Using Gradient Flow to Renormalise Matrix Elements for B Meson Mixing and Lifetimes

Matthew Black

In collaboration with:
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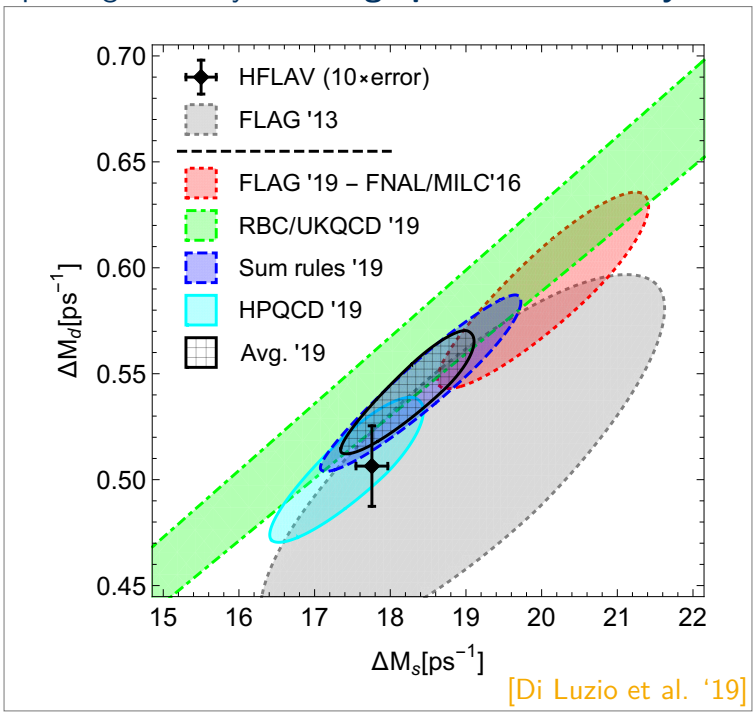
- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➡ Key observables for probing New Physics ➡ **high precision in theory needed!**



[Di Luzio et al. '19]

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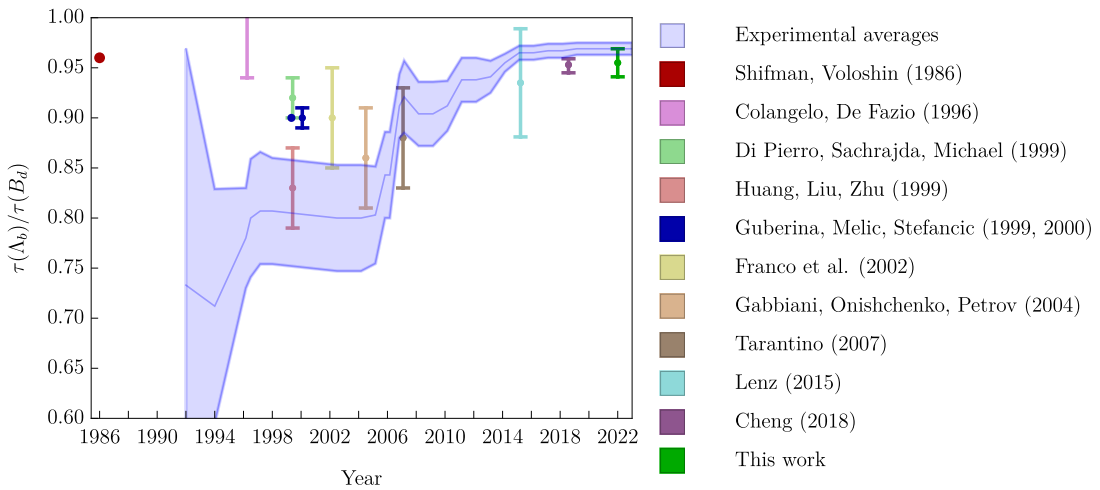

[Gratrex et al. '23]

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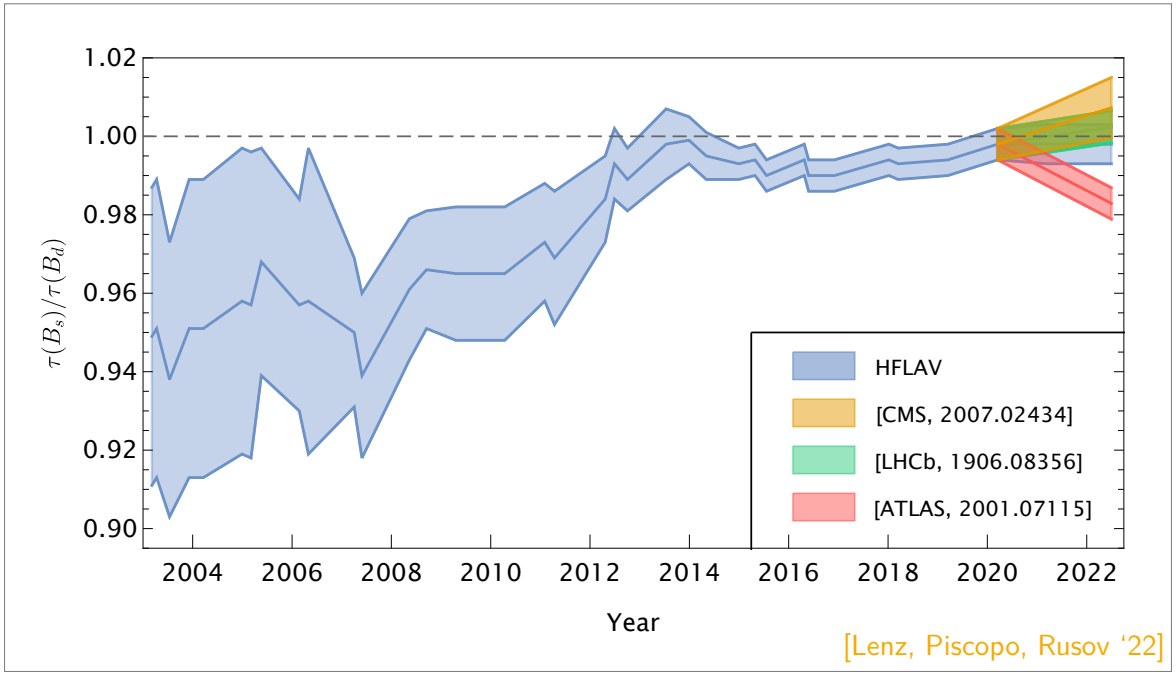
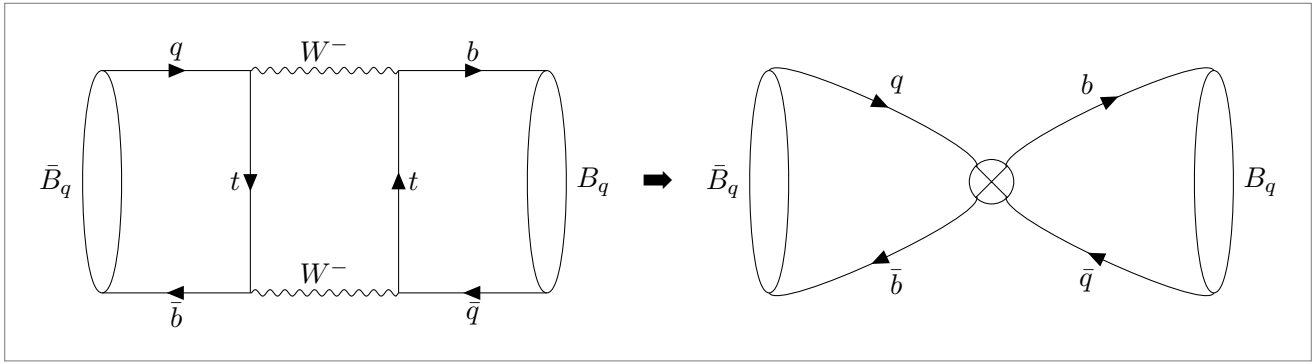


- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➡ Key observables for probing New Physics ➡ **high precision in theory needed!**
- For lifetimes and decay rates, we use the **Heavy Quark Expansion**

$$\Gamma_{B_q} = \Gamma_3 \langle \mathcal{O}_{D=3} \rangle + \Gamma_5 \frac{\langle \mathcal{O}_{D=5} \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_{D=6} \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_{D=6} \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_{D=7} \rangle}{m_b^4} + \ldots \right]$$



- Factorise observables into ➡ perturbative QCD contributions
  ➡ **Non-Perturbative Matrix Elements**

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing, but some tension
  - ➥ $\Delta K = 2$ for Kaon mixing already studied with gradient flow [Suzuki et al. '20]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago + new developments for lifetime ratios [Lin, Detmold, Meinel '22]
  - ➥ contributions from disconnected diagrams
  - ➥ mixing with lower dimension operators in renormalisation

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1. Verify procedure with $\Delta B = 2$ matrix elements against other calculations
2. Pioneer **connected** $\Delta B = 0$ matrix element calculation with gradient flow renormalisation scheme
3. Resolve disconnected contributions

- Mass difference of neutral mesons $\Delta M_q\,(q = d, s)$ governed by $\Delta B = 2$ four-quark operators
- Standard 'SUSY' operator basis

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q,$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_2^q \rangle = \langle \bar{B}_q | \mathcal{O}_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$\mathcal{O}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1-\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_3^q \rangle = \langle \bar{B}_q | \mathcal{O}_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$\mathcal{O}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \qquad \langle \mathcal{O}_4^q \rangle = \langle \bar{B}_q | \mathcal{O}_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b+m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$\mathcal{O}_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1+\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_5^q \rangle = \langle \bar{B}_q | \mathcal{O}_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b+m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

- Matrix elements parameterised in terms of **decay constant** f_{B_q} and **bag parameters** B_i^q

- HPQCD and FNAL/MILC choose perturbative renormalisation + matching schemes
- RBC/UKQCD set up a non-perturbative renormalisation (NPR) ➡ transform operator basis

$$
\begin{aligned}
\mathcal{Q}_1^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \\
\mathcal{Q}_2^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1+\gamma_5) q^\beta, \\
\mathcal{Q}_3^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \\
\mathcal{Q}_4^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \\
\mathcal{Q}_5^q &= \frac{1}{4} \bar{b}^\alpha \sigma_{\mu\nu} (1-\gamma_5) q^\alpha \; \bar{b}^\beta \sigma_{\mu\nu} (1-\gamma_5) q^\beta
\end{aligned}
$$

$$
\begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ \mathcal{O}_3^+ \\ \mathcal{O}_4^+ \\ \mathcal{O}_5^+ \end{pmatrix}
=
\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}
\begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \mathcal{Q}_3^+ \\ \mathcal{Q}_4^+ \\ \mathcal{Q}_5^+ \end{pmatrix}
$$

- Advantages for both lattice calculation and the NPR procedure
- Only colour-singlet operators now appear
- We are only concerned with parity-even components which then can be transformed back to SUSY basis

- Matrix elements are calculated directly from lattice simulations

Decay constants well-known independently

$$\langle \mathcal{O}_i^q \rangle \quad \Rightarrow \quad f_{B_q}^2 B_i^q \quad \Rightarrow \quad B_i^q$$

- [FLAG '21] reports on $\langle \mathcal{O}_1^q \rangle$ ➡ **tension between** most recent $2+1$ and $2+1+1$ calculations:

$$N_f = 2+1: \quad f_{B_s}\sqrt{\hat{B}_1^s} = 274(8)\,\mathrm{MeV}, \text{ [FNAL/MILC '16]}$$

$$N_f = 2+1+1: \quad f_{B_s}\sqrt{\hat{B}_1^s} = 256.1(5.7)\,\mathrm{MeV} \text{ [HPQCD '19A]}$$

- $\langle \mathcal{O}_{2-5}^{d,s} \rangle$ determined for $N_f = 2$ [ETM '13] and $N_f = 2+1$ [FNAL/MILC '16]
  - ➡ Work in progress by RBC/UKQCD + JLQCD at $N_f = 2+1$ [Boyle et al. '18], [Boyle et al. '21]
    - ➡ **we use same setup as RBC/UKQCD for comparisons later**
- First lattice calculations for dim-7 $\langle R_{2,3}^q \rangle$ and $\langle \widetilde{R}_{2,3} \rangle$ from [HPQCD '19B]
  - ➡ Suffers from large uncertainties e.g. from matching to continuum regularisation scheme

- Formulated by [Lüscher '10], [Lüscher '13] ➡ scale setting, RG β-function, **renormalisation**...
- Introduce auxiliary dimension, **flow time** t as a way to regularise the UV
- Extend gauge and fermion fields in flow time and express dependence with first-order differential equations:

$$\partial_t B_\mu(t,x) = \mathcal{D}_\nu(t) G_{\nu\mu}(t,x), \quad B_\mu(0,x) = A_\mu(x),$$
$$\partial_t \chi(t,x) = \mathcal{D}^2(t)\chi(t,x), \quad \chi(0,x) = q(x).$$

- Re-express effective Hamiltonian in terms of 'flowed' operators:

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(t)\widetilde{\mathcal{O}}_n(t).$$

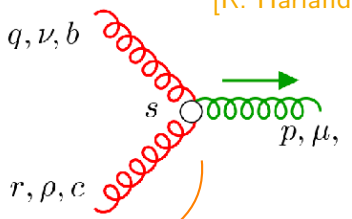
- Relate to regular operators in 'small-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(t) = \sum_m \zeta_{nm}(t)\mathcal{O}_m + O(t)$$

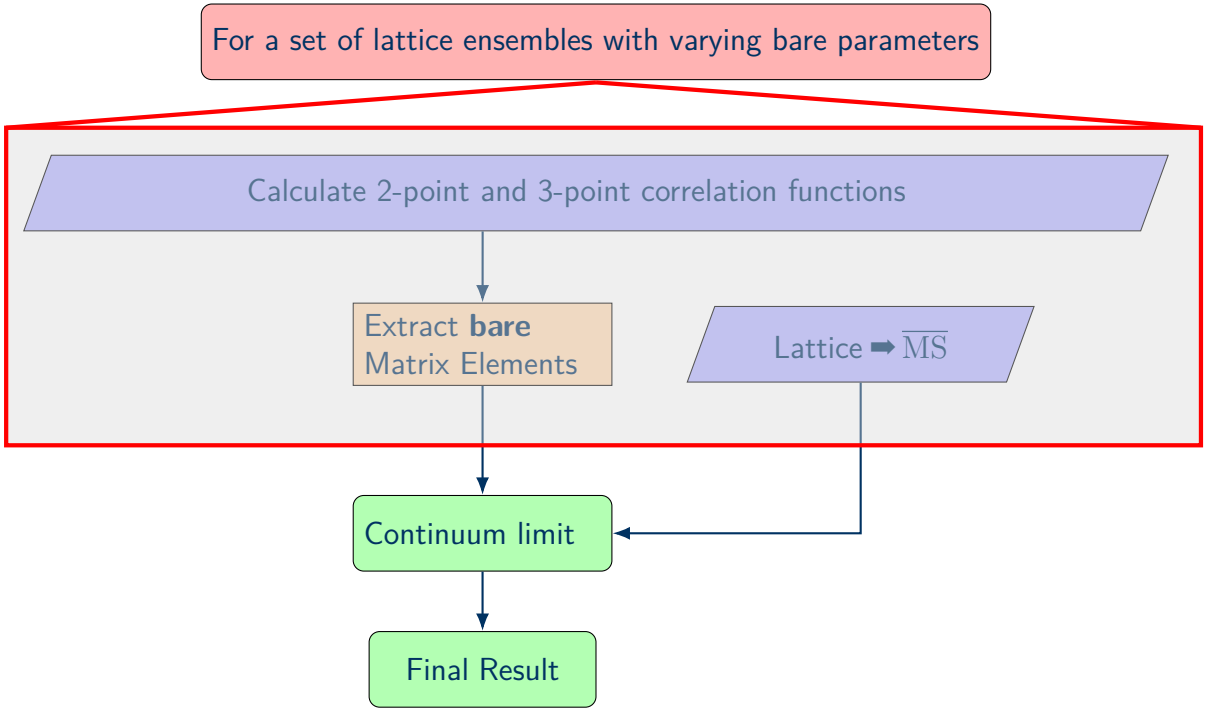
'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

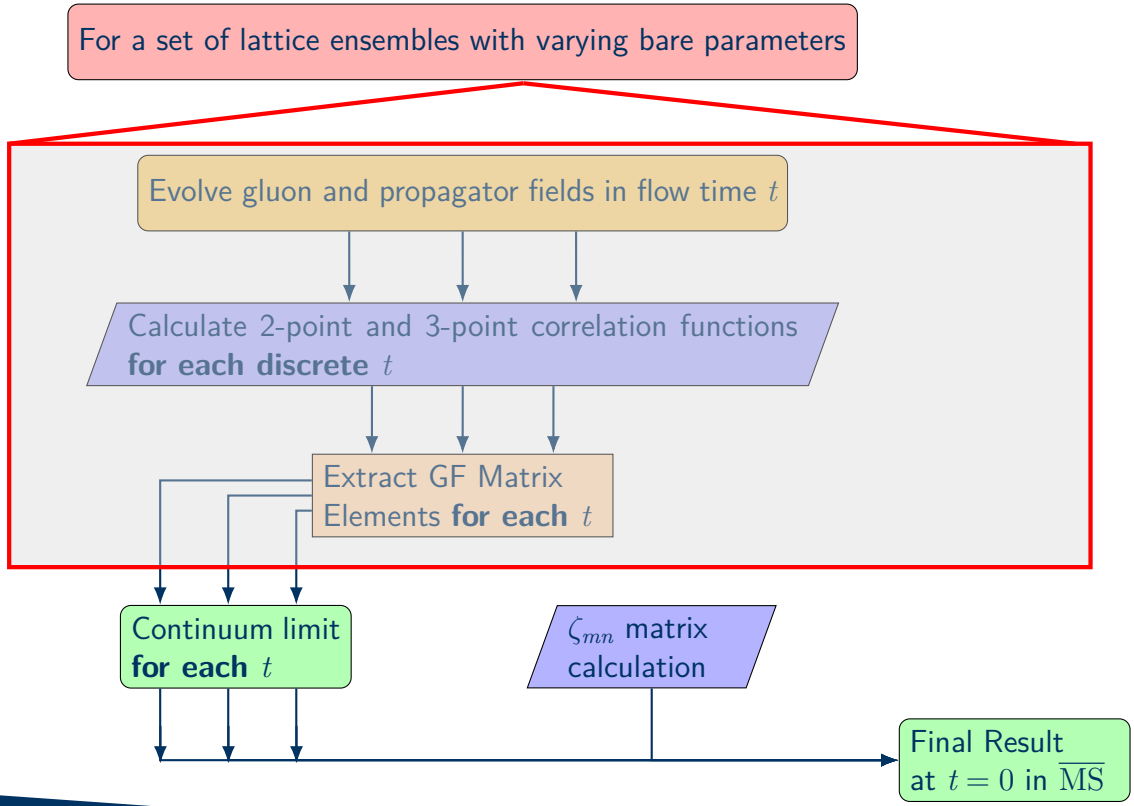
matching matrix calculated perturbatively

[R. Harlander]



new Feynman diagrams


For a set of lattice ensembles with varying bare parameters
Calculate 2-point and 3-point correlation functions
Extract **bare** Matrix Elements
Lattice ➡ $\overline{\mathrm{MS}}$
Continuum limit
Final Result


For a set of lattice ensembles with varying bare parameters
Evolve gluon and propagator fields in flow time t
Calculate 2-point and 3-point correlation functions **for each discrete** t
Extract GF Matrix Elements **for each** t
Continuum limit **for each** t
ζ_{mn} matrix calculation
Final Result at $t = 0$ in $\overline{\mathrm{MS}}$

- We will consider RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles

	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs $\times$ N$_{\text{conf}}$
C1	24	64	1.785	0.005	0.040	340	32×101
C2	24	64	1.785	0.010	0.040	433	
M1	32	64	2.383	0.004	0.030	302	
M2	32	64	2.383	0.006	0.030	362	
M3	32	64	2.383	0.008	0.030	411	
F1S	48	96	2.785	0.002144	0.02144	267	

[Allton et al. '08]
[Aoki et al. '10]
[Blum et al. '14]
[Boyle et al. '17]

- Exploratory simulations only on C1 with single set of valence parameters so far

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- For heavy b quarks, $am_q > 1$ ➡ large discretisation effects ✗
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- Z2 wall sources for all quark propagators [Boyle et al. '08]
  - ➥ Sources for strange propagators are also Gaussian smeared [Allton et al. '93]
- Calculate **non-eye** weak 3-point functions

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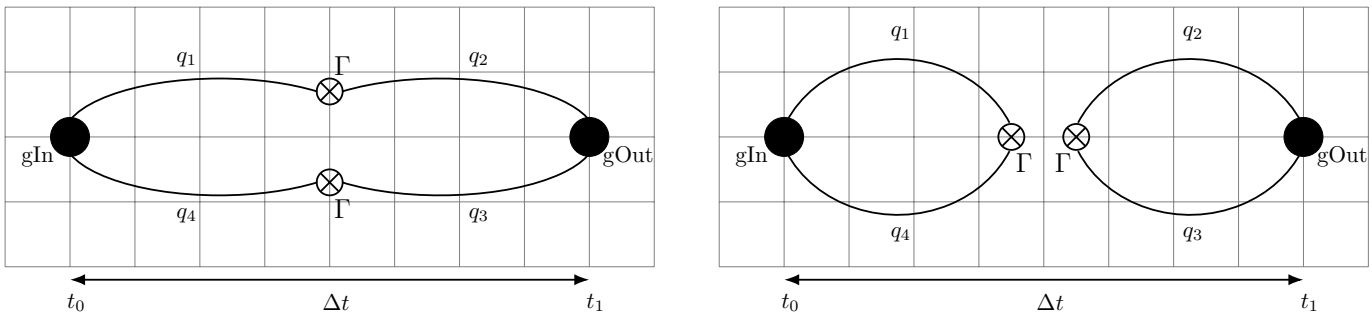


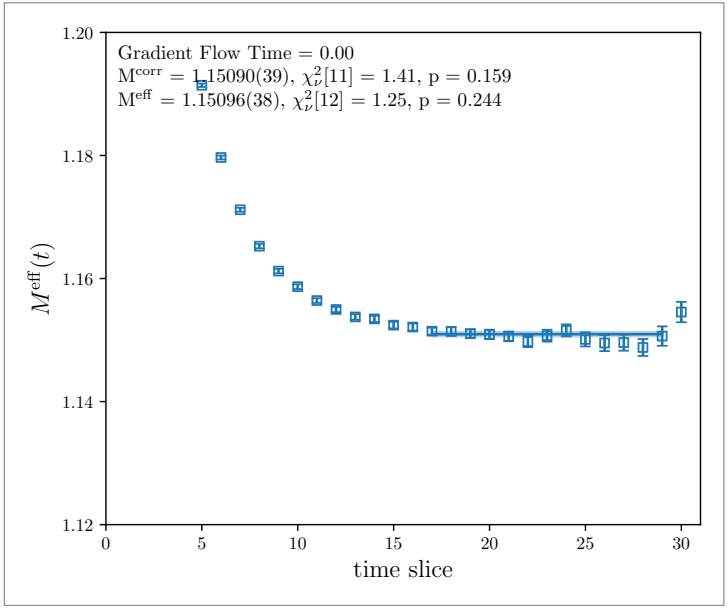
- Valence simulations carried out using Hadrons [Portelli et al. '22]
- Gauge Flow
  - Runge-Kutta scheme for small step size ϵ
  - Pre-existing implementation of Wilson action gauge flow in Grid [Boyle et al. '15]
  - Re-implemented into Hadrons also with Symanzik and Zeuthen actions
- Fermion Flow
  - Uses gauge flow implementation with Wilson action
  - Evolve propagators with 4D Laplacian in Runge-Kutta scheme
- Gauge and fermion fields evolved with $\epsilon = 0.01$
- Measurements taken every 10 steps for $t/a^2 < 5$

- We want to find **suitable window** in flow time
  - May be different for different quantities
- Look at different quantities and their behaviour with the flow
  - 2-point functions ➡ effective mass, decay amplitude

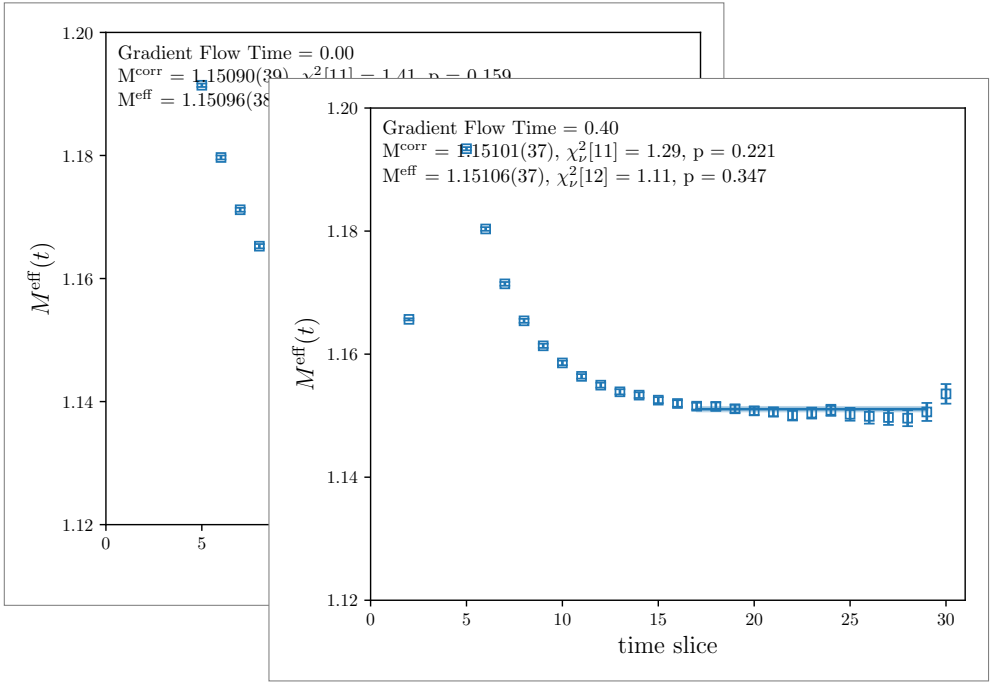
$$M^{\text{eff}}(t) = \cosh^{-1}\left(\frac{C_{PP}(t) + C_{PP}(t+2)}{2\,C_{PP}(t+1)}\right) \qquad \Phi^{\text{eff}}(t) = \sqrt{2}\frac{|C_{AP}(t)|}{\sqrt{C_{PP}(t)e^{-Mt}}}$$

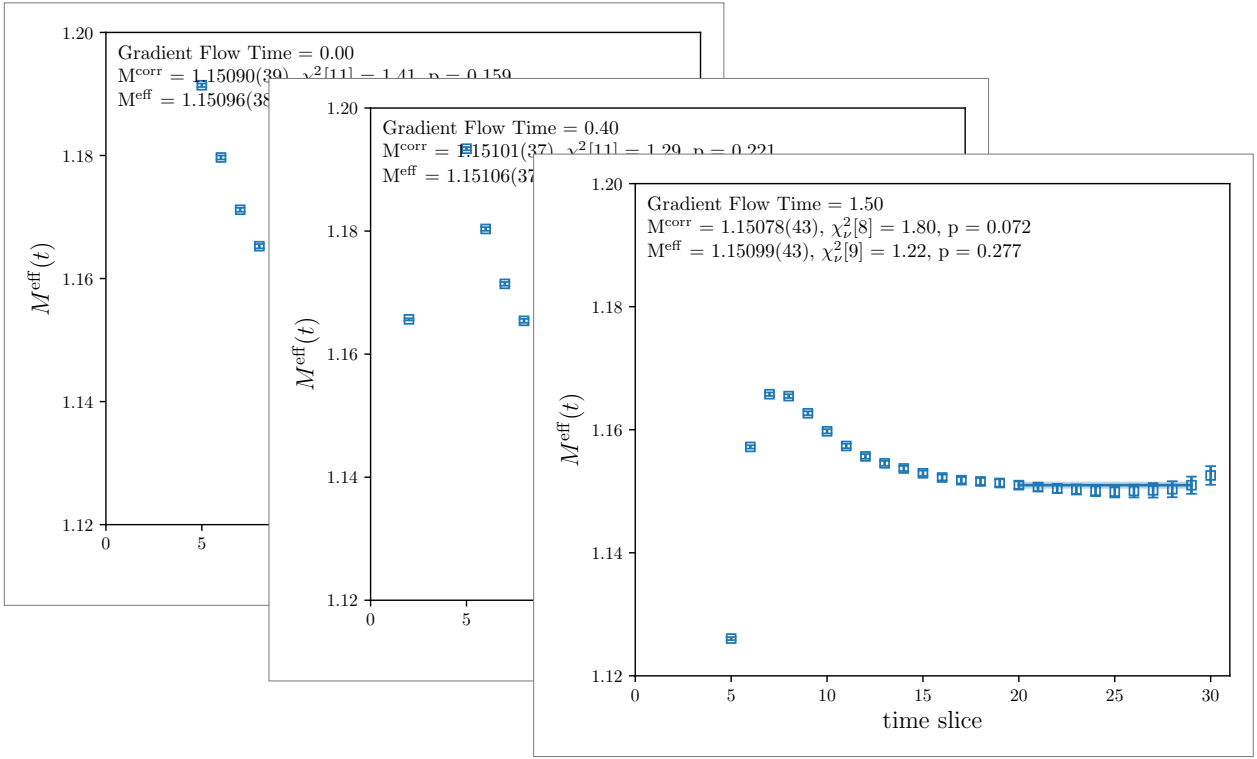
  - 3-point functions
  - 3-point/2-point ratios ➡ approximate to Bag parameters

$$R_1 = \frac{C^{3\text{pt}}_{\mathcal{O}_1}(t,\Delta t)}{C^{2\text{pt}}_{AP}(t)\,C^{2\text{pt}}_{PA}(\Delta T - t)} \qquad R_i = \frac{C^{3\text{pt}}_{\mathcal{O}_i}(t,\Delta t)}{C^{2\text{pt}}_{PP}(t)\,C^{2\text{pt}}_{PP}(\Delta T - t)}, i = 2 \to 5$$

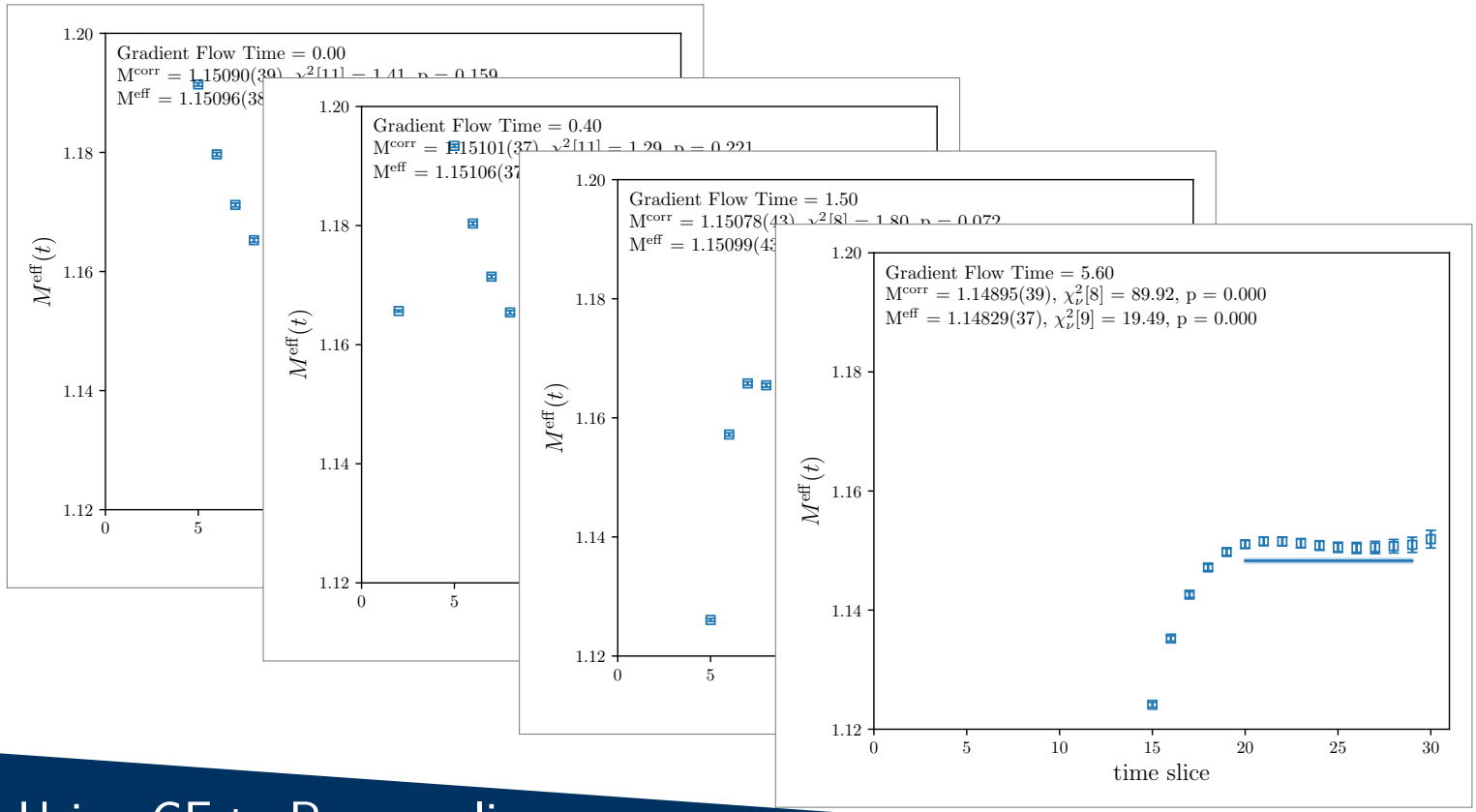


- Flow acts on effective mass as sink smearing
- Excited states suppressed at earlier time slices
- Ground state effective mass should not change
- Large flow time will destroy the ground state


Gradient Flow Time = 0.00
Gradient Flow Time = 0.40
M^{corr} = 1.15101(37), $\chi^2_\nu[11]$ = 1.29, p = 0.221
M^{eff} = 1.15106(37), $\chi^2_\nu[12]$ = 1.11, p = 0.347
$M^{\text{eff}}(t)$
time slice
1.20
1.18
1.16
1.14
1.12
0
5
10
15
20
25
30


Gradient Flow Time = 0.00
Gradient Flow Time = 0.40
Gradient Flow Time = 1.50
$M^{\mathrm{corr}} = 1.15078(43)$, $\chi^2_\nu[8] = 1.80$, p = 0.072
$M^{\mathrm{eff}} = 1.15099(43)$, $\chi^2_\nu[9] = 1.22$, p = 0.277
$M^{\mathrm{eff}}(t)$
time slice

First Look – 2-point function (effective mass)



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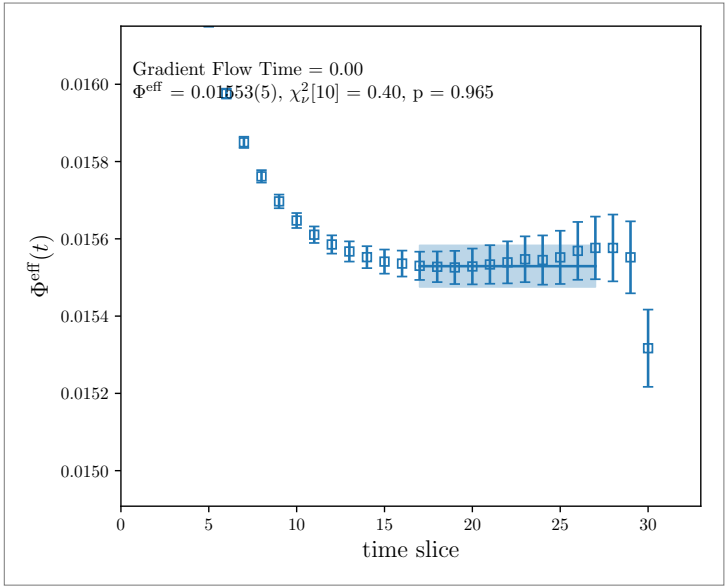
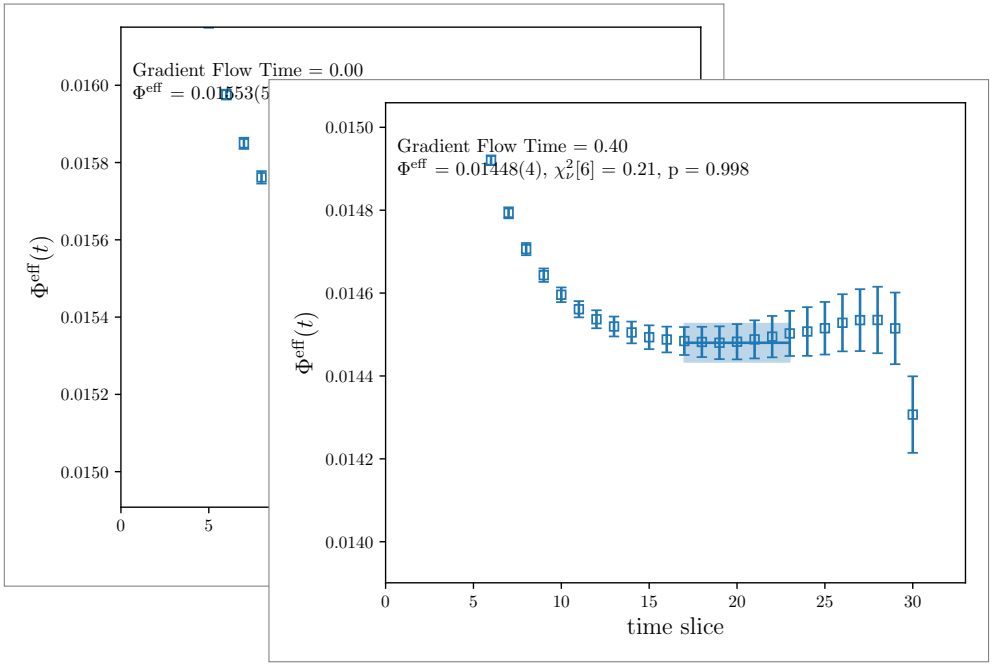
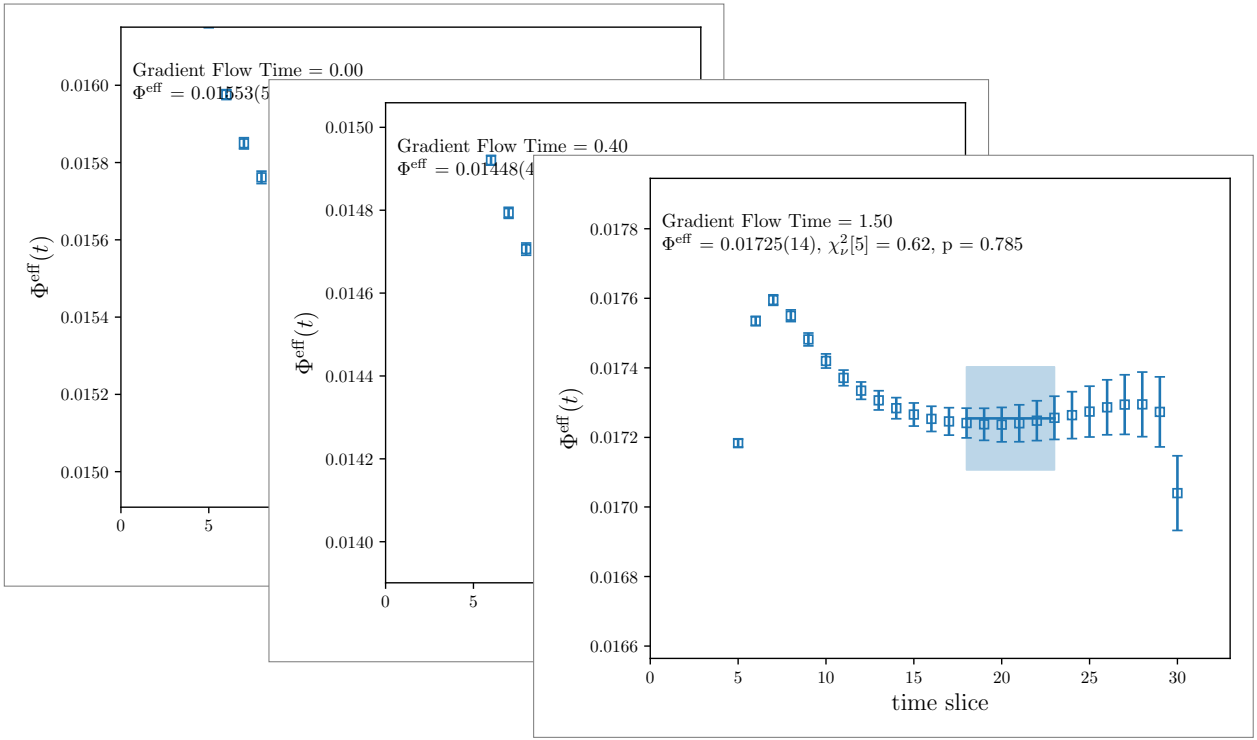
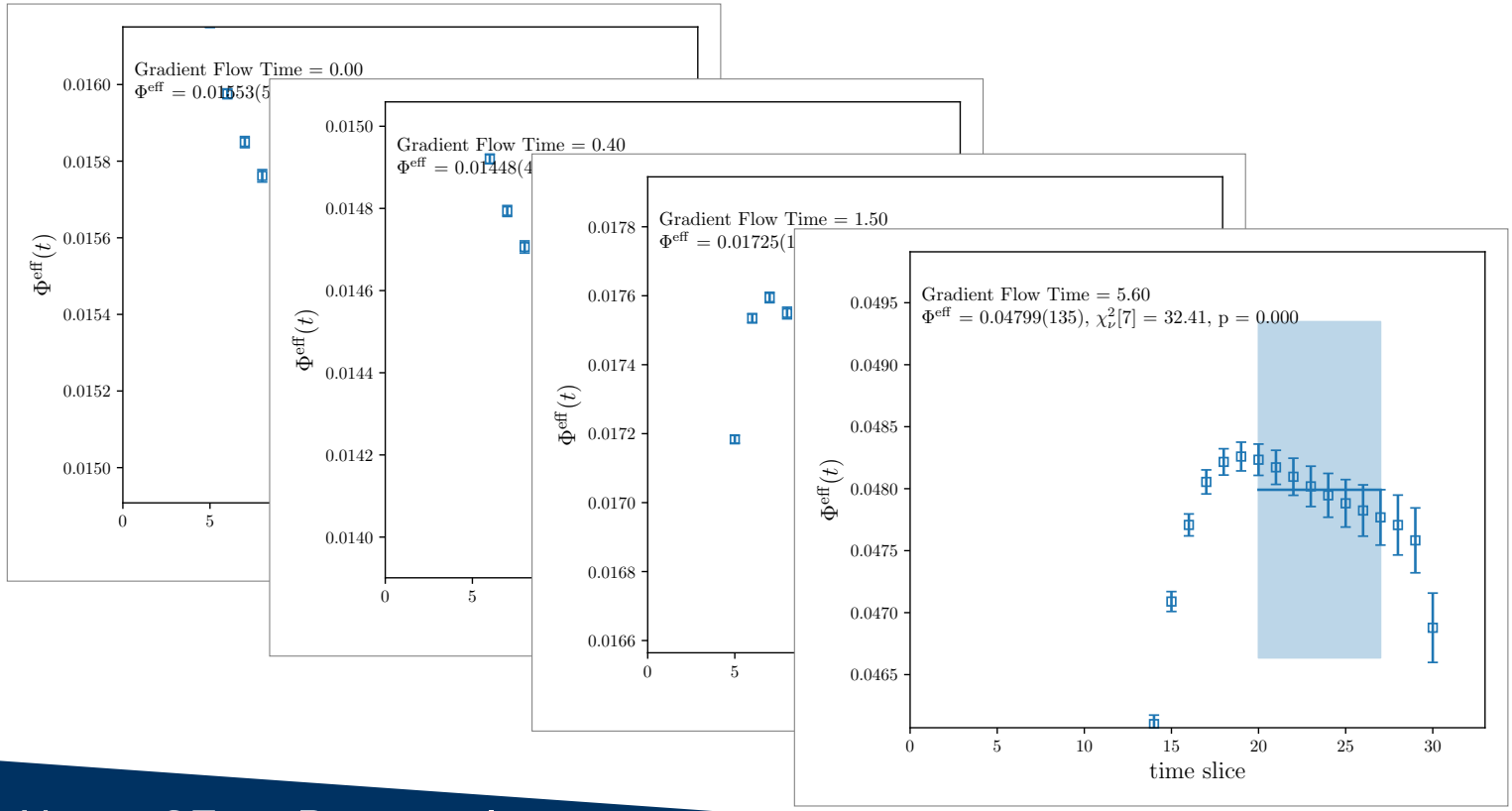


- Expect similar behaviour to effective mass
- Value of ground state decay amplitude **will** change


Gradient Flow Time = 0.00
$\Phi^{\text{eff}} = 0.01553(5$
$\Phi^{\text{eff}}(t)$
0.0160
0.0158
0.0156
0.0154
0.0152
0.0150
0
5
Gradient Flow Time = 0.40
$\Phi^{\text{eff}} = 0.01448(4)$, $\chi^2_\nu[6] = 0.21$, p = 0.998
$\Phi^{\text{eff}}(t)$
0.0150
0.0148
0.0146
0.0144
0.0142
0.0140
0
5
10
15
20
25
30
time slice


Gradient Flow Time = 0.00
Gradient Flow Time = 0.40
Gradient Flow Time = 1.50
$\Phi^{\text{eff}} = 0.01725(14)$, $\chi^2_\nu[5] = 0.62$, p = 0.785
$\Phi^{\text{eff}}(t)$
time slice



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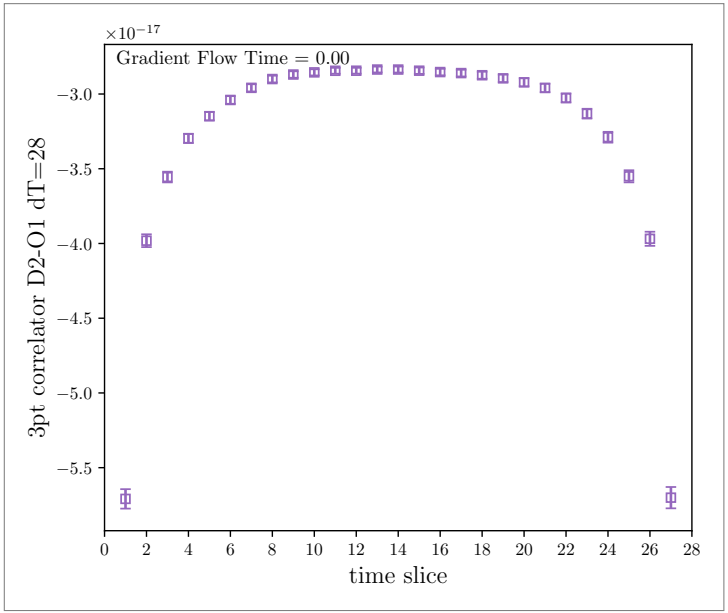
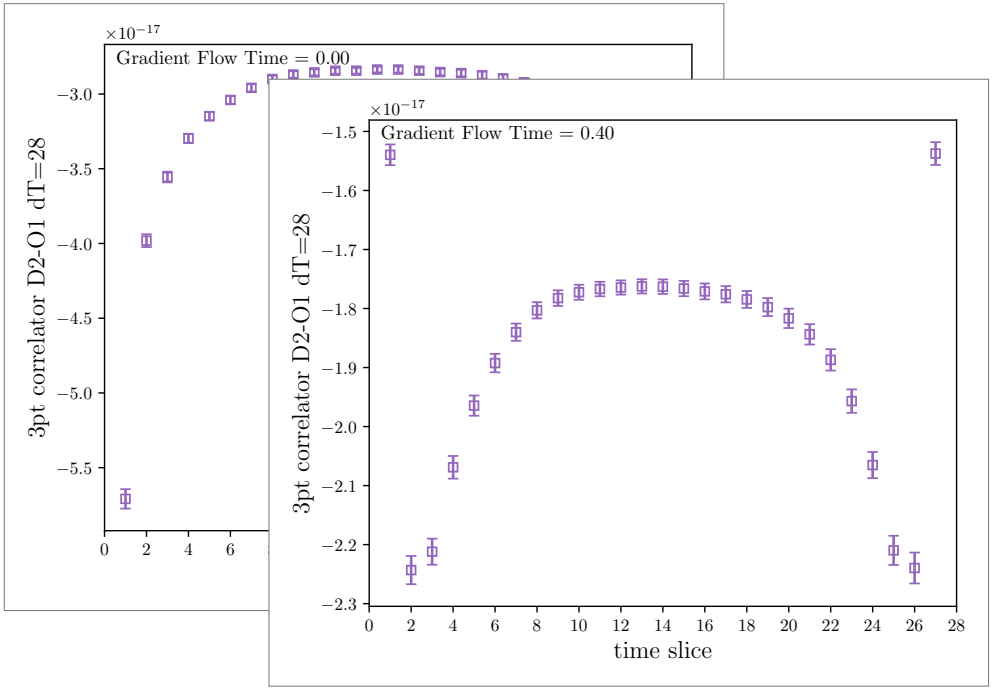
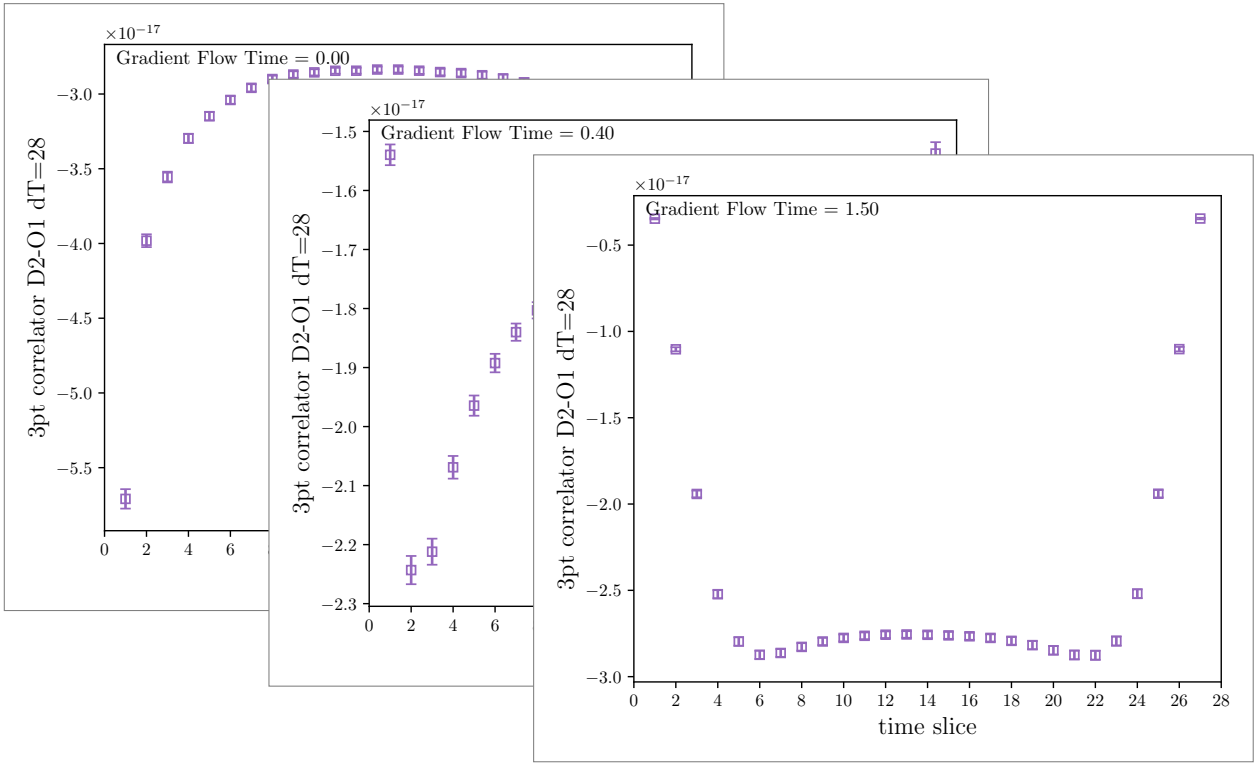
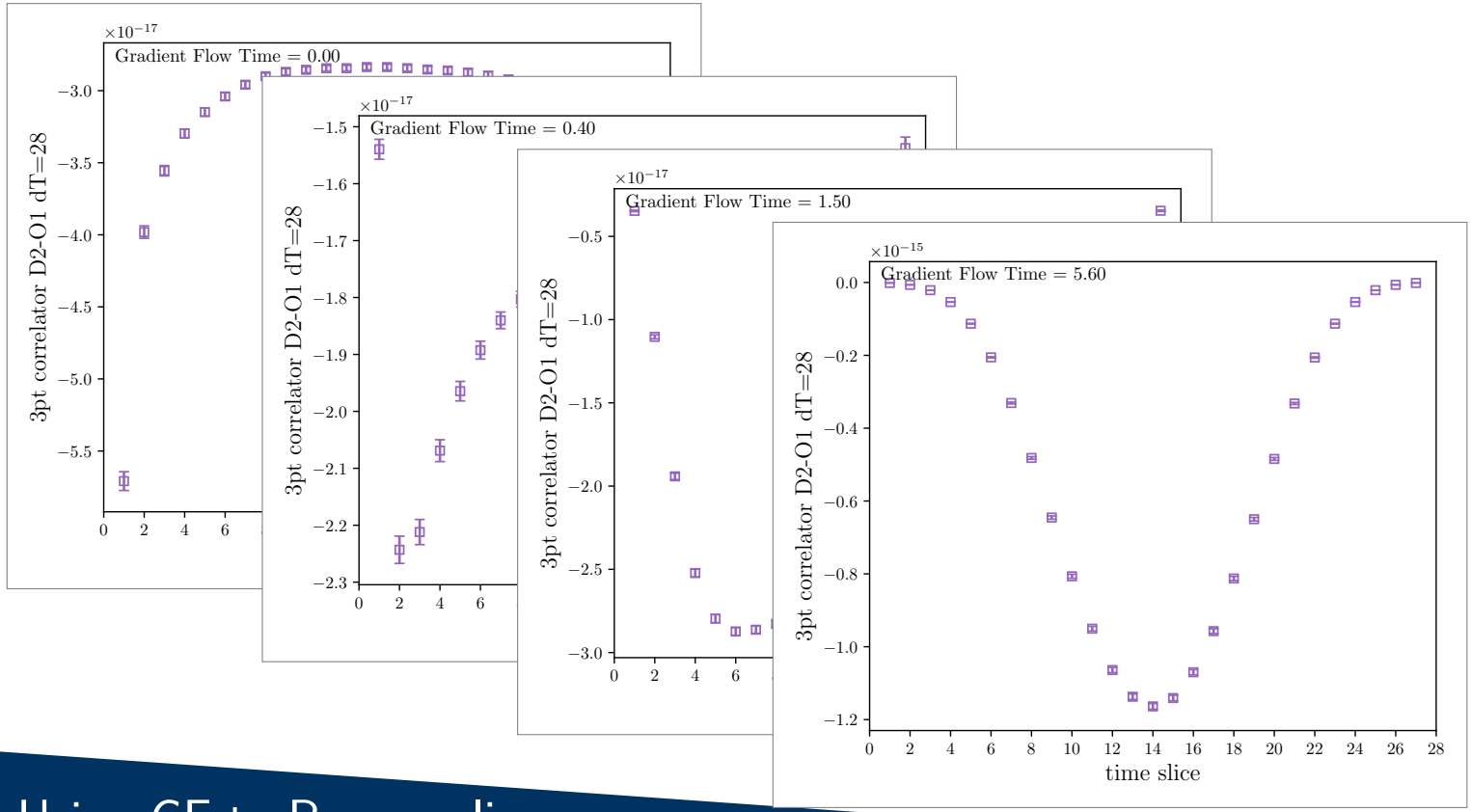


- Ground state should change **but how?**
- At large flow time, smearing of sources overlaps with 3pt ground state
- Different ΔT will have different windows


×10^−17
Gradient Flow Time = 0.00
3pt correlator D2-O1 dT=28
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
0
2
4
6
×10^−17
Gradient Flow Time = 0.40
3pt correlator D2-O1 dT=28
−1.5
−1.6
−1.7
−1.8
−1.9
−2.0
−2.1
−2.2
−2.3
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
time slice


×10^−17
Gradient Flow Time = 0.00
3pt correlator D2-O1 dT=28
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
0 2 4 6
×10^−17
Gradient Flow Time = 0.40
3pt correlator D2-O1 dT=28
−1.5
−1.6
−1.7
−1.8
−1.9
−2.0
−2.1
−2.2
−2.3
0 2 4 6
×10^−17
Gradient Flow Time = 1.50
3pt correlator D2-O1 dT=28
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
time slice



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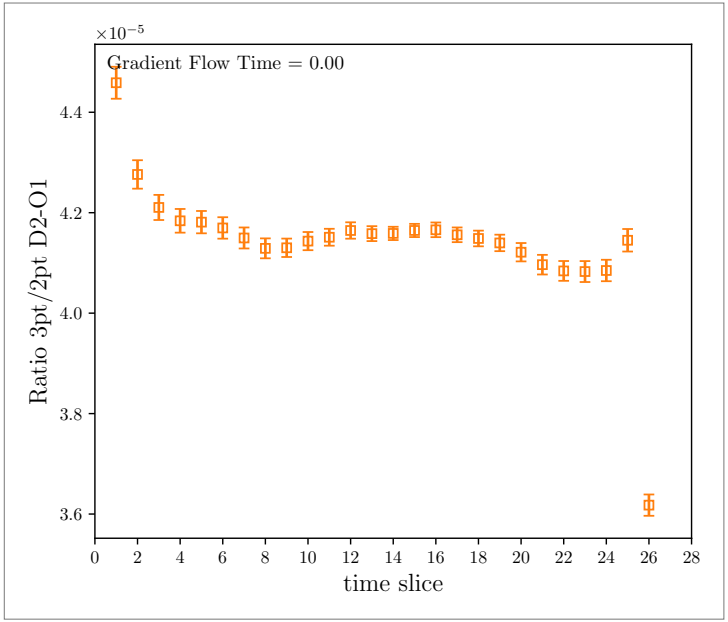
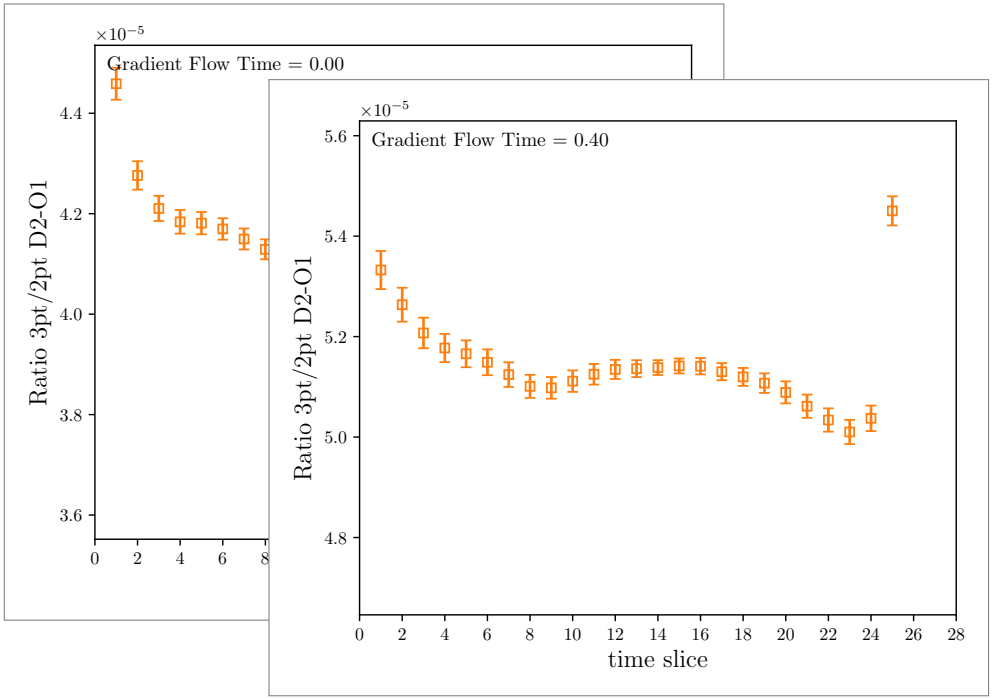
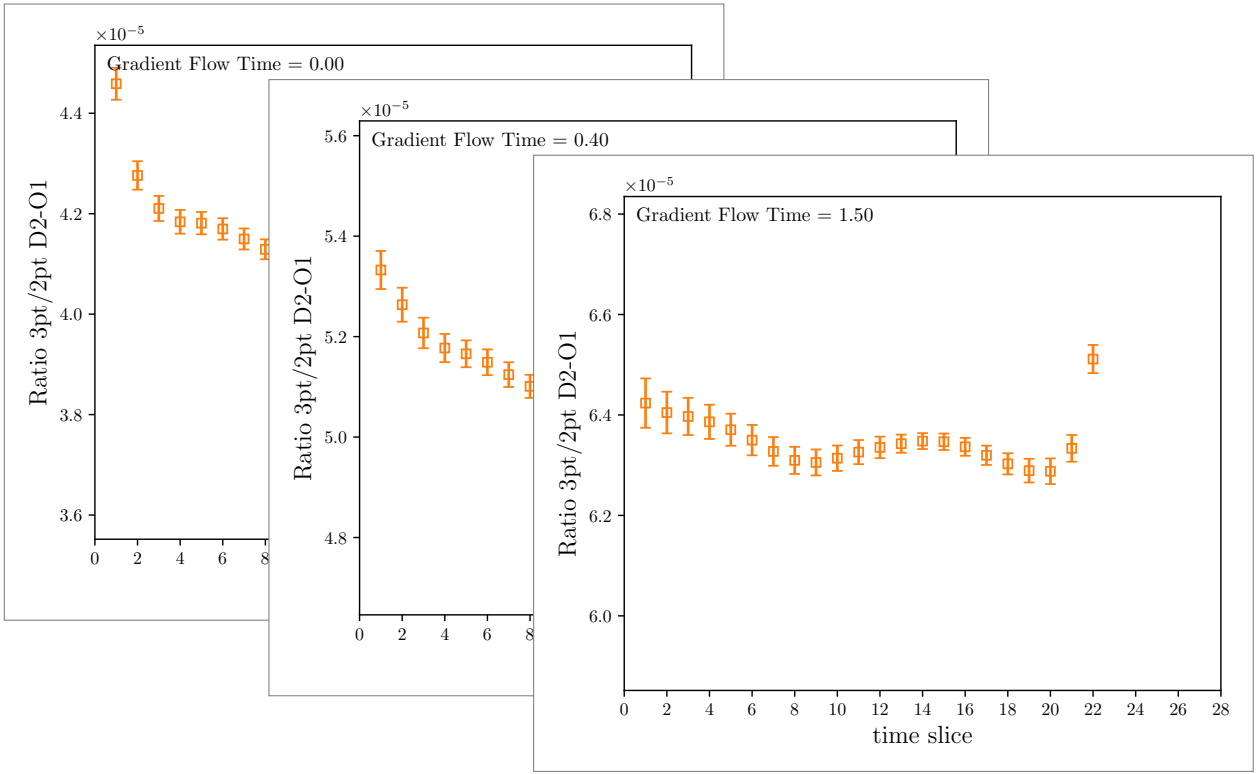
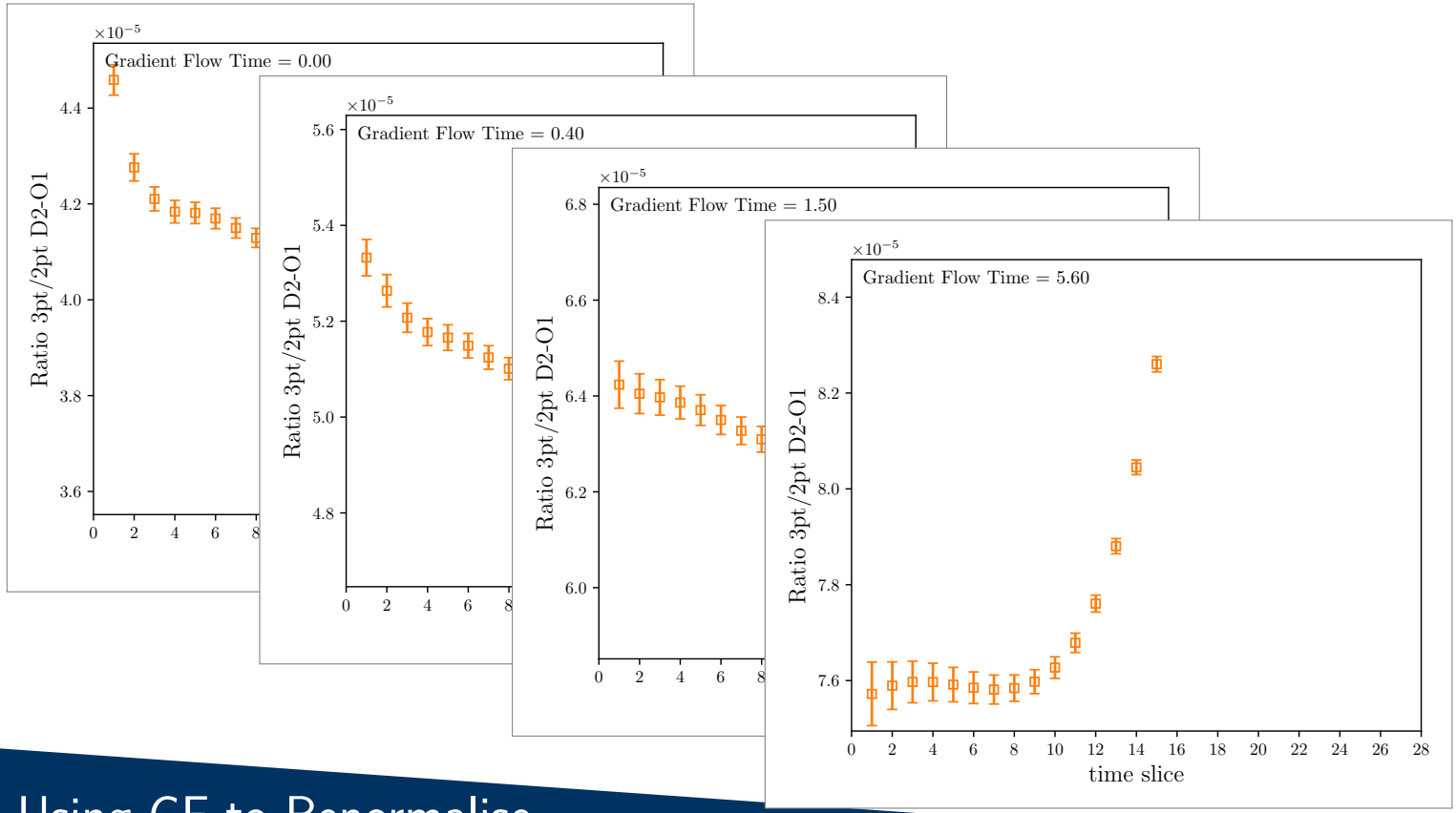

×10⁻⁵
Gradient Flow Time = 0.00
Ratio 3pt/2pt D2-O1
4.4
4.2
4.0
3.8
3.6
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
time slice


×10⁻⁵
Gradient Flow Time = 0.00
Ratio 3pt/2pt D2-O1
4.4
4.2
4.0
3.8
3.6
0
2
4
6
8
×10⁻⁵
Gradient Flow Time = 0.40
Ratio 3pt/2pt D2-O1
5.6
5.4
5.2
5.0
4.8
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
time slice


×10⁻⁵
Gradient Flow Time = 0.00
Ratio 3pt/2pt D2-O1
4.4
4.2
4.0
3.8
3.6
0 2 4 6 8
×10⁻⁵
Gradient Flow Time = 0.40
Ratio 3pt/2pt D2-O1
5.6
5.4
5.2
5.0
4.8
0 2 4 6 8
×10⁻⁵
Gradient Flow Time = 1.50
Ratio 3pt/2pt D2-O1
6.8
6.6
6.4
6.2
6.0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
time slice



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- $\Delta B = 0$ four-quark matrix elements are strongly-desired quantities needed in predictions of B meson lifetimes
  - ➡ Standard renormalisation introduces mixing with operators of lower mass dimension
  - ➡ We aim to use the fermionic gradient flow as a non-perturbative renormalisation procedure
- Testing method first with well-studied $\Delta B = 2$ matrix elements for B meson mixing and decay constants
- Implemented the fermionic flow in Hadrons, plus gauge flow for Symanzik and Zeuthen actions
- Shown first simulations for $\Delta B = 2$

Next Steps:

- Simulate at multiple heavy quark masses on all ensembles
- Extrapolate to physical heavy mesons + continuum limits at each t in 'small-flow-time' region
- Combine with perturbative matching matrix ➡ final results at $t = 0$ in $\overline{\mathrm{MS}}$
- Repeat analysis for quark-line connected $\Delta B = 0$ matrix elements
- Consider disconnected contributions

- For lifetimes, the dimension-6 $\Delta B = 0$ operators are:

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5)(T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5)(T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5)(T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5)(T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we again want these all to be colour-singlet operators:

$$\begin{aligned} \mathcal{Q}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\ \mathcal{Q}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1+\gamma_5) b^\beta) \\ \tau_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\ \tau_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \end{aligned}$$

$$\begin{pmatrix} Q_1^+ \\ Q_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \tau_1^+ \\ \tau_2^+ \end{pmatrix}$$