Probing Gluon Bose Correlations in DIS

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Motivation

Can we probe correlations at EIC, or more generally in DIS.

Inclusive DIS - just collinear PDF - total number of partons, very average characteristic of the hadron WF.

SIDIS and such - a little more detailed, but still single particle operators.

Correlations are generic. A while ago we (CGC community) were actively discussing correlations in relation to "Ridge correlations". Although this may not be relevant for $p-A$ collisions after all, the correlations should be there. With EIC approved it is natural to ask, can we find an observable which probes such correlations in DIS.

The rough idea

We want specifically to probe Bose-Einstein correlations in the hadronic wave function.

The observable we came up with is the production of three jets - a dijet in the photon moving direction, and an additional jet separated from them in rapidity.

The basic idea: we need an observable that probes two gluons from the target. Suppose one of these gluons is absorbed by the projectile (γ^*) while the other is produced directly in the final state. Then we should observe correlations between the momentum of produced gluon and the total momentum of the system going in the γ^* direction.

The cartoon

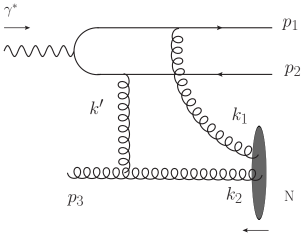


Figure: Schematic diagram showing the trijet production in $\gamma^* N$ collisions. Bose-Einstein correlations in the hadron wave function lead to the increase in the cross-section of the trijet production, when the transverse momenta $\vec{p}_3 \approx \pm(\mathbf{p}_1 + \mathbf{p}_2)$.

Trifles

We choose a diffractive configuration of the dijet (i.e. color singlet) to minimize Sudakov radiation on the γ^* side and in general the cross talk between the remnants of γ^* and the gluon jet on the nucleus side.

In the letter we used the dilute-dilute approximation with an impressionistic use of Q_s as a quick and dirty proxy for the actual calculation, and saw interesting correlation structure reflecting Bose-Einstein correlation.

Here - the full dilute-dense framework including multiple scattering.

We also change the frame - now we treat he gluon jet is actually also emitted from the dipole - it is mathematically equivalent.

The setup

We use the standard dilute-dense formulation:

The observable:

$$\mathcal{O}(p_1^+,\mathbf{p}_1;p_2^+,\mathbf{p}_2;p_3^+,\mathbf{p}_3) = \sum_{\{c\},\{\sigma\}} \langle\psi_F|\hat{d}^\dagger_{c_1,\sigma_1}(p_1)\hat{d}_{c_1,\sigma_1}(p_1)\hat{b}^\dagger_{c_2,\sigma_2}(p_2)\hat{b}_{c_2,\sigma_2}(p_2)\hat{a}_i^{c_3\dagger}(p_3)\hat{a}_i^{c_3}(p_3)|\psi_F\rangle$$

The state:

$$|\psi_F\rangle = \hat{C}_D^\dagger\hat{S}\hat{C}_D|\gamma^*\rangle\otimes|N\rangle$$

$\hat{C}_D$ - the coherent dressing of the dipole.

$$\hat{C}_D = \exp\left\{i\int d^2\mathbf{x}\,\mathcal{B}_i^a(\mathbf{x})\int_{\Lambda^+e^{-\Delta y}}^{\Lambda^+}\frac{dk^+}{\sqrt{2\pi}|k^+|}\left(\hat{a}_i^{a\dagger}(k^+,\mathbf{x})+\hat{a}_i^a(k^+,\mathbf{x})\right)\right\} \qquad (1)$$

$\mathcal{B}_i^a(\mathbf{x})$ is the Weiszacker-Williams (WW) field generated by the dipole.

The S-matrix is eikonal

$$\hat{S} = \exp\left[i\int d^2\mathbf{x}\hat{j}^a(\mathbf{x})\alpha_T^a(\mathbf{x})\right].$$

$\hat{j}^a$ - color charge of the dipole+emitted gluon, α_T - the target field.

The expression

Long story - short:

$$\mathcal{O}_{\rm diff}(p_1^+,\mathbf{p}_1;p_2^+,\mathbf{p}_2;p_3^+,\mathbf{p}_3) =$$

$$\frac{g^2}{N_c}\sum_{\sigma_1,\sigma_2,j}\int d^2\mathbf{z}_1 d^2\mathbf{z}_1' e^{i\mathbf{p}_1\cdot(\mathbf{z}_1-\mathbf{z}_1')}\int d^2\mathbf{z}_2 d^2\mathbf{z}_2' e^{i\mathbf{p}_2\cdot(\mathbf{z}_2-\mathbf{z}_2')}\int d^2\mathbf{z}_3 d^2\mathbf{z}_3' e^{i\mathbf{p}_3\cdot(\mathbf{z}_3-\mathbf{z}_3')}$$

$$\times(8\pi)\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,\mathbf{z}_1;p_2^+,\mathbf{z}_2)[\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,\mathbf{z}_1';p_2^+,\mathbf{z}_2')]^*$$

$$\times\left[\partial_j\phi(\mathbf{z}_3'-\mathbf{z}_2')-\partial_j\phi(\mathbf{z}_3'-\mathbf{z}_1')\right]\left[\partial_j\phi(\mathbf{z}_3-\mathbf{z}_2)-\partial_j\phi(\mathbf{z}_3-\mathbf{z}_1)\right]$$

$$\times\Big\{[\mathcal{U}^\dagger(\mathbf{z}_3)\mathcal{U}(\mathbf{z}_3')]^{ae}\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_2)t^a\mathcal{S}^\dagger(\mathbf{z}_1)\right]\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_1')t^e\mathcal{S}^\dagger(\mathbf{z}_2')\right]$$

$$-\mathcal{U}^{c_3e}(\mathbf{z}_3')\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_2)\mathcal{S}^\dagger(\mathbf{z}_1)t^{c_3}\right]\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_1')t^e\mathcal{S}^\dagger(\mathbf{z}_2')\right]$$

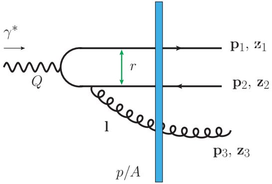
$$-\mathcal{U}^{\dagger ac_3}(\mathbf{z}_3)\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_2)t^a\mathcal{S}^\dagger(\mathbf{z}_1)\right]\mathrm{Tr}\left[t^{c_3}\mathcal{S}(\mathbf{z}_1')\mathcal{S}^\dagger(\mathbf{z}_2')\right]$$

$$+\mathrm{Tr}\left[\mathcal{S}(\mathbf{z}_2)\mathcal{S}^\dagger(\mathbf{z}_1)t^{c_3}\right]\mathrm{Tr}\left[t^{c_3}\mathcal{S}(\mathbf{z}_1')\mathcal{S}^\dagger(\mathbf{z}_2')\right]\Big\}.$$

U, S - adjoint and fundamental Wilson lines.

High momentum limit

Now what? Too tough to calculate in full glory.
Try to simplify for high momenta $|p_1|, |p_2|, |p_3| \gg Q_s$



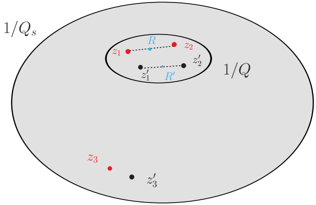


Figure: A typical coordinate space configuration corresponding to high momentum limit for $|\mathbf{p}_1|, |\mathbf{p}_2|, |\mathbf{p}_3| \sim Q \gg Q_s$

Analytics

We can expand in $|z_1 - z_2|$, $z_1 - z_1'|$ etc., but not $|z_1 - z_3|$ and such. In the end things simplify a bit, and we are faced with:

$$\begin{aligned}
&\left\langle A_a^i(\mathbf{R}) A_b^{i'}(\mathbf{R}') \left[U(\mathbf{z}_3) U^\dagger(\mathbf{R}) - 1\right]^{ca} \left[U(\mathbf{z}_3') U^\dagger(\mathbf{R}') - 1\right]^{cb} \right\rangle \\
=&\left\langle A_a^i(\mathbf{R}) A_b^{i'}(\mathbf{R}') \left[U(\mathbf{z}_3) U^\dagger(\mathbf{R})\right]^{ca} \left[U(\mathbf{z}_3') U^\dagger(\mathbf{R}')\right]^{cb} \right\rangle \\
&- \left\langle A_a^i(\mathbf{R}) A_b^{i'}(\mathbf{R}') \left[U(\mathbf{z}_3') U^\dagger(\mathbf{R}')\right]^{ab} \right\rangle - \left\langle A_a^i(\mathbf{R}) A_b^{i'}(\mathbf{R}') \left[U(\mathbf{z}_3) U^\dagger(\mathbf{R})\right]^{ba} \right\rangle \\
&+ \left\langle A_c^i(\mathbf{R}) A_c^{i'}(\mathbf{R}') \right\rangle .
\end{aligned}$$

$\mathbf{R} = \frac{1}{2}(\mathbf{z}_1 + \mathbf{z}_2)$ and $\mathbf{R}' = \frac{1}{2}(\mathbf{z}_1' + \mathbf{z}_2')$

Factorizing.

Better, but still complicated. We assume factorization:

$$
\begin{aligned}
&\left\langle A^i_a(\mathbf{R}) A^{i'}_b(\mathbf{R}') \left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})\right]^{ca} \left[U(\mathbf{z}'_3)U^\dagger(\mathbf{R}')\right]^{cb} \right\rangle \\
\simeq &\left\langle A^i_a(\mathbf{R}) A^{i'}_b(\mathbf{R}')\right\rangle \left\langle \left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})\right]^{ca} \left[U(\mathbf{z}'_3)U^\dagger(\mathbf{R}')\right]^{cb} \right\rangle \\
&+ \left\langle A^i_a(\mathbf{R}) \left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})\right]^{ca} \right\rangle \left\langle A^{i'}_b(\mathbf{R}') \left[U(\mathbf{z}'_3)U^\dagger(\mathbf{R}')\right]^{cb} \right\rangle \\
&+ \left\langle A^i_a(\mathbf{R}) \left[U(\mathbf{z}'_3)U^\dagger(\mathbf{R}')\right]^{cb} \right\rangle \left\langle A^{i'}_b(\mathbf{R}') \left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})\right]^{ca} \right\rangle .
\end{aligned}
$$

And that we can calculate in MV model!

Averages I.

$$
\begin{aligned}
&\left\langle A_a^i(\mathbf{R})A_b^{i'}(\mathbf{R}')\right\rangle\left\langle\left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})\right]^{ca}\left[U(\mathbf{z}_3')U^\dagger(\mathbf{R}')\right]^{cb}\right\rangle\\
=&G_{WW}^{ii'}(\mathbf{R},\mathbf{R}')\left\langle\mathrm{Tr}\left[U(\mathbf{z}_3)U^\dagger(\mathbf{R})U(\mathbf{R}')U^\dagger(\mathbf{z}_3')\right]\right\rangle\\
\simeq&\frac{1}{N_c^2-1}G_{WW}^{ii'}(\mathbf{R},\mathbf{R}')\left\langle\mathrm{Tr}\left[U(\mathbf{z}_3)U^\dagger(\mathbf{z}_3')\right]\right\rangle\left\langle\mathrm{Tr}\left[U(\mathbf{R})U^\dagger(\mathbf{R}')\right]\right\rangle\\
=&(N_c^2-1)G_{WW}^{ii'}(\mathbf{R},\mathbf{R}')D_g(\mathbf{z}_3,\mathbf{z}_3')D_g(\mathbf{R},\mathbf{R}')\,.
\end{aligned}
\tag{2}
$$

Here we used the factorized approximation for the average of the adjoint quadrupole. This is well justified in our kinematics, since for typical configurations we have the hierarchy of distances $|\mathbf{\Delta}|\sim|\mathbf{R}-\mathbf{R}'|\sim|\mathbf{z}_3-\mathbf{z}_3'|\ll|\mathbf{z}_3-\mathbf{R}|\sim Q_s$. Corrections to this factorization are of order Q_s^2/Δ^2.

Averages II

More interesting:

$$\left\langle A^i_a(\mathbf{R})\left[U(\mathbf{z}'_3)U^\dagger(\mathbf{R}')\right]^{cb}\right\rangle$$

$$= T^a_{cb}\frac{2\pi Q_s^2}{N_c g^2}\left[e^{2Q_s^2\hat{\Gamma}_{\mathbf{R}',\mathbf{z}'_3}}\right]\frac{(-ig)\partial^i_{\mathbf{R}}[\Gamma_{\mathbf{R},\mathbf{z}'_3}-\Gamma_{\mathbf{R},\mathbf{R}'}]}{Q_s^2(\hat{\Gamma}_{\mathbf{R},\mathbf{R}'}+\hat{\Gamma}_{\mathbf{R},\mathbf{z}'_3}-\hat{\Gamma}_{\mathbf{z}'_3,\mathbf{R}'})}\left[e^{Q_s^2(\hat{\Gamma}_{\mathbf{R},\mathbf{R}'}+\hat{\Gamma}_{\mathbf{R},\mathbf{z}'_3}-\hat{\Gamma}_{\mathbf{z}'_3,\mathbf{R}'})}-1\right.$$

$$\simeq T^a_{cb}\frac{4\pi Q_s^2}{N_c g^2}D_g(\mathbf{R}',\mathbf{z}'_3)(-ig)\partial^i_{\mathbf{R}}[L_{\mathbf{R},\mathbf{z}'_3}-L_{\mathbf{R},\mathbf{R}'}]. \tag{3}$$

MV model:

$$\langle A^{+a}(x^-,\mathbf{x})A^{+b}(y^-,\mathbf{y})\rangle = \delta^{ab}\delta(x^- - y^-)g^2\mu^2(x^-)L(\mathbf{x},\mathbf{y}) \tag{4}$$

$$L_{\mathbf{x},\mathbf{y}} \equiv L(\mathbf{x},\mathbf{y}) = \frac{1}{\nabla^4}(\mathbf{x},\mathbf{y}) \propto \frac{1}{\mathbf{p}^4}$$

$$\Gamma_{\mathbf{x},\mathbf{y}} = \pi\hat{\Gamma}_{\mathbf{x},\mathbf{y}} = 2L(\mathbf{x},\mathbf{y}) - L(\mathbf{x},\mathbf{x}) - L(\mathbf{y},\mathbf{y})$$

Averages III

Putting everything together, the event-averaged diffractive trijet production in the high momentum limit is

$$\langle|\mathcal{M}_{\rm diff}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)|^2\rangle = (2\pi)\frac{4g^2}{N_c}\int\frac{d^2\mathbf{l}}{(2\pi)^2}\frac{d^2\mathbf{l}'}{(2\pi)^2}\frac{\mathbf{l}\cdot\mathbf{l}'}{l^2 l'^2}$$

$$\partial_{\mathbf{P}^i_\perp}\left[\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,p_2^+,\mathbf{P}_\perp+\mathbf{l}/2)-\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,p_2^+,\mathbf{P}_\perp-\mathbf{l}/2)\right]$$

$$\times\,\partial_{\mathbf{P}^{i'}_\perp}\left[\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,p_2^+,\mathbf{P}_\perp+\mathbf{l}'/2)-\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+,p_2^+,\mathbf{P}_\perp-\mathbf{l}'/2)\right]^*$$

$$\times\left(S_\perp(2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\frac{(N_c^2-1)g^2}{4}\int\frac{d^2\mathbf{k}}{(2\pi)^2}G^{ii'}_{WW}(-\boldsymbol{\Delta}-\mathbf{l}-\mathbf{k})D_g(\mathbf{k})D_g(\mathbf{l}-\mathbf{p}_3)\right.$$

$$-\,(4\pi Q_s^2)^2\frac{N_c}{4}S_\perp\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}\frac{-\mathbf{q}_1^i(-\boldsymbol{\Delta}-\mathbf{p}_3+\mathbf{q}_1)^{i'}}{\mathbf{q}_1^4(-\boldsymbol{\Delta}-\mathbf{p}_3+\mathbf{q}_1)^4}\left[D_g(\boldsymbol{\Delta}+\mathbf{l}-\mathbf{q}_1)-D_g(\mathbf{l}-\mathbf{p}_3)\right]$$

$$\left.\times\left[D_g(\mathbf{p}_3-\mathbf{l}'-\mathbf{q}_1)-D_g(\mathbf{p}_3-\mathbf{l}')\right]\right).$$

Note the poles at $\mathbf{q}=0$ and $-\boldsymbol{\Delta}-\mathbf{p}_3+\mathbf{q}_1=0$ - those are not regularized by Q_s in the MV model.

Summary of analytics.

$$\frac{dN}{d^2\mathbf{p}_1 d^2\mathbf{p}_2 d^2\mathbf{p}_3} \simeq \int \frac{d^2\mathbf{l}}{(2\pi)^2}\frac{d^2\mathbf{l}'}{(2\pi)^2}\sigma^{ii'}(\mathbf{P}_\perp, \mathbf{l}, \mathbf{l}')G^{ii'}(\mathbf{\Delta}, \mathbf{p}_3, \mathbf{l}, \mathbf{l}').$$

with

$$\sigma^{ii'}(\mathbf{P}_\perp, \mathbf{l}, \mathbf{l}') = (2\pi)\frac{g^4}{N_c}\frac{\mathbf{l}\cdot\mathbf{l}'}{l^2 l'^2}\partial_{\mathbf{P}^i_\perp}\Big[\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+, p_2^+, \mathbf{P}_\perp + \mathbf{l}/2) - \Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+, p_2^+, \mathbf{P}_\perp - \mathbf{l}/$$
$$\times\, \partial_{\mathbf{P}^{i'}_\perp}\Big[\Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+, p_2^+, \mathbf{P}_\perp + \mathbf{l}'/2) - \Psi^{\gamma^*\to q\bar{q}}_{\sigma_1\sigma_2}(p_1^+, p_2^+, \mathbf{P}_\perp - \mathbf{l}'/2)\Big]^*$$

and

$$G^{ii'}(\mathbf{\Delta}, \mathbf{p}_3, \mathbf{l}, \mathbf{l}') = S_\perp(2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\frac{(N_c^2-1)g^2}{4}\int\frac{d^2\mathbf{k}}{(2\pi)^2}G^{ii'}_{WW}(-\mathbf{\Delta}-\mathbf{l}-\mathbf{k})D_g(\mathbf{k})D_g(\mathbf{l} -$$
$$-\,(4\pi Q_s^2)^2\frac{N_c}{4}S_\perp\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}\frac{-\mathbf{q}_1^i(-\mathbf{\Delta}-\mathbf{p}_3+\mathbf{q}_1)^{i'}}{\mathbf{q}_1^4(-\mathbf{\Delta}-\mathbf{p}_3+\mathbf{q}_1)^4}\Big[D_g(\mathbf{\Delta}+\mathbf{l}-\mathbf{q}_1) - D_g(\mathbf{l}-\mathbf{p}_3)\Big]$$
$$\times\Big[D_g(\mathbf{p}_3-\mathbf{l}'-\mathbf{q}_1) - D_g(\mathbf{p}_3-\mathbf{l}')\Big].$$

Numerics

We are interested in the angular correlations between $\mathbf{p}_3$ and $\mathbf{\Delta}$, so we integrate over the orientation of the momentum $\mathbf{P}_\perp$ keeping $|\mathbf{P}_\perp|$ fixed. We also integrate over the direction of $\mathbf{p}_3$ (denoted by β_3)

$$C(\mathbf{p}_3, \mathbf{\Delta}) = \frac{1}{\mathcal{N}} \int_0^{2\pi} d\beta_3 \int d\phi_{P_\perp} \frac{dN}{d^2\mathbf{p}_1 d^2\mathbf{p}_2 d^2\mathbf{p}_3}$$

The normalization factor $1/\mathcal{N}$ represents normalization over all the angles between $[-\pi/4, \pi/4]$.
Strictly speaking one needs to regularize the $1/l^2$,

$$\frac{1}{l^2} \longrightarrow \frac{1}{l^2 + \Lambda^2_{\mathrm{QCD}}}.$$

The pole itself contributes only in the terms that do not lead to same side correlations, and even in these terms the residue of the pole is exponentially small. Thus our results practically do not depend on the exact value of the regulator.

Numerical results

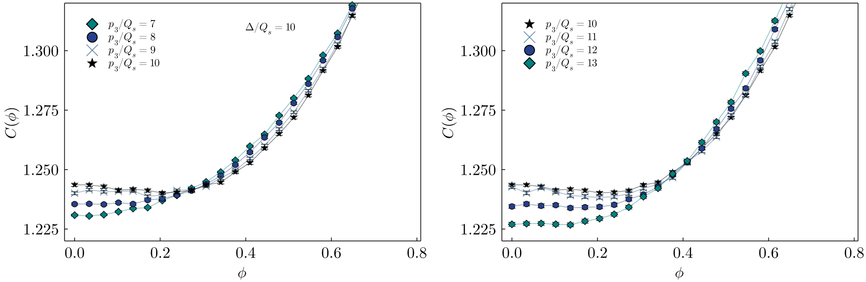


Figure: Angular correlation function for different p_3, and $P_\perp/Q_s = 15$, $\Delta/Q_s = 10$, $z = 1/2$, $Q/Q_s = 8$, and transverse photon.

Numerical results

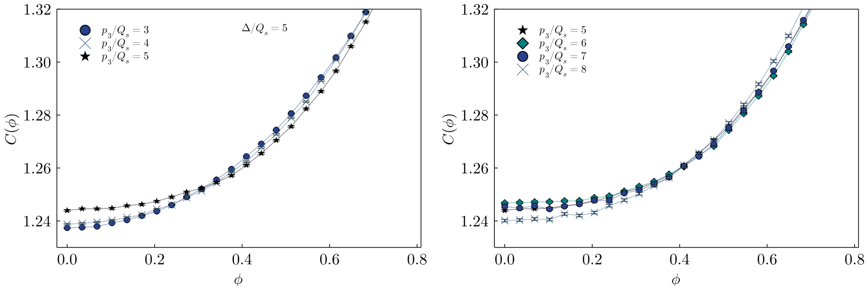


Figure: The same for $P_\perp/Q_s = 10$, $\Delta/Q_s = 5$, $z = 1/2$, $Q/Q_s = 8$.

Discussion

The signal is there, although perhaps not very strong, for $.8 \lesssim p_3/\Delta \lesssim 1.2$

The angular width of the maximum is about $\Delta\phi \sim .1$. Consistent with the"primordial" sharp Bose correlations smeared by Q_s.

In the letter - signal is more pronounced with growing Q_s. Here not. Why?

We understand: in the letter we regulated $\langle A_i(q)A_i(-q)\rangle$ by Q_s. Physically: expect $1/Q_s$ to be the scale for color neutralization. But not so in MV model- no color neutralization. Evolution from MV does lead to suppression at low q - so we expect the effect to be real, but missed by the MV - type averaging. We expect our current results to be an underestimate of the effect.

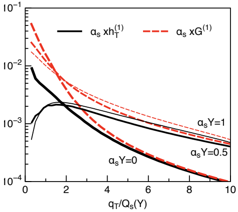


Figure: From A. Dumitru, T. Lappi and V. Skokov, Phys. Rev. Lett. 115, 252301 (2015)