19.05.2023,
ECT*, Trento.

CGC at the EIC

NCBJ

Propagators in Background field at Next to Eikonal order

Swaleha Mulani
(NCBJ, Poland)

In Collaboration with:

Tolga Altinoluk and Guillaume Beuf

(Ongoing work)

Outline

- Eikonal Approximation and relaxing it
- Gluon Propagator in gluon background field
    - At eikonal order
    - Next to eikonal corrections
    - Application
- Propagators in quark background field
    - SIDIS in Quark background field
    - Single Inclusive Production

Introduction

- In high energy scattering, for projectile-target collision:

Projectile: dilute proton   Target: dense nucleus (described by CGC)

- For simplification, we have

Projectile: parton   Target: gluon background field $A_\mu(x)$

There is hierarchy between components of $A_\mu(x)$ with respect to Lorentz boost factor γ_t of the target:

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

Eikonal Approximation : leading order energy term (leading term in γ_t)

Eikonal

Zero Width

1.Highly boosted background field (target) is localised in the longitudinal direction $x^+ = 0$ (zero width).

Leading Component

2.Only leading component of target (–component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

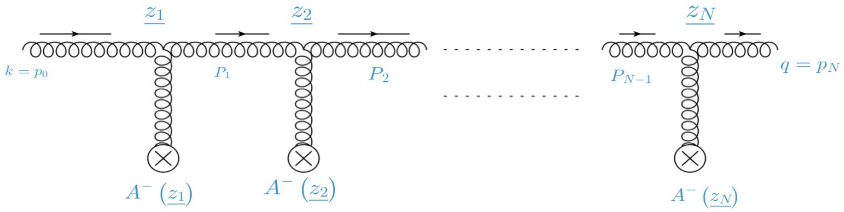
x^- independence

3.Dynamics of the target are neglected (x^- dependence of target neglected).

Background field of target is : $A^\mu\left(x^-, x^+, \boldsymbol{x}\right) \approx \delta^{\mu-}\delta\left(x^+\right) A^-\left(\boldsymbol{x}\right)$

Gluon Propagator at eikonal order:

- To calculate gluon propagator at eikonal order, we re-sum multiple interaction diagrams of Gluon background field as shown in figure below.
- Only “–” component (leading component) of classical gluon background field is considered.



Similar for quarks in Altinoluk,Beuf, Czajka, Tymowska [2012.03886] , Altinoluk, Beuf [arXiv:2109.01620]

Gluon Propagator at eikonal order:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y)\big|_{Eik} &= i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\,\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]\\
&+\left\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\ \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+}\left[2\pi\ \delta(k^+ - q^+)\right]\right.\\
&\times\left[-g^{\mu\nu}+\frac{\check{k}^\mu\eta^\nu}{k^+}+\frac{\eta^\mu\check{q}^\nu}{q^+}-\frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left.\left[\int d^2z_\perp\ e^{-i(q_\perp - k_\perp)z_\perp}\right]\right\}\\
&\times\left[\theta(x^+ - y^+)\ \theta(k^+)\ \mathcal{U}_A(x^+,y^+;z_\perp)-\theta(y^+ - x^+)\ \theta(-k^+)\ \mathcal{U}_A^\dagger(x^+,y^+;z_\perp)\right]
\end{aligned}
$$

This is general expression in pure A^- background field at eikonal order for any x^+ and y^+.

(........and of course... we are going to use it in almost every calculation!)

Gluon Propagator at eikonal order:

$$G_F^{\mu\nu}(x,y)|_{Eik} = i\delta^2(x_\perp - y_\perp)\;\delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\,\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$

$$+\Bigg\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\;\frac{e^{-ix\cdot\check{q}}\;e^{iy\cdot\check{k}}}{2k^+}\left[2\pi\;\delta(k^+ - q^+)\right]$$

$$\times\left[-\,g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left[\int d^2z_\perp\,e^{-}\right.$$

$$\times\left[\theta(x^+ - y^+)\;\theta(k^+)\;\mathcal{U}_A(x^+,y^+;z_\perp) - \theta(y^+ - x^+)\;\theta(-k^+)\;\mathcal{U}_A^\dagger(x^+,y^+;z_\perp)\right]$$

Summations are included in Wilson lines,Not dependent on z^-

Where, $\mathcal{U}_A(x^+,y^+;z_\perp) = 1+\sum_{N=1}^{\infty}\frac{1}{N!}\mathcal{P}_+\left[-ig\int_{y^+}^{x^+}dz^+A^-(z^+,z_\perp)\cdot T\right]^N$

Gluon Propagator at eikonal order:

$$G_F^{\mu\nu}(x,y)|_{Eik} = i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\,\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$

$$+\Bigg\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\,\frac{e^{\ ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+}\Big[2\pi\ \delta(k^+ - q^+)\Big]$$

$$\times\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\Big[\int d^2z_\perp\, e^{-}$$

$$\times\Big[\theta(x^+ - y^+)\ \theta(k^+)\ \mathcal{U}_A(x^+,y^+;z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ \mathcal{U}_A^\dagger(x^+,y^+;z_\perp)\Big]$$

Summations are included in Wilson lines,Not dependent on z^-

Where, $\mathcal{U}_A(x^+,y^+;z_\perp) = 1+\sum_{N=1}^{\infty}\frac{1}{N!}\mathcal{P}_+\left[-ig\int_{y^+}^{x^+}dz^+A^-(z^+,z_\perp)\cdot T\right]^N$

Eikonal and beyond it

Zero Width

1.Highly boosted background field (target) is localised in the longitudinal direction $x^+ = 0$ (zero width).

Leading Component

2.Only leading component of target (-component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

x^- independence

3.Dynamics of the target are neglected (x^- dependence of target neglected).

Background field of target is : $A^{\mu}\left(x^{-}, x^{+}, \boldsymbol{x}\right) \approx \delta^{\mu-} \delta\left(x^{+}\right) A^{-}(\boldsymbol{x})$

Beyond Eikonal:

- In very high energy accelerators (γ_t~1000 order), next-to-eikonal (NEik) order terms are negligible while calculating observables.
- But to analyze the data from RHIC and future electron ion collider (EIC)(γ_t~ 10-100 order), NEik order terms might be sizable!

Eikonal and beyond it

Zero Width

1.Highly boosted background field (target) is localised in the longitudinal direction $x^+ = 0$ (zero width).

Leading Component

2.Only leading component of target (- component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

x^- independence

3.Dynamics of the target are neglected (x^- dependence of target neglected).

Background field of target is : $A^{\mu}\left(x^{-},x^{+},\boldsymbol{x}\right)\approx\delta^{\mu-}\delta\left(x^{+}\right)A^{-}\left(\boldsymbol{x}\right)$

To go beyond Eikonal:

Finite Width

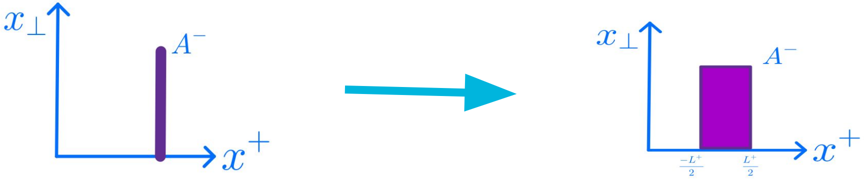
1.Instead of infinite thin shockwave as a target, we consider finite width of a target.

Transverse Component

2.Instead of neglecting sub-leading components, we include transverse component of background field.

x^- dependence

3.We take into account corrections coming due to the x^- dependence of a target (consider background field is x^- dependent).



From now on, we will consider the case where $x^+ > y^+$ such that $x^+ > L^+/2$ and $y^+ < -L^+/2$

Next to eikonal corrections:

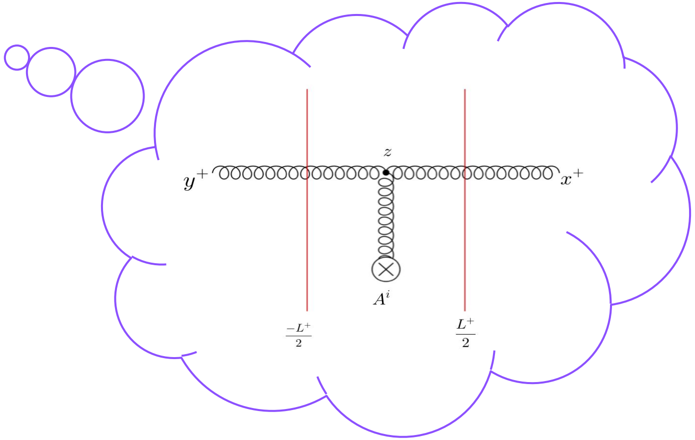
Due to relaxing the eikonal approximation we will get three kinds of corrections:

- **Due to finite longitudinal width of the target**
    - Expanding the transverse motion of the parton around the eikonal trajectory (around the fixed transverse position)

Next to eikonal corrections:

Due to relaxing the eikonal approximation we will get three kinds of corrections:

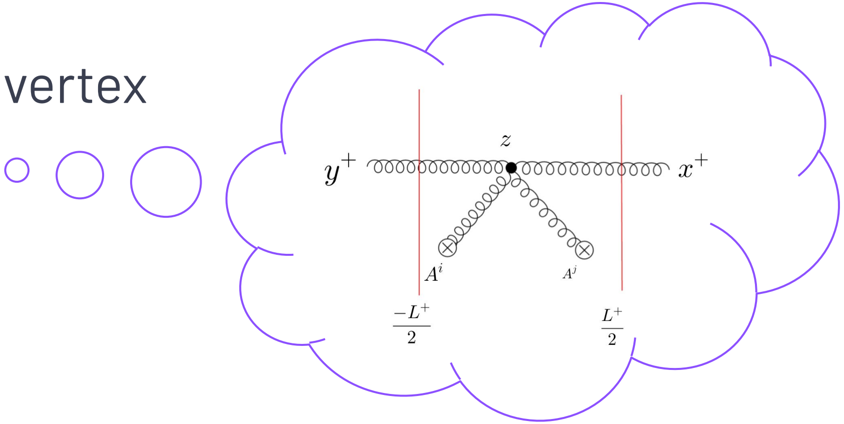
- **Due to finite longitudinal width of the target**
- **Considering interactions with transverse component of gluon background field:**
  - Due to single three gluon vertex



Next to eikonal corrections:

Due to relaxing the eikonal approximation we will get three kinds of corrections:

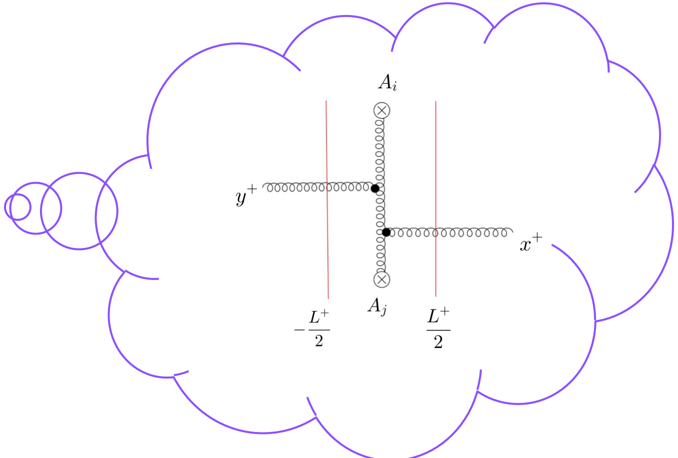
- **Due to finite longitudinal width of the target**
- **Considering interactions with transverse component of gluon background field:**
  - Due to single three gluon vertex
  - **Due to four gluon vertex**



Next to eikonal corrections:

Due to relaxing the eikonal approximation we will get three kinds of corrections:

- **Due to finite longitudinal width of the target**
- **Considering interactions with transverse component of gluon background field:**
  - Due to single three gluon vertex
  - Due to four gluon vertex
  - **Due to double three gluon vertices at the same z^+**



Next to eikonal corrections: Transverse component

We will calculate NEik corrections to gluon propagator due to interaction with subleading components background field by evaluating following kinds of equations:

$$\delta G_F^{\mu\nu}(x,y) = \int d^4z \; G_F^{\mu\mu'}(x,z)\big|_{Eik} \boxed{X^{3g}_{\mu'\nu'}(\underline{z})} G_F^{\nu'\nu}(z,y)\big|_{Eik}$$

Insertion factor (different for different background field insertion)

Next to eikonal corrections:

Due to relaxing the eikonal approximation we will get three kinds of corrections:

- **Due to finite longitudinal width of the target**
- **Considering interactions with transverse component of gluon background field:**
    - Due to single three gluon vertex
    - Due to four gluon vertex
    - Due to double three gluon vertices at the same z^+
- **Taking into account dynamics of target**
    - By performing gradient expansion of $A^-(z)$ with respect to z^-

Total Gluon propagator at NEik order

Total gluon propagator upto NEik order travelling through the entire medium (dynamic gluon background field) for the case $x^+ > L^+/2$ and

$y^+ < -L^+/2$ with $x^+ > y^+$ is:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y) = & \int \frac{d^3\underline{q}}{(2\pi)^3}\; e^{-ix\cdot\check{q}}\;\theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3}\; e^{iy\cdot\check{k}}\;\theta(k^+)\;\frac{1}{q^+ + k^+} \\
& \times \left[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \right] \int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
& \times \int dz^-\; e^{i(q^+-k^+)z^-}\; \mathcal{U}_A(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-) \\
& + \int \frac{d^3\underline{q}}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{e^{iy\cdot\check{k}}}{2k^+}\;\theta(k^+)\; 2\pi\delta(q^+ - k^+) \\
& \times \int d^2z_\perp\; e^{-iz_\perp(q_\perp - k_\perp)} \Bigg\{ \left(-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \right) \\
& \times \Bigg(-\frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big) \\
& \times \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big] - i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\; \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big]\Bigg) \\
& + \left(g^{\mu j} g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j} k^i \eta^\nu}{q^+} + \frac{\eta^\mu \eta^\nu k^i q^j}{q^+ q^+} \right) \\
& \times \left(\int dz^+\; \mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\; gT\cdot F_{ij}\; \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big) \right) \Bigg\}
\end{aligned}
$$

Total Gluon propagator at NEik order

Total gluon propagator upto NEik order travelling through the entire medium (dynamic gluon background field) for the case $x^+ > L^+/2$ and $y^+ < -L^+/2$ with $x^+ > y^+$ is:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y) = &\int \frac{d^3\underline{q}}{(2\pi)^3}\; e^{-ix\cdot\check{q}}\; \theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3}\; e^{iy\cdot\check{k}}\; \theta(k^+)\; \frac{1}{q^+ + k^+} \\
&\times \Big[- g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu \eta^\nu}{q^+ k^+}(\check{q}\cdot\check{k}) \Big] \int d^2 z_\perp e^{-i(q_\perp - k_\perp) z_\perp} \\
&\times \int dz^-\; e^{i(q^+ - k^+) z^-}\, \mathcal{U}_A\big(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-\big) \\
&+ \int \frac{d^3\underline{q}}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{e^{iy\cdot\check{k}}}{2k^+}\; \theta(k^+)\; 2\pi\delta(q^+ - k^+) \\
&\times \int d^2 z_\perp\; e^{-iz_\perp(q_\perp - k_\perp)} \Bigg\{ \Bigg(- g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu \eta^\nu}{q^+ k^+}(\check{q}\cdot\check{k}) \Bigg) \\
&\times \Bigg(- \frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big) \\
&\times \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big) \Big] - i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overleftarrow{D}_{z^j} \overrightarrow{D}_{z^j}\Big)\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big) \Big] \Bigg) \\
&+ \Bigg(g^{\mu j} g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j} k^i \eta^\nu}{q^+} + \frac{\eta^\mu \eta^\nu k^i q^j}{q^+ q^+} \Bigg) \\
&\times \Bigg(\int dz^+\, \mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\; gT\cdot F_{ij}\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big) \Bigg) \Bigg\}
\end{aligned}
$$

Application: Gluon scattering on the background field

The simplest process to compute using obtained gluon propagator is gluon-target scattering (as a building block for a forward pA collision).

- To compute it first we obtain s-matrix using LSZ-type reduction formula, given as:

$$S_{g_2 \leftarrow g_1} = \lim_{x^+ \to +\infty} (-1)(2p_2^+) \int d^2x \int dx^- \, e^{+ix\cdot\check{p}_2} \, \epsilon_\mu^{\lambda_2}(\underline{p_2})^*$$

$$\times \lim_{y^+ \to -\infty} (-1)(2p_1^+) \int d^2y \int dy^- \, e^{-iy\cdot\check{p}_1} \, \epsilon_\nu^{\lambda_1}(\underline{p_1}) \boxed{G_F^{\mu\nu}(x,y)_{a_2 \, a_1}}$$

Application: Gluon scattering on the background field

Squaring the amplitudes, we get partonic level gluon production cross-section at NEik accuracy for forward pA collision:

$$
\begin{aligned}
\frac{d\sigma^{gA\to g+X}}{dP.S.} = & \frac{1}{2(N_c^2-1)} \int d^2z'_\perp \int d^2z_\perp \; e^{-i(q_\perp - k_\perp)(z_\perp - z'_\perp)} \\
& \times \Bigg\{ \int d\Delta z^- \; e^{i(k^+-q^+)\Delta z^-} 2 \; \mathrm{Tr}\Bigg[\mathcal{U}_A^\dagger\Big(z'_\perp, z'^+, \frac{-\Delta z^-}{2} \Big) \, \mathcal{U}_A\Big(z_\perp, z^+, \frac{\Delta z^-}{2} \Big) \Bigg] \\
& + 2\pi\delta(k^+ - q^+) \mathrm{Tr}\Bigg[\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \, \mathcal{U}_A^\dagger(z'_\perp) \, \mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp \Big) \\
& \times \Big(-\frac{q^j+k^j}{2} (\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}) - i(\overleftarrow{D}_{z^j} \overrightarrow{D}_{z^j}) \Big) \, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp \Big) \\
& + \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz'^+ \, \mathcal{U}_A^\dagger\Big(z'^+, -\frac{L^+}{2}; z'_\perp \Big) \, \Big(-\frac{q^l+k^l}{2} (\overrightarrow{D}_{z'^l} - \overleftarrow{D}_{z'^l}) + i(\overleftarrow{D}_{z'^l} \overrightarrow{D}_{z'^l}) \Big) \\
& \times \mathcal{U}_A^\dagger\Big(\frac{L^+}{2}, z'^+; z'_\perp \Big) \mathcal{U}_A(z_\perp) \Bigg] \Bigg\}
\end{aligned}
$$

Including Quark background Field

In addition to previous computation,
we also need to take into account effect of quark background field.

Kovchegov et al. [arXiv:1511.06737], G. V. Chirilli [arXiv:1807.11435] have included this effect.

Quark Background Field: Basics

- Due to large boost of the target along x^-: its localized in longitudinal x^+ direction around small support.
- Components of quark background field in terms of bilinear currents scale as:

$$\bar{\Psi}(z)\gamma^-\Psi(z) = \mathcal{O}(\gamma_t) \text{ and } \bar{\Psi}(z)\gamma^+\Psi(z) = \mathcal{O}(1/\gamma_t)$$

- If we consider projections on quark background field then,

$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\psi(z) + \frac{\gamma^-\gamma^+}{2}\psi(z) = \Psi^-(z) + \Psi^+(z)$$

Guillaume's Talk

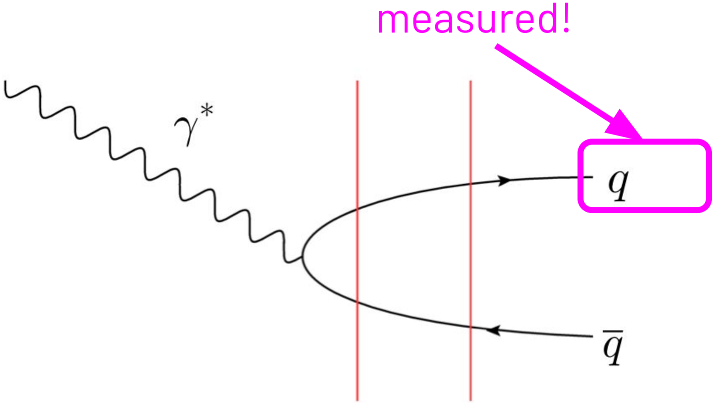
$$\boxed{\mathcal{O}(\sqrt{\gamma_t})} \qquad \boxed{\mathcal{O}(1/\sqrt{\gamma_t})}$$

- For NEik corrections, only - component considered and + component is neglected (contribute at NNEik only).

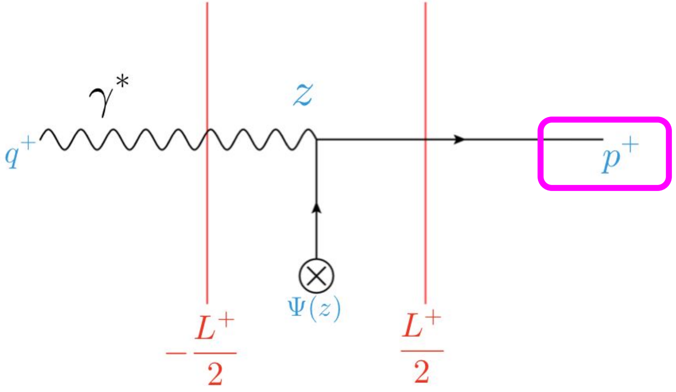
Semi Inclusive Deep Inelastic Scattering (SIDIS):

- In CGC, for this process: two kinds of contributions!
- Each of them are expected to be dominant in different kinematic regions.

1



2



- Contribution (1) is studied by Marquet, Xiao, Yuan [arXiv:0906.1454].
- In this talk contribution coming due to (2) is discussed.

SIDIS: S-matrix computation

- S-matrix is calculated at NEik order: only $\Psi^{-}(z)$ of component considered

$$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \, e^{i\check{p}\cdot x} \int d^4z \, \epsilon_\mu^\lambda(q) \, e^{-iq\cdot z} \, \overline{u}(p,h) \, \gamma^+ \, S_F(x,z)|_{Eik}^{IA}(-iee_f\gamma^\mu \boxed{)\Psi^-(z)}$$

- Two polarizations of photons are considered:
  - Longitudinal Polarization: no contribution at NEik order
  - Transverse Polarization: Contribution at NEik order

Finally, S-matrix for SIDIS process:

Similar calculations in case of q-g dijets are done in Altinoluk et al. (arXiv:2303.12691)

SIDIS: S-matrix computation

- S-matrix is calculated at NEik order: only $\Psi^{-}(z)$ of component considered
- Two polarizations of photons are considered.
- We get S-matrix for SIDIS process:

$$\boxed{\mathcal{O}(1/\gamma_t)}$$

$$S_{\gamma_T^* \to q} = 2\pi\delta(q^+ - p^+) \int dz^+ \int d^2 z_\perp \; e^{i(q_\perp - p_\perp)z_\perp} \; \overline{u}(p,h)$$

$$\times\, \epsilon_\lambda^j (iee_f) U_F(\infty, z^+, z_\perp) \; \underline{\left(\frac{\gamma^j \gamma^+ \gamma^-}{2}\right)\psi(z)}$$

$$\boxed{\mathcal{O}(\sqrt{\gamma_t})}$$

SIDIS: Cross-Section

Squaring amplitudes, we get cross-section for SIDIS process, in terms of Wilson lines:

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2p_\perp} = \frac{e^2e_f^2}{(2\pi)^2}\frac{1}{2}\frac{1}{2q^+}\int d^2z'_\perp \int d^2z_\perp\; e^{i(q_\perp-p_\perp)(z_\perp-z'_\perp)}\int dz'^+\int dz^+$$
$$\times\left\langle \overline{\psi}(z')\gamma^-\, \mathcal{U}_F^\dagger(\infty,z'^+,z'_\perp)\;\mathcal{U}_F(\infty,z^+,z_\perp)\;\psi(z)\right\rangle$$

Over all suppression of $\mathcal{O}(1/\gamma_t)$: NEik order

SIDIS: Relation at small-x between CGC and TMD calculations

- Any color operator $\mathcal{O}$, the CGC-like target average $\langle \mathcal{O} \rangle$ is proportional to the quantum expectation value in the momentum state of target.

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

- Using this relation, we can relate obtained cross-section with unpolarized transverse momentum dependent (TMD) quark distribution.

SIDIS: Relation at small-x between CGC and TMD calculations

Tolga's Talk

- unpolarized transverse momentum dependent (TMD) quark distribution:

$$f_1^q(x, k_\perp) = \frac{1}{(2\pi)^3} \int_{b_\perp} e^{ik_\perp b_\perp} \int_{z^+} e^{-iz^+ x P_{tar}^-} \left\langle P_{tar} \middle| \overline{\Psi}(z^+, b_\perp) \frac{\gamma^-}{2} \mathcal{U}_F^\dagger(\infty, z^+; b_\perp) \mathcal{U}_F(\infty, 0; 0) \Psi(0,0) \middle| P_{tar} \right\rangle$$

By comparing, we get cross-section in terms of TMD distribution:

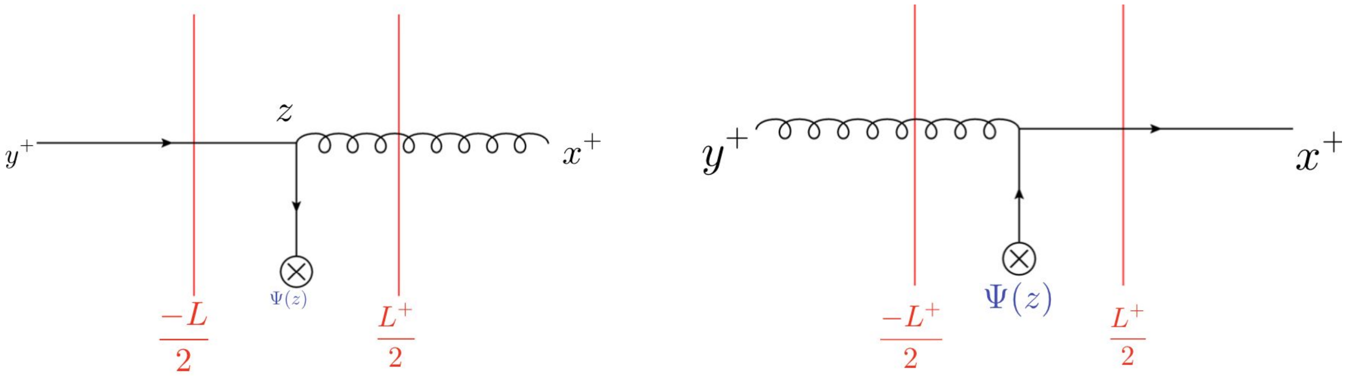
$$\frac{d^2\sigma^{\gamma_T^* \to q}}{d^2 p_\perp} = \frac{\pi e^2 e_f^2}{W^2} f_1^q(x = 0, p_\perp - q_\perp)$$

Suppression by centre of mass energy $1/W^2$ characterizes **NEik contribution** in terms of exchange t channel quark!

Single Inclusive Production with considering Quark background field

- Effect of quark background field is taken into account: To calculate single inclusive gluon production cross section at NEik order.
  - Four kinds of contributions.
  - Only $\Psi^-(z)$ component of the target is considered.

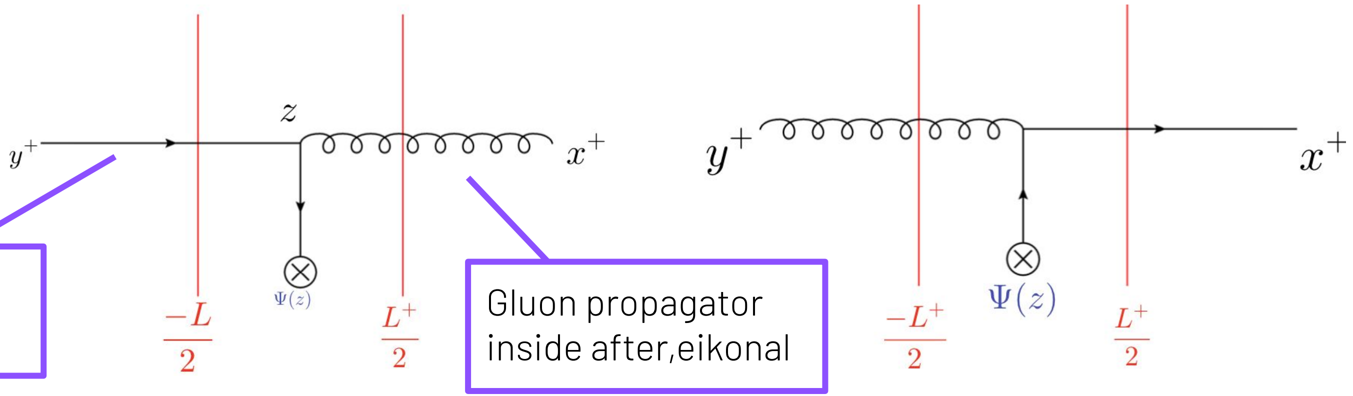
1



Single Inclusive Production with considering Quark background field

- Effect of quark background field is taken into account: To calculate single inclusive gluon production cross section at NEik order.
  - Four kinds of contributions
  - Only $\Psi^{-}(z)$ component of the target is considered.

1

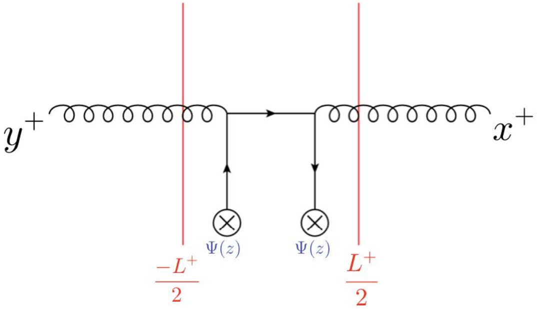


Quark propagator before inside, eikonal order

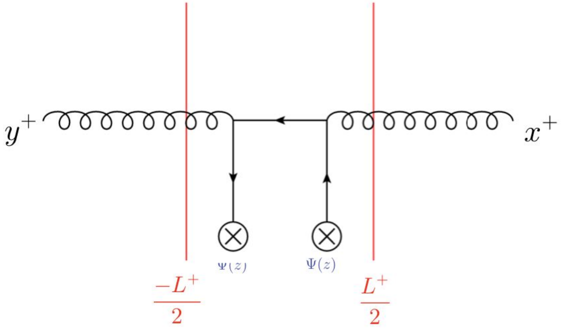
Gluon propagator inside after,eikonal

Single Inclusive Production with considering Quark background field

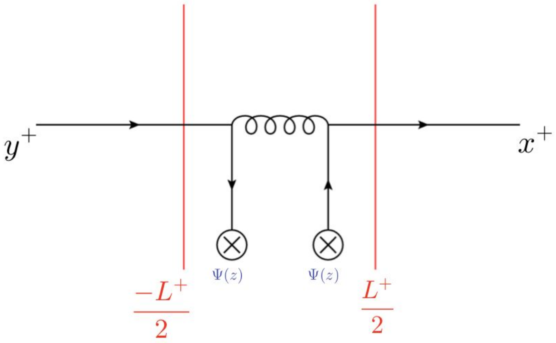
2



3



4



Summary and Conclusion

- Gluon propagator at Next-to-eikonal order in dynamic gluon background field is calculated.
  - Correction due to finite width, transverse component of target and x^- dependence of target field included.
- Obtained expression of gluon propagator is of general form therefore of general use: can be used for different scattering processes.
- SIDIS cross-section is calculated at NEik order by including quark background field.
  - Compared with TMD quark distribution function.
- Quark background field contributions at NEik order will be included to obtain full single inclusive particle production at NEik accuracy in pA.

Thank You!