3D proton structure: from partons to strong fields

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2001.06449, 2006.14569, 2112.01412 [hep-ph]

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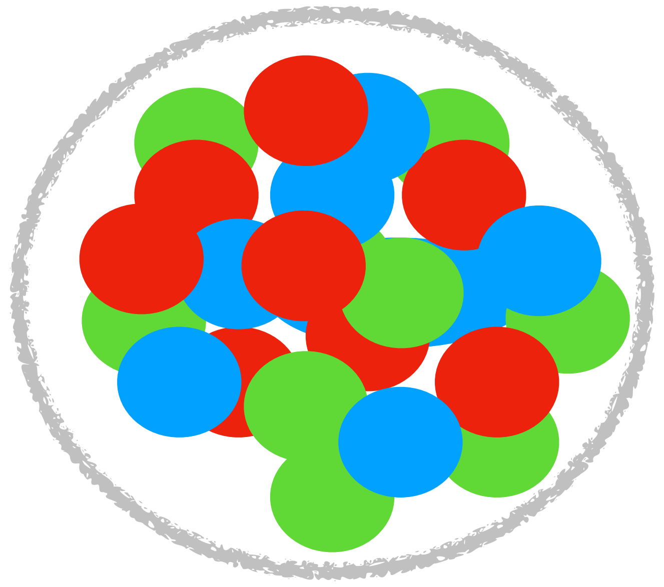
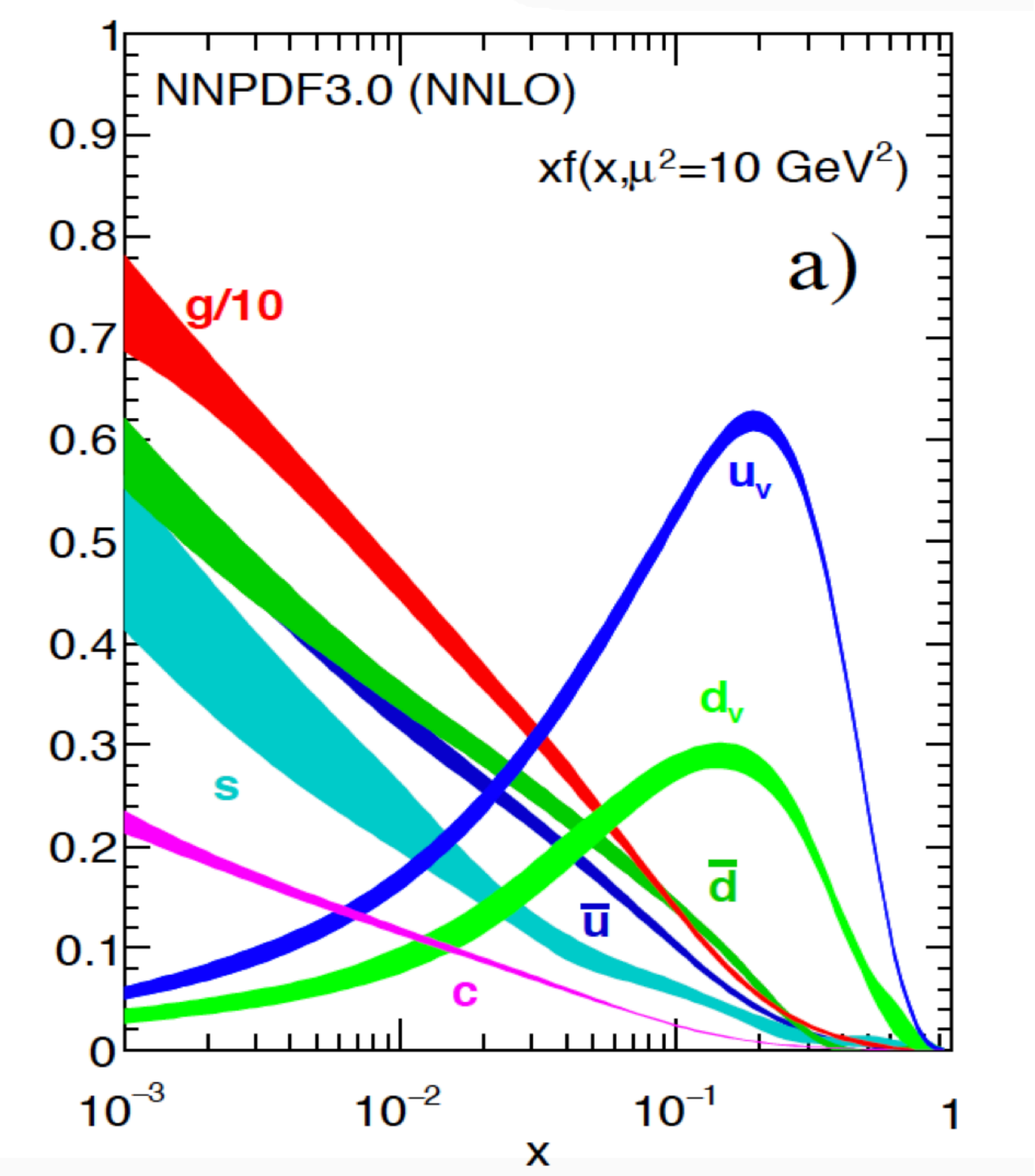
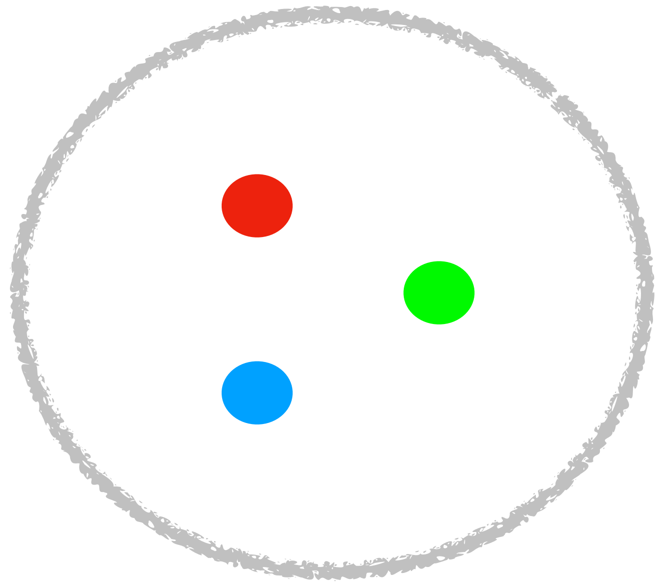
Motivation

Parton distribution function



Strong fields $A^\mu \sim 1/g$





Partons

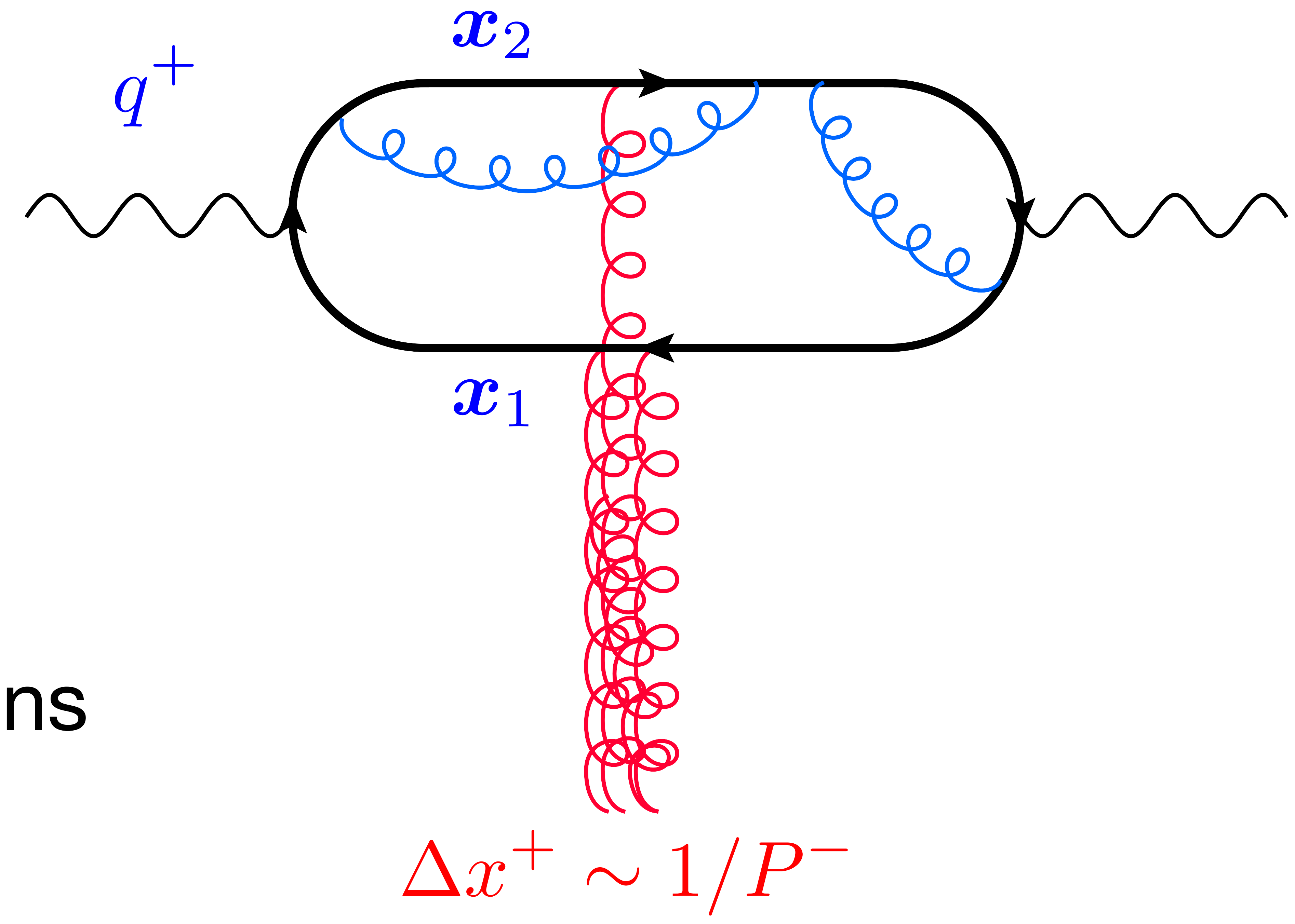
- The 3D imaging of the nucleon largely based on the weakly interacting parton picture
- Saturation: Parton picture expected to break down at small x. Relevant d.o.f.'s: strong classical fields
- Connection of these two pictures not clear but crucial to properly interpret experimental results

Can we construct a framework that encompasses the two regimes of QCD?

High energy factorization

- Building block: path ordered exponential (Wilson line)

$$U_{\boldsymbol{x}} \equiv [+\infty, -\infty]_{\boldsymbol{x}} = P \exp\left[ig \int_{-\infty}^{+\infty} \mathrm{d}x^{+} A^{-}(x^{+}, \boldsymbol{x}) \right]$$



- Coherent scattering off longitudinally polarized gluons

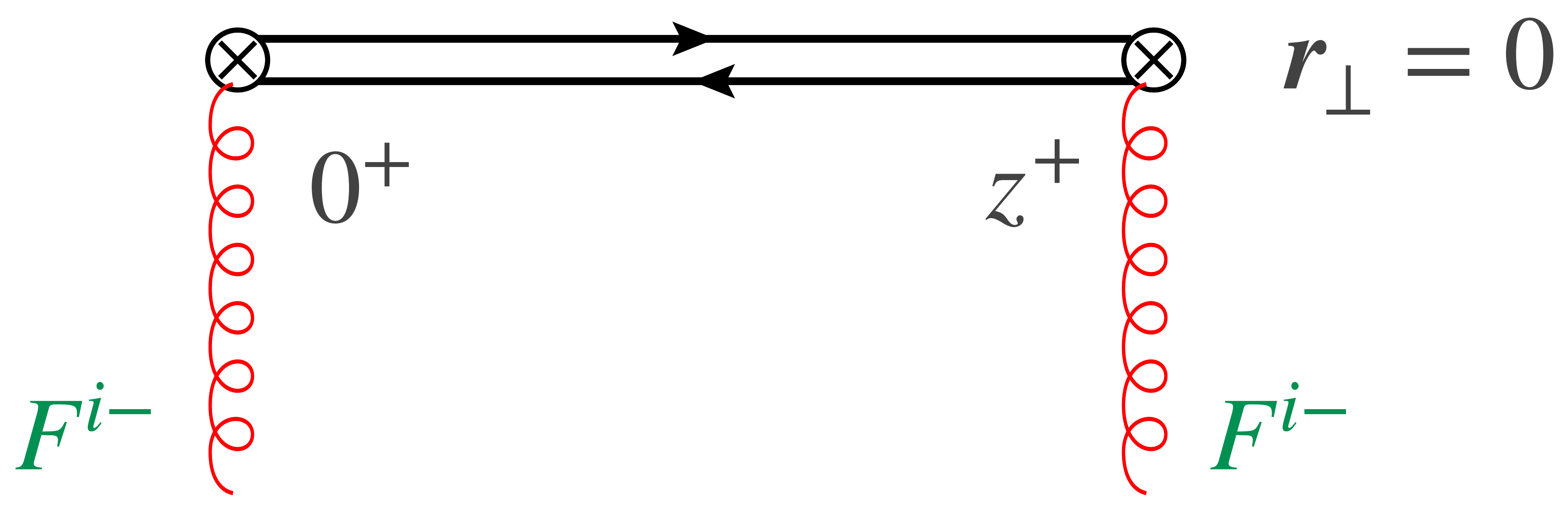
- Gluon distribution encoded in dipole operator

$$G(\boldsymbol{k}_{\perp}) \sim \int \mathrm{d}^{2}\boldsymbol{k}_{\perp} \langle P | \, \mathrm{Tr}\ U_{\boldsymbol{r}}\, U_{\boldsymbol{0}}^{\dagger} \, | P \rangle \ \mathrm{e}^{-i\boldsymbol{r}_{\perp}\cdot \boldsymbol{k}_{\perp}}$$

Two kinds of distributions

Gluon PDF at moderate x

$$\langle P | F^{i-}(z^+)\, W\, F^{i-}(0^+)\, W^\dagger | P \rangle\, e^{iz^+ x P^-}$$

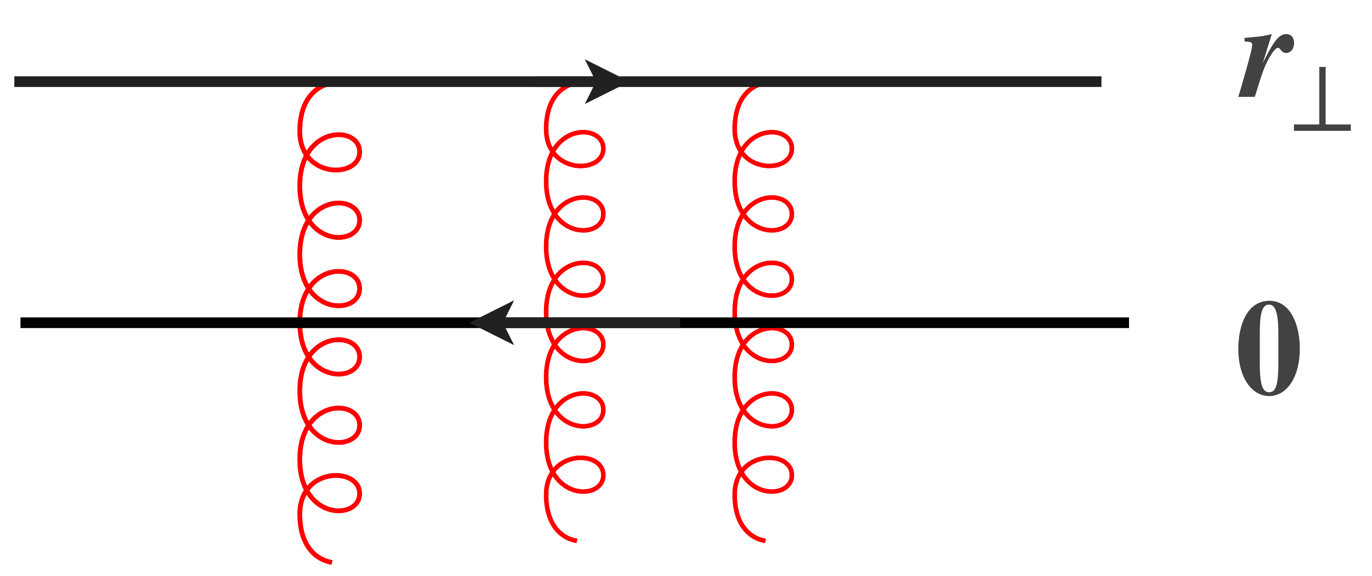


$$F^{i-} \equiv \partial^i A^- - \partial^- A^i - ig[A^i, A^-]$$

on-shell gluon

Dipole gluon distribution at small x

$$G(\boldsymbol{k}_\perp) \sim \int \mathrm{d}^2\boldsymbol{k}_\perp \langle P | \mathrm{Tr}\; U_{\boldsymbol{r}}\, U_{\boldsymbol{0}}^\dagger | P \rangle \;\; e^{-\boldsymbol{r}_\perp \cdot \boldsymbol{k}_\perp}$$



$$A^-$$

off-shell gluon, strong background field

Diagnosing small x

- Dipole distribution evaluated in the strict $x = 0$ limit
- Hard part integrated over x

Bjorken limit

$$s \sim Q^2$$

$$\sigma \sim \int \mathrm{d}x\, H(x) f(x) + O(Q^{-2})$$

Regge limit

$$s \gg Q^2$$

$$\sigma \sim f(0, k_\perp) \int \mathrm{d}x\, H(x, k_\perp) + O(s^{-1})$$

- Works only if $f(x) \sim \mathrm{const}$ however $f(x) \sim x^{-\Delta}$ (via quantum evolution)

Diagnosing small x

- In the Regge limit distributions (operators) evaluated strictly at x=0

$$f(x = 0\,, k_\perp)$$

- No x dependence at LO: quantum evolution generates rapidity dependence. Ambiguous connection to x.

- The dipole model is inconsistent with exact x kinematics: breakdown of dipole size conservation

Bialas, Navelet and Peschanski (2000)

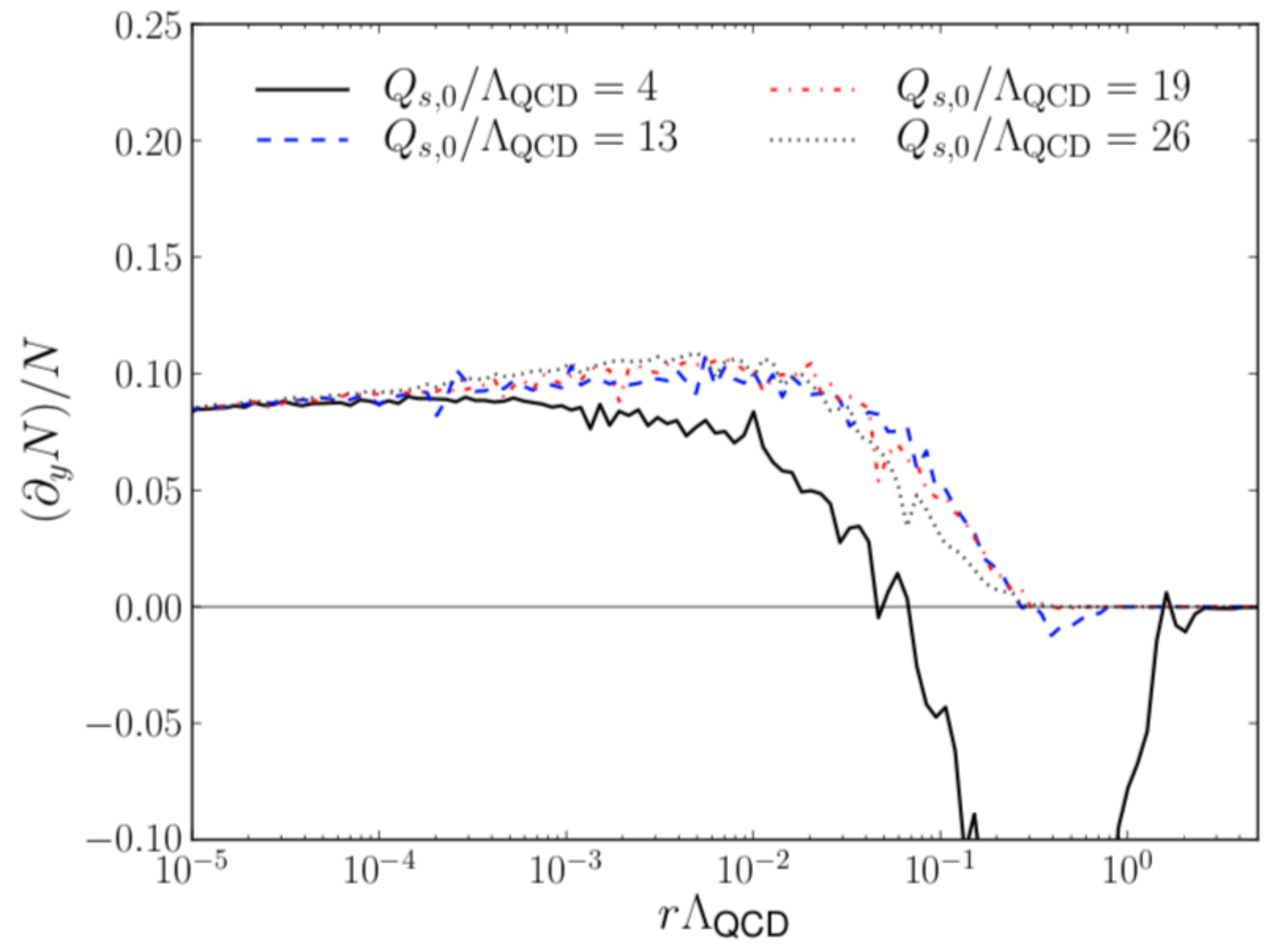
Our main conclusion is that, although equivalent at the leading logarithm level, the extensions of the dipole model and those based on k_T factorization to the next-to-leading order lead to results *which are not compatible with each other.* This conclusion emphasizes the urgent need for the full next order calculation[2] which would settle the question of validity of the two most

Diagnosing small x

- Instability of NLO BFKL/BK: rapidity $Y \equiv \log(q^+/\Lambda^+)$ evolves independently from $r_\perp$ violating $k^- = xP^-$ ordering (producing large collinear logs)

Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)

Lappi and Mäntysaari (2015)



- Similar issues in forward hadron production in pA and forward dijet production in DIS

- Ad hoc solutions: restoring kinematic constraint in k^-, resummation, better choice of evolution variable $\eta \sim \log k^-$

Salam (1998), Shi, Wang, Wei, Xiao (2021) Liu, Xie, Kang, Liu, (2022) Caucal, Salazar, Schenke, Venugopalan, (2022) Taels, Altinoluk, Beuf, Marquet (2022)

Why go beyond Shock Wave?

→ Systematic treatment of the collinear corner of phase-space

→ Smooth connection with Bjorken limit (leverage the phenomenological success of the parton model)

→ Spin dependent observables at small x

Beyond shock wave approximation

- Sub-eikonal expansion around the shock wave $\delta(x^+)$ [Agostini, Altinoluk, Armesto, Beuf, Martinez, Moscoso, Salgado] [Kovchegov, Sievert,Pitonyak]
- Expansion in the boost parameter [Chirilli] ; [Altinoluk, Beuf, Czajka, Tymowska]
- Addition of a single additional hard scattering [Jalilian-Marian]

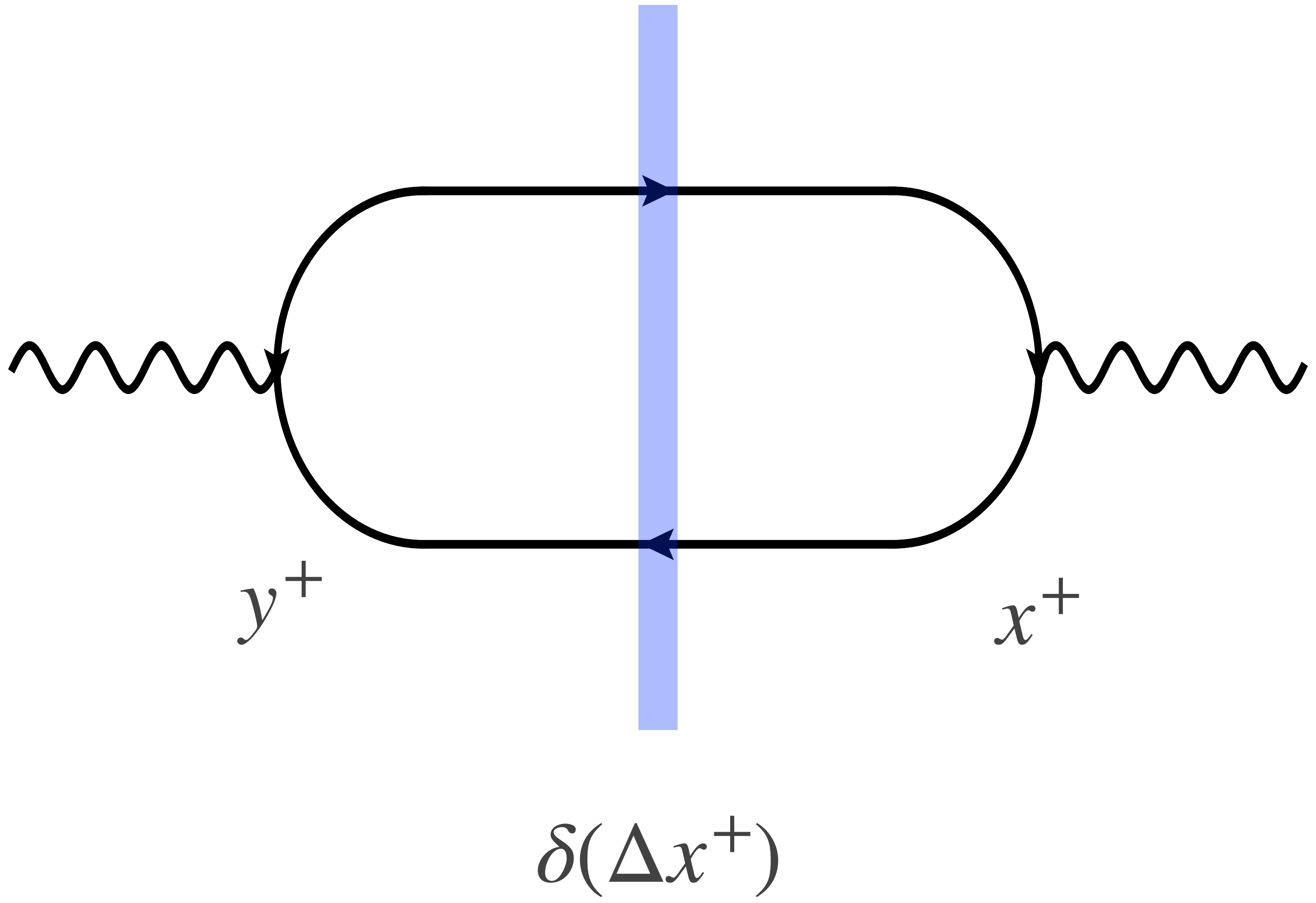
Our approach:

- revisit the shock wave factorization scheme to restore the x dependence of the gluon distribution - factorization in k^+ [Balitsky-Tarasov]
- perform a partial twist expansion to connect Regge and Bjorken limits

$$f(k_\perp, x) + O\left(\frac{x_{\mathrm{Bj}}}{Q^2}\right)$$

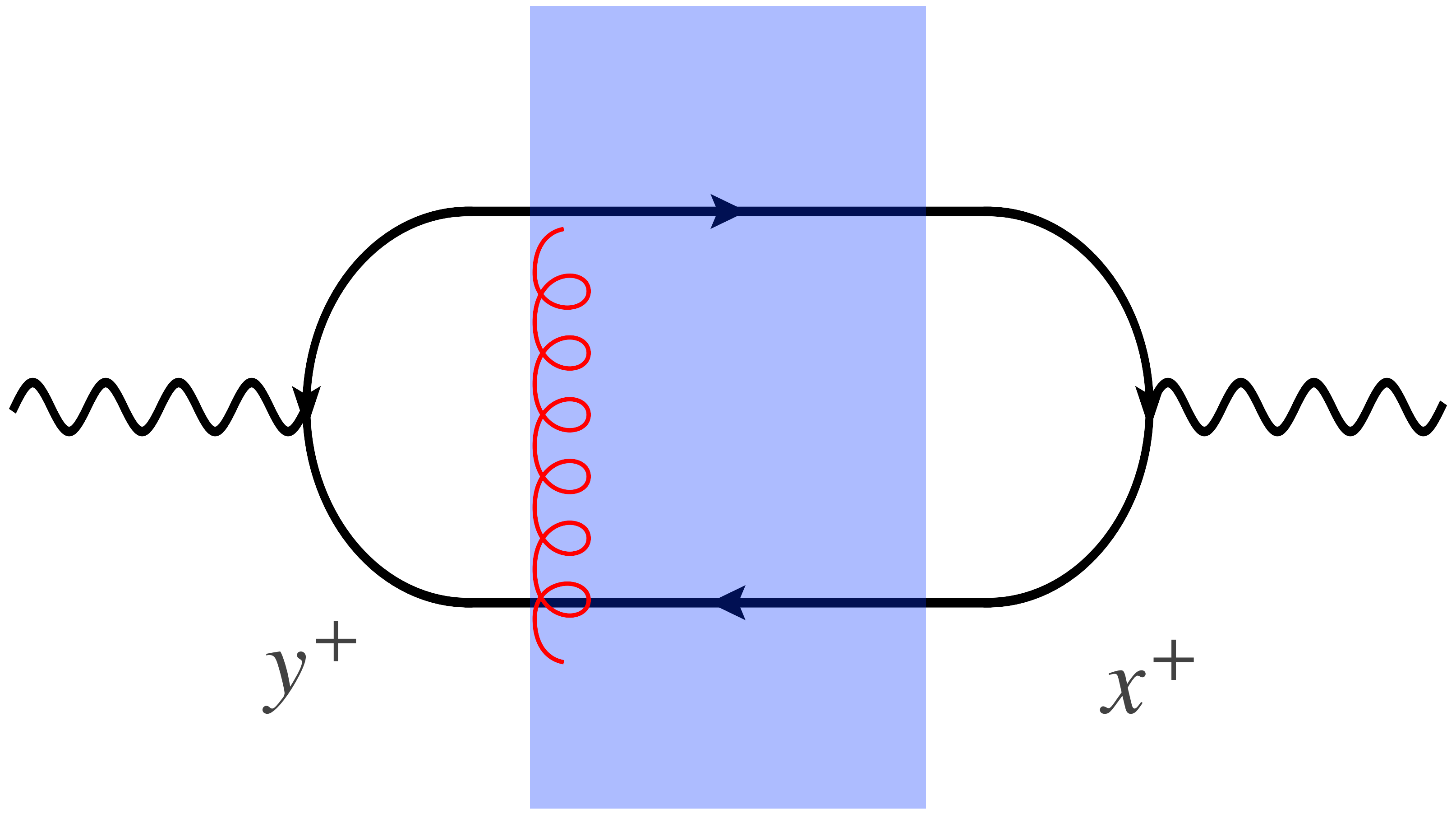
Longitudinal structure of the target

- **Decoupling of time integrals in the shock wave approximation at leading power in s**
  - times of the photon splitting into quark antiquark pair are integrated form $0 < x^+ < +\infty$ and $-\infty < y^+ < 0$
  - Times of the background field in the target integrated from $-\infty$ to $+\infty$



Longitudinal structure of the target

- **To restore x dependence: need to keep time ordering and convolution**
  - To do so: identify the first and last interaction with the target
  - There are 4 contributions: 2 for the initial time + 2 for the finial time



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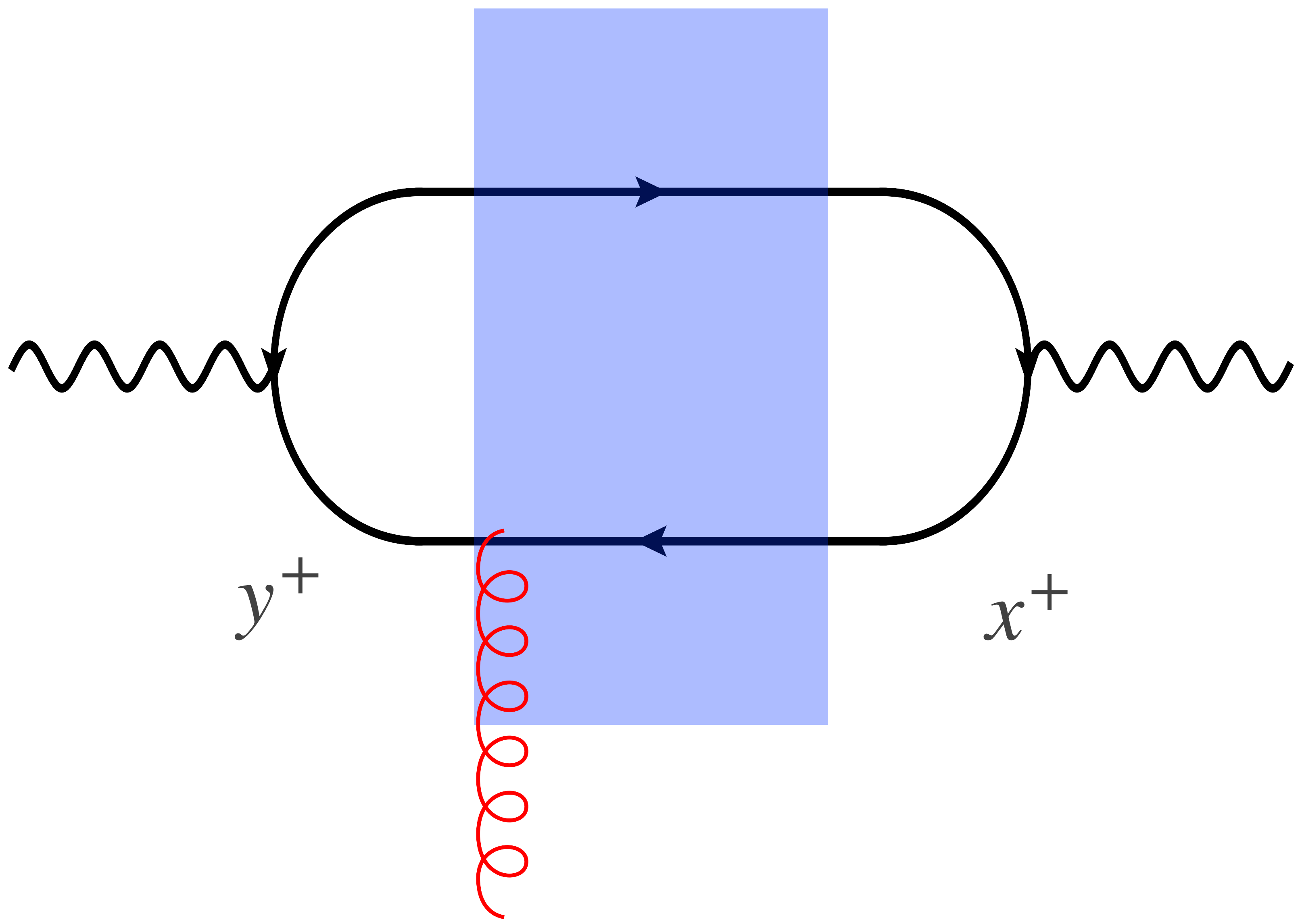

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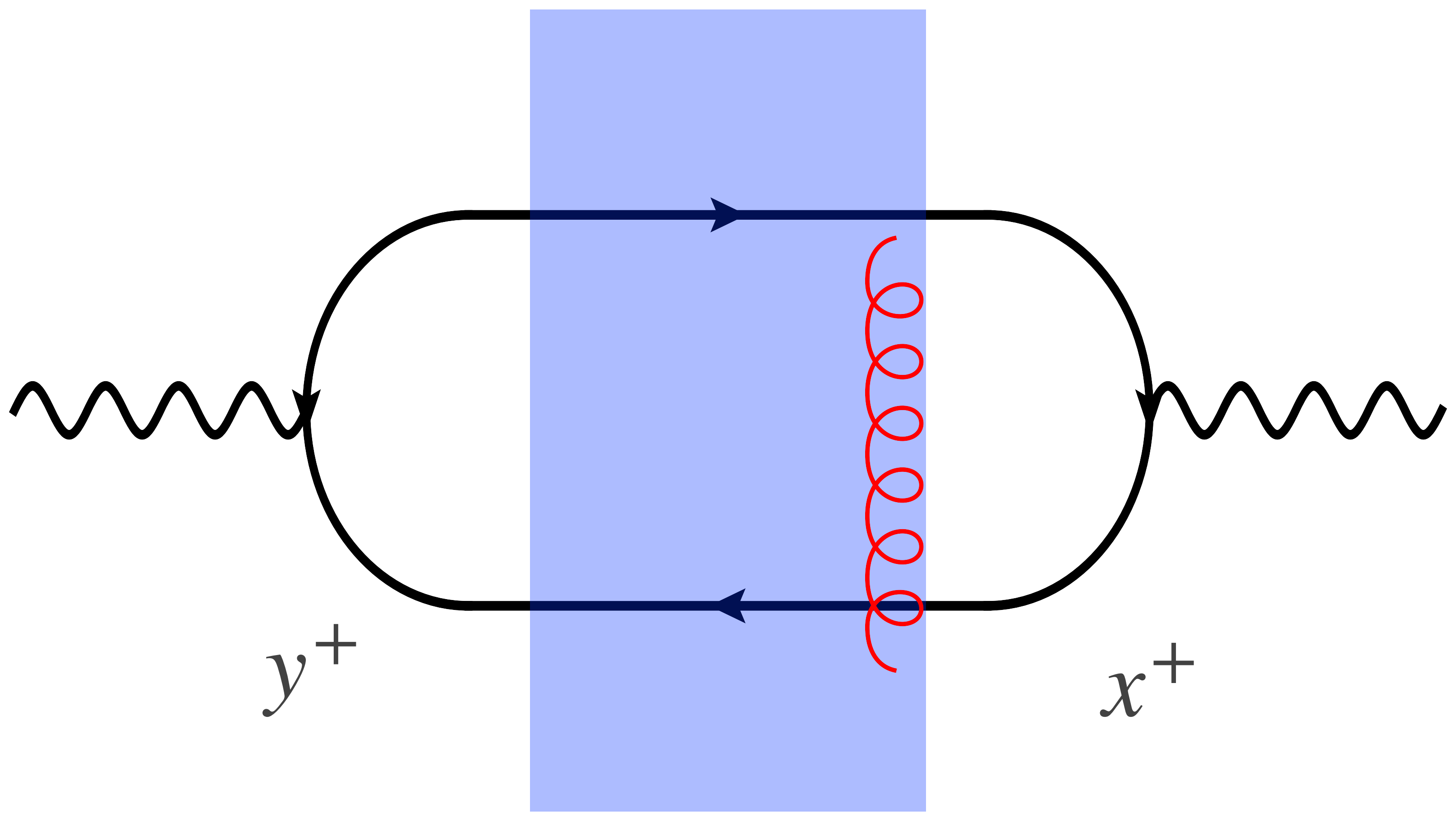

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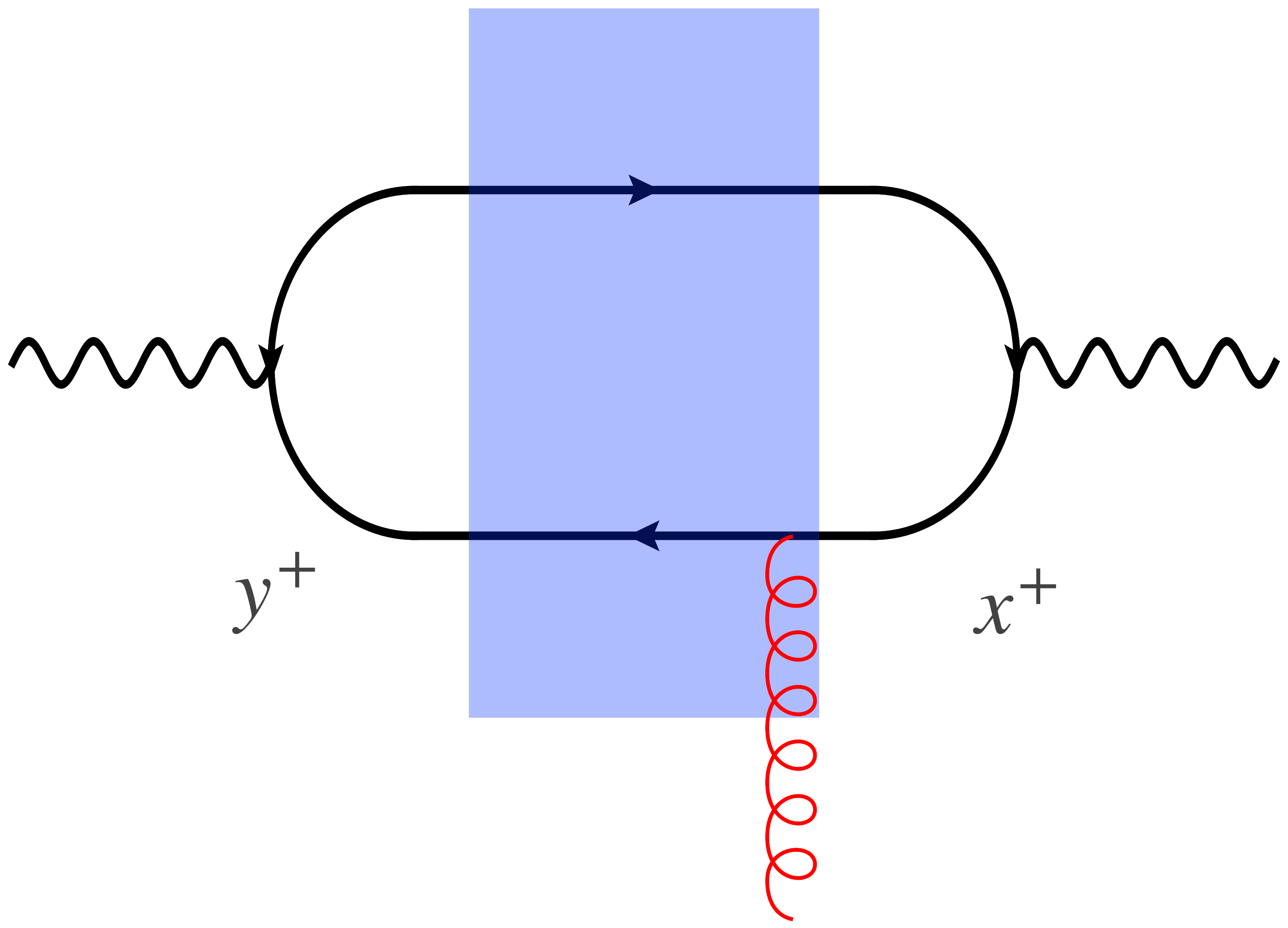
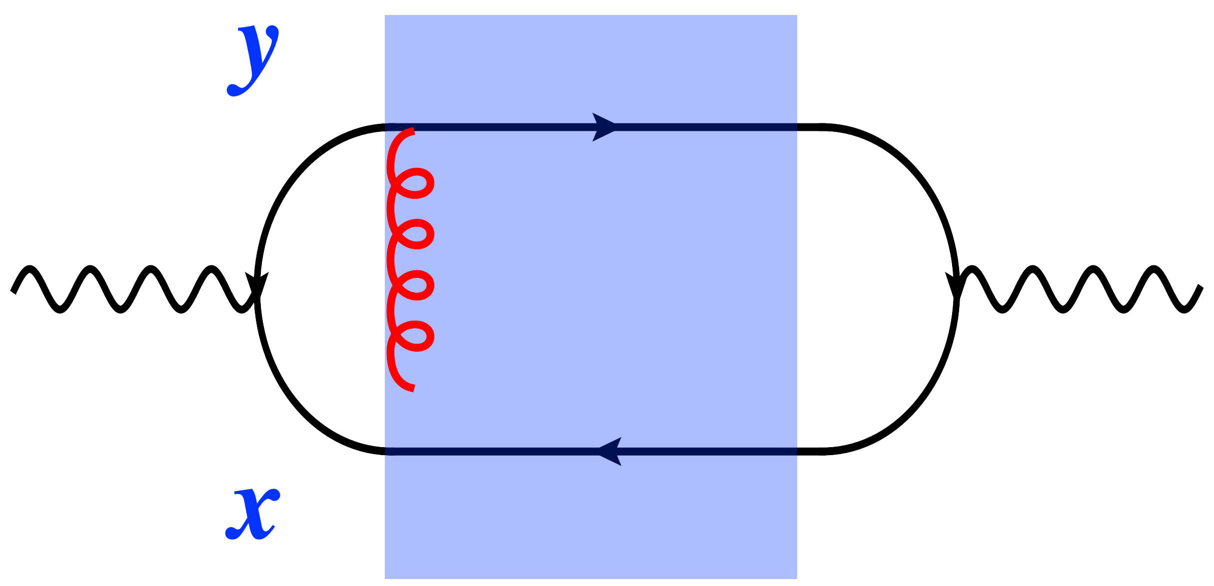


Longitudinal structure of the target

- **To restore x dependence: need to keep time ordering and convolution**
  - To do so: identify the first and last interaction with the target
  - There are 4 contributions: 2 for the initial time + 2 for the finial time
  - 4 terms combine into one! (restores explicit gauge invariance as well)

$$A^-(\boldsymbol{x}) - A^-(\boldsymbol{y}) = \int_0^1 \mathrm{d}s\, r^i \partial^i A^-(\boldsymbol{y} + s\boldsymbol{r}) = \int_0^1 \mathrm{d}z^i\, F^{i-}(\boldsymbol{z})$$



$\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$

Parallel transport in transverse direction - generates transverse gauge links

Longitudinal structure of the target

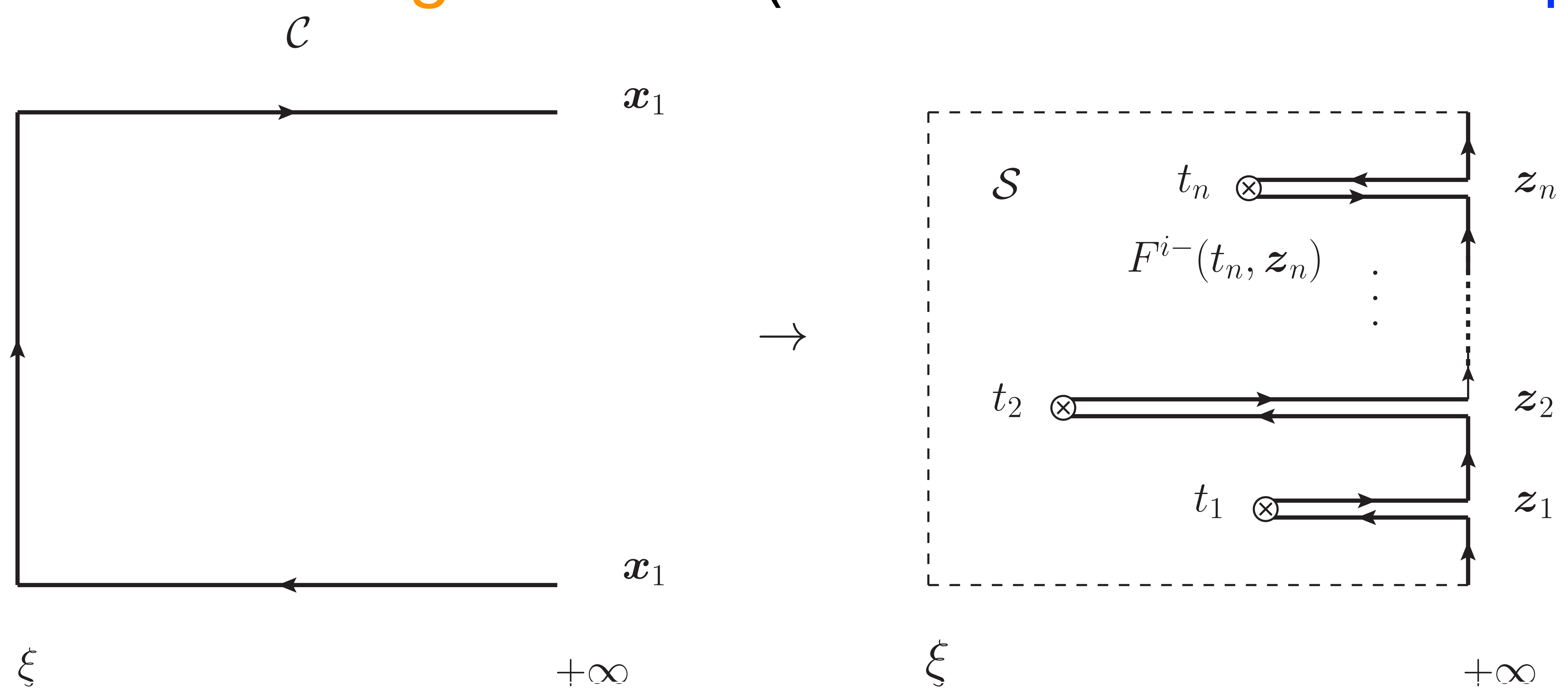
- **To restore x dependence: need to keep time ordering and convolution**
  - To do so: identify the first and last interaction with the target
  - There are 4 contributions: 2 for the initial time + 2 for the finial time
  - More generally we have

$$\frac{\partial^+}{\partial x^+}[y^+,x^+]_{\mathbf{x}}[x^+,y^+]_{\mathbf{y}} = \int_{\mathbf{z}} [y^+,x^+]_{\mathbf{x}}\, F^{i-}(x^+,\mathbf{z})\, [x^+,y^+]_{\mathbf{y}}$$

- ***which extracts the leading twist!***

Dipole ↔ parton distribution equivalence

- non-Abelian Stokes' theorem: the dipole operator can be written as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)

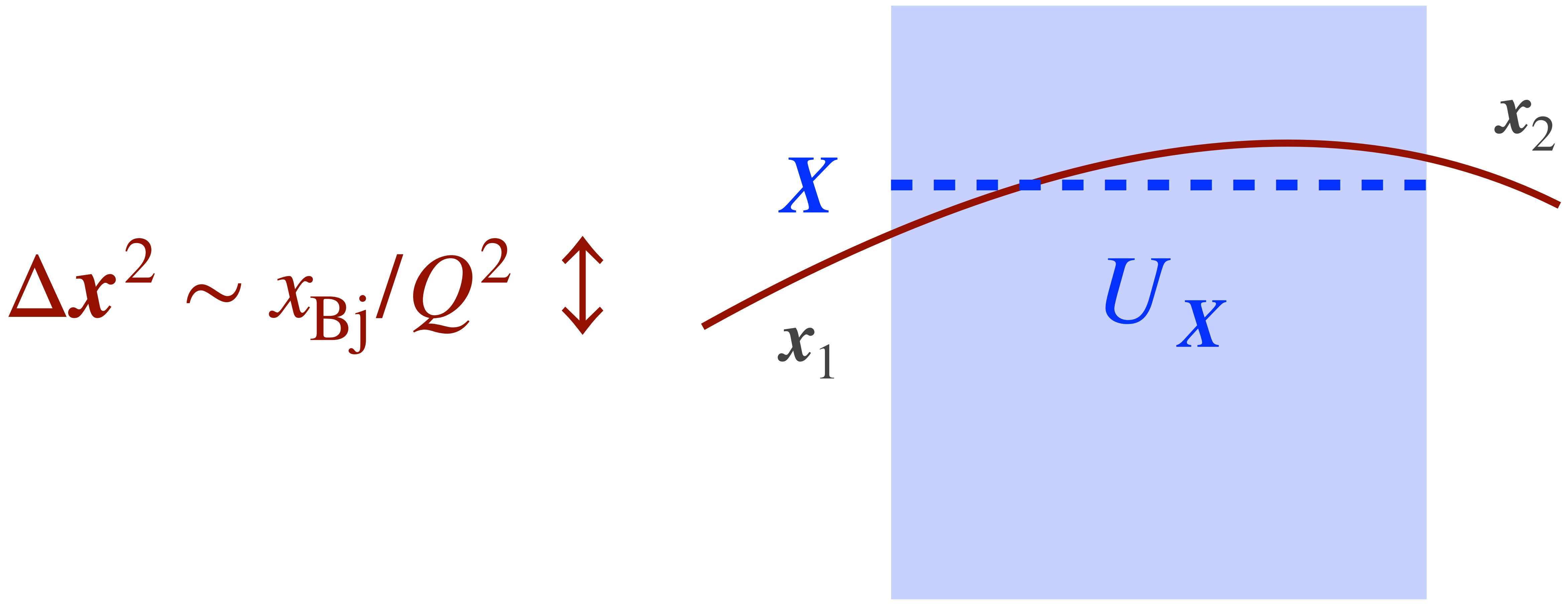


[Fishbane, Gasiorowicz, Kaus (1981) Wiedemann (2000) YMT, Boussarie (2020)]

$$U_{\boldsymbol{x}_2}U^\dagger_{\boldsymbol{x}_1} \equiv P\exp\left[-ig\int_S \mathrm{d}t\mathrm{d}\boldsymbol{z}\,[+\infty, x^+]_{\boldsymbol{x}}\, F^{i-}(x^+,\boldsymbol{x})\, [x^+, +\infty]_{\boldsymbol{x}}\right]$$

Partial twist expansion

- We aim at resumming higher twists $O(r_\perp^n) \sim O(1/Q^n)$ that are relevant at small x
- Neglect twist associated with x^- dependence of the gauge field $A^-(x^+, x^- = 0, x_\perp)$
- Neglect transverse recoil of high energy partons in the target - expand around classical trajectory



$$G_{p^+}(x^+, \boldsymbol{x}_2; y^+, \boldsymbol{x}_1) = G_0(\boldsymbol{x}_2 - \boldsymbol{x}_1, x_2^+ - y_1^+)\, U_X(x_2^+, x_1^+) + O(|\boldsymbol{x}_2 - \boldsymbol{x}_1|/X)$$

quantum phase Wilson line

Partial twist expansion (PTE)

- Quantum phase instrumental to restore x dependence

$$G_0 \sim \mathrm{e}^{-i\frac{k_\perp^2}{2p^+}\Delta x^+} \sim \mathrm{e}^{-ixP^-\Delta x^+}$$

- In strict shock wave limit: $p^+ \to +\infty$ and $G_0 \to 1$

$$G_{p^+}(x^+, \boldsymbol{x}_2; y^+, \boldsymbol{x}_1) = \delta(\boldsymbol{x}_2 - \boldsymbol{x}_1)\ U_X(x_2^+, x_1^+) + \ldots$$

quantum phase Wilson line

Revisiting rapidity factorization in DIS

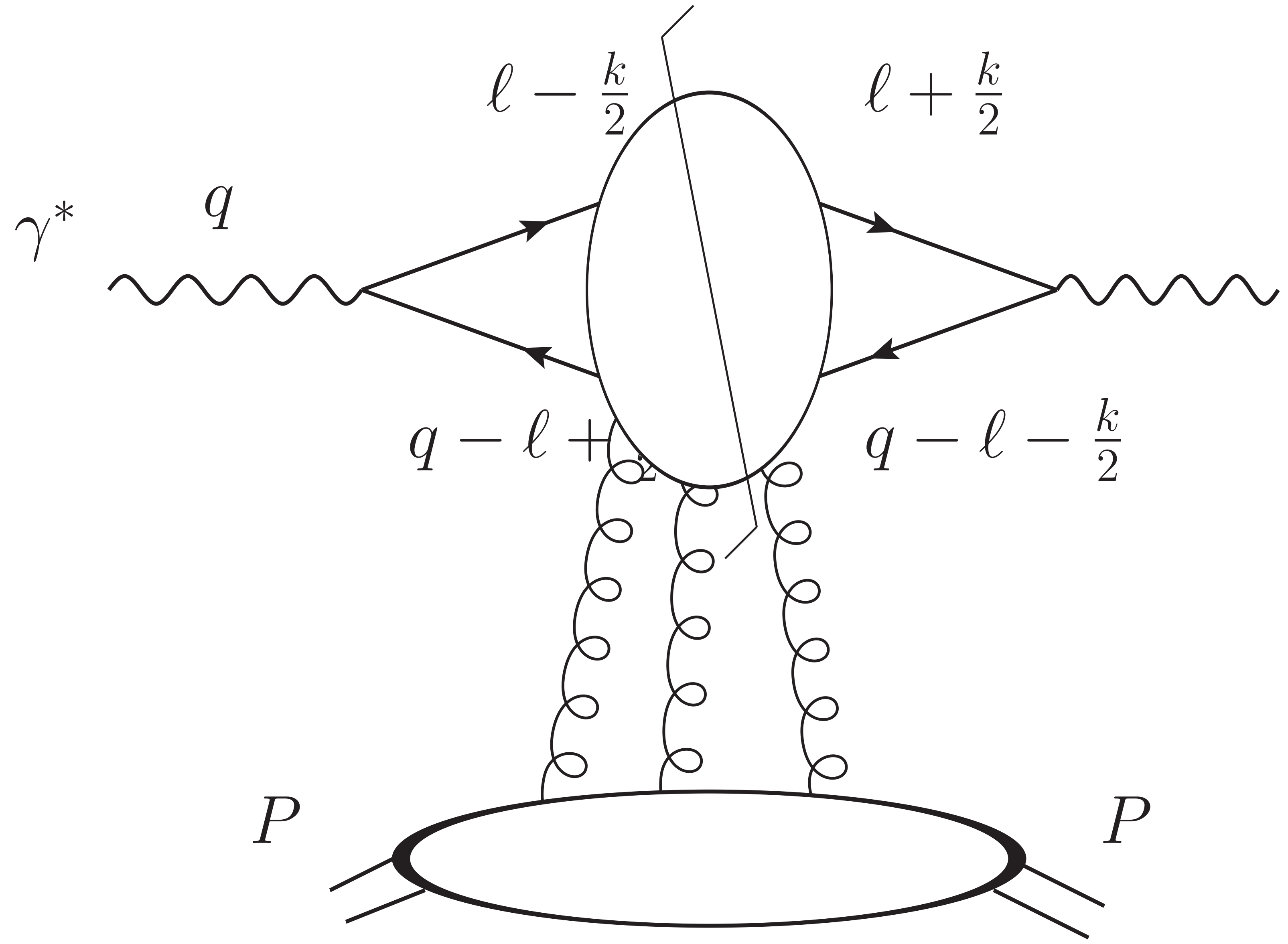
- After applying partial PTE to leading power we obtain the factorization formula (for the transverse photon cross-section), in momentum space,

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}x \int_{\boldsymbol{\ell},\boldsymbol{k}} \partial^i \varphi\left(\boldsymbol{\ell} - \frac{\boldsymbol{k}}{2}\right) \partial^j \varphi^*\left(\boldsymbol{\ell} + \frac{\boldsymbol{k}}{2}\right) \delta\left(x - x_{Bj} - \frac{\boldsymbol{\ell}^2}{2z\bar{z}q^+}\right)$$

$$\times\, xG^{ij}(x, \boldsymbol{k}) \;+\; O\left(k_\perp^2/s\right)$$

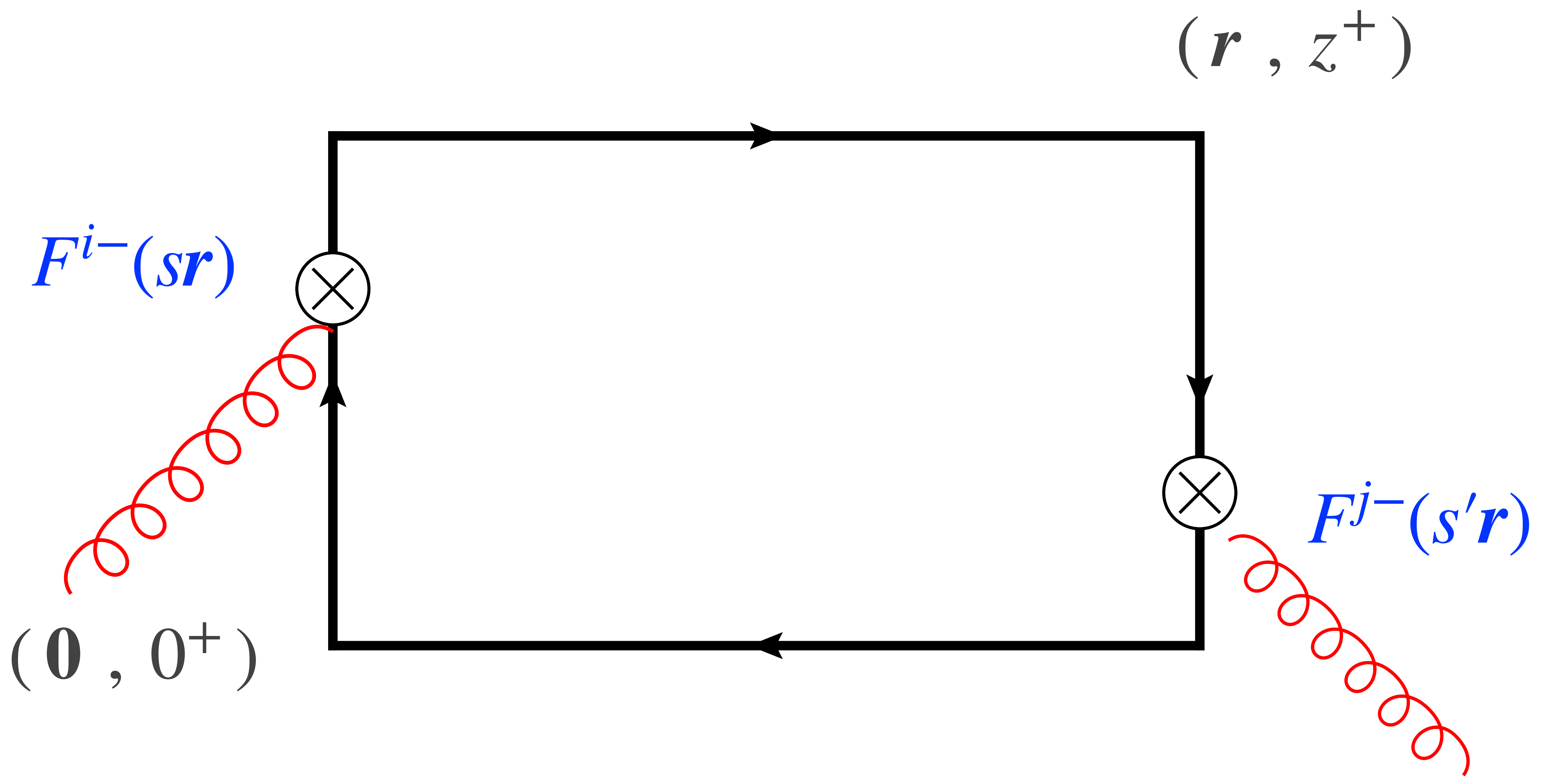
kinematic constraint

- Same wave functions as small x!
- The delta function relates x in the gluon distribution to x_{Bj} (kinematic constraint in momentum space)
- Gluon distribution different than small x



3D gluon distribution

$$xG^{ij}(x,k_\perp) \equiv 2\int_{s,s'}\int \frac{dz^+ d\boldsymbol{r}}{(2\pi)^3 P^-} e^{\,i\,x\,P^- z^+ - i\boldsymbol{k}\cdot\boldsymbol{r}} \,\langle P\,|\,\mathrm{Tr}\,[\,0\,,z^+]_{\boldsymbol{r}}\,F^{j-}(z^+, s'\boldsymbol{r})\,[\,z^+\,,0]_{\boldsymbol{0}}\,F^{i-}(\,0\,,s\boldsymbol{r})\,|\,P\rangle$$

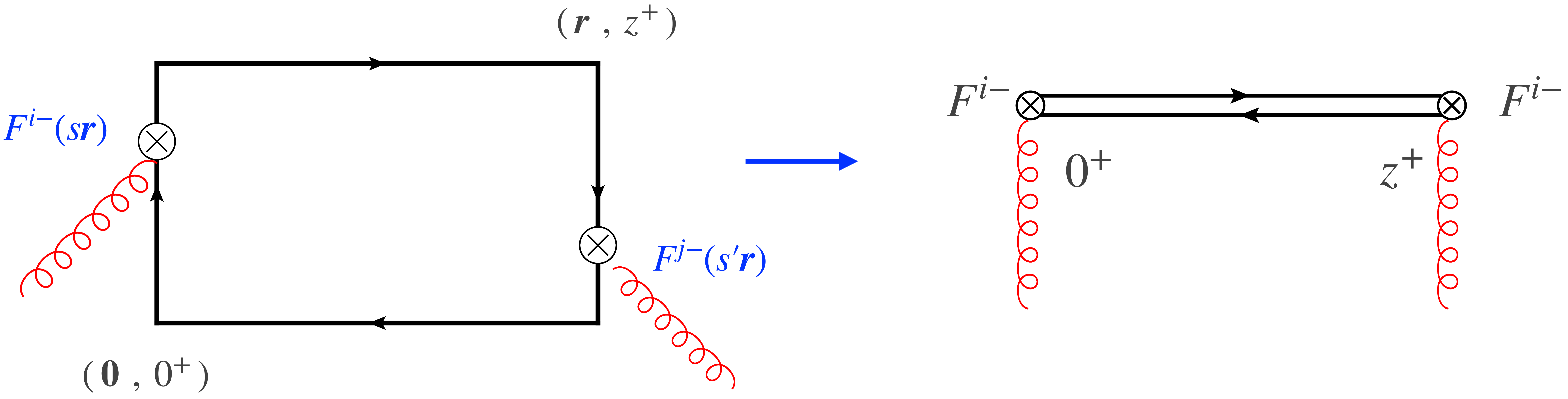


- Note that this uPDF involves finite Wilson lines in contrast with gluon TMD's such as Weizsacker-Williams

Bjorken limit

- Collinear limit: Integrating over $k_\perp$ yields $r_\perp = 0$ and we recover the gluon PDF

$$\int \mathrm{d}\boldsymbol{k}\, G^{ii}(x,\boldsymbol{k}) = xg(x) = 2\int \frac{\mathrm{d}\xi^+}{(2\pi)P^-}\, \mathrm{e}^{ixP^-\xi^+} \langle P|\,\mathrm{Tr}\,[0,z^+]\,F^{i-}(\xi^+)\,[z^+,0]\,F^{i-}(0)\,|P\rangle$$



Regge limit

- At small x. Neglect the phase $x = 0$ and we recover the dipole operator

$$r^i r^j G^{ij}(x = 0\,, \boldsymbol{r}) \;\to\; \langle P \,|\, \mathrm{Tr}\; U_{\boldsymbol{r}}\; U^{\dagger}_{\boldsymbol{0}} \,|\, P \rangle$$

- PTE provides the interpolation between the leading twist term in the Bjorken limit and the eikonal term in the Regge limit

DIS and DVCS (see Renaud's talk)

Interpolating scheme for exclusive Compton scattering

Overarching scheme

$$\int \mathrm{d}x \int \mathrm{d}^d k\, G^{ij}(x, \xi, k, \Delta) H^{ij}(x, \xi, k, \Delta)$$

Bjorken limit

$$\int \mathrm{d}x H^{ij}(x, \xi, 0, \Delta) \times [\int \mathrm{d}^d k\, G^{ij}(x, \xi, k, \Delta)]$$

Regge limit

$$\lim_{\xi \to 0} \int \mathrm{d}^d k\, G^{ij}(0, \xi, k, \Delta) \times \int \mathrm{d}x H^{ij}_{\mathrm{cut}}(x, \xi, k, \Delta)$$

We found an interpolating scheme

Bjorken limit for DIS

- In the collinear limit $Q^2 \to \infty$, we reproduce the 1-loop contribution to the DIS structure function

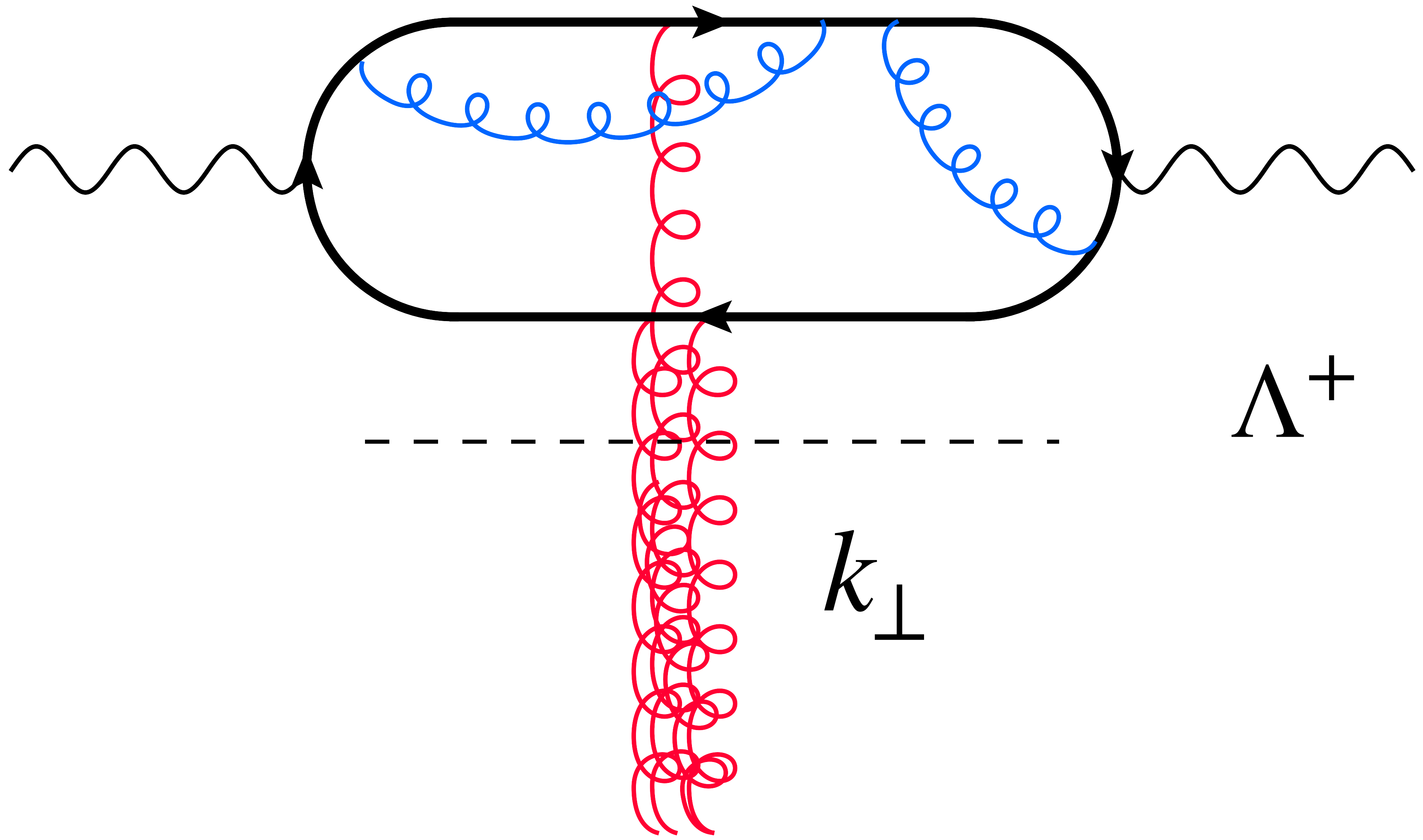
$$F_T(x_{Bj}, Q^2) = \frac{\alpha_s}{\pi} \sum_f q_f^2 \int_{x_{Bj}}^1 dy\ xg(x_{Bj}/y, \mu^2)$$

$$\times \left[\frac{1}{\epsilon} \left(\frac{e^{\gamma_E}}{4\pi} \right)^{\epsilon} P_{qg}(y) + \left[(1-y)^2 + y^2 \right] \log \left[\frac{Q^2(1-y)}{\mu^2 y} \right] - 1 + 4y(1-y) \right]$$

Rapidity evolution (work in progress)

- Evolution according to k^+ : increase the space of the gluon kt-distribution from 1 to 2 variables to account for the ordering of k^-

$$\varphi_G(\boldsymbol{k};\Lambda^+) \equiv \frac{1}{N_c}\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{r}\cdot\boldsymbol{k}} \langle \mathrm{tr}\, U_{\boldsymbol{0}} U_{\boldsymbol{r}}^{\dagger}\rangle_{\Lambda^+} \quad \rightarrow \quad G^{ij}_{\Lambda^+}(x,\boldsymbol{k})$$



Summary and outlook

- Minimal correction to the semi-classical approach to small x to restore x dependence using a partial twist expansion instead of the shock wave approximation - top down approach to collinear region of phase space
- In the case of inclusive DIS: while the hard part is unchanged we find a new (gauge invariant) 3D gluon distribution that interpolates between the dipole operator at small x and the gluon PDF at leading twist
- Outlook:
    - Investigate DVCS, TCS, DDVCS (Renaud's talk), TMD's
    - Compute quantum evolution of the 3D gluon distribution
    - Explore on the Lattice the emergence of the saturation scale?