The Gradient Flow in QCD and
OTHER S.C. Field Theories
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Fixed point and anomalous dimensions in a Composite Higgs theory

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PRELIMINARY

- Composite Higgs and (especially) partially composite top quark
- Fairly economical model **M6** discussed by Ferretti (1404.7137,1604.06467)
  - $\mathrm{SU}(4)$ gauge theory, quartet fermions and sextet fermions
- Variant model: Add fermions, look for large anomalous dimensions near the conformal sill

COMPOSITE HIGGS + PARTIALLY COMPOSITE top (Georgi & Kaplan 1984) (Kaplan 1991)

- $\mathrm{SU}(4)$ gauge theory — *hypercolor* — new strong sector with scale $\Lambda_{\mathrm{HC}} \sim 5$ TeV
- N_6 Majorana fermions Q in sextet rep $\Longrightarrow$ composite Higgs field in Goldstone sector
- N_4 Dirac fermions q in quartet $\square$ rep $\Longrightarrow$ $B = Qqq$ is a chimera baryon of HC

Partial compositeness: Mix massless t quark with B via

$$V_{\mathrm{top}}^{\mathrm{HC}} = G_R \bar{t}_L B_R + G_L \bar{t}_R B_L + \mathrm{h.c.} \quad — \quad t \equiv \text{top quark}, B \equiv Qqq$$

tB is really $tQqq$ — a four-Fermi interaction from a gauge theory — EHC — at a much higher scale Λ_{EHC}:

$$G_{L,R} \sim g_{\mathrm{EHC}}^2 / \Lambda_{\mathrm{EHC}}^2$$

THE PROBLEM: Λ_{EHC} is large (*flavor violations!*) $\Longrightarrow$ $G_{L,R}$ are much too small
unless Qqq has a large anomalous dimension γ, and then

$$\Lambda_{\mathrm{EHC}}^{-2} \longrightarrow \Lambda_{\mathrm{EHC}}^{-(2-\gamma)}$$

THE HOPE: Large γ appears near the sill of the conformal window $\Longrightarrow$ Choose N_4, N_6 appropriately.

THE MODEL:

$\mathrm{SU}(4)$ gauge theory with massless Wilson fermions ($\kappa_r = \kappa_r^{(c)}$) — $4\times$sextet (Dirac) and $4\times$quartet

THE (gauge) FLOW:

$$\frac{\partial B_\mu}{\partial t} = -\frac{\partial S_g}{\partial B_\mu}, \qquad B_\mu|_{t=0} = A_\mu$$

$$g^2(t) = Ct^2 \langle E(t) \rangle$$

FLOW as RG:

$g^2(t)$ is the running coupling at scale $\mu^2 = 1/t$, and

$$\beta(g^2) = -t\frac{dg^2}{dt}$$

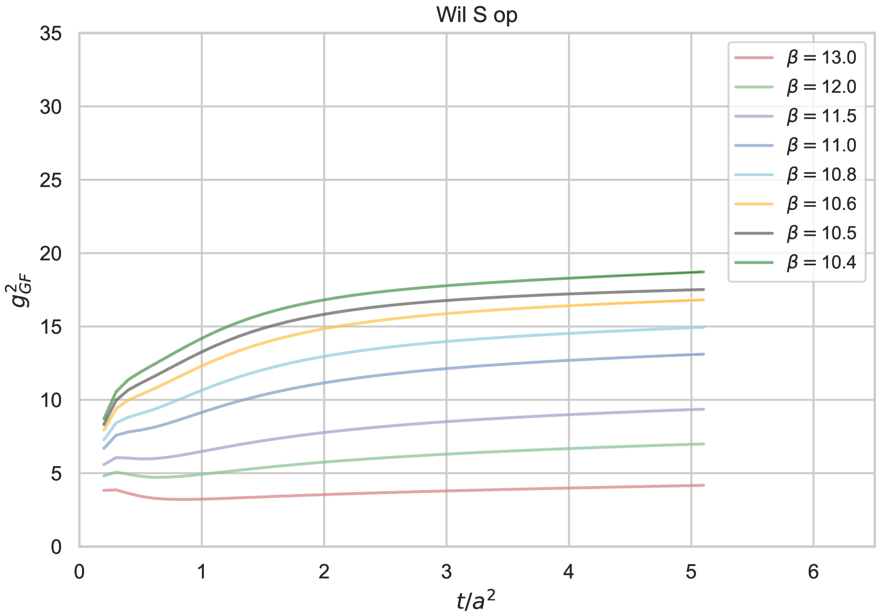
DISCRETIZATIONS:

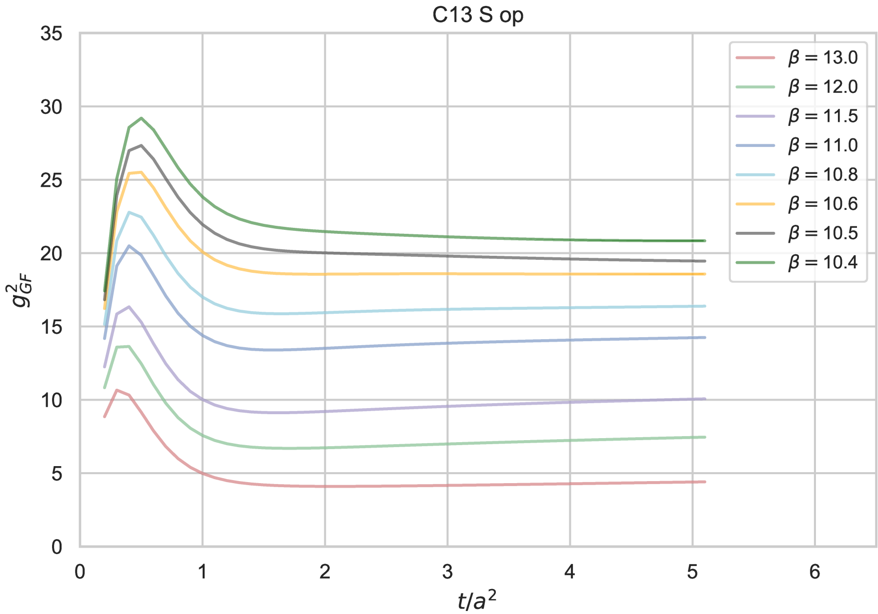
$$S_g = c_p S_p + c_r S_r \ , \qquad c_p + 8c_r = 1$$

Wilson flow: $c_r = 0$
Symanzik flow: $c_r = -1/12$
“C13” flow: $c_r = +1/12$

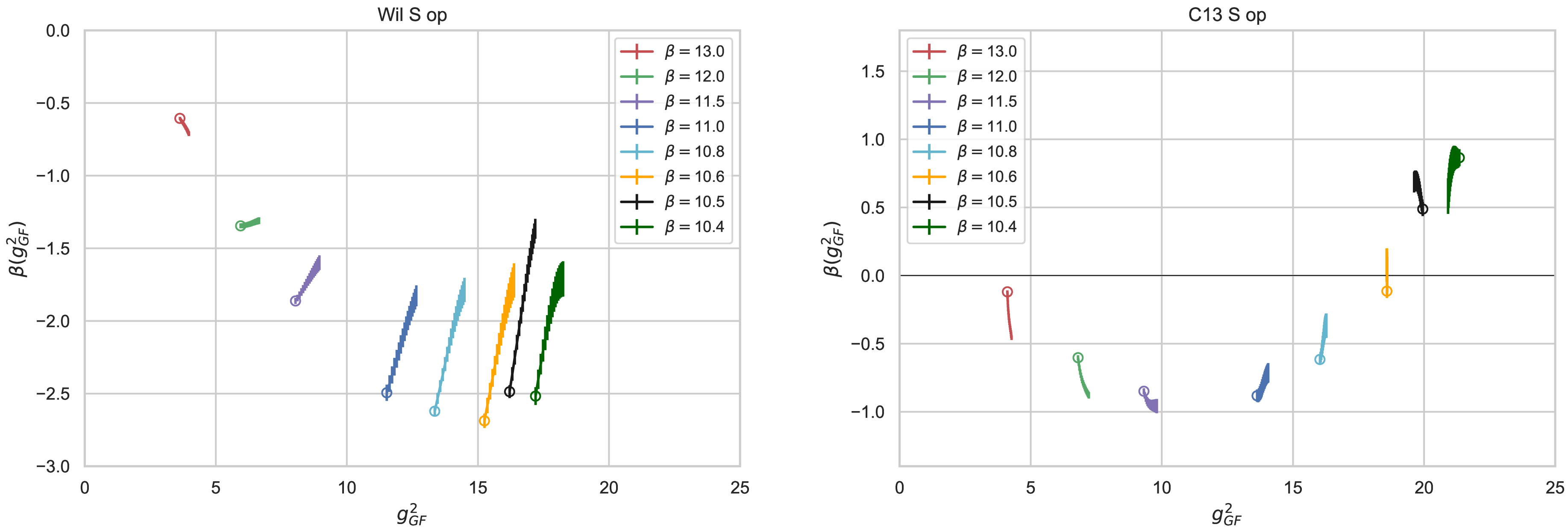
$E(t)$ operators: Wilson, Symanzik, Clover

RAW FLOWS – Wilson vs. C13 flow





RAW β functions (slopes) – Wilson vs. C13 flow

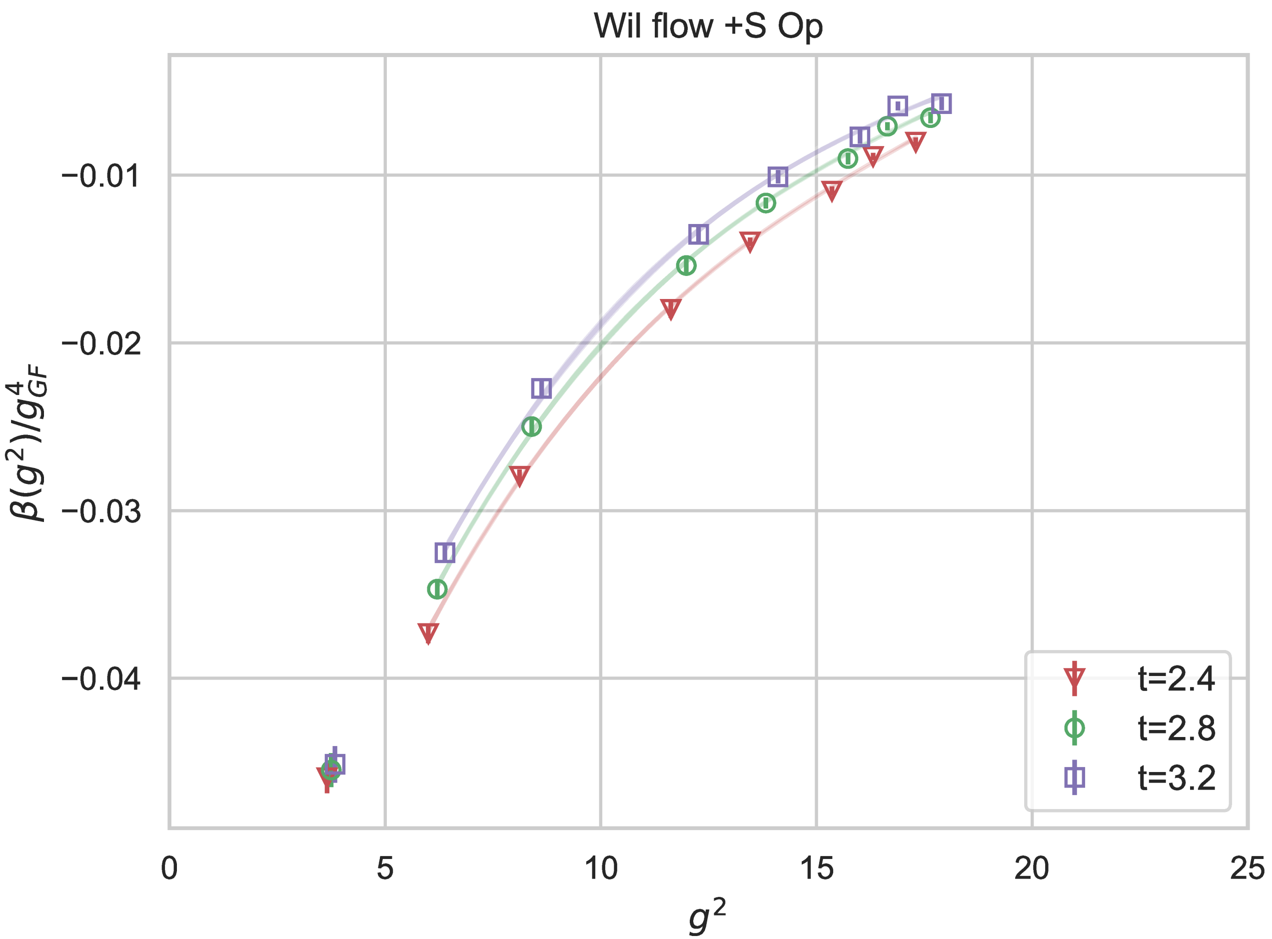


$\beta > 0?$

Continuum limit: Take $a^2/t \to 0$ at fixed g^2 — same as $t/a^2 \to \infty$

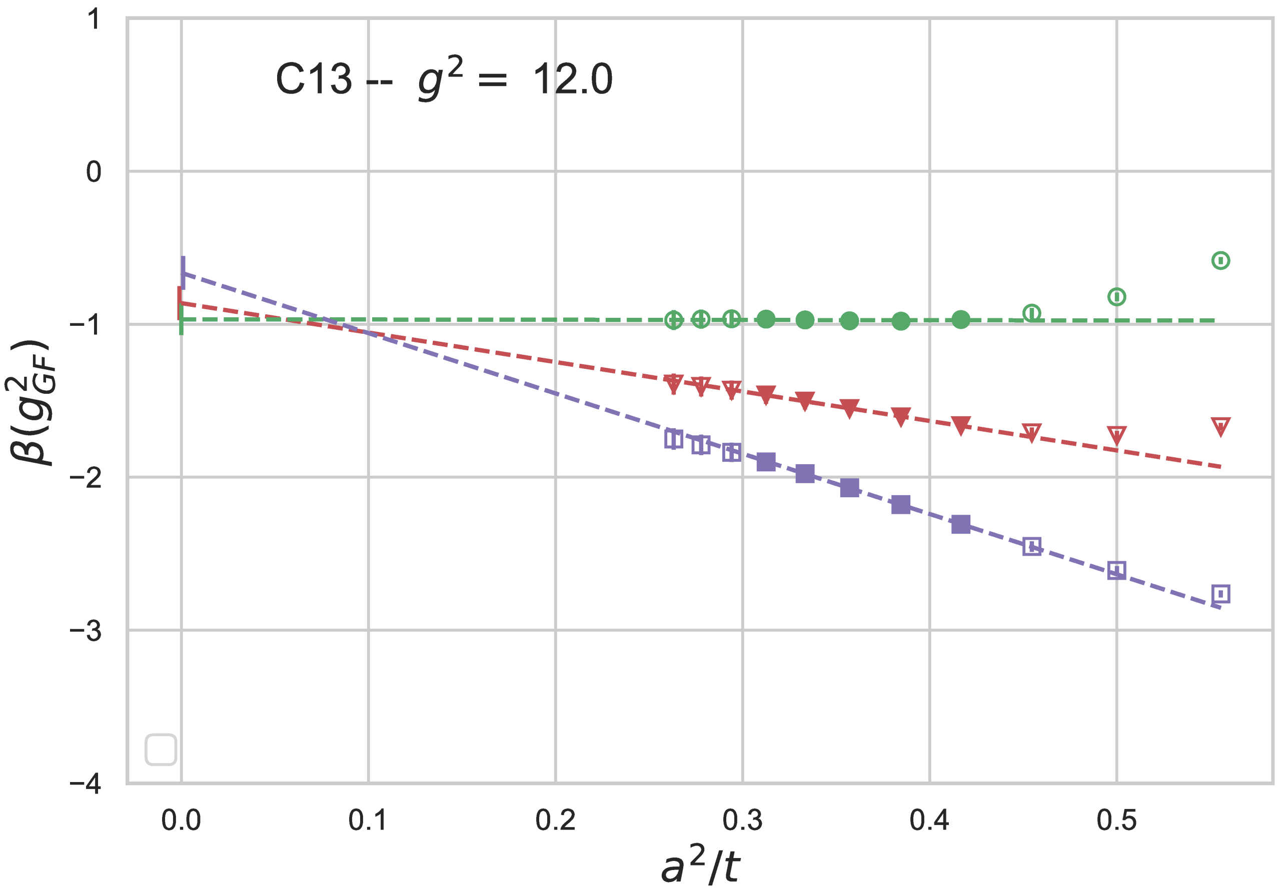
$\Longrightarrow$ we need continuous $\beta(g^2)$ at arbitrary t ...

Interpolations for $\beta(g^2)$ at arbitrary t



$\Longrightarrow$ giving $\beta(g^2)$ at any g^2 as a fn of t. Now extrapolate $t/a^2 \to \infty \ldots$

CONTINUUM EXTRAPOLATION $a^2/t \to 0$



Symanzik operator

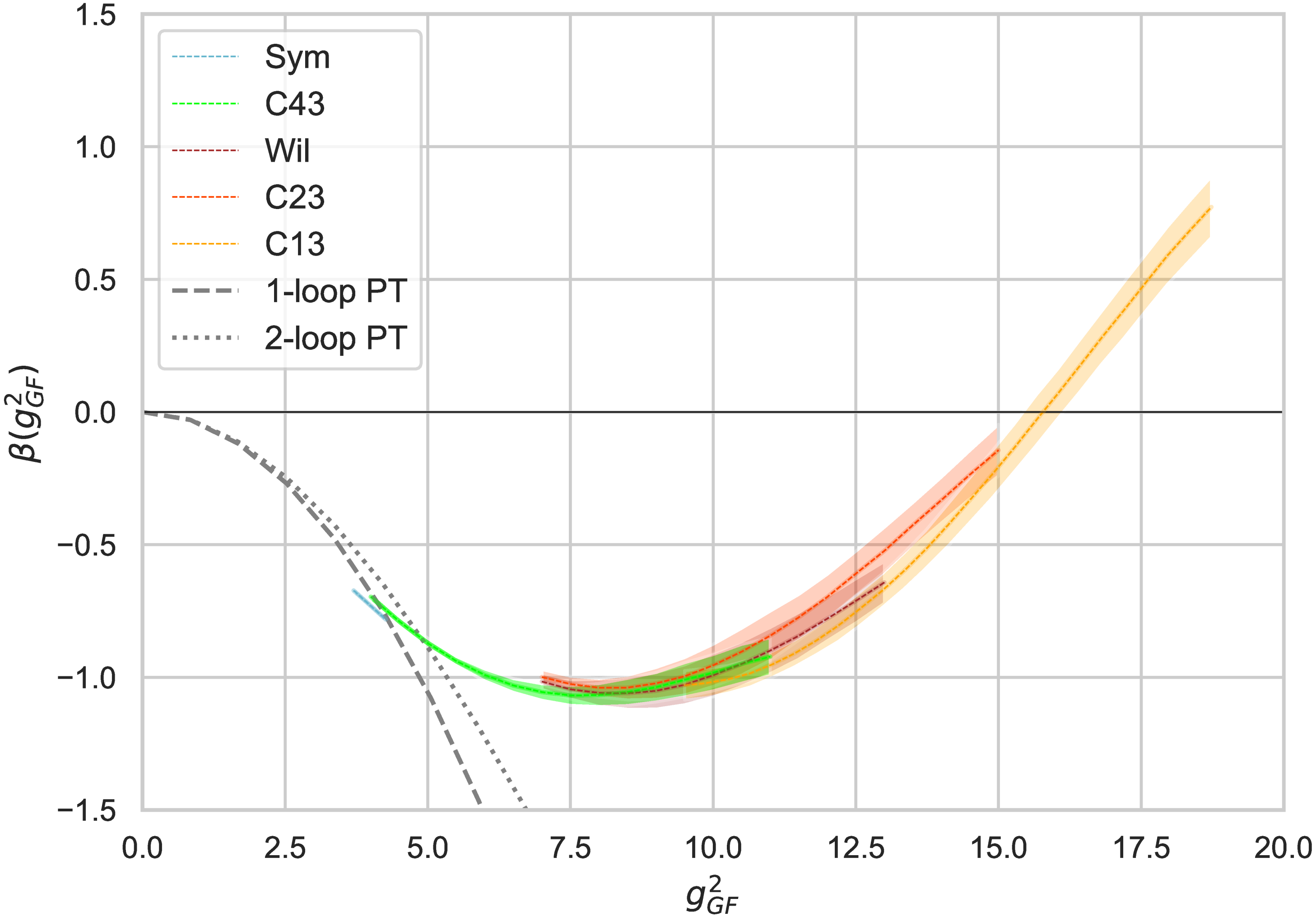
Wilson operator

Clover operator

Consistent continuum limit: For given g^2, demand that S and W extrapolations agree.

(Clover op always has large slope in continuum extrap)

CONTINUUM LIMIT OF $\beta(g^2)$ — IRFP at $g^2 \simeq 16$



Now for ANOMALOUS DIMENSIONS:

GAUGE FLOW — as before

FERMION FLOW

$$\frac{\partial \chi}{\partial t} = \Delta \chi \; , \qquad \chi|_{t=0} = \psi$$

FLOWED MESON CORRELATORS

$$C_X(x_4, t) = \langle X(0) X'(x_4, t) \rangle$$

source $X = \bar{\psi} \Gamma \psi$ (not flowed; Gaussian point-split)
sink $X' = \bar{\chi} \Gamma \chi$ (flowed; point operator)

Correct for η of ψ field — divide by conserved current

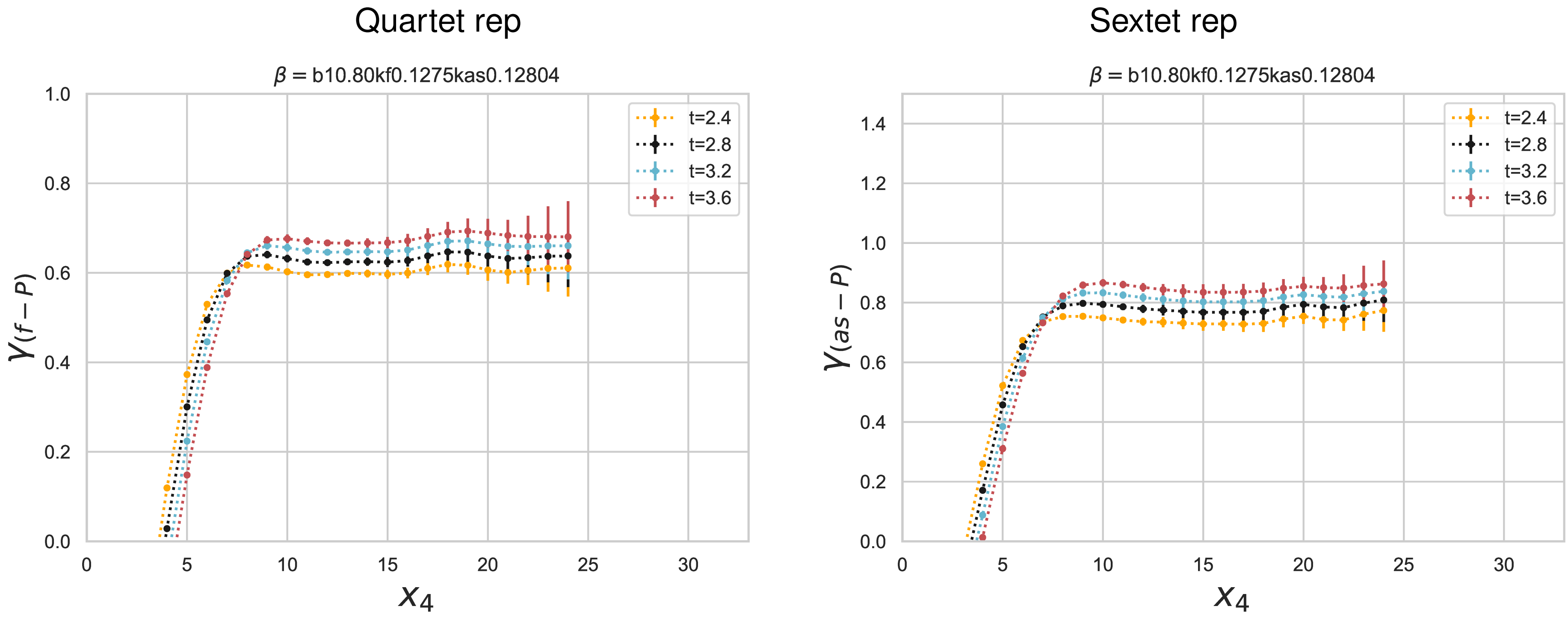
$$R(x_4, t) = \frac{\langle X(0) X'(x_4, t) \rangle}{\langle V(0) V'(x_4, t) \rangle}$$

ANOMALOUS DIMENSION

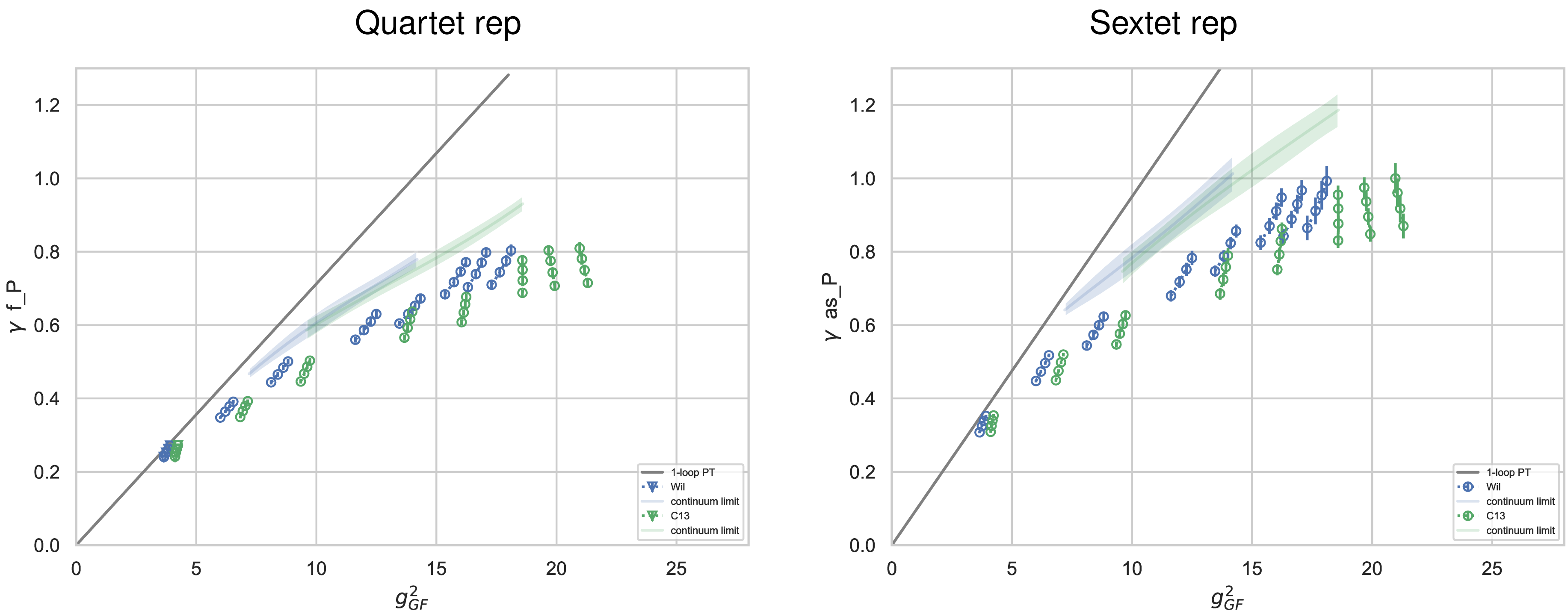
$$\gamma_X = -2 \frac{t}{R} \frac{\partial R}{\partial t}$$

x_4: look for plateau in x_4-dependence
t: take limit $a^2/t \to 0$ at fixed g^2, as for $\beta(g^2)$

x_4 PLATEAU of $\gamma_P(x_4, t)$ — pseudoscalar operator in each fermion rep

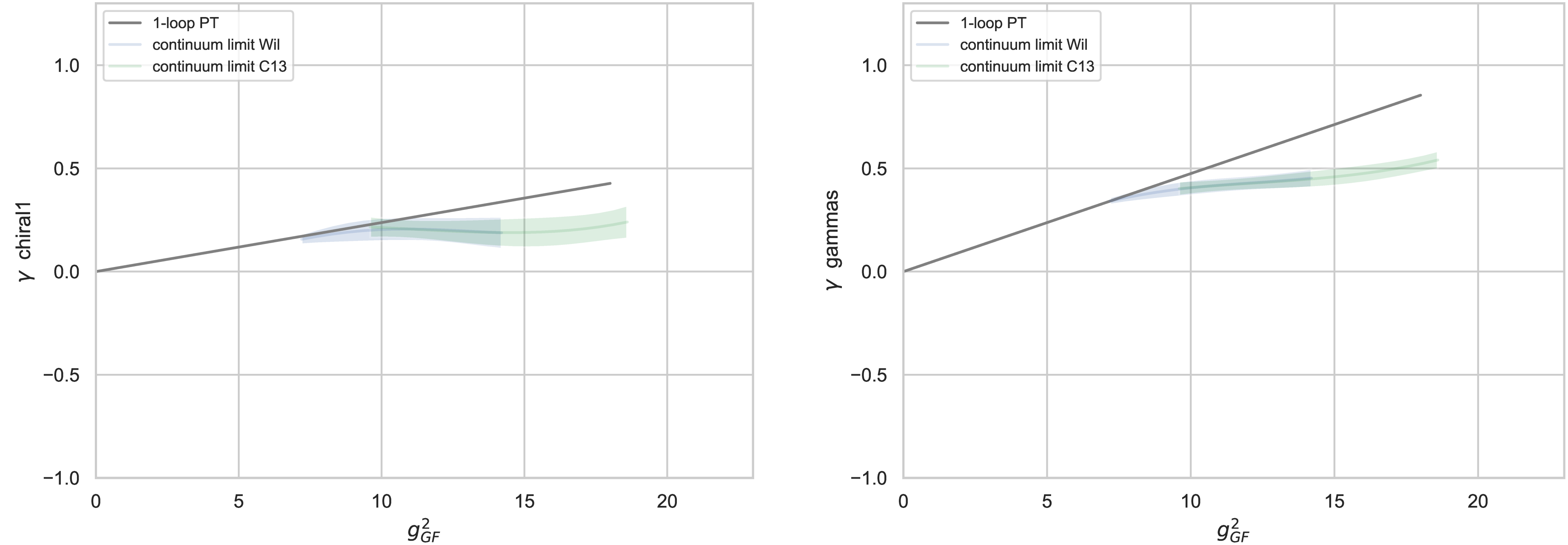


ANOMALOUS DIMENSION $\gamma_m(t)$ — raw data and extrapolation $a^2/t \to 0$



Recall fixed point at $g^2 \simeq 16$: γ_m is large in both reps

Finally, the CHIMERA (showing two of the three operators)



NOT large — trouble for partial compositeness, at least in this model.

Keep looking?

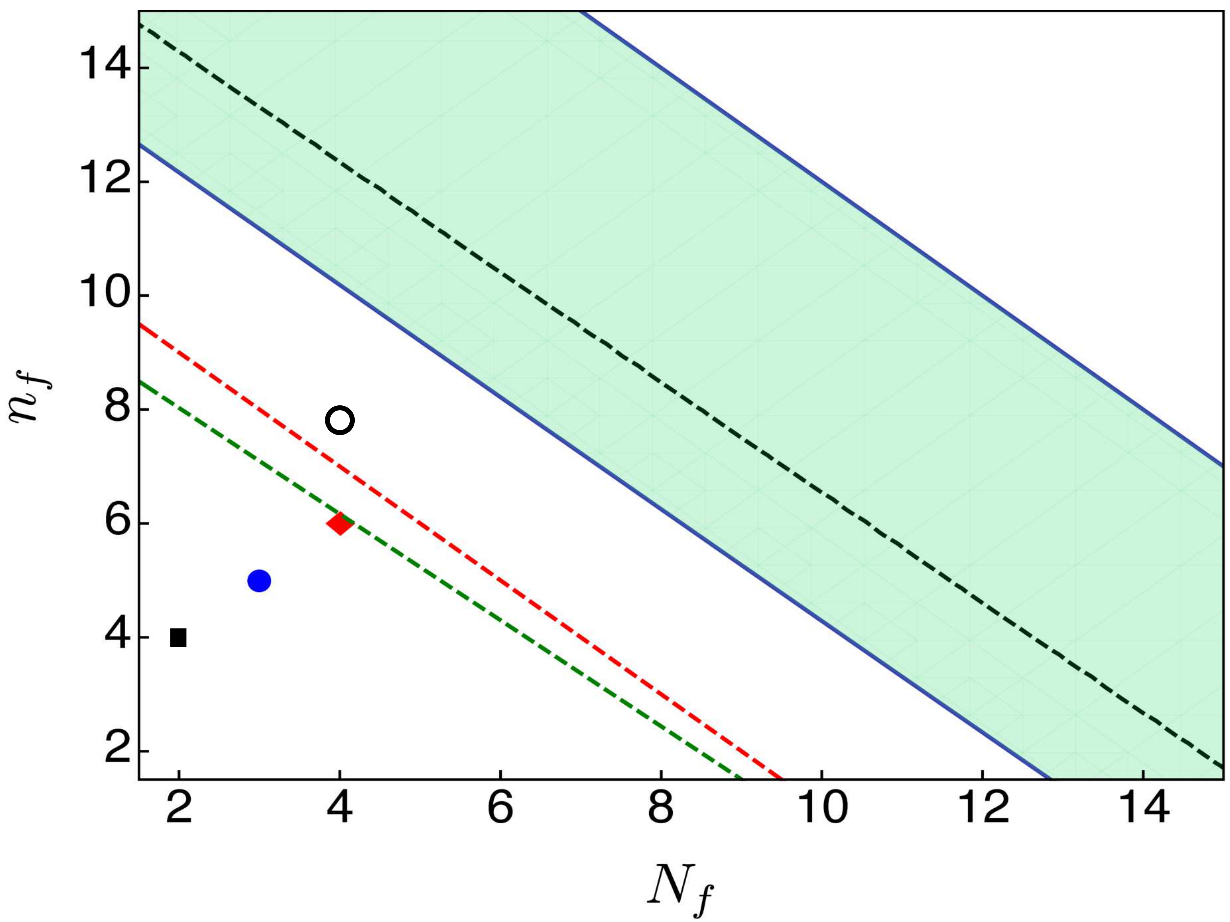
Epilogue: HOW DO WE REACH SUCH STRONG COUPLINGS?

1. Improved action: clover + nHYP smearing (and dislocation suppression by new gauge term)
2. Comparison of different flows $\Rightarrow$ C13 flow
3. Pauli–Villars fields
   - N_{PV} scalar fields with fermion action

     $$S_{PV} = \Phi^\dagger (D_W + M)\Phi$$

     with $Ma = 1$ (we choose $N_{PV} = 3 \times N_f$ for each fermion species)
   - speeds up running of coupling at short distances $L \to a$ and decouples at $L \gg a$
     $\Rightarrow$ weakens bare coupling for given IR coupling
     $\Rightarrow$ allows strong IR coupling with smaller cutoff effects

Epilogue: THE CONFORMAL WINDOW



(from B. S. Kim, D. K. Hong and J. W. Lee, 2001.02690)