Probing quark TMDs in the CGC: quark-gluon dijets in DIS

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Outline

- Introduction
- S-matrix and general jet kinematics
- Back-to-back limit and quark TMD
- Suppressing background processes
- Discussion: Consistent high-energy expansion in powers?

TMD vs CGC: from gluons to quark

For a process with a hard $\mathbf{P}$ and a not so hard $\mathbf{k}$ transverse momenta:

- TMD factorization: leading power (twist 2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$

Consistency of both approaches shown in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$, with gluon TMDs (Dominguez, Marquet, Xiao, Yuan, 2011) or low x sea quark TMDs (Marquet, Xiao, Yuan, 2009)

Gluon-driven TMDs dominant in this double limit because the target is described by a gluon background field $\mathcal{A}^{-}(x)$ in the (eikonal) CGC

Quark background field of the target should be included as well beyond the eikonal limit.

$\Rightarrow$ Possibility to recover the quark TMDs from (non-eikonal) CGC, including valence?

Power counting for the quark background field $\Psi(z)$

- Under a boost of the target of parameter γ_t along the "$-$" direction, a current associated with the target should behave as

$$J^-(z) \propto \gamma_t\,, \quad J^j(z) \propto (\gamma_t)^0\,, \quad J^+(z) \propto (\gamma_t)^{-1}\,,$$

- The quark background field of the target can be split as $\Psi(z) = \Psi^{(-)}(z) + \Psi^{(+)}(z)$, with

$$\Psi^{(-)}(z) \equiv \frac{\gamma^+\gamma^-}{2}\,\Psi(z)\,, \quad \Psi^{(+)}(z) \equiv \frac{\gamma^-\gamma^+}{2}\,\Psi(z)\,.$$

  Then, the components of the background quark current write

$$\begin{aligned}
\overline{\Psi}(z)\,\gamma^-\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^-\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^j\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^j\,\Psi^{(+)}(z) + \overline{\Psi^{(+)}}(z)\,\gamma^j\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^+\,\Psi(z) &= \overline{\Psi^{(+)}}(z)\,\gamma^-\,\Psi^{(+)}(z)\,.
\end{aligned}$$

Under a boost of the target, the projections $\Psi^{(-)}(z)$ and $\Psi^{(+)}(z)$ should thus scale as

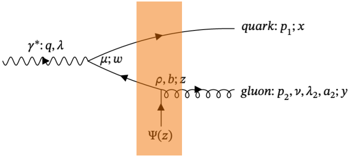
$$\Psi^{(-)}(z) \propto (\gamma_t)^{\frac{1}{2}}\,, \quad \Psi^{(+)}(z) \propto (\gamma_t)^{-\frac{1}{2}},$$

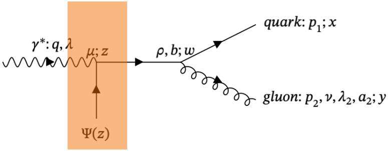
$\Rightarrow$ Let us keep only the leading components $\Psi^{(-)}(z)$ of $\Psi(z)$

See also Kovchegov *et al.* (2016-2023), and Chirilli (2019).

Contributions to $\gamma^* \to qg$ dijets from quark background

qg dijet production in DIS: a simple process sensitive to the quark background beyond eikonal CGC





$$S^{\text{bef}}_{\gamma\to q_1 g_2} = \lim_{x^+,y^+\to+\infty} \int_{\mathbf{x},\mathbf{y}} \int_{x^-,y^-} e^{ip_1\cdot x}\, \bar{u}(1)\gamma^+\, e^{ip_2\cdot y} \epsilon_\nu^{\lambda_2}(p_2)^*(-2p_2^+)$$
$$\times \int_{w,z} e^{-iq\cdot w} \epsilon_\mu^\lambda(q)\, G_F^{\nu\rho}(y,z)_{a_2 b} S_F(x,w)(-iee_f)\gamma^\mu S_F(w,z)(-ig)\gamma_\rho t^b \Psi(z),$$

$$S^{\text{in}}_{\gamma\to q_1 g_2} = \lim_{x^+,y^+\to+\infty} \int_{\mathbf{x},\mathbf{y}} \int_{x^-,y^-} e^{ip_1\cdot x}\, \bar{u}(1)\gamma^+\, e^{ip_2\cdot y} \epsilon_\nu^{\lambda_2}(p_2)^*(-2p_2^+)$$
$$\times \int_{w,z} e^{-iq\cdot z} \epsilon_\mu^\lambda(q)\, G_{F,0}^{\nu\rho}(y,w)_{a_2 b} S_{F,0}(x,w)(-ig)\gamma_\rho t^b S_F(w,z)(-iee_f)\gamma^\mu \Psi(z)$$

Propagators from inside to outside a gluon background

Eikonal quark propagator from y (inside the target) to x (after the target):

$$S_F(x,y)\Big|_{\text{Eik.}}^{\text{IA,q}} = \int \frac{d^2\mathbf{p}}{(2\pi)^2}\frac{dp^+}{(2\pi)}\frac{\theta(p^+)}{2p^+}\, e^{-ix\cdot p}\,(\not{p}+m)\,\mathcal{U}_F(+\infty,y^+;\mathbf{y})\,\left[1-\frac{\gamma^+\gamma^i}{2p^+}\, i\,\overleftarrow{\mathcal{D}^F_{\mathbf{y}^i}}\right]\, e^{iy^-p^+}\, e^{-i\mathbf{y}\cdot\mathbf{p}}$$

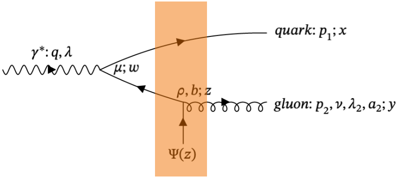
Eikonal quark propagator from y (inside the target) to x (before the target):

$$S_F(x,y)\Big|_{\text{Eik.}}^{\text{IB,q}} = \int \frac{d^2\mathbf{p}}{(2\pi)^2}\frac{dp^+}{(2\pi)}\frac{\theta(p^+)}{2p^+}\, e^{ix\cdot p}\,(\not{p}-m)\,(-1)\mathcal{U}_F^\dagger(y^+,-\infty;\mathbf{y})\,\left[1+\frac{\gamma^+\gamma^i}{2p^+}\, i\,\overleftarrow{\mathcal{D}^F_{\mathbf{y}^i}}\right]\, e^{-iy^-p^+}\, e^{i\mathbf{y}\cdot\mathbf{p}}$$

Eikonal gluon propagator from y (inside the target) to x (after the target):

$$G_F^{\mu\nu}(x,y)\Big|_{\text{Eik.}}^{\text{IA,}g} = \int \frac{d^2\mathbf{p}}{(2\pi)^2}\frac{dp^+}{(2\pi)}\frac{\theta(p^+)}{2p^+}\, e^{-ix\cdot p}\left[-g^{\mu j}+\frac{\mathbf{p}^j}{p^+}g^{\mu +}\right]\mathcal{U}_A(+\infty,y^+;\mathbf{y})\left[g^{\nu}{}_{j}+\frac{g^{\nu+}}{p^+}\left(\mathbf{p}^j+i\,\overleftarrow{\mathcal{D}^A_{\mathbf{y}^j}}\right)\right] e^{iy^-p^+}e^{-i\mathbf{y}\cdot\mathbf{p}}$$

Contribution with γ^* splitting before the target



$$S^{\text{bef}}_{\gamma_{T,L}\to q_1 g_2} = iee_f g 2\pi\delta(p_1^+ + p_2^+ - q^+)\int_{\mathbf{v},\mathbf{z}} e^{-i\mathbf{v}\cdot\mathbf{p}_1 - i\mathbf{z}\cdot\mathbf{p}_2}\int \frac{d^2\mathbf{K}}{(2\pi)^2}\frac{e^{i(\mathbf{v}-\mathbf{z})\cdot\mathbf{K}}}{\left[\mathbf{K}^2 + m^2 + \frac{p_1^+ p_2^+}{(q^+)^2}Q^2\right]}$$

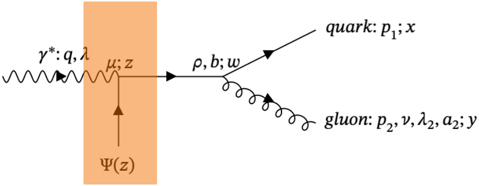
$$\times \bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\text{bef}}_{T,L}\int_{z^+} U_A\left(+\infty, z^+; \mathbf{z}\right)_{a_2 b} U_F(\mathbf{v}) U_F^\dagger\left(z^+, -\infty; \mathbf{z}\right) t^b \Psi(z^+, \mathbf{z})$$

with

$$\Gamma_L^{\text{bef}} = 2\frac{p_1^+ p_2^+}{(q^+)^2} Q \varepsilon^{j*}_{\lambda_2}\gamma^j \qquad \Gamma_T^{\text{bef}} = \varepsilon^i_\lambda\, \varepsilon^{j*}_{\lambda_2}\left\{\mathbf{K}^l\left[\left(\frac{p_1^+ - p_2^+}{q^+}\right)\delta^{il} - \frac{[\gamma^i, \gamma^l]}{2}\right] + m\gamma^i\right\}\gamma^j$$

Similar to the case of $\gamma^* \to q\bar{q}$ dijet production in the the dipole picture/CGC at eikonal accuracy, but with a different color structure

Contribution with γ^* conversion inside the target



$$S^{\rm in}_{\gamma_T\to q_1 g_2} = i\frac{ee_f g 2\pi\delta(p_1^+ + p_2^+ - q^+)}{\left[\left(\mathbf{p}_1 - \frac{p_1^+}{p_2^+}\mathbf{p}_2\right)^2 + m^2\right]}\int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{p}_1+\mathbf{p}_2)}\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma^{\rm in}_T \int_{z^+} t^{a_2} U_F\left(+\infty, z^+; \mathbf{z}\right)\Psi(z^+, \mathbf{z})$$

with

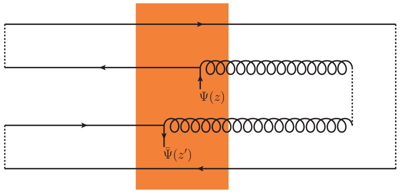
$$\Gamma^{\rm in}_T = \varepsilon^{l*}_{\lambda_2}\varepsilon^{j}_{\lambda}\left\{\left[\mathbf{p}_1^i - \frac{p_1^+}{p_2^+}\mathbf{p}_2^i\right]\left[-\left(\frac{2p_1^+ + p_2^+}{p_2^+}\right)\delta^{il} + \frac{[\gamma^i, \gamma^l]}{2}\right] + m\gamma^l\right\}\gamma^j$$

Note: $S^{\rm in}_{\gamma_L\to q_1 g_2} = 0$ at NEik in Light-cone gauge, because $\not\epsilon_L(q)\,\Psi^{(-)}(z) = \frac{Q}{q^+}\,\gamma^+\,\Psi^{(-)}(z) = 0.$

Color operators at cross section level

From the two diagrams contributing to the S-matrix, one can calculate the cross section for general kinematics of the jets.

$\rightarrow$ Various color structures are obtained at cross section level:



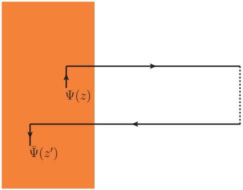
From the square of the diagram with splitting before the target:

Quadrupole with quark background field insertions, converting from fundamental to adjoint a portion of the quadrupole.

Color operators at cross section level

From the two diagrams contributing to the S-matrix, one can calculate the cross section for general kinematics of the jets.

$\rightarrow$ Various color structures are obtained at cross section level:



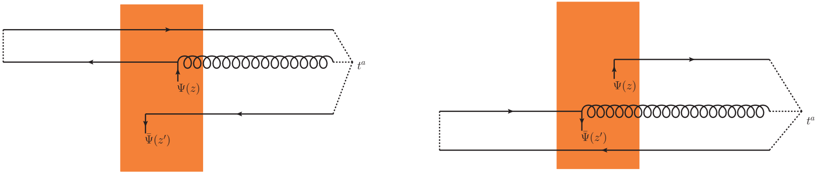
From the square of the diagram with splitting inside the target:

Quark bilinear with future staple.

Color operators at cross section level

From the two diagrams contributing to the S-matrix, one can calculate the cross section for general kinematics of the jets.

$\rightarrow$ Various color structures are obtained at cross section level:



From the interference of the two diagrams:

More exotic decorated multipole operators with quark field insertions!

Back-to-back jets: *inside* diagram

Back-to-back limit of dijets are conveniently expressed in terms of:

(dijet momentum imbalance) $\mathbf{k} = \mathbf{p}_1 + \mathbf{p}_2$ and (relative momentum) $\mathbf{P} = (1-z)\mathbf{p}_1 - z\mathbf{p}_2$

$z = p_1^+/(p_1^+ + p_2^+)$ and $(1-z) = p_2^+/(p_1^+ + p_2^+)$

S-matrix for *in* contribution in the back-to-back limit: upon renaming $\mathbf{z} \to \mathbf{b}$ (No limit required!!)

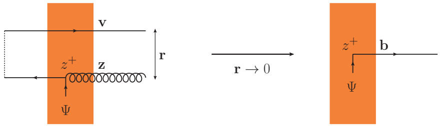
$$S^{\text{in}}_{\gamma_T \to q_1 g_2} = i\frac{ee_f g\, 2\pi\delta(p_1^+ + p_2^+ - q^+)}{[\mathbf{P}^2 + (1-z)^2 m^2]}\,(1-z)^2\,\bar{u}(1)\frac{\gamma^+\gamma^-}{2}\Gamma_T^{\text{in}}\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\int_{z^+} t^{a_2} U_F\left(+\infty, z^+; \mathbf{b}\right)\Psi(z^+, \mathbf{b})$$

Back-to-back jets: *before* diagram

S-matrix for *bef* contribution in the back-to-back limit: define $\mathbf{b} = z\mathbf{v} + (1-z)\mathbf{z}\,, \quad \mathbf{r} = \mathbf{z} - \mathbf{v}$

- phase: $e^{-i\mathbf{v}\cdot\mathbf{p}_1 - i\mathbf{z}\cdot\mathbf{p}_2 + i(\mathbf{v}-\mathbf{z})\cdot\mathbf{K}} \to e^{-i\mathbf{k}\cdot\mathbf{b} + i\mathbf{r}\cdot(\mathbf{P}-\mathbf{K})}$
- back-to-back limit: $\mathbf{P}^2 \gg \mathbf{k}^2 \Rightarrow \mathbf{r}^2 \ll \mathbf{b}^2$ (because of the phase factor), the color structure simplifies:

$$U_A\left(+\infty, z^+; \mathbf{z}\right)_{a_2 b} U_F(\mathbf{v}) U_F^\dagger\left(z^+, -\infty; \mathbf{z}\right) t^b \Psi(z^+, \mathbf{z}) \to U_A\left(+\infty, z^+; \mathbf{b}\right)_{a_2 b} U_F(\mathbf{b}) U_F^\dagger\left(z^+, -\infty; \mathbf{b}\right) t^b \Psi(z^+, \mathbf{b})$$
$$= t^{a_2} U_F\left(+\infty, z^+; \mathbf{b}\right) \Psi(z^+, \mathbf{b})$$



$$S^{\text{bef}}_{\gamma_{T,L}\to q_1 g_2} \simeq i\frac{e e_f g\, 2\pi\delta(p_1^+ + p_2^+ - q^+)}{\left[\mathbf{P}^2 + \bar{Q}^2\right]}\, \bar{u}(1) \frac{\gamma^+\gamma^-}{2} \Gamma^{\text{bef}}_{T,L} \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} t^{a_2} U_F\left(+\infty, z^+; \mathbf{b}\right) \Psi(z^+, \mathbf{b})$$

Back-to-back qg dijet cross sections

In the back-to-back limit, the photon-target dijet cross section reads

$$(2\pi)^6(2p_1^+)(2p_2^+)\,\frac{d\sigma^{\gamma_{T,L}\to q_1 g_2}}{dp_1^+ d^2\mathbf{p}_1 dp_2^+ d^2\mathbf{p}_2}\bigg|_{\text{corr.lim.}} = 2\pi\delta(p_1^+ + p_2^+ - q^+)\,(4\pi)^2\alpha_{\text{em}}\alpha_s C_F e_f^2\,\mathcal{H}_{T,L}(\mathbf{P},z,Q)\mathcal{T}(\mathbf{k})$$

with the hard factors for the longitudinal and the transverse photon polarizations

$$\mathcal{H}_L = \frac{4Q^2 z^3(1-z)^2}{[\mathbf{P}^2+\bar{Q}^2]^2} \qquad\qquad \bar{Q}^2 \equiv m^2 + z(1-z)Q^2$$

$$\mathcal{H}_T = z\left\{\frac{(1+z^2)\mathbf{P}^2+(1-z)^4m^2}{[\mathbf{P}^2+(1-z)^2m^2]^2} + \frac{[z^2+(1-z)^2]\mathbf{P}^2+m^2}{[\mathbf{P}^2+\bar{Q}^2]^2} - \frac{2z^2\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2][\mathbf{P}^2+(1-z)^2m^2]}\right\}$$

and the target averaged color operator

$$\mathcal{T}(\mathbf{k}) = \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \bar{\Psi}(z'^+,\mathbf{b}')\gamma^- U_F^\dagger(+\infty,z'^+;\mathbf{b}')U_F(+\infty,z^+;\mathbf{b})\Psi(z^+,\mathbf{b})\right\rangle$$

with a generalized CGC target average $\langle\ldots\rangle$ over both the quark and gluon background fields.

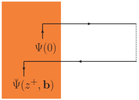
Not yet known how to explicitly perform this target average!

Recovering the unpolarized quark TMD

$$\mathcal{T}(\mathbf{k}) = \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \Big\langle \bar{\Psi}(z'^+,\mathbf{b}')\gamma^- U_F^\dagger(+\infty,z'^+;\mathbf{b}')U_F(+\infty,z^+;\mathbf{b})\Psi(z^+,\mathbf{b})\Big\rangle$$

Can be related to the unpolarized quark TMD (with future staple gauge link):

$$f_1^q(x,\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}} \int_{z^+} e^{-iz^+xP^-_{tar}} \langle P_{tar}|\bar{\Psi}(z^+,\mathbf{b})\frac{\gamma^-}{2}U_F^\dagger(+\infty,z^+;\mathbf{b})U_F(+\infty,0;\mathbf{0})\Psi(0,\mathbf{0})|P_{tar}\rangle$$



Indeed, the CGC-like target average $\langle\cdots\rangle$ is an effective model for the quantum expectation value in the target state $\langle P_{tar}|\cdots|P_{tar}\rangle$, but with a normalization $\langle 1\rangle = 1$.

$\Rightarrow$ Both expectation values can be related as

$$\langle\mathcal{O}\rangle = \lim_{P'_{tar}\to P_{tar}} \frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{\langle P'_{tar}|P_{tar}\rangle} = \lim_{P'_{tar}\to P_{tar}} \frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{2P^-_{tar}\,(2\pi)^3\,\delta(P'^-_{tar}-P^-_{tar})\,\delta^{(2)}(\mathbf{P'}_{tar}-\mathbf{P}_{tar})}$$

With this relation, and after performing translations of the whole operator in $\mathcal{T}(\mathbf{k})$ in the $+$ and transverse directions, one finds

$$\mathcal{T}(\mathbf{k}) = \frac{(2\pi)^3}{P^-_{tar}} f_1^q(x=0,\mathbf{k})$$

Factorized cross section with the quark TMD

Alternatively, in the correlation limit we can write the cross section in terms of $\mathbf{k}$, z and the dijet mass M_{jj}

$$\boxed{M_{jj}^2 \equiv (p_1+p_2)^2 = \frac{\mathbf{P}^2}{z(1-z)} + \frac{m^2}{z}}$$

The cross section in the back-to-back limit in terms of the dijet mass:

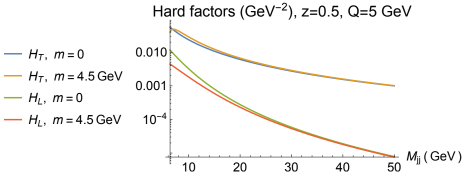
$$\frac{d\sigma^{\gamma_{T,L}\to q_1 g_2}}{dz\, dM_{jj}^2\, d^2\mathbf{k}}\bigg|_{\text{corr.lim.}} = (2\pi)\, \frac{\alpha_{\text{em}}\, \alpha_s\, C_F\, e_f^2}{W^2}\, \widetilde{\mathcal{H}}_{T,L}(M_{jj}, z, Q) f_1^q(x=0,\mathbf{k})$$

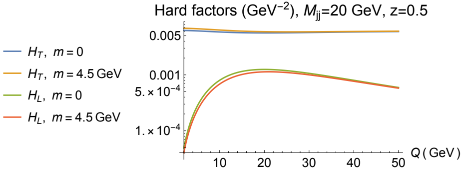
with the γ^*-target center of mass energy $W \simeq \sqrt{2q^+ P_{tar}^-}$ and the new hard factors

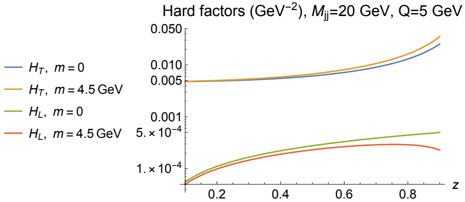
$$\widetilde{\mathcal{H}}_L(M_{jj}, z, Q) = \frac{4Q^2\, z(1-z)^2}{\Big[(1-z)(M_{jj}^2+Q^2)+m^2\Big]^2}$$

$$\widetilde{\mathcal{H}}_T(M_{jj}, z, Q) = \frac{(1+z^2)}{(1-z)}\frac{1}{\Big[M_{jj}^2-m^2\Big]} + \frac{(1-2z)}{\Big[(1-z)(M_{jj}^2+Q^2)+m^2\Big]} + \frac{(1-z)\,\Big[2m^2-\big(z^2+(1-z)^2\big)Q^2\Big]}{\Big[(1-z)(M_{jj}^2+Q^2)+m^2\Big]^2}$$
$$-\frac{2m^2}{\Big[M_{jj}^2-m^2\Big]^2} + \frac{2z(1-z)m^2}{\Big[M_{jj}^2-m^2\Big]\Big[(1-z)(M_{jj}^2+Q^2)+m^2\Big]}$$

Behavior of the hard factors

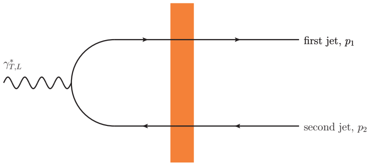






Background from eikonal $q\bar{q}$ dijet

First background process to qg dijet production:
(Eikonal) $q\bar{q}$ dijet production in DIS



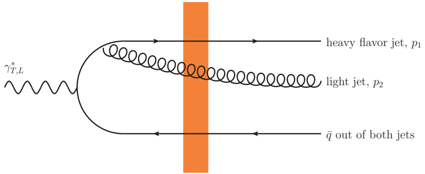
Distinguishing these two processes depends on our ability to distinguish quark and gluon jets

- Very challenging in general, for light quarks
- Heavy quark jets can be distinguished from light quark or gluon jets with heavy flavor tagging techniques

$\Rightarrow$ Focus on the heavy quark case to be able to separate qg dijets from $q\bar{q}$ dijets

Background from eikonal $q\bar{q}g$ production

Second background process to qg dijet production:
(Eikonal) $q\bar{q}g$ production in DIS, reconstructed as a qg dijet, and $\bar{q}$ outside of both jets



Main difference with our qg process:

Emitted $\bar{q}$ will take away some $+$ momentum and some transverse momentum

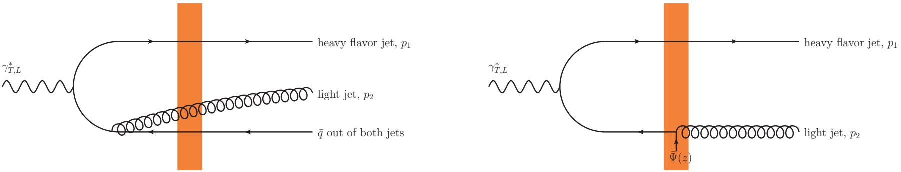
$\Rightarrow$ Background can be suppressed by imposing cuts on such momentum leaks :

1. Impose $(q^+ - p_1^+ - p_2^+) \ll q^+$
2. Impose $\mathbf{k}^2 \ll \mathbf{P}^2 = (1-z)\left[z\, M_{jj}^2 - m^2\right]$ (back-to-back limit)

Applicability of the Eikonal approximation

Eikonal approximation: performed mostly at propagator level for each parton of the projectile, or radiated by the projectile.

$\rightarrow$ Requires each of these partons to be far from the fragmentation region of the target



Ex: If final antiquark on the left diagram is close to the target fragmentation region, Eik approx not valid

$\rightarrow$ Contribution to be accounted for by the diagram on the right instead!

Consistent high-energy expansion in powers

For generic kinematics of the jets, both types of contributions are potentially important and should be included (NEik with quark background fields, and Eik $q\bar{q}g$).

However, non-trivial interplay and risks of double counting!

$\Rightarrow$ Need to understand what are the precise kinematical restrictions to be applied to each unmeasured parton on the projectile side so that Eikonal approximation is valid.

Remark: It is a very general issue, relevant to other processes:

- SIDIS : needed to include both
  - Sea quarks from gluon splitting (Marquet, Xiao, Yuan, 2009)
  - Contribution from quark background (see talk by S. Mulani on friday)
- DIS : related issues in the aligned jet part of the phase space

Summary

- The quark background field of the target is relevant in CGC beyond the eikonal approximation
- We have calculated the qg dijet production in DIS in NEik CGC, driven by interaction with this quark background field
- TMD factorization of the cross section in the back-to-back limit: quark TMD (including valence) obtained from a (non-eikonal) CGC calculation!
- qg dijet in DIS: new way to probe the unpolarized quark TMD, in particular at the EIC
- This process can be distinguished from background processes at least in the heavy quark case, by required heavy flavor tagging on a single jet of the dijet, and using appropriate kinematical cuts.
- In general: need to build a consistent scheme to treat unmeasured parton in the Eikonal expansion, to obtain correct high-energy expansion of observables without double counting