Diffractive DIS from nonlinear evolutions: analytical and numerical predictions

Anh-Dung Le

Based on :
- work in preparation with T. Lappi and H. Mäntysaari
- ADL, Mueller and Munier, PRD 104, 034026 (2021) & PRD 103, 054031 (2021)

CGC at the EIC, Trento 2023

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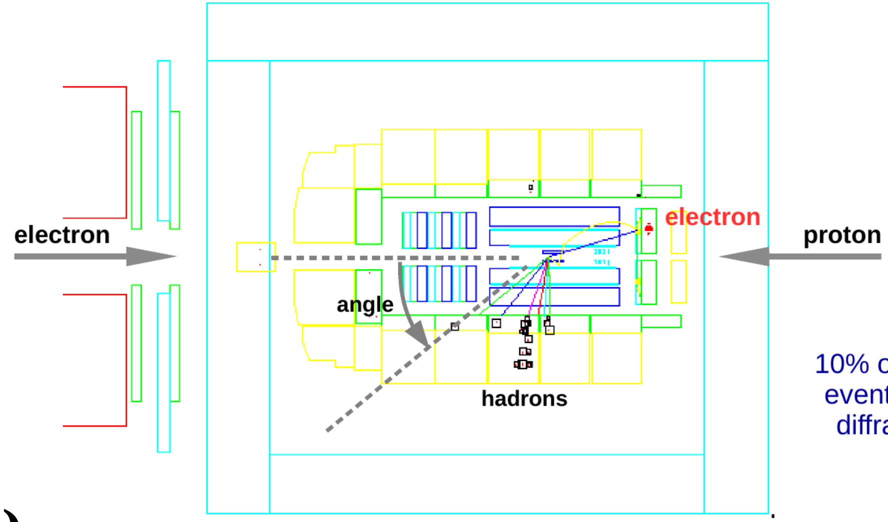
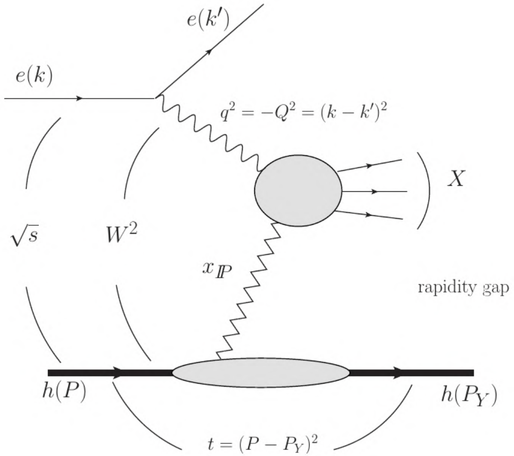
Diffractive dissociation in DIS

- **Single inclusive diffraction:**
  - Hadron (p/A) intact
  - Dissociated inclusive system X
  - Large rapidity gap



10% of HERA events were diffractive!

- **Striking observation at HERA (~ 10%)**
  - Various available data



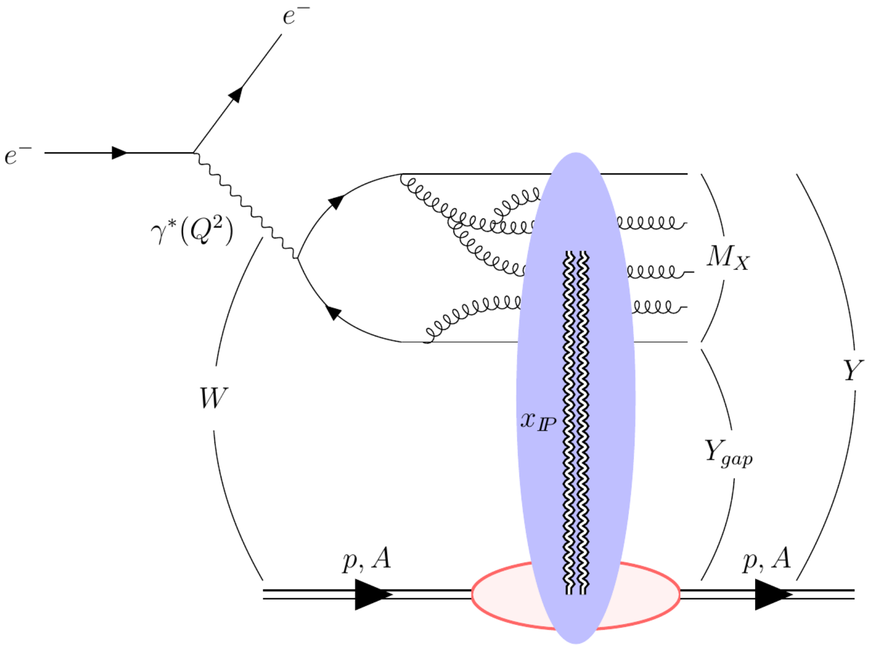
- **One of key measurements at EIC**
  - Sensitive to gluon saturation

(EIC White Paper (2012))

Outlines

- Dipole picture + CGC for diffraction
- Asymptotic solutions to Kovchegov – Levin evolution for diffraction
- Numerical solutions to Kovchegov – Levin equation:
  - Comparison to HERA diffractive data
  - EIC predictions
  - GBW comparison

Dipole picture



$$x_{IP}=\frac{Q^2+M_X^2}{Q^2+W^2}, \quad \beta=\frac{Q^2}{Q^2+M_X^2}$$

$$x=x_{IP}\beta$$

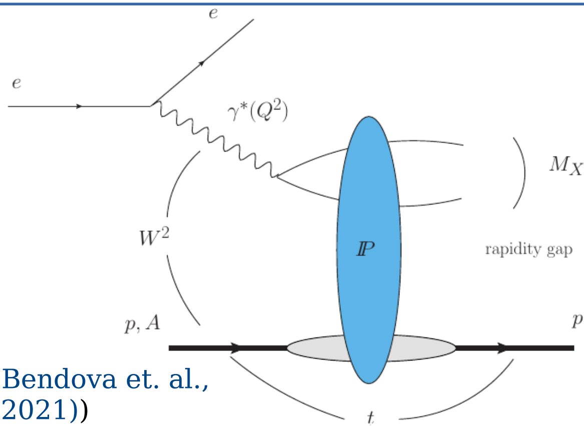
- M_X: dissociation of the quantum Fock state of the virtual photon (qq + qqg +….)

- Dipole factorization:

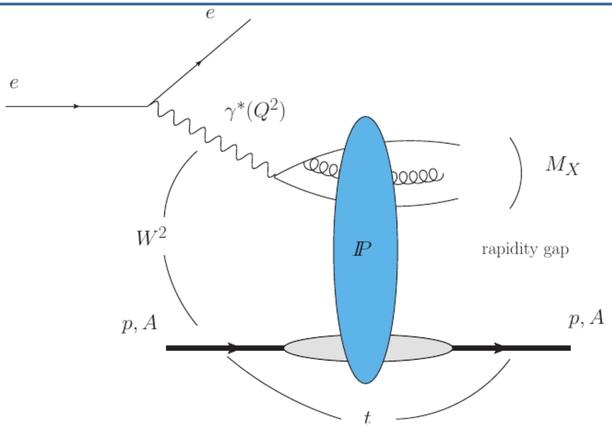
$$\sigma_{tot(T/L)}^{\gamma^* h}(x,Q^2) = \left|\Psi_{T/L}^{\gamma^*\to q\bar{q}}\right|^2 \otimes \sigma_{tot}^{q\bar{q}h}(x,r)$$

$$\frac{d\,\sigma_{D,(T/L)}^{\gamma^* h}(x_{IP},\beta,Q^2)}{d\ln(1/\beta)} = \left|\Psi_{T/L}^{\gamma^*\to q\bar{q}}\right|^2 \otimes \frac{d\,\sigma_{D}^{q\bar{q}h}(x_{IP},\beta,r)}{d\ln(1/\beta)}$$

Dipole picture: Golec-Biernat-Wusthoff (GBW) result



(image credit: Bendova et. al., EPJC 81, 221 (2021))



e. g.,
Wusthoff, PRD 56, 4311 (1997);
Golec-Biernat and Wusthoff, PRD 60, 114023 (1999);
Munier and Shoshi, PRD 69, 074022 (2004);
Kowalski et. al., PRC 78, 045201 (2008);
…

- Gluon emissions are suppressed by power of α_s
  $\Rightarrow$ (**qq + qqg**) contributions are dominant at medium to large β.
- **qq** component ~ N^2 (N: forward dipole-hadron elastic amplitude)
- **qqg** component:
  - Known at exact kinematics (NLO) (G. Beuf et. al., PRD 106, 094014 (2022))
  - Well-known, and widely used, result at large Q^2 (Wusthoff's limit):

$$F^{D(3)}_{q\bar{q}g,T} \sim (2N - N^2)^2 \quad (gluon\ \ dipole-hadron\ \ elastic\ \ cross\ \ section)$$

N(**r**, x, **b**): solution to Balitsky-Kovchegov (BK) evolution (**b**: impact parameter)

$$\frac{\partial N(\boldsymbol{r},x,\boldsymbol{b})}{\partial \ln(1/x)} = \boldsymbol{K}(\boldsymbol{r},\boldsymbol{r_1},\boldsymbol{r_2}) \otimes [N(\boldsymbol{r_1},x,\boldsymbol{b_1}) + N(\boldsymbol{r_2},x,\boldsymbol{b_2}) - N(\boldsymbol{r},x,\boldsymbol{b}) - N(\boldsymbol{r_1},x,\boldsymbol{b_1})\,N(\boldsymbol{r_2},x,\boldsymbol{b_2})]$$

Balitsky, NPB 463, 99 (1996)
Kovchegov, PRD 60, 034008 (1999)

Dipole picture: Kovchegov - Levin (KL) evolution

- At small β, need to resum soft gluon contributions → KL evolution:

$$\frac{\partial N_I(\boldsymbol{r},Y,Y_0,\boldsymbol{b})}{\partial Y}=\boldsymbol{K}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2)\otimes[N_I(\boldsymbol{r}_1,Y,Y_0,\boldsymbol{b}_1)+N_I(\boldsymbol{r}_2,Y,Y_0,\boldsymbol{b}_2)-N_I(\boldsymbol{r},Y,Y_0,\boldsymbol{b})-N_I(\boldsymbol{r}_1,Y,Y_0,\boldsymbol{b}_1)N_I(\boldsymbol{r}_2,Y,Y_0,\boldsymbol{b}_2)]$$

Kovchegov and Levin, NPB 577, 221 (2000)

Kovchegov, PLB 710, 192 (2012)

$$Y = \ln\frac{1}{x}$$
$$Y_{gap} = \ln\frac{1}{x_{IP}}$$
$$Y_0 = min(Y_{gap})$$

  - $N_D(\mathbf{r}, Y, Y_0, \mathbf{b}) = 2N(\mathbf{r}, Y, \mathbf{b}) - N_I(\mathbf{r}, Y, Y_0, \mathbf{b})$: diffractive dipole-target cross section with a minimal gap Y_0 (per impact parameter).
  - Initial condition: $N_D(\mathbf{r}, Y=Y_0, Y_0, \mathbf{b}) = N^2(\mathbf{r}, Y_0, \mathbf{b})$.
  - One needs to BK evolve the forward elastic amplitude N to Y_0 from Y = 0.

- LO kernel :

$$\boldsymbol{K}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) \rightarrow \boldsymbol{K}_{LO}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{N_C\,\alpha_s^{fx}}{2\pi^2}\frac{r^2}{r_1^2 r_2^2}$$

- Running coupling kernel:

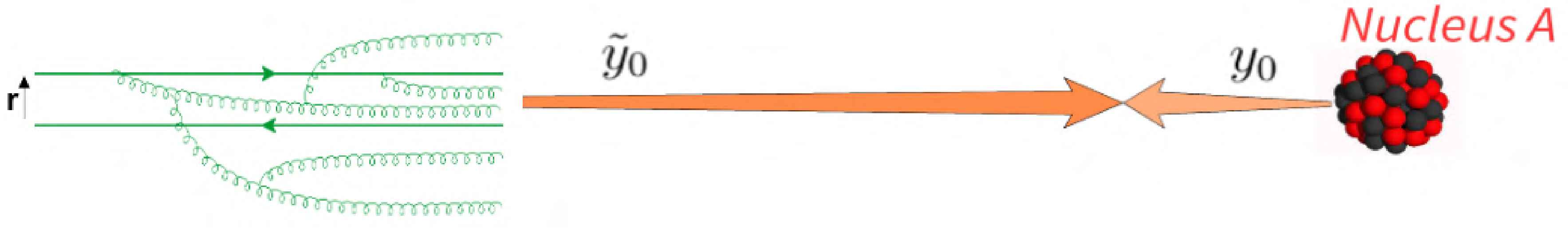
$$\boldsymbol{K}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) \rightarrow \boldsymbol{K}_{rc}^{Bal}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{N_C\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2 r_2^2}+\frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)}-1\right)+\frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)}-1\right)\right],$$

$$\alpha_s(r^2)=\frac{12\pi}{(33-2N_f)\ln\frac{4C^2}{r^2\Lambda_{QCD}^2}}$$

Balitsky, PRD 75, 014001 (2007)

- NLO KL is known: Lublinsky, PLB 735,200 (2014)

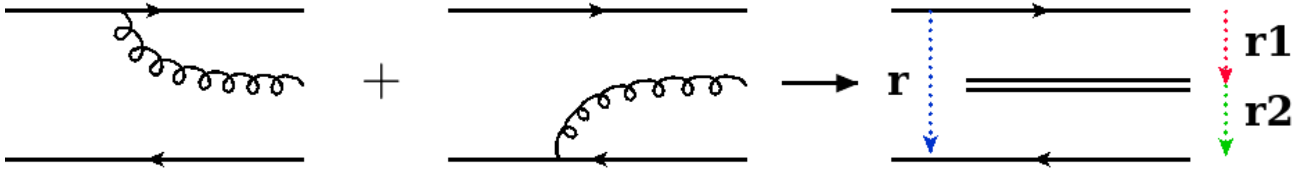
Asymptotic solutions to b-independent KL equation at LO



Let's assume :

- Total relative rapidity Y is large : $Y \gg 1$
- Dipole and target share Y : $Y = Y_0 + \tilde{Y}_0 \quad (Y_0, \tilde{Y}_0 \gg 1)$
- Dipole size is small : $1 \ll \ln \frac{1}{r^2 Q_s^2(Y)} \ll \sqrt{Y}$ → geometric scaling windows

Wave function of a dipole at small-x



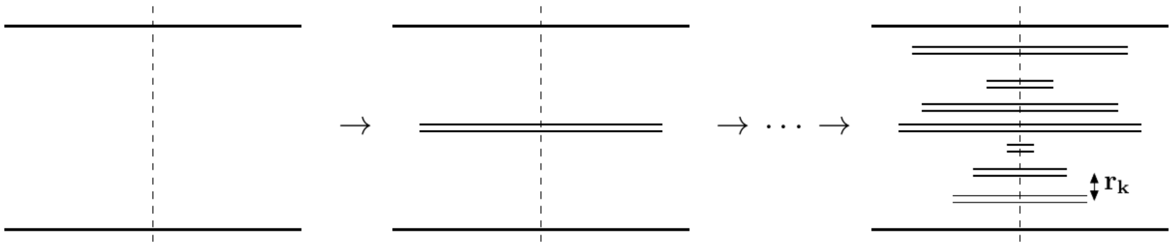
- A soft-gluon emission from a dipole can be intepreted as a 1-to-2 dipole branching at large N_c with the rate

$$dP(\boldsymbol{r} \to \boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{d^2 r_1}{2\pi} \frac{r^2}{r_1^2 r_2^2} dy, \quad dy \equiv \frac{\alpha_s N_c}{\pi} dY$$

A. H. Mueller, NPB 415 (1994)

- Subsequent gluon emissions → subsequent dipole splittings

- Dipole's Fock state at small-x and large N_c ≃ stochastic set of dipoles (random number of dipoles with random transverse size)



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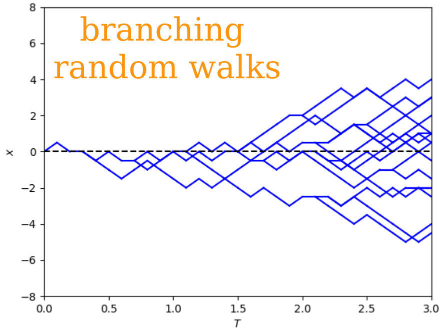

Nuclear scattering of dipole's Fock state

Assuming that we know well the random dipole density: $n(\rho', \tilde{y}_0 \mid \rho)$, with

- $\rho \equiv \ln 1/[r^2 Q_{s0}^2]$: log-size of the parent dipole
- $\rho' \equiv \ln 1/[r'^2 Q_{s0}^2]$: log-size of a dipole in the Fock state of the parent dipole

S-matrix element of the nuclear scattering of a random dipole's Fock state :

$$\mathfrak{S}(\rho, y_0, \tilde{y}_0) = \prod_{\rho'} [S(\rho', y_0)]^{n(\rho', \tilde{y}_0|\rho)d\rho'} \to \exp\left[-\int d\rho' n(\rho', \tilde{y}_0|\rho) \ln \underbrace{\frac{1}{S(\rho', y_0)}}\right]$$

S-matrix element of the nucl. scattering of an elementary dipole ρ', S = 1 - N

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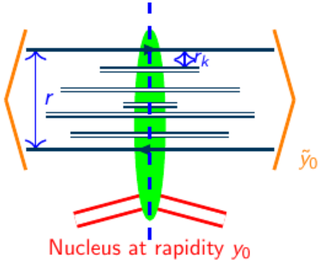
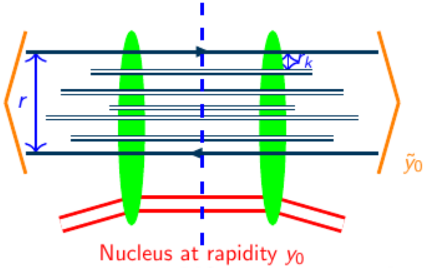




$$\sigma_{tot}^{q\bar{q}A}(\rho, y) = 2\left[1 - \left\langle \mathfrak{S}(\rho, y_0, \tilde{y}_0) \right\rangle_{\tilde{y}_0}\right]$$

Total cross section

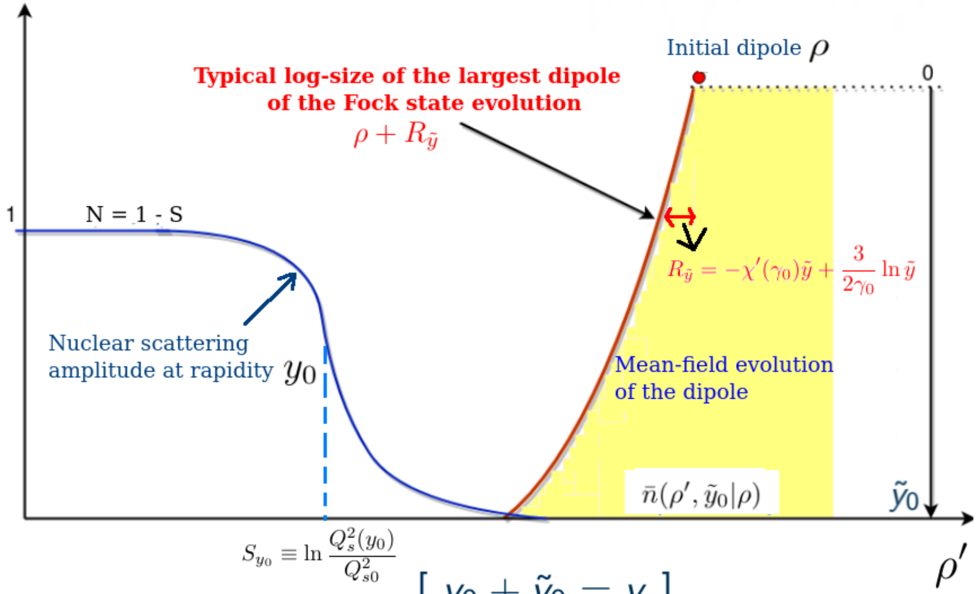
$$\sigma_D^{q\bar{q}A}(\rho, y; y_0) = \left\langle \left[1 - \mathfrak{S}(\rho, y_0, \tilde{y}_0)\right]^2 \right\rangle_{\tilde{y}_0}$$

Diffractive cross section with a minimal gap y_0

Nuclear scattering of a small dipole with typical evolution

Typical evolution : deterministic with no large fluctuation, with mean-field dipole density given by the solution to BFKL with a cut-off

$$\bar{n}(\rho', \tilde{y}_0|\rho) = C_1(\rho' - \rho - R_{\tilde{y}_0}) e^{\gamma_0(\rho' - \rho - R_{\tilde{y}_0})} \exp\left[-\frac{(\rho' - \rho - R_{\tilde{y}_0})^2}{2\chi''(\gamma_0)\tilde{y}_0}\right]$$



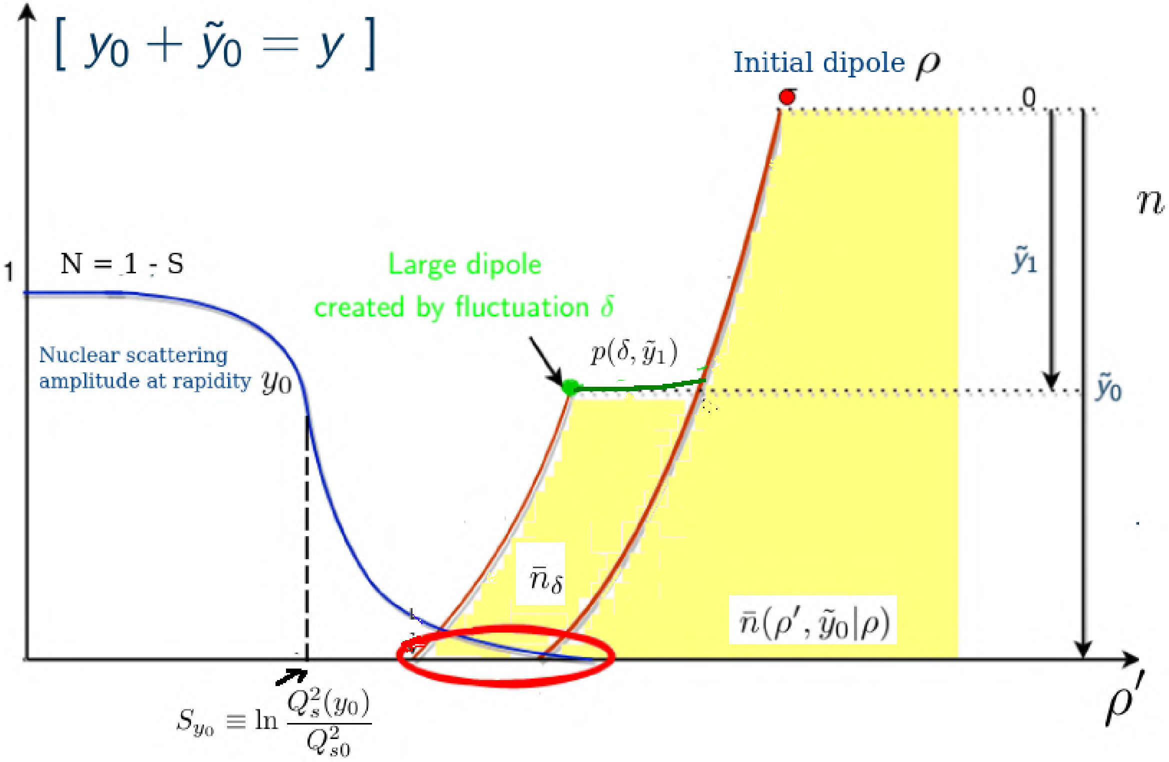
$[\; y_0 + \tilde{y}_0 = y \;]$

Sub-leading contribution

$$N(\rho', y_0) = C_2\,(\rho' - S_{y_0})\, e^{-\gamma_0(\rho' - S_{y_0})} \exp\left[-\frac{(\rho' - S_{y_0})^2}{2\chi''(\gamma_0)y_0}\right] \quad [1 \ll \rho' - S_{y_0} \ll \sqrt{y_0}]$$

Nuclear scattering of a small dipole with large fluctuation

- Now introduce a **large fluctuation δ** at some intermediate rapidity of the evolution (lowest cost).
- The evolution of the large dipole from large fluctuation is deterministic.
- Distribution of **δ** solves BK in the dilute regime



$$n \otimes \ln \frac{1}{S} = (\bar{n} + \bar{n}_\delta) \otimes \ln \frac{1}{1-N} \approx \bar{n}_\delta \otimes N$$

$$\left\langle \cdots \right\rangle_{\tilde{y}_0} \simeq \int_0^{\tilde{y}_0} d\tilde{y}_1 \int d\delta p(\delta, \tilde{y}_1) \cdots$$

Note : largest dipole of the dominant configurations will not cross the saturation boundary

Nuclear scattering of a small dipole with large fluctuation

Results (small dipole in the scaling regime, large rapidity variables):

Diffractive cross section:

$$\frac{\sigma_D^{q\bar{q}A}(r,y;y_0)}{\sigma_{tot}^{q\bar{q}A}(r,y)} = \frac{\ln 2}{\gamma_0}\left[\frac{1}{\ln\frac{1}{r^2Q_s^2(y)}} + \sqrt{\frac{2}{\pi\chi''(\gamma_0)}}\frac{1}{\sqrt{y_0}}\right]$$

$$\left(y = \frac{\alpha_s N_c}{\pi} Y \quad y_0 = \frac{\alpha_s N_c}{\pi} Y_0 \right)$$

[Compare to GBW estimation : $\frac{\sigma_D^{\gamma^* h}}{\sigma_{tot}^{\gamma^* h}}(Q^2,x) \propto \frac{\ln 2}{2}\frac{1}{\ln\frac{Q^2}{Q^2_{s,gbw}(x)} + \gamma_E + \frac{1}{2}}$ (large Q^2)]

(see Golec-Biernat & Wusthoff, PRD 59 (1998))

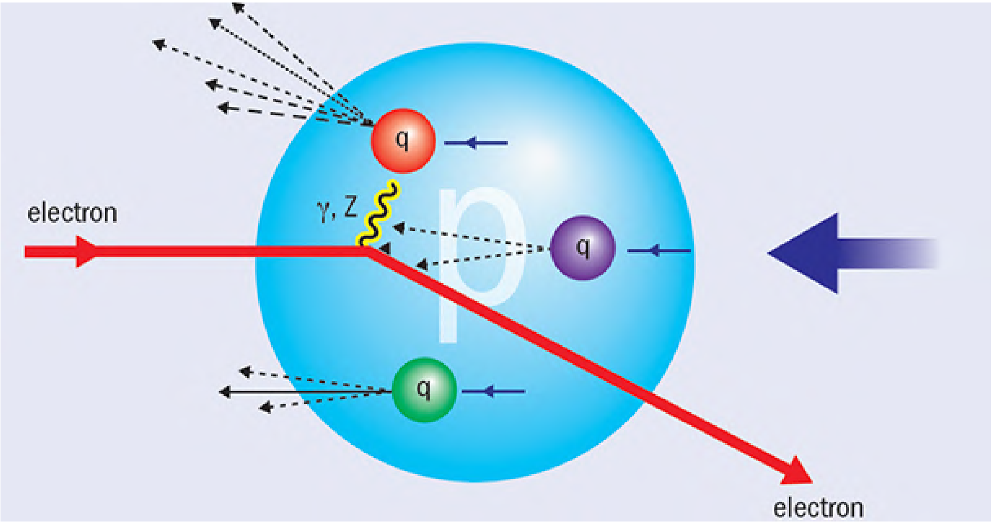
Gap distribution :

$$\frac{1}{\sigma_{tot}^{q\bar{q}A}}\frac{d\sigma_D^{q\bar{q}A}}{dy_0}\Big|_{y_0=y_{\rm gap}}(r,y;y_{\rm gap}) = \frac{\ln 2}{\gamma_0}\frac{1}{\sqrt{2\pi\chi''(\gamma_0)}}\left(\frac{y}{y_{\rm gap}(y-y_{\rm gap})}\right)^{3/2}\exp\left[-\frac{\ln^2[r^2Q_s^2(y)]}{2\chi''(\gamma_0)(y-y_{\rm gap})}\right]$$

No free parameters : all constants above known from BFKL !

Numerical predictions from b-dependent KL equation with running coupling

Scattering off proton :
confronting the HERA data



Impact parameter treament

- **b**-dependence is factorized in both total and diffractive cross-sections:

$$\sigma_{tot}^{q\bar{q}h} = 2\int d^2\boldsymbol{b}\, N(\boldsymbol{r},Y,\boldsymbol{b}) = 2\int d^2\boldsymbol{b}\, N(r,Y)\, T_p(\boldsymbol{b}) = \sigma_0 N(r,Y)$$

$$\sigma_{D}^{q\bar{q}h} = \int d^2\boldsymbol{b}\, N_D(\boldsymbol{r},Y,Y_0\boldsymbol{b}) = N_D(r,Y,Y_0)\underbrace{\int d^2\boldsymbol{b}\, T_p^2(\boldsymbol{b})}_{\stackrel{\text{def}}{=}\sigma_0^D}$$

  - $\sigma_0/2$: effective transverse area, from fit to the HERA inclusive data (Lappi and Mäntysaari, PRD 88, 114020 (2013))
  - σ_0^D: depends on proton shape
  - $N(r, Y)$ and $N_D(r, Y, Y_0)$ obey (b-independent) BK and KL evolutions.

- I.P. profile for proton:

$$T_p(\boldsymbol{b}) = \frac{\Gamma\left(\frac{1}{\omega}, \frac{2\pi b^2}{\sigma_0 \omega}\right)}{\Gamma\left(\frac{1}{\omega}\right)}$$

  - ω: steepness parameter
  - ω = 1: gaussian; ω = 0: hard disk (step function)

Initial conditions for the evolution

- Initial condition for the first-step (BK) evolution at $x=x_{init}= 0,01$:

$$\mathcal{N}(r) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(e \cdot e_c + \frac{1}{r\Lambda_{\mathrm{QCD}}}\right)\right]$$

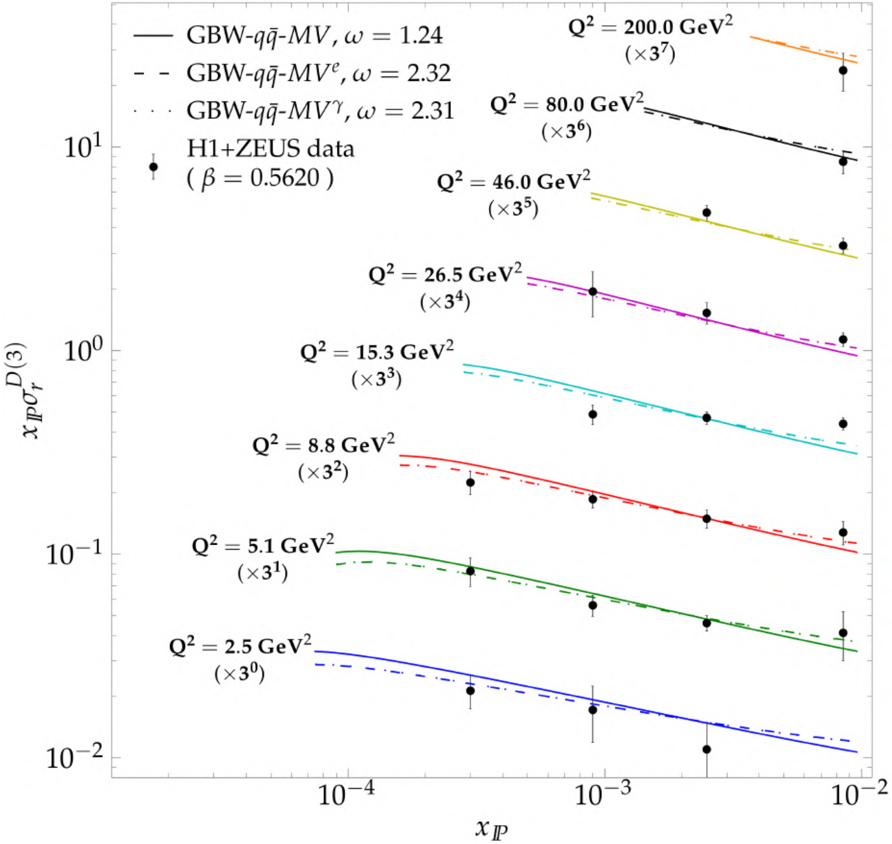
  - (original) **MV**: $\gamma = e_c = 1$, Q_{s0} is free param.
  - **MV^e**: $\gamma = 1$, Q_{s0} and e_c are free params.
  - **MV$^\gamma$**: $e_c = 1$, Q_{s0} and γ are free params.

- Free parameters obtained from fits (light quarks only) to HERA inclusive data

Parametrization	$Q_{s0}^2(\mathrm{GeV}^2)$	γ	e_c	$\sigma_0/2$ (mb)	C^2
MV	0.104	1	1	18.81	14.5
MV$^\gamma$	0.159	1.129	1	16.35	7.05
MV^e	0.060	1	18.9	16.36	7.2

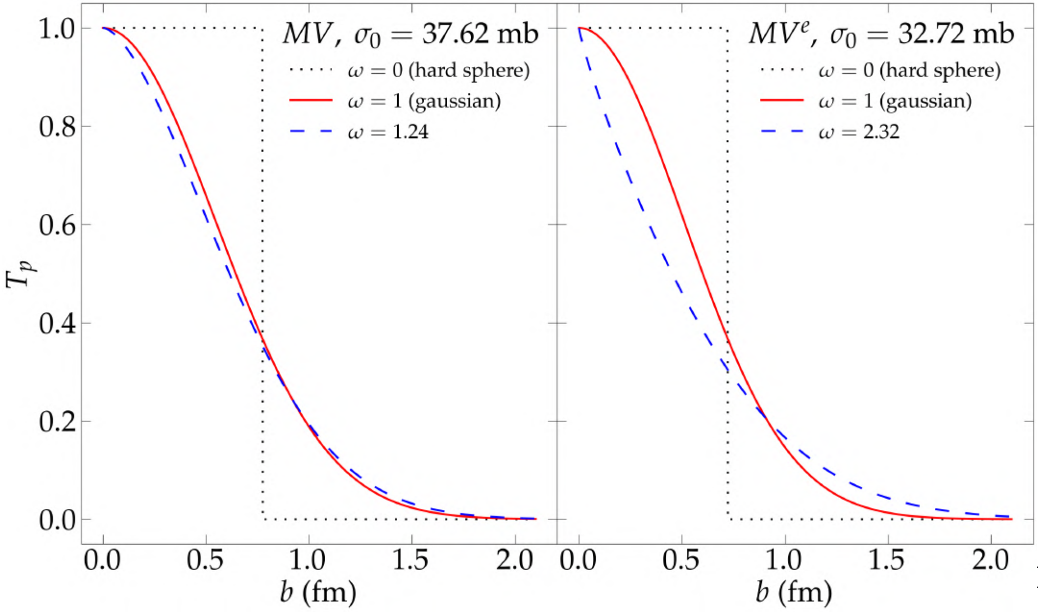
Lappi and Mäntysaari, PRD 88, 114020 (2013)

Proton shape: Optimal steepness



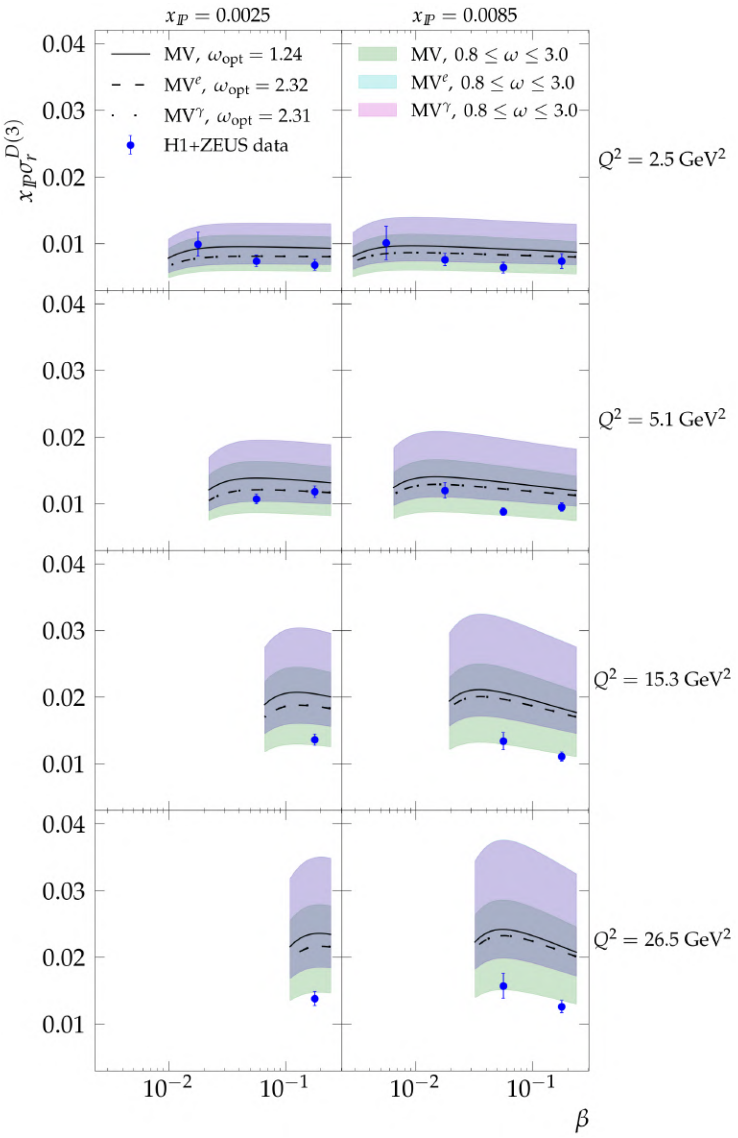
Combined data from
H1, ZEUS collaboration, EPJC 72, 2175 (2012)

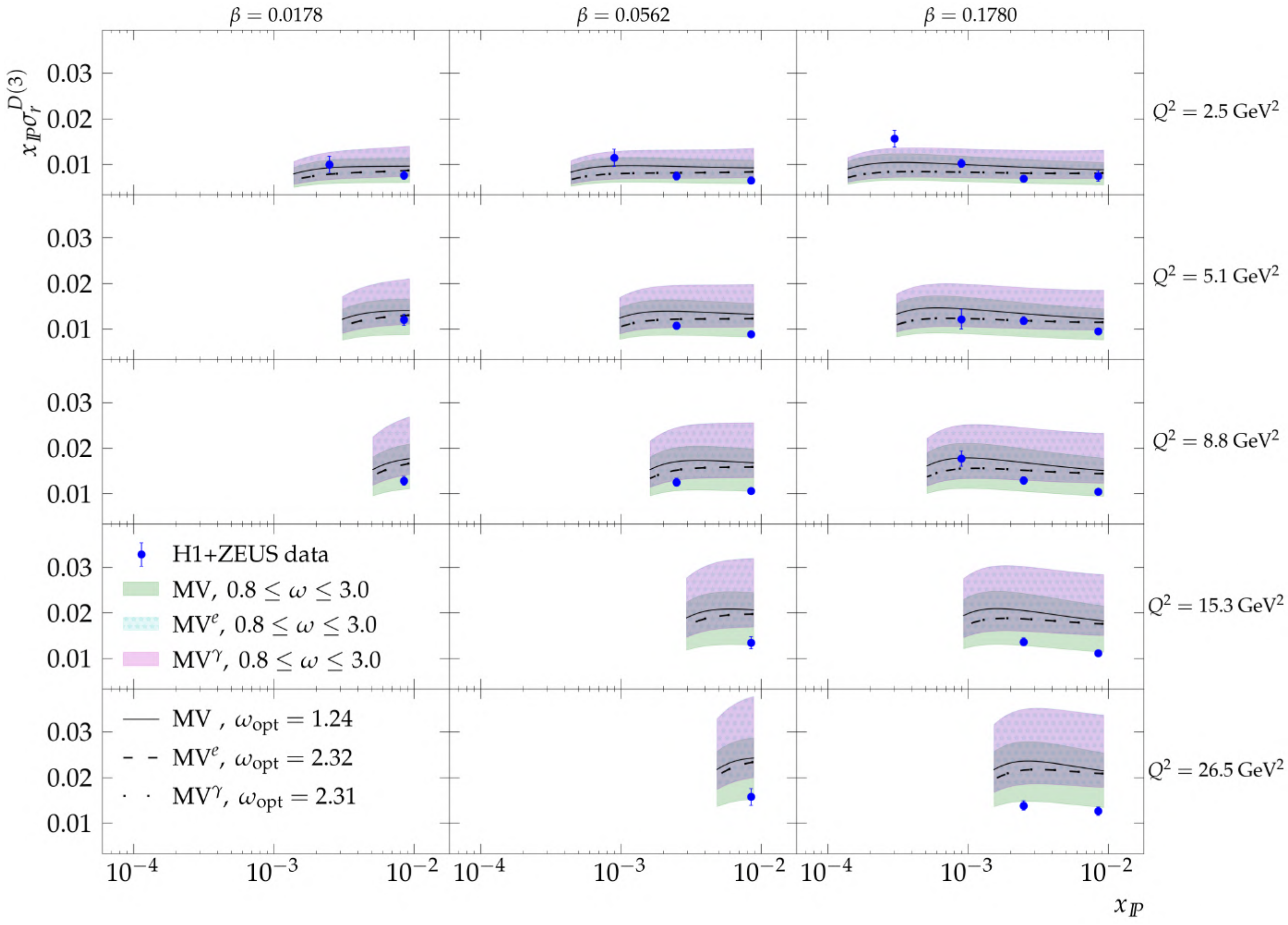
- **GBW-qq** result is fitted to diffractive HERA combined data for β>0.5 (24 data points) using the chosen proton shape.
- Optimal steepness:
  - ω = 1.24 (χ^2/dof ≈ 1.87) for **MV**
  - ω = 2.32 (χ^2/dof ≈ 1.08) for **MV^e**
  - ω = 2.31 (χ^2/dof ≈ 1.09) for **MV$^\gamma$**



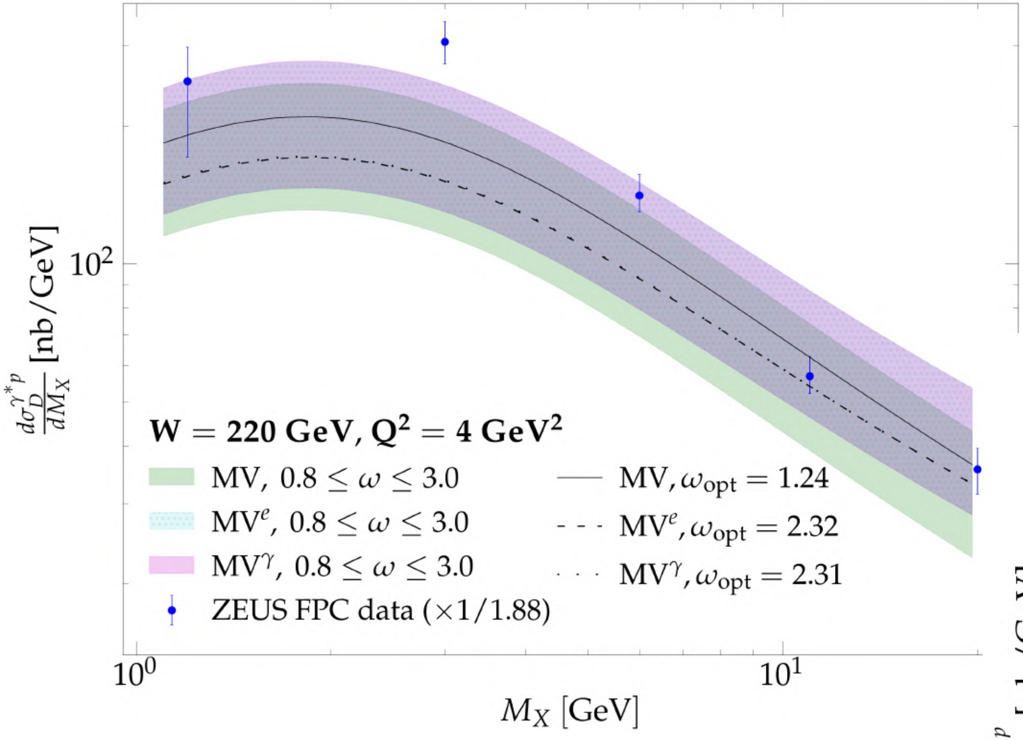
$$T_p(\boldsymbol{b}) = \frac{\Gamma\left(\frac{1}{\omega}, \frac{2\pi b^2}{\sigma_0 \omega}\right)}{\Gamma\left(\frac{1}{\omega}\right)}$$

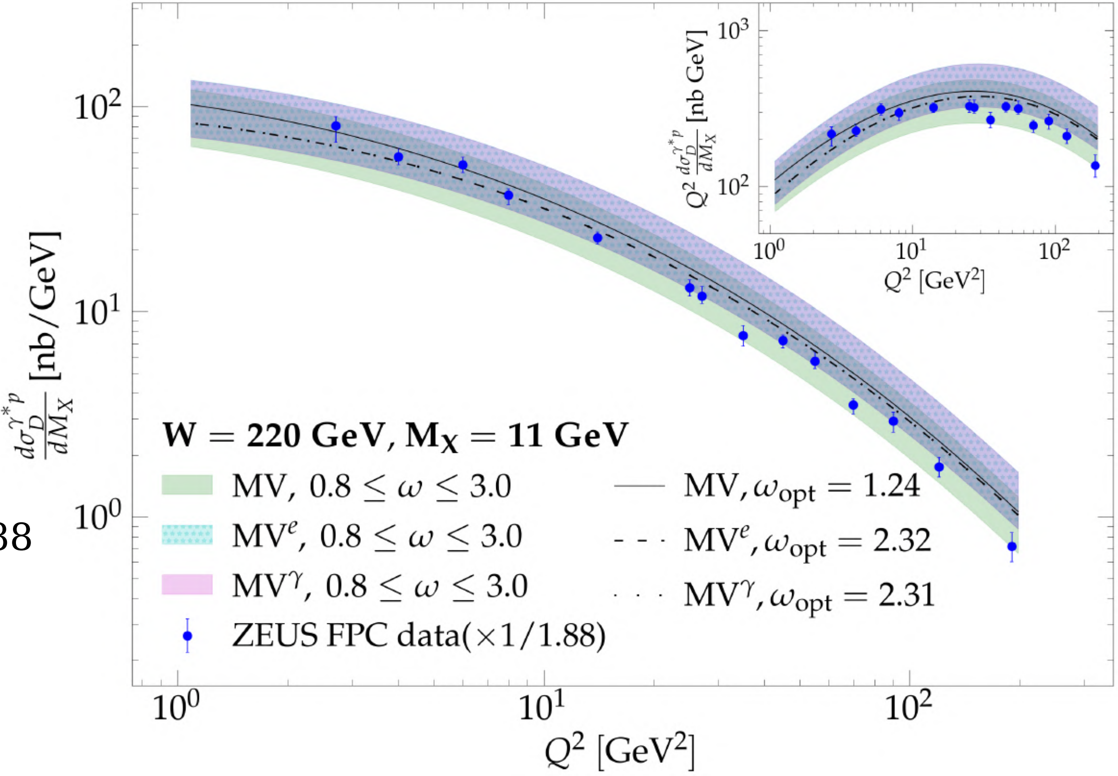
Comparison to HERA combined data



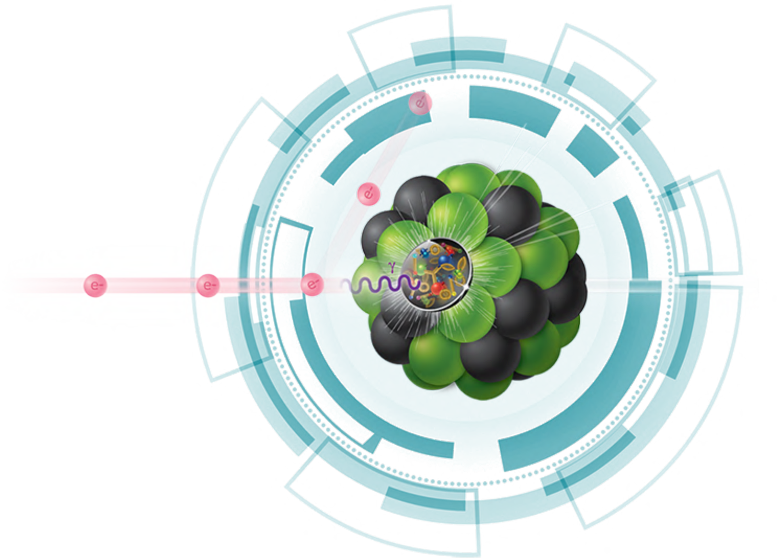


Comparison to ZEUS FPC data





(ZEUS FPC data are scaled down by a factor 1.88 for correcting for the incoherent diffraction)



Scattering off nucleus:
EIC prediction

Impact parameter treatment and initial conditions

- Neglect the dependence on the orientation of transverse vectors (**r** and **b**)
- BK & KL evolutions are solved for each **b** independently.
- Initial condition (optical Glauber):

$$N_A(r,b) = 1-\exp\left[-AT_A(b)\frac{\sigma_0}{2}\frac{(r^2Q_{s0}^2)^\gamma}{4}\ln\left(e\cdot e_c+\frac{1}{r\Lambda_{QCD}}\right)\right]$$

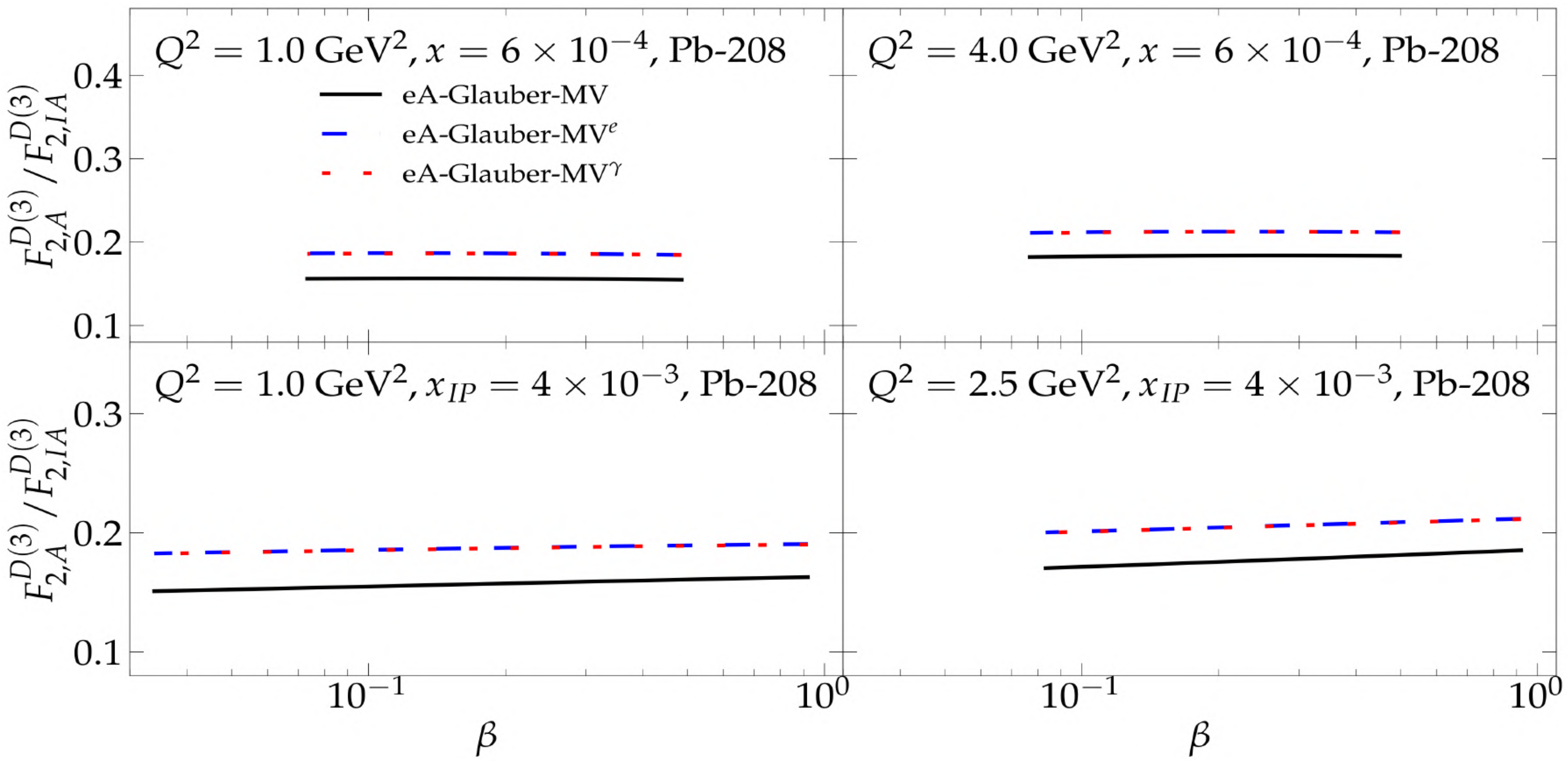
  - $T_A(b)$: Wood-Saxon I.P. profile, $\int d^2\mathbf{b}\, T_A(b) = 1$.

- Unphysical increase of gluon density at $|\mathbf{b}| \gtrsim b_c \sim 6.3$ fm → Assuming no nuclear (gluon) modification for total cross section at such large $|\mathbf{b}|$:

$$N_A(r,Y,b>b_c) = AT_A(b)\frac{\sigma_0 N(r,Y)}{2}$$

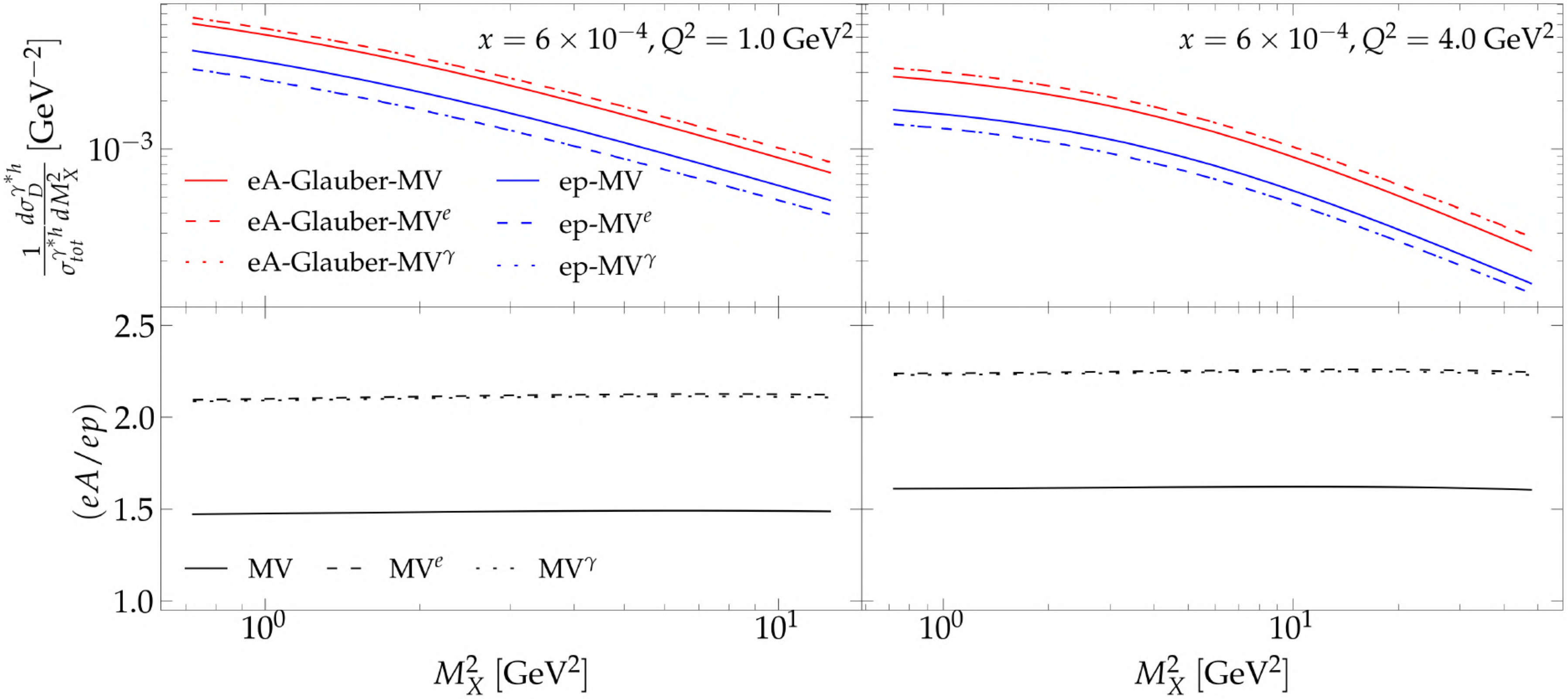
$$N_{D,A}(r,Y,Y_0,b>b_c) = A^2 T_A^2(b)\frac{\sigma_0^2 N_D(r,Y,Y_0)}{4}$$

Nuclear suppression : KL / IA (impulse approximation)



- Huge nuclear suppression
- Very mild dependence on β

Nuclear enhancement : double ratio



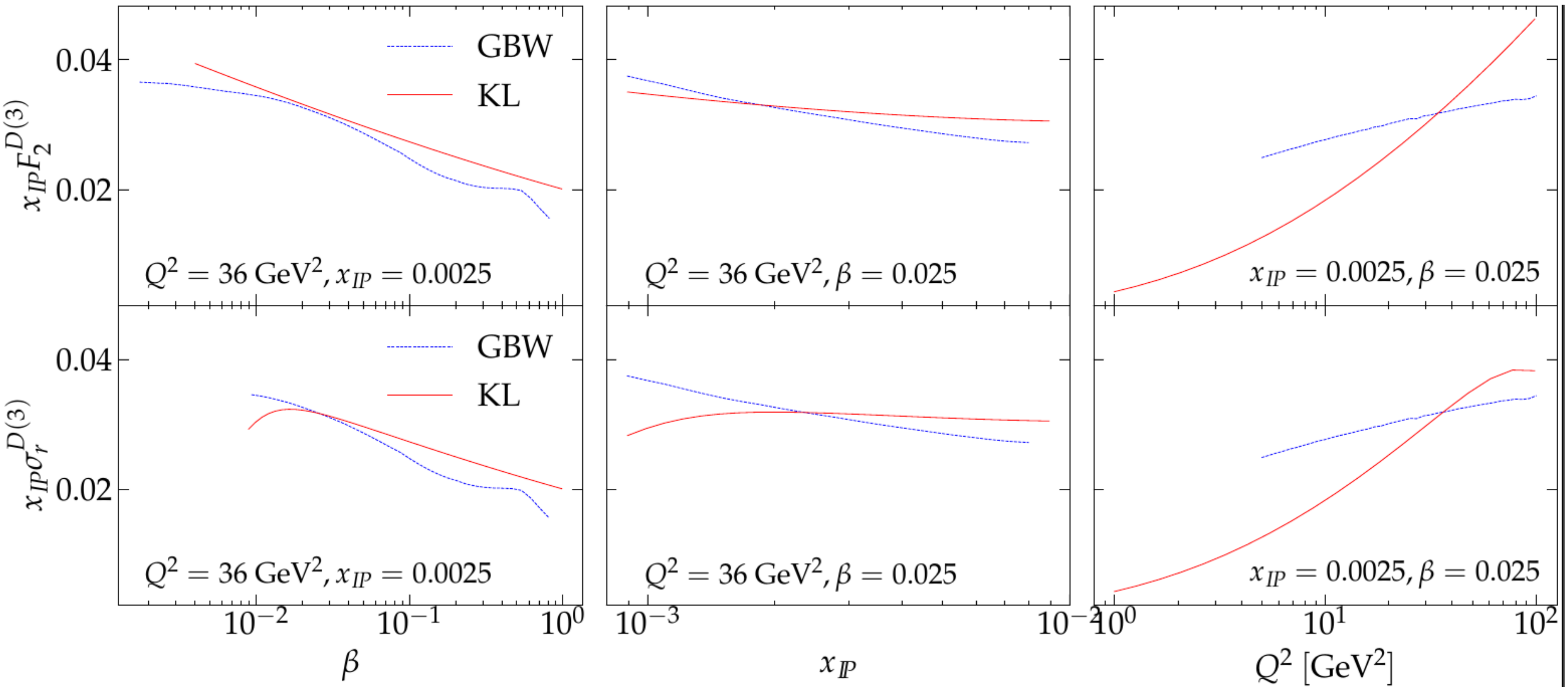
- Larger enhancement compared to the prediction of the GBW approach (~ 1.1 ... 1.7)

(see EIC White Paper (2012))

KL vs GBW

KL vs. GBW (1)

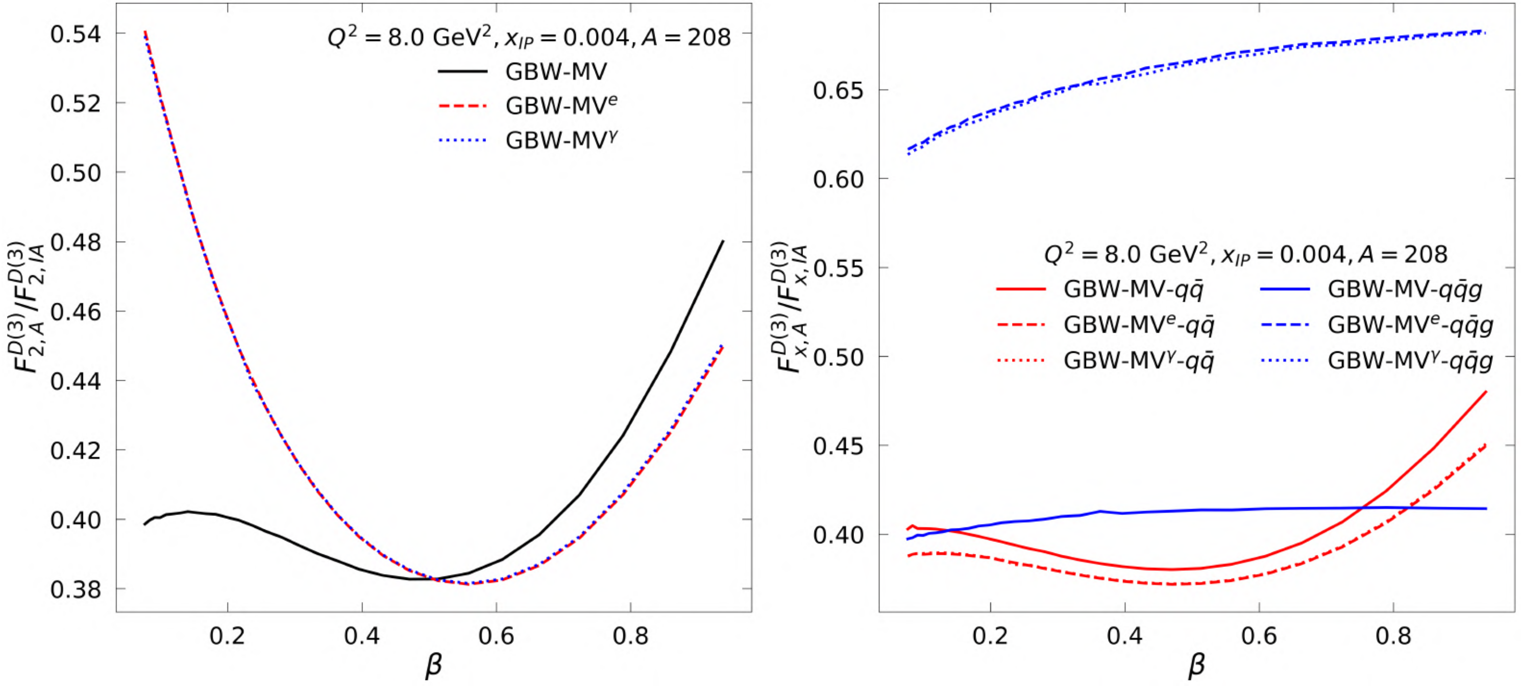
(proton scattering)



Note : Significant differences in x_{IP} and Q^2 dependences

KL vs. GBW (2)

(Nuclear scattering)



Notes :

- Less suppression observed in GBW (reminder : KL suppression ~ 0.15…0.25)

- GBW : β spectra of the nuclear suppression ratio not flat for (qq+qqg), different components have different behaviors

- GBW-qqg : similar qualitative behavior to KL

KL vs. GBW (3)

What make the differences ?

- GBW uses N^2, while it is used as the initial condition for KL evolution
  - KL evolution changes the anomalous dimension in the dilute regime .

$$N_D \sim (r^2 Q_s^2)^{2\gamma_c} \xrightarrow{\text{KL}} (r^2 Q_s^2)^{\gamma_c}$$

  - GBW : small dipoles are suppressed, large dipoles ~ $1/Q_s$ dominate.
  - KL : dipole sizes ~ 1/Q more important
- In KL, one should use the « diffractive » (modified) saturation scale, which depends on x, and slightly depends on x_{IP} (due to a small delay of BK front at Y_0).

e.g., nuclear suppression :

- KL : (large Q^2)

$$\frac{\sigma_D^{\gamma^* A}}{\sigma_{IA,D}^{\gamma^* A}} \sim \frac{\int d^2 b (Q_{s,A}^2/Q^2)^{\gamma_c}}{(\sigma_0/2)^2 A^{4/3} (Q_{s,p}^2/Q^2)^{\gamma_c}} \sim A^{-1/3-\delta(\gamma_c)} \left(\frac{\sigma_0}{2}\right)^{\gamma_c - 2}$$

$$(\delta(\gamma_c) \approx 0.11 \quad if \quad \gamma_c = 0.85)$$

- GBW-qq: (large Q^2)

$$\frac{\sigma_D^{\gamma^* A}}{\sigma_{IA,D}^{\gamma^* A}} \sim \frac{\int d^2 b (Q_{s,A}^2/Q^2)}{(\sigma_0/2)^2 A^{4/3} (Q_{s,p}^2/Q^2)} \sim A^{-1/3} \left(\frac{\sigma_0}{2}\right)^{-1}$$

→ smaller suppression wrt to KL

Final remarks

- Analytical asymptotic solutions to the LO KL for dipoles of small (but not tiny) size

- Reasonable description to the diffractive HERA data at $\beta \lesssim 0.1$
  - Steeper-than-gaussian profiles are favored

- Huge nuclear effect compared to the GBW approach

- Some signatures suggest the applicability of KL for inclusive diffraction at $\beta \lesssim 0.1$ at the future colliders (EIC/LHeC)
  - Some probes for KL : Q^2 dependency, nuclear suppression

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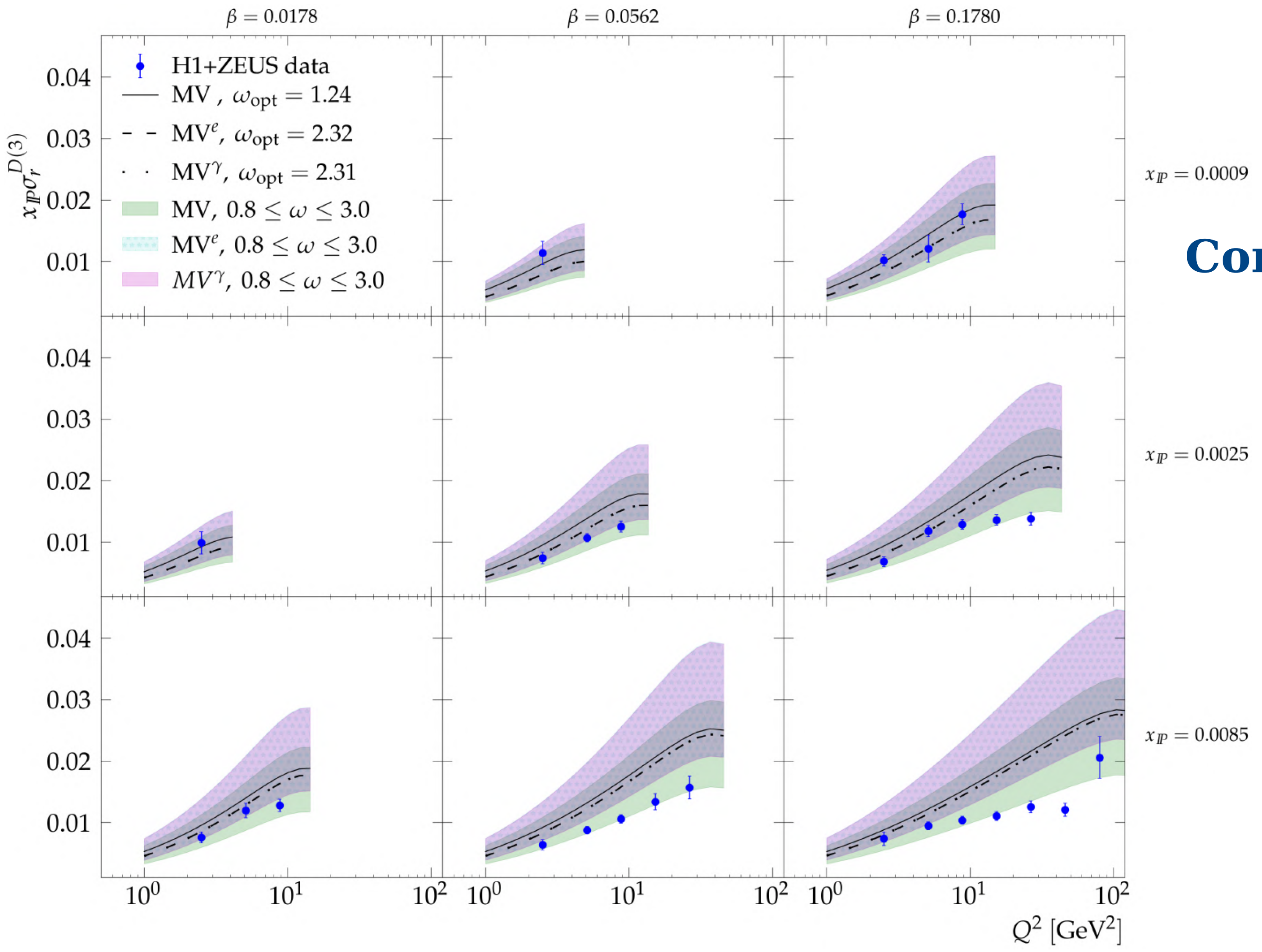
Thank You

BACK-UP

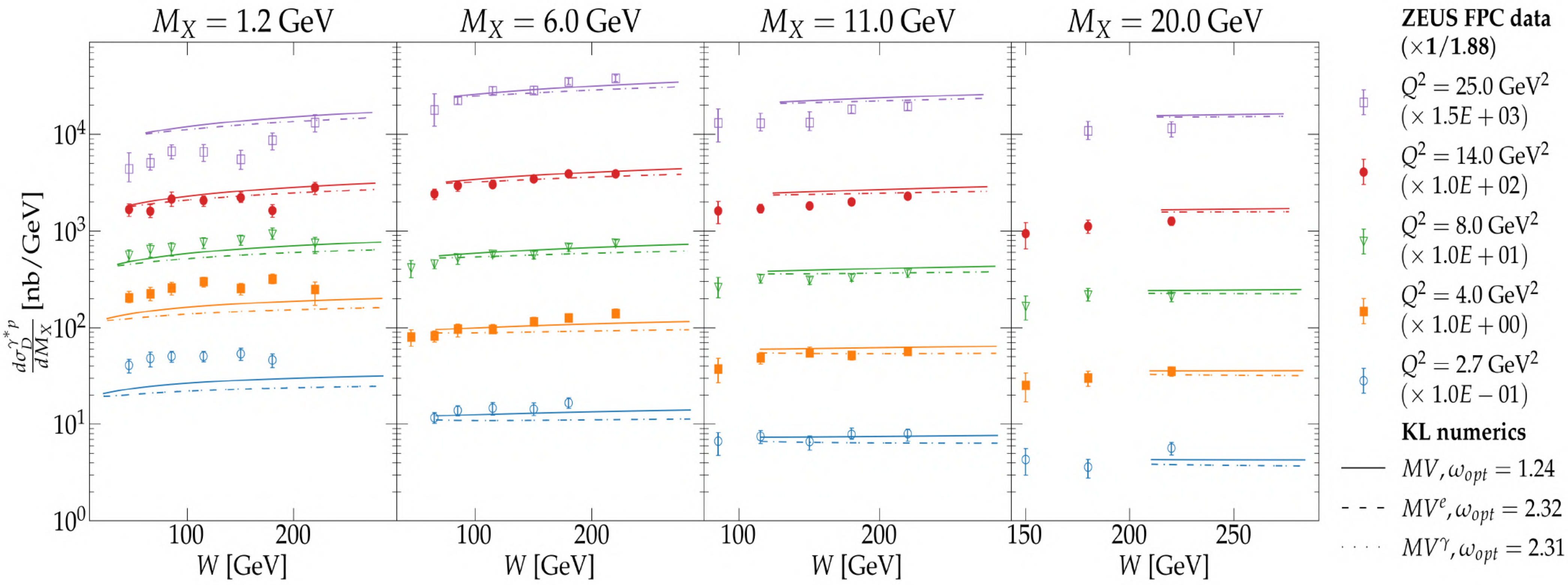


Comparison to HERA combined data

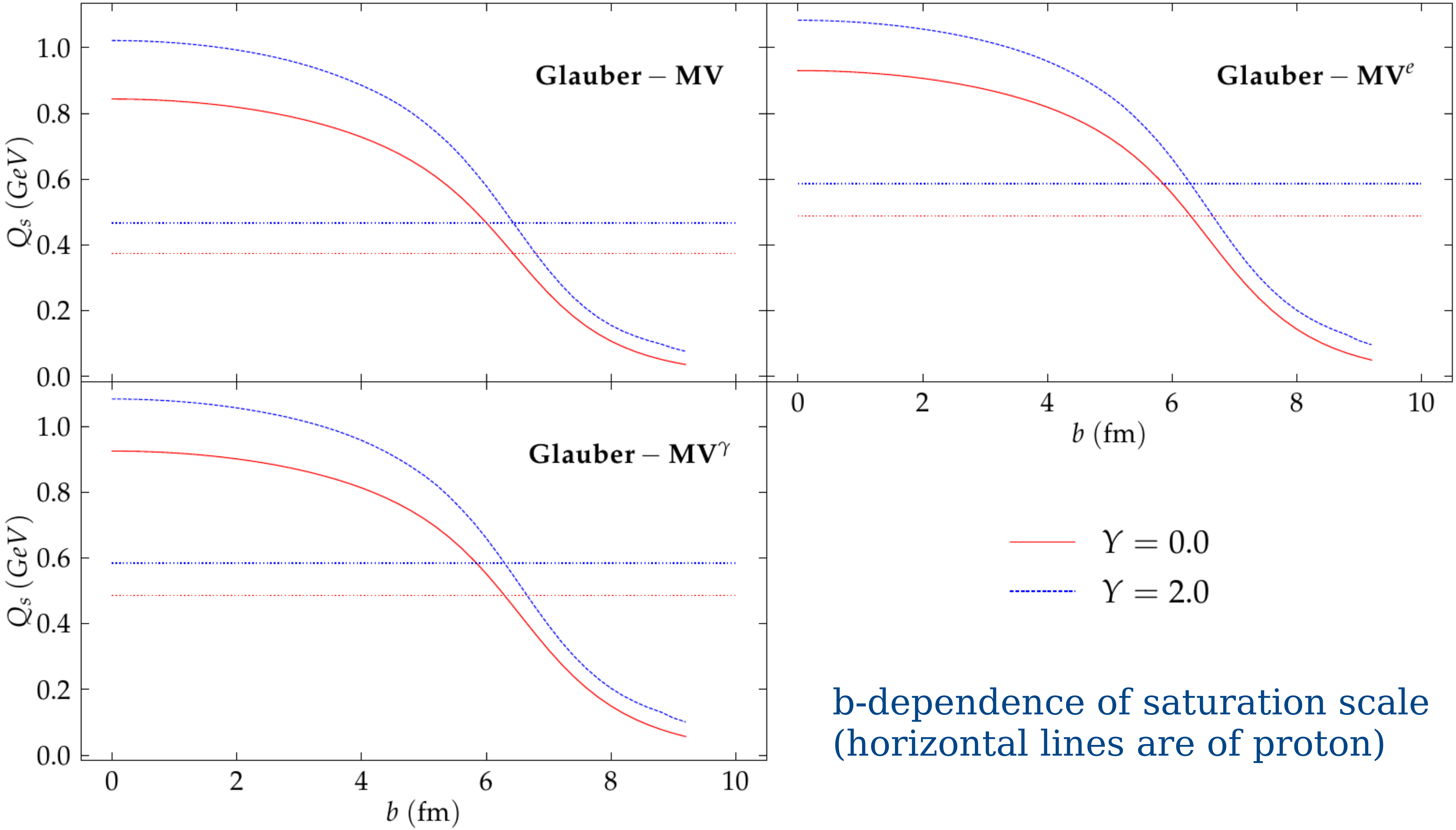
(Q2 dependence)

Comparison to ZEUS FPC data (cont)

$$\frac{d\sigma_D^{\gamma^* p}}{dM_x}$$



FPC data from
ZEUS collaboration, NPB 713, 3 (2005) & NPB 800, 1 (2008)



b-dependence of saturation scale
(horizontal lines are of proton)

$$\widetilde{T}^2(|\boldsymbol{t}|) = \left| \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}}\, T(b) \right|^2$$