Imaging the odderon from exclusive η_c production at the EIC

Sanjin Benić (University of Zagreb)

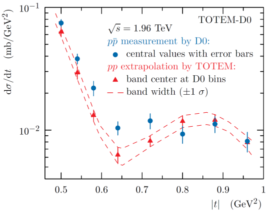
SB, Horvatić, Kaushik, Vivoda, in preparation

Color Glass Condensate at the Electron Ion Collider, ECT*
Trento, May 18, 2023

Odderon

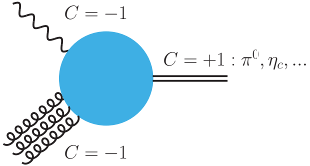
- 50 years ago Lukaszuk, Nicolescu: odderon is a $C = -1$ exchange
  Lukaszuk, Nicolescu, LNC 8 (1973) 405
  Joynson, Leader, Nicolescu, Lopez, NCA 30 (1975) 345
  Ewerz, 0306137

- QCD: three gluons $\mathrm{tr}\left[\{A^\mu, A^\nu\}A^\rho\right] \sim d_{abc}$



- elastic pp vs $p\bar{p}$ cross section

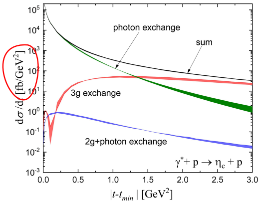
TOTEM, D0, PRL 127 (2021) 6, 062003



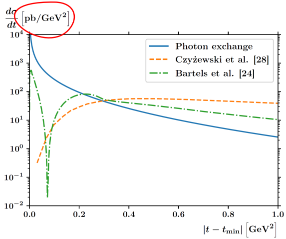
- exclusive productions to fix the C parity of the final state
  Czyzewski, Kwiecinski, Motyka, Sadzikowski, PLB 398 400 (1997)
  Bartels, Braun, Colferai, Vacca, EPJC 20 323 (2001)

Odderon in $ep \to ep\eta_c$

- no experimental measurement so far
  $\to$ EIC, LHC UPCs?
  (null result from H1 at HERA for exclusive π^0)
  H1, PLB 544 (2002) 35-43
- more recent computations at moderate $x \sim 0.1$ lead to somewhat lower cross sections than the original estimates



Dumitru, Stebel, PRD 99 (2019) 9, 094038



Jia, Mo, Pan, Zhang, 2207.14171

- this work: take into account evolution effects and consider nuclear targets

Odderon in the CGC

- high energy collisions → Wilson lines

$$V(\boldsymbol{z}_\perp) = \mathcal{P}\exp\left[ig\int_{-\infty}^{\infty} dy^- A^+(y^-, \boldsymbol{z}_\perp)\right]$$

- Color Glass Condensate (CGC): emergence of a saturation scale $Q_S^2 \sim A^{1/3}$

  → better theoretical control for a large nuclei $Q_S^2 \gg \Lambda_{\rm QCD}^2$

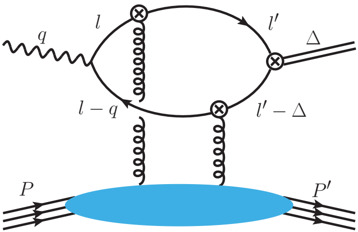
- odderon as imaginary part of the dipole

$$\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \equiv -\frac{1}{2iN_c}\mathrm{tr}\left\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - V(\boldsymbol{y}_\perp)V^\dagger(\boldsymbol{x}_\perp)\right\rangle$$

- C-odd: $\boldsymbol{x}_\perp \leftrightarrow \boldsymbol{y}_\perp$ $\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \to -\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$

  Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
  Hatta, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

Amplitude



$$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$$

$$\boldsymbol{b}_\perp = \frac{\boldsymbol{x}_\perp + \boldsymbol{y}_\perp}{2}$$

- CGC vertex

$$\tau(p,p') = (2\pi)\delta(p^- - p'^-)\gamma^-\mathrm{sgn}(p^-)\int_{\boldsymbol{z}_\perp} e^{-i(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp)\cdot\boldsymbol{z}_\perp} V^{\mathrm{sgn}(p^-)}(\boldsymbol{z}_\perp)$$

$$\langle\mathcal{M}_\lambda\rangle = eq_c\int_{\boldsymbol{r}_\perp}\int_{ll'}(2\pi)\delta(l^- - l'^-)\theta(l^-)\theta(q^- - l^-)e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp - \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot\boldsymbol{r}_\perp}$$
$$\times(-iN_c)\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{\Delta}_\perp)\mathrm{tr}\left[S(l)\not{\epsilon}(\lambda,q)S(l-q)\gamma^- S(l'-\Delta)(i\gamma_5)S(l')\gamma^-\right]$$

Amplitude

- only transverse photon polarizations $\lambda = \pm 1$ survive in the eikonal approximation

$$\langle \mathcal{M}_\lambda \rangle = q^- \lambda e^{i\lambda\phi_\Delta} \langle \mathcal{M} \rangle \qquad \frac{d\sigma}{d|t|} = \frac{1}{16\pi} |\langle \mathcal{M} \rangle|^2$$

- polarization independent part of the amplitude

$$\langle \mathcal{M} \rangle = 8\pi i e q_c N_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp dr_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \times \mathcal{A}(\boldsymbol{r}_\perp) \left[J_{2k}(r_\perp \delta_\perp) - \frac{2k+1}{r_\perp \delta_\perp} J_{2k+1}(r_\perp \delta_\perp) \right] .$$

- result proportional to m_c

$$\mathcal{A}(\boldsymbol{r}_\perp) = (-1) \frac{\sqrt{2} m_c}{2\pi} \frac{1}{z\bar{z}} \left[K_0(\varepsilon r_\perp) \partial_{r_\perp} \phi_{\mathcal{P}}(z, r_\perp) - \varepsilon K_1(\varepsilon r_\perp) \phi_{\mathcal{P}}(z, r_\perp) \right]$$

Dumitru, Stebel, PRD 99 (2019) 9, 094038
SB, Horvatić, Kaushik, Vivoda, in preparation

BK equation

- odderon $\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$ is explicitly $b_\perp$-dependent

  $\rightarrow$ in principle need to solve the fully impact parameter dependent Balitsky-Kovchegov (BK) equation

$$\frac{\partial \mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{D}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}) \mathcal{D}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}) - \mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\right]$$

$$\boldsymbol{r}_{2\perp} = \boldsymbol{r}_\perp - \boldsymbol{r}_{1\perp} \quad \boldsymbol{b}_{1\perp} = \boldsymbol{b}_\perp + \frac{1}{2}(\boldsymbol{r}_\perp - \boldsymbol{r}_{1\perp}) \quad \boldsymbol{b}_{2\perp} = \boldsymbol{b}_\perp - \frac{1}{2}\boldsymbol{r}_{1\perp}$$

$$\mathcal{D}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \frac{1}{N_c} \mathrm{tr} \left\langle V\left(\boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2}\right) V^\dagger \left(\boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}\right)\right\rangle$$
$$= 1 - \mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) + i\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$$

- in general non-local in $\boldsymbol{b}_\perp$

- this work: local approximation $\rightarrow$ $b_\perp$ becomes an external parameter

  Kowalski, Lappi, Marquet, Venugopalan, PRC 78 (2008) 045201
  Lappi, Mäntysaari, PRD 88 (2013) 114020
  Le, Tuesday, May 16, 11:20

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  Lappi, Mäntysaari, PRD 88 (2013) 114020
  Le, Tuesday, May 16, 11:20

BK equation

- non-linear terms couple the Pomeron-Odderon system

$$\frac{\partial\mathcal{N}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2}\int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2\boldsymbol{r}_{2\perp}^2}\Big[\mathcal{N}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)+\mathcal{N}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp)-\mathcal{N}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp)\Big]$$

$$\frac{\partial\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2}\int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2\boldsymbol{r}_{2\perp}^2}\Big[\mathcal{O}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)+\mathcal{O}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp)-\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp) - \mathcal{N}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp},\boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp},\boldsymbol{b}_\perp)\Big]$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
Hatta, Itakura, McLerran, NPA 760 (2005) 172-207
Motyka, PLB 637, 185 (2006)
Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)
Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

- small $r_\perp$ limit: $\mathcal{N} \to 0$ (linear) $\to \mathcal{O} \sim e^{-\# Y}$
- large $r_\perp$ limit: $\mathcal{N} \to 1$ (saturation) $\to \mathcal{O} \sim e^{-\# Y}$
- in numerical computations we are replacing $\frac{\alpha_S N_c}{2\pi^2}\frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2\boldsymbol{r}_{2\perp}^2}$ by a running coupling kernel with Balitsky's prescription

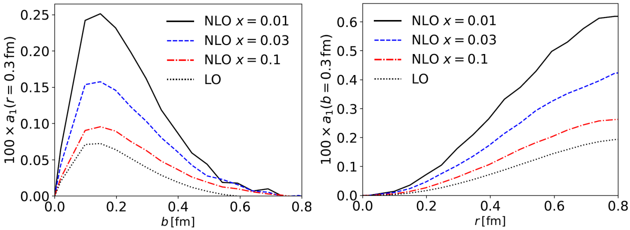
Initial condition for proton

- $\mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$: HERA fit

$$\mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = 1 - \exp\left[-\frac{1}{4}\boldsymbol{r}_\perp^2\, T_p(\boldsymbol{b}_\perp)\frac{\sigma_0}{2}Q_{S,0}^2 \log\left(\frac{1}{r_\perp \Lambda_{\mathrm{QCD}}} + e_c e\right)\right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

- $\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$: recent computation starting from quark light-cone wavefunctions at NLO by Dumitru, Mäntysaari, Paatelainen (DMP)



Dumitru, Mäntysaari, Paatelainen, PRD 105 (2022) 3, 036007
Dumitru, Mäntysaari, Paatelainen, PRD 107 (2023) 1, L011501

Initial conditions for nuclei

- $\mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$ same HERA fit + optical Glauber

$$\mathcal{N}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = 1 - \exp\left[-\frac{1}{4}\boldsymbol{r}_\perp^2 A T_A(\boldsymbol{b}_\perp)\frac{\sigma_0}{2} Q_{S,0}^2 \log\left(\frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e\right)\right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

- for $\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$ we use the Jeon-Venugopalan (JV) model

$$W[\rho] = \exp\left[-\int_{\boldsymbol{x}_\perp}\left(\frac{\delta^{ab}\rho^a\rho^b}{2\mu^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa}\right)\right]$$

Jeon, Venugopalan, PRD 71 (2005) 125003

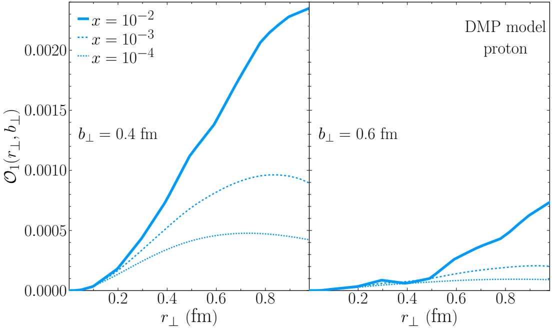
$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \frac{\lambda}{8}\left[A^{2/3}\frac{dT_A(\boldsymbol{b}_\perp)}{db_\perp} R_A \frac{\sigma_0}{2}\right] A^{1/2}(Q_{S,0} r_\perp)^3 (\hat{\boldsymbol{r}}_\perp \cdot \hat{\boldsymbol{b}}_\perp)$$
$$\times \log\left(\frac{1}{r_\perp \Lambda_{QCD}} + e_c e\right) \exp\left[-\frac{1}{4}\boldsymbol{r}_\perp^2 A T_A(\boldsymbol{b}_\perp)\frac{\sigma_0}{2} Q_{S,0}^2 \log\left(\frac{1}{r_\perp \Lambda_{\text{QCD}}} + e_c e\right)\right]$$

- in the JV model $\lambda_{\text{JV}} = -\frac{3}{16}\frac{N_c^2-4}{(N_c^2-1)^2}\frac{(Q_{S,0}R_A)^3}{\alpha_S^3 A^{3/2}}$

Kovchegov, Sievert, PRD 86 (2012) 034028
Boer, Van Daal, Mulders, Petreska, JHEP 07, 140 (2018)
SB, Horvatić, Kaushik, Vivoda, in preparation

Numerical solutions: proton

- rapid drop of the odderon with evolution
- does not obey geometric scaling



Motyka, PLB 637, 185 (2006)
Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)
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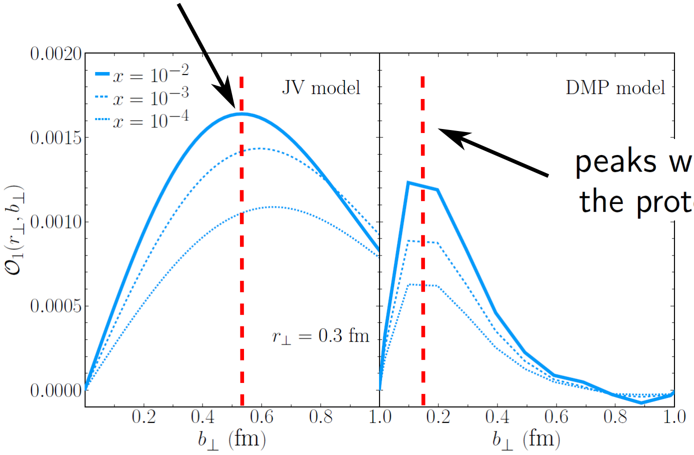
SB, Horvatić, Kaushik, Vivoda, in preparation

DMP vs JV for proton

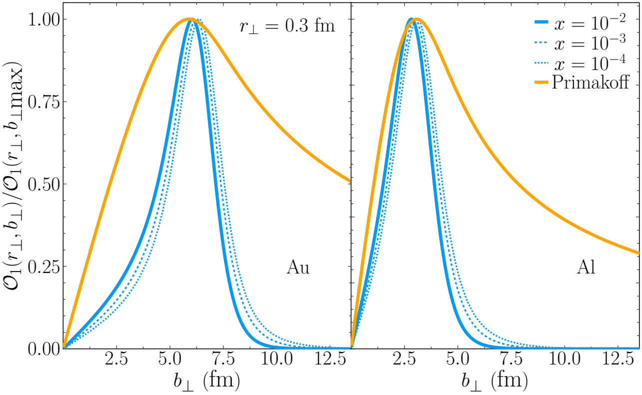
peaks at the edge of the proton $\sim \dfrac{dT_p}{db_\perp}$



peaks within the proton

SB, Horvatić, Kaushik, Vivoda, in preparation

Nuclear odderon vs Primakoff photon



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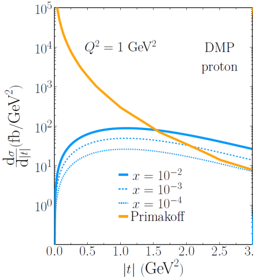
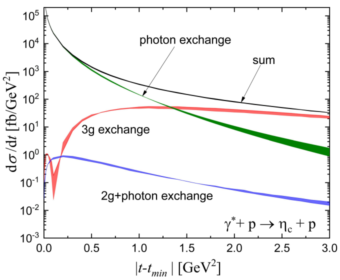
Results: proton target





$x \sim 10^{-2} - 10^{-4}$ $x \sim 0.1$

- weak $|t|$-dependence: coupling of three gluons to three different quarks

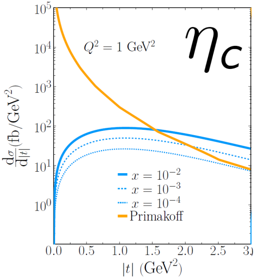
  $\rightarrow$ not changed by evolution to small-x

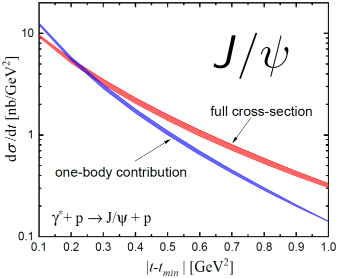
  $\rightarrow$ probes the odderon in the high $|t| > 1.5$ GeV^2 region
- confirming the general conclusions by Dumitru and Stebel

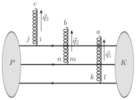
  Dumitru, Stebel, PRD 99 (2019) 9, 094038

  SB, Horvatić, Kaushik, Vivoda, in preparation

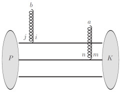
Contrast with J/Ψ







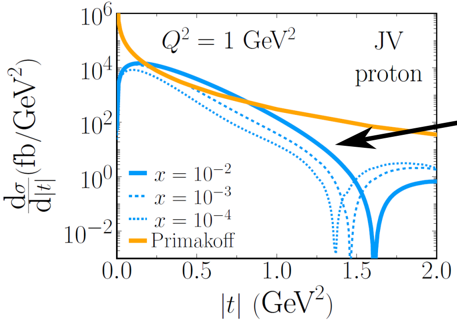
weak $|t|$ dependence



strong $|t|$ dependence

Czyzewski, Kwiecinski, Motyka, Sadzikowski, PLB 398 400 (1997)
Dumitru, Stebel, PRD 99 (2019) 9, 094038
SB, Horvatić, Kaushik, Vivoda, in preparation

Contrast with JV model for proton



strong $|t|$ dependence

- t-slope roughly controlled by the peak of the odderon
- DMP odderon about two orders of magnitude smaller than JV at small t

$$r = \lim_{\Delta_\perp \to 0} \frac{\langle \mathcal{M} \rangle_{\text{DMP}}}{\langle \mathcal{M} \rangle_{\text{JV}}} \simeq 0.022$$

SB, Horvatić, Kaushik, Vivoda, in preparation

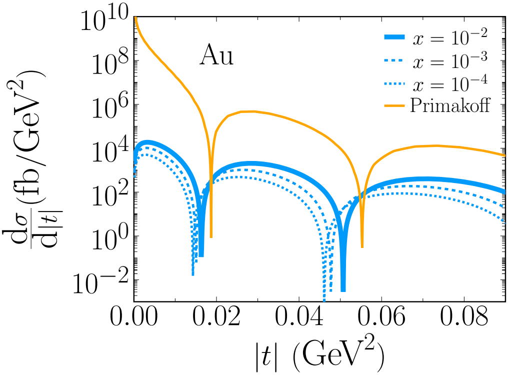
Nuclear targets: preliminary comments

- assumption: JV model captures the A-dependence, but odderon coupling has some uncertainty

  $\rightarrow$ comparison to the DMP computation suggests it is overestimated by about $r \simeq 0.022$

- ultimately to be determined experimentally
- lets consider rescaling of the odderon by r but also as a free parameter

Results: Au target



- odderon contribution has a shifted diffractive pattern with respect to the Primakoff cross section
- becomes more pronounced at small-x

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Origin of the shift?

- the odderon cross section at leading twist

$$\frac{d\sigma}{d|t|} \simeq \frac{9\pi q_c^2 \alpha \alpha_S^6 A^2 C_{3F}^2 \mathcal{R}_{\mathcal{P}}^2(0)}{N_c m_c^5} \frac{|t|\, T_A^{\text{strong}}(\sqrt{|t|})^2}{m_c^4}$$

SB, Horvatić, Kaushik, Vivoda, in preparation

- contrast with the Primakoff cross section

$$\frac{d\sigma}{d|t|} \simeq \frac{\pi q_c^4 \alpha^3 Z^2 N_c \mathcal{R}_{\mathcal{P}}^2(0)}{m_c^5} \frac{T_A^{\text{charge}}(\sqrt{|t|})^2}{|t|}$$

Jia, Mo, Pan, Zhang, 2207.14171

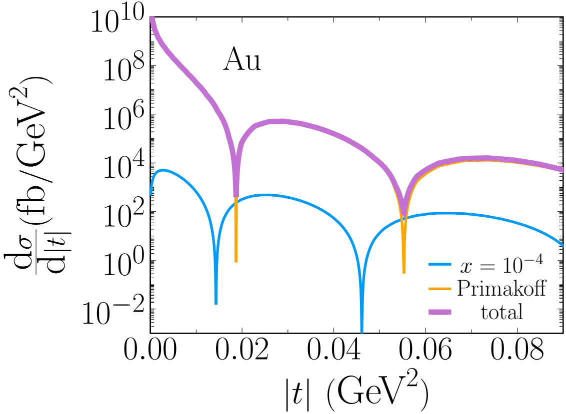
- in our Woods-Saxon parametrizations

$$R_A^{\text{strong}} \simeq R_A^{\text{charge}} \qquad d_A^{\text{strong}} \simeq d_A^{\text{charge}}$$

$\rightarrow$ shift is not seen at leading twist

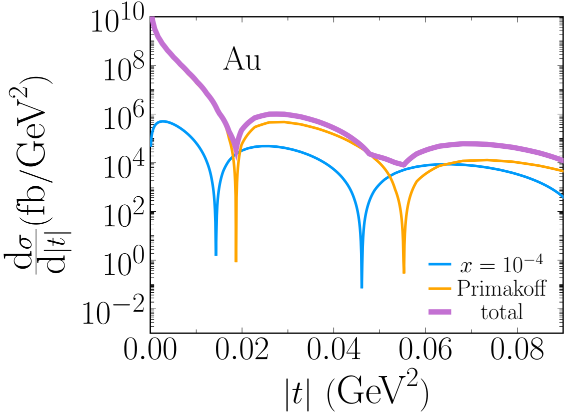
$\rightarrow$ origin is the multiple scatterings in the odderon distribution

Consider the odderon coupling as a free parameter



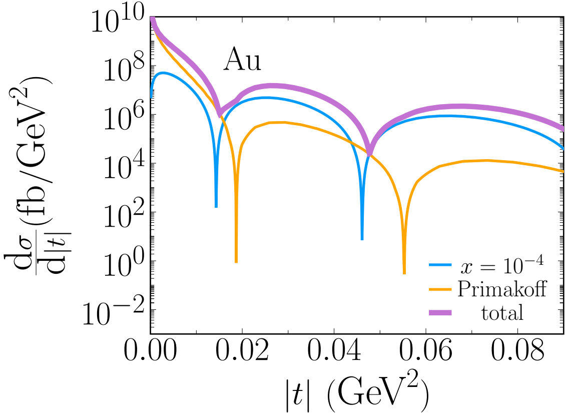
- original case

Consider the odderon coupling as a free parameter



- $10 \times \langle \mathcal{M}_{\text{odderon}} \rangle$

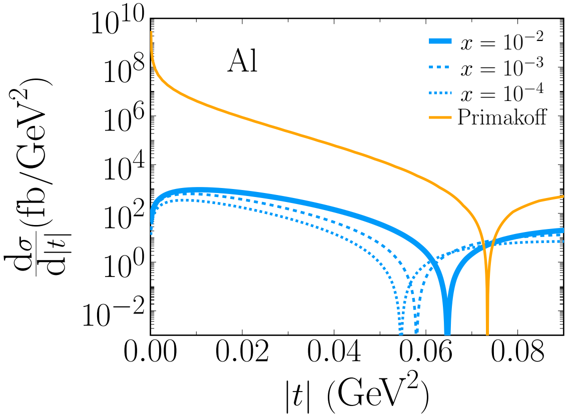
Consider the odderon coupling as a free parameter



- $100 \times \langle \mathcal{M}_{\text{odderon}} \rangle$

SB, Horvatić, Kaushik, Vivoda, in preparation

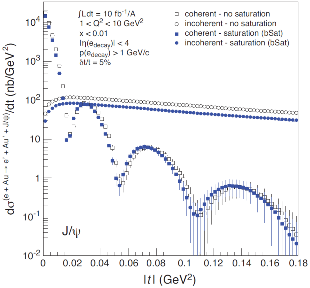
Consider different nuclei species

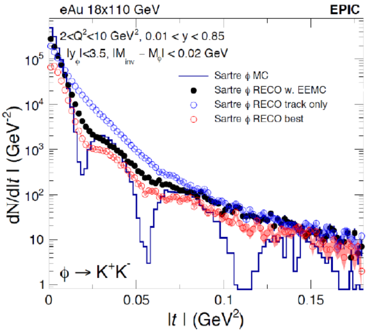


- can this be measured?
- need fine t-binning + large number of events

SB, Horvatić, Kaushik, Vivoda, in preparation

A parallel with J/ψ





- an initiative to measure the diffractive pattern with J/ψ at the EIC

Toll, Ulrich, PRC 87 (13) 0249
Accardi et al. (EIC White paper) EPJA 52 (2016) 9, 268
Van Hulse, Monday, May 15, 09:40

Comments

- can η_c be measured at the EIC (or LHC)?
  $\rightarrow$ results generic, should work for other C-even quarkonia states/light mesons

- forward limit $|t| \rightarrow 0$ cross section vanishes
  $\rightarrow$ consider the spin-flip contribution from the gluon Sivers function
  Boussarie, Hatta, Szymanowski, Wallon, PRL 124 (2020) 17, 172501

- can we "remove" the Primakoff background?
  $\rightarrow$ exclusive production off a neutron?
  (Primakoff vanishes at $t = 0$)

The Primakoff photon

- photon has $C = -1 \to$ important background for extracting the odderon
  - replace QCD with QED Wilson line

$$U(\boldsymbol{x}_\perp) = e^{ieq_c\Lambda(\boldsymbol{x}_\perp)} \qquad e\Lambda(\boldsymbol{x}_\perp) = 4\pi Z\alpha \int_{\boldsymbol{k}_\perp} \frac{\mathcal{T}_A(\boldsymbol{k}_\perp)}{\boldsymbol{k}_\perp^2} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

  - get the photon distribution in $(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$ space

$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) \to -\frac{1}{2i}\left[U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{y}_\perp) - U(\boldsymbol{y}_\perp)U^\dagger(\boldsymbol{x}_\perp)\right] \equiv \Omega(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)$$

- at all orders

$$\Omega(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \sin(\boldsymbol{r}_\perp\cdot\boldsymbol{b}_\perp\mathcal{F}(b_\perp)) \quad \mathcal{F}(b_\perp) \equiv \frac{4\pi Zq_c\alpha}{b_\perp^2}\int_0^{b_\perp} b'_\perp db'_\perp \mathcal{T}_A(b'_\perp)$$

  - Fourier modes: $\Omega_{2k+1}(r_\perp, b_\perp) = (-1)^k J_{2k+1}(r_\perp b_\perp \mathcal{F}(b_\perp))$

SB, Horvatić, Kaushik, Vivoda, in preparation