The Chromomagnetic Dipole Operator in the Gradient Flow

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The Gradient Flow in QCD and other Strongly Coupled Field Theories
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Introduction

Relavance

- CP-Violation required for baryon asymmetry
- Neutron EDM violates P- and T-symmetry ⇒ CP-Violation

Current status

- Experimental bound on neutron EDM: [Abel *et al.* 2020]

$$d_n = (0.0 \pm 1.1_{\text{stat}} \pm 0.2_{\text{sys}}) \times 10^{-26} e\,\text{cm}$$

  Future experiments (PSI, TRIUMF, ...) may improve sensitivity even further [nEDM collaboration 2021; TUCAN collaboration 2022]

- SM estimates from CKM matrix: [Seng 2015]

$$d_n \sim 10^{-32} e\,\text{cm}$$

⇒ Large window for BSM contributions [Gavela *et al.* 1994]

Describe BSM effects with effective theory:

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SU(N)}} + \sum_{n\geq 5} \frac{C_n}{\Lambda^n} \mathcal{O}^{(n)}$$

- CP-odd dimension-five operator:

$$\mathcal{O}_{\mathrm{CE}} = \overline{\psi} \sigma_{\mu\nu} \gamma_5 G_{\mu\nu} \psi$$

- γ_5 complicated in dimensional regularization $\Rightarrow$ CP-even analogue:

$$\mathcal{O}_{\mathrm{CM}} = \overline{\psi} \sigma_{\mu\nu} G_{\mu\nu} \psi$$

$\mathcal{O}_{\mathrm{CM}}$ relevant on its own: [D'Ambrosio, Isidori, Martinelli 1999]

- Strangeness-changing CM operators contribute to CP-violating kaon decays
- Neutral kaon mixing

Applying the Gradient Flow

Lattice calculations $\Rightarrow$ Flowed operators:

- Flow equations + initial conditions [Lüscher 2010; Lüscher 2013; Narayanan, Neuberger 2006]
- 4+1-dimensional field theory [Lüscher, Weisz 2011]
- Replace fields by flowed fields:

$$\mathcal{O}_{\mathrm{CM}}(\overline{\psi}, \psi, A) \to \tilde{\mathcal{O}}_{\mathrm{CM}}(\overline{\chi}(t), \chi(t), B(t))$$

Relate to physical operators $\Rightarrow$ Small-t expansion (SFTE): [Lüscher, Weisz 2011]

$$\tilde{\mathcal{O}}_{\mathrm{CM}}(t, x) = \sum_n c_n(t)\mathcal{O}_n(x) + \dots$$

- $\tilde{\mathcal{O}}_{\mathrm{CM}}$ gets contributions from operators of smaller mass dimension: $c_n(t) \sim \log t, \quad c_n(t) \sim \frac{1}{t^n}$
- Other contributions vanish as $t \to 0$

$\tilde{\mathcal{O}}_{\mathrm{CM}}$ mixes with

- 6 dimension-5 operators:

$$\mathcal{O}_{\mathrm{CM}} = \overline{\psi}\sigma_{\mu\nu}G_{\mu\nu}\psi\,, \qquad \mathcal{O}_{\mathrm{m^5}} = m^5\,,$$
$$\mathcal{O}_{\mathrm{mD}} = m\overline{\psi}\not{D}\psi\,, \qquad \mathcal{O}_{\mathrm{mG}} = mG_{\mu\nu}G_{\mu\nu}\,,$$
$$\mathcal{O}_{\mathrm{mE}} = m\overline{\psi}(\not{D}+m)\psi\,, \qquad \mathcal{O}_{\mathrm{DE}} = \overline{\psi}\not{D}(\not{D}+m)\psi$$

- 2 dimension-3 operators:

$$\mathcal{O}_S = \overline{\psi}\psi\,, \quad \mathcal{O}_{\mathrm{m^3}} = m^3$$

- 1 dimension-1 operator:

$$\mathcal{O}_{\mathrm{m}} = m$$

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Invert SFTE $\Rightarrow$ Need full matching matrix:

$$\tilde{\mathcal{O}}_i(t,x) = \sum_j \zeta_{ij}(t)\mathcal{O}_j(x)$$

Now express regular in terms of flowed operators:

$$\mathcal{O}_{\mathrm{CM}}(x) = \sum_i \zeta^{-1}_{\mathrm{CM},i}(t)\tilde{\mathcal{O}}_i(t,x)$$

How do we obtain ζ_{ij}?

Method of Projectors

Method of Projectors [Gorishny, Larin, Tkachov 1983; Gorishny, Larin 1987]

- Define orthonormal set of projectors P_k such that

$$P_k[\mathcal{O}_j] = \delta_{kj}$$

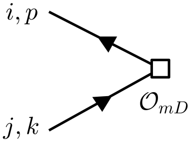
- Apply on both sides of the SFTE:

$$P_k[\tilde{\mathcal{O}}_i(t)] = \sum_j \zeta_{ij}(t)\, \underbrace{P_k[\mathcal{O}_j]}_{\delta_{kj}} = \zeta_{ik}(t)$$

- How to define the P_k?

Example: P_{mD}

- Find a simple Feynman rule of $\mathcal{O}_{\text{mD}} = m\overline{\psi}\not{D}\psi$:



$$= \mathrm{i}\delta_{ij} m \not{p}$$

[All diagrams created with FeynGame]

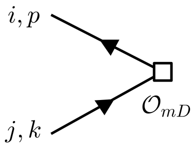
- Take expectation value with corresponding external states:

$$P_{\text{mD}}[X] = \ldots \langle 0 | X | \overline{\psi}_i(p)\psi_j(k) \rangle \; + \; \ldots$$

$\Rightarrow$ tree-level + loop diagrams

- Apply derivatives, set all massive scales except t to 0
- If X unflowed, loops become massless tadpoles and vanish

Only need tree level diagram (i.e., the Feyman rule):



$= \mathrm{i}\delta_{ij} m \not{p}$

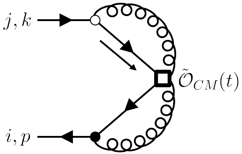
Now project ...

- ... in Dirac space: $\mathrm{Tr}\gamma_\mu \not{p} = 4p_\mu$
- ... onto momenta and mass: $\frac{\partial}{\partial p_\mu} p_\mu = D = 4 - 2\epsilon\,, \quad \frac{\partial}{\partial m} m = 1$
- ... in color space: $\delta_{ij}\delta_{ij} = N_C$

$$\Rightarrow \quad P_{\mathrm{mD}}[X] = \frac{-\mathrm{i}\delta_{ij}}{4DN_C} \frac{\partial^2}{\partial m \partial p_\mu} \mathrm{Tr}\gamma_\mu \langle 0 | X | \overline{\psi}_i(p)\psi_j(k) \rangle \Big|_{p,k,m_B=0} + \ldots$$

Apply to flowed operators:

$$P_{\mathrm{mD}}[\tilde{\mathcal{O}}_{\mathrm{CM}}] \ni \frac{-\mathrm{i}\delta_{ij}}{4DN_C}\frac{\partial^2}{\partial m \partial p_\mu}\mathrm{Tr}\gamma_\mu \left(\right.$$



$$\left. \right)\Bigg|_{p,k,m_B=0}$$

$\Rightarrow$ Left with scalar integrals depending only on t, e.g.

$$P_{\mathrm{mD}}[\tilde{\mathcal{O}}_{\mathrm{CM}}] \overset{\mathrm{NNLO}}{\ni} t\int_0^1 du\, u \int_{p,k} \frac{e^{[2up^2+2k^2]t}}{p^4 k^2}(p+k)^2$$

Then: integrals $\xrightarrow{\text{IBP relations}}$ master integrals $\xrightarrow{\text{evaluation}}$ result

Plan:

- Find projector for every operator: $\mathcal{O}_i \to P_i$
- Ensure that $P_i[\mathcal{O}_j] = \delta_{ij}$
- Calculate $\zeta_{ij} = P_j[\tilde{\mathcal{O}}_i]$

Problem:

- Equations of Motion (EoM) $\Rightarrow$ Relations between operators
- Basis not linearly independent
- Have to get rid of redundant operators

Equations of Motion

Equations of Motion:

$$\mathcal{O}_{\mathrm{mE}} = m\overline{\psi}(\not{D} + m)\psi \;= \mathcal{O}_{\mathrm{mD}} + \underbrace{m^2\overline{\psi}\psi}_{\mathcal{O}_{\mathrm{m^2S}}} = 0\,,$$

$$\mathcal{O}_{\mathrm{DE}} = \overline{\psi}\not{D}(\not{D} + m)\psi \;= \underbrace{\overline{\psi}D^2\psi}_{\mathcal{O}_{\mathrm{D^2}}} - \frac{\mathrm{i}}{2}\mathcal{O}_{\mathrm{CM}} + \mathcal{O}_{\mathrm{mD}} = 0\,.$$

SFTE coefficients **ill-defined** on-shell: [Harlander, Kluth, Lange 2019]

$$\begin{aligned}\tilde{\mathcal{O}}_{\mathrm{CM}} &= c_{\mathrm{m^2S}}\mathcal{O}_{\mathrm{m^2S}} + c_{\mathrm{mD}}\mathcal{O}_{\mathrm{mD}} + \dots \\ &= \left(c_{\mathrm{m^2S}} - c_{\mathrm{mD}}\right)\mathcal{O}_{\mathrm{m^2S}} + \dots \\ &= \left(\left(c'_{\mathrm{m^2S}} + \frac{1}{\epsilon}\right) - \left(c'_{\mathrm{mD}} + \frac{1}{\epsilon}\right)\right)\mathcal{O}_{\mathrm{m^2S}} + \dots\end{aligned}$$

$\Rightarrow$ Remove EoM-redundant operators from the basis

This affects P_{CM} and P_{mD}:

- Projectors can still project on EoM operators
- With

$$P'_{\mathrm{mD}}[X] = \frac{-\mathrm{i}\delta_{ij}}{4DN_C} \frac{\partial^2}{\partial m \partial p_\mu} \mathrm{Tr}\gamma_\mu \langle 0 | X | \bar{\psi}_i(p)\psi_j(k)\rangle \Big|_{p,k,m_B=0}$$

we get

$$P'_{\mathrm{mD}}\left[\bar{\psi}\not{D}(\not{D}+m)\psi\right] = 1\,, \qquad P'_{\mathrm{mD}}\left[m\bar{\psi}(\not{D}+m)\psi\right] = 1\,,$$

$$\Rightarrow \quad P_{\mathrm{mD}} := P'_{\mathrm{mD}} - P_{\mathrm{m^2S}} - P_{\mathrm{D^2}}$$

Analogously:

$$P'_{\mathrm{CM}}\left[\bar{\psi}\not{D}(\not{D}+m)\psi\right] = -\frac{\mathrm{i}}{2} \quad \Rightarrow \quad P_{\mathrm{CM}} = P'_{\mathrm{CM}} + \frac{\mathrm{i}}{2}P_{\mathrm{D^2}}$$

Automation

There is much to calculate at NNLO:

Known (4-dim. subset → check) [Harlander, Lange, Neumann 2020], New

	P_{m}	P_{m^3}	P_S	P_{m^5}	P_{mD}	P_{mG}	P_{CM}
$\tilde{\mathcal{O}}_{\mathrm{S}}$	S_{m}/t	S_{m^3}	S_{S}	$\mathcal{O}(t)$	$\mathcal{O}(t)$	$\mathcal{O}(t)$	$\mathcal{O}(t)$
$\tilde{\mathcal{O}}_{\mathrm{mD}}$	D_{m}/t^2	D_{m^3}/t	$\mathcal{O}(m)$	D_{m^5}	D_{mD}	D_{mG}	$\mathcal{O}(m)$
$\tilde{\mathcal{O}}_{\mathrm{mG}}$	G_{m}/t^2	G_{m^3}/t	$\mathcal{O}(m)$	G_{m^5}	G_{mD}	G_{mG}	$\mathcal{O}(m)$
$\tilde{\mathcal{O}}_{\mathrm{CM}}$	C_{m}/t^2	C_{m^3}/t	C_{S}/t	C_{m^5}	C_{mD}	C_{mG}	C_{CM}

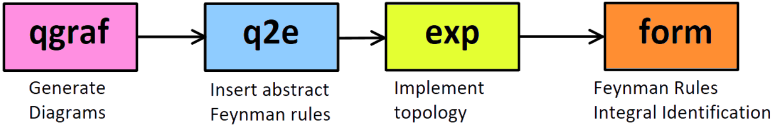
Many diagrams:

- C_{CM} : 3375 (P_{CM})+ 3375 (P_{D^2})
- C_{mD} : 226 (P_{mD})+ 226 $(P_{\mathrm{m}^2\mathrm{S}})$ + 226 (P_{D^2})
- ...

⇒ Automation with q2e-exp

Automatic computation with `q2e-exp`:

- `qgraf`: Graphical rules [Nogueira 1991]
- `q2e`: Abstract rules [Harlander, Seidensticker, Steinhauser 1998; Seidensticker 1999]
- `exp`: Topology, momentum conservation [Harlander, Seidensticker, Steinhauser 1998; Seidensticker 1999]
- `form`: Feynman rules, expansion, integral identification [Vermaseren 1991]



Integral reduction:

- Generate IBP relations [Tkachov 1981; Chetyrkin, Tkachov 1981; Artz, Harlander, Lange, Neumann, Prausa 2019]
- Reduce to master integrals with `kira+FireFly` [Maierhöfer, Usovitsch, Uwer 2018; Klappert, Lange, Maierhöfer, Usovitsch 2021; Klappert, Lange 2020; Klappert, Klein, Lange 2021]
- Evaluate master integrals analytically or with `pySecDec` [Borowka, Heinrich *et al.*]

Renormalization:

- Renormalize unflowed operators:

$$\mathcal{O}_i = Z_{ij}\mathcal{O}_j^R$$

- Insert into SFTE:

$$\tilde{\mathcal{O}}(t) \;=\; \zeta(t)\cdot\mathcal{O} \;=\; \zeta(t)Z^{-1}\cdot Z\mathcal{O}$$

- Renormalized SFTE matrix:

$$\zeta^R(t) = \zeta(t)Z^{-1}$$

Check Z:

- Z known [Spiridonov 1984; Chetyrkin, Spiridonov 1988; Gorbahn, Haisch, Misiak 2005]

Results for the massless case:

[Mereghetti, Monahan, Rizik, Shindler, Stoffer 2022; JB, Harlander, Rizik, Shindler 2022]

Single-flavor massless QCD, $\overline{\text{MS}}$ + ringed fermions, $a_s = \frac{\alpha_s}{\pi}$, $l_{\mu t} = \log(8\pi\mu^2 t)$

$$
\begin{aligned}
C_{\text{CM}} &= 1 + a_s\left(-4.023 + 0.166 l_{\mu t}\right) \\
&\quad + a_s^2\left(-11.611 - 10.147 l_{\mu t} + 0.229 l_{\mu t}^2\right)
\end{aligned}
$$

$$
C_S/\mathrm{i} = -2a_s + a_s^2\left(6.136 + 3.167 l_{\mu t}\right)
$$

$$
S_S = 1 + a_s\left(-2.690 - l_{\mu t}\right) + a_s^2\left(-4.546 - 8.328 l_{\mu t} - 0.792 l_{\mu t}^2\right)
$$

Full NNLO massive case in preparation

Outlook:

- Full NNLO massive case
  - Projections onto dimension-4 times mass operators $\rightarrow$ 2-loop
  - NNLO VEVs $\rightarrow$ 3-loop
- Massless case ready for combination with BSM Wilson coefficients
- Apply the same procedure to $\mathcal{O}_{\text{CE}} = \overline{\psi}\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\psi$

  $\Rightarrow$ Neutron EDM