

Instanton Identification Using Gradient Flow

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Yang-Mills Theory On Finite Volume

- Finite volumes (toroidal geometries) and non-thermal boundary conditions can be used to explore the dynamics of Yang-Mills theories.
- Specifically $L_s \ll 1/\Lambda$ (weak coupling regime) are interesting:
 - Confinement persist at small coupling.
 - Semi-classical methods available.

R^4 with PBC



$R \times T^3$ with TBC



R^4 strong coupling regime

$R^3 \times T^1$ SYM

[Bergner, Piemonte, Ünsal 2018]



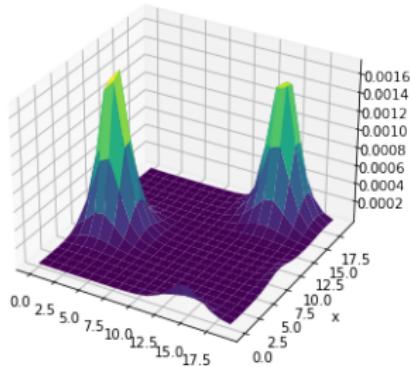
- Recently gain insights in resurgence gives hope these methods can be applied to Yang-Mills.

Confinement in Yang-Mills

- Phenomenology: Chromoelectric flux form tubes, creating a linear potential (effectively like superconductivity).
- Experimental measurement: Lattice
- What are the relevant microscopical degrees of freedom?
 - Magnetic Monopoles [T'Hooft 1975; Mandelstam 1976]
 - Magnetic Bions [Ünsal 2009]
 - Fractional Instantons [González-Arroyo, Martínez, Montero 1995]
- Need tools to control/monitor these microscopical degrees of freedom
 - Gradient Flow [Narayanan-Neuberger 2006, Lüscher 2010]
 - Adjoint Filtering Method [González-Arroyo, Kirchner 2005]
 - Polyakov Loops

Fractional Instantons On The Lattice

Ground state of YM with Twisted Boundary Conditions (TBC)



Action density of two fractional instantons with $Q = 1/2$ on a $4^2 \times 20^2$ lattice ($T^2 \times R^2$).

$$U_\mu(\vec{x} + L/a \vec{\nu}) = \Omega_\nu U_x(x) \Omega_\nu^\dagger$$

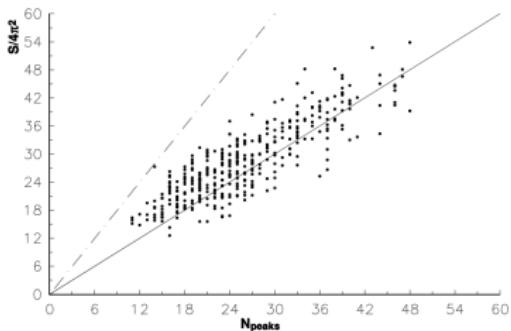
Topological charge fractionalize

$$Q = -\frac{\lambda}{N_c} + n, \quad \nu \in \mathbb{Z}, \quad \lambda \in \mathbb{Z} \bmod N_c.$$

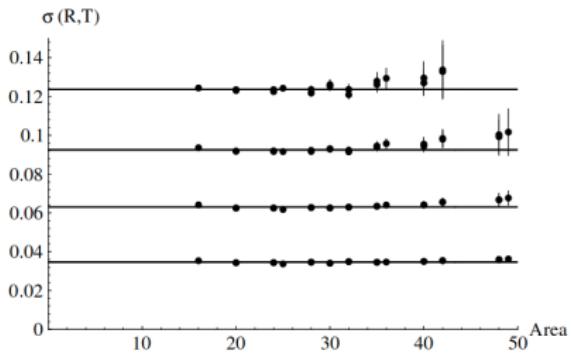
Self-dual solutions of YM, $F_{\mu\nu} = \pm \tilde{F}_{\mu\nu}$

$$Q = \frac{1}{16\pi^2 t_R} \int_M \text{Tr}(F_{\mu\nu} \tilde{F}_{\mu\nu}),$$
$$S = 8\pi^2 |Q|.$$

Fract. Inst. Role In Confinement?



Action density of MC cooled configurations normalized to fractional stanton density. Lattice with $V = 8^3 \times 64$, $\beta = 2.325$ and TBC. [González-Arroyo, Martínez, Montero 1995]



String tension σ as a function of the area $R \times T$ of the Wilson loops. Lattice with $V = 16^3 \times 8$, $\beta = 2.235$, PBC and cooling with 10,20,40,100 steps [González-Arroyo, Montero 1996].

Gradient Flow

Flow the gauge fields according to:

$$\begin{aligned}B_\mu(x, 0) &= A_\mu(x), \\ \frac{dB_\mu(x, t)}{dt} &= D_\nu G_{\nu u}(x, t), \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu].\end{aligned}$$

Since $D_\nu G_{\nu u}(x, t) \sim -\delta S_{YM}/\delta B_\mu$ the gauge field is flowed towards a classical solution \rightarrow UV fluctuations are suppressed.

Gradient Flow

The flow is a local operation, at leading order in perturbation theory one obtains

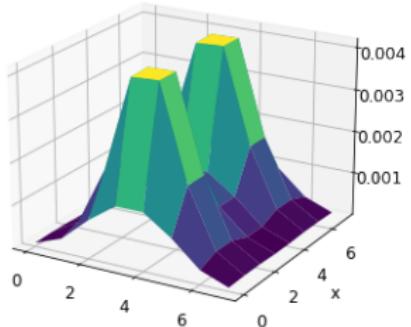
$$B_\mu(t, x) = g \int d^4y K_t(x - y) A_\mu(y), \quad K_t(z) = \frac{e^{-z^2/4t}}{(4\pi t)^2}.$$

The flow has an effective radius of smearing $r_{smear} = \sqrt{8t}$

One expects the IR physics to stay unaffected by the flow up to:

- Lattice artifacts.
- Processes that may occur during the flow.

GF Applied To Fract. Inst.



2D slice of topological density of the starting configuration.

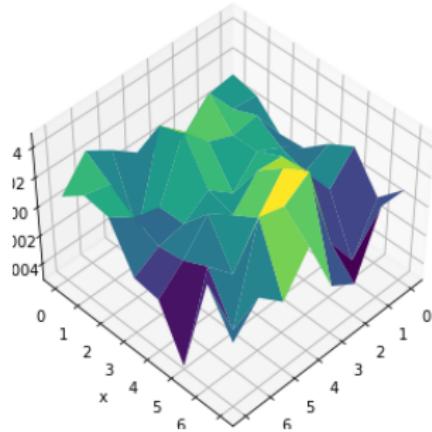
1) Construct smooth configuration:

- $V = 8 \times 4^3$, twist and $Q = 1/2$.
- Replicate in spatial directions to undo twist.
- $V = 8^4$, no twist, $Q=4$, 8 fract. inst.

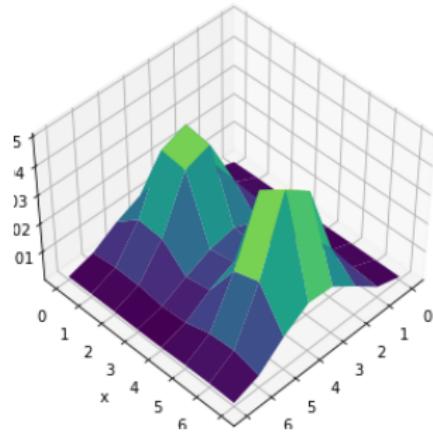
2) Heat the configuration.

3) Apply Gradient flow.

Gradient Flow



Topological density after heating.



Topological density after GF.

Lattice Artifacts

Take an improved action for the gradient flow

$$\begin{aligned} S(\{c_i\}) &\equiv \sum_x \text{Tr}\{c_0 \langle 1 - \square \rangle + 2c_1 \langle 1 - \square \square \rangle\} = \\ &= -\frac{a^4}{2}(c_0 + 8c_1) \sum_{x,\mu,\nu} \text{Tr}(F_{\mu\nu}^2(x)) + \frac{a^6}{12}(c_0 + 20c_1) \sum_{x,\mu,\nu} \text{Tr}(\mathcal{D}_\mu F_{\mu\nu}(x))^2. \end{aligned}$$

And substitute an infinite volume continuum instanton solution with width ρ

$$A_\mu(x) = -i \frac{\eta_{\mu\nu}^a x_\nu \sigma_a}{x^2 + \rho^2}.$$

[Perez et al. 1996].

Lattice Artifacts

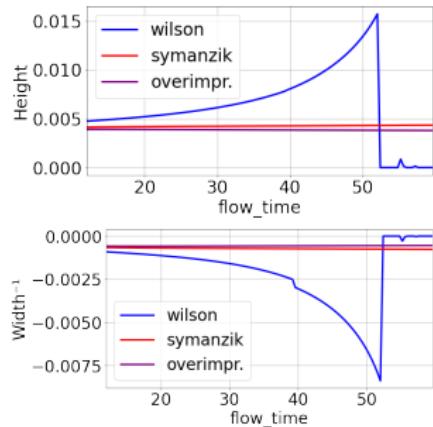
One instanton action (with $c_0 + 8c_1 = 1$)

$$S = 8\pi^2 \left\{ 1 - \frac{(1 + 12c_1)}{5} (a/\rho)^2 + \mathcal{O}(a/\rho)^4 \right\}.$$

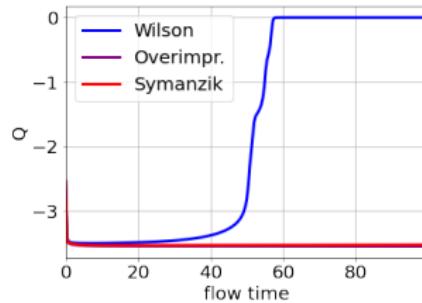
The stability of the solution (i.e whether the action decreases with decreasing ρ) depends on the lattice artifacts:

- Underimproved $c_1 < -1/12 \rightarrow$ unstable.
- Symanzik $c_1 = -1/12 \rightarrow$ depends on $\mathcal{O}(a/\rho)^4$.
- Overimproved $c_1 > -1/12 \rightarrow$ stable.

Lattice Artifacts



Instanton size and $(\text{width})^{-1}$ evolution by 3 point fit to a parabola. For overimproved flow $c_1 = -1/8$.



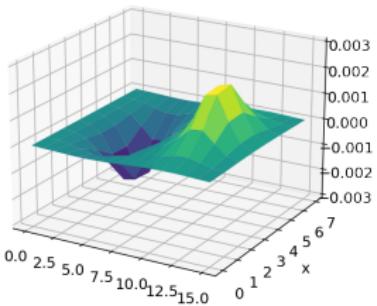
Topological charge evolution.

Inst./Anti-Inst. Annihilation

Physical processes can occur during the gradient flow

Inst./Anti-Inst. Annihilation

Physical processes can occur during the gradient flow



2D slice of an Instanton/Anti-Instanton configuration

- Okay for $r_{smear} < d_{II}$
- Dense scenario?
 - The density of objects increases towards the infinite volume
 - Use some tool that does not modify configuration.

Adjoint Filtering Method

For a configuration with A_μ solution of the classical e.o.m there exists

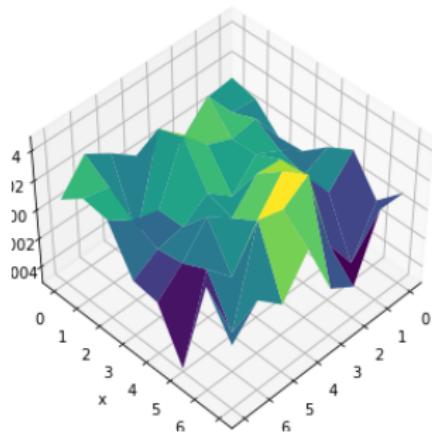
$$\psi^a(x) = \frac{1}{8} F_{\mu\nu}^a(x) [\gamma_\mu, \gamma_\nu] V \rightarrow \text{Supersymmetric zero-mode}$$

- Zero-mode of the Dirac operator in the adjoint representation
- For $V = (1, 0, 0, 0)$, vanishing $\text{Re}(\psi^a)$ everywhere \rightarrow Distinguishable
- It's density reproduces the dual part of the action $|\psi|^2 = |F_{\mu\nu} + \tilde{F}_{\mu\nu}|^2$

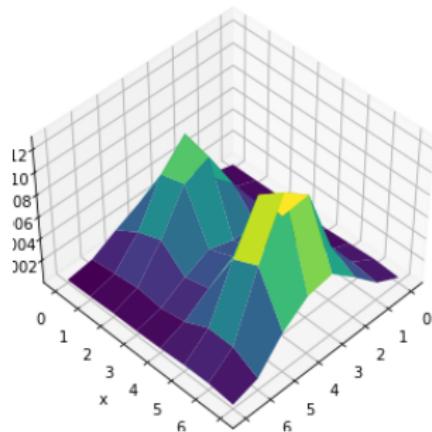
$$\psi_+^a = i \begin{pmatrix} \frac{B_3^a + E_3^a}{2} \\ \frac{B_1^a + E_1^a}{2} - i \frac{B_2^a + E_2^a}{2} \\ 0 \\ 0 \end{pmatrix}$$

[González-Arroyo, Kirchner 2005].

AFM Using Overlap Operator



Topological density after heating



Topological density after AFM

Conclusion And Outlook

- Lattices on “non-trivial” manifolds can help us understand phenomena of the Yang-Mills vacuum
 - Tests for conjectures like Adiabatic Continuity.
 - Numerical evidences to false/justify underlying confinement mechanisms.
- On the lattice we have some tools to monitor the topological structures
 - Gradient Flow.
 - Adjoint Filtering Method.
 - Polyakov Loops.
- We are applying them to MC configuration with different topologies, volumes and larger N.