GPDs at small-x

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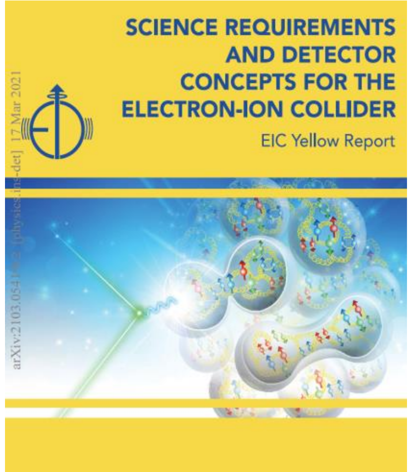
Color Glass Condensate at the electron-ion collider, ECT*, May 15-19, 2023

Nucleon tomography



Multidimensional tomography is one of the main scientific goals of the EIC.

Wigner $W(x, \vec{k}_\perp, \vec{b}_\perp)$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2)\gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$

$\int d\vec{b}_\perp$ → **TMD** $f(x, \vec{k}_\perp)$

$\int d\vec{k}_\perp$ → **GPD** $f(x, \vec{b}_\perp)$

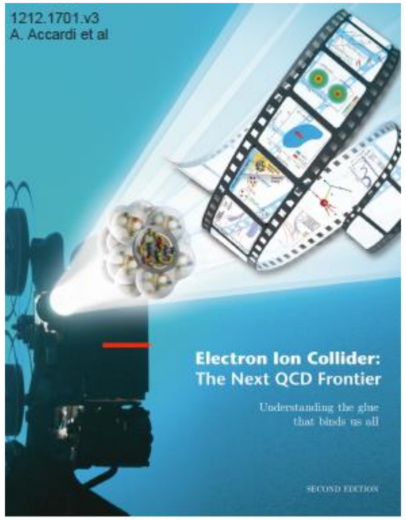
$f(x, \vec{k}_\perp)$ → $\int d\vec{k}_\perp$ → $f(x)$ **PDF**

$f(x, \vec{b}_\perp)$ → $\int d\vec{b}_\perp$ → $f(x)$ **PDF**

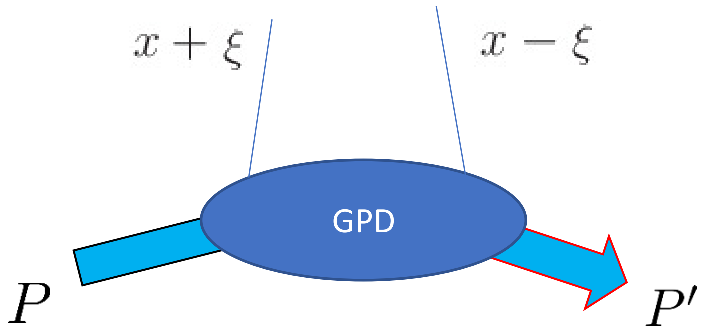
$f(x, \vec{b}_\perp)$ → $\int dx$ → $F(\vec{b}_\perp)$ **Form factor**

$f(x)$ → $\int dx$ → Q **charge**

$F(\vec{b}_\perp)$ → $\int d\vec{b}_\perp$ → Q **charge**



Generalized Parton Distribution



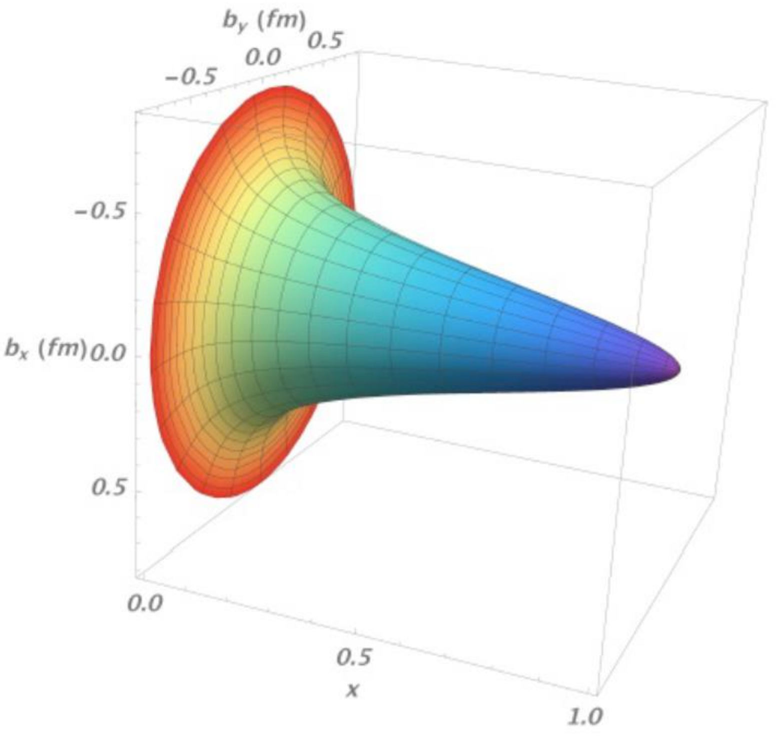
3D partonic imaging encoded in generalized parton distributions (GPDs)

$$f(x) \to f(x,\xi,t) \qquad \xi = \frac{P^+ - P'^+}{P^+ + P'^+} \quad \text{skewness}$$

$$t = (P' - P)^2 \approx -\Delta_\perp^2$$

Fourier transform $\Delta_\perp \to b_\perp$

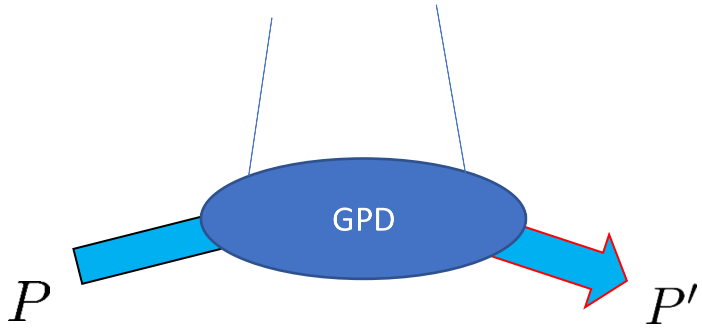
Distribution of partons in impact parameter space



First moment → elemag/axial form factors

Second moment → gravitational form factors

GPD definitions



Twice as many GPDs as PDFs

`Unpolarized' quark GPDs

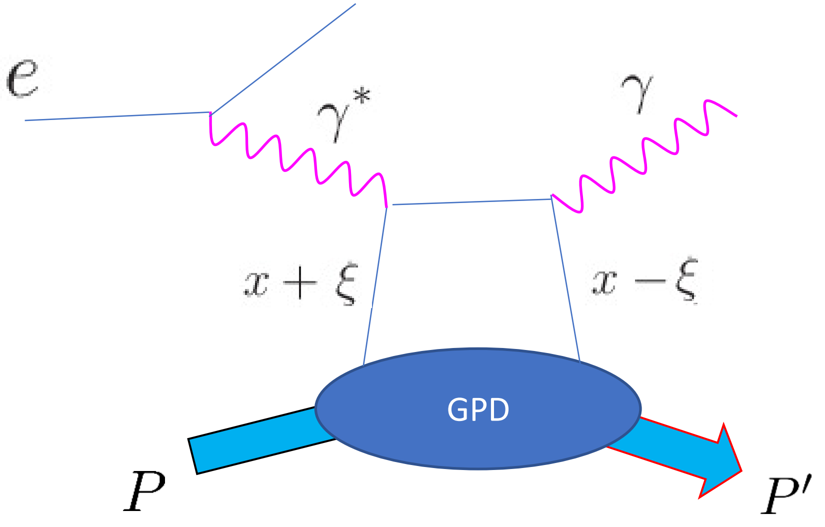
$$\bar{P}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{q}(-z/2)\gamma^+ q(z/2) | P \rangle = \bar{u}(P') \left[\gamma^+ H_q(x,\xi,t) + \frac{i\sigma^{+\nu}\Delta_\nu}{2m_N} E_q(x,\xi,t) \right] u(P)$$

`Polarized' quark GPDs

$$\bar{P}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{q}(-z/2)\gamma_5\gamma^+ q(z/2) | P \rangle = \bar{u}(P') \left[\gamma_5\gamma^+ \tilde{H}_q(x,\xi,t) - \frac{\gamma_5\Delta^+}{2m_N} \tilde{E}_q(x,\xi,t) \right] u(P)$$

Beware, `polarized' GPDs also contribute to unpolarized processes.

Deeply Virtual Compton Scattering



QCD factorization Collins, Freund (1998); Ji, Osborne (1998)

$$i\int d^4y e^{iqy}\langle P'|T\{J^\mu(y)J^\nu(0)\}|P\rangle$$

$$= -(g^{\mu+}g^{\nu-}+g^{\nu+}g^{\mu-}-g^{\mu\nu})\int\frac{dx}{2}\left(\frac{1}{x+\xi-i\epsilon}+\frac{1}{x-\xi+i\epsilon}\right)H_q(x,\xi,\Delta)\bar{u}(P')\gamma^+u(P)+\cdots$$

Compton form factor

Theory frontier

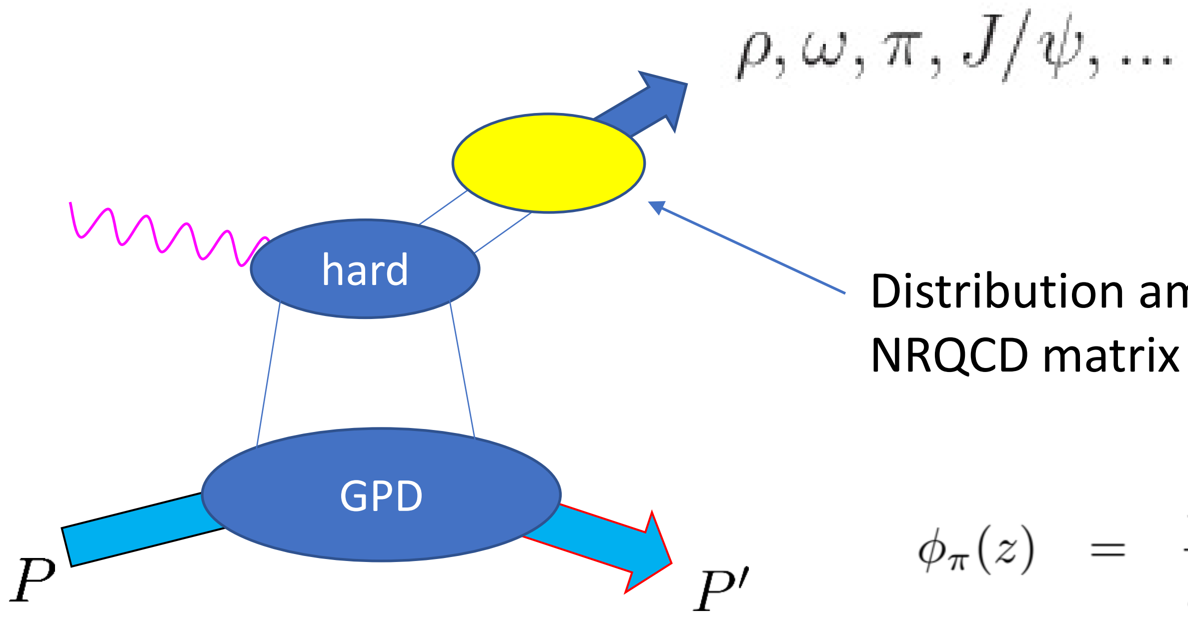
2-loop coefficient functions (singlet, unpol) Braun, Ji, Schenleber (2022)
3-loop evolution equation (nonsinglet) Braun, Manashov, Moch, Strohmaier (2017)

In principle, the ingredients for NNLO global analysis will be ready in near future
In practice, complete NLO global analysis is not achieved yet (but close).

Deeply Virtual Meson Production (DVMP)

QCD factorization when $Q^2 \to \infty$ — Collins, Frankfurt, Strikman (1996)

$M_{QQ} \to \infty$ — Ivanov, Schafer, Szymanowski, Krasnikov (2004)



Distribution amplitude (DA) for light mesons
NRQCD matrix element for heavy quarkonia

$$\phi_\pi(z) = \frac{-i}{f_\pi}\int \frac{dx^+}{2\pi} e^{-i(1-z)p^- x^+} \langle \pi(p)|\bar{\psi}(0)\gamma_5\gamma^- \frac{\tau^3}{2}\psi(x^+)|0\rangle$$

Small-x and GPD: general remarks

Not many discussions in the literature, the two communities usually don't talk to each other…

Diehl (2003 review paper, Section 4.4)
Balitsky, Kuchina (2000), Goeke, Guzey, Siddikov (2007); YH, Xiao, Yuan (2017)

At high energy, gluon GPDs are most important.

Amplitude dominantly imaginary, sensitive to GPDs at $x = \xi$

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x,\xi,t) \sim i\pi H_g(\xi,\xi,t)$$

In the context of GPDs, it is more correct to speak of "small- ξ "

Assume weak dependence on skewness $H_g(x,\xi,t) \approx H_g(x,0,t)$

In the eikonal approximation, $\xi \approx 0$

Twist-two unpolarized gluon GPDs

nucleon helicity-flip

$$\delta_{ij}\int\frac{dz^-}{2\pi\bar{P}^+}e^{ix\bar{P}^+z^-}\langle P'|F_a^{+i}(-z/2)F_a^{+j}(z/2)|P\rangle=\frac{1}{2\bar{P}^+}\bar{u}(P')\left(H_g\gamma^+ + E_g\frac{i\sigma^{+\nu}\Delta_\nu}{2m_N}\right)u(P)$$

nucleon helicity
non-flip

open indices

$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$

$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+\cdots$$

gluon transversity GPD
→ photon helicity-flip
→ $\cos 2(\phi_{P'}-\phi_l)$ asymmetry in DVCS

Spin and orbital angular momentum at small-x?

Ji sum rule

$$J_g = \frac{1}{2}\int_0^1 dx\, x\, [H_g(x,\xi) + E_g(x,\xi)]$$

OAM distribution in Jaffe-Manohar sum rule YH, Yoshida (2012)

$$\mathcal{L}_g(x) = x\int_x^1 \frac{dx'}{x'}(H_g(x') + E_g(x')) - x\int_x^1 \frac{dx'}{x'^2}\Delta G(x') + \cdots$$

Is there a significant contribution from small-x in spin sum rules?

$$H_g(x,0) = G(x) \sim \frac{1}{x^{1+\alpha(Q^2)}}$$

$\alpha(Q^2) \sim 0.3$ in the pQCD regime (from HERA)

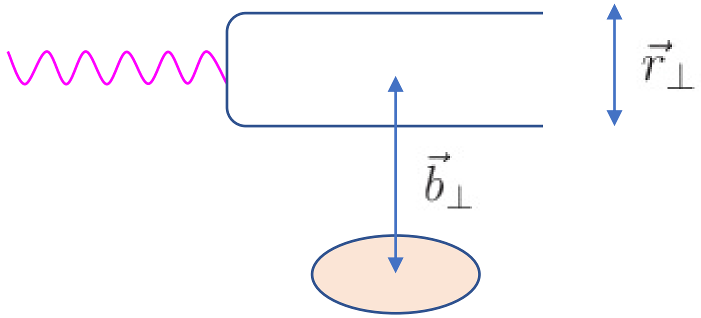
Small-x region likely important for H_g

What about E_g ?

Prejudice: nucleon helicity-flip amplitudes are suppressed at high energy (small-x)

Color dipole at small-x

$$S_x(\vec{b}_\perp, \vec{r}_\perp) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger \left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$



Direct connection to gluon Wigner distribution at small-x

YH, Xiao, Yuan (2016)

$$xW(x, \vec{q}_\perp, \vec{b}_\perp) \approx \frac{2N_c}{\alpha_s} \int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{r}_\perp} \left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp, \vec{r}_\perp)$$

Color dipole → Mother distribution of all unpol gluon GPDs

Fourier transform → gluon GTMD

$$F_x(q_\perp, \Delta_\perp) = F_0(|q_\perp|, |\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) F_\epsilon(|q_\perp|, |\Delta_\perp|) + \cdots$$

Elliptic Wigner

H_g, E_{Tg} in the color dipole picture

YH, Xiao, Yuan (2017)

$$\frac{1}{P^+}\int \frac{d\zeta^-}{2\pi} e^{ixP^+\zeta^-}\langle p'|F^{+i}F^{+j}|p\rangle \approx$$

$$= \frac{2N_c}{\alpha_s}\left(\frac{\delta^{ij}}{2}\int d^2q_\perp q_\perp^2 F_0 + \frac{1}{\Delta_\perp^2}\left(\Delta_\perp^i\Delta_\perp^j - \frac{\delta^{ij}\Delta_\perp^2}{2}\right)\int d^2q_\perp q_\perp^2 F_\epsilon\right)$$

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0\,,$$

$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

Elliptic Wigner → Gluon transversity GPD

$E_g(x)$ in the color dipole picture

YH, Zhou (2022)

Parameterization of color dipole for transversely polarized proton → GTMD

$$S(b_\perp, r_\perp) = \langle P' S_\perp | \frac{1}{N_c} \mathrm{tr} U\left(b_\perp - \frac{r_\perp}{2}\right) U\left(b_\perp + \frac{r_\perp}{2}\right) | P S_\perp \rangle$$

F.T.

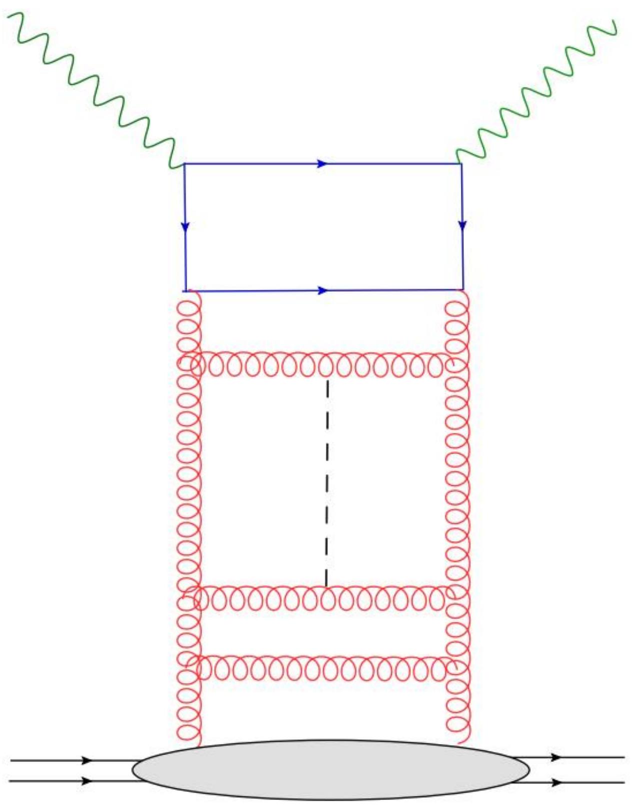
$$\frac{\pi g^2}{2N_c k_\perp^2}\left[f_{1,1} - i\frac{k_\perp \times S_\perp}{M^2}\left(\frac{k_\perp \cdot \Delta_\perp}{M^2} f_{1,2} + i g_{1,2}\right) + i\frac{\Delta_\perp \times S_\perp}{2M^2}(2f_{1,3} - f_{1,1})\right]$$

spin-dependent Odderon

$$xE_g(x) = \int d^2k_\perp \left[-f_{1,1}(k_\perp) + 2f_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2} f_{1,2}(k_\perp)\right]$$

Small-x evolution of $H_g(x)$

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0 \quad \sim \left(\frac{1}{x}\right)^{4\ln 2\bar{\alpha}_s}$$



Satisfies the Balitsky-Lipatov-Kuraev-Fadin (BFKL) equation at small-x

At even smaller-x, Balitsky-Kovchegov (BK) equation → gluon saturation

$$\partial_\tau S(\vec{x},\vec{y}) = \int \frac{d^2\vec{z}}{2\pi}\mathcal{M}_{xy}(\vec{z})\left(\langle S(\vec{x},\vec{z})S(\vec{z},\vec{y})\rangle - S(\vec{x},\vec{y})\right)$$

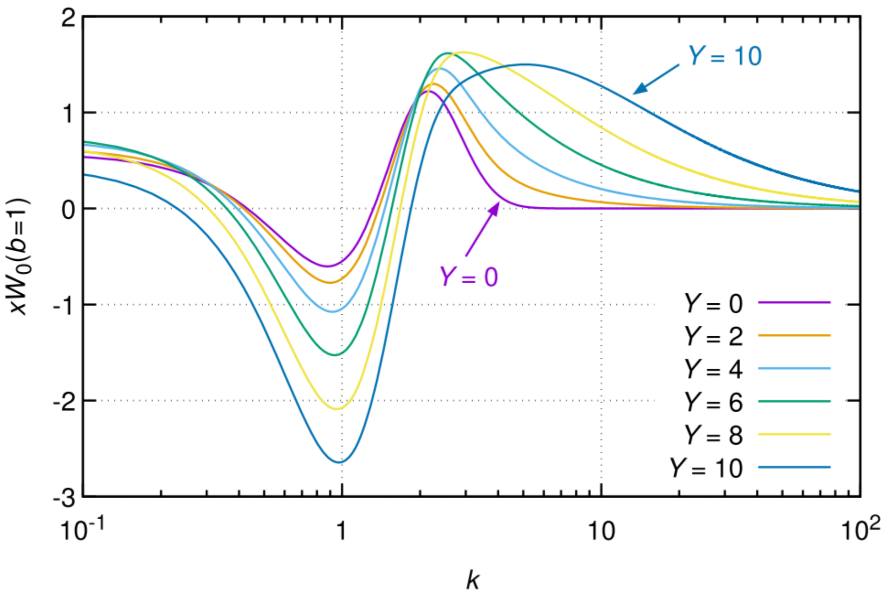
Small-x evolution of $E_{Tg}(x)$

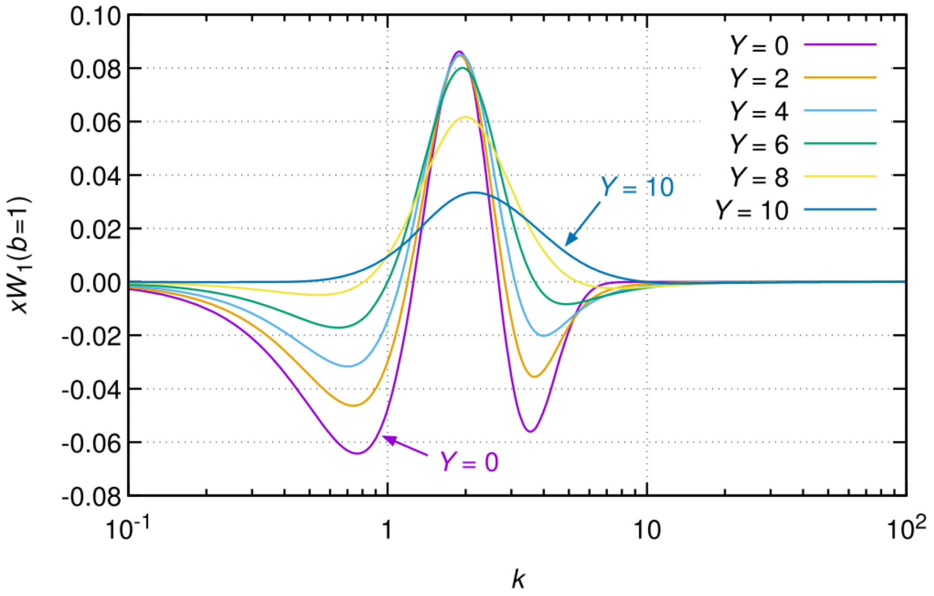
$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_c M^2}{\alpha_s \Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

$$F_x(q_\perp,\Delta_\perp) = F_0(|q_\perp|,|\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp})F_\epsilon(|q_\perp|,|\Delta_\perp|) + \cdots$$

Evolution of the angular dependent part from impact parameter dependent BK

Hagiwara, YH, Ueda (2016)





Much slower growth with rapidity compared to the angular independent part. No traveling wave

Small-x evolution of $E_g(x)$

YH, Zhou (2022)

$$S(b_\perp, r_\perp) = \langle P' S_\perp | \frac{1}{N_c} \mathrm{tr} U\left(b_\perp - \frac{r_\perp}{2}\right) U\left(b_\perp + \frac{r_\perp}{2}\right) | P S_\perp \rangle$$

F.T. $\Rightarrow$ $$\frac{\pi g^2}{2N_c k_\perp^2}\left[f_{1,1} - i\frac{k_\perp \times S_\perp}{M^2}\left(\frac{k_\perp \cdot \Delta_\perp}{M^2} f_{1,2} + i g_{1,2}\right) + i\frac{\Delta_\perp \times S_\perp}{2M^2}(2f_{1,3} - f_{1,1})\right]$$

From BK, one can derive coupled equations $f_{1,1}, f_{1,2}, f_{1,3}$

$$\partial_Y \mathcal{F}_{1,2}(k_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2}\left[-\frac{k_\perp^2}{2k'^2_\perp}\mathcal{F}_{1,2}(k_\perp) + \frac{2(k_\perp \cdot k'_\perp)^2 - k_\perp^2 k'^2_\perp}{(k_\perp^2)^2}\mathcal{F}_{1,2}(k'_\perp)\right] - 4\pi^2\alpha_s^2 \bar{\mathcal{F}}_{1,1}(k_\perp)\mathcal{F}_{1,2}(k_\perp),$$

$$\partial_Y \mathcal{F}_{1,3}(k_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2}\left[-\frac{k_\perp^2}{2k'^2_\perp}\mathcal{F}_{1,3}(k_\perp) + \frac{k_\perp^2 k'^2_\perp - (k_\perp \cdot k'_\perp)^2}{k_\perp^2}\frac{\mathcal{F}_{1,2}(k'_\perp)}{M^2} + \mathcal{F}_{1,3}(k'_\perp)\right] - 4\pi^2\alpha_s^2 \bar{\mathcal{F}}_{1,1}(k_\perp)\mathcal{F}_{1,3}(k_\perp).$$

Equations diagonalize in terms of

YH, Zhou (2022)

$$k_\perp^2(\partial^2/\partial k_\perp^\alpha \partial k_\perp^\alpha)\mathcal{E} = -f_{1,1} + 2f_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2} f_{1,2}(k_\perp)$$

$$\frac{\partial}{\partial \ln 1/x}\mathcal{E}(k_\perp) = \underbrace{\frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp}\mathcal{E}(k_\perp)\right]}_{\text{BFKL equation}} - 4\pi^2\alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{E}(k_\perp)$$

coupling with H_g

$$xE_g(x) = \int d^2k_\perp[-f_{1,1}(k_\perp) + 2f_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2}f_{1,2}(k_\perp)$$

$$\sim \left(\frac{1}{x}\right)^{4\ln 2\bar{\alpha}_s}$$

BFKL Pomeron behavior,
the same as unpol gluon PDF!

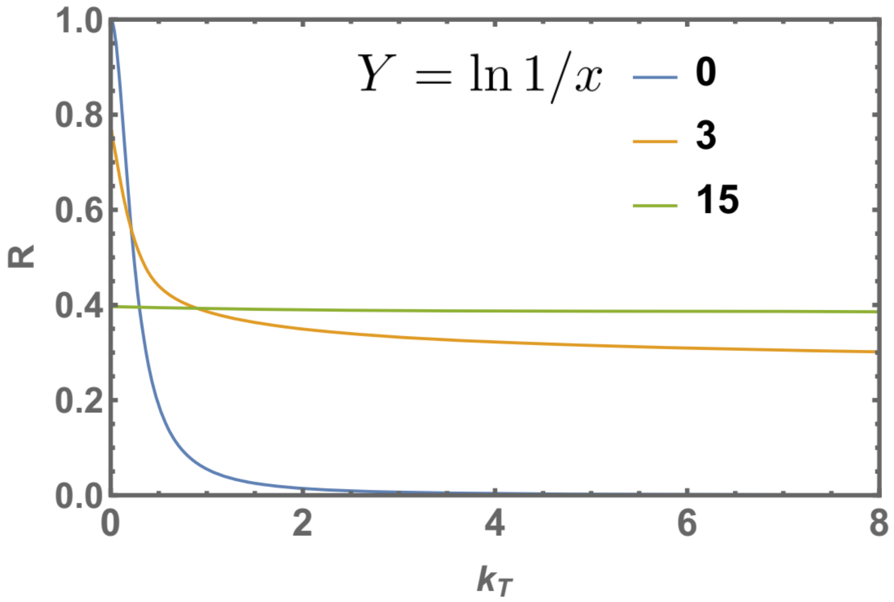
Gluon saturation of $E_g(x)$

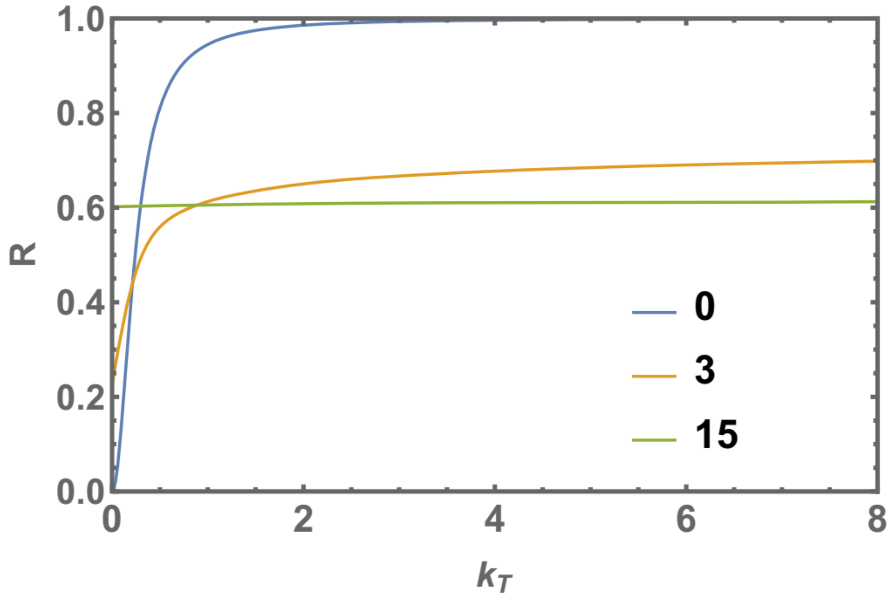
The ratio

$$R \equiv \frac{\mathcal{E}(x, k_\perp)}{\mathcal{F}_{1,1}(x, k_\perp)} \sim \frac{E_g(x)}{H_g(x)}$$

becomes constant at small-x.

$xE_g(x)$ gets saturated in the same way as $xG(x)$





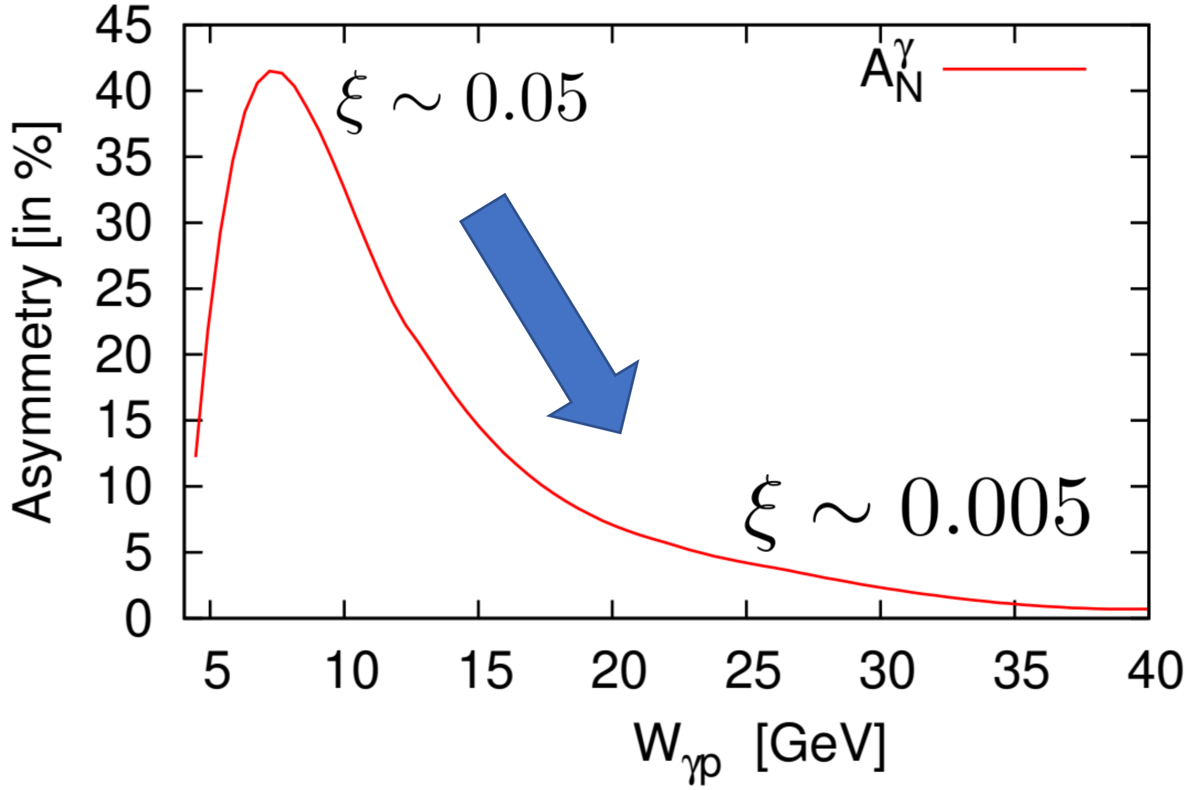
Application: Single spin asymmetry of J/ψ in UPC at RHIC

Koempel, Kroll, Metz, Zhou (2012)
Lansberg, Massacrier, Szymanowski, Wagner (2018)

STAR collaboration plans to measure SSA of J/ψ in ultraperipheral pA collisions at RHIC in order to access the gluon GPD $E_g(x)$

$$A_N \sim \frac{Im(\mathcal{H}_g^* \mathcal{E}_g)}{|\mathcal{H}_g|^2} \sim \frac{Re\mathcal{E}_g}{Im\mathcal{H}_g}$$

$$\sim \xi \sim \frac{M_{J/\psi}^2}{2W_{\gamma p}^2}$$



Shape of A_N at high-$W_{\gamma p}$ can constrain the x-dependence of $E_g(x)$

DVCS in the color dipole picture

Kowalski, Motyka, Watt (2006)
Goeke, Guzey, Siddikov (2007)
YH, Xiao, Yuan (2017)

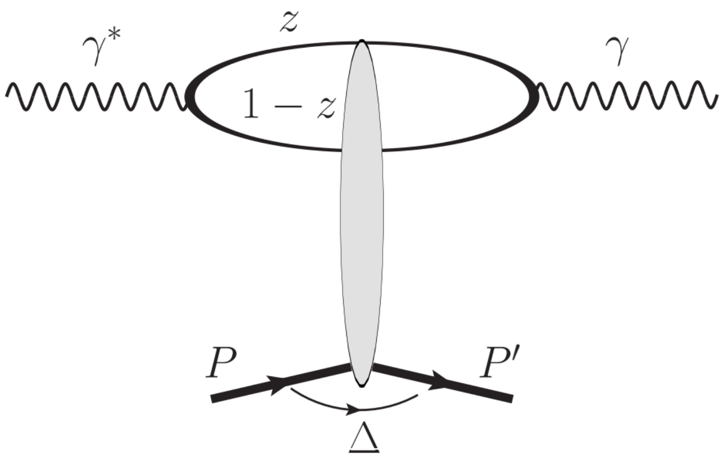
Helicity non-flip

Photon helicity flip

$$\frac{d\sigma(ep \to e'\gamma p')}{dx_B dQ^2 d^2\Delta_\perp} = \frac{\alpha_{em}^3}{\pi x_{Bj} Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)(\mathcal{A}_0^2+\mathcal{A}_2^2)+2(1-y)\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l})\right.$$
$$\left.+(2-y)\sqrt{1-y}(\mathcal{A}_0+\mathcal{A}_2)\mathcal{A}_L\cos\phi_{\Delta l}+(1-y)\mathcal{A}_L^2\right\}.$$

L→ T transition

$$\mathcal{F}_x(\tilde{q}_\perp, \Delta_\perp, z) = \int \frac{d^2r_\perp d^2b'_\perp}{(2\pi)^4} e^{i\Delta_\perp\cdot b'_\perp + i\tilde{q}_\perp\cdot r_\perp} e^{-i\delta_\perp\cdot r_\perp} S_x\left(b'_\perp + \frac{r_\perp}{2}, b'_\perp - \frac{r_\perp}{2}\right)$$



Beware the phase factor $\delta_\perp \equiv \frac{1-2z}{2}\Delta_\perp$

YH, Xiao, Yuan (2017)

Helicity non-flip amplitude

$$\mathcal{A}_0 = -\sum_q \frac{e_q^2 N_c}{\pi} \int dz d^2 q_\perp d^2 q_{1\perp} \frac{(z^2+(1-z)^2) q_{1\perp}\cdot k_\perp}{q_{1\perp}^2 (k_\perp^2 + \epsilon_q^2)} F_x(q_\perp, \Delta_\perp)$$

Collinear limit $Q \gg q_\perp$ →

$$\sum_q \frac{2\pi e_q^2 \alpha_s}{Q^2} \int \frac{d^2 k'_\perp}{(2\pi)^2} \frac{1}{k'^2_\perp} x H_g(x)$$

Consistent with the 1-loop evolution of quark GPD

$$xH_q(x,\xi,\Delta_\perp^2) = \frac{\alpha_s}{2\pi}\frac{1}{2}\int_x^1 d\zeta \frac{\zeta^2 + (1-\zeta)^2 - \frac{\xi^2}{x^2}\zeta^2}{(1-\frac{\xi^2}{x^2}\zeta^2)^2} x' H_g(x',\xi,\Delta_\perp^2) \int \frac{dk_\perp^2}{k_\perp^2}$$

Caveat: Reproduces only the imaginary part of the Compton form factor

Photon helicity-flip

$$\frac{d\sigma}{d\phi_{\Delta_\perp}} \sim \mathcal{A}_0 \mathcal{A}_2 \cos 2(\phi_{l'_\perp} - \phi_{\Delta_\perp})$$

$$\mathcal{A}_2 = -2 \sum_q e_q^2 N_c \int dz d\alpha d^2 q_\perp \frac{z(1-z)\alpha}{\alpha \tilde{q}_\perp^2 + \epsilon_q^2} \frac{2(\tilde{q}_\perp \cdot \Delta_\perp)^2 - \tilde{q}_\perp^2 \Delta_\perp^2}{\Delta_\perp^2} F_x(q_\perp, \Delta_\perp)$$

Collinear limit →

$$- \sum_q \frac{e_q^2 N_c}{Q^2} \int d^2 q_\perp q_\perp^2 F_\epsilon(q_\perp, \Delta_\perp) = - \frac{e_q^2 \alpha_s \Delta_\perp^2}{4Q^2 M^2} E_{Tg}(x, \Delta_\perp)$$

Consistent with Ji Hoodbhoy (1998) in collinear factorization

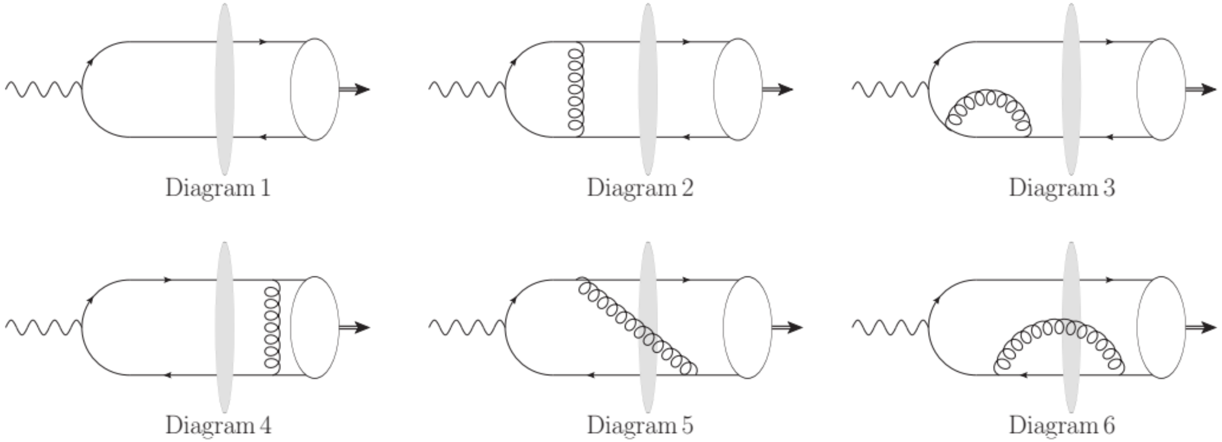
$$\mathcal{A}_2 = \sum_q \frac{e_q^2 \alpha_s \Delta_\perp^2}{8\pi Q^2 M^2} \xi \, \mathrm{Im} \left[\int dx \left(\frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right) E_{Tg}(x, \xi) \right]$$

Vector meson production at NLO

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)
Mantisaari, Penttala (2022)

Connect color dipole approach to GPD factorization with meson DA

$$\mathcal{A}^{\eta}_{LO} \equiv -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx\, \varphi\left(x, \mu_F\right) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right) \times \Phi_0^\beta\left(x, \vec{p}_1, \vec{p}_2\right) \left[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c\right]\left(\vec{p}_1, \vec{p}_2\right).$$



Divergences absorbed into the evolution of meson DA (ERBL equation) and the JIMWLK equation for the dipole amplitude. → factorization works at 1-loop level

Nonzero k_T cuts off the endpoint singularities. Saturation momentum Q_s provides a hard scale.
→ Applicable to a larger class of observables

Conclusions

- Two approaches to exclusive processes
  moderate energy→ GPD factorization,
  very high energy → color dipole, kt factorization

- Small-x behavior of leading twist gluon GPDs $H_g(x)$ $E_g(x)$ $E_{Tg}(x)$

- Matching to collinear GPD factorization

- More interaction with GPD community in future?