Use of the gradient flow in matching the QED 2+1 Hamiltonian and Lagrangian

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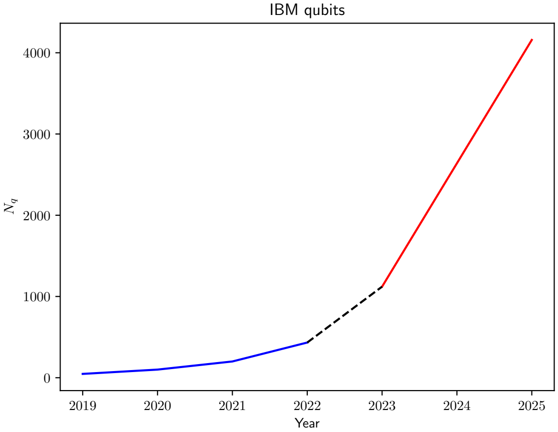
EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Quantum simulations

Resources

NISQ era: $O(10-100)$ noisy qubits.



Introduction

QED 2+1

- Toy model for QCD (confinement, dynamical mass generation)

$$V(r) = \frac{\alpha}{r} + \sigma r + b \log r$$

- Cheaper than QCD: $N_c = 1$, $d = 3 \rightarrow$ Hamiltonian simulations

Introduction

QED 2+1

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- Hamiltonian simulations: either $\beta \ll 1$ (electric basis) or $\beta \gg 1$ (magnetic basis).
  Small volumes $\rightarrow$ significant Finite Volume Effects
- Lagrangian simulations: can go to large L, but $\beta \sim O(1)$.
- $\rightarrow$ we need to match the 2 formalisms to cover all values of β.

The gradient flow provides a natural choice for setting the scale

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Hamiltonian simulations

Advantages

- Real time dynamics

$$|\psi(t)\rangle = e^{-iHt}\,|\psi(0)\rangle$$

- No sign problem (finite baryon density, topological terms, ...)
- and out-of-equilibrium dynamics
- No critical slowing down

But at present...

At the moment only toy models [1]

- (1+1) and (2+1) dimensions
- Small volumes
- Low spectrum cutoff

Hamiltonian-Lagrangian matching

Motivation

In the continuum Lagrangian and Hamiltonian are equivalent (!)
$\rightarrow$ at finite a_s the need to be matched

Pure gauge theory [2]:

$$H \sim \sum_{\vec{x}} \frac{1}{\beta} E^2(\vec{x}) + \beta \,\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(\vec{x})$$

$$S_W \sim -\beta \sum_{t,\vec{x}} \mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(t,\vec{x})$$

But β in H is not the same as in L!
H sums are only spatial...

Question: How do I find β_H from β_L?

Hamiltonian-Lagrangian matching

Hamiltonian limit

We need to find the "Hamiltonian limit", i.e. send a_t while keeping a function of a_s fixed.

- Usual approach: Send the anisotropy $a_t/a_s \to 0$ while a_s is constant:

$$O^H = \lim_{a_t \to 0, a_s = \text{const.}} O^L$$

- Brute force: choose a set of matching observables and find the relations among the couplings

$$\vec{g}^H = f(\vec{g}^L)$$

- Look at a renormalized quantity, and match it up to lattice artifacts.

Matchable observables

In pure gauge theory

- Static potential

$$V(r) = \frac{\alpha}{r} + \sigma r + b \log r$$

- Plaquette:

$$\langle P \rangle \sim \sum_x \sum_{i,j} P_{ij}(x)$$

- Mass gap of the theory and ground state:

$$\langle \psi | H | \psi \rangle \to E_0$$

- First excited states: $E_1, \ldots$

QED 2+1 observables

- States beyond the ground state:
  - noisy correlators
  - truncation bias in the Hamiltonian simulations
- Static potential:
  - At small volume FVEs are large $\rightarrow$ really bad signal-to-noise ratio
  - Very small volumes on the Hamiltonian $\rightarrow$ in practice we can compute only $V(a)$ and $V(a\sqrt{2})$.

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- Mass gap (glueball 0^{--}): ok but λ_{Compton} must fit the spatial box $\rightarrow$ large volumes with the MC simulations
  $\rightarrow$ match with Hamiltonian requires extrapolation to small L.
- Plaquette expectation value

Matching steps

$$S_W = \frac{\beta}{\xi_0} \sum_{x,i} \mathrm{Re}\,(1 - P_{0i}(x)) + \beta\xi_0 \sum_{x,i>j} \mathrm{Re}\,(1 - P_{ij}(x))$$

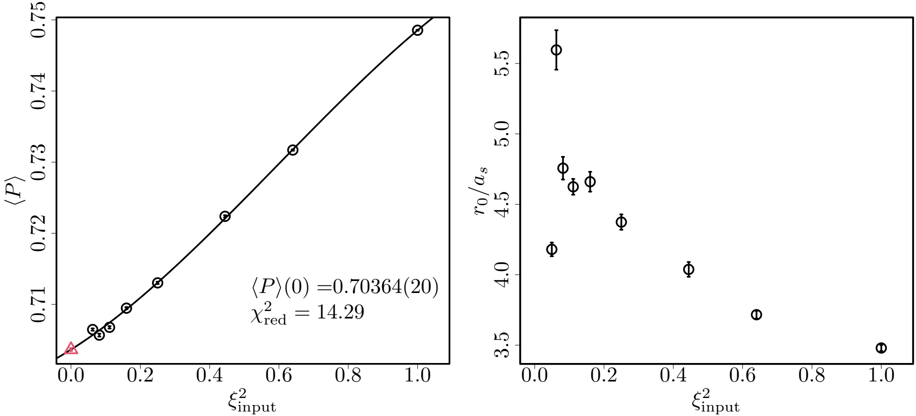
Note: $\xi_R = a_t/a_s \neq \xi_0$

- Start at a given β and $\xi_0 = 1$.
  Find r_0/a_s.
- Change ξ_0 ($\leq$ previous one), and change β such that r_0/a_s stays constant.
- Repeat

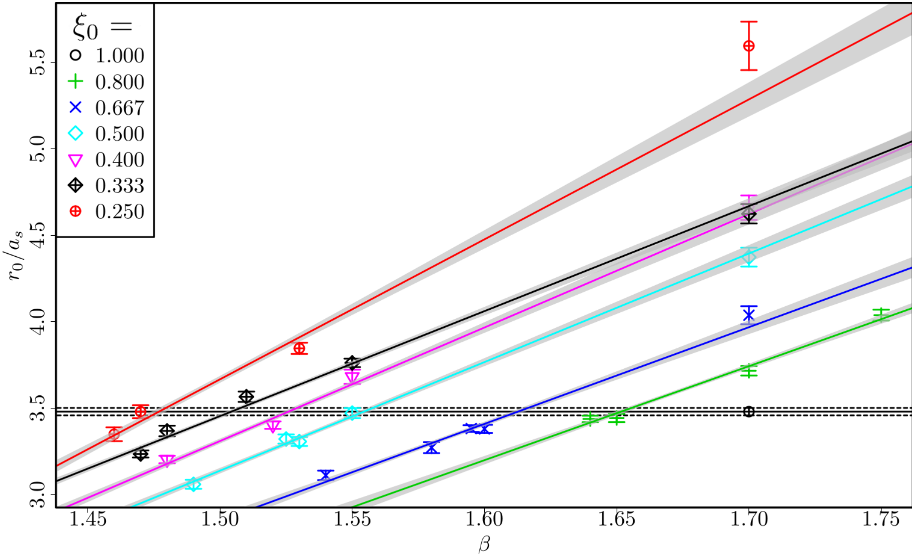
Up to $O(a_s)$ artifacts:

$$\lim_{a_t \to 0, a_s = \text{const}} \ldots = \lim_{\xi_R \to 0, r_0/a_s = \text{const}} \ldots$$

Naive Hamiltonian limit

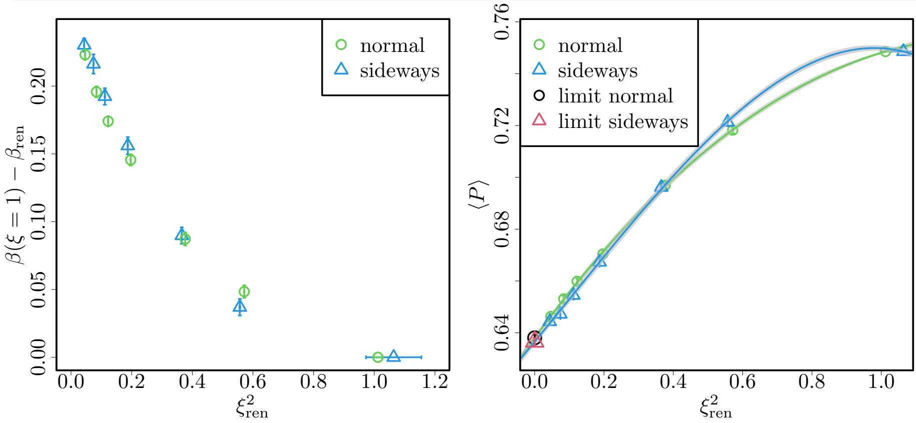


Matching steps (continued)



$\xi_R \to 0$ extrapolation

ξ_R found from the static potential



ξ_R from the Gradient Flow

Gradient flow evolution [3]

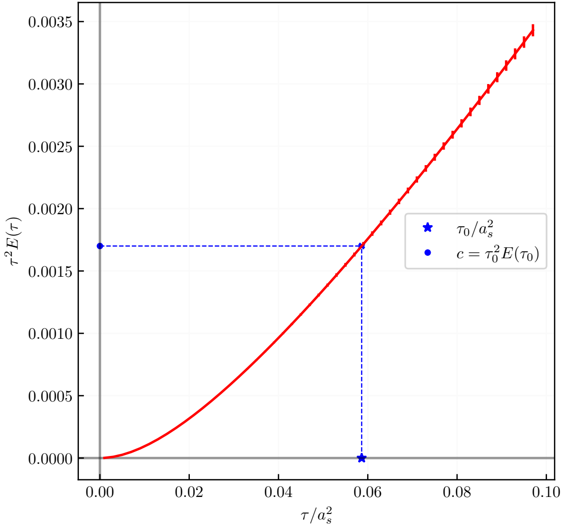
$$\dot{U}_\mu(x,\tau) = -\frac{1}{\beta}\left[\nabla_\mu(x) S_W(U)\right] U_\mu(x,\tau)\,,$$

Scale setting

$$E(\tau) = 2\sum_x \sum_{\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - P_{\mu\nu}(x,\tau)\right]\,.$$

$$\tau^2 E(\tau)|_{\tau=\tau_0} = c\,,$$

Scale setting



Renormalized anisotropy

Plaquette energy contributions

$$E(\tau) = 2 \sum_x \sum_{\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - P_{\mu\nu}(x,\tau)\right] \, .$$

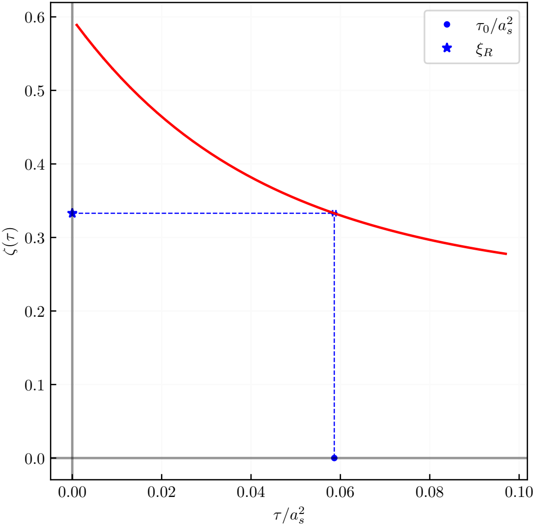
$$E_{ts} \sim a_t^2 a_s^2 \sum_x \sum_i F_{0i}^2(x,\tau) = a_t^2 a_s^2 V(d-1)\tilde{E}_{ts} \, ,$$

$$E_{ss} \sim a_s^4 \sum_x \sum_{i\neq j} F_{ij}^2(x,\tau) = a_s^4 V \frac{(d-1)(d-2)}{2} \tilde{E}_{ss} \, ,$$

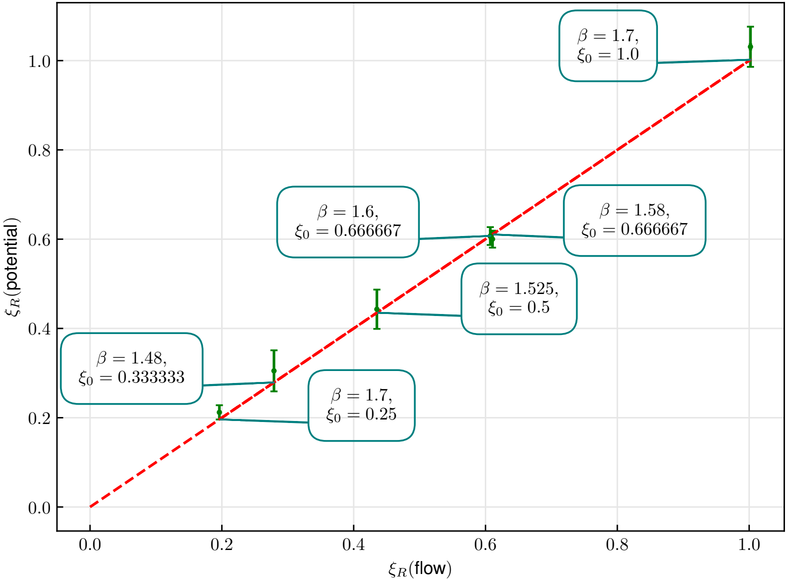
$$\zeta(\tau) = \sqrt{\frac{d-2}{2} \frac{E_{ts}(\tau)}{E_{ss}(\tau)}}$$

Note: this is analogous to what done for QCD in [4]

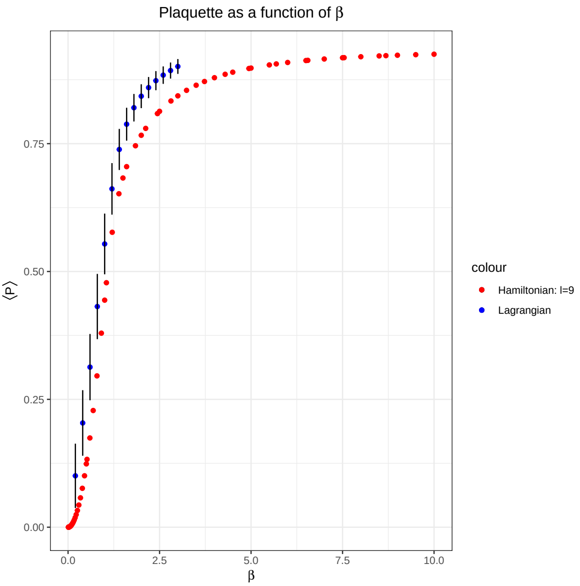
ξ_R from the flow



Wilson flow VS static potential



plot $\langle P \rangle$ VS β



Step scaling function

Goal

Find the Λ parameter of QED 2+1 by matching to 1-loop lattice perturbation theory.
Be build numerically the step scaling function:

$$\sigma(\alpha(r)) = \alpha(s\,r)$$

where s is the scaling factor.

We match in the weak coupling region:

$$V(r) \approx \frac{\alpha}{r}$$

Step scaling function

Steps

1. Start at small g in the Hamiltonian simulations and compute:

$$\alpha(r_0) = u$$

Note: on th lattice we find $\alpha(r_0/a_0, g_0)$.

2. Go to $r_1 = sr_0$, and adjust g such that:

$$\alpha(r_1/a_1, g_1) = \alpha(sr_0/a_0, g_0)$$

and $r_1/a_1 = r_0/a_0$.

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   and $r_1/a_1 = r_0/a_0$.

3. By construction, $\alpha(sr_0) = \alpha(r_1)$.
   In the perturbative regime the running coupling function is invertible $\implies r_1 = sr_0$ in physical units.

4. Continue until one can match to the Lagrangian simulations and make contact to phenomenology.

Step scaling function

$$
\begin{array}{rcccc}
 & g_0 & g_1 & g_2 & \cdots \\
r_0 : & \alpha(r_0, g_0^2) & & & \\
2r_0 : & \alpha(2r_0, g_0^2) = & \alpha(r_1, g_1^2) & & \\
4r_0 : & & \alpha(2r_1, g_1^2) = & \alpha(r_2, g_2^2) & \\
\vdots & & & &
\end{array}
$$

Step scaling function

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\vdots & & & &
\end{array}
$$

At this point (say in step p), we can determine r_p in physical units:

1. Go to large L on the Lagrangian simulation $\rightarrow$ neglect FVEs.
2. Find a_p with the gradient flow.
3. Find r_p in physical units and each $r_i = r_p / s^{p-i}$

$\Longrightarrow$ we have determined the running of the coupling α.

Conclusion

- We can connect β in the Lagrangian with the β in the Hamiltonian
- Support for both periodic and open boundary conditions
- We have the setup for finding the Λ parameter of QED 2+1

Next steps

- Include fermions on the Hamiltonian sector:
- Account for $O(a_s)$ corrections in ξ_R from the Wilson flow

Topics for discussion

- Value of c in $\tau_0^2 E(\tau_0) = c$: we find $c = 1.628(91) \cdot 10^{-3}$
- Λ parameter (see yesterday talks): can we extract it from $\alpha(\tau)$?

Thank you for your attention!

Main references

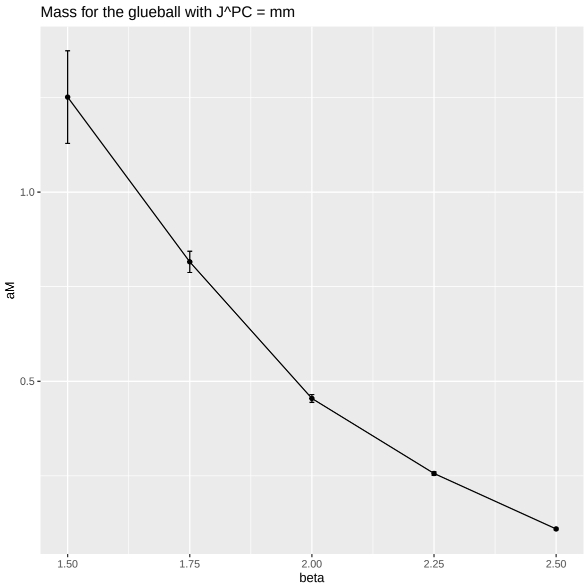
[1] Lena Funcke et al. "Review on Quantum Computing for Lattice Field Theory". In: *arXiv preprint arXiv:2302.00467* (2023) (cit. on p. 6).

[2] Angus Kan et al. "Investigating a $(3+1)\mathrm{D}$ topological θ-term in the Hamiltonian formulation of lattice gauge theories for quantum and classical simulations". In: *Phys. Rev. D* 104 (3 Aug. 2021), p. 034504. DOI: 10.1103/PhysRevD.104.034504. URL: https://link.aps.org/doi/10.1103/PhysRevD.104.034504 (cit. on p. 7).

[3] Martin Lüscher. "Properties and uses of the Wilson flow in lattice QCD". In: *Journal of High Energy Physics* 2010.8 (2010), pp. 1–18 (cit. on p. 17).

[4] Szabolcs Borsányi et al. "Anisotropy tuning with the Wilson flow". In: *arXiv preprint arXiv:1205.0781* (2012) (cit. on p. 19).

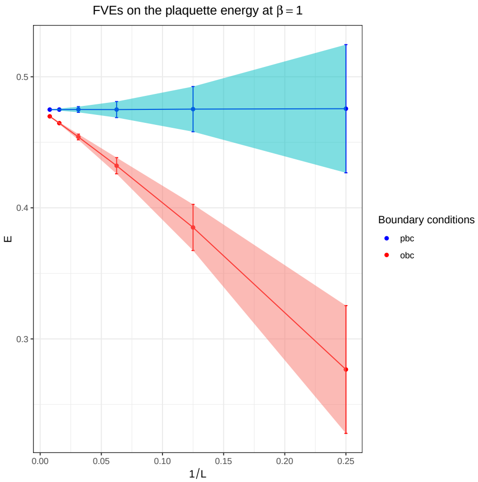
plot M_{0--} VS β



Effect of boundary conditions

Overview plots

In this section are shown the FVEs on the plaquette energy for the different choices of boundary conditions.



Effect of boundary conditions

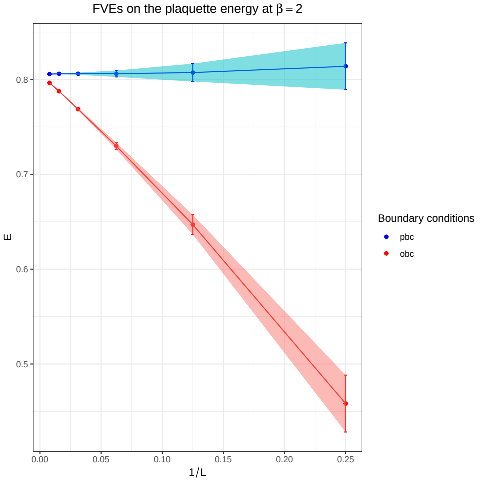


Figure 2: Volume dependence of the plaquette energy expectation value.