Small-x TMD factorization @ NLO

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UCLA/LBL $\longrightarrow$ INT UW

CGC for EIC

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Small-x TMD factorization @ NLO

based on

JHEP 11 (2021) 222

JHEP 11 (2022) 169

preprint 2304.03304

+ work in progress

in collaboration with

Paul Caucal, Raju Venugopalan

Björn Schenke & Tomasz Stebel

Two-particle correlation in the CGC

CGC cross-section / amplitudes

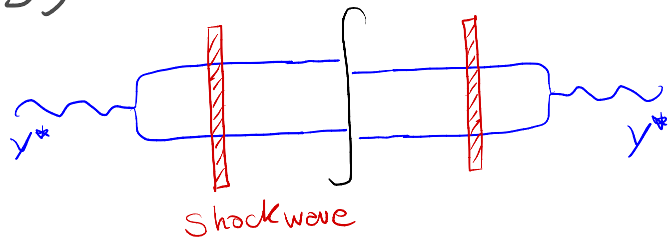
= convolution of perturbative factor (LC wavefunctions)

with correlator of Wilson lines

For example: $\gamma^* + A \to \text{dijet} + X$ (@ LO)

Gelis & Jalilian Marian (PRD 2003)

$$d\sigma_{LO} \propto \int_{\substack{x_\perp, x_\perp' \\ y_\perp, y_\perp'}} e^{-i k_{1\perp}\cdot(x_\perp - x_\perp')} e^{-i k_{2\perp}\cdot(y_\perp - y_\perp')} \psi(z, Q; r_\perp)\, \psi^*(z, Q; r_\perp') \left[1 - S_Y(x_\perp, y_\perp) - S_Y(x_\perp', y_\perp') + Q_Y(x_\perp, y_\perp; x_\perp', y_\perp')\right]$$



Difficult to evaluate numerically, let alone do phenomenology!
grasp analytically

First numerical evaluation by Mäntysaari, Mueller, Salazar, Schenke (PRL 2020)

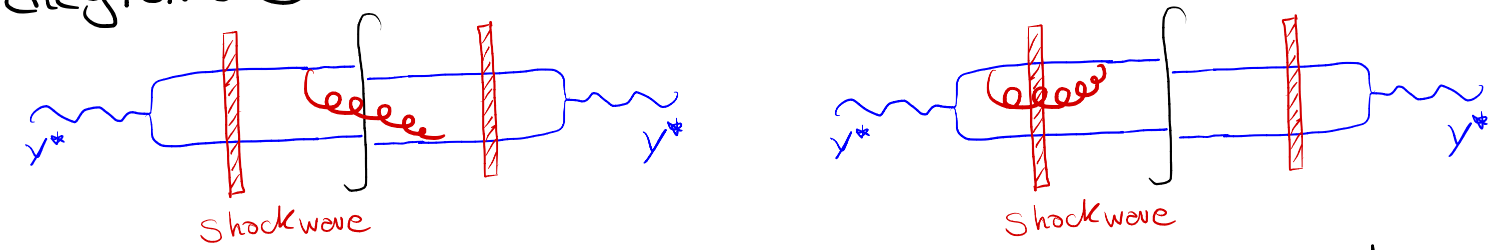
For incoherent dijets see Dionysios' talk

Dijet production DIS in CGC @ NLO

Caucal, Salazar, Venugopalan (JHEP 2021)

First complete NLO calculation for inclusive two-particle correlations in the CGC: $\gamma^* + A \to \text{dijet} + X$ @ NLO

Ex of diagrams @ NLO



real correction

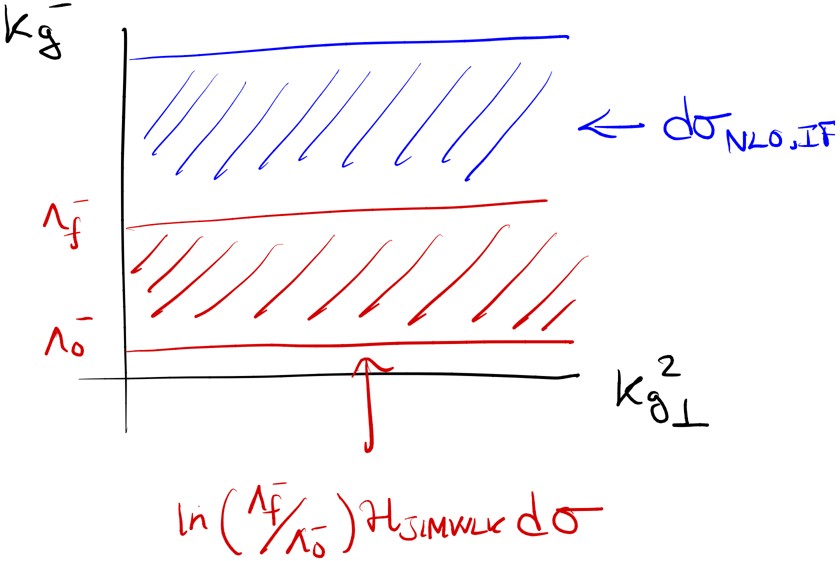
virtual correction

As usual introduce a lower cut-off for momenta modes (along projectile direction) $\Lambda_0^- = z_0 q^-$ + dim reg in "$\perp$" components

$$d\sigma_{NLO} = \underbrace{\ln\left[\frac{k_{1,2}^-}{\Lambda_0^-}\right] H_{JIMWLK}\, d\sigma_{LO}}_{\text{Proof of LL JIMWLK factorization}} + \underbrace{d\sigma_{NLO,IF}}_{\text{expressed as convolution of LC with correlator of Wilson lines}}$$

Dijet production DIS in CGC @ NLO

Caucal, Salazar, Venugopalan (JHEP 2021)



Impact factor

involves color correlators D, Q, DD, DQ & QQ & convolutions with part factors

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2,sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times C_F \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{r}_{xx'}^2 R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2',sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times \Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(4\alpha_s C_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{xx'}}}{(\boldsymbol{k}_{g\perp}-\frac{z_g}{z_1}\boldsymbol{k}_{1\perp})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)}\right.$$
$$\left.-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right]\right.$$
$$\left.\times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\}$$
$$+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}$$
$$\alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)\right.$$
$$+\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)$$
$$+\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$-\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$\left.+(1\leftrightarrow 2)+c.c.\right\}-\frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{\alpha_s C_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{k}_{2\perp}{}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})$$
$$\times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy})\right.\right.$$
$$\left.-\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right]$$
$$\left.+K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow 2)\right\}\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}$$
$$\times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}\right.$$
$$-\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2 e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}}$$
$$-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)\right.$$
$$\left.\left.-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}+(1\leftrightarrow 2)\right\}+c.c.\,.$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi}$$
$$\times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right.$$
$$\left.+\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

← only long pol case shown

For diff dijet see Boussarie et al (JHEP 2016)

For dihadrons see Jamal's talk

Extremely difficult to evaluate numerically

$\Rightarrow$ no phenomenology any time soon!

Small-x TMD factorization @ LO

Dominguez, Marquet, Xiao & Yuan (PRD 2011)

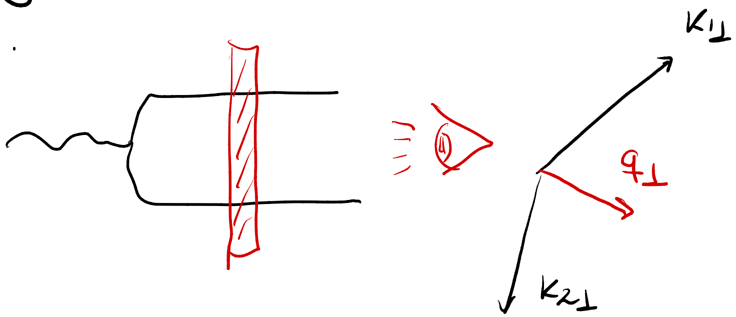
proved small-x TMD factorization @ LO

In the correlation limit ($q_\perp, Q_s \ll P_\perp$) and

↳ forward jets

but close to back-to-back in the transverse plane

high-energy limit ($P_\perp \ll W$)



$q_\perp = k_{1\perp} + k_{2\perp}$

$P_\perp \simeq \frac{1}{2}(k_{1\perp} - k_{2\perp})$

$\simeq k_{1\perp}$

$$d\sigma \propto H_{LO}(z, Q, P_\perp)\, G(Y, q_\perp) \leftarrow$$

"TMD" built from Wilson line correlators

↳ saturation Q_s implicit

Lot of phenomenology e.g.

Dihadron correlations EIC

See Charlotte's talk

extensions such as ITMD see e.g.

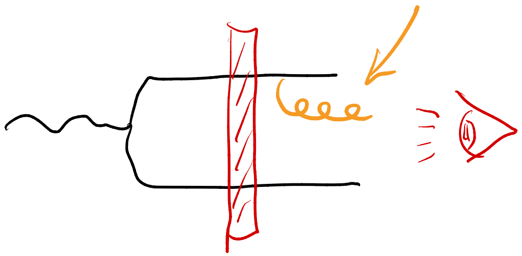
Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (JHEP 2015)

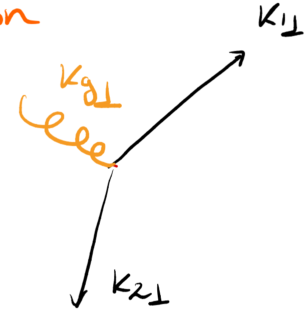
Boussarie, Mäntysaari, Salazar, Schenke (JHEP 2021)

Beyond LO: soft gluon resummation

Mueller, Xiao, Yuan (PRL 2012 & PRD 2013)

soft gluon emission





$k_{g\perp}$ generates imbalance

MXY postulated the joint resummation of small-x & double Sudakov logs

$$d\sigma \propto H_{LO}(z, Q, P_\perp) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G(Y, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)}$$

obeys small-x evolution BK-JIMWLK / DMMX eqs

Dominguez, Mueller, Munier, Xiao (PLB 2011)

Sudakov form factor:

double log

$$S_{Sud}(P_\perp, b_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{1}{2} \ln\left(\frac{P_\perp^2}{\mu^2}\right)$$

"large logs incomplete cancellation between virtual & real emissions"

Our goal:

$$d\sigma \propto \underset{LO+NLO}{H^{ij}(z, Q, P_\perp)} \int_{b_\perp} e^{-i b_\perp \cdot q_\perp} G^{ij}(Y, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)}$$

I) What evolution equation governs the Y evolution of $G(Y, b_\perp)$? Kinematically constrained JIMWLK / DMMX? Is this equation related to non-local RG eq's known in small literature?

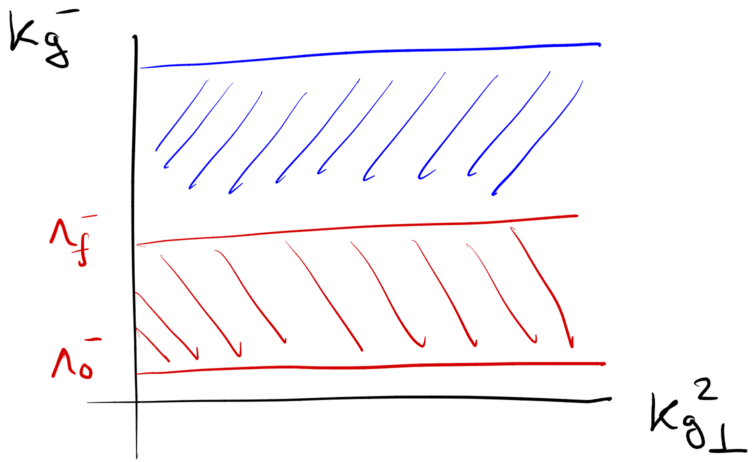
II) Is it possible to determine Sudakov double & single logs?

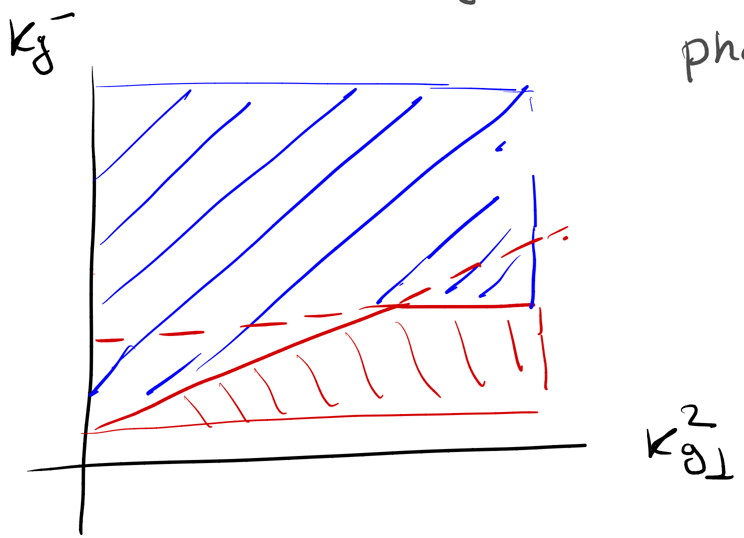
III) Can one prove small-x TMD factorization? I.e is it possible to express all NLO contributions (in the impact factor) in terms of the WW gluon dist (unpolarized & polarized)?

IV) Put all ingredients together to produce complete numerical results for dijet production in DIS @ NLO

The need for a Kinematically constrained evolution (See Pieter's talk)

Taels, Altinoluk
Marquet & Beuf (JHEP2022)
photoproduction





$k_g^- < \Lambda_f^-$

$\Downarrow$

Sudakov double log with opposite sign!

$k_g^- \leq \Lambda_f^-$ & $k_g^+ \geq \Lambda_f^+$

$\Updownarrow$

$k_g^- \leq \dfrac{\Lambda_f^-}{Q_f^2} k_{g\perp}^2$

$\Downarrow$

Correct Sudakov double log

Conclusion:

Separating small-x & Sudakov logs requires ammending small-x evolution eq with a Kinematic constraint (ordering in + & - components)

Sudakov factor : single & double @ finite N_c

Caucal, Salazar, Schenke & Venugopalan (JHEP 2022)

obeys small-x evol with kinematic constraint

$$\Theta\left(\frac{z_f}{z_g Q_f^2} - r_{zb}^2\right)$$

$$d\sigma \propto H_{LO}(z, Q, P_\perp) \int_{b_\perp} e^{-i b_\perp \cdot q_\perp} G(Y, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)} + \text{finite pieces } \mathcal{O}(\alpha_s)$$

$$S_{Sud}(P_\perp, b_\perp) = \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \left[\frac{1}{2} \ln\left(P_\perp^2/\mu^2\right) + \frac{C_F}{N_c} S_0 - S_f \right]$$

$$S_0 = \ln\left(\frac{1}{z_1 z_2 R^2}\right), \quad S_f = \ln\left[\frac{P_\perp^2}{z_1 z_2 c_0^2} \cdot \frac{z_f}{Q_f^2}\right]$$

observed in coll fact see e.g. Hatta, Xiao, Yuan, Zhou (PRD 2021)

intriguing contribution

Factorization of finite pieces into WW

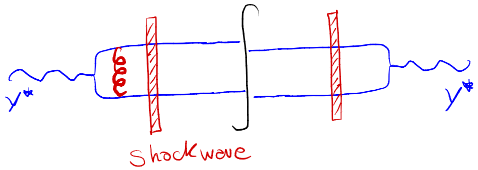
Caucal, Salazar, Schenke, Stebel & Venugopalan (2304.03304)

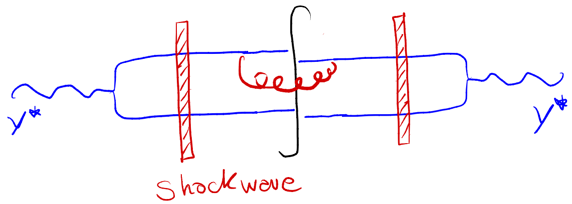
finite pieces :

(I) diagrams gluon does not interact with shock-wave give terms proportional to WW

↳ trivially (as LO case)

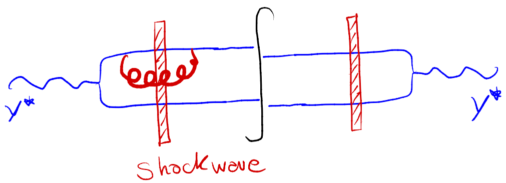
e.g

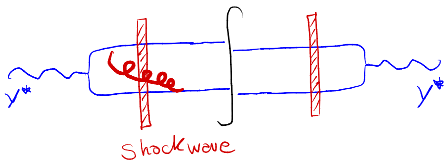




⇒ same color correlator as LO

(II) diagrams gluon interacts with the shockwave ⇒ additional $z_\perp$ coordinat

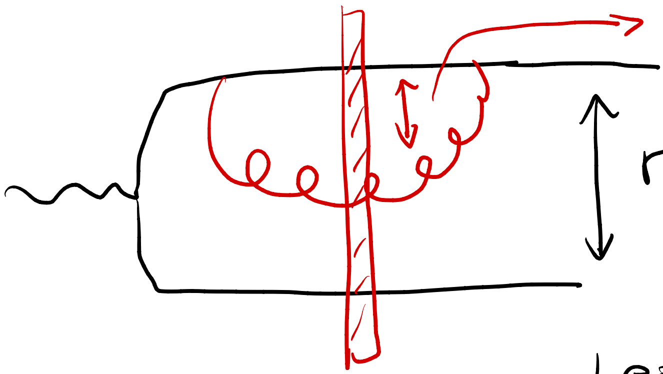




⇒ more complex color correlators involving $z_\perp$ coordinate

usual correlation limit $r_\perp << b_\perp$. What about $r_{zb\perp}$?
($q\bar{q}$ size) (qg size)

Factorization of finite pieces into WW



$r_\perp \sim 1/P_\perp$ fourier transform

Explicit expansion in $r_{zb\perp}$ comes with powers $1/P_\perp$!

Leading contribution involves only WW distrib.

Physically, gluon radiation is dominated by hard emissions

$$1/r_{zb\perp} \sim k_{g\perp} \sim P_\perp$$

Real emission crossing SW does not contribute @ Leading Power (LP)

↳ All finite corrections @ LP can be absorbed by defining new NLO hard factor H_{NLO}

* By product : fixes $\frac{z_f}{Q_f^2} = \frac{e\, c_0^2\, z_1 z_2}{P_\perp^2 + \bar{Q}^2}$ in order to have convergent LP

↳ $S_f = 1 - \ln\left[\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right]$

$\underbrace{\hphantom{S_f = 1 - \ln[\ldots]}}_{\text{consistent with HXYZ 2021}}$

The missing "β_0" Sudakov single log

One-loop correction to background field

Ayala, Jalilian-Marian, McLerran & Venugopalan (PRD 1996)

$$A_\perp^i(x_\perp) = A_{\perp cl}^{(0),i}(x_\perp) + \underbrace{A_\perp^{(1),i}(x_\perp)}_{\text{quantum correction}}$$

$$A_\perp^{(1),i}(x_\perp) = 1/\varepsilon_{UV}\Big[\frac{\alpha_s N_c}{2\pi}\underset{\uparrow}{\beta_0} + \text{finite pieces}\Big] A_\perp^{(0),i}(x_\perp)$$

running of the coupling

$$\hookrightarrow G(Y, b_\perp; \mu) = \frac{-2}{\alpha_s(\mu)} \mathrm{Tr}\left[V \partial^i V^\dagger(b_\perp)\, V \partial^i V^\dagger(0_\perp)\right]$$

$\leftarrow$ renormalized, natural scale $\mu = P_\perp$

$$\hookrightarrow G(Y, b_\perp; \mu) = \underbrace{\left[e^{\frac{\alpha_s N_c}{\pi}\beta_0 \int_{c_0^2/b_\perp^2}^{P_\perp^2} d\mu^2/\mu^2}\right]} G(Y, b_\perp; c_0^2/b_\perp^2)$$

missing "β_0" Sudakov $\rightarrow$

See Zhou PRD 2018 for an alternative approach

Small-x TMD factorization @ NLO

Caucal, Salazar, Schenke, Stebel & Venugopalan (2304.03304)

$$d\sigma \propto \mathcal{H}_{LO+NLO}(z, Q, P_\perp; z_f) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G(Y_f, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)}$$

fully analytic
contains $\ln(\bar{Q}/P_\perp)$ & Li_2

satisfies K.C DMMX eq

$Y_0 = \ln(z_0) \downarrow Y_f = \ln(z_f)$

residual z_f dependence
$\frac{\alpha_s N_c}{\pi} \ln(z_f)$

$$S_{Sud}(P_\perp, b_\perp) = \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \Big[\underbrace{\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right) + \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R}\right) - \frac{\pi \beta_0}{N_c} + \ln\left(\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right)}_{\text{same as in CSS}} - 1 \Big]$$

see e.g. HXYZ 2021

discrepancy!

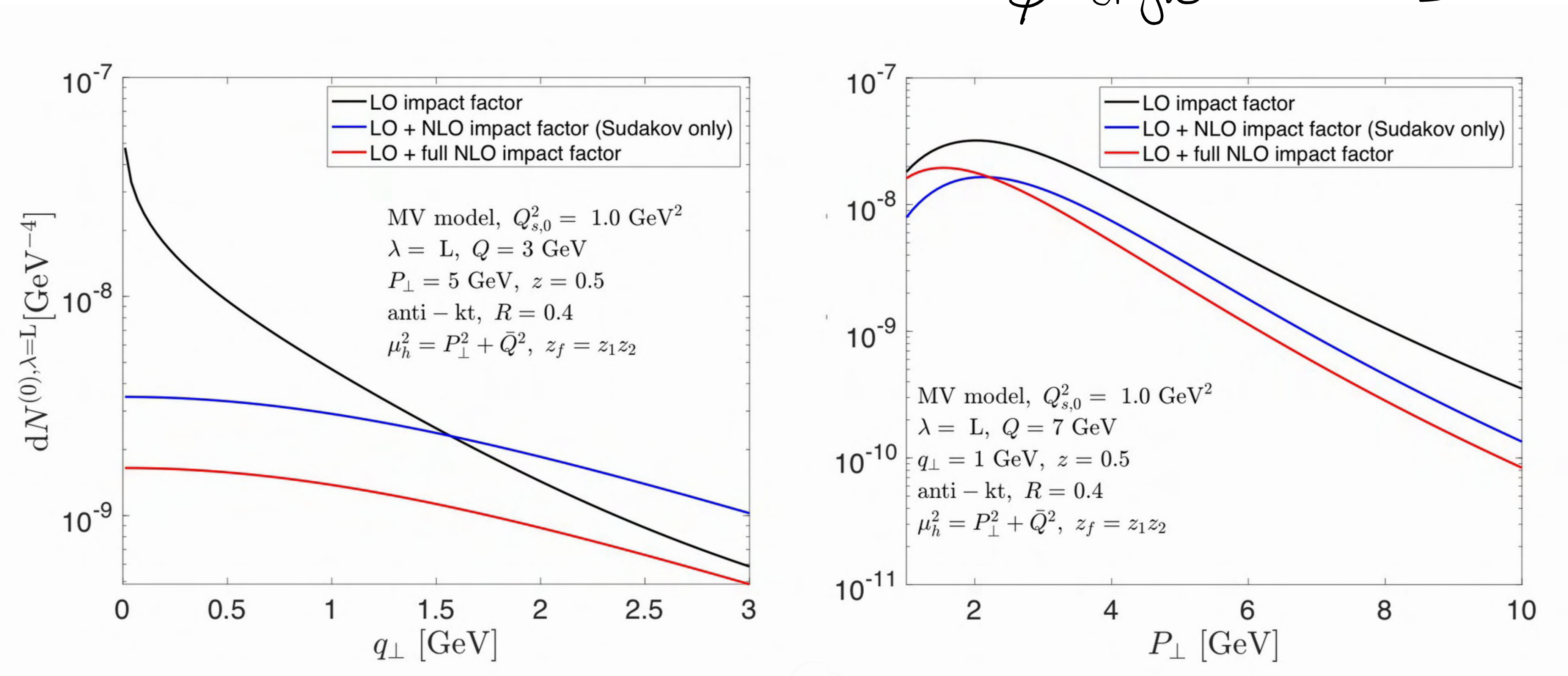
Preliminary numerical results

Caucal, Salazar, Schenke, Stebel & Venugopalan (2304.03304)

* No evolution (only initial conditions)
* Added non-perturbative Sudakov to avoid Landau pole

$$d\sigma = S_\perp dN^0 \left[1 + v_2 \cos 2\phi + v_4 \cos 4\phi\right]$$

ϕ angle between $P_\perp$ & $q_\perp$



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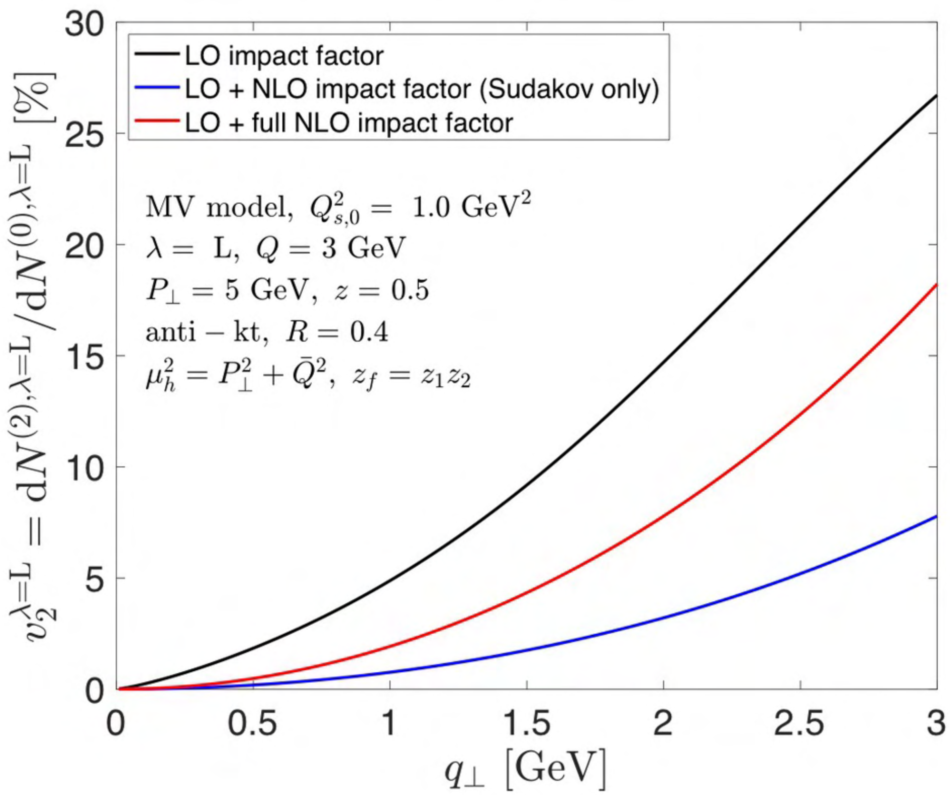
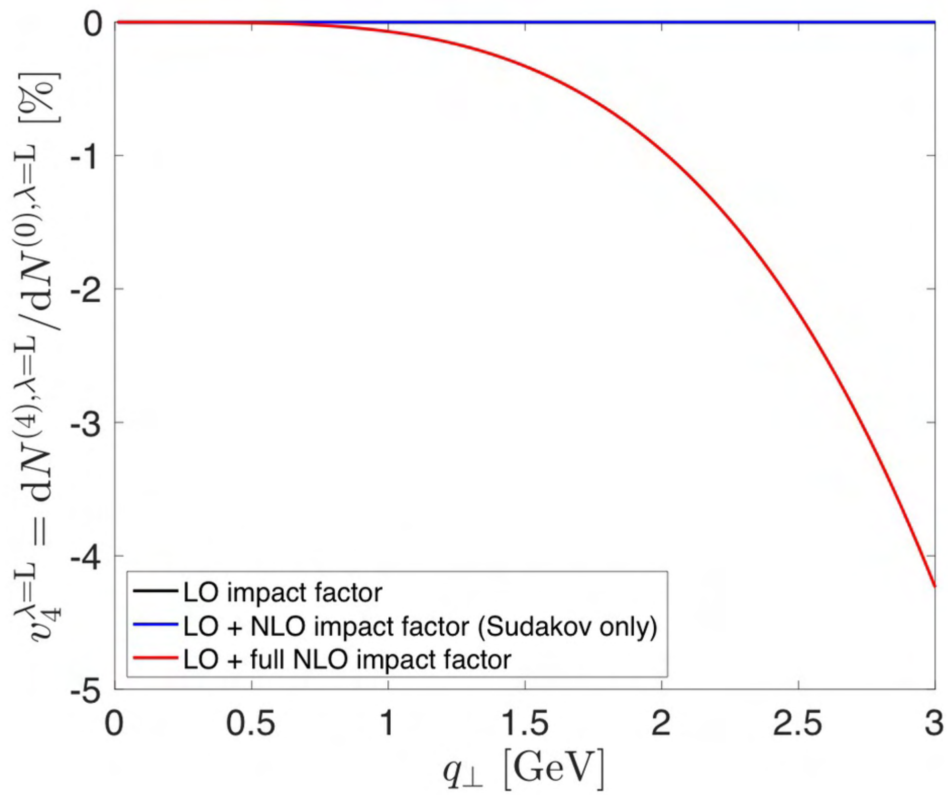




A new insight: from Y to η evolution

Let's assume we can use the Gaussian approximation $\quad \mu_\perp^2 = \dfrac{M_{q\bar{q}}^2 + Q^2}{c_0^2 e}$

For BK our kinematic constraint implies $\quad r_<^2 \equiv \min(r_{zb'}^2, r_{zb}^2)$

$$\frac{\partial S(r_{bb'})}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z_\perp}{(2\pi)} \, \Theta\left(-Y - \ln(r_<^2 \mu_\perp^2)\right) \frac{r_{bb'}^2}{r_{zb}^2 r_{zb'}^2} \left[S_Y(r_{zb}) S(r_{zb'}) - S_Y(r_{bb'}) \right]$$

Can be cast as

$$\frac{\partial \mathcal{S}(r_{bb'})}{\partial \eta} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z_\perp}{(2\pi)} \, \Theta\left(\eta - \ln\left(\frac{r_{bb'}^2}{r_<^2}\right)\right) \frac{r_{bb'}^2}{r_{zb}^2 r_{zb'}^2}$$

Same eq as Duclové, Iancu, Mueller, Soyez, Triantafyllopoulos (JHEP 2019)

$$\rightarrow \left[\mathcal{S}_{\eta - \ln(r_{bb'}^2/r_{zb}^2)}(r_{zb}) \, \mathcal{S}_{\eta - \ln(r_{bb'}^2/r_{zb'}^2)}(r_{zb'}) - \mathcal{S}_\eta(r_{bb'}) \right]$$

non-local equation

Relation between Y & η evolution: $\quad S_{Y_f} = \mathcal{S}_{\eta_f} \quad$ with $\quad \eta_f = Y_f + \ln(r_{bb'}^2 Q^2) - \ln(x_{Bj}/x_0)$

Note when $Y_f = -\ln(r_{bb'}^2 \bar{\mu}_\perp^2) \rightarrow \eta_f = \ln\left[\dfrac{x_0}{x_f}\right], \; x_f = \dfrac{\mu_\perp^2}{W^2} \leftarrow$ natural choice

Will cancel Sudakov single log -1!

Caucal, Salazar, Schenke, Stebel & Venugopalan (in progress)

Small-x TMD factorization @ NLO ordered in η

$$d\sigma \propto \mathcal{H}_{LO+NLO}(z, Q, P_\perp) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G(\eta_f, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)}$$

fully analytic
contains $\ln(\bar{Q}/P_\perp)$
& dilogarithms

Satisfies K.C
non-local DMMX eq

$\eta_0 = 0$
to
$\eta_f = \ln(x_0/x_f)$

$$S_{Sud}(P_\perp, b_\perp) = \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \Big[\underbrace{\frac{1}{2}\ln\left(P_\perp^2/\mu^2\right) + \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R}\right) - \frac{\pi\beta_0}{N_c} + \ln\left(\frac{P_\perp^2+\bar{Q}^2}{P_\perp^2}\right)}_{\text{same as in CSS}} \Big]$$

see. eg.
HXYZ 2021

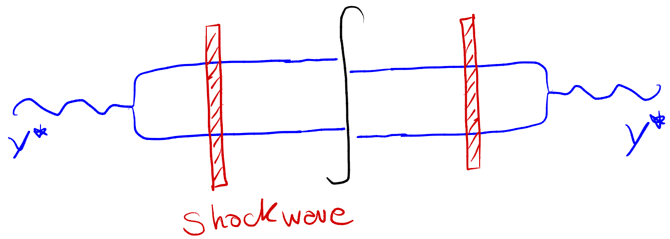
Caucal, Salazar, Schenke, Stebel
& Venugopalan (in progress)

Outlook

* Implement K.C. evolution to our numerical results

* Study dihedron prod & UPCs

* Extend to other two-particle correlation (e.g in pA)

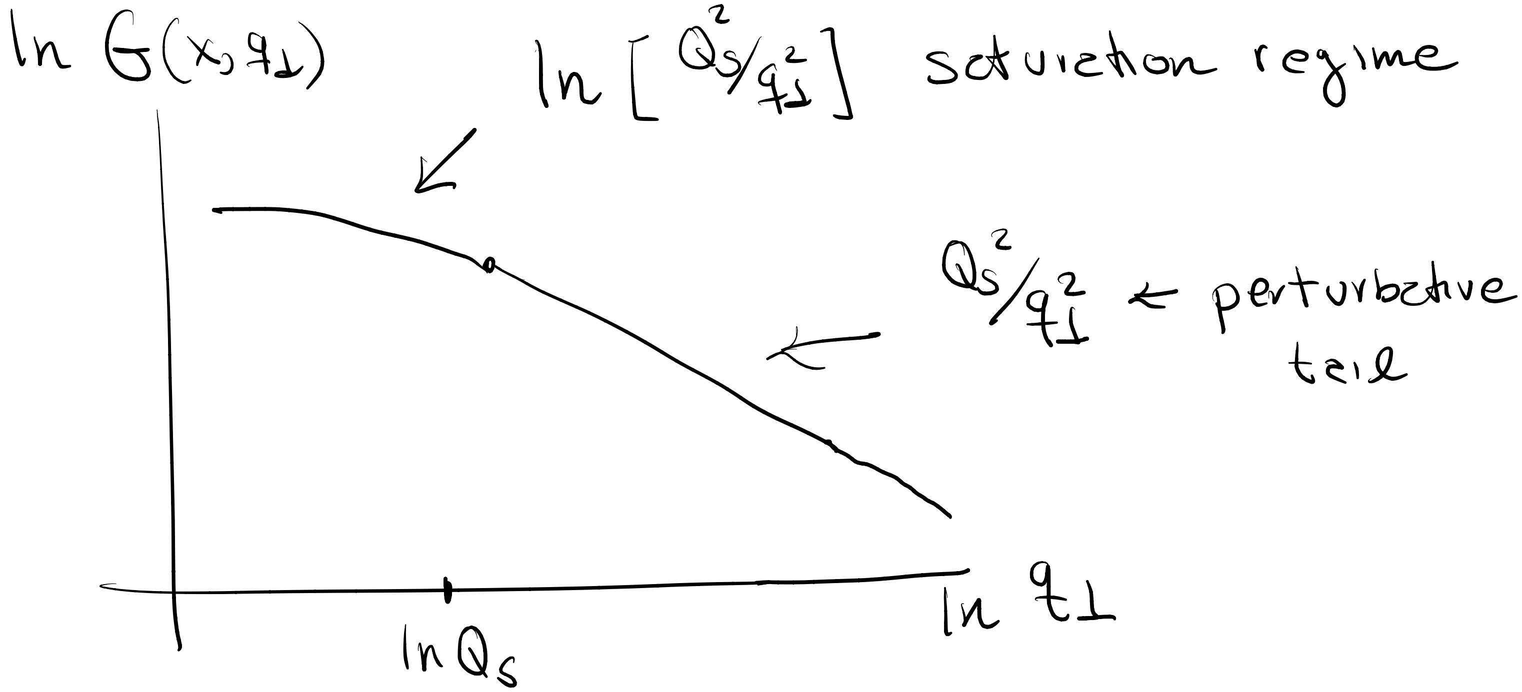


γ*
γ*
Shockwave

Back-up

Slides

Small-x WW distribution



$$K_0\left[QX\right]$$

$$X^2 = z_1 z_2 (x-y)^2 + z_1 z_3 (x-z)^2 + z_2 z_3 (y-z)^2$$

$$= z_1 z_2\, r_\perp^2 + z_3 (b-z)^2$$

$$= z_1 z_2 \left[r_\perp^2 + \frac{z_3}{z_1 z_2} (b-z)^2 \right]$$

$$r_\perp \sim 1/p_\perp \qquad \frac{z_3}{z_1 z_2} (b-z)^2 \ll r_\perp^2$$

$$\hookrightarrow \quad \frac{z_3 P_\perp^2}{z_1 z_2} \ll k_{g\perp}^2$$