DIS dijet production at next-to-eikonal accuracy and its back-to-back limit

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CGC at the EIC

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Eikonal approximation in the CGC

In the CGC framework two approximations adopted:

(i) Semi-classical approximation $\rightarrow$ dense target is represented by a strong semiclassical gluon field $\mathcal{A}^\mu(x)$

(ii) Eikonal approximation $\rightarrow$ can be understood as the limit of **infinite boost** of $\mathcal{A}^\mu(x)$:

- Under a boost of parameter γ_t along the "$-$" direction, strong ordering between the components of the field:

$$\mathcal{A}^- = O(\gamma_t) \;\gg\; \mathcal{A}_\perp = O(1) \;\gg\; \mathcal{A}^+ = O(1/\gamma_t)$$

  ⋆ Only the enhanced component of the background field ($\mathcal{A}^-$) is kept.

- Lorentz contraction of the background field $\mathcal{A}^\mu(x)$ (shockwave limit)

  ⋆ background field is localized around $x^+ = 0$ (no transverse motion within the target)

- $\mathcal{A}^\mu(x)$ independent on x^- (static limit) due to Lorentz time dilation

  ⋆ dynamics of the target is neglected (no p^+ transfer from the target).

Background field in the eikonal limit

$$\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu-}\mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$$

Eikonal interaction between the projectile and the target:

• each parton picks up a Wilson line during the interaction

$$U_{\mathcal{R}}(\mathbf{x}) = \mathcal{P}_+ \exp\left[ig\int dx^+ T^a_{\mathcal{R}} A^-_a(x^+, \mathbf{x})\right]$$

• dipole operator appears in the observable

$$d_{\mathcal{R}}(\mathbf{x}, \mathbf{y}) = \frac{1}{D_{\mathcal{R}}}\mathrm{tr}\left[U_{\mathcal{R}}(\mathbf{x})U^\dagger_{\mathcal{R}}(\mathbf{y})\right]$$

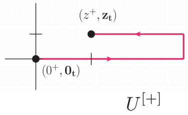
From dipole operators to gluon TMDs

[Collins (2002) / Belitsky, Ji, Yuan (2002) / Ji,Yuan (2002)]

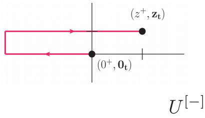
The unpolarized TMDs are defined as the FT of forward matrix elements of bilocal products gluon field strength tensor:

$$\mathcal{F}(x_2,k_t) \propto \int dz^+\, d^2z_\perp e^{ix_2 p_A^- z^+ - ik_t\cdot z_\perp} \langle p_A|\mathrm{tr}\big[F_0^{i-}\, U^{[C]}_{(0,z)}\, F_z^{i-}\, U^{[C']}_{(z,0)}\big]|p_A\rangle$$

$U^{[C]}_{(0,z)}$: gauge staples connecting the points $(0^+, 0_\perp)$ and $(z^+, z_\perp)$ to ensure gauge invariance.



future pointing



past pointing

• different choices to connect the points! → different TMDs enter different processes!

[Kotko, Kutak, Marquet, Sapeta, van Hameran (2015)]

• in the large k_t limit: the process dependence of the gauge links disappear! At small-x, all the TMDs share a universal perturbative tail (Unintegrated gluon distribution):

$$\mathcal{F}_{g/A}(x_2,k_t) = \mathrm{UGD}(x_2,k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

Gauge links from Wilson lines

[Dominguez, Marquet, Xiao, Yuan (2011)]

gluon TMDs at small-x: average over the state $|p_A\rangle$ is replaced with CGC averaging:

$$\frac{\langle p_A|\cdots|p_A\rangle}{\langle p_A|p_A\rangle} \to \langle\cdots\rangle_{x_2}$$

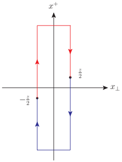
The Wilson line operator

$$U(-\infty,+\infty;x) = \mathcal{P}\exp\Big[ig\int_{-\infty}^{+\infty} dx^+ A^-(x^+,x)\Big]$$

Derivative of the Wilson line

$$\partial^i U(x) = ig\int dx^+ U(-\infty,x^+,x) F^{i-}(x^+,x) U(x^+,+\infty;x)$$

Dipole TMD

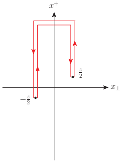


$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) \propto \int dz^+ d^2z\, e^{ix_2 p_A^- z^+ + ik_t z}\langle p_A|\mathrm{tr}\big[F^{i-}(\tfrac{z}{2})U^{[-]\dagger}F^{i-}(-\tfrac{z}{2})U^{[+]}\big]|p_A\rangle$$

<u>in the small-x limit:</u>

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) \to \int d^2z\, e^{ik_t z}\Big\langle \mathrm{tr}\Big\{\Big[\partial^i U^\dagger(\tfrac{z}{2})\Big]\Big[\partial^i U(-\tfrac{z}{2})\Big]\Big\}\Big\rangle_{x_2}$$

Weizsäcker-Williams TMD



$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) \propto \int dz^+ d^2z\, e^{ix_2 p_A^- z^+ + ik_t z}\langle p_A|\mathrm{tr}\big[F^{i-}(\tfrac{z}{2})U^{[+]\dagger}F^{i-}(-\tfrac{z}{2})U^{[+]}\big]|p_A\rangle$$

<u>in the small-x limit:</u>

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) \to \int d^2z\, e^{ik_t z}\Big\langle \mathrm{tr}\big\{\big[\partial^i U(\tfrac{z}{2})\big]U^\dagger(-\tfrac{z}{2})\big[\partial^i U(-\tfrac{z}{2})\big]U^\dagger(\tfrac{z}{2})\big\}\Big\rangle_{x_2}$$

Correlation limit in the CGC

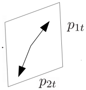
Dominguez, Marquet, Xiao, Yuan (2011)
Marquet, Petreska, Roiesnel (2016)
Petreska (2018)

How do we get the derivates of the Wilson lines?

Consider production of two hard jets:

$|p_1| \sim |p_2| \gg Q_s$ & total momenta comes from the target: $|p_1 + p_2| \sim Q_s$

Two typical transverse scale that appears:
$k_t = p_1 + p_2$: *total momentum of the produced jets*
$\mathbf{P} = z_2 p_1 - z_1 p_2$: *momentum imbalance of the two jets*



$k_t \ll \mathbf{P}$: *jets fly almost back-to-back (correlation limit).* $\Rightarrow$ small transverse size

We can perform a Taylor expansion of the Wilson and lines get access to TMDs

$$U_{b+\frac{r}{2}} U_{b-\frac{r}{2}} - 1 = \frac{r^i}{2}\left[(\partial^i U_b) U_b - U_b(\partial^i U_b)\right] + O(r^2)$$

in the small-x limit of TMDs: phase drops - only longitudinal dependence is in staple gauge links.

in the correlation limit of the CGC: expansion around small dipole size $\rightarrow$ derivatives of the Wilson lines

small-x limit of TMD factorization $\equiv$ correlation limit of the CGC

Small-x improved TMD factorization

Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015-2016)
TA, Boussarie, Kotko (2019)

$$d\sigma \propto f(x_1) \sum_c H_{(c)}(\mathbf{P}, k_t) \times \mathrm{TMD}_{(c)}(x_2, k_t)$$

- $f(x_1)$ → Standard collinear PDF for the large-x projectile
- $H_{(c)}(\mathbf{P}, k_t)$ → Off-shell hard factors
- $\mathrm{TMD}_{(c)}(x_2, k_t)$ → Several gluon TMDs for the small-x target

$$\mathrm{iTMD} \propto f(x_1) \sum_c \left[H_{(c)}(\mathbf{P},0)_{(c)} + \mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(c)} \right] \times \left[\mathrm{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2}{k_t^2}\right)_{(c)} \right]$$

- $H_{(c)}(\mathbf{P},0)_{(c)}$ → Leading-twist hard factor
- $\mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)$ → Kinematic higher twists
- $\mathrm{UGD}(x_2, k_t)$ → Universal perturbative tail
- $\mathcal{O}\left(\frac{Q_s^2}{k_t^2}\right)$ → Leading-twist saturation corrections

$$\mathrm{TMD} \propto f(x_1) \sum_c \left[H_{(c)}(\mathbf{P},0)_{(c)} + \cancel{\mathcal{O}\left(\frac{k_t^2}{\mathbf{P}^2}\right)_{(c)}} \right] \times \left[\mathrm{UGD}(x_2, k_t) + \mathcal{O}\left(\frac{Q_s^2}{k_t^2}\right)_{(c)} \right]$$

- $\mathrm{iTMD} + (Q_s/\mathbf{P})^n \to \mathrm{CGC}$ — TA, Boussarie (2019) / Boussarie, Mehtar-Tani (2020)

What about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

⋆ NEik corrections arise from relaxing either of the three approximations:

1. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$.

2. Target with finite longitudinal width $\Rightarrow$ transverse motion of the parton within the medium.

3. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation:

   - Treated as gradient expansion around a common x^- value:

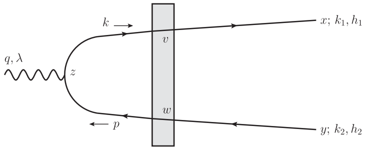
$$\boxed{\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)}$$

$\Rightarrow$ allows possibility of (small) p^+ exchange with the target

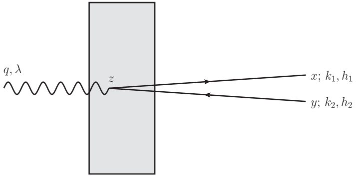
DIS dijet production at NEik accuracy

Two different diagrams contribute to the DIS dijet production at NEik accuracy:

Photon splitting to $q\bar{q}$ pair before the medium



Photon splitting to $q\bar{q}$ pair inside the medium



$$S^{\text{in}}_{q_1\bar{q}_2\leftarrow\gamma^*} \propto \epsilon^{\lambda}_{\mu}(q)\,\bar{u}(1)\Big[\,\#^{j}_{1}\,\gamma^{+}\,\gamma^{j}\,\gamma^{\mu}\;+\#^{j}_{2}\,\gamma^{\mu}\,\gamma^{+}\,\gamma^{j}+\#^{ij}_{3}\;g^{\mu+}\,\gamma^{+}\,\gamma^{i}\,\gamma^{j}\,\Big]v(2)$$

longitudinal polarization vector $\epsilon^{L}_{\mu}(q) = g_{\mu}{}^{+}Q/q^{+}$, then we get: $\gamma^{+}\gamma^{+} = 0$ & $\{\gamma^{+},\gamma^{j}\} = 0$ & $\epsilon^{+}_{\lambda}(q) = 0$

$\Rightarrow$ Contribution from the photon splitting inside the medium vanishes for dijet production via longitudinal photon.

$\star$ Dijet production via longitudinal photon $\rightarrow$ only splitting to $q\bar{q}$ pair before the medium contributes

$\star$ Dijet production via transverse photon $\rightarrow$ both splitting to $q\bar{q}$ pair before the medium and inside the medium contribute.

Disclaimer: We will only consider the DIS dijet production via longitudinal photon.

DIS dijet at NEik accuracy

[TA, Beuf, Czajka, Tymowska (2022)]

S-matrix element at NEik accuracy (longitudinal photon polarization)

$$S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{Gen. Eik}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dyn. target}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } q} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } \bar{q}}$$

with

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{Gen. Eik}} = -\,2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2}\,\theta(q^+ + k_1^+ - k_2^+)\,\theta(q^+ + k_2^+ - k_1^+) \times \int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\, \mathrm{K}_0\Big(\hat{Q}\,|\mathbf{w}-\mathbf{v}|\Big)\int db^-\, e^{ib^-(k_1^+ + k_2^+ - q^+)}\Big[\mathcal{U}_F\Big(\mathbf{v},b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{w},b^-\Big) - 1\Big]$$

zeroth order term in the expansion around a common value $b^- = (v^- + w^-)/2$

Its form resembles the strict eikonal term with extra b^- dependence.

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dyn. target}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\; iQ\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+v(2)\,\frac{(k_1^+ - k_2^+)}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2} \times \Big[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right) - \frac{(\bar{Q}^2 - m^2)}{2\bar{Q}}\,|\mathbf{w}-\mathbf{v}|\,\mathrm{K}_1\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)\Big]\Big[\mathcal{U}_F\Big(\mathbf{v},b^-\Big)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\Big(\mathbf{w},b^-\Big)\Big]\Big|_{b^-=0}$$

first term in the expansion of the around the common value b^-.

DIS dijet at NEik accuracy

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } q} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\;\bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_F(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}^\dagger_F(\mathbf{w})\,v(2)$$

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}\Big|_{\text{dec. on } \bar{q}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_1^+}{(q^+)^2}\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\,e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\;\bar{u}(1)\gamma^+\left[\mathcal{U}_F(\mathbf{v})\left(\frac{[\gamma^i,\gamma^j]}{4}\mathcal{U}^{(3)\dagger}_{F;ij}(\mathbf{w}) - i\,\mathcal{U}^{(2)\dagger}_F(\mathbf{w}) + \left(\frac{i}{2}\overleftarrow{\partial_{\mathbf{v}^j}} - \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2}\right)\mathcal{U}^{(1)\dagger}_{F;j}(\mathbf{w})\right)\right]v(2)$$

Stem from finite width and the interaction with the transverse component of the background field.

<u>decorated Wilson lines:</u>

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right)\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}}\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$
$$\mathcal{U}^{(2)}_F(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right)\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$
$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right) g t\cdot\mathcal{F}_{ij}(\underline{v})\,\mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$

Rewriting NEik corrections as $\mathcal{F}^{\mu\nu}$ insertions

The relation between derivatives of the Wilson lines and field strength insertions:

$$\partial_\mu \mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) + igt\cdot\mathcal{A}_\mu(x^+,\mathbf{v},v^-)\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) - ig\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-)t\cdot\mathcal{A}_\mu(y^+,\mathbf{v},v^-)$$
$$= -ig\int_{y^+}^{x^+} dv^+ \mathcal{U}_F(x^+,v^+;\mathbf{v},v^-)t\cdot\mathcal{F}_\mu{}^-(v)\mathcal{U}_F(v^+,y^+;\mathbf{v},v^-) \qquad \text{for } \mu\neq +$$

DIS dijet production cross section at NEik accuracy written in terms of field strength insertions!

Expressions are lengthy before considering the back-to-back limit! e.g.

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{NEik corr.}}^{\text{dec. on } q} = (2q^+)\,2\pi\delta(k_1^+ + k_2^+ - q^+)\,8k_1^+k_2^+Q^2\left(\frac{ee_f}{2\pi}\right)^2\frac{k_1^+k_2^+}{(q^+)^3}\,\frac{k_2^+}{2(q^+)^3}$$
$$\times\, 2\text{Re}\int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})}\,e^{i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\text{K}_0\left(\bar{Q}\,|\mathbf{w}'-\mathbf{v}'|\right)\text{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\,\text{Tr}\Big\langle\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}') - 1\Big]\Big[\Big(-i\mathcal{U}_F^{(2)}(\mathbf{v}) + \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\Big)\mathcal{U}_F^\dagger(\mathbf{w}) + \frac{i}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\;\partial_{\mathbf{w}^j}\mathcal{U}_F^\dagger(\mathbf{w})\Big]\Big\rangle$$

with

$$\mathcal{U}_F^{(2)}(\mathbf{v}) = \int_{z^+,z'^+}(z^+ - z'^+)\,\theta(z^+ - z'^+)\mathcal{U}_F(+\infty,z^+,\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z^+,z'^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z'^+,\mathbf{v})]\mathcal{U}_F(z'^+,-\infty;\mathbf{v})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{v}) = -2\int_{z^+} z^+\,\mathcal{U}_F(+\infty,z^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z^+,-\infty;\mathbf{v})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\;\partial_{\mathbf{w}^j}\mathcal{U}_F^\dagger(\mathbf{w}) = -2\int_{z^+,w^+} z^+\,\mathcal{U}_F(+\infty,z^+;\mathbf{v})[-igt\cdot\mathcal{F}_j{}^-(z^+,\mathbf{v})]\mathcal{U}_F(z^+,-\infty;\mathbf{v})$$
$$\times\,\mathcal{U}_F^\dagger(w^+,-\infty;\mathbf{w})[igt\cdot\mathcal{F}_j{}^-(w^+,\mathbf{w})]\mathcal{U}_F^\dagger(+\infty,w^+;\mathbf{w})$$

Remark: Terms with $\mathcal{F}_{ij}$ insertions cancel at cross section level for γ_L^*, but survive for γ_T^*

Back-to-back limit

Back-to-back limit of dijets are conveniently expressed in terms of:

(total dijet momentum) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$$\boxed{\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}} \qquad \boxed{\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}}$$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

$$\boxed{\mathbf{v} = \mathbf{b} + z_2\,\mathbf{r}} \qquad \boxed{\mathbf{w} = \mathbf{b} - z_1\,\mathbf{r}}$$

back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

perform a small $\mathbf{r}$ expansion at the level of the squared amplitude

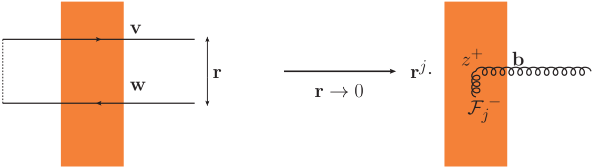
Small $\mathbf{r}$ expansion for the eikonal contribution

We perform small $\mathbf{r}$ expansion and use the following identity to simplify our results:

$$\Big[U_R\, T_R^a\, U_R^\dagger\Big]_{ij} = \Big[T_R^b\Big]_{ij}\Big(U_A\Big)_{ab}$$

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\Big[\mathcal{U}_F\Big(\mathbf{v}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{w}, b^-\Big) - 1\Big] = -\frac{\mathbf{r}^j}{2}\Big[\mathcal{U}_F\Big(\mathbf{b}, b^-\Big)\overleftrightarrow{\partial_{\mathbf{b}^j}}\mathcal{U}_F^\dagger\Big(\mathbf{b}, b^-\Big)\Big] + O(\mathbf{r}^2)$$
$$= \mathbf{r}^j(-igt^a)\int_{z^+}\mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab}\,\mathcal{F}_j^{b\,-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$

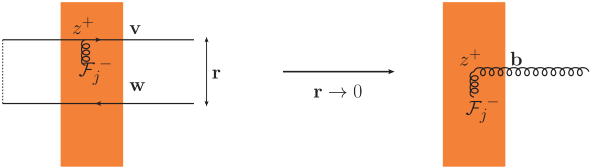


Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order needed

Small $\mathbf{r}$ limit for the NEik corrections

Open decorated dipole with $\mathcal{U}^{(1)}_{F;j}$:

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{v})\mathcal{U}^{\dagger}_{F}\Big(\mathbf{w}\Big) = -2\int_{z^+} z^+\,\mathcal{U}_F\Big(+\infty, z^+;\mathbf{v}\Big)(-ig)t\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{v})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{v}\Big)\mathcal{U}^{\dagger}_{F}(\mathbf{w})$$
$$= 2igt^a\int_{z^+} z^+\,\mathcal{U}_A\Big(+\infty, z^+;\mathbf{b}\Big)_{ab}\,\mathcal{F}^{b\;-}_{j}(z^+,\mathbf{b}) + O(|\mathbf{r}|)$$



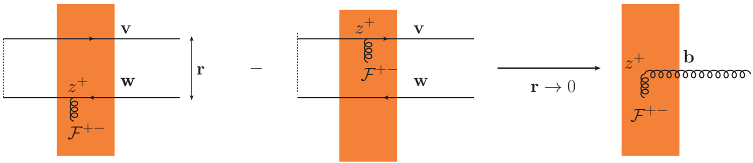
- Nontrivial result already at 0th order in the small $\mathbf{r}$ expansion due to the decoration
- Similar result as for the Generalized Eikonal contribution, except for the z^+ factor

Small $\mathbf{r}$ limit for the NEik corrections

Open decorated dipole from the dynamics of the target:

$$\left[\mathcal{U}_F\Big(\mathbf{v},b^-\Big)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\Big(\mathbf{w},b^-\Big)\right]\Big|_{b^-=0} = \int_{z^+}\Big\{\mathcal{U}_F(\mathbf{v})\mathcal{U}_F^\dagger\Big(z^+,-\infty;\mathbf{w}\Big)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{w})\mathcal{U}_F^\dagger\Big(+\infty,z^+;\mathbf{w}\Big)$$
$$-\mathcal{U}_F\Big(+\infty,z^+;\mathbf{v}\Big)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{v})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{v}\Big)\mathcal{U}_F^\dagger(\mathbf{w})\Big\}$$
$$= 2igt^a\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b}) + O(|\mathbf{r}|)$$

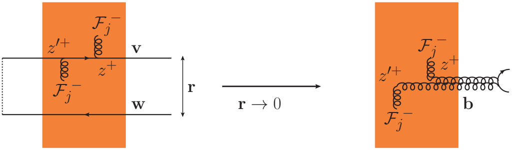
$\Rightarrow$ Involves the longitudinal chromoelectric field $\mathcal{F}^{+-}$ instead of the transverse field $\mathcal{F}_j^{\;-}$



Small $\mathbf{r}$ limit for the NEik corrections

Open decorated dipole with $\mathcal{U}_F^{(2)}$:

$$\mathcal{U}_F^{(2)}(\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w}) = \int_{z^+,z'^+}(z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_F(+\infty,z^+,\mathbf{v})[-igt\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{v})]$$
$$\times\,\mathcal{U}_F(z^+,z'^+;\mathbf{v})[-igt\cdot\mathcal{F}_j^{\;-}(z'^+,\mathbf{v})]\mathcal{U}_F(z'^+,-\infty;\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})$$
$$= -g^2(t^at^b)\int_{z^+,z'^+}(z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{aa'}\mathcal{F}_j^{a'\;-}(z^+,\mathbf{b})$$
$$\times\,\mathcal{U}_A\Big(+\infty,z'^+;\mathbf{b}\Big)_{bb'}\mathcal{F}_j^{b'\;-}(z'^+,\mathbf{b})+O(|\mathbf{r}|)$$



2 Field strength insertions at amplitude level
$\Rightarrow$ At least 3 at cross section level (beyond TMDs)

Back-to-back cross section: (Generalized) Eikonal piece

The dijet cross section for the longitudinal photon in the back-to-back correlation limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen. Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{NEik corr.}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen.Eik}} = 2q^+\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}(ee_f)^2(q^++k_1^+-k_2^+)^2(q^+-k_1^++k_2^+)^2\frac{2k_1^+k_2^+}{(q^+)^6}\frac{Q^2\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4}$$
$$\times g^2T_F\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\bigg\langle\mathcal{F}_i^{a\,-}\Big(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\Big[\mathcal{U}_A^\dagger\Big(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\Big)\Big]_{ab}$$
$$\times\mathcal{F}_j^{b\,-}\Big(z^+,\mathbf{b},\frac{\Delta b^-}{2}\Big)\bigg\rangle$$

On the other hand:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Gen. Eik}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Strict Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{NEik corr. from Gen. Eik}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Strict.Eik}} = 2q^+2\pi\delta(k_1^++k_2^+-q^+)(ee_f)^2g^2T_F32z_1^3z_2^3Q^2\frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle\mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\Big[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\Big]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\Big\rangle$$

- **Strict eikonal contribution: Twist-2 gluon TMDs** (both linearly polarized and unpolarized)
- *NEik corr. from Gen. Eik: Either 2 or 3-body terms: twist 4?* (study is still in progress!)

Back-to-back cross section: NEik terms

Explicit NEik corrections to the dijet cross section for the longitudinal photon in the back-to-back correlation limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\bigg|_{\text{NEik corr.}}^{\text{corr. lim.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\bigg|_{\text{dyn. target}}^{\text{corr. lim.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}}$$

with

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\bigg|_{\text{dyn. target}}^{\text{corr. lim.}} = -(2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F \frac{(k_1^+k_2^+)^2(k_1^+-k_2^+)}{(q^+)^6}\frac{16Q^2\mathbf{P}^i}{(\mathbf{P}^2+\bar{Q}^2)^3}\left[1-\frac{(\bar{Q}^2-m^2)}{\mathbf{P}^2+\bar{Q}^2}\right]2\text{Re}$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})\right\rangle$$

$\Rightarrow$ NEik. correction stemming from the dynamics of the target is a **twist-3 gluon TMD**. [Mulders, Rodrigues (2000)]

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 16 z_1^2 z_2^2 Q^2\frac{\mathbf{P}^i(2\mathbf{P}^j-(z_2-z_1)\mathbf{k}^j)}{(\mathbf{P}^2+\bar{Q}^2)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}$$
$$\times\int_{z^+,z'^+} i(z^+-z'^+)\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$
$$+\,3\,\text{body contributions}$$

- The term proportional to k^j is a kinematical twist 3 contribution.
- **The main contribution from this term is a contribution to twist-2 gluon TMDs.**

Twist 2 term from NEik corrections

Leading twist contributions from Strict Eik. and NEik (dec. on $q+\bar{q}$) terms can be combined:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict Eik}}^{\text{corr. lim.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}} \simeq 2\pi(2q^+)\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 32 z_1^3 z_2^3 Q^2 \frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left[1 + i(z^+ - z'^+)\frac{\mathbf{P}^2+\bar{Q}^2}{2q^+z_1z_2}\right] \Big\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}') \Big[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\Big]_{ab} \mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\Big\rangle$$

On the other hand, the " $-$ " momentum extracted from the target can be defined from the conservation relation:

$$xP^-_{tar.} \equiv \check{k}_1^- + \check{k}_2^- - q^- = \frac{\mathbf{k}_1^2+m^2}{2k_1^+} + \frac{\mathbf{k}_2^2+m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{\mathbf{P}^2+\bar{Q}^2}{2q^+z_1z_2} + \frac{\mathbf{k}^2}{2q^+}$$

- $\mathbf{k}^2$ term is a kinematical twist 4 contribution (can be neglected to our accuracy!)

The leading twist contribution can be summed into a phase! $\Rightarrow$ x dependence of the twist 2 gluon TMDs

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict Eik}}^{\text{corr. lim.}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{dec. on } q+\bar{q}}^{\text{corr. lim.}} \simeq 2\pi(2q^+)\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 T_F 32 z_1^3 z_2^3 Q^2 \frac{\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} e^{i(z^+-z'^+)xP^-_{tar.}} \Big\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}') \Big[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\Big]_{ab} \mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\Big\rangle$$

Summary

$\star$To further study the interplay between CGC and TMD frameworks, we studied the back-to-back limit of the DIS dijet production at NEik accuracy.

We obtained various contributions:

- Leading twist term: interpreted as the first order expansion of the x phase from the gluon TMD definition. (stems from strict eikonal term together with the "$\mathrm{dec.\ on}\ q+\bar{q}$")
- Kinematical twist 3 terms. (stems from the "$\mathrm{dec.\ on}\ q+\bar{q}$" term)
- Twist 3 gluon TMD: This term has one $\mathcal{F}^{+-}$ as an insertion instead of $\mathcal{F}_i^{\ -}$. (stems from the "$\mathrm{dyn.\ target}$" contribution)
- Correlators of 3 field strengths: These terms stem from the expansion of the "Generalized Eikonal" terms after expanding it around $b^- = 0$ (correction to the strict eikonal limit in this specific contribution)