

Heavy Quark Diffusion with Gradient Flow

Viljami Leino
Helmholtz Institute Mainz, JGU Mainz

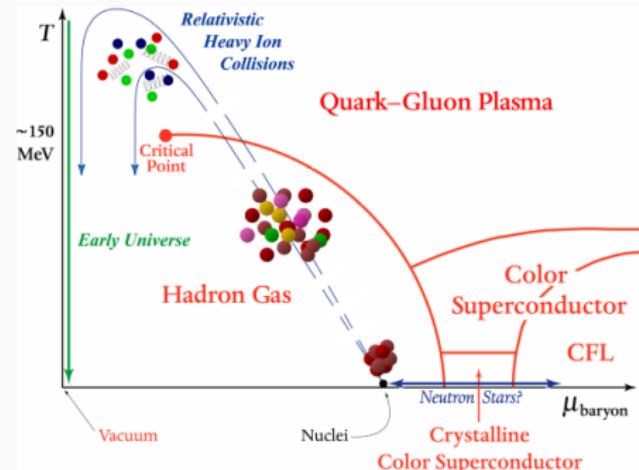
Based on:

Nora Brambilla¹, V.L., Julian Mayer-Stedte¹, Péter Petreczky²: hep-lat/2206.02861

ECT*, Trento
21.03.2023

Setting

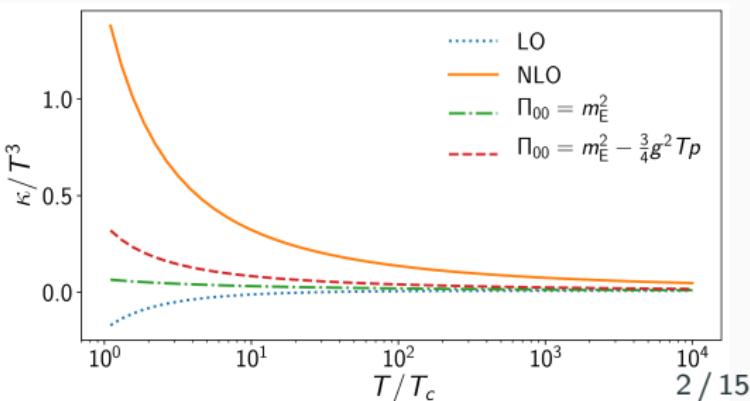
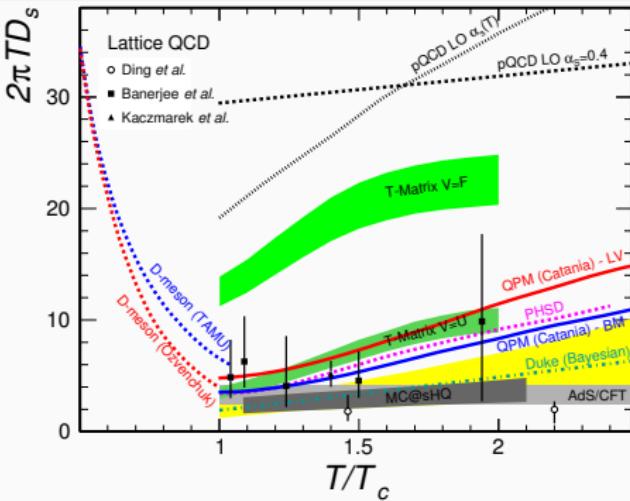
- We aim to understand the strongly coupled Quark Gluon Plasma (QGP)
- QGP generated at particle accelerators such as LHC/RHIC



- The QGP can be described in terms of transport coefficients
- In this talk we focus on the heavy quark momentum diffusion coefficient κ
- Focus on the gradient flow usage for these results

Motivation

- Nuclear modification factor R_{AA} and elliptic flow v_2 described by spatial diffusion coefficient D_x
- Multiple theoretical models predicting wide range of values
- Perturbative series unreliable
- HTL has too strict assumption $m_E \ll T$
- Non-perturbative lattice simulations needed



Heavy Quark diffusion

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
→ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t), \quad \langle \xi(t) \xi(t') \rangle = \kappa \delta(t - t')$$

- Heavy quark momentum diffusion coefficient κ related also to:

Spatial diffusion coefficient $D_s = 2T^2/\kappa$,

Drag coefficient $\eta_D = \kappa/(2MT)$,

Heavy quark relaxation time $\tau_Q = \eta_D^{-1}$

- Considering full Lorentz force:

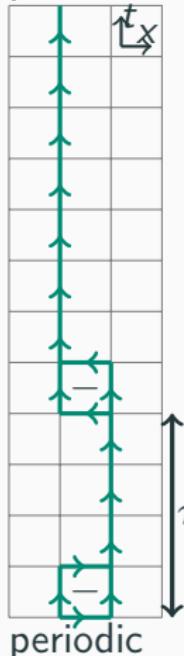
$$F(t) = \dot{p} = q(E + v \times B)(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3} \langle v^2 \rangle \kappa_B$$

Heavy quark diffusion from lattice: Euclidean Correlator

periodic



- Traditional approach uses HQ current-current correlators:
Problem: Transport peak at zero

- HQEFT inspired Euclidean correlator is peak free

$$G_E(\tau) = -\frac{1}{3} \sum_{i=1}^3 \frac{\langle \text{Re Tr } [U(\beta, \tau) g E_i(\tau, 0) U(\tau, 0) g E_i(0, 0)] \rangle}{\langle \text{Re Tr } [U(\beta, 0)] \rangle}$$

$$G_B(\tau) = \sum_{i=1}^3 \frac{\langle \text{Re Tr } [U(1/T, \tau) B_i(\tau, 0) U(\tau, 0) B_i(0, 0)] \rangle}{3 \langle \text{Re Tr } U(1/T, 0) \rangle}$$

- Field strength tensor components need discretization
- Choose corner discretization for this study
- On lattice there is a self-energy contribution that generates a multiplicative renormalization
- For chromomagnetic fields there is also a finite anomalous dimension and renormalization is required

Connecting $G_{E,B}$ to κ

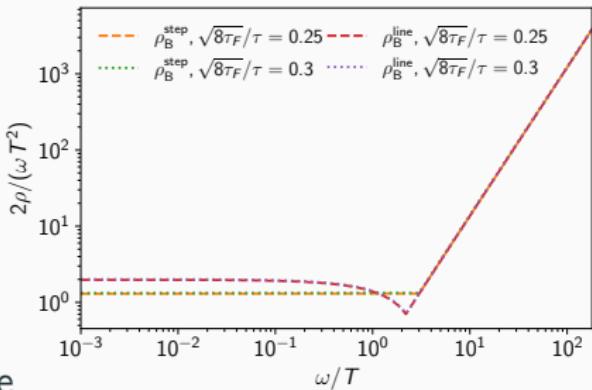
$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh \frac{\omega}{2T}} \quad \kappa = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho(\omega) \quad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- Simply compare lattice data to a model $\rho(\omega)$
 - $\rho(\omega)$ known at IR and UV
 - Connect IR and UV with i) step ii) line

$$\rho_{E,B}^{\text{step}}(\omega, T) = \rho_{E,B}^{\text{IR}}(\omega, T) \theta(\Lambda - \omega) + \rho_{E,B}^{\text{UV}, T=0}(\omega, T) \theta(\omega - \Lambda),$$

$$\begin{aligned} \rho_{E,B}^{\text{line}}(\omega, T) = & \rho_{E,B}^{\text{IR}}(\omega, T) \theta(\omega^{\text{IR}} - \omega) + \\ & \left[\frac{\rho_{E,B}^{\text{IR}}(\omega^{\text{IR}}, T) - \rho_{E,B}^{\text{UV}}(\omega^{\text{UV}}, T)}{\omega^{\text{IR}} - \omega^{\text{UV}}} (\omega - \omega^{\text{IR}}) + \rho_{E,B}^{\text{IR}}(\omega^{\text{IR}}, T) \right] \\ & \times \theta(\omega - \omega^{\text{IR}}) \theta(\omega^{\text{UV}} - \omega) + \rho_{E,B}^{\text{UV}}(\omega, T) \theta(\omega - \omega^{\text{UV}}), \end{aligned}$$

- Related: γ not measured yet



Lattice simulations

- Pure gauge simulations with Wilson action
- Few different temperatures (this talk focuses on $1.5 T_c$).
Scale setting with [\(Francis et.al. PRD91 \(2015\)\)](#)
- Temporal sizes $20 - 34$, spatial size $48^3 - 68^3$
- Gradient flow with Symanzik action
 - + Automatically renormalizes gauge invariant observables
 $\Rightarrow Z_E = 1$ for sufficiently large flow time
 - + Can be used un-quenched (This work: quenched)
- Default order of limits
 1. Continuum
 2. Zero flow time
 3. spectral reconstruction

Renormalization and spectral function: G_E

- Normalize the data with perturbative LO result (also tree-level improve)

$$G_{E,B}^{\text{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- On Lattice E has non-physical self-energy contribution

Removed by the flow. Previous studies used the 1-loop result:

$$Z_E = 1 + g_0^2 \times 0.137718569 \dots + \mathcal{O}(g_0^4) \quad (\text{Christensen and Laine PLB02 (2016)})$$

- Still need additional normalization, normalize to 1 at small τT
- 1-loop GF behavior of G_E partially known [A.M. Eller \(2021\)](#)
- Model the spectral function by connecting known IR and UV behavior with ansatz:

$$\rho_{\text{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$$\rho_{\text{QCD,naive}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_c \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- Set scale such that NLO UV contribution vanishes

Renormalization and spectral function: G_B

- κ_B more complicated, requires renormalization

$$G_B^{\text{flow,UV}}(\tau, \tau_F) = (1 + \gamma_0 g^2 \ln(\mu \sqrt{8\tau_F}))^2 Z_{\text{flow}} G_B^{\overline{\text{MS}}, \text{UV}}(\tau, \mu) + h_0 \cdot (\tau_F/\tau),$$

- Normalize at finite flow time with G^{flow}
- Assuming $h_0 = 0$ for simplicity. We see very little flow time dependence even with this assumption.
- We know Z_E wasn't enough, so we determine Z_{flow} similarly, i.e., just normalize at a point.
- Use same tree-level improvement as for E-correlator
- IR model the same:

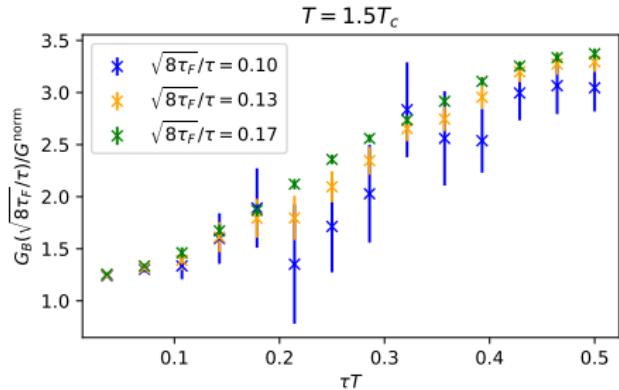
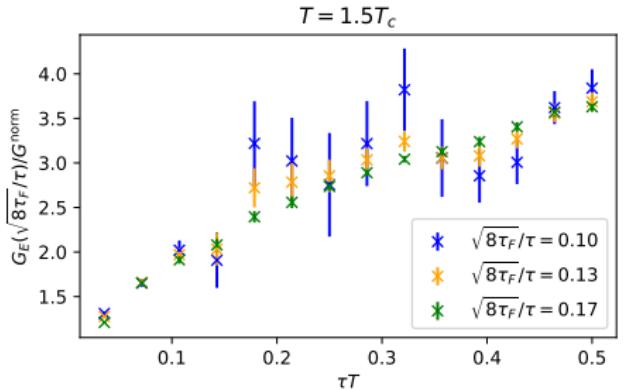
$$\rho_{\text{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

- UV now depends on flow time

$$\rho_B^{\text{UV}}(\omega, \tau_F) = Z_{\text{flow}} \frac{g^2(\mu)\omega^3}{6\pi} (1 + g^2(\mu)(\beta_0 - \gamma_0) \ln(\mu^2/(A\omega^2)) + g^2(\mu)\gamma_0 \ln(8\tau_F\mu^2))$$

- Again, set the scale such that the second term vanishes

Raw (normalized*, tree-level improved) lattice data

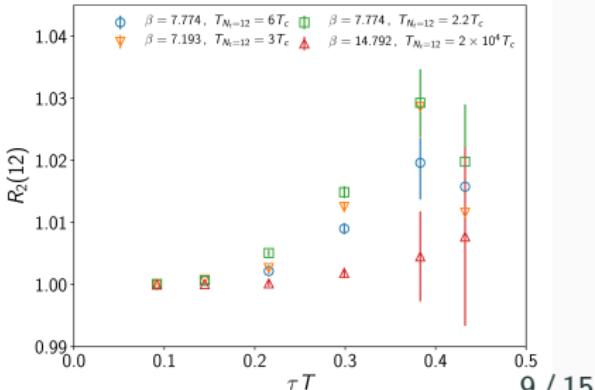


G_E

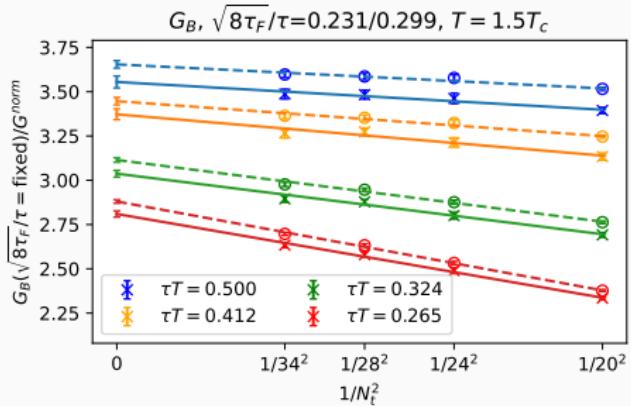
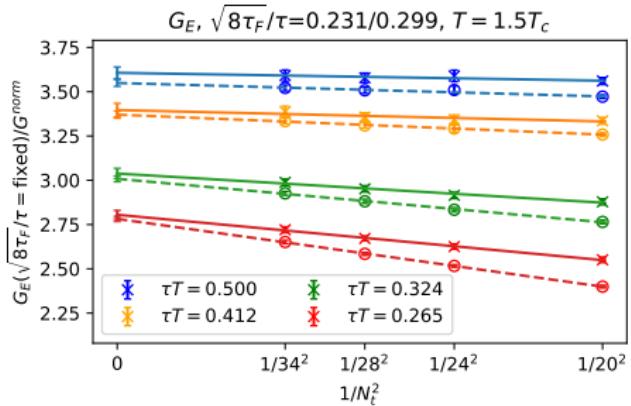
- Similar shapes for electric and magnetic correlators
- Thermal effects dominant in forming the shape

$$R_2(N_t) = \frac{G_E(N_t, \beta)}{G_E^{\text{norm}}(N_t)} \Big/ \frac{G_E(2N_t, \beta)}{G_E^{\text{norm}}(2N_t)}.$$

G_P



Continuum limit



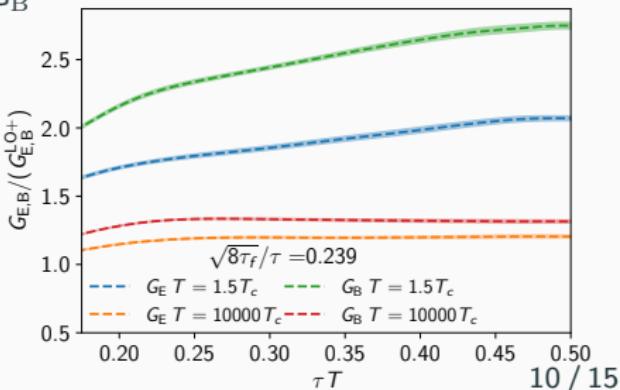
G_E

- Good limits for valid ranges of τT and τ_F

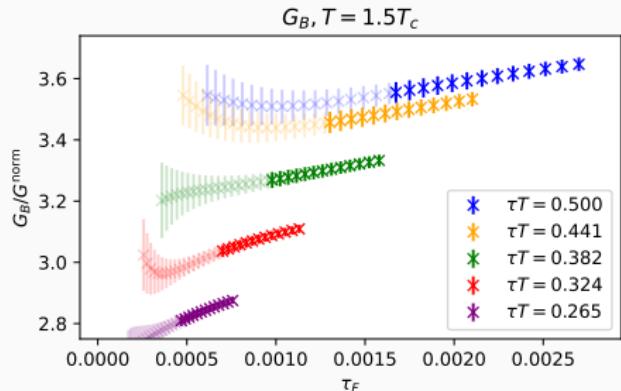
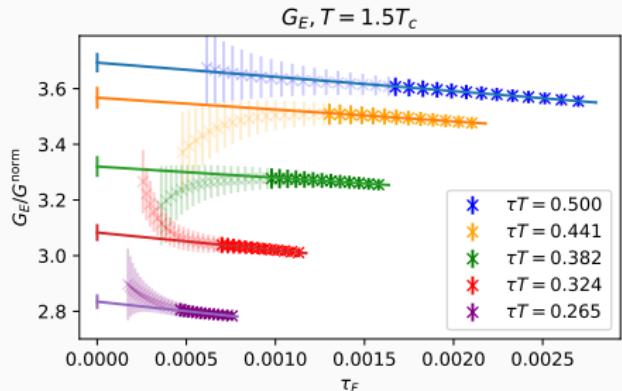
$$\frac{a}{\tau} \leq \frac{\sqrt{8\tau_F}}{\tau} \leq \frac{1}{3}$$

- Spatial volume scaling negligible

G_B



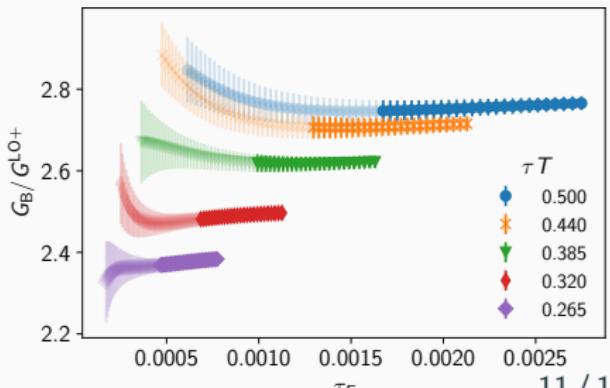
Flow time dependence of G_E and G_B



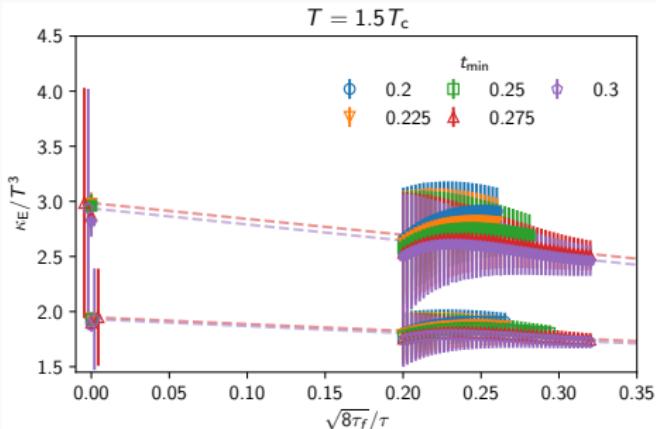
G_E

- We observe different small flow time scaling between G_E and G_B
- Flow dependent spectral function seems to remove most of flow dependence from G_B

G_B

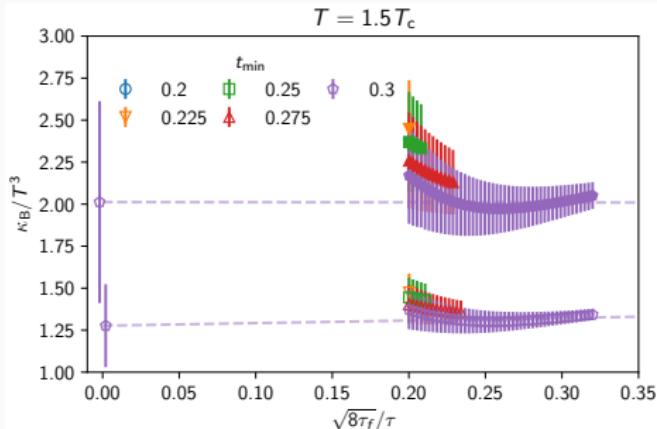


Flow time κ and order of limits

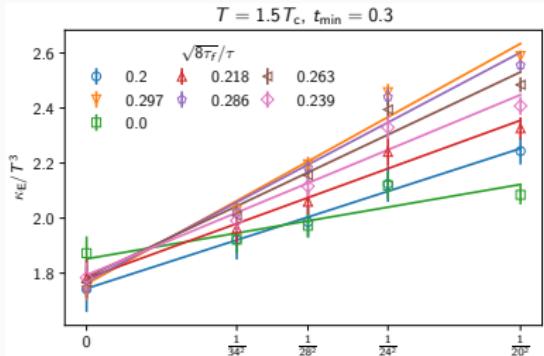


G_E

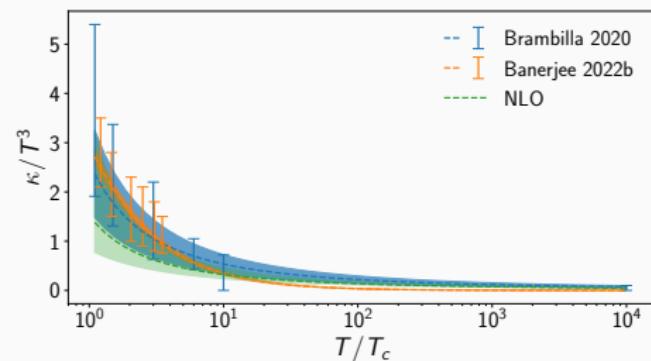
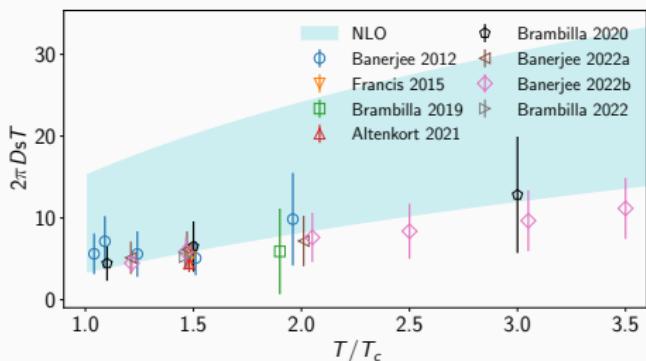
- Very little dependence on flow time
- Ordering of Continuum limit, zero flow time limit, and spectral function inversion doesn't seem to matter much



G_B



κ_E results



- Our results:
(Brambilla et.al. hep-lat/2206.02861)

$$1.7 \leq \kappa_E / T^3 \leq 3.12, \quad T = 1.5$$

$$0.02 \leq \kappa_E / T^3 \leq 0.16, \quad T = 10^4$$

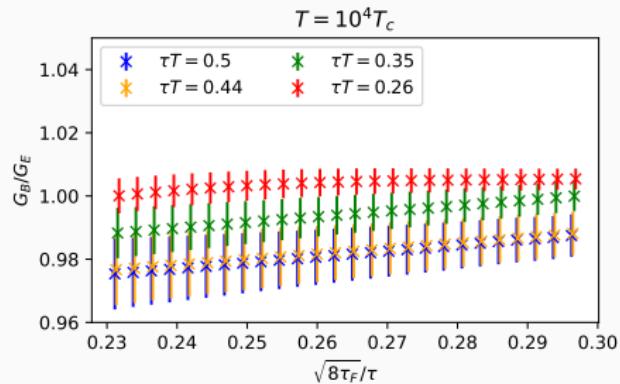
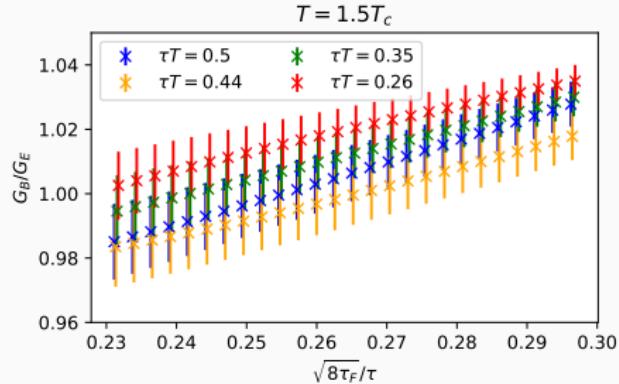
- Can fit temperature dependence:

$$\frac{\kappa^{\text{NLO}}}{T^3} = \frac{g^4 C_F N_c}{18\pi} \left[\ln \frac{2T}{m_E} + \xi + C \frac{m_E}{T} \right].$$

- Other similar quenched lattice studies

Meyer NJP13 (2011),
Ding et.al. JPG38 (2011),
Banerjee et.al. PRD85 (2012),
Francis et.al. PRD92 (2015)
Brambilla et.al. PRD102 (2020)
Altenkort et.al. PRD103 (2021)
Banerjee et.al. JHEP08 (2022)
Banerjee et.al. hep-lat/2206.15471

κ_B Results



- We get $1.03 \leq \kappa_B/T^3 \leq 2.61$ (In agreement: $1.0 \leq \kappa_B/T^3 \leq 2.1$)
Brambilla et.al. hep-lat/2206.02861 Banerjee et.al. JHEP08 (2022)

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3} \langle v^2 \rangle \kappa_B$$

- Using $\langle v^2 \rangle$ from Petreczky et.al. Eur. Phys. J. C62 (2009)
- $\langle v_{\text{charm}}^2 \rangle \simeq 0.51$ and $\langle v_{\text{bottom}}^2 \rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark diffusion coefficient is 34% and 20% for the charm and bottom quarks respectively.

Conclusions and Future prospects

- Gradient flow helps with heavy quark diffusion quark studies
- Good agreement with previous results
- Measured $1/M$ corrections
 - Were able to flow with the anomalous dimension
 - Good agreement with other recent study
 - The mass correction indicated to be 20 to 30% for bottom and charm quarks
- Future prospects and ongoing work:
 - Measure γ from our data
 - Check adjoint representation correlator to study quarkonium

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Thank you for your attention!