Remarks on the Langevin formulation of JIMWLK equation, toward NLO.

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Overview

Introduction

- Increasing accuracy
- JIMWLK Equation
- Langevin formulation
- Remark on longitudinal dependence

Bulk of the presentation

- The NLO evolution kernel
- The LO double counting
- Renormalization
- Recap of the approach

Disclaimer: Equations in this presentation probably satisfy $1 = -1 = i = \pi \ldots$

Introduction

Introduction - Increasing the accuracy

We've seen a look of progress this week...

Higher order in perturbation theory

- NLO impact factors
- Running coupling kernel, NLO evolution kernel: either BFKL, BK, JIMWLK

Improving eikonal accuracy

- Propagators at sub^n-eikonal accuracy
- Evolution at sub-eikonal accuracy

My question here: *How does one get numbers?*

Status: (To my knowledge)

Forefront (unpolarized)

@eik	BK	JIMWLK
LO	✓	✓
r.c.	✓	✓
NLO	✓	✗
N^2LO	✗	✗

Personal bias: (polarized)

@sub-eik	Large-N_C	Large-N_c&N_f	All-N_c
LO	✓	✓ / ✗	✗
r.c.	✗	✗	✗

There is still a lot of fun to have!

Expectation of an operator made of Wilson Lines (WL) at a given rapidity

Rapidity Y — $\alpha \equiv$ the large component of the gauge field

$$\langle \hat{\mathcal{O}}_\alpha \rangle_Y = \frac{\int \mathcal{D}\alpha \ \hat{\mathcal{O}} \ \mathcal{W}_Y[\alpha]}{\int \mathcal{D}\alpha \ \mathcal{W}_Y[\alpha]} \qquad \int \mathcal{D}\alpha \ \mathcal{W}_Y[\alpha] = 1$$

target weight functional

target average

Evolution $Y \to Y + \delta Y$

$$\partial_Y W_Y[\alpha] = H_{JIMWLK} \cdot W_Y[\alpha] \tag{1}$$

Evolution Hamiltonian *I think this is the first time we see it written down this week?*

$$H_{JIMWLK} \equiv \frac{\alpha_s}{\pi^2} \int \mathrm{d}^2\underline{x}\, \mathrm{d}^2\underline{y}\, \mathrm{d}^2\underline{z} \ \frac{(\underline{x}-\underline{z})\cdot(\underline{y}-\underline{z})}{(\underline{x}-\underline{z})^2(\underline{y}-\underline{z})^2}$$
$$\times \left(U_{\underline{z}} - \tfrac{1}{2}U_{\underline{x}} - \tfrac{1}{2}U_{\underline{y}}\right)^{ba} (ig)^{-2} \frac{\delta}{\delta\alpha^a(x^- < 0, \underline{x})} \frac{\delta}{\delta\alpha^b(y^- > 0, \underline{y})} \tag{2}$$

Goal: Get a formulation which can be used in numerical evaluation (all N_c)

Notations

Functional derivative: ($\Leftrightarrow$ charge density ρ $\Leftrightarrow$ the Lie derivative $J_{L/R}$)

$$\frac{\delta}{\delta\alpha^a(x^- \lessgtr 0, \underline{x})} \equiv \delta^a_{\lessgtr,\underline{x}} \Leftrightarrow \rho(\underline{x}) \Leftrightarrow J_{R/L}(\underline{x}) \tag{3}$$

Separations in the transverse plane:

$$\underline{X}^i \equiv (\underline{x} - \underline{z})^i, \qquad \underline{Y}^i \equiv (\underline{y} - \underline{z})^i \tag{4}$$

Let's evolve an operator $\hat{\mathcal{O}}[\alpha]$ with $Y \to Y + \delta Y$

$$\hat{\mathcal{O}} \to \exp\left\{-\alpha_s \delta Y \int_{\underline{x}\underline{y}\underline{z}} \frac{\underline{X}\cdot\underline{Y}}{\underline{X}^2\underline{Y}^2}\left[\delta_{<\underline{y}}\delta_{<\underline{x}} + \delta_{>\underline{y}}\delta_{>\underline{x}} - 2U^{ba}_{\underline{z}}\delta^b_{>\underline{y}}\delta^a_{<\underline{x}}\right]\right\}\hat{\mathcal{O}} \tag{5}$$

$\rightarrow$ It can be factorized!

Introduce the auxiliary noise $\nu_i^a(\underline{z})$. It is Gaussian distributed and the expectation of two noises is

$$\left\langle \nu_{j>}^b(\underline{y})\ \nu_{i<}^a(\underline{x}) \right\rangle = (U_{\underline{x}}[>,<])^{ba}\delta_{ji}\delta^{(2)}\left(\underline{y}-\underline{x}\right) \tag{6}$$

Moving a noise across the s.w. is simply:

$$\nu_{j>}^b(\underline{x}) = (U_{\underline{x}}[>,<])^{ba}\nu_{j<}^a(\underline{x}) \tag{7}$$

The evolution kernel can be written ($\mathcal{G}$ is the Gaussian weight)

$$\int \mathcal{D}\nu\ \mathcal{G}[\nu]\left[\sqrt{\alpha_s\delta Y}\int_{\underline{y},\underline{z}'}\frac{\underline{Y}'^j}{\underline{Y}'^2}\left(\nu^b_{j,<,\underline{z}'}\delta^b_{<,\underline{y}}-\nu^b_{j,>,\underline{z}'}\delta^a_{>,\underline{y}}\right)\right] \times\left[\sqrt{\alpha_s\delta Y}\int_{\underline{x},\underline{z}}\frac{\underline{X}^i}{\underline{X}^2}\left(\nu^a_{i,<,\underline{z}}\delta^a_{<,\underline{x}}-\nu^a_{i,>,\underline{z}}\delta^a_{>,\underline{x}}\right)\right] \sim \langle (e\cdot\nu\delta)^2\rangle_\nu \tag{8}$$

where we identify:

$$e^i_{\underline{x},\underline{z}} \equiv \sqrt{\alpha_s\delta Y}\,\frac{\underline{X}^i}{\underline{X}^2} \tag{9}$$

→ Let's return to our operator

The evolution of the previous operator is given by

$$\hat{\mathcal{O}} \to \exp\left\{-\alpha_s \delta Y \int_{\underline{x},\underline{y},\underline{z}} \frac{\underline{X}\cdot\underline{Y}}{\underline{X}^2\underline{Y}^2}\left[\delta_{<\underline{y}}\delta_{<\underline{x}} + \delta_{>\underline{y}}\delta_{>\underline{x}} - 2U^{ba}_{\underline{z}}\delta^b_{>\underline{y}}\delta^a_{<\underline{x}}\right]\right\}\hat{\mathcal{O}}$$

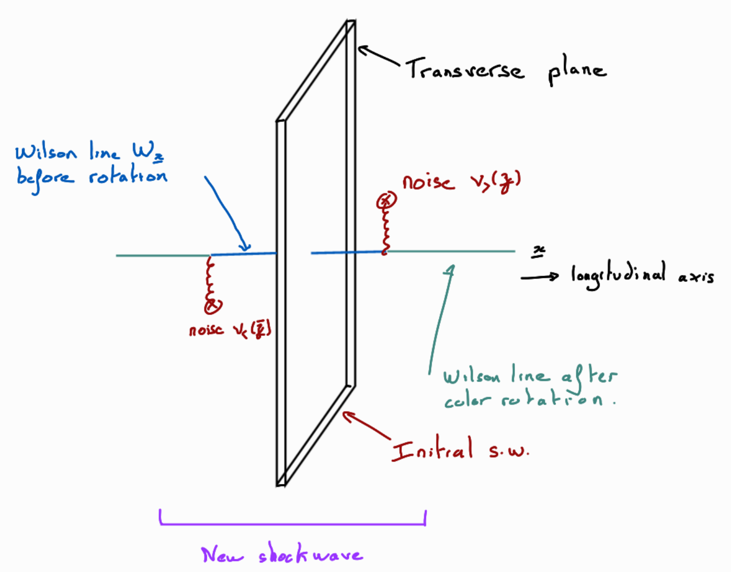
$$= \exp\left\{-\int \mathcal{D}\nu\ \mathcal{G}[\nu]\left[\int_{\underline{x},\underline{z}} e^i_{\underline{x},\underline{z}}\left(\nu^a_{i,<,\underline{z}}\delta^a_{<,\underline{x}} - \nu^a_{i,>,\underline{z}}\delta^a_{>,\underline{x}}\right)\right]^2\right\}\hat{\mathcal{O}} \quad (10)$$

$$= \int \mathcal{D}\nu\ \mathcal{G}[\nu]\ \exp\left\{(\pm i)\left[\int_{\underline{x},\underline{z}} e^i_{\underline{x},\underline{z}}\left(\nu^a_{i,<,\underline{z}}\delta^a_{<,\underline{x}} - \nu^a_{i,>,\underline{z}}\delta^a_{>,\underline{x}}\right)\right]\right\}\hat{\mathcal{O}} \quad (11)$$

Remarks

- Use the property of a Gaussian weight in the last line
- Can be discretized $\delta Y \to \sum_n \delta y$.
- Several iterations of H, use pairwise expectation value (Gaussian distribution)

After one infinitesimal step of evolution δy:



How does one obtain the JIMWLK evolution kernel? Let's focus on the *real* contribution only

- Take the wave function for an extra gluon in the Fock state

$$\left|\psi^{LO}_{g\,\rho}\right\rangle \propto ig \int_{\Lambda}^{\Lambda e^{\delta Y}} \frac{dk^+}{\sqrt{k^+}} \int \mathrm{d}^2\underline{x}\,\mathrm{d}^2\underline{z}\, \frac{\underline{X}^i}{\underline{X}^2}\rho^a(\underline{x}) \left|g^i_a(k^+,\underline{z})\right\rangle \tag{12}$$

Then the real contribution is

$$\left\langle \psi^{LO}_{g\,\rho}\right| \hat{S} \left|\psi^{LO}_{g\,\rho}\right\rangle \propto \alpha_s \int_{\Lambda}^{\Lambda e^{\delta Y}} \frac{dk^+}{k^+} \int \mathrm{d}^2\underline{x}\,\mathrm{d}^2\underline{y}\,\mathrm{d}^2\underline{z}\, \frac{\underline{X}\cdot\underline{Y}}{\underline{X}^2\underline{Y}^2} U^{ba}_{\underline{z}}\delta^b_{>,\underline{y}}\delta^a_{<,\underline{x}} \tag{13}$$

Proposal Just undo the integral in the kernel. Motivated by

(a) NLO observations, in favor of kinematic constrain / lifetime ordering /...
(b) DLA evolution (polarized operator). One needs to enforce specific orderings for the log.
(c) Anticipating: NLO consistency

Impact for the Langevin formulation: (at LO)

- $|g^a_i(k^+,\underline{z})\rangle \longrightarrow \nu^a_i(\underline{x}) \longrightarrow \nu^a_i(k^+,\underline{x})$
- Replace the simple exponential by a path order exponential in k^+:

$$e^{-\delta Y H_{JIMWLK}} \to P_{k^+}\exp\left\{-\int_{\Lambda}^{\Lambda e^{\delta Y}} \frac{k^+}{k^+} H_{JIMWLK}\right\}$$

Going to NLO

The NLO evolution kernel

Starting point: The NLO JIMWLK Hamiltonian defined by

$$
\begin{aligned}
H^{NLO}_{JIMWLK} = & \int_{\underline{x},\underline{y},\underline{z}} K_{JSJ}(\underline{x},\underline{y},\underline{z}) \left[J^a_L(\underline{x}) J^a_L(\underline{y}) + J^a_R(\underline{x}) J^a_R(\underline{y}) - 2 J^a_L(\underline{x}) S^{ab}_A(\underline{z}) J^a_R(\underline{y})\right] \\
& + \int_{\underline{x},\underline{y},\underline{z},\underline{z}'} K_{JSSJ}(\underline{x},\underline{y},\underline{z},\underline{z}') \left[f^{abc} f^{def} J^a_L(\underline{x}) S^{ab}_A(\underline{z}) S^{cf}_A(\underline{z}') J^d_R(\underline{y}) - N_c J^a_L(\underline{x}) S^{ab}_A(\underline{z}) J^b_R(\underline{y})\right] \\
& + \int_{\underline{x},\underline{y},\underline{z},\underline{z}'} K_{q\bar{q}}(\underline{x},\underline{y},\underline{z},\underline{z}') \left[2 J^a_L(\underline{x}) tr(S^\dagger(\underline{z}) t^a S(\underline{z}') t^b) J^b_R(\underline{y}) - J^a_L(\underline{x}) S^{ab}_A(\underline{z}) J^b_R(\underline{y})\right] \\
& + \int_{\underline{w},\underline{x},\underline{y},\underline{z},\underline{z}'} K_{JJSSJ}(\underline{w},\underline{x},\underline{y},\underline{z},\underline{z}') f^{abc} \Big[J^d_L(\underline{x}) J^e_L(\underline{y}) S^{dc}_A(\underline{z}) S^{eb}_A(\underline{z}') J^a_R(\underline{w}) \\
& - J^a_L(\underline{w}) S^{cd}(\underline{z}) S^{be}(\underline{z}') J^d_R(\underline{x}) J^e_R(\underline{y})) + \frac{1}{3}\Big(J^c_L(\underline{x}) J^b_L(\underline{y}) J^a_L(\underline{w}) - J^c_R(\underline{x}) J^b_R(\underline{y}) J^c_R(\underline{w})\Big)\Big] \\
& + \int_{\underline{w},\underline{x},\underline{y},\underline{z}} K_{JJSJ}(\underline{w},\underline{x},\underline{y},\underline{z}) f^{bde} \Big[J^d_L(\underline{x}) J^e_L(\underline{y}) S^{ba}_A(\underline{z}) J^a_R(\underline{w}) \\
& - J^a_L(\underline{w}) S^{ab}(\underline{z}) J^d_R(\underline{x}) J^e_R(\underline{y})) + \frac{1}{3}\Big(J^d_L(\underline{x}) J^e_L(\underline{y}) J^b_L(\underline{w}) - J^d_R(\underline{x}) J^e_R(\underline{y}) J^b_R(\underline{w})\Big)\Big]
\end{aligned}
\tag{14}
$$

For the full glory, see e.g. [JHEP 05 (2017) 097]

Aim: Formulation *à la Langevin* -equation for further numerical evaluations.

Today's discussion: Kernel in red and green: LO^2- subtraction, and UV-renormaliation

Removing the double counting with two iteration of LO kernel

Consider the kernel K_{JSSJ}. In complete analogy with the introduction of $e^i_{\underline{x},\underline{z}}$ at LO, introduce

$$T^{ij,abc}_{\underline{x};\underline{z},\underline{z}';k^+,\xi} = \frac{(ig)^3}{(2\pi)^2}\left[if^{abc}\right]\frac{1}{\sqrt{\xi(1-\xi)}}\left\{\frac{\xi(1-\xi)}{(1-\xi)\underline{X}'^2+\xi\underline{X}^2}\frac{\delta^{ij}}{2\underline{Z}^2}(\underline{X}^2-\underline{X}'^2)\right.$$
$$\left.+\frac{(1-\xi)\underline{X}'^2}{(1-\xi)\underline{X}'^2+\xi\underline{X}^2}\frac{\underline{X}'^j}{\underline{X}'^2}\left[\frac{\underline{Z}}{\underline{Z}^2}+\frac{1}{2}\frac{\underline{X}}{\underline{X}^2}\right]^i+\frac{\xi\underline{X}^2}{(1-\xi)\underline{X}'^2+\xi\underline{X}^2}\frac{\underline{X}^i}{\underline{X}^2}\left[\frac{\underline{Z}}{\underline{Z}^2}-\frac{1}{2}\frac{\underline{X}'}{\underline{X}'^2}\right]^j\right\} \tag{15}$$

where $\xi = k_1^+/(k_1^+ + k_2^+)$. This is simply:

$$|\psi_{gg\rho}\rangle = \int_{\underline{x},\underline{z},\underline{z}'}\int\frac{dk^+}{\sqrt{k^+}}d\xi\, T^{ij,abc}_{\underline{x};\underline{z},\underline{z}';k^+,\xi}\rho^a(\underline{x})\left|g^b_i(\xi,\underline{z})g^c_j((1-\xi)k^+,\underline{z}')\right\rangle \sim \int T|gg\rangle\rho \tag{16}$$

Which still contains emission from two iterations of LO. This is easily seen by taking the square

$$\int\frac{dk^+}{k^+}\int\frac{d\xi}{\xi(1-\xi)}\sim\mathcal{O}[(\delta Y)^2] \tag{17}$$

Remark

- The overlap still contain genuine NLO contributions which we want to leave untouched.
- Longitudinal dependence: $(k^+,\xi)\leftrightarrow(k_1^+,k_2^+)$
- Longitudinal correlations $\rightarrow$ thick slice of plus-momenta?

Removing the double counting with two iteration of LO kernel

Introduce the auxiliary noise ν_{gg} such that

$$|\psi_{gg\rho}\rangle \sim \int T|gg\rangle\rho \longrightarrow \int T\nu_{gg}\rho = \int_{\underline{x},\underline{z},\underline{z}'} \int dk^+ d\xi\; T^{ij,abc}_{\underline{x};\underline{z},\underline{z}';k^+,\xi}\rho^a(\underline{x})\,[\nu_{gg}]_{ij,bc}\,(\underline{z},\underline{z}',k^+,\xi) \quad (18)$$

Noise property: Consider the noise ν_{gg} to be distributed according to a Gaussian distribution whose moment is obtained by:

$$\langle\nu_{gg}(k^+,\xi)\nu_{gg}(\bar{k}^+,\bar{\xi})\rangle_\tau \propto \frac{1}{k^+}\delta(k^+-\bar{k}^+)\delta(\xi-\bar{\xi}) \times \text{" A bunch of WL and Dirac delta "} \quad (19)$$

After average over the noise ν_{gg} configurations, one recovers the non-subtracted result

$$\int \mathcal{D}\nu_{gg}\mathcal{G}_{gg}[\nu_{gg}]\left[\int T\nu_{gg}\rho\right]^2 = \langle\psi_{gg\rho}|\hat{S}|\psi_{gg\rho}\rangle = \Sigma_{JSSJ} \quad (20)$$

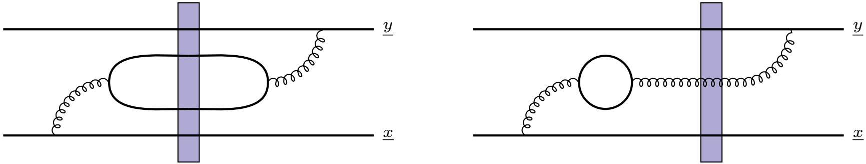
Subtraction. In order to remove the two iterations of LO evolution kernel, perform the following replacement

$$\delta(\xi-\bar{\xi}) \to \delta_\epsilon(\xi-\bar{\xi}) \equiv \delta(\xi-\bar{\xi})\left\{\theta(1-\epsilon>\xi>\epsilon) - \log\left(\frac{1-\epsilon}{\epsilon}\right)[\xi\delta(\xi)+(1-\xi)\delta(1-\xi)]\right\} \quad (21)$$

Works like a plus-distribution, but symmetric under $\xi \leftrightarrow 1-\xi$.

Renormalization - The N_f-part

Consider the kernel $K_{q\bar{q}}$ and the N_f-dependent part of the K_{JSJ} kernel.



Goal

- UV-renormaliation
- Scheme suitable to numerical evaluation *à la Langevin*
- Recover the Evolution kernel after average over the auxiliary noise

Notations:

$$\bar{\xi} \equiv 1-\xi, \qquad \underline{\chi} \equiv \frac{1}{\xi-\bar{\xi}}\left(\xi \underline{z}' - \bar{\xi}\underline{z}\right), \qquad \underline{Z} \equiv \underline{z} - \underline{z}', \qquad \underline{\Delta} \quad \equiv \underline{\Delta} \equiv (\underline{\chi} - \underline{x}) + \underline{Z}\frac{\xi^2+\bar{\xi}^2}{\xi-\bar{\xi}}$$

Wave function for the quark-antiquark

$$\begin{aligned} |\psi_{q\bar{q}\ \rho}\rangle = & -\sum_{\lambda_1\lambda_2 f}\int_{\Lambda}^{\Lambda e^{\delta Y}} dk^+ \int_0^1 d\xi \int \frac{\mathrm{d}^2\underline{k}}{(2\pi)^2}\frac{\underline{k}^i}{\underline{k}^2}\int \mathrm{d}^2\underline{x}\mathrm{d}^2\underline{\chi}\mathrm{d}^2\underline{Z}\ e^{i\,\underline{x}\cdot\underline{k}} e^{-i\,\underline{k}\cdot\underline{\chi}-i\,\underline{Z}\cdot\underline{k}\frac{\xi^2+\bar{\xi}^2}{\xi-\bar{\xi}}} \\ & \times \left|\bar{q}^{\beta f}_{\lambda_2}\left(\bar{\xi}k^+, \underline{\chi}+\frac{\bar{\xi}}{\xi-\bar{\xi}}\underline{Z}\right), q^{\alpha f}_{\lambda_1}\left(\xi k^+, \underline{\chi}+\frac{\xi}{\xi-\bar{\xi}}\underline{Z}\right)\right\rangle \left(2\pi g^2 t^a_{\alpha\beta}\rho^a(\underline{x})\right) \\ & \times \int \frac{\mathrm{d}^2\underline{p}}{(2\pi)^2}\ e^{-i\,\underline{p}\cdot\underline{Z}}\frac{1}{\underline{p}^2+\xi\bar{\xi}\underline{k}^2} \times \left\{\underline{p}^j\left[(\bar{\xi}-\xi)\lambda_1\lambda_2\delta^{ij} - i\epsilon^{ij}\lambda_1\delta_{\lambda_1\lambda_2}\right] + 2\xi\bar{\xi}\delta^{ij}\underline{k}^j\lambda_1\lambda_2\right\} \end{aligned} \tag{22}$$

$\Rightarrow$ Go into mixed space $(k^+, \underline{z})$

$$\begin{aligned} |\psi_{q\bar{q}\ \rho}\rangle = & -\sum_{\lambda_1\lambda_2 f}\int_{\Lambda}^{\Lambda e^{\delta Y}} dk^+ \int_0^1 d\xi \int \mathrm{d}^2\underline{x}\mathrm{d}^2\underline{\chi}\mathrm{d}^2\underline{Z}\ e^{i\,\underline{x}\cdot\underline{k}} e^{-i\,\underline{k}\cdot\underline{\chi}-i\,\underline{Z}\cdot\underline{k}\frac{\xi^2+\bar{\xi}^2}{\xi-\bar{\xi}}} \\ & \times \left|\bar{q}^{\beta f}_{\lambda_2}\left(\bar{\xi}k^+, \underline{\chi}+\frac{\bar{\xi}}{\xi-\bar{\xi}}\underline{Z}\right), q^{\alpha f}_{\lambda_1}\left(\xi k^+, \underline{\chi}+\frac{\xi}{\xi-\bar{\xi}}\underline{Z}\right)\right\rangle \left(g^2 t^a_{\alpha\beta}\rho^a(\underline{x})\right) \\ & \times \left\{\left[(\bar{\xi}-\xi)\lambda_1\lambda_2\delta^{ij} - i\epsilon^{ij}\lambda_1\delta_{\lambda_1\lambda_2}\right]\frac{-\underline{Z}^j\underline{\Delta}^i}{\underline{Z}^2} + 2\xi\bar{\xi}\delta^{ij}\lambda_1\lambda_2\right\}\frac{1}{(2\pi)^3}\frac{1}{\xi\bar{\xi}\underline{Z}^2+\underline{\Delta}^2} \end{aligned} \tag{23}$$

Take the square $\Longrightarrow$ UV singularity $|\underline{p}| \to \infty$ or $|\underline{Z}| \to 0$

Wave function for the one gluon N_f-part @NLO

$$|\psi_g^1\rangle = -\sum_{f,\lambda_1,\lambda_2}\int_{\Lambda}^{\Lambda e^{\delta Y}} dk^+ \int_0^{k^+} dp^+ \int \frac{\mathrm{d}^2\underline{k}\mathrm{d}^2\underline{p}}{(2\pi)^4}\, \frac{g^3\rho^a(-\underline{k})}{32\pi^{3/2}\sqrt{k^+}} \times \frac{\underline{k}^i}{\underline{k}^4}\, \frac{(1-4\xi\bar{\xi})\underline{p}^i\underline{p}^j + \epsilon^{i\ell}\epsilon^{jk}\underline{p}^\ell\underline{p}^k}{\xi\bar{\xi}\ \left[\xi\bar{\xi}\underline{k}^2+\underline{p}^2\right]}\,|g_j^a(k)\rangle \tag{24}$$

Go into mixed space $(k^+, \underline{z})$ - Stop before using dim-reg for $\mathrm{d}^2\underline{p}$ integral!

$$|\psi_g^1\rangle = -\frac{g^3 N_f}{2\sqrt{\pi}}\frac{dk^+}{\sqrt{k^+}}\int\frac{d\xi}{\xi\bar{\xi}}\int\frac{\mathrm{d}^2\underline{k}}{(2\pi)^2}\frac{\underline{k}^i}{\underline{k}^2}\left[(1-4\xi\bar{\xi})\delta^{ik}\delta^{jl}+\epsilon_\perp^{ik}\epsilon_\perp^{j\ell}\right]\int e^{i\underline{k}\cdot\underline{x}}\rho^a(\underline{x})$$
$$\times\int \mathrm{d}^2\underline{\chi}\, e^{-i\underline{\chi}\cdot\underline{k}}|g_j^a(k^+,\underline{\chi})\rangle\frac{1}{\underline{k}^2}\int \mathrm{d}^2\underline{Z}\,\frac{1}{2\pi}K_0\left(\sqrt{\underline{k}^2\xi\bar{\xi}\underline{Z}^2}\right)\left[i\nabla_{\underline{Z}}^k i\nabla_{\underline{Z}}^\ell\delta^2(\underline{Z})\right] \tag{25}$$

Focus on the Green part.

Isolate the singular behavior:

$$K_0(m\,Z) \to \underbrace{\left[K_0(m\,Z)+\log(\Lambda\,Z)\right]}_{UV-finite} + \underbrace{\left[-\log(\Lambda\,Z)\right]}_{UV-singular} \tag{26}$$

Remarks

- Need to cancel the UV-singular piece against the $\langle\psi_{gg\,\rho}|\psi_{gg\,\rho}\rangle$ UV-singular piece.
- Recall: looking for $\sqrt{H^{NLO}}\cdot\nu$.

Renormalization - The N_f-part - Proposed solution

Introduce The auxiliary $\nu_{q\bar{q}}$ such that

$$|\psi_{q\bar{q}}\rangle^{(N_f)} \sim \int Q\ |q\bar{q}\rangle\rho \longrightarrow \int Q\ \nu_{q\bar{q}}\rho \tag{27}$$

Focusing on the $\underline{Z}$ -dependence of the Gaussian distribution for $\nu_{q\bar{q}}$, write

$$\int \mathcal{D}\nu_{q\bar{q}}\mathcal{G}_{q\bar{q}}[\nu_{q\bar{q}}]\ \nu_{q\bar{q}}(\underline{z},\underline{z}')\nu_{q\bar{q}}(\bar{\underline{z}},\bar{\underline{z}}') \propto \delta^2(\underline{z}-\bar{\underline{z}})\delta^2(\underline{z}'-\bar{\underline{z}}')\left[1-\#\log(\Lambda Z)\ \underline{Z}^2\delta^2(\underline{Z})\right] \times \text{" A bunch of WL and delta "} \tag{28}$$

The integrals of interest are

$$\int d^2\underline{Z}\ \left(\frac{1}{\xi\bar{\xi}\underline{Z}^2+\underline{\Delta}^2}\right)^2 \#\log(\Lambda Z)\underline{Z}^2\delta^2(\underline{Z}) \times 1 \to 0 \tag{29a}$$

$$\int d^2\underline{Z}\ \left(\frac{1}{\xi\bar{\xi}\underline{Z}^2+\underline{\Delta}^2}\right)^2 \#\log(\Lambda Z)\underline{Z}^2\delta^2(\underline{Z}) \times \frac{\underline{Z}\cdot\underline{\Delta}}{\underline{Z}^2} \to 0 \tag{29b}$$

$$\int d^2\underline{Z}\ \left(\frac{1}{\xi\bar{\xi}\underline{Z}^2+\underline{\Delta}^2}\right)^2 \#\log(\Lambda Z)\underline{Z}^2\delta^2(\underline{Z}) \times \frac{\underline{Z}\cdot\underline{\Delta}}{\underline{Z}^2}\frac{\underline{Z}\cdot\underline{\Delta}}{\underline{Z}^2} \to UV-singular \tag{29c}$$

Remark

- Fix # to cancel the UV-singular part in $\langle\psi_g^{(LO)}|\psi_g^{N_f}\rangle + \langle\psi_g^{N_f}|\psi_g^{LO}\rangle$
- Property of $\nu_{q\bar{q}}$ dependent on the scheme for the N_f-part in the NLO gluon w.f.

Recap of the approach

Transparent and systematic approach:

Higher order in α_s: New Fock states available when opening phase space. Taken into account by the introduction of corresponding new auxiliary noises.

Use Gaussian distributions in order to reproduce the properties of $\hat{S}$ in the eikonal limit.
(Should also work when expanding around the eikonal approximation, with some caveat)

Subtraction of LO. The scheme is applied by choosing the property of the two point function of the auxiliary noises.

Renormalization shares similar concerns as the LO subtraction. Renormalization schemes are applied at the level of the Hamiltonian not the wave function. Therefore, the renormalization scheme is encoded in the properties of the noises.

Use distribution properties to write the equation in a *linear* form wrt noises.

A very last slide

Conclusion

- Introduction of several auxiliary noises (high dimensional)
- Double counting, and renormalization both taken into account by the properties of those noises.

Prospect

- Checks, checks, ..., then write!
- Simplification of the formulation for practical implementation.
  (Reducing the vector space? Can we really not use Dim-reg? ...)
- Application of those tools to subeikonal accuracy is twofold.
  (a) Evolution Hamiltonian at subeikonal accuracy, (b) Reformulation *à la Langevin*.