Diffractive structure functions at NLO in the dipole picture

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Color Glass Condensate at the electron-ion collider

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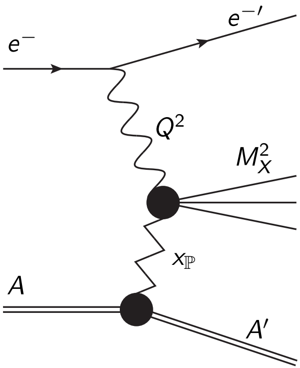
Diffractive structure functions

- Definition of diffraction
  - Experimental: A rapidity gap in the final state
  - Theoretical: No color exchange
- HERA: Almost 10% of DIS events diffractive
- Sensitive to gluon structure at high energy: $\sigma^D \sim [xg(x)]^2$
- Inclusive diffraction: final states with a definite invariant mass M_X^2 and momentum transfer $t = -\mathbf{\Delta}^2$



Diffractive structure functions

$$x_\mathbb{P} F_\lambda^{D(4)}(x_\mathbb{P}, Q^2, M_X^2, t) = \frac{Q^2}{(2\pi)^2 \alpha_{\mathrm{em}}} \frac{Q^2}{\beta} \frac{\mathrm{d}\sigma_\lambda^D}{\mathrm{d}|t|\,\mathrm{d}M_X^2}$$

$x_\mathbb{P} \approx \dfrac{M_X^2 + Q^2}{W^2 + Q^2}$, $\beta \approx \dfrac{Q^2}{Q^2 + M_X^2}$, $Q^2 =$ photon virtuality, $\lambda =$ photon polarization (L or T)

Comparison to similar processes

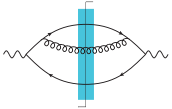
- Lots of progress in recent years for calculating $\gamma^* + A$ processes at NLO
  - Inclusive DIS — talk by Risto Paatelainen

    Beuf: 1606.00777, 1708.06557; Hänninen et al: 1711.08207; Beuf et al: 2103.14549, 2112.03158, 2204.02486
  - Exclusive vector meson production

    Boussarie et al: 1612.08026; Mäntysaari and JP: 2104.02349.2203.16911, 2204.14031
  - Dijet production — talk by Pieter Taels

    Boussarie et al: 1405.7676, 1606.00419; Caucal et al: 2108.06347, 2208.13872, 2304.03304; Taels et al: 2204.11650
  - Dihadron production — talk by Jamal Jalilian-Marian

    Bergabo and Jalilian-Marian: 2207.03606, 2301.03117; Fucilla et al: 2211.05774
- The main difference: the final state
- Inclusive diffraction: The final state is fully perturbative
  - The only nonperturbative part is the interaction with the target



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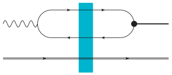
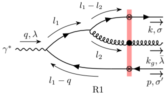
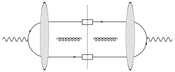




2108.06347



2211.05774

Inclusive diffraction in the high-energy limit

High-energy limit leads to factorization:

Inclusive diffraction cross section at LO

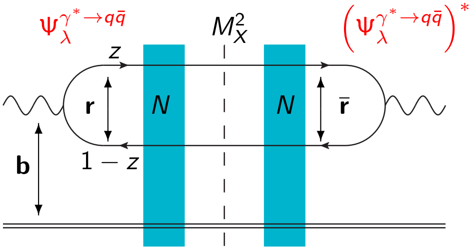
$$\frac{\mathrm{d}\sigma_\lambda^{\mathrm{D}}}{\mathrm{d}M_X^2} = \frac{N_C}{(4\pi)^2} \int \mathrm{d}z\, \mathrm{d}^2\mathbf{r}\, \mathrm{d}^2\bar{\mathbf{r}}\, \mathrm{d}^2\mathbf{b}\, J_0\left(M_X |\mathbf{r} - \bar{\mathbf{r}}| \sqrt{z(1-z)}\right) N(\mathbf{r}, \mathbf{b}) N(\bar{\mathbf{r}}, \mathbf{b}) \Psi_\lambda^{\gamma^* \to q\bar{q}}(\mathbf{r}, z) \left(\Psi_\lambda^{\gamma^* \to q\bar{q}}(\bar{\mathbf{r}}, z)\right)^*$$

- $\Psi_\lambda^{\gamma^* \to q\bar{q}}$: Photon wave function for the $q\bar{q}$ state

  Calculable perturbatively
- N: Dipole-target scattering amplitude

  Energy dependence by the JIMWLK equation
- Eikonal interaction with target:

  Convenient to work in the mixed space $(\mathbf{r}, z)$

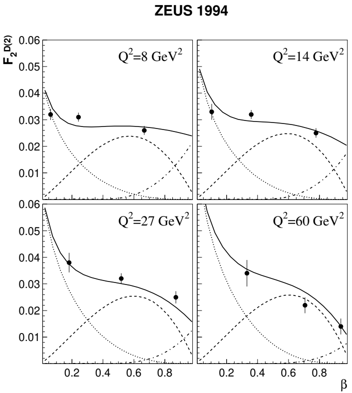


The need for an NLO calculation of DDIS

- LO calculation not enough to describe the data small $\beta \ll 1$
- Gluons start to dominate – appear only at higher orders
- Have been calculated at various different limits:

  Bartels:1999, Kovchegov:1999, Kopeliovich:1999, Kovchegov:2001, Munier:2003, Golec-Biernat:2005, Wusthoff:1997, GolecBiernat:1999, GolecBiernat:2001
- The full NLO calculation in general kinematics still missing
  - Contribution from initial-state gluon emission calculated in

    Beuf et al., 2206.13161
- How important are the loop corrections?

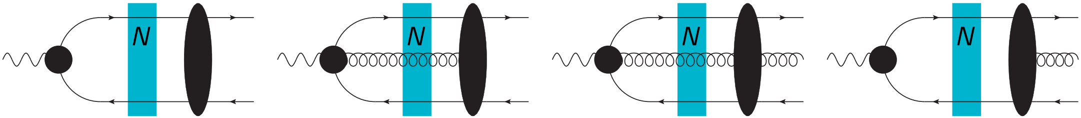


Golec-Biernat, Wusthoff, hep-ph/9903358

Dashed and dot-dashed: LO

Dotted: $q\bar{q}g$ at large Q^2

Next-to-leading order: different contributions



$\mathcal{M}_{(a)}$ $\mathcal{M}_{(b)}$ $\mathcal{M}_{(c)}$ $\mathcal{M}_{(d)}$

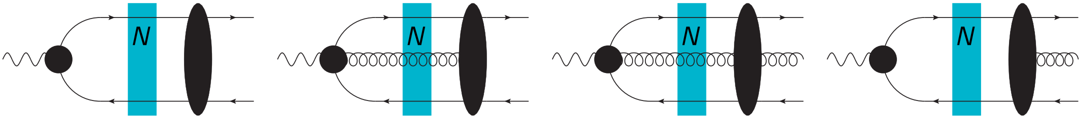
Inclusive diffraction cross section at NLO

$$i\mathcal{M}_n = \sum_m \int \mathrm{d}[\mathrm{PS}]_m \, 2q^+ (2\pi)\delta(q^+ - p_n^+) e^{-i\mathbf{b}\cdot\mathbf{\Delta}} \Psi_{\mathrm{in}}^{\gamma^* \to m} (\Psi_{\mathrm{out}}^{n\to m})^* N^m$$

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma_\lambda^* + A}}{\mathrm{d}^2\mathbf{\Delta}\,\mathrm{d}M_X^2} = \sum_{\text{color-singlet } n} \int \mathrm{d}[\mathrm{PS}]_n \, 2q^+ (2\pi)\delta(q^+ - p_n^+)\delta(M_X^2 - M_n^2)\delta^2(\mathbf{\Delta} - (\mathbf{p}_n - \mathbf{q}))|\mathcal{M}_n|^2$$

where at NLO we need $\mathcal{M}_{q\bar{q}} = \mathcal{M}_{(a)} + \mathcal{M}_{(b)}$ and $\mathcal{M}_{q\bar{q}g} = \mathcal{M}_{(c)} + \mathcal{M}_{(d)}$.

Next-to-leading order: different contributions



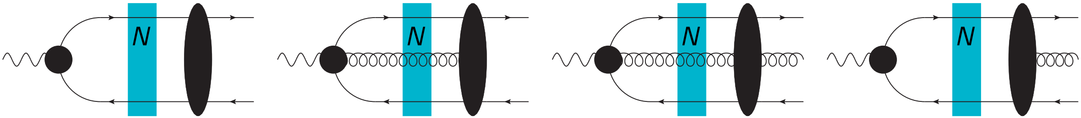
$\mathcal{M}_{(a)}$ $\mathcal{M}_{(b)}$ $\mathcal{M}_{(c)}$ $\mathcal{M}_{(d)}$

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip-trip}}$$

Divide the cross section into three parts based on the Wilson line structure of $\mathcal{M} \times \mathcal{M}^*$:

- dip: dipole $\times$ dipole*
- trip: tripole $\times$ tripole*
- dip-trip: dipole $\times$ tripole* $+$ tripole $\times$ dipole*

Next-to-leading order: different contributions



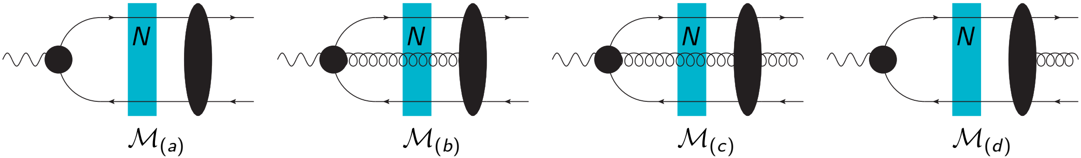
$\mathcal{M}_{(a)}$ $\mathcal{M}_{(b)}$ $\mathcal{M}_{(c)}$ $\mathcal{M}_{(d)}$

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip\text{-}trip}}$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip}} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{|(a)|^2} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{|(d)|^2} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{UV}}$$

- $|(a)|^2$ and $|(d)|^2$: Contain UV divergences that are made finite by the UV subtraction term

Next-to-leading order: different contributions

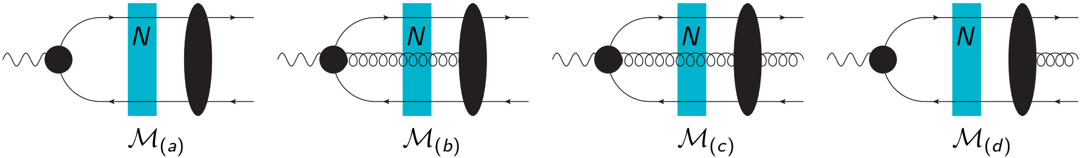


$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip-trip}}$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{trip}} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{|(c)|^2}$$

- $|(c)|^2$: Finite
- This contribution has already been calculated in Beuf et al., 2206.13161

Next-to-leading order: different contributions



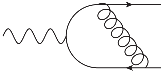
$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip-trip}}$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{dip-trip}} = 2\,\mathrm{Re}\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{(b)\times(a)^*} + 2\,\mathrm{Re}\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{(c)\times(d)^*} - \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\text{UV}}$$

- $(b) \times (a)^*$: Contains a UV divergence that is made finite by the UV subtraction term
- JIMWLK evolution arises from the combination $(b) \times (a)^* - \mathrm{UV}$
- $(c) \times (d)^*$: Finite

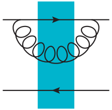
Divergences at NLO

Regularization scheme: dim. reg. for transverse coordinates, cutoff α for plus momentum k^+

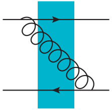


1. Corrections to the initial state: UV ε and $\log \alpha$ divergences

   Included in the photon wave function $\Psi_\lambda^{\gamma^* \to q\bar{q}}$

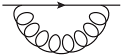


2. UV divergences from gluon loops over the shock wave



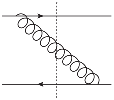
3. Rapidity divergence for gluons with small plus momentum over the shock wave:

   $\log \alpha$ divergence regularized by JIMWLK (BK) equation



4. Self-energy diagrams: IR and UV divergences

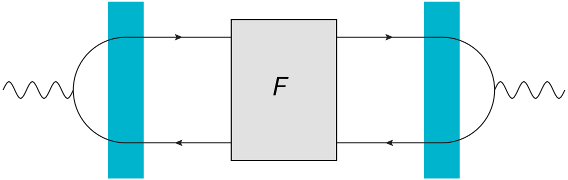
   Cancel in dimensional regularization with one ε



5. Corrections to the final state: IR and $\log \alpha$ divergences

   Complicated!

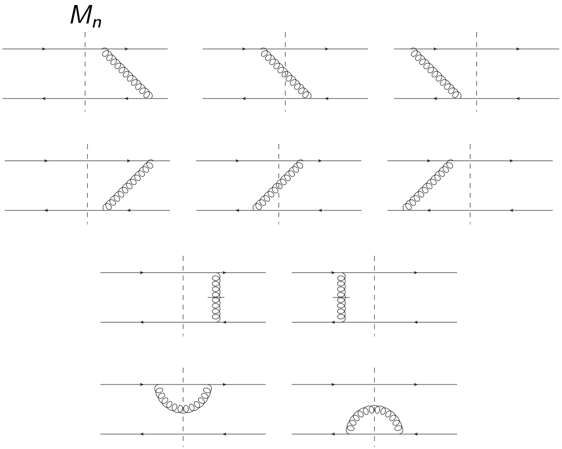
Final-state corrections



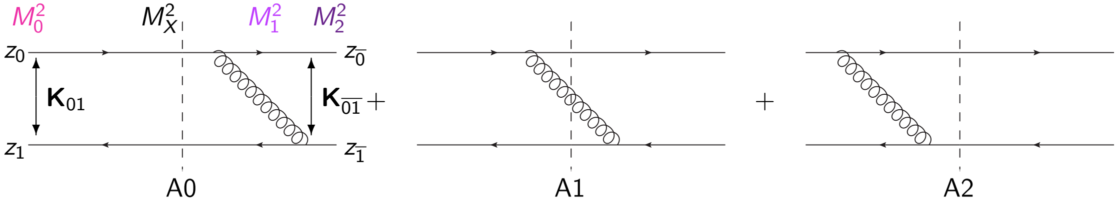
F = sum of final-state corrections

- Lots of pesky diagrams to calculate
- Cut introduces a delta function $\delta(M_X^2 - M_n^2)$

  – Hard to integrate!
- Strategy:
  1. Sum the diagrams in momentum space
  2. Fourier transform to mixed space



Final state: Regular gluon exchange

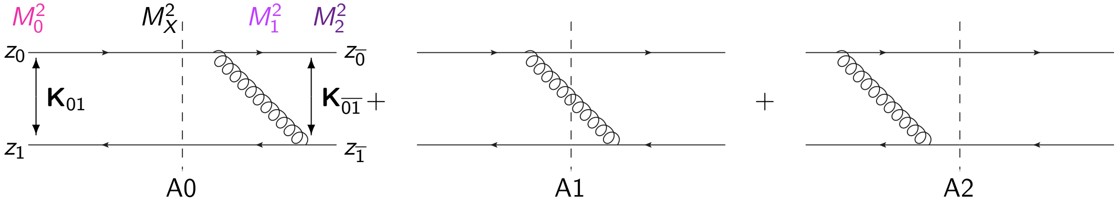


$$F_{A0} + F_{A1} + F_{A2} \propto \int \frac{\mathrm{d}^2\mathbf{K}_{01}\,\mathrm{d}^2\mathbf{K}_{\overline{01}}}{(2\pi)^4} e^{i\mathbf{K}_{\overline{01}}\cdot\mathbf{x}_{\overline{01}} - i\mathbf{K}_{01}\cdot\mathbf{x}_{01}} \frac{1}{z_g^3} \times (z_{\bar{0}}\mathbf{K}_{01} - z_0\mathbf{K}_{\overline{01}}) \cdot (z_{\bar{1}}\mathbf{K}_{01} - z_1\mathbf{K}_{\overline{01}})$$

$$\times \left\{ \frac{\delta(M_0^2 - M_X^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)} + \frac{\delta(M_1^2 - M_X^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta(M_2^2 - M_X^2)}{(M_2^2 - M_0^2 + i\delta)(M_2^2 - M_1^2 + i\delta)} \right\}$$

- Divergences from gluon's plus-momentum fraction $z_g = z_0 - z_{\bar{0}}$ going to zero
- This expression can be simplified by rewriting:
  1. The sum of the energy denominators
  2. The numerator

Final state: Regular gluon exchange



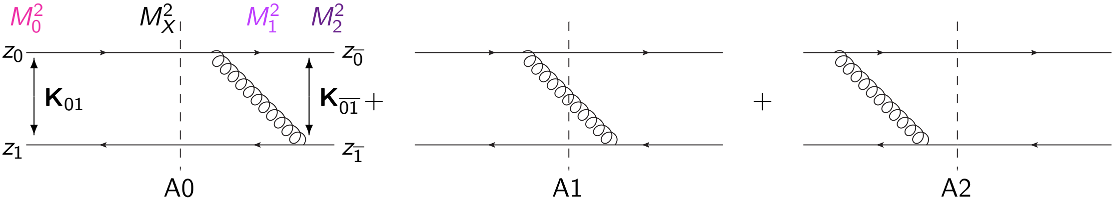
Sum of the energy denominators:

$$\frac{\delta(M_0^2 - M_X^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)} + \frac{\delta(M_1^2 - M_X^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta(M_2^2 - M_X^2)}{(M_2^2 - M_0^2 + i\delta)(M_2^2 - M_1^2 + i\delta)}$$

$$= \frac{1}{2\pi i}\left[\frac{1}{(M_X^2 - M_0^2 - i\delta)(M_X^2 - M_1^2 - i\delta)(M_X^2 - M_2^2 - i\delta)} - \frac{1}{(M_X^2 - M_0^2 + i\delta)(M_X^2 - M_1^2 + i\delta)(M_X^2 - M_2^2 + i\delta)}\right]$$

- This combines divergences from different graphs
- Note: the signs of the infinitesimals $i\delta$ important!

Final state: Regular gluon exchange

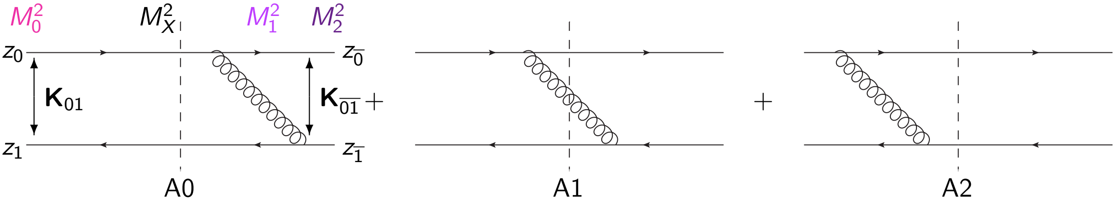


Rewrite the numerator as:

$$
\begin{aligned}
&(z_{\overline{0}}\mathbf{K}_{01} - z_0\mathbf{K}_{\overline{01}}) \cdot (z_{\overline{1}}\mathbf{K}_{01} - z_1\mathbf{K}_{\overline{01}}) \\
&= \frac{1}{2} z_g \left[z_0 z_{\overline{1}}(M_X^2 - M_0^2) + z_0 z_{\overline{1}}(M_X^2 - M_2^2) - (z_{\overline{0}} z_1 + z_0 z_{\overline{1}})(M_X^2 - M_1^2) - z_g M_X^2 \right]
\end{aligned}
$$

- Written in terms of the energy denominators

Final state: Regular gluon exchange

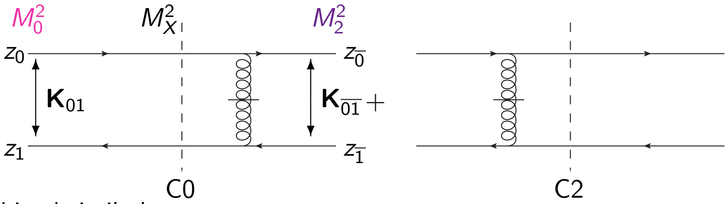


$$F_{A0} + F_{A1} + F_{A2} \propto \int \frac{\mathrm{d}^2\mathbf{K}_{01}\,\mathrm{d}^2\mathbf{K}_{\overline{01}}}{(2\pi)^4} e^{i\mathbf{K}_{\overline{01}}\cdot\mathbf{x}_{\overline{01}} - i\mathbf{K}_{01}\cdot\mathbf{x}_{01}} \frac{1}{z_g^2} \times \left[z_0 z_{\bar{1}} D_{01} + z_0 z_{\bar{1}} D_{12} - (z_{\bar{0}} z_1 + z_0 z_{\bar{1}}) D_{02} - z_g M_X^2 D_{012} \right]$$

$$D_{ij} = \frac{1}{2\pi i} \left[\frac{1}{(M_X^2 - M_i^2 - i\delta)(M_X^2 - M_j^2 - i\delta)} - \frac{1}{(M_X^2 - M_i^2 + i\delta)(M_X^2 - M_j^2 + i\delta)} \right]$$

- $D_{01}, D_{12} \sim z_g \log z_g$: a logarithmic divergence $\log^2 \alpha$
- D_{02}: a power divergence $1/\alpha$
- $D_{012} \sim z_g \log z_g$: no divergences

Final state: Instantaneous gluon exchange



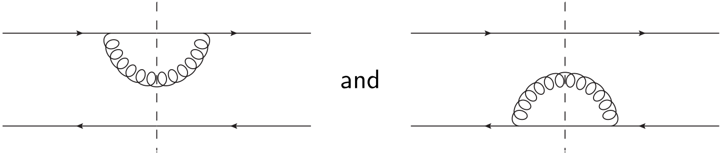
These can be combined similarly:

$$F_{C0} + F_{C2} \propto \int \frac{\mathrm{d}^2\mathbf{K}_{01}\,\mathrm{d}^2\mathbf{K}_{\overline{01}}}{(2\pi)^4} e^{i\mathbf{K}_{\overline{01}}\cdot\mathbf{x}_{\overline{01}} - i\mathbf{K}_{01}\cdot\mathbf{x}_{01}} \frac{1}{z_g^2} \times \left\{ \frac{\delta(M_X^2 - M_2^2)}{M_0^2 - M_2^2 - i\delta} + \frac{\delta(M_X^2 - M_0^2)}{M_2^2 - M_0^2 + i\delta} \right\}$$

$$\frac{\delta(M_X^2 - M_2^2)}{M_0^2 - M_2^2 - i\delta} + \frac{\delta(M_X^2 - M_0^2)}{M_2^2 - M_0^2 + i\delta} = \frac{1}{2\pi i}\left[\frac{1}{(M_X^2 - M_0^2 + i\delta)(M_X^2 - M_2^2 + i\delta)} - \frac{1}{(M_X^2 - M_0^2 - i\delta)(M_X^2 - M_2^2 - i\delta)}\right] = -D_{02}$$

- Combined with the previous diagrams, these cancel the power divergence $1/\alpha$
- Some $\log^2 \alpha$ divergences still left

Final state: Cut gluon loops

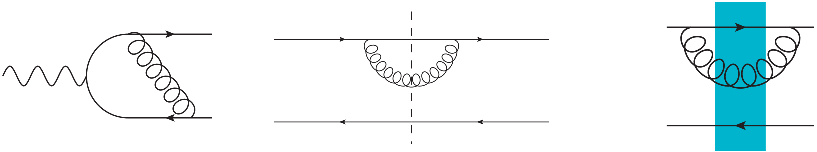


- Can be calculated analytically without additional Feynman/Schwinger integrals
- Contain IR (collinear) and $\log^2 \alpha$ divergences
- Self-energy diagrams: IR $\rightarrow$ UV divergences
- $\log^2 \alpha$ divergence cancels with the other final-state diagrams
- In total: final-state corrections have UV ε and $\log \alpha$ divergences
- **The same divergence structure as the NLO wave function for $\gamma^* \rightarrow q\bar{q}$**

Cancellation of divergences

- UV divergences:

$$\text{NLO } \gamma^* \to q\bar{q} \; + \; \text{final-state corrections} \; + \; \text{gluon loops crossing the shock wave} \; = \; \text{finite}$$



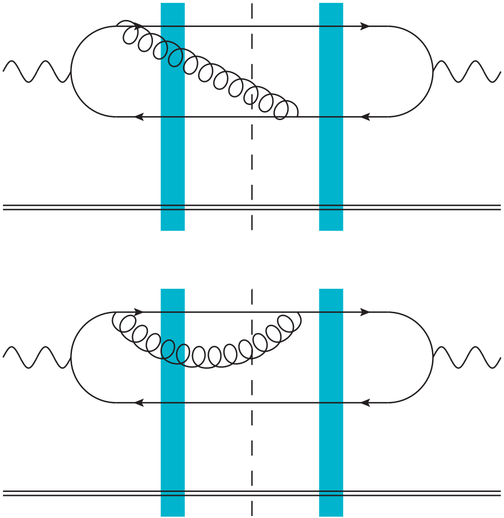
- Remaining log α divergence:

  Absorbed into the JIMWLK evolution of the Wilson lines

  $\Rightarrow$ All of the divergences cancelled: finite result!

Remaining finite pieces

- Cross-terms with gluon emission from initial and final states
- Finite:
  - Cut regulates UV region
  - No IR divergences due to the energy denominator structure:
    gluon emission in the initial state from an *off-shell* quark – cannot go on-shell ⇒ no divergence
- Related to the diagrams where the gluon is absorbed before the final state
  – Simplifications in their sum



Final result

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2} = \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{trip}} + \left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip\text{-}trip}}$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip}} = 2\pi\alpha_{\mathrm{em}}N_c\sum_f e_f^2\int[\mathrm{dPS}]_{\mathrm{dip}}\left[\mathcal{G}^{\mathrm{LO}}_{\lambda,\mathrm{dip}} + \left(\frac{\alpha_s C_F}{2\pi}\right)\mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{dip}}\right]\left(1-\hat{S}^{(2)}_{01}\right)\left(1-\hat{S}^{(2)}_{\overline{01}}\right)^\dagger$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{trip}} = 2\pi\alpha_{\mathrm{em}}N_c\sum_f e_f^2\int[\mathrm{dPS}]_{\mathrm{trip}}\left(\frac{\alpha_s C_F}{2\pi}\right)\mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{trip}}\left(1-\hat{S}^{(3)}_{012}\right)\left(1-\hat{S}^{(3)}_{\overline{012}}\right)^\dagger$$

$$\left[\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\gamma^*_\lambda+A}}{\mathrm{d}^2\boldsymbol{\Delta}\,\mathrm{d}M_X^2}\right]_{\mathrm{dip\text{-}trip}} = 2\pi\alpha_{\mathrm{em}}N_c\sum_f e_f^2\int[\mathrm{dPS}]_{\mathrm{dip\text{-}trip}}\frac{\alpha_s C_F}{2\pi}$$
$$\times\left\{\mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{dip\text{-}trip}}2\,\mathrm{Re}\left(1-\hat{S}^{(3)}_{012}\right)\left(1-\hat{S}^{(2)}_{\overline{01}}\right)^\dagger - \mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{dip\ sub}}2\,\mathrm{Re}\left(1-\hat{S}^{(2)}_{01}\right)\left(1-\hat{S}^{(2)}_{\overline{01}}\right)^\dagger\right\}$$

$$\mathcal{G}^{\mathrm{LO}}_{\lambda,\mathrm{dip}} = \mathcal{F}_\lambda(z_0,z_1,z_0,z_1)J_0(|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\overline{M}_X)$$

$$\mathcal{F}_\lambda(z_0,z_1,z_{\overline{0}},z_{\overline{1}}) = \begin{cases} 4[z_0z_1z_{\overline{0}}z_{\overline{1}}]^{3/2}Q^2K_0(|\mathbf{x}_{01}|Q\sqrt{z_0z_1})K_0(|\mathbf{x}_{\overline{01}}|Q\sqrt{z_{\overline{0}}z_{\overline{1}}}) & \lambda = L \\ z_0z_1z_{\overline{0}}z_{\overline{1}}(z_0z_{\overline{0}}+z_1z_{\overline{1}})\frac{\mathbf{x}_{01}\cdot\mathbf{x}_{\overline{01}}}{|\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|}Q^2K_1(|\mathbf{x}_{01}|Q\sqrt{z_0z_1})K_1(|\mathbf{x}_{\overline{01}}|Q\sqrt{z_{\overline{0}}z_{\overline{1}}}) & \lambda = T \end{cases}$$

$$\mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{dip}} = f^{\mathrm{UV}}\mathcal{G}^{\mathrm{LO}}_{\lambda,\mathrm{dip}} + \int_0^1\mathrm{d}z_{\overline{0}}\,\mathrm{d}z_{\overline{1}}\,\delta(1-z_{\overline{0}}-z_{\overline{1}})f^{\mathrm{final\ state}}\mathcal{F}_\lambda(z_0,z_1,z_{\overline{0}},z_{\overline{1}})$$

$$f^{\mathrm{UV}} = (2\log(z_0z_1)-3)$$
$$\times\left[\log\left(\frac{M_Xe^{\gamma_E}|\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|}{2|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|}\right)J_0\left(\sqrt{z_0z_1}M_X|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\right)+\frac{\pi}{2}Y_0\left(\sqrt{z_0z_1}M_X|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\right)\right]$$
$$+\left[11-\pi^2+\log^2 z_0+\log^2 z_1-4\log z_0\log z_1\right]J_0\left(\sqrt{z_0z_1}M_X|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\right)$$

$$f^{\mathrm{final\ state}} = F_t + F_{\mathrm{sub}} + F^{02} + F^{012}_{\mathrm{A}} + F^{012}_{\mathrm{B}}$$

$$F_{\mathrm{non\text{-}soft}} = \int_0^1\frac{\mathrm{d}t}{t}\frac{1}{|z_0-z_{\overline{0}}|}\frac{z_0z_1+z_{\overline{0}}z_{\overline{1}}}{\sqrt{z_0z_1z_{\overline{0}}z_{\overline{1}}}}$$
$$\times\left\{\theta(z_0-z_{\overline{0}})f_{\mathrm{non\text{-}soft}}\left(t, M_X\sqrt{\frac{z_1}{z_0}}|z_{\overline{0}}\mathbf{x}_{\overline{01}}-z_0\mathbf{x}_{01}|, M_X|\mathbf{x}_{\overline{01}}|\sqrt{(z_0-z_{\overline{0}})\frac{z_{\overline{0}}}{z_0}}\right)\right.$$
$$+\theta(z_0-z_{\overline{0}})f_{\mathrm{non\text{-}soft}}\left(t, M_X\sqrt{\frac{z_{\overline{0}}}{z_{\overline{1}}}}|z_{\overline{1}}\mathbf{x}_{\overline{01}}-z_1\mathbf{x}_{01}|, M_X|\mathbf{x}_{01}|\sqrt{(z_0-z_{\overline{0}})\frac{z_1}{z_{\overline{1}}}}\right)$$
$$+\theta(z_{\overline{0}}-z_0)f_{\mathrm{non\text{-}soft}}\left(t, M_X\sqrt{\frac{z_0}{z_1}}|z_{\overline{1}}\mathbf{x}_{\overline{01}}-z_1\mathbf{x}_{01}|, M_X|\mathbf{x}_{\overline{01}}|\sqrt{(z_{\overline{0}}-z_0)\frac{z_{\overline{1}}}{z_1}}\right)$$
$$\left.+\theta(z_{\overline{0}}-z_0)f_{\mathrm{non\text{-}soft}}\left(t, M_X\sqrt{\frac{z_{\overline{1}}}{z_{\overline{0}}}}|z_{\overline{0}}\mathbf{x}_{\overline{01}}-z_0\mathbf{x}_{01}|, M_X|\mathbf{x}_{01}|\sqrt{(z_{\overline{0}}-z_0)\frac{z_0}{z_{\overline{0}}}}\right)\right\}$$

$$F_{\mathrm{sub}} = \int_0^1\frac{\mathrm{d}z_2'}{z_2'}\frac{1}{2}\frac{z_0z_1+z_{\overline{0}}z_{\overline{1}}}{\sqrt{z_0z_1z_{\overline{0}}z_{\overline{1}}}}$$
$$\times\left\{4\delta(z_0-z_{\overline{0}})f_{\mathrm{sub}}\left(M_X\sqrt{z_0z_1}|\mathbf{x}_{\overline{01}}-\mathbf{x}_{01}|, M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_2'}\right)\right.$$
$$-\delta(z_2'-(z_0-z_{\overline{0}}))f_{\mathrm{sub}}\left(M_X\sqrt{\frac{z_1}{z_0}}|z_{\overline{0}}\mathbf{x}_{\overline{01}}-z_0\mathbf{x}_{01}|, M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_2'\frac{z_{\overline{0}}}{z_0}}\right)$$
$$-\delta(z_2'-(z_0-z_{\overline{0}}))f_{\mathrm{sub}}\left(M_X\sqrt{\frac{z_{\overline{0}}}{z_{\overline{1}}}}|z_{\overline{1}}\mathbf{x}_{\overline{01}}-z_1\mathbf{x}_{01}|, M_X|\mathbf{x}_{01}|\sqrt{z_2'\frac{z_1}{z_{\overline{1}}}}\right)$$
$$-\delta(z_2'-(z_{\overline{0}}-z_0))f_{\mathrm{sub}}\left(M_X\sqrt{\frac{z_0}{z_1}}|z_{\overline{1}}\mathbf{x}_{\overline{01}}-z_1\mathbf{x}_{01}|, M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_2'\frac{z_{\overline{1}}}{z_1}}\right)$$
$$\left.-\delta(z_2'-(z_{\overline{0}}-z_0))f_{\mathrm{sub}}\left(M_X\sqrt{\frac{z_{\overline{1}}}{z_{\overline{0}}}}|z_{\overline{0}}\mathbf{x}_{\overline{01}}-z_0\mathbf{x}_{01}|, M_X|\mathbf{x}_{01}|\sqrt{z_2'\frac{z_0}{z_{\overline{0}}}}\right)\right\}$$

$$f_{\mathrm{non\text{-}soft}}(t,a,b) = [1-J_0(tb)]\,J_0\left(a\sqrt{1-t^2}\right),\ \text{and}$$
$$f_{\mathrm{sub}}(a,b) = J_0(a)\log\left(\frac{b^2e^{\gamma_E}}{2a}\right)+\frac{\pi}{2}Y_0(a)$$

$$F^{02} = \pi\frac{z_0z_1+z_{\overline{0}}z_{\overline{1}}}{\sqrt{z_0z_1z_{\overline{0}}z_{\overline{1}}}}$$
$$\times\left[J_0\left(M_X|\mathbf{x}_{01}|\sqrt{z_0z_1}\right)Y_0\left(M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_{\overline{0}}z_{\overline{1}}}\right)+Y_0\left(M_X|\mathbf{x}_{01}|\sqrt{z_0z_1}\right)J_0\left(M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_{\overline{0}}z_{\overline{1}}}\right)\right]$$

$$F^{012}_{\mathrm{A}} = -\frac{1}{2}\int_0^1\mathrm{d}z_2'\,\delta\left(z_2'-(z_0-z_{\overline{0}})\right)\int_0^1\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}v\,\delta(1-t-u-v)$$
$$\times\frac{z_0z_1+z_{\overline{0}}z_{\overline{1}}}{[z_0z_{\overline{1}}v+z_2'tu]^{3/2}}\sqrt{v|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|^2+z_2'\frac{\mathbf{x}_{01}^2}{z_{\overline{0}}z_{\overline{1}}}(u+z_{\overline{1}}v)+z_2'\frac{\mathbf{x}_{\overline{01}}^2}{z_0z_1}(t+z_0v)}$$
$$\times J_1\left(M_X\sqrt{\frac{z_0z_1z_{\overline{0}}z_{\overline{1}}}{z_0z_{\overline{1}}v+z_2'tu}}\sqrt{v|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|^2+z_2'\frac{\mathbf{x}_{01}^2}{z_{\overline{0}}z_{\overline{1}}}(u+z_{\overline{1}}v)+z_2'\frac{\mathbf{x}_{\overline{01}}^2}{z_0z_1}(t+z_0v)}\right)$$

$$F^{012}_{\mathrm{B}} = -\frac{1}{2}\int_0^1\mathrm{d}z_2'\,\delta\left(z_2'-(z_{\overline{0}}-z_0)\right)\int_0^1\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}v\,\delta(1-t-u-v)$$
$$\times\frac{z_0z_1+z_{\overline{0}}z_{\overline{1}}}{[z_1z_{\overline{0}}v+z_2'tu]^{3/2}}\sqrt{v|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|^2+z_2'\frac{\mathbf{x}_{01}^2}{z_{\overline{0}}z_{\overline{1}}}(u+z_{\overline{0}}v)+z_2'\frac{\mathbf{x}_{\overline{01}}^2}{z_0z_1}(t+z_1v)}$$
$$\times J_1\left(M_X\sqrt{\frac{z_0z_1z_{\overline{0}}z_{\overline{1}}}{z_1z_{\overline{0}}v+z_2'tu}}\sqrt{v|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|^2+z_2'\frac{\mathbf{x}_{01}^2}{z_{\overline{0}}z_{\overline{1}}}(u+z_{\overline{0}}v)+z_2'\frac{\mathbf{x}_{\overline{01}}^2}{z_0z_1}(t+z_1v)}\right)$$

$$\mathcal{G}^{\mathrm{NLO}}_{L,\mathrm{trip}} = 8z_0z_1Q^2K_0(QX_{012})K_0(QX_{\overline{012}})\times\frac{M_X}{Y_{012}}J_1(M_XY_{012})$$
$$\times\left\{z_1^2\left[2z_0(z_0+z_2)+z_2^2\right]\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{\overline{20}}}{\mathbf{x}_{20}^2\mathbf{x}_{\overline{20}}^2}+z_0^2\left[2z_1(z_1+z_2)+z_2^2\right]\frac{\mathbf{x}_{21}\cdot\mathbf{x}_{\overline{21}}}{\mathbf{x}_{21}^2\mathbf{x}_{\overline{21}}^2}\right.$$
$$\left.-z_0z_1\left[z_0(1-z_0)+z_1(1-z_1)\right]\left[\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{\overline{21}}}{\mathbf{x}_{20}^2\mathbf{x}_{\overline{21}}^2}+\frac{\mathbf{x}_{21}\cdot\mathbf{x}_{\overline{20}}}{\mathbf{x}_{21}^2\mathbf{x}_{\overline{20}}^2}\right]\right\}$$

$$\mathcal{G}^{\mathrm{NLO}}_{T,\mathrm{trip}} = 2z_0z_1Q^2K_1(QX_{012})K_1(QX_{\overline{012}})\frac{1}{X_{012}}\frac{1}{X_{\overline{012}}}\times\frac{M_X}{Y_{012}}J_1(M_XY_{012})$$
$$\times\left\{\Upsilon^{|(b)|^2}_{\mathrm{reg.}}+\Upsilon^{|(c)|^2}_{\mathrm{reg.}}+\Upsilon^{(d)}_{\mathrm{inst.}}+\Upsilon^{(e)}_{\mathrm{inst.}}+\Upsilon^{(b)\times(c)}_{\mathrm{interf.}}\right\}$$

$$\mathcal{G}^{\mathrm{NLO}}_{L,\mathrm{dip\text{-}trip}} = \frac{8}{z_2}Q^2K_0(X_{012}Q)$$
$$\times\left[z_1^2(1-z_1)K_0\left(|\mathbf{x}_{\overline{01}}|\overline{Q}_0\right)\right.$$
$$\times\left(z_1\left(2z_0(z_0+z_2)+z_2^2\right)\frac{1}{\mathbf{x}_{20}^2}-z_0\left(z_0(1-z_0)+z_1(1-z_1)\right)\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{\mathbf{x}_{20}^2\mathbf{x}_{21}^2}\right)$$
$$\times J_0\left(\overline{M}_{X0}\sqrt{\left(\mathbf{x}_{0+2;1}-\mathbf{x}_{\overline{01}}\right)^2+\omega_0\mathbf{x}_{20}^2}\right)$$
$$+z_0^2(1-z_0)K_0\left(|\mathbf{x}_{\overline{01}}|\overline{Q}_1\right)$$
$$\times\left(z_0\left(2z_1(z_1+z_2)+z_2^2\right)\frac{1}{\mathbf{x}_{21}^2}-z_1\left(z_0(1-z_0)+z_1(1-z_1)\right)\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{\mathbf{x}_{20}^2\mathbf{x}_{21}^2}\right)$$
$$\left.\times J_0\left(\overline{M}_{X1}\sqrt{\left(\mathbf{x}_{\overline{01}}-\mathbf{x}_{0;1+2}\right)^2+\omega_1\mathbf{x}_{21}^2}\right)\right]$$

$$\mathcal{G}^{\mathrm{NLO}}_{T,\mathrm{dip\text{-}trip}} = \frac{2}{z_2}\frac{Q}{X_{012}|\mathbf{x}_{\overline{01}}|}K_1(X_{012}Q)$$
$$\times\left[z_1\overline{Q}_0K_1\left(|\mathbf{x}_{\overline{01}}|\overline{Q}_0\right)\times J_0\left(\overline{M}_{X0}\sqrt{\left(\mathbf{x}_{0+2;1}-\mathbf{x}_{\overline{01}}\right)^2+\omega_0\mathbf{x}_{20}^2}\right)\right.$$
$$\times\left(\Upsilon^{\mathrm{U1}}+\Upsilon^{\mathrm{U2}}+\Upsilon^{\mathrm{U3}}+\Upsilon^{\mathrm{U4}}\right)$$
$$+z_0\overline{Q}_1K_1\left(|\mathbf{x}_{\overline{01}}|\overline{Q}_1\right)\times J_0\left(\overline{M}_{X1}\sqrt{\left(\mathbf{x}_{\overline{01}}-\mathbf{x}_{0;1+2}\right)^2+\omega_1\mathbf{x}_{21}^2}\right)$$
$$\left.\times\left(\Upsilon^{\mathrm{V1}}+\Upsilon^{\mathrm{V2}}+\Upsilon^{\mathrm{V3}}+\Upsilon^{\mathrm{V4}}\right)\right]$$

$$\mathcal{G}^{\mathrm{NLO}}_{\lambda,\mathrm{dip\ sub}} = \frac{2}{z_2(1-z_1)^2}\left[z_0^2+(1-z_1)^2\right]\frac{1}{\mathbf{x}_{20}^2}\exp\left\{-\frac{\mathbf{x}_{20}^2}{\mathbf{x}_{01}^2e^{\gamma_E}}\right\}$$
$$\times\mathcal{F}_\lambda(1-z_1,z_1,1-z_1,z_1)J_0\left(|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\overline{M}_{X0}\right)$$
$$+\frac{2}{z_2(1-z_0)^2}\left[z_1^2+(1-z_0)^2\right]\frac{1}{\mathbf{x}_{21}^2}\exp\left\{-\frac{\mathbf{x}_{21}^2}{\mathbf{x}_{01}^2e^{\gamma_E}}\right\}$$
$$\times\mathcal{F}_\lambda(z_0,1-z_0,z_0,1-z_0)J_0\left(|\mathbf{x}_{01}-\mathbf{x}_{\overline{01}}|\overline{M}_{X1}\right)$$

Conclusions and future considerations

- Diffractive structure functions are a good probe for saturation effects
- Previously: leading log Q^2 calculated at NLO
- We are completing the full NLO calculation
  - Explicit cancellation of divergences
  - Results suitable for numerical calculations
- Future:
  - Numerical implementation of the full NLO result
  - Comparisons to the existing HERA data and predictions for the EIC