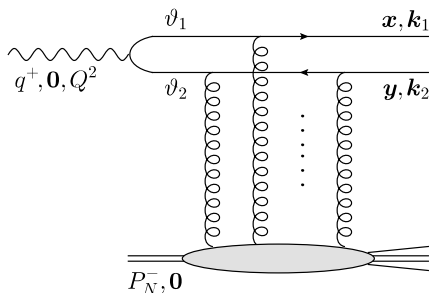
H. Mäntysaari, N. Mueller, F. Salazar, B. Schenke, 1912:05586, PRL

- Dijets in electron DIS off nuclei
- Incoherent diffractive process
- CGC correlator at 4-gluon exchange and in correlation limit
- “Factorization” into hard \times semi-hard factors
- Cross section and angular correlations
- Next to leading kinematic twists
- 2+1 jets contribution

DIJETS IN THE DIPOLE PICTURE OF DIS AT HIGH ENERGY

Right moving (RM) virtual photon, left moving (LM) hadron/nucleus

γ^* decay to RM $q\bar{q}$ pair, scattering off strong color field

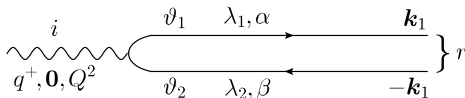


Size $\mathbf{r} = \mathbf{x} - \mathbf{y}$, center of energy $\mathbf{b} = v_1 \mathbf{x} + v_2 \mathbf{y}$

Dijet imbalance $\Delta = \mathbf{k}_1 + \mathbf{k}_2$, relative momentum $\mathbf{P} = v_2 \mathbf{k}_1 - v_1 \mathbf{k}_2$

VIRTUAL PHOTON WAVEFUNCTION

No scattering \rightsquigarrow no \mathbf{b} dependence



$q\bar{q}$ component of transverse γ^* in coordinate space

$$|\gamma_T^i(q)\rangle_{q\bar{q}} = \sum_{\lambda_{1,2}=\pm 1/2} \sum_{\alpha,\beta=1}^{N_c} \delta_{\alpha\beta} \int_0^1 d\vartheta_1 d\vartheta_2 \delta(1 - \vartheta_1 - \vartheta_2) \int d^2\mathbf{x} d^2\mathbf{y} \\ \times \tilde{\psi}_{\lambda_1\lambda_2}^i(\vartheta_1, \mathbf{r}) |q_{\lambda_1}^\alpha(\vartheta_1, \mathbf{x}) \bar{q}_{\lambda_2}^\beta(\vartheta_2, \mathbf{y})\rangle$$

with wavefunction (WF)

$$\tilde{\psi}_{\lambda_1\lambda_2}^i(\vartheta, \mathbf{r}) = -\sqrt{\frac{q^+}{2}} \frac{ieef}{(2\pi)^2} \varphi_{\lambda_1\lambda_2}^{il}(\vartheta) \frac{\bar{Q}r^l}{r} K_1(\bar{Q}r), \quad \bar{Q}^2 = \vartheta_1\vartheta_2Q^2$$

φ contains EM vertex helicity structure

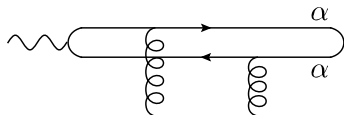
Eikonal multiple scattering of quark off target color field

$$V(\mathbf{x}) = \text{T exp} \left[ig \int dx^+ t^a A_a^-(x^+, \mathbf{x}) \right]$$

For the $q\bar{q}$ pair let

$$\delta_{\alpha\beta} \rightarrow [V(\mathbf{x})V^\dagger(\mathbf{y}) - \mathbf{1}]_{\alpha\beta}$$

Diffraction: close dipole color line in DA (and separately in CCA)



$$\begin{aligned} [V(\mathbf{x})V^\dagger(\mathbf{y}) - \mathbf{1}]_{\alpha\beta} &\rightarrow \left\{ \frac{1}{N_c} \text{tr}[V(\mathbf{x})V^\dagger(\mathbf{y})] - 1 \right\} \delta_{\alpha\beta} \\ &\equiv [S(\mathbf{x}, \mathbf{y}) - 1] \delta_{\alpha\beta} = -T(\mathbf{x}, \mathbf{y}) \delta_{\alpha\beta} \end{aligned}$$

In transverse sector

$$\frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{\Delta}} = \frac{\alpha_{em} N_c}{2\pi^2} \left(\sum e_f^2 \right) \delta(1-\vartheta_1-\vartheta_2) (\vartheta_1^2 + \vartheta_2^2) \int \frac{d^2\mathbf{b}}{2\pi} \frac{d^2\bar{\mathbf{b}}}{2\pi} \frac{d^2\mathbf{r}}{2\pi} \frac{d^2\bar{\mathbf{r}}}{2\pi} \\ \times e^{-i\mathbf{\Delta}\cdot(\mathbf{b}-\bar{\mathbf{b}})-i\mathbf{P}\cdot(\mathbf{r}-\bar{\mathbf{r}})} \frac{\mathbf{r}\cdot\bar{\mathbf{r}}}{r\bar{r}} \bar{Q}^2 K_1(\bar{Q}r) K_1(\bar{Q}\bar{r}) \langle T(\mathbf{x}, \mathbf{y}) T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle$$

QCD dynamics in correlator $\langle T(\mathbf{x}, \mathbf{y}) T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle$

Target average to be taken with CGC wave-function

- $\langle T(\mathbf{x}, \mathbf{y}) \rangle \langle T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle \rightarrow$ Coherent diffraction
- $\langle T(\mathbf{x}, \mathbf{y}) T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle - \langle T(\mathbf{x}, \mathbf{y}) \rangle \langle T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle \rightarrow$ Incoherent diffraction

Homogeneous target: coherent diffraction $\sim \delta^2(\mathbf{\Delta})$ (smeared to $1/R_A$)

Negligible momentum transfer and dijet imbalance

Variance of scattering amplitude

$$\mathcal{W}_D = \langle T(\mathbf{x}, \mathbf{y})T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle - \langle T(\mathbf{x}, \mathbf{y}) \rangle \langle T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle$$

- Pomeron loops: particle number fluctuations in target
- Hot spots (Demirci, Lappi, Schlichting)
- $1/N_c^2$ color fluctuations (MV model, JIMWLK) (Marquet, Weigert)

Homogeneous target: $\mathbf{r}, \bar{\mathbf{r}}, \mathbf{B} \equiv \mathbf{b} - \bar{\mathbf{b}}$ independent variables

$$\begin{aligned} \frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\Delta} = \dots & \frac{S_\perp}{2\pi} \int \frac{d^2\mathbf{B}}{2\pi} \frac{d^2\mathbf{r}}{2\pi} \frac{d^2\bar{\mathbf{r}}}{2\pi} e^{-i\Delta \cdot \mathbf{B} - i\mathbf{P} \cdot (\mathbf{r} - \bar{\mathbf{r}})} \\ & \times \frac{\mathbf{r} \cdot \bar{\mathbf{r}}}{r\bar{r}} \bar{Q}^2 K_1(\bar{Q}r) K_1(\bar{Q}\bar{r}) \mathcal{W}_D(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{B}) \end{aligned}$$

↪ Non-zero momentum transfer and dijet imbalance

- Energy conservation

$$x_{\text{gap}} P_N^- = \frac{1}{2q^+} \left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{2\perp}^2}{\vartheta_2} + Q^2 \right) \implies x_{\text{gap}} = \frac{Q^2}{W^2} \left(1 + \underbrace{\frac{P_\perp^2}{\bar{Q}^2}}_{\mathcal{O}(1)} + \underbrace{\frac{\Delta_\perp^2}{Q^2}}_{\ll 1} \right)$$

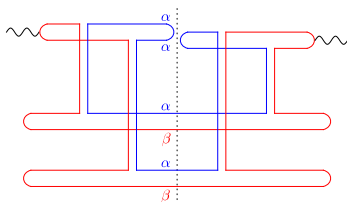
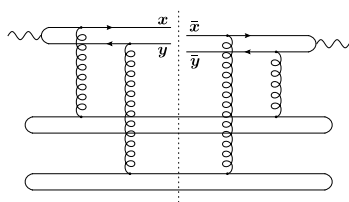
Consider Y_{gap} as independent of P_\perp

- Symmetric splitting $\vartheta_1 \sim \vartheta_2 \sim 1/2$

$$\frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{\Delta}} = \underbrace{\vartheta_1 \vartheta_2}_{\sim 1/4} \frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}X}}{d\eta_1 d\eta_2 d^2\mathbf{P} d^2\mathbf{\Delta}}$$

CGC CORRELATOR AT 4-GLUON EXCHANGE

Assume Gaussian CGC WF, only pieces connecting DA with CCA survive:



$$\frac{g^4}{4N_c^2} \sum_{a,b} \langle \alpha_x^a \alpha_y^b \rangle \langle \alpha_y^a \alpha_x^b \rangle = \frac{g^4}{4N_c^2} \sum_a \langle \alpha_x^a \alpha_y^a \rangle \langle \alpha_y^a \alpha_x^a \rangle = \sum_{a,b} \frac{g^4}{4N_c^2(N_c^2-1)} \langle \alpha_x^a \alpha_y^a \rangle \langle \alpha_y^b \alpha_x^b \rangle$$

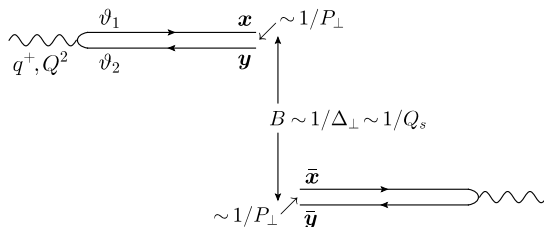
Target colorless substructures exchange only one gluon in DA (CCA)

Put together all 16 terms, add/subtract "same point correlators"

$$\begin{aligned} \mathcal{W}_D(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{B}) &\simeq \frac{1}{2(N_c^2-1)} [\mathcal{T}(\mathbf{x}, \bar{\mathbf{y}}) + \mathcal{T}(\mathbf{y}, \bar{\mathbf{x}}) - \mathcal{T}(\mathbf{x}, \bar{\mathbf{x}}) - \mathcal{T}(\mathbf{y}, \bar{\mathbf{y}})]^2 \\ &\simeq \frac{1}{2(N_c^2-1)} r^i \bar{r}^j r^k \bar{r}^l \partial^i \partial^j \mathcal{T}(\mathbf{B}) \partial^k \partial^l \mathcal{T}(\mathbf{B}) \end{aligned}$$

Incoherent scattering $\leftrightarrow 1/N_c^2$ suppression

Extend above to correlation limit $P_\perp \gg \Delta_\perp, Q_s \longleftrightarrow r, \bar{r} \ll B, 1/Q_s$



$\langle TT \rangle$ known at finite- N_c , expand for small r, \bar{r} , let B arbitrary

$$\mathcal{W}_D(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{B}) \simeq \frac{C_F}{2N_c^3} r^i \bar{r}^j r^k \bar{r}^l \Phi(\mathcal{S}_g(B)) [\partial^i \partial^j \ln \mathcal{S}_g(B)] [\partial^k \partial^l \ln \mathcal{S}_g(B)]$$

Same structure as in 4 gluon exchange approximation

Angular correlation between \mathbf{r} and \mathbf{B} , hence between \mathbf{P} and Δ

Hard integrals over $\mathbf{r}, \bar{\mathbf{r}}$ factorize from semi-hard over \mathbf{B}

Factorization holds at cross section (not at amplitude) level

$$\frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{\Delta}} = \frac{S_{\perp} \alpha_{em} N_c}{4\pi^3} \left(\sum e_f^2 \right) \delta(1-\vartheta_1-\vartheta_2) (\vartheta_1^2 + \vartheta_2^2) \frac{C_F}{2N_c^3} \left| \mathcal{A}_D^T \right|^2$$

Reduced cross section

$$\left| \mathcal{A}_D^T \right|^2 = H_T^{iks}(\mathbf{P}, \bar{\mathbf{Q}}) H_T^{jls^*}(\mathbf{P}, \bar{\mathbf{Q}}) \mathcal{G}_D^{ij,kl}(\mathbf{\Delta})$$

Hard factor trivial to calculate

$$H_T^{iks}(\mathbf{P}, \bar{\mathbf{Q}}) = -\frac{2i}{(P_{\perp}^2 + \bar{Q}^2)^2} (\delta^{ik} P^s + \delta^{is} P^k + \delta^{ks} P^i) + \frac{8i P^i P^k P^s}{(P_{\perp}^2 + \bar{Q}^2)^3}$$

Scales like $1/P_{\perp}^3$ for $\bar{\mathbf{Q}} \sim P_{\perp}$

Semi-hard factor with dimension of mass squared

$$\mathcal{G}_D^{ij,kl}(\Delta) = \int \frac{d^2 \mathbf{B}}{2\pi} e^{-i\Delta \cdot \mathbf{B}} \Phi(\mathcal{S}_g(B)) [\partial^i \partial^j \ln \mathcal{S}_g(B)] [\partial^k \partial^l \ln \mathcal{S}_g(B)]$$

Square of structure appearing in WW gluon TMD

$$\partial^i \partial^j \ln \mathcal{S}_g(B) = \underbrace{\frac{\delta^{ij}}{2} \frac{1}{B} \frac{\partial}{\partial B} \left(B \frac{\partial \ln \mathcal{S}_g}{\partial B} \right)}_{F_+(B)} + \left(\frac{B^i B^j}{B^2} - \frac{\delta^{ij}}{2} \right) \underbrace{B \frac{\partial}{\partial B} \left(\frac{1}{B} \frac{\partial \ln \mathcal{S}_g}{\partial B} \right)}_{F_-(B)}$$

$\mathcal{G}_D^{ij,kl}(\Delta)$ decomposed into terms \propto to $\delta^{ij} \delta^{kl}$ or $\delta^{ij} \Delta^k \Delta^l$ or $\Delta^i \Delta^j \Delta^k \Delta^l$

\rightsquigarrow Four coefficients $\mathcal{G}_D^{(n)}(\Delta_\perp)$: “distributions” (cf. xG, xh in inclusive)

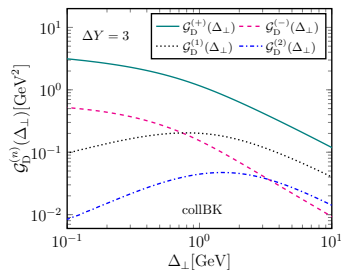
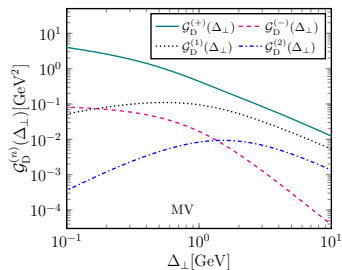
$$\mathcal{G}_D^{[+,-,1,2]} = \int \frac{d^2 \mathbf{B}}{2\pi} e^{-i\Delta \cdot \mathbf{B}} \Phi(\mathcal{S}_g(B)) [F_+^2, F_-^2, F_+ F_- \cos 2\phi_{\Delta B}, F_-^2 \cos 4\phi_{\Delta B}]$$

Compact analytic expression in correlation limit $P_{\perp} \gg \Delta_{\perp}, Q_s$

$$\begin{aligned} |A_D^T|^2 = & \frac{4P_{\perp}^2(3\bar{Q}^4 + P_{\perp}^4)}{(P_{\perp}^2 + \bar{Q}^2)^6} \mathcal{G}_D^{(+)}(\Delta_{\perp}) + \frac{8\bar{Q}^4 P_{\perp}^2}{(P_{\perp}^2 + \bar{Q}^2)^6} \mathcal{G}_D^{(-)}(\Delta_{\perp}) \\ & + \frac{16\bar{Q}^2 P_{\perp}^2 (\bar{Q}^2 - P_{\perp}^2) \cos 2\phi}{(P_{\perp}^2 + \bar{Q}^2)^6} \mathcal{G}_D^{(1)}(\Delta_{\perp}) - \frac{8\bar{Q}^2 P_{\perp}^4 \cos 4\phi}{(P_{\perp}^2 + \bar{Q}^2)^6} \mathcal{G}_D^{(2)}(\Delta_{\perp}) \end{aligned}$$

- Simple 1D integration (Bessel transform) to get $\mathcal{G}_D^{(n)}(\Delta_{\perp})$
- Dependence on P_{\perp} , Δ_{\perp} and also on $\phi = \angle(\mathbf{P}, \mathbf{\Delta})$
- For $\bar{Q} \sim P_{\perp}$ all hard factors $\sim 1/P_{\perp}^6$, like in exclusive dijets
- One-to-one correspondence between angles $\phi_{\Delta B}$ and ϕ :
 $\cos n\phi_{\Delta B} \rightsquigarrow \cos n\phi$

MOMENTUM DEPENDENCE OF “DISTRIBUTIONS”



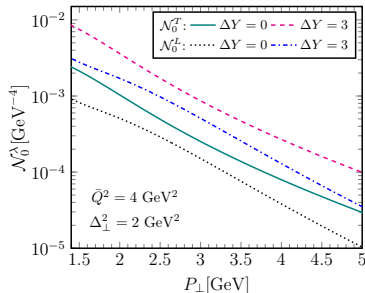
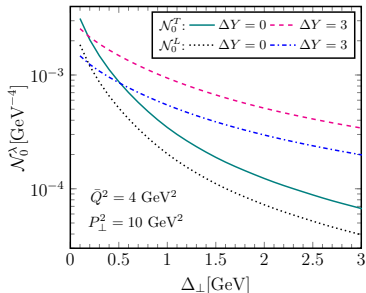
Analytic estimates in MV model for $\Delta_\perp \gg Q_s$ and $\Delta_\perp \sim Q_s$

$$\mathcal{G}_D^{(+)} \simeq \frac{2Q_A^4}{\Delta_\perp^2} \ln \frac{\Delta_\perp^2}{\Lambda^2}, \quad \mathcal{G}_D^{(-)} \simeq \frac{Q_A^6}{3\Delta_\perp^4}, \quad \mathcal{G}_D^{(1)} \simeq \frac{Q_A^4}{\Delta_\perp^2} \ln \frac{\Delta_\perp^2}{\Lambda^2}, \quad \mathcal{G}_D^{(2)} \simeq \frac{2Q_A^4}{\Delta_\perp^2}$$

$$\mathcal{G}_D^{(+)} \sim Q_s^2, \quad \mathcal{G}_D^{(-)} \sim \frac{Q_s^2}{\rho_A^2}, \quad \mathcal{G}_D^{(1)} \sim \frac{Q_s^2}{\rho_A}, \quad \mathcal{G}_D^{(2)} \sim \frac{Q_s^2}{\rho_A^2}$$

Overall $\mathcal{G}_D^{(+)} > \mathcal{G}_D^{(1)} \gg \mathcal{G}_D^{(2)}$, while $\mathcal{G}_D^{(-)}$ may be neglected

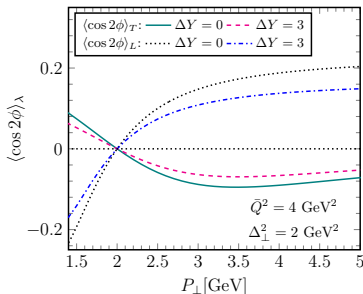
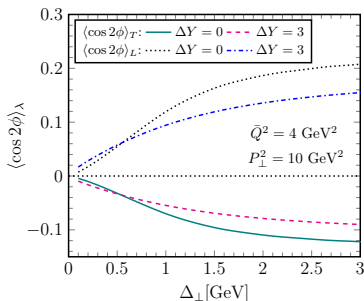
AVERAGED CROSS SECTION



$$\frac{d\sigma_D^{\gamma_\lambda^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2P d^2\Delta} \propto \frac{\alpha_{\text{em}} S_\perp}{N_c} \frac{1}{P_\perp^6} \frac{Q_s^4}{\Delta_\perp^2}$$

- If $\Delta_\perp \sim Q_s$, double suppression $1/N_c^2 \times Q_s^2/P_\perp^2$ w.r.t. inclusive

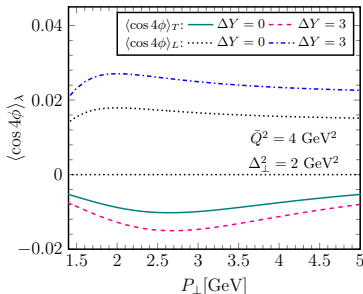
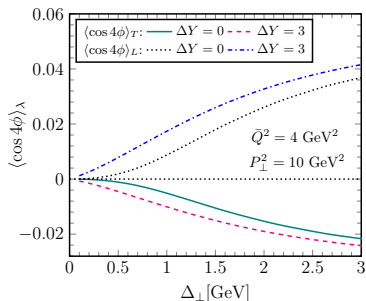
ANGULAR CORRELATION $\langle \cos 2\phi \rangle$



$$\langle \cos 2\phi \rangle_T \simeq \frac{2\bar{Q}^2(\bar{Q}^2 - P_\perp^2)}{3\bar{Q}^4 + P_\perp^4} \frac{\mathcal{G}_D^{(1)}(\Delta_\perp)}{\mathcal{G}_D^{(+)}(\Delta_\perp)}, \quad \langle \cos 2\phi \rangle_L \simeq \frac{4P_\perp^2(P_\perp^2 - \bar{Q}^2)}{\bar{Q}^4 - 2\bar{Q}^2P_\perp^2 + 5P_\perp^4} \frac{\mathcal{G}_D^{(1)}(\Delta_\perp)}{\mathcal{G}_D^{(+)}(\Delta_\perp)}$$

- Saturation leads to suppression of anisotropy
- Both T and L vanish at $P_\perp = \bar{Q}$

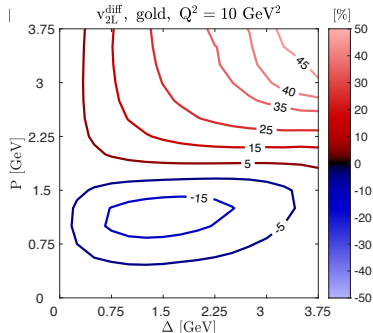
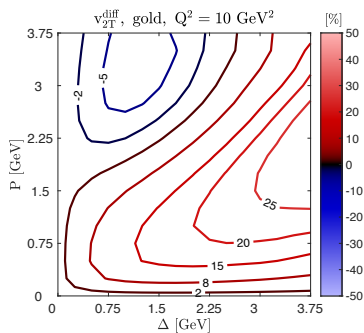
ANGULAR CORRELATION $\langle \cos 4\phi \rangle$



$$\langle \cos 4\phi \rangle_T \simeq -\frac{\bar{Q}^2 P_\perp^2}{3\bar{Q}^4 + P_\perp^4} \frac{\mathcal{G}_D^{(2)}(\Delta_\perp)}{\mathcal{G}_D^{(+)}(\Delta_\perp)}, \quad \langle \cos 4\phi \rangle_L \simeq \frac{2P_\perp^4}{\bar{Q}^4 - 2\bar{Q}^2 P_\perp^2 + 5P_\perp^4} \frac{\mathcal{G}_D^{(2)}(\Delta_\perp)}{\mathcal{G}_D^{(+)}(\Delta_\perp)}$$

- Saturation leads to suppression of anisotropy
- Same twist as $\langle \cos 2\phi \rangle$, suppressed semi-hard factor

Exact numerical solution in for arbitrary P_{\perp} , Δ_{\perp} and Q_s (Mäntysaari et al)



- Very good agreement in longitudinal sector
- Puzzling in the transverse sector:
 - zero not at $P_{\perp} = \bar{Q}$
 - for fixed $P_{\perp} > \bar{Q}$, minimum as a function of Δ_{\perp}

Three scales, classify corrections (Altinoluk, Boussarie, Fujii, Kotko, Marquet, Mehtar-Tani, Watanabe...)

- Genuine saturation twists $(Q_s^2/P_\perp^2)^n$: numerical implementation
- Kinematic twists $(\Delta_\perp^2/P_\perp^2)^n$: resummation \rightsquigarrow improved TMD
Done for inclusive dijets (Boussarie, Mäntysaari, Salazar, Schenke)

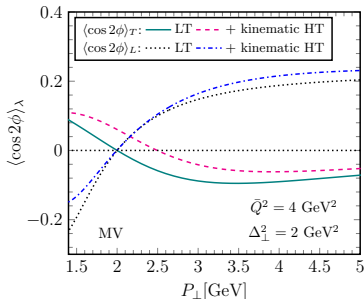
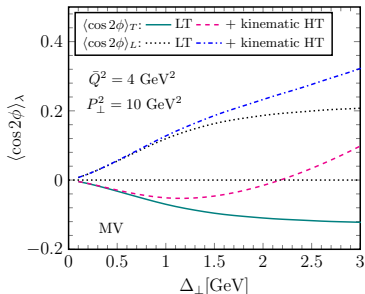
Calculate just first kinematic twist

$$\delta|\mathcal{A}_D^T|^2 = \frac{1}{6} H_T^{ikmns}(\mathbf{P}, \bar{Q}) H_T^{jls^*}(\mathbf{P}, \bar{Q}) \mathcal{G}_D^{ij,klmn}(\Delta)$$

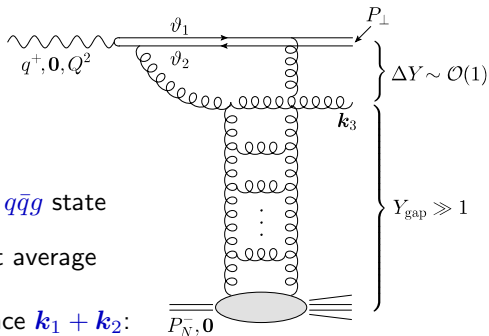
$$\mathcal{G}_D^{ij,klmn}(\Delta) = \int \frac{d^2\mathbf{B}}{2\pi} e^{-i\Delta\cdot\mathbf{B}} \Phi((S_g(B)))[\partial^i \partial^j \ln S_g(B)][\partial^k \partial^l \partial^m \partial^n \ln S_g(B)]$$

- leading twist $\mathcal{W}_D \sim r^4 \partial^2 \ln \mathcal{S} \partial^2 \ln \mathcal{S} \sim r^4 Q_A^4$
- NL kin twist $\delta\mathcal{W}_D \sim r^6 \partial^2 \ln \mathcal{S} \partial^4 \ln \mathcal{S} \sim r^6 Q_A^4/B^2$
- $\delta\mathcal{W}_D/\mathcal{W}_D \sim r^2/B^2 \sim \Delta_\perp^2/P_\perp^2 \checkmark$

$\langle \cos 2\phi \rangle$ WITH NL KINEMATIC TWIST



- No qualitative change in longitudinal sector ✓
- New features in transverse sector
 - for fixed P_\perp , minimum as a function of Δ_\perp ✓
 - sign change at P_\perp value which increases with Δ_\perp ✓



- Diffractive projection to $q\bar{q}g$ state
- Connected part in target average

Two sources for dijet imbalance $\mathbf{k}_1 + \mathbf{k}_2$:

- Momentum transfer from nucleus
- Kick due to gluon emission \mathbf{k}_3

If $k_{3\perp} \ll P_{\perp}$, then gluon dipole configuration scatters

Keep $1/N_c^2$ piece of $\langle T_g T_g \rangle$ (Kovchegov, Wertepny)

- Simple, but illustrative case: $P_{\perp} \gg k_{3\perp} \gg \Delta_{\perp}, Q_s$

$\langle T_g T_g \rangle$ simplifies, analytical calculation

$$\frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}gX}}{d^2P d^2\Delta d^2k_3} \propto \frac{1}{P_{\perp}^4} \frac{Q_s^4}{\Delta_{\perp}^2} \frac{1}{k_{3\perp}^4}$$

- Gluon integration dominated by lower limit

Eventually exact integration should be determined by $k_{3\perp} \sim \Delta_{\perp}$

$$\boxed{\frac{d\sigma_D^{\gamma_T^* A \rightarrow q\bar{q}gX}}{d^2P d^2\Delta} \propto \frac{1}{P_{\perp}^4} \frac{Q_s^4}{\Delta_{\perp}^4}}$$

Larger than $q\bar{q}$ component

Thanks to everybody for the lively participation