From BSM toward new gradient flow based lattice determination of the strong QCD coupling at the Z-pole

collaborators at various stages of the project

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University of California, San Diego

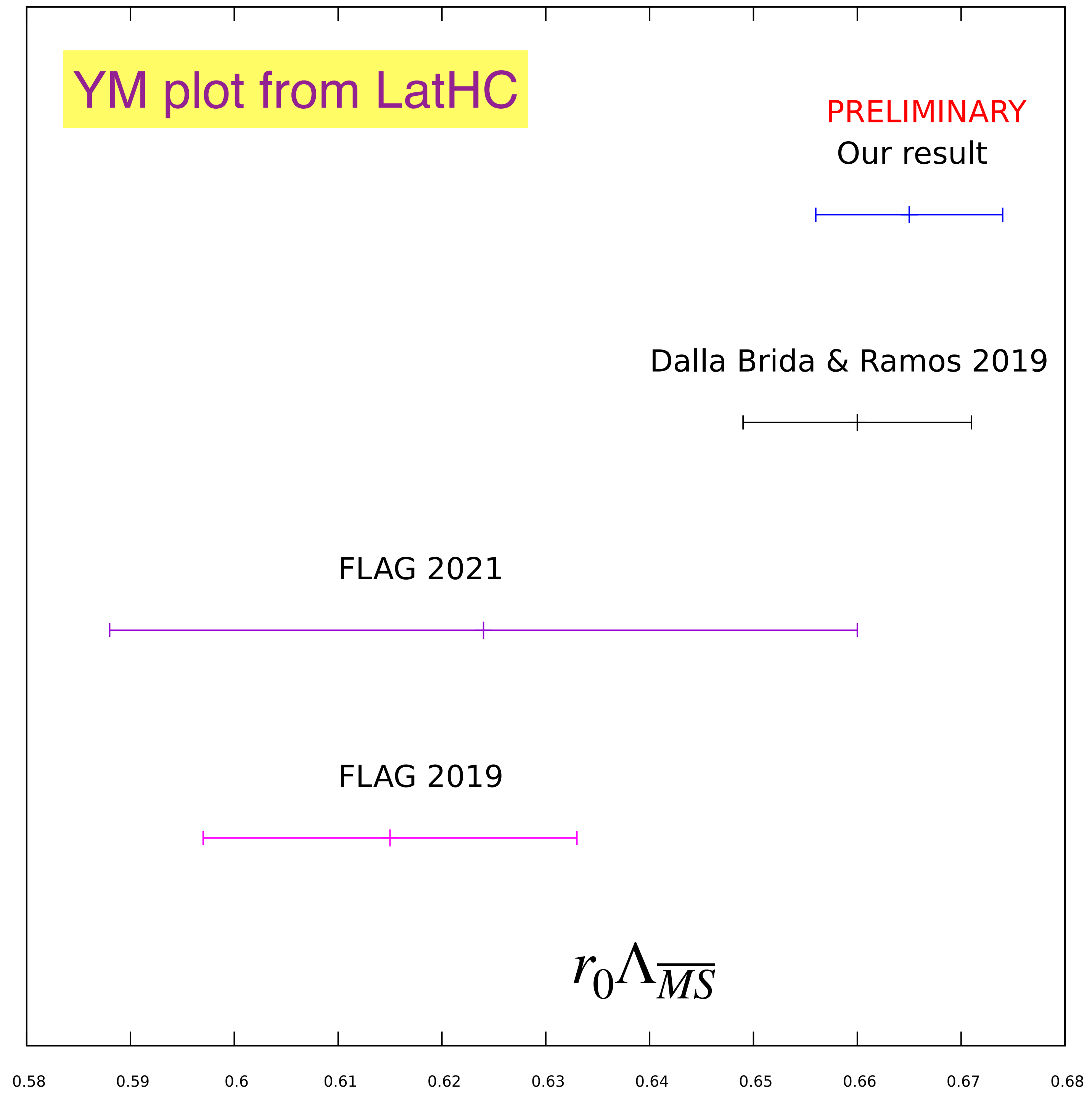
THE GRADIENT FLOW IN QCD AND OTHER STRONGLY COUPLED FIELD THEORIES

March 20, 2023 - March 24, 2023

ETC* Trento, Italy

Why are we doing this?

Wong et al. e-Print: 2301.06611 [hep-lat] Lattice 2022



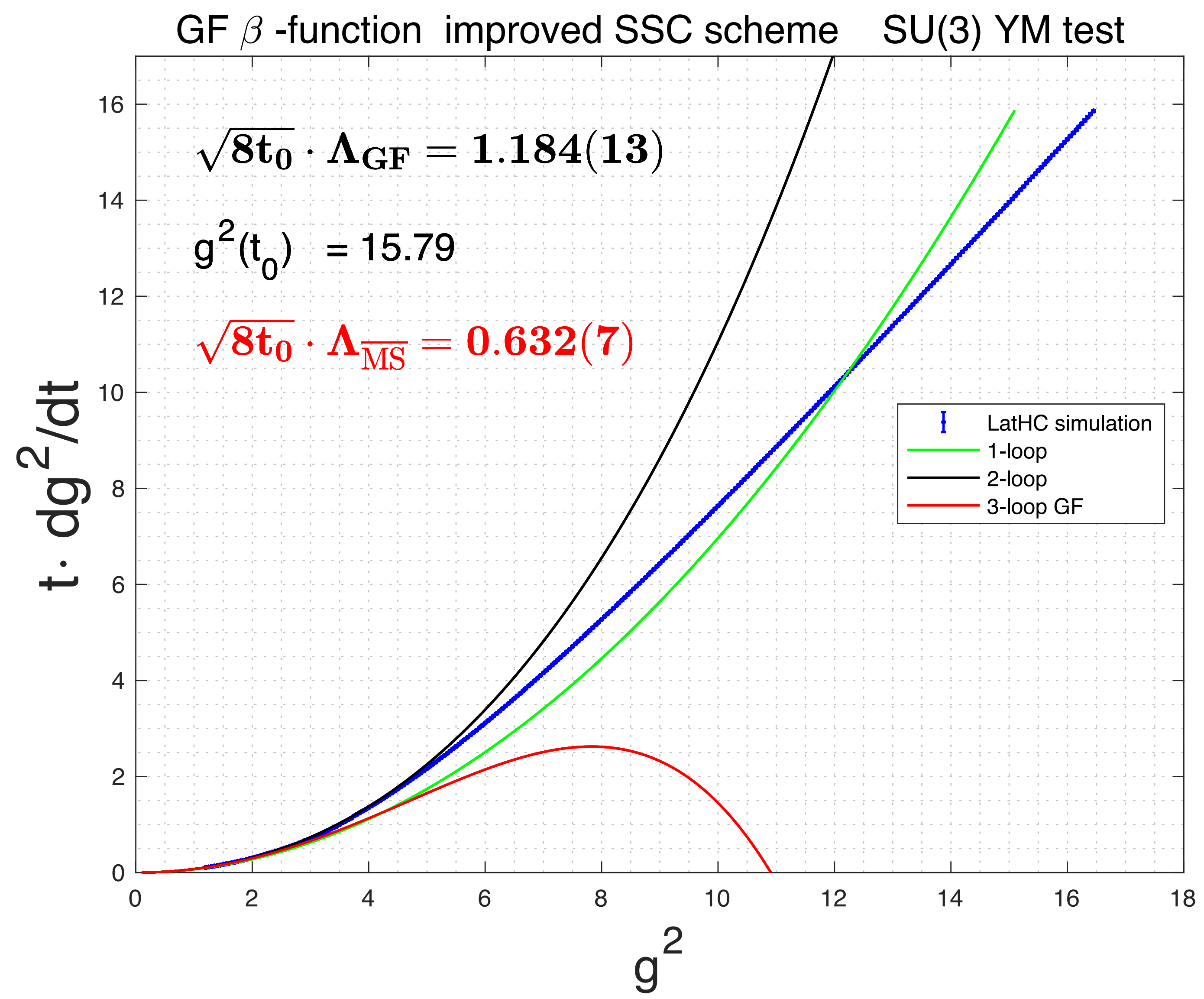
Foreword:

- After careful high precision calculations of the Alpha collaboration who needs another QCD strong coupling?

- Even the simplest SU(3) Yang-Mills model shows considerable tension after repeated FLAG reviews

  — FLAG 2019 was without first high precision GF result
  — combined FLAG 2021 error analysis partially hides the tension
  — the two new GF results increase the tension against other methods

- Our YM analysis is based on the Harlander-Neumann 3-loop beta-function over infinite Euclidean volume in the continuum limit — to be explained



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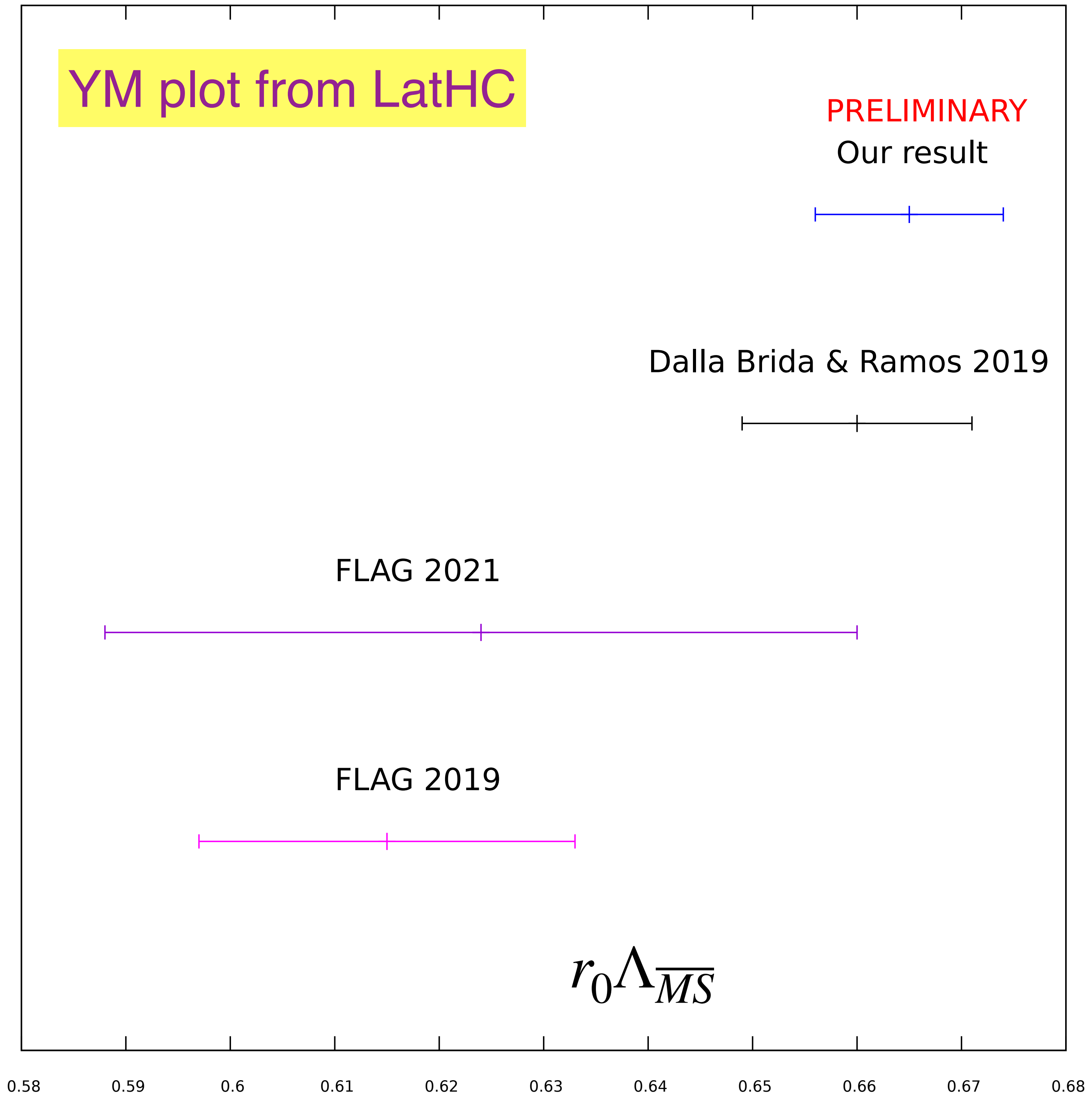
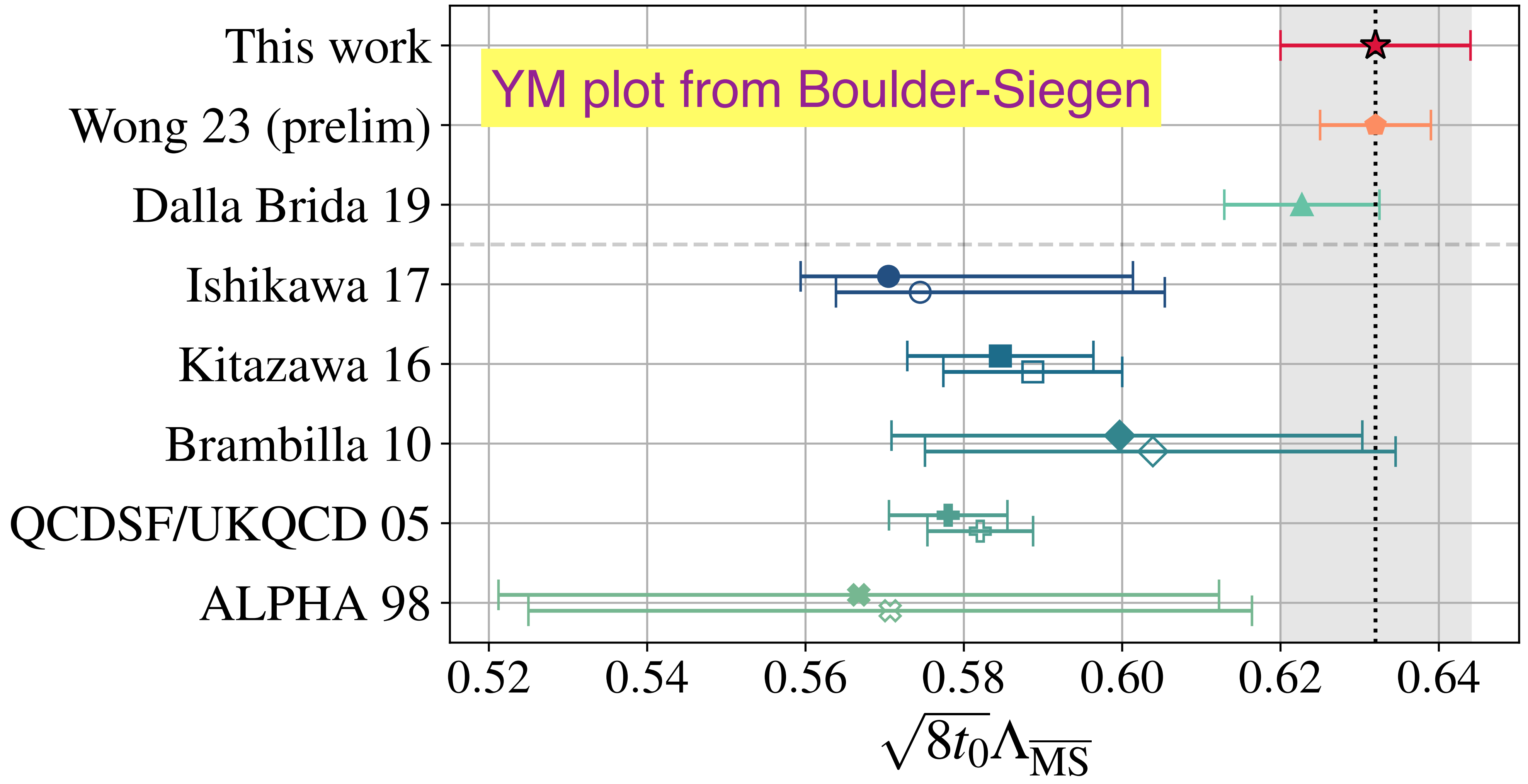


A. Hasenfratz et al. e-Print 2303.00704 [hep-lat]



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  - — the two new GF results increase the tension against other methods
- Our YM analysis is based on the Harlander-Neumann 3-loop beta-function over infinite Euclidean volume in the continuum limit — to be explained
- Similar results in Boulder-Siegen project
- Recent YM developments have an interesting history — useful to recall
- Toward an ambitious goal: to create similar competing result in QCD with three massless fermions? — to be explained

timeline and results

Lattice 2017

1711.04833 LatHC

sextet beta-function $\beta_{GF}(g^2_{GF}(t)) = tdg^2/dt$ in the massless fermion limit over infinite Euclidean volume where the scale $\mu = 1/\sqrt{8t}$ is implicit from target choice g^2_{GF}

— only one target $g^2_{GF} = 6.7$ at strong coupling motivated by the BSM CW controversy

— massless fermion limit of infinite volume tdg^2/dt is approached from chiral symmetry breaking phase (like in QCD with three massless fermions)

— getting rid of zero mode in direct m=0 L $\to$ ∞ implementation in the strong coupling phase?
problem with exactly massless pions?
intrinsic scale? worth trying?

— contact with 3-loop perturbation theory would be more accurate than reach from χSB phase

— before 2019 experimenting with various m=0 direct implementations of tdg^2/dt for infinite volume limit at strong coupling
unpublished tests before 2019 nf=10 and nf=12 models
limited understanding of fitting in a^4/L^4, or a^2/L^2 for $L \to \infty$ limit, weak coupling, frozen topology, other hidden effects from gapless spectrum?

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direct calculation with two massless fermions presented going public for the first time 1910.06408 1911.11531 [hep-lat] A. Hasenfratz and O. Witzel

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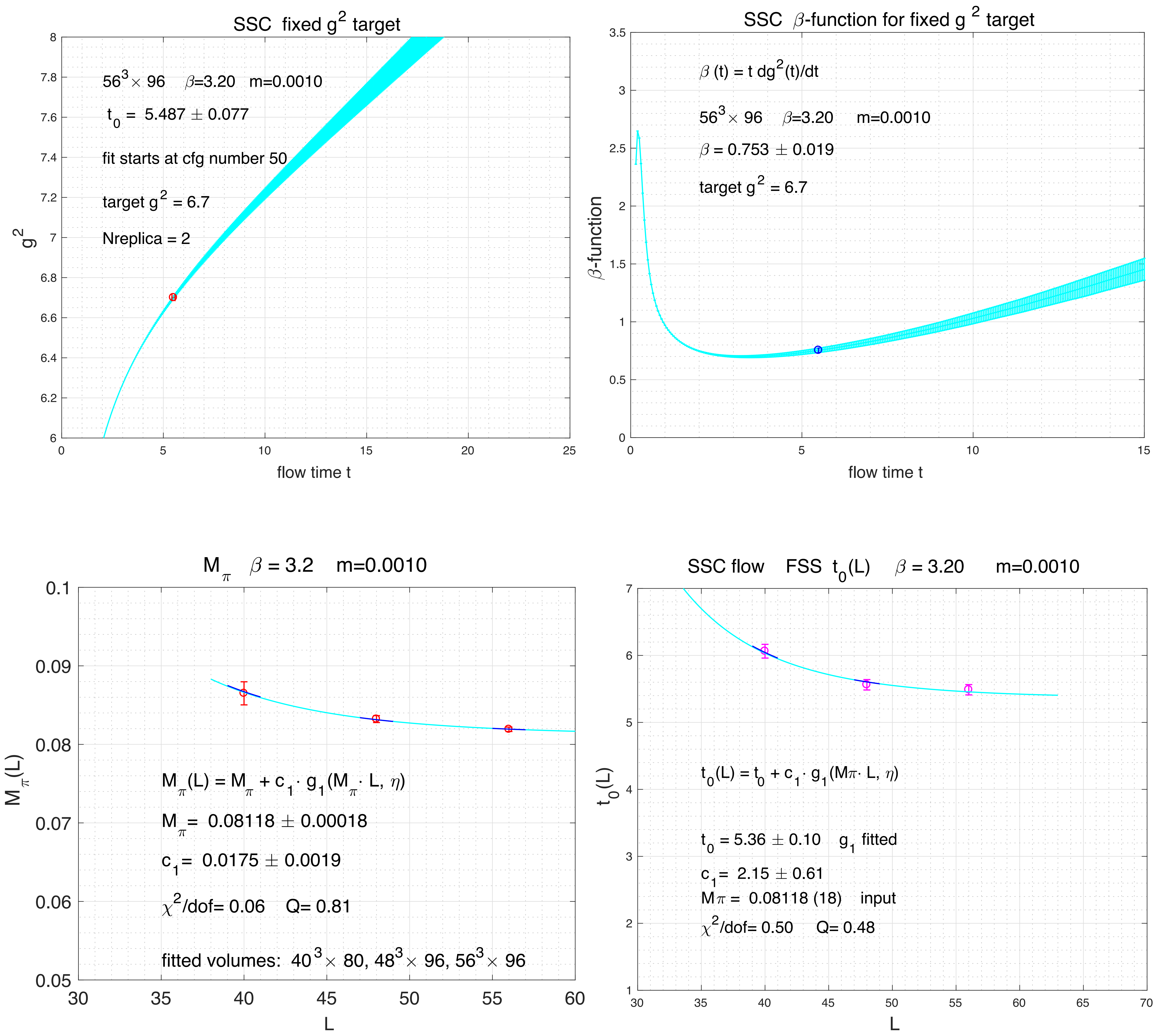
First YM result: PoS **LATTICE2021** (2022) 174 e-Print: 2109.09720 , A. Hasenfratz, C. Peterson, J. Van Sickle and O. Witzel

Lattice 2022

YM test of infinite volume β-function is easier with gap in the spectrum
High precision YM results presented by LatHC at Lattice 2022 and the results published January, 2023. Boulder-Siegen result followed almost immediately

$$[-g^2(t+2\epsilon)+8g^2(t+\epsilon)-8g^2(t-\epsilon)+g^2(t-2\epsilon)]/(12\epsilon) = dg^2/dt + O(\epsilon^4)$$

checked by 7-point stencil $\quad \frac{1}{30} f^{(5)}(x)\cdot\epsilon^4$



original sextet algorithm

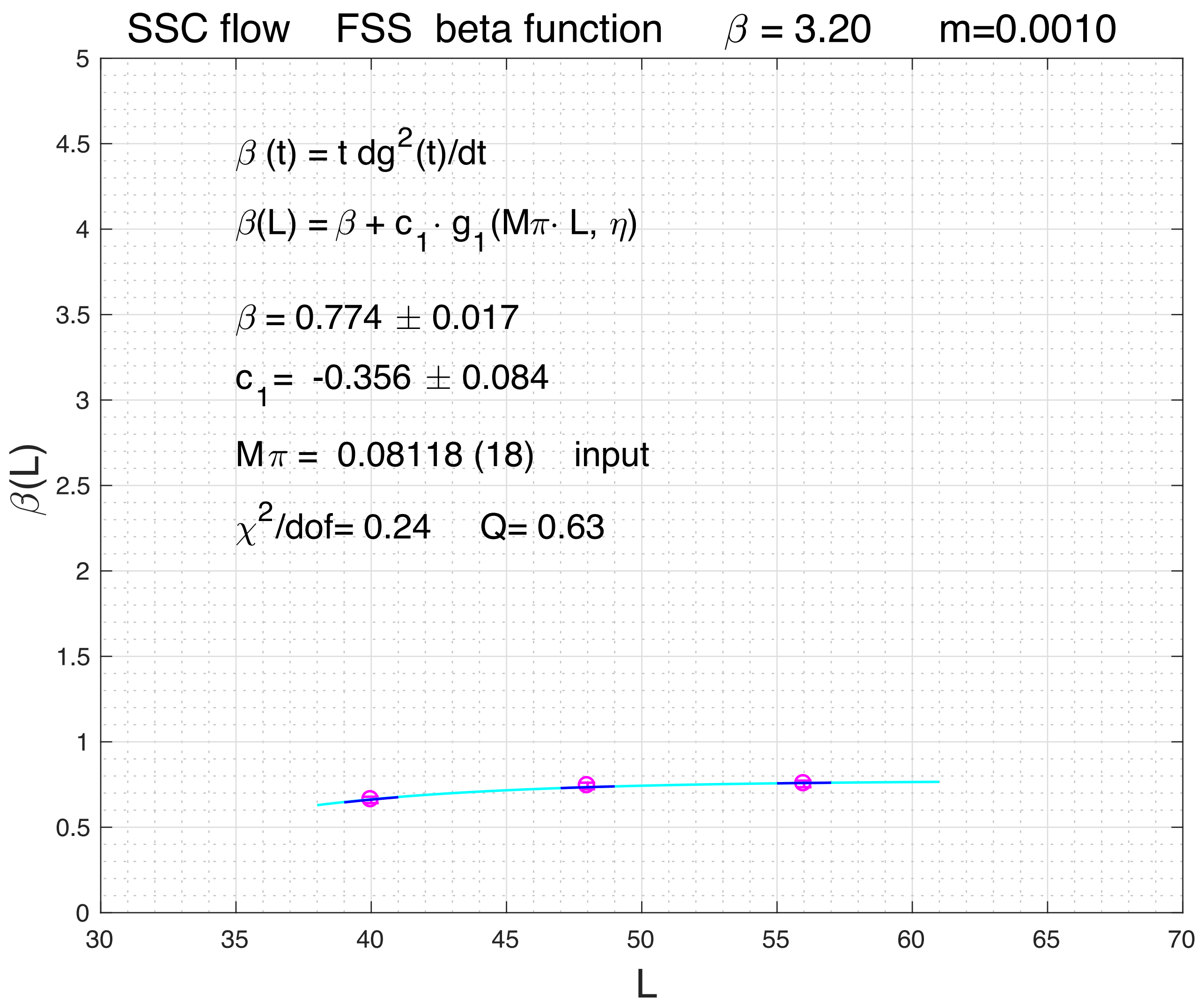
sextet model beta=function infinite volume approach from p-regime

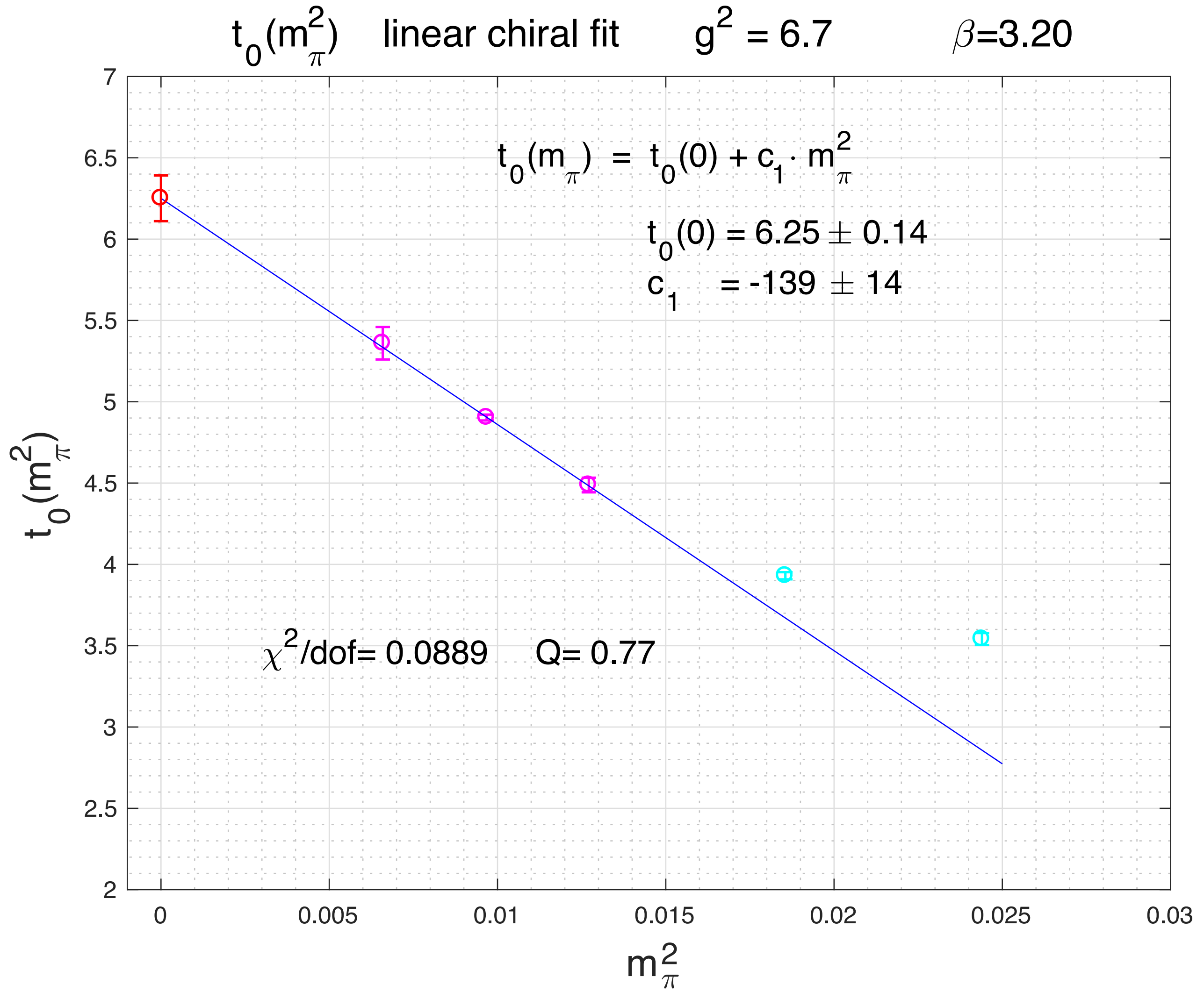
step 1 select target coupling g^2 at some lattice spacing (several m fermion masse, several L at each m)

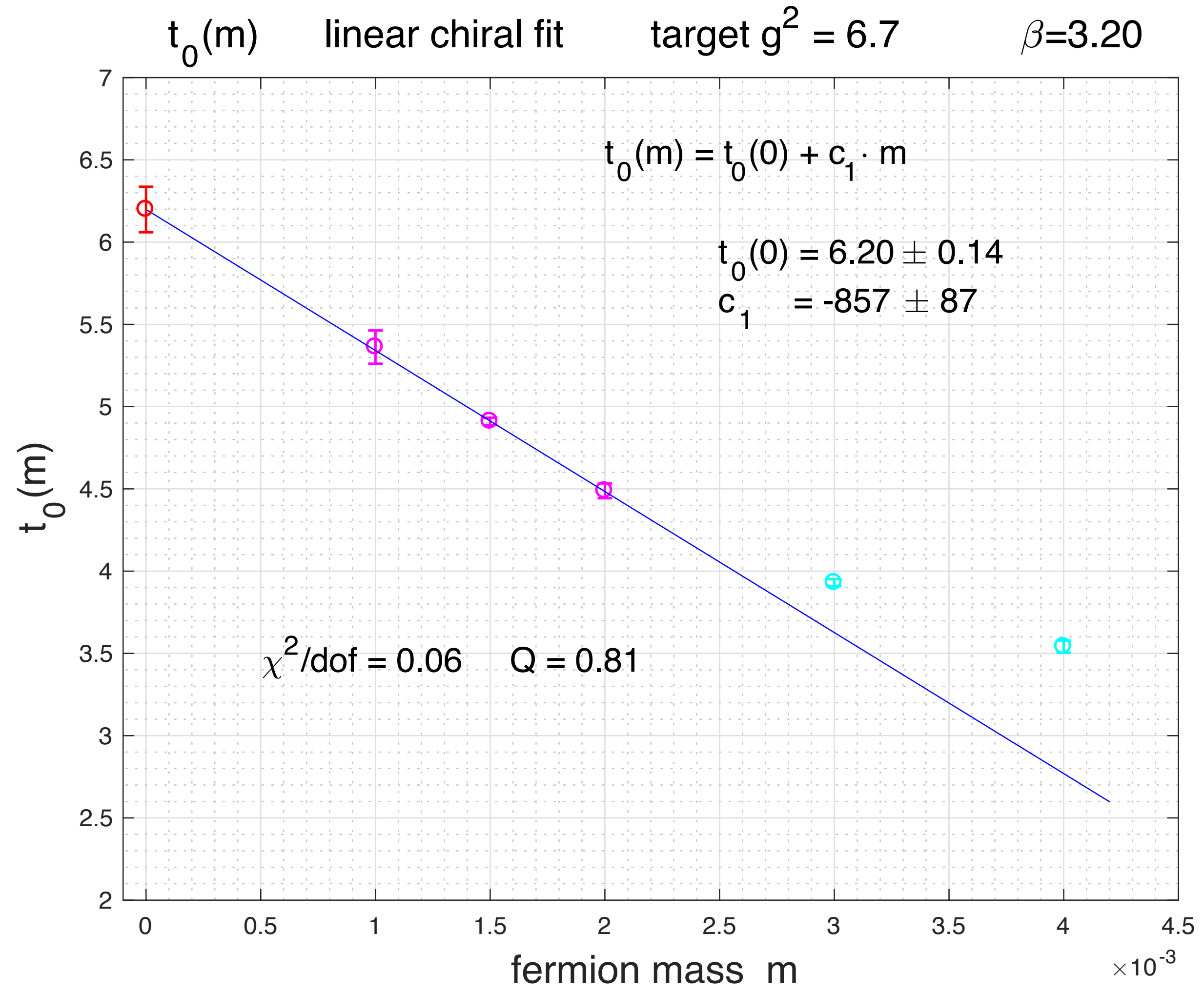
step 2 at each m take L → ∞ limit $\quad M_\pi,\ t_0,\ tdg^2/dt$

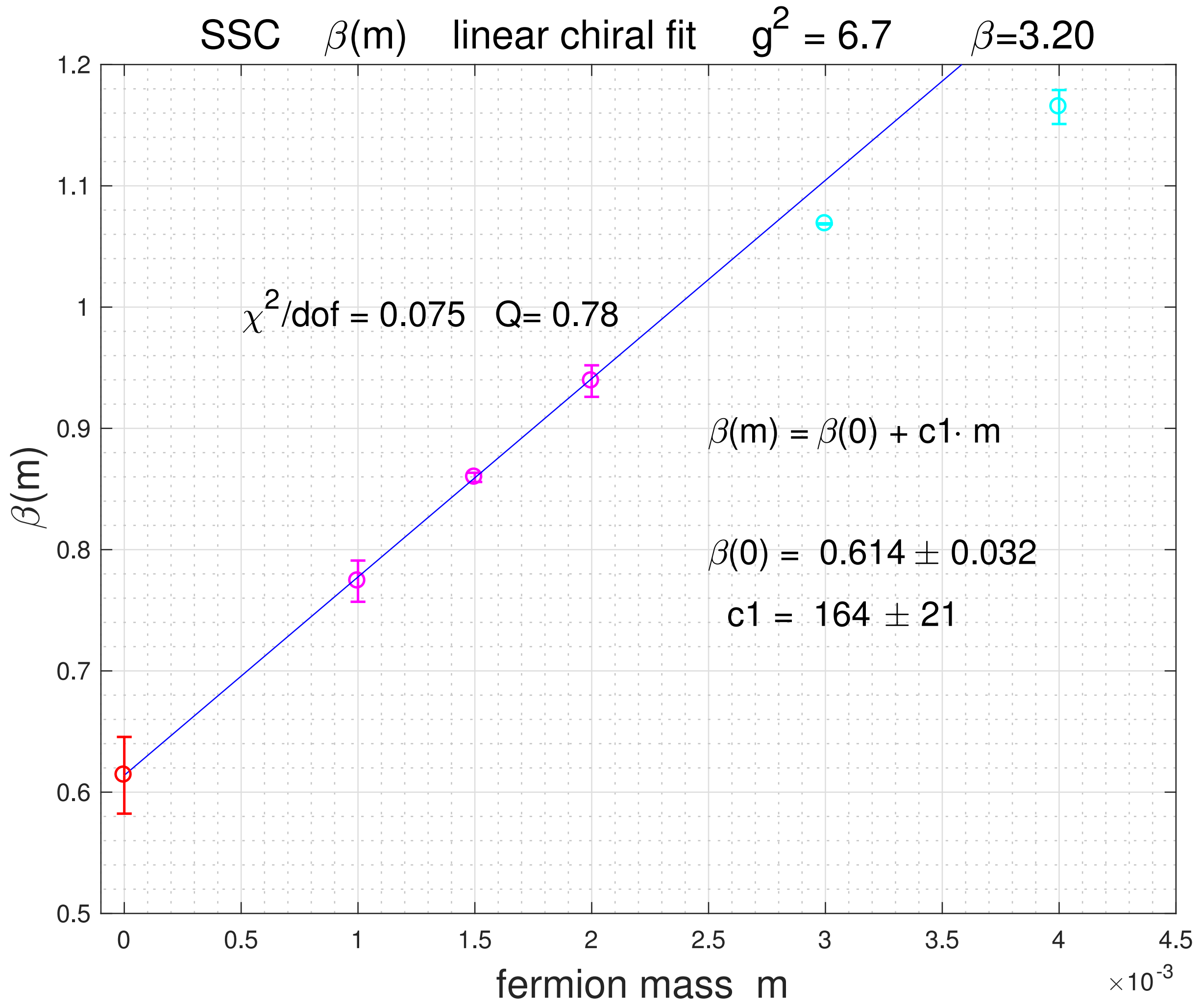
step 3 chiral limit $m \to 0$ at fixed a and g^2 for $t_0, tdg^2/dt$

step 4 repeat for 3 a values and take $a^2/t \to 0$ continuum limit of tdg^2/dt









$$t_0 = t_{0,\mathrm{ch}}\left(1 + k_1\frac{M_\pi^2}{(4\pi f)^2} + k_2\frac{M_\pi^4}{(4\pi f)^4}\log\left(\frac{M_\pi^2}{\mu^2}\right) + k_3\frac{M_\pi^4}{(4\pi f)^4}\right)$$

Golterman, Baer

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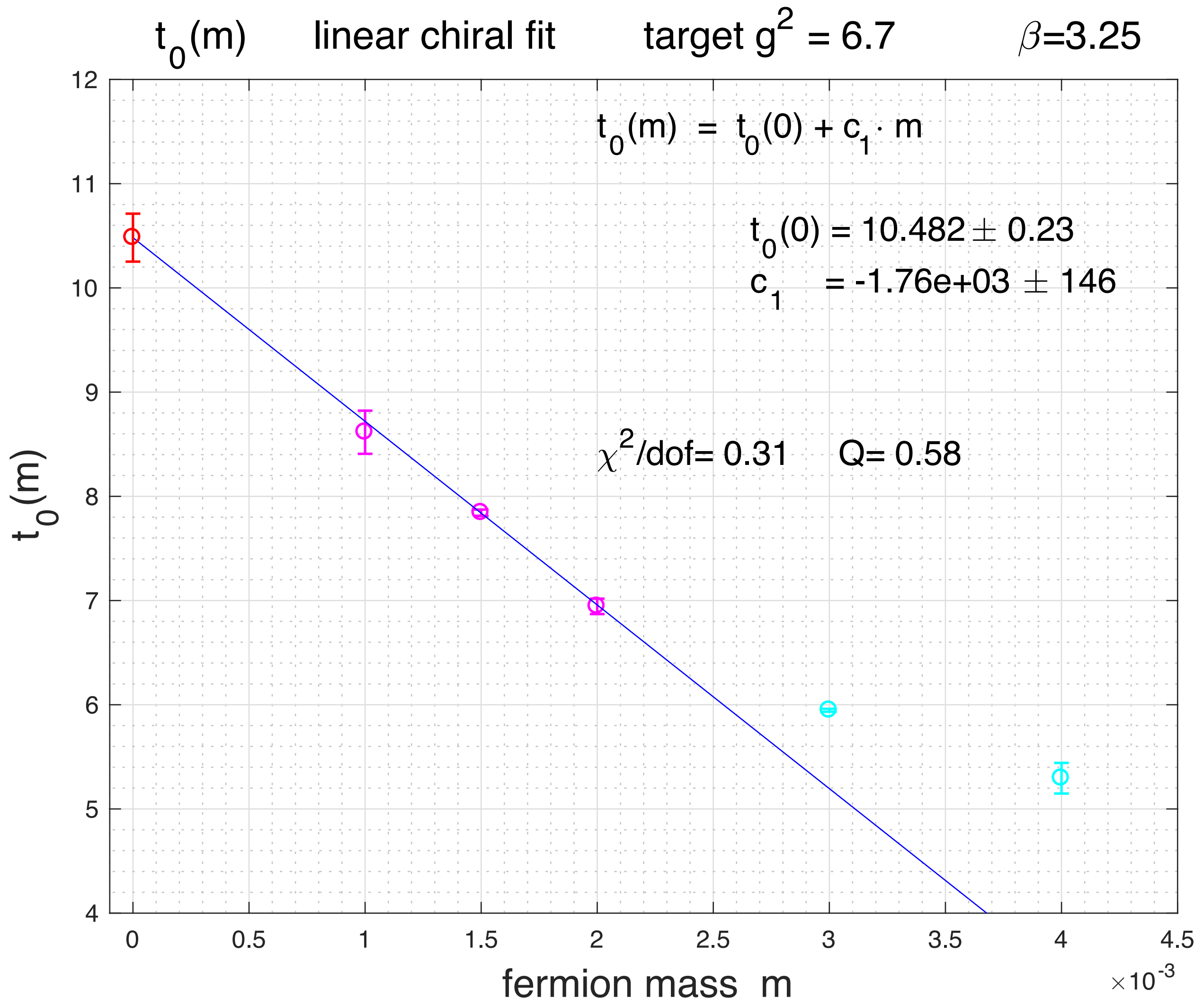

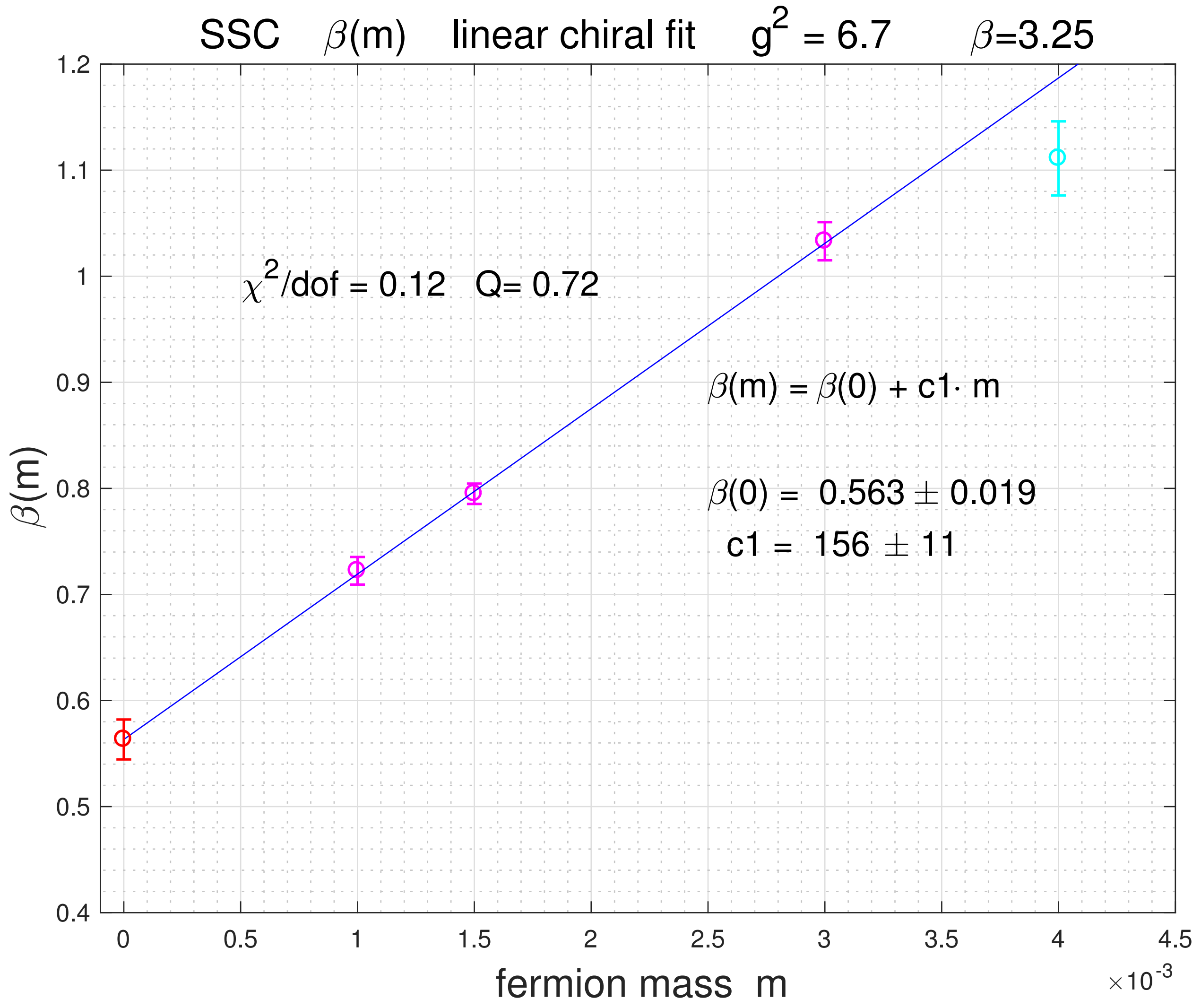

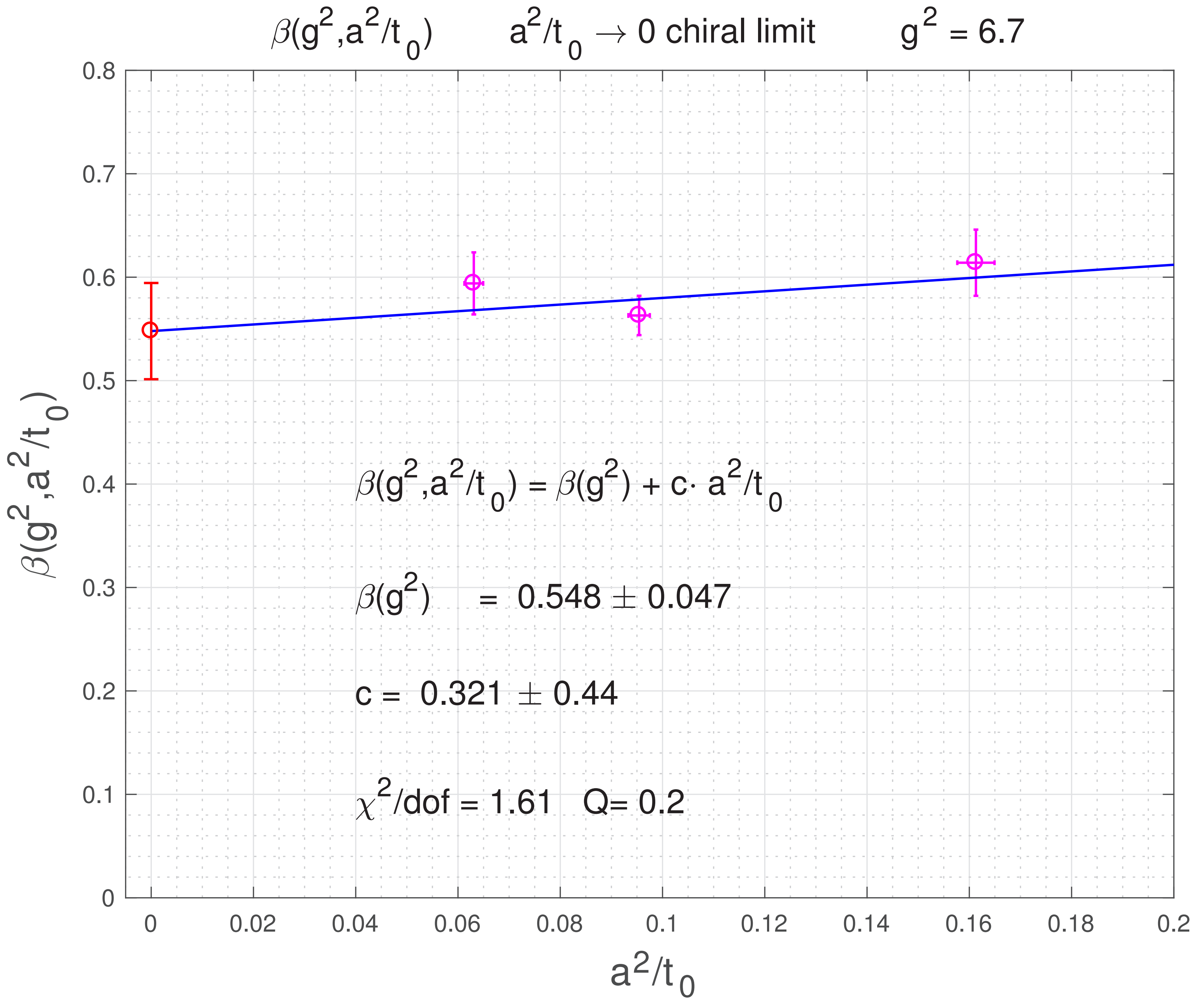

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Yang-Mills project

lattice ensembles

SU(3) lattice Yang-Mills gauge action

Symanzik tree-level improved with bare lattice gauge coupling $6/g_0^2$

periodic gauge field boundary conditions (the notorious zero mode!)

smearing with 4 stout steps $\rho = 0.12$

multi-platform over-relaxation + heat bath code

lattice ensembles at 45 lattice gauge couplings in $6/g_0^2$ = 4.39 - 11.5 range

several volumes at each lattice gauge coupling including L=32,36,40,48,56,64,80,96,112,128,160

not all L-values exist at every gauge coupling

L=80,96,112,128,160 for master field analysis (plan for increased precision)

gradient flow

Symanzik tree-improved SU(3) Yang-Mills gauge action

we use Wilson flow and Symanzik flow (Zeuthen flow not used)

tree-level improvement

two operators are clover and the Symanzik lattice action

SSC, WSC, SSS, WSS scheme
SSC stands for S(flow)S(action)C(clover)

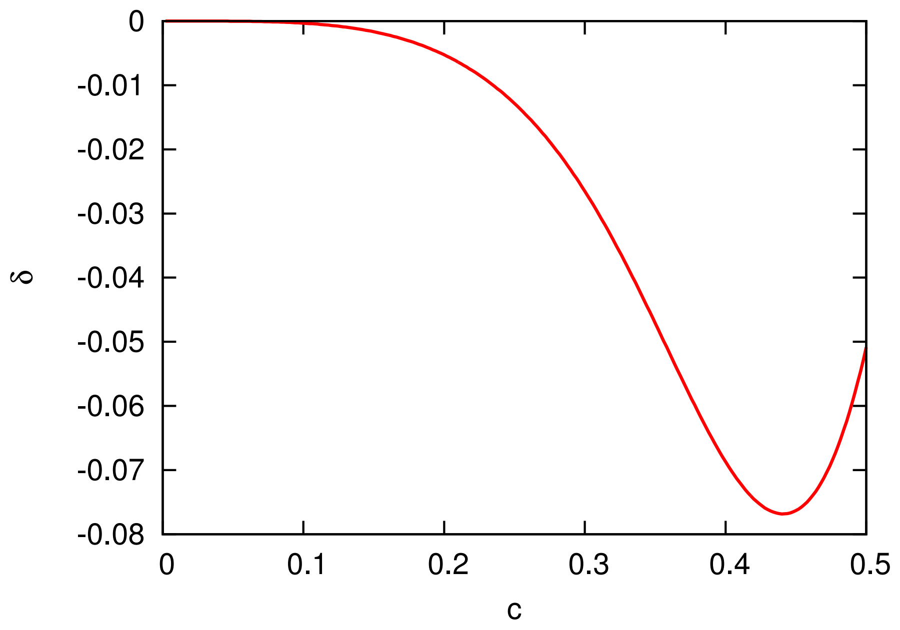
adaptive Runge-Kutta integration on the flow

strategies for getting $\beta(g^2(t)) = t dg^2/dt$ over infinite Euclidean volume will be discussed in the YM analysis and in the QCD analysis with massless fermions

$$c = \sqrt{8t}/L \qquad g_c^2 = \frac{16\pi^2 \langle t^2 E(t) \rangle}{3(1+\delta(c))}$$

$$\delta(c) = \vartheta^4(e^{-1/c^2}) - 1 - \frac{\pi^2 c^4}{3}$$

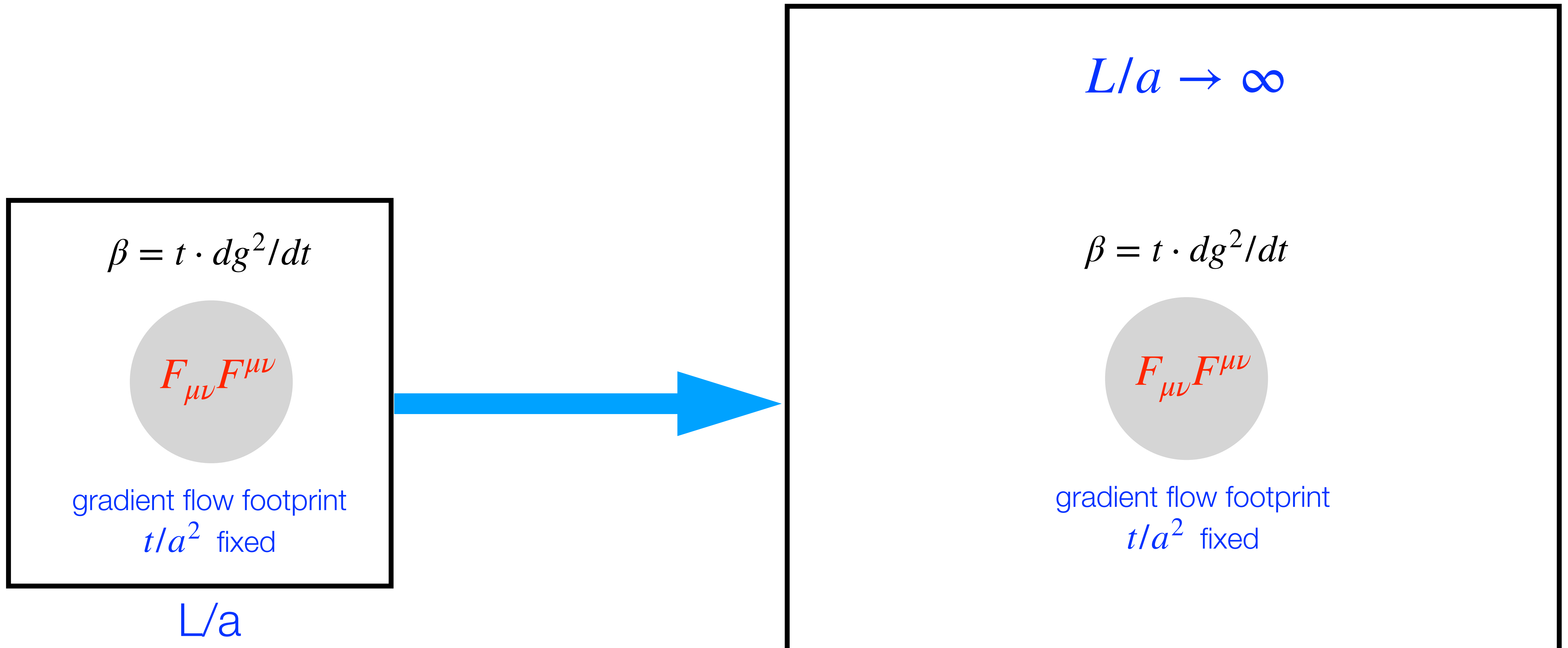
zero mode becomes irrelevant in $c \to 0$ limit

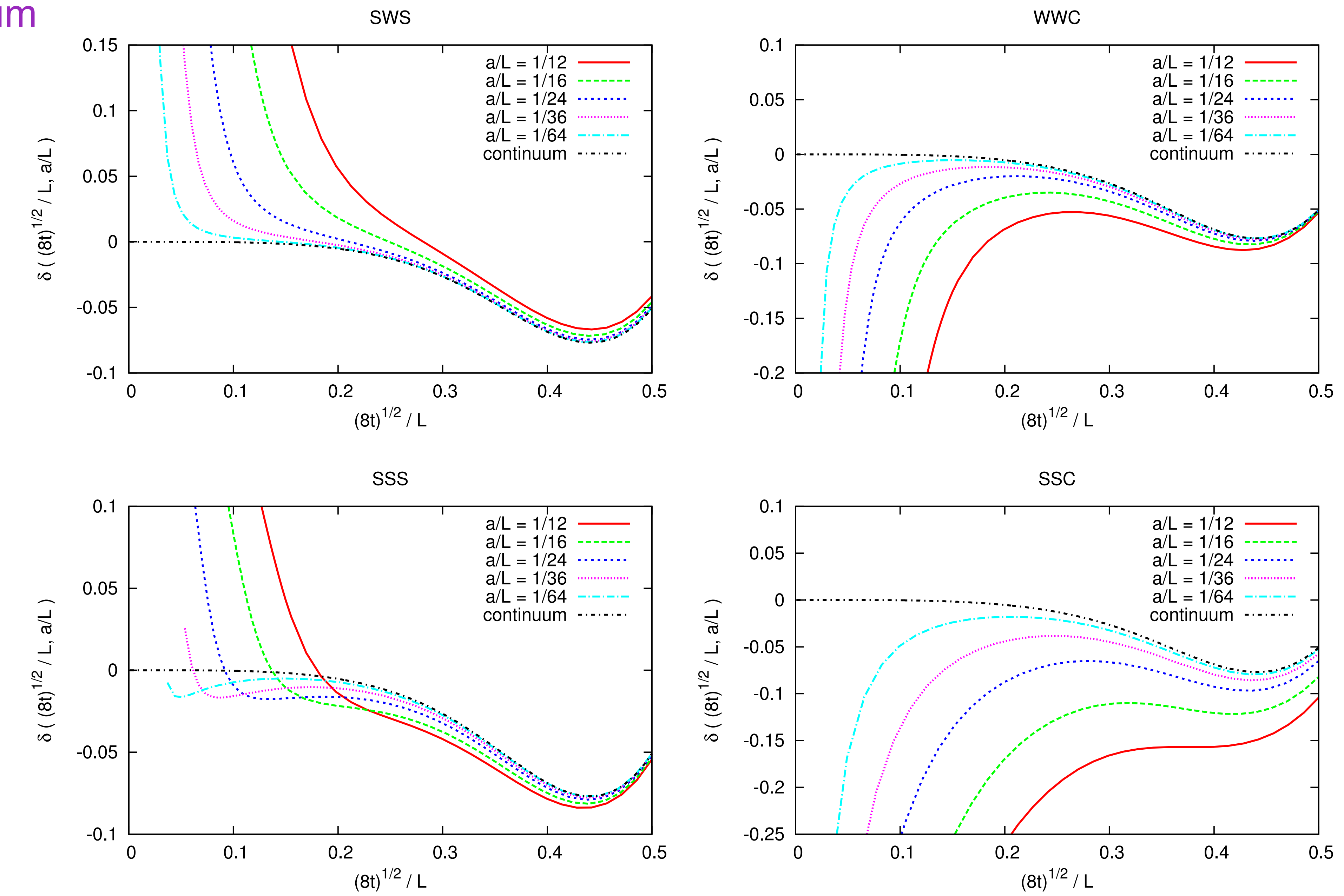


tree-level lattice improvement at finite L on the lattice:

$$\langle t^2 E(t) \rangle = g^2 \frac{3(N^2-1)}{128\pi^2} C(a^2/t, \sqrt{8t}/L)$$

$$C(a^2/t, \sqrt{8t}/L) = \underbrace{\frac{128\pi^2 t^2}{3L^4}}_{\text{zero-mode}} + \frac{64\pi^2 t^2}{3L^4} \underbrace{\sum_{n_\mu=0,\ n^2\neq 0}^{L/a-1}}_{\text{finite lattice sum}} \mathrm{Tr}\left(e^{-t(\mathcal{S}^f+\mathcal{G})}(\mathcal{S}^g+\mathcal{G})^{-1}e^{-t(\mathcal{S}^f+\mathcal{G})}\mathcal{S}^e\right)$$





at weak coupling:

$$g_c^2 = g_{\overline{\rm MS}}^2\left(1 - a_1(c) g_{\overline{\rm MS}} + O(g_{\overline{\rm MS}}^2)\right)$$

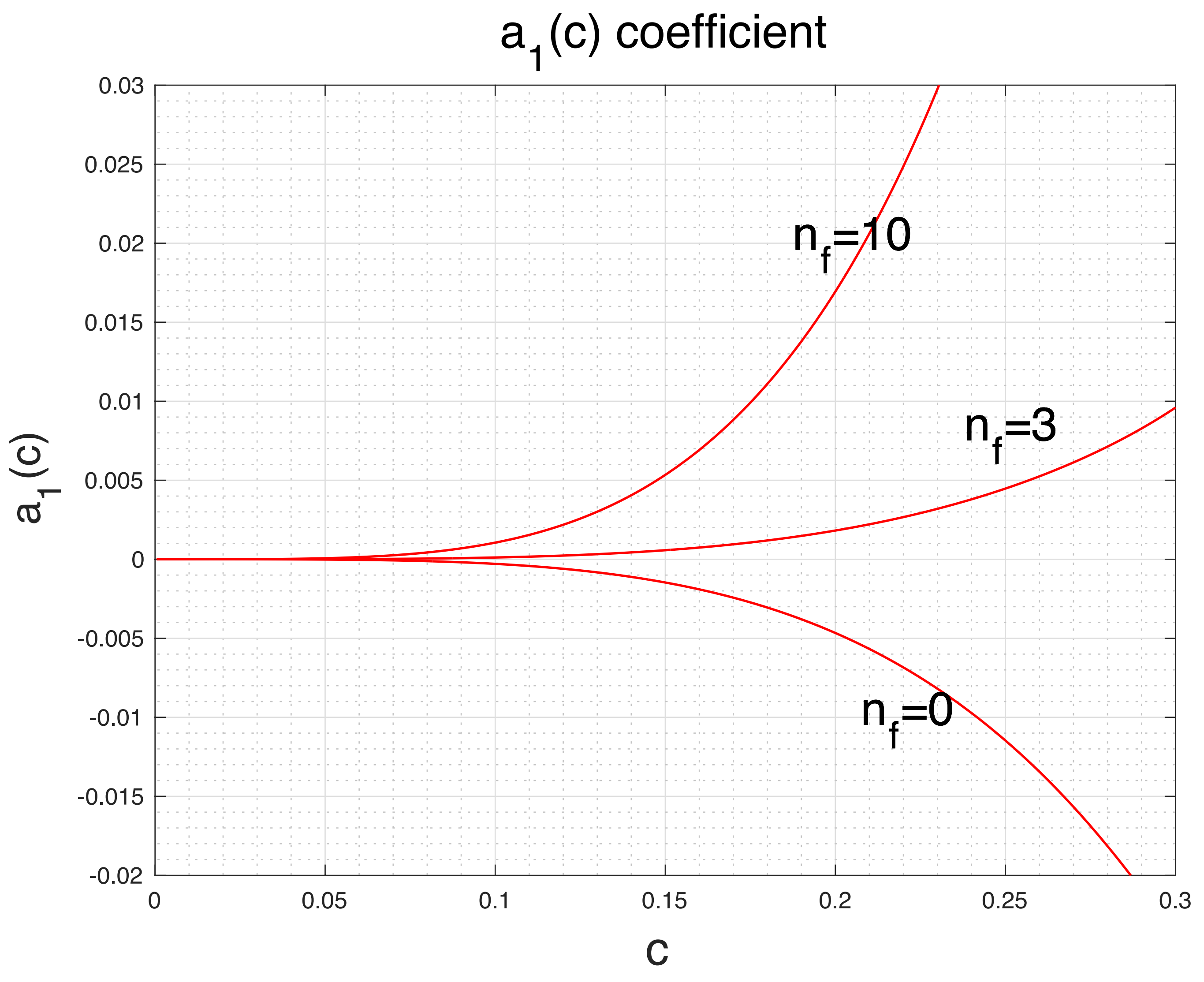
initiated by Dani Nogradi
JK cross-checked

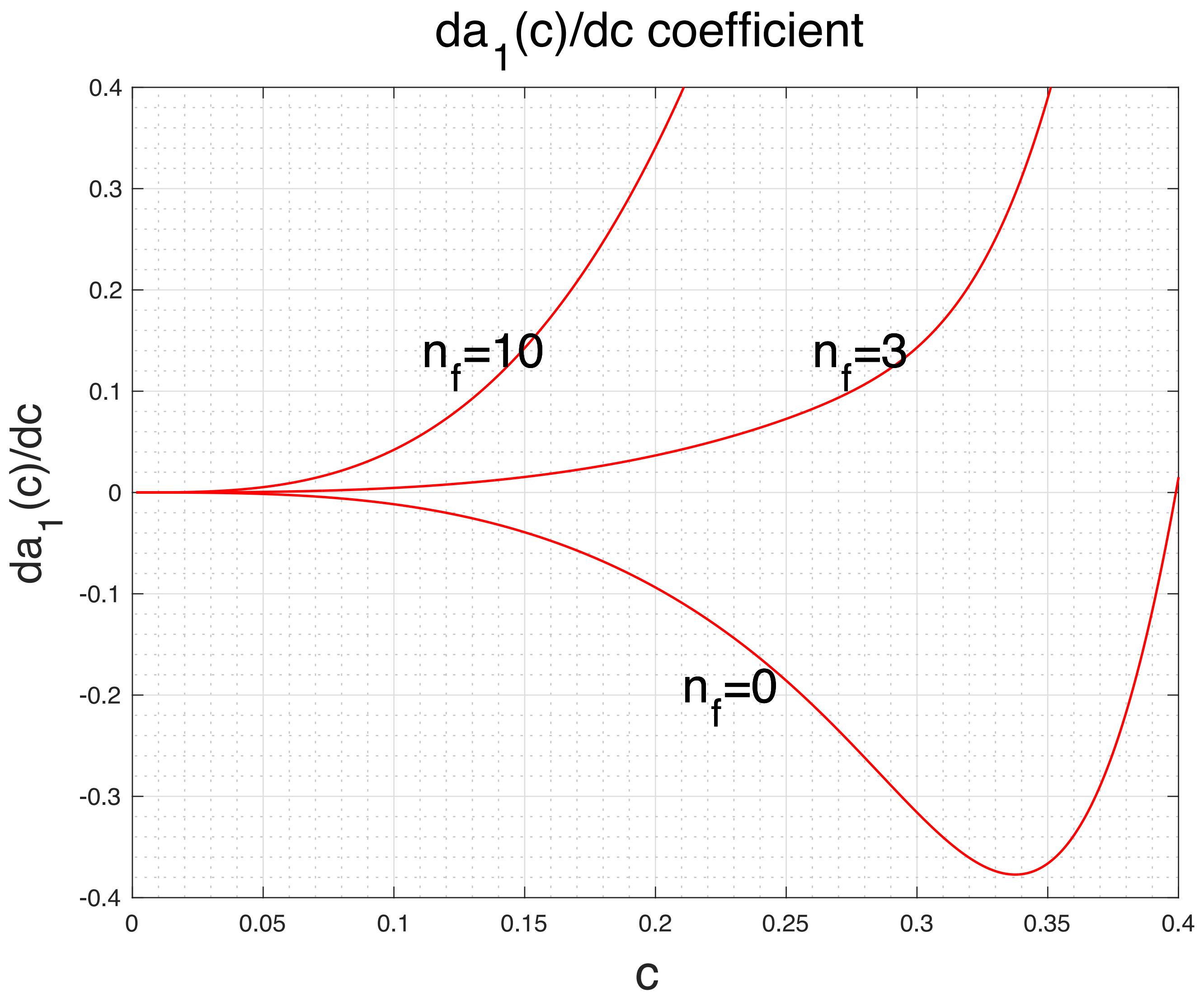
$$\tilde{g}_c^2 = g_c^2(1 + a_1(c) g_c)$$

fixes β-function to two loop

but we want 3-loop accuracy

and at finite lattice spacing





lattice $a_1(c, a^4/L^4)$ has been now calculated

$a_1(c, a^4/L^4)$ is being incorporated in
weak coupling $a^4/L^4 \to 0$ analysis

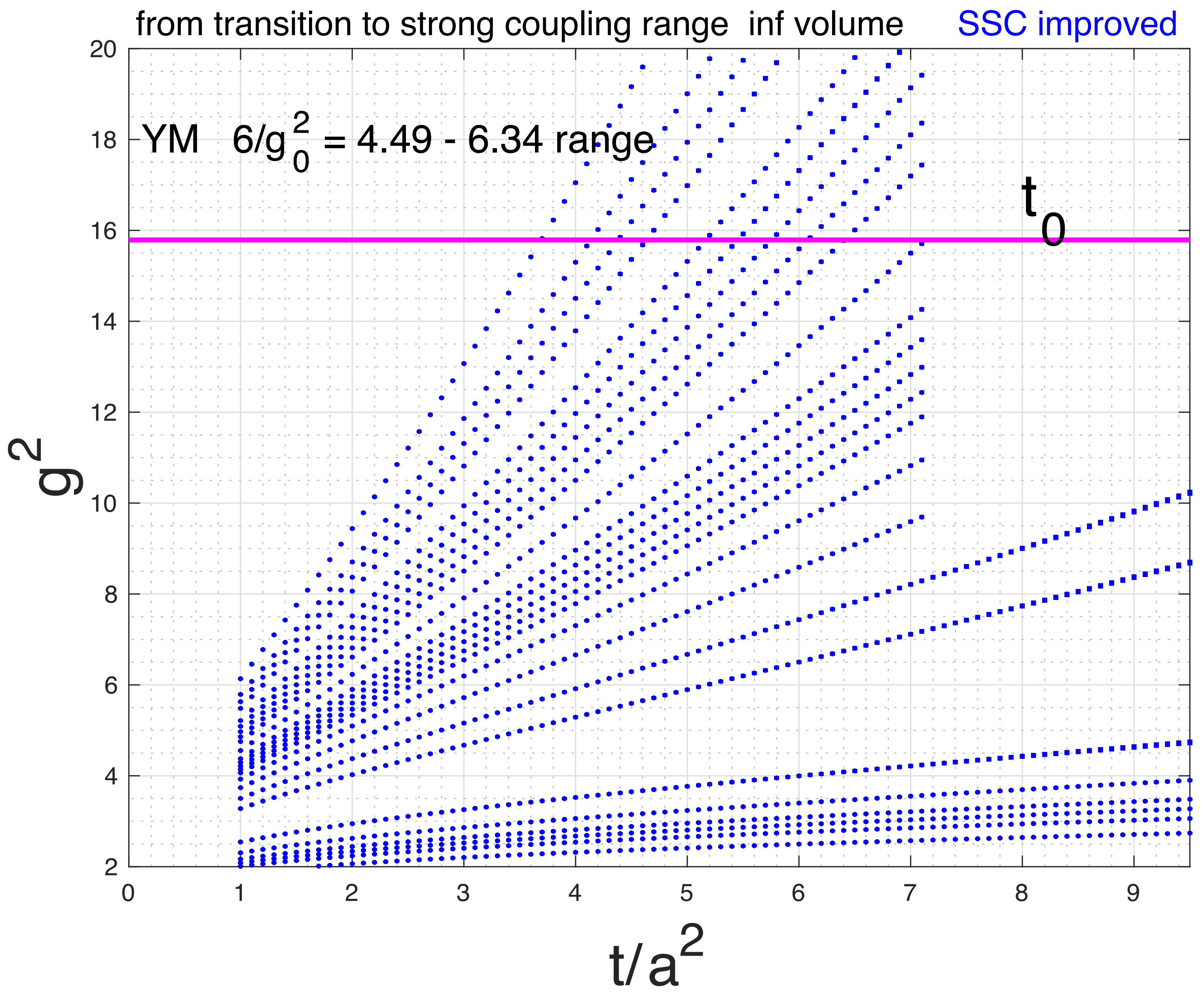
$a_1 \approx c^4$ for small c

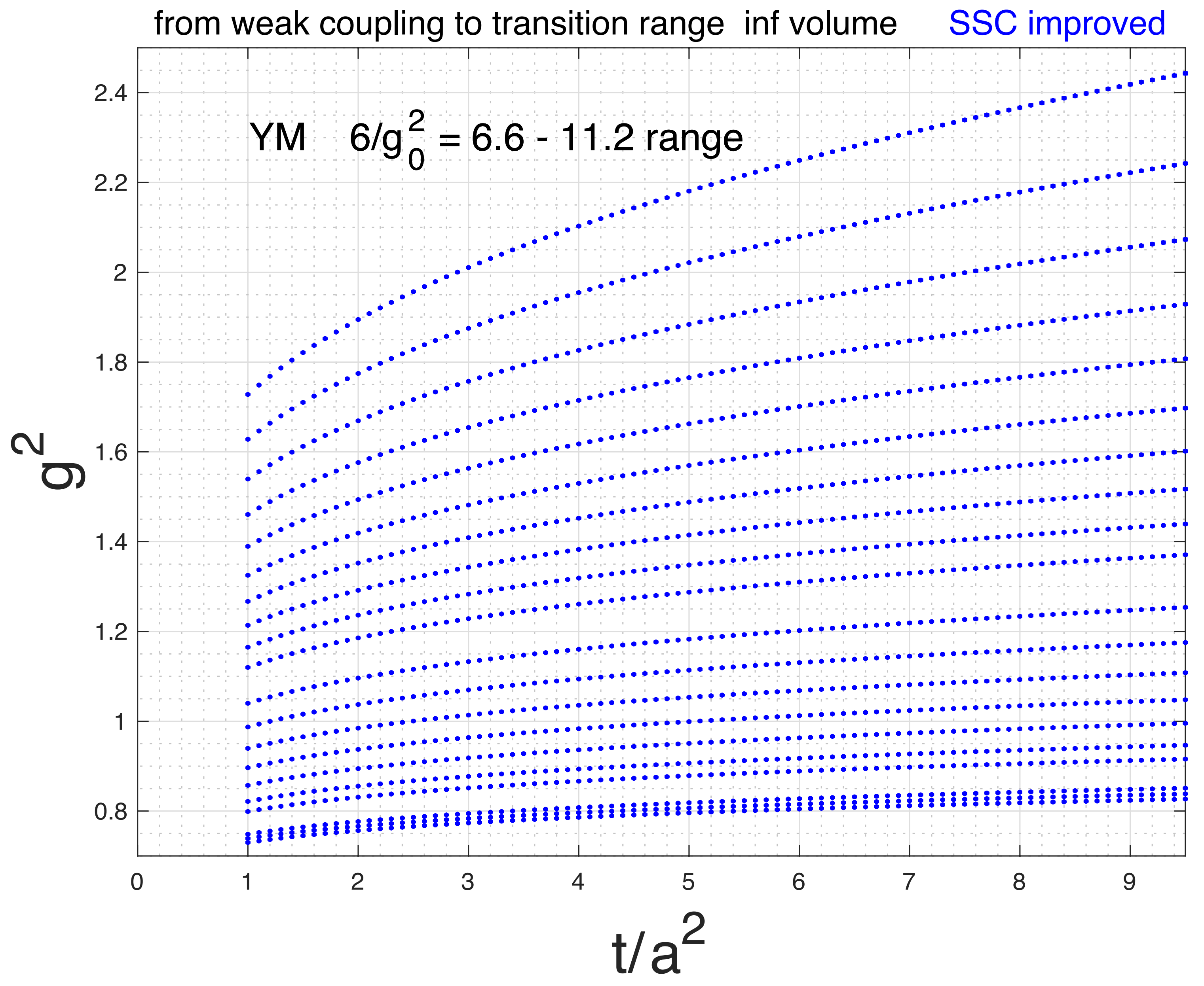
reaching the continuum over infinite Euclidean volume requires three steps:

(1) For each L-value at fixed $6/g_0^2$ we calculate $g^2|_L$ and $t \cdot dg^2/dt|_L$ on the gradient flow at each flow time t/a^2 and take the $L \to \infty$ limit on g^2 and $t \cdot dg^2/dt$ from a sequence of L-values ($c \to 0$) at each flow time and each bare coupling.

(2) We select (target) some g^2 in the infinite volume limit at finite cutoff, with flow time $t(g^2)$ set by $6/g_0^2$ and the target g^2. At this flow time $t(g^2)$ we calculate the value of $t \cdot dg^2/dt$. The errors in g^2, $t(g^2)$, and $t \cdot dg^2/dt$ propagate.

(3) From a sequence of $6/g_0^2$ values we take the continuum limit $a^2/t(g^2) \to 0$ of the $t \cdot dg^2/dt$ sequence set by available bare gauge couplings.

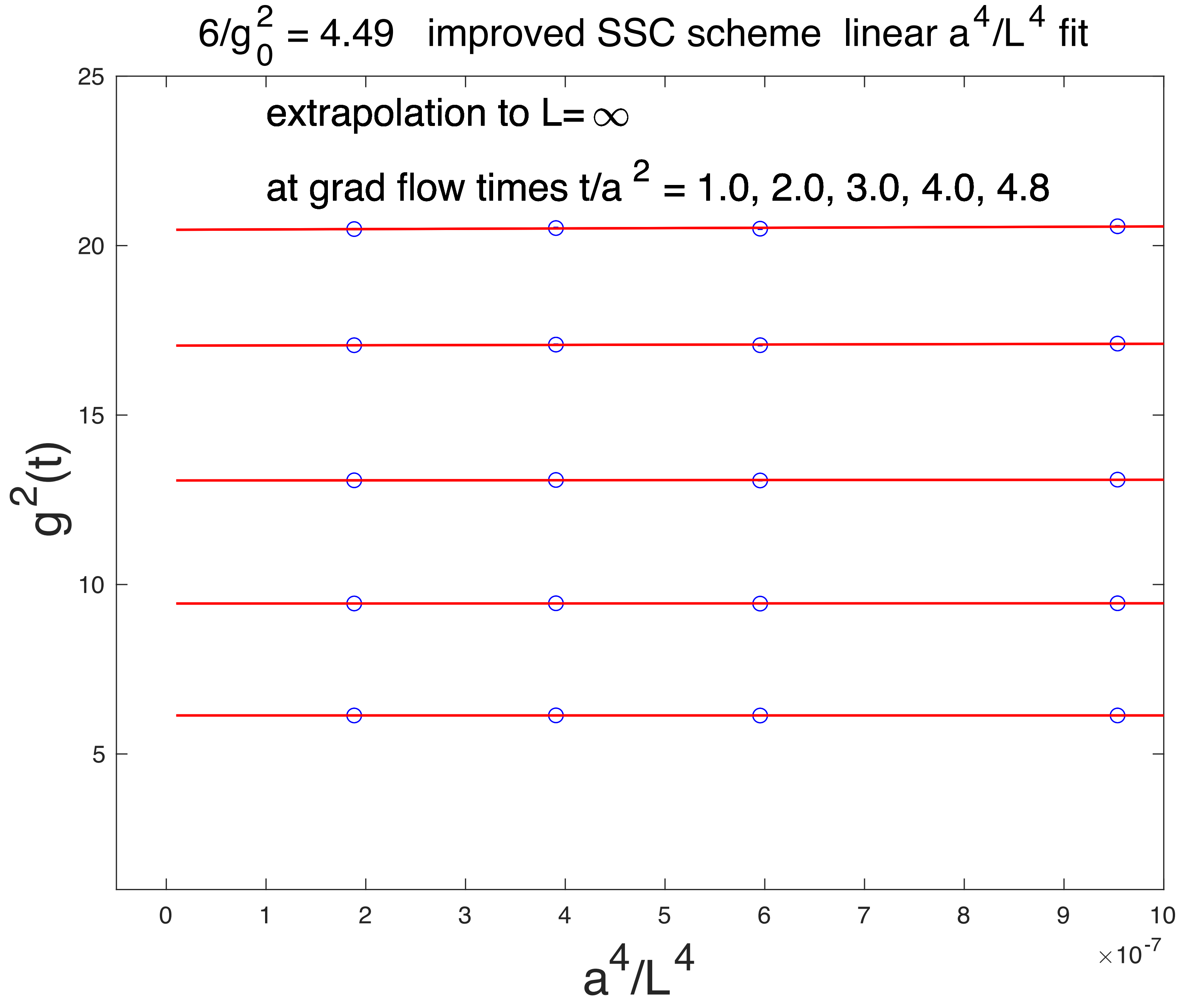




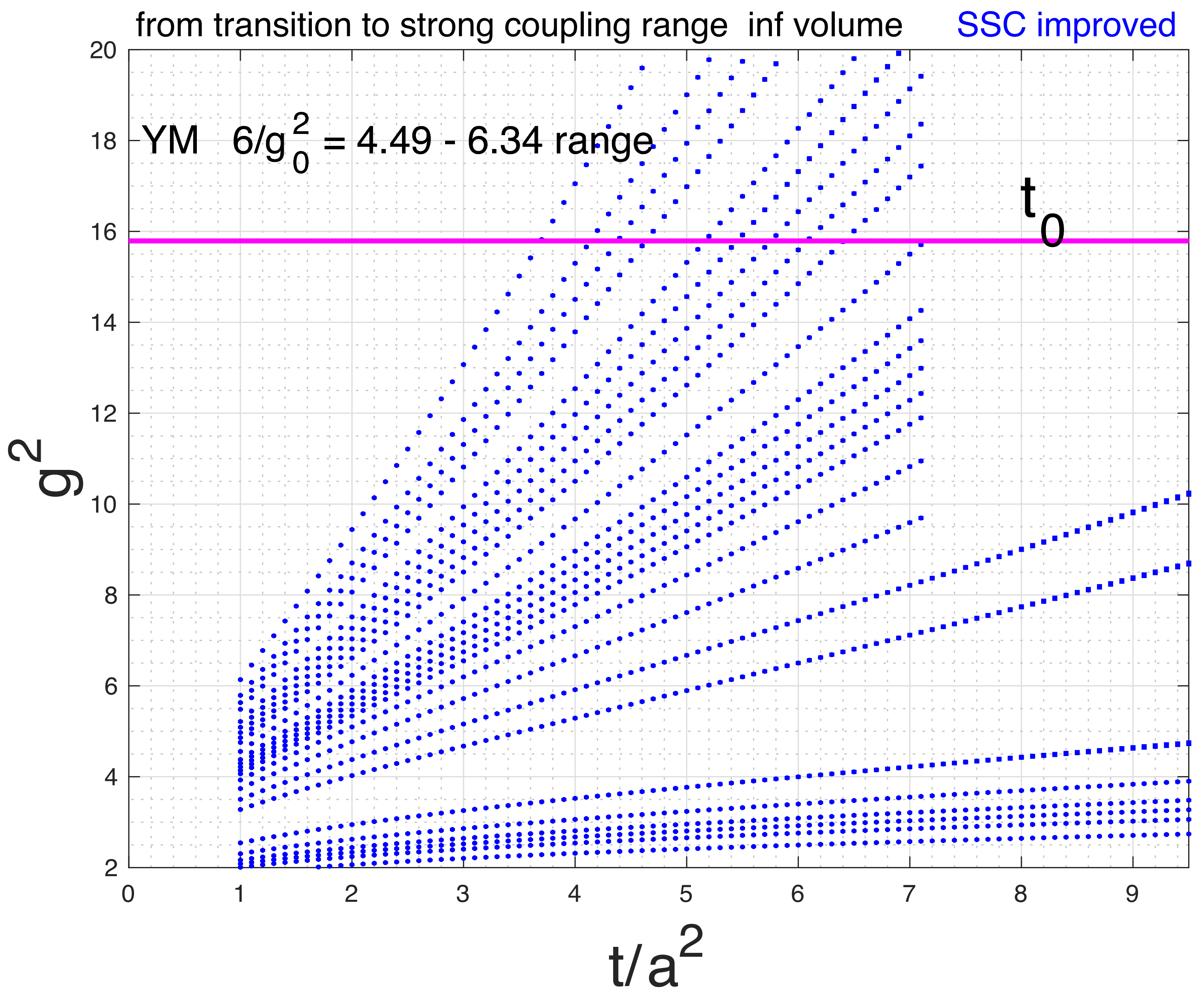
only samples of all t fits are shown at fixed $6/g_0^2$

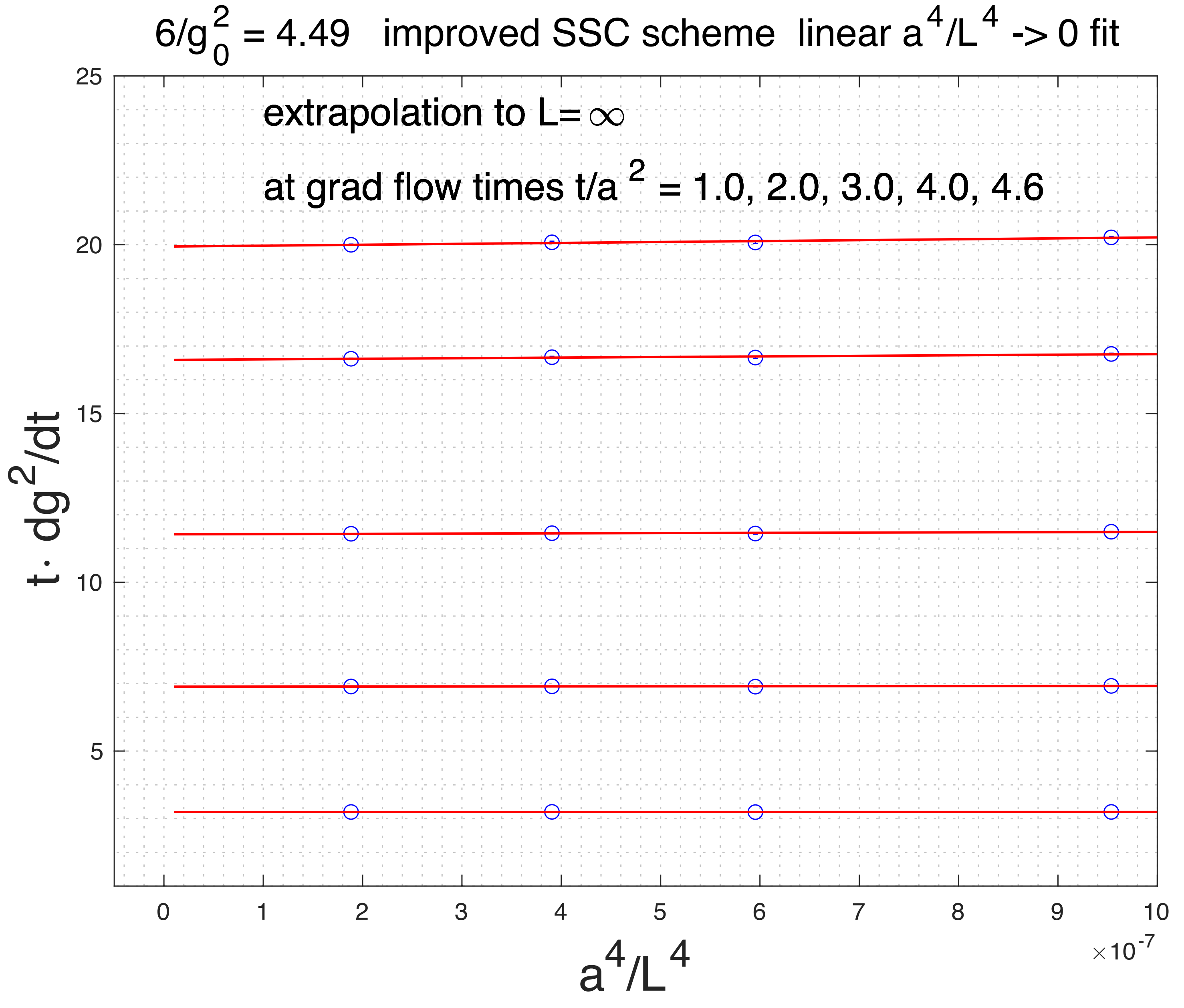
infinite volume limit for each $6/g_0^2$ value after using several L at the same bare gauge coupling

step 1 illustrated



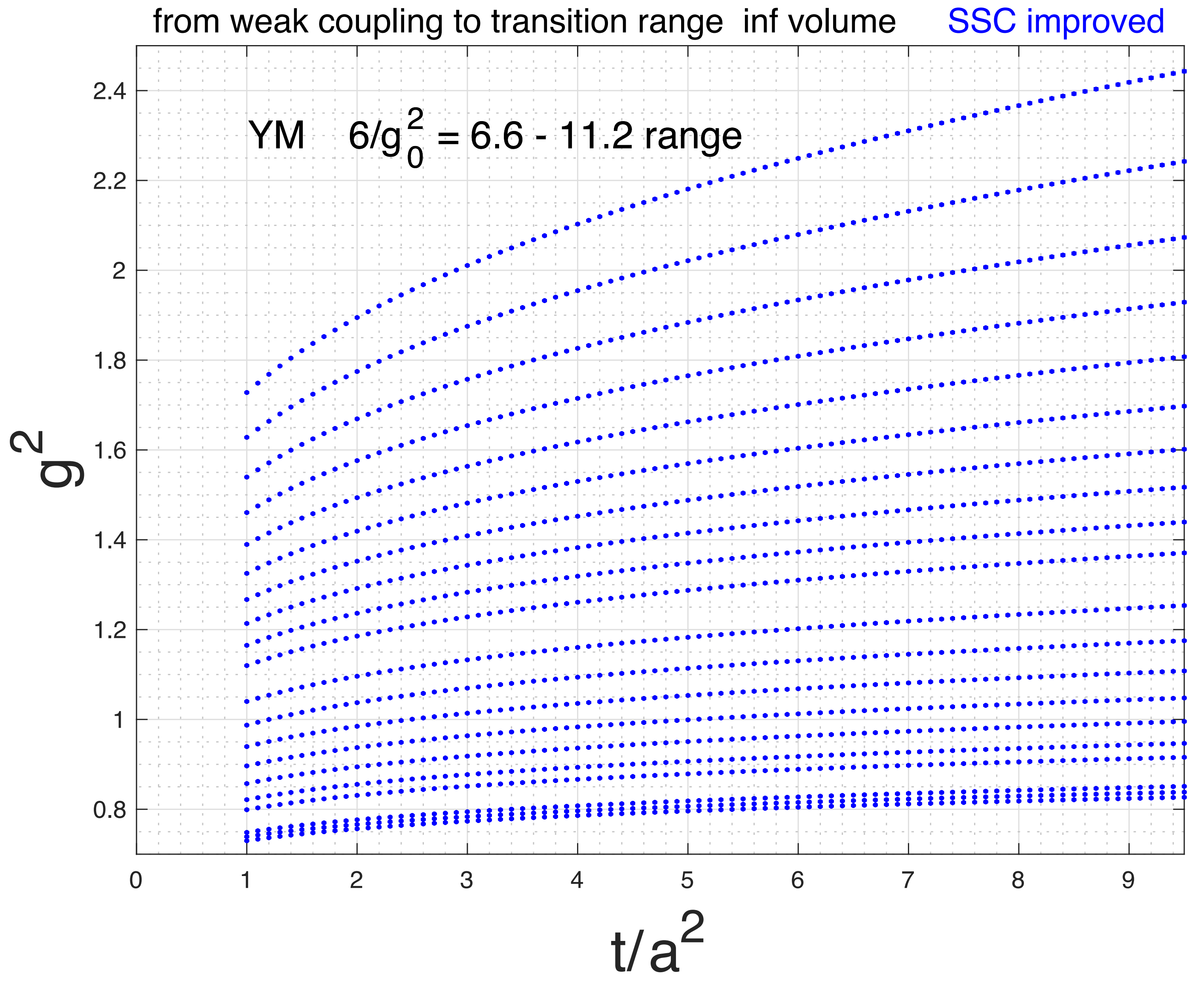
repeated for each $6/g_0^2$ at all flow rime steps using four L values in the fits





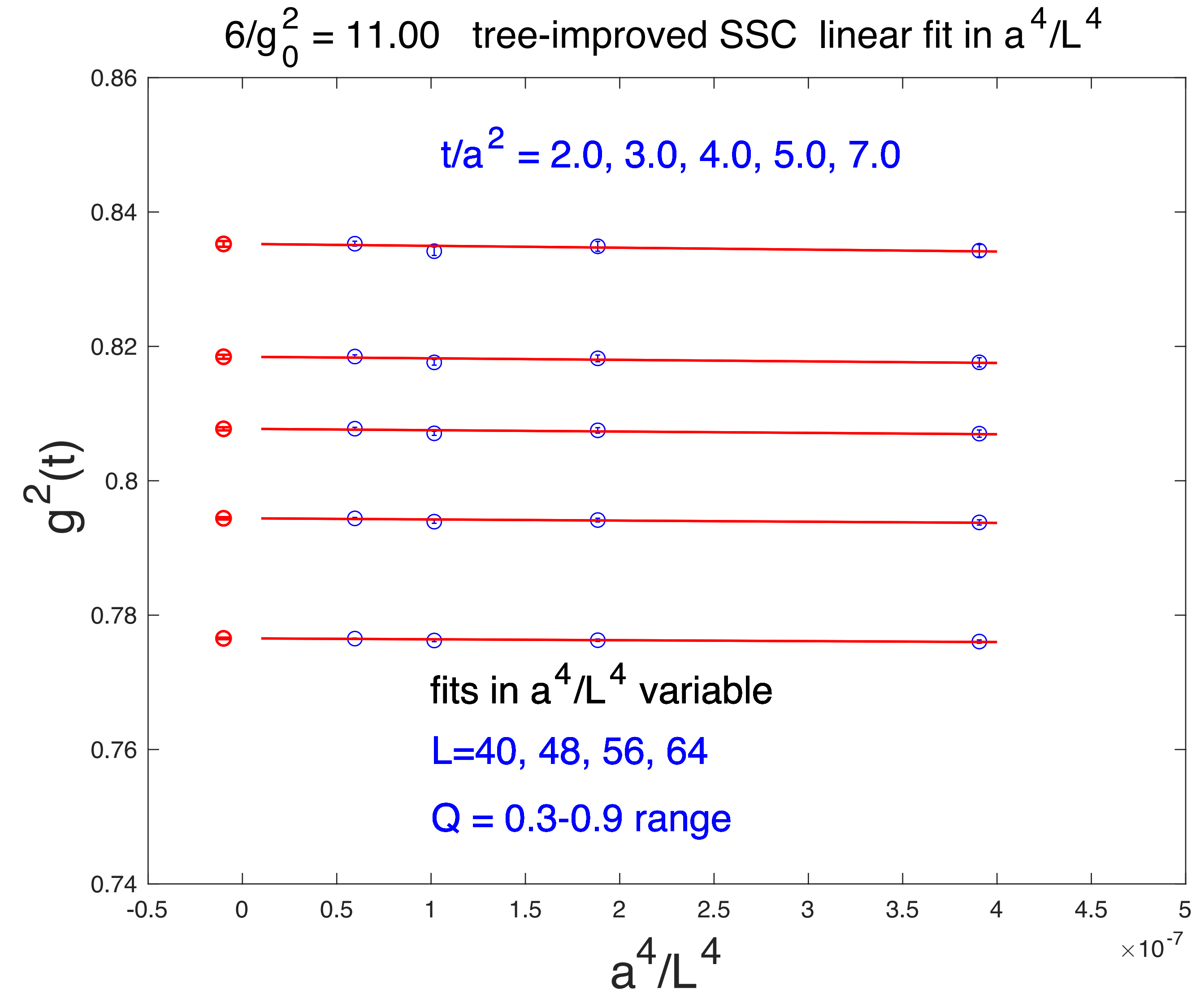
similarly, though not shown, we generate plots for $t \cdot dg^2/dt$ at each $6/g_0^2$ and at each flow time step

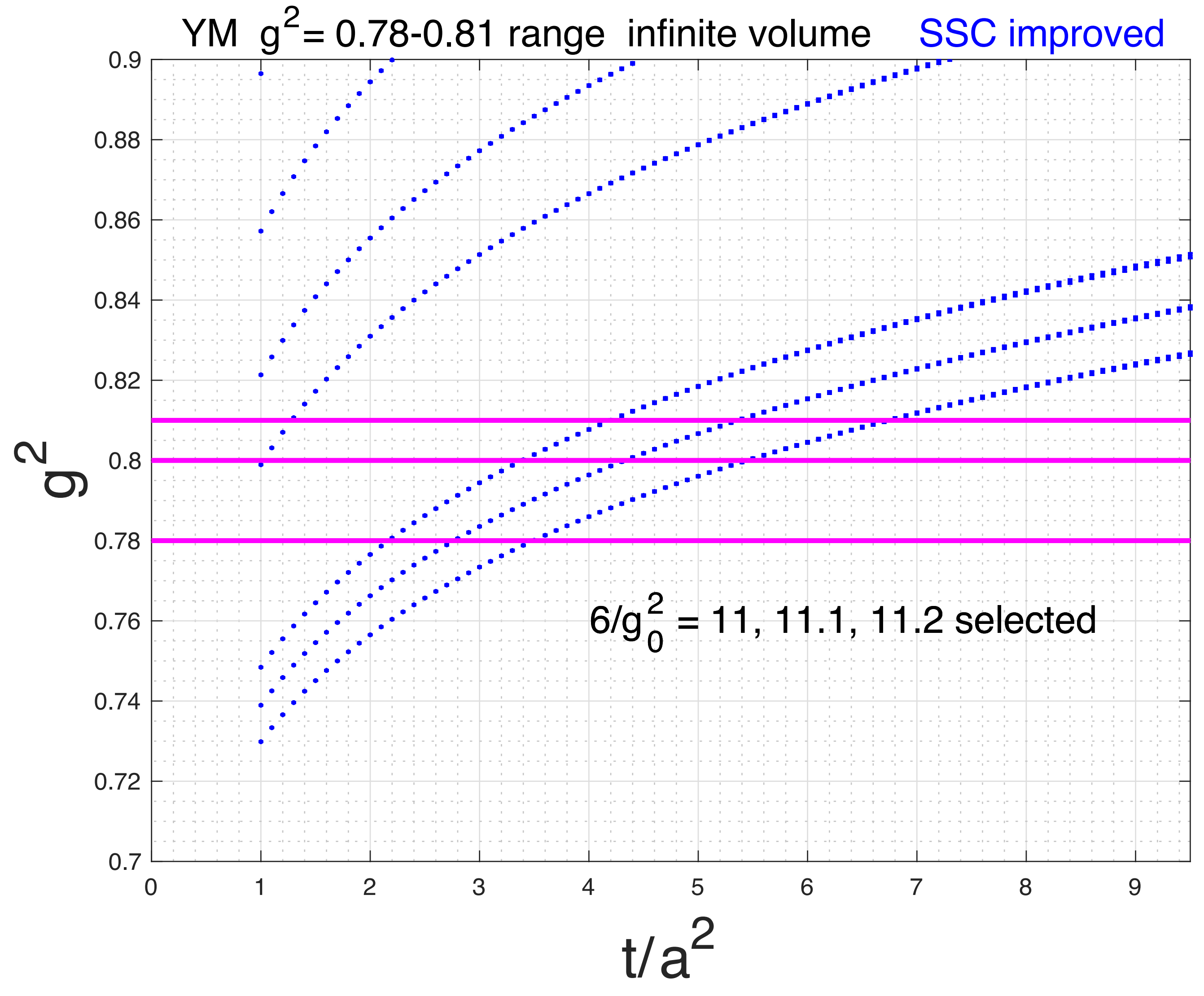
ready to go to step 2 with $g^2(t), t \cdot dg^2/dt$ available in the the infinite volume limit at each $6/g_0^2$

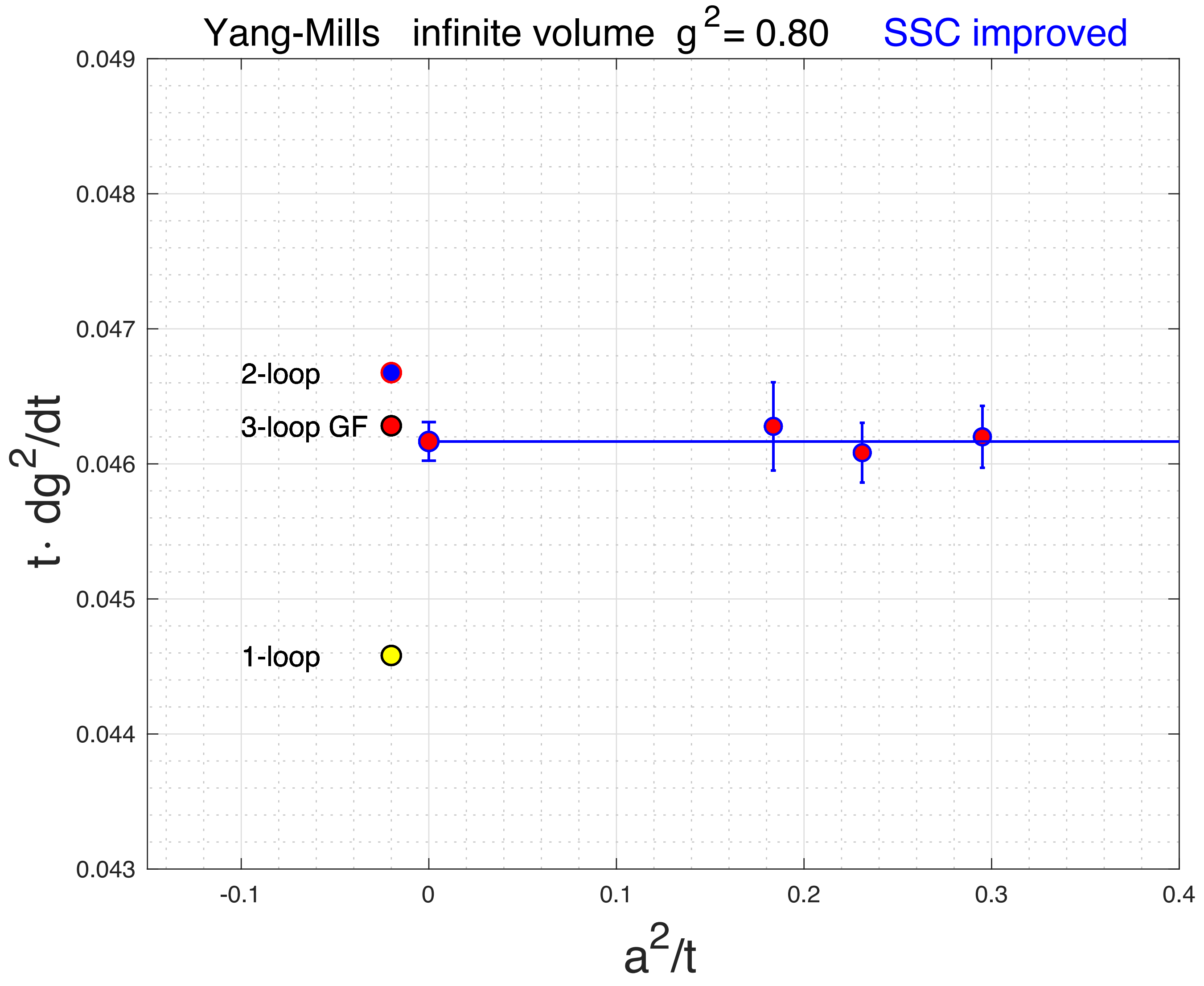


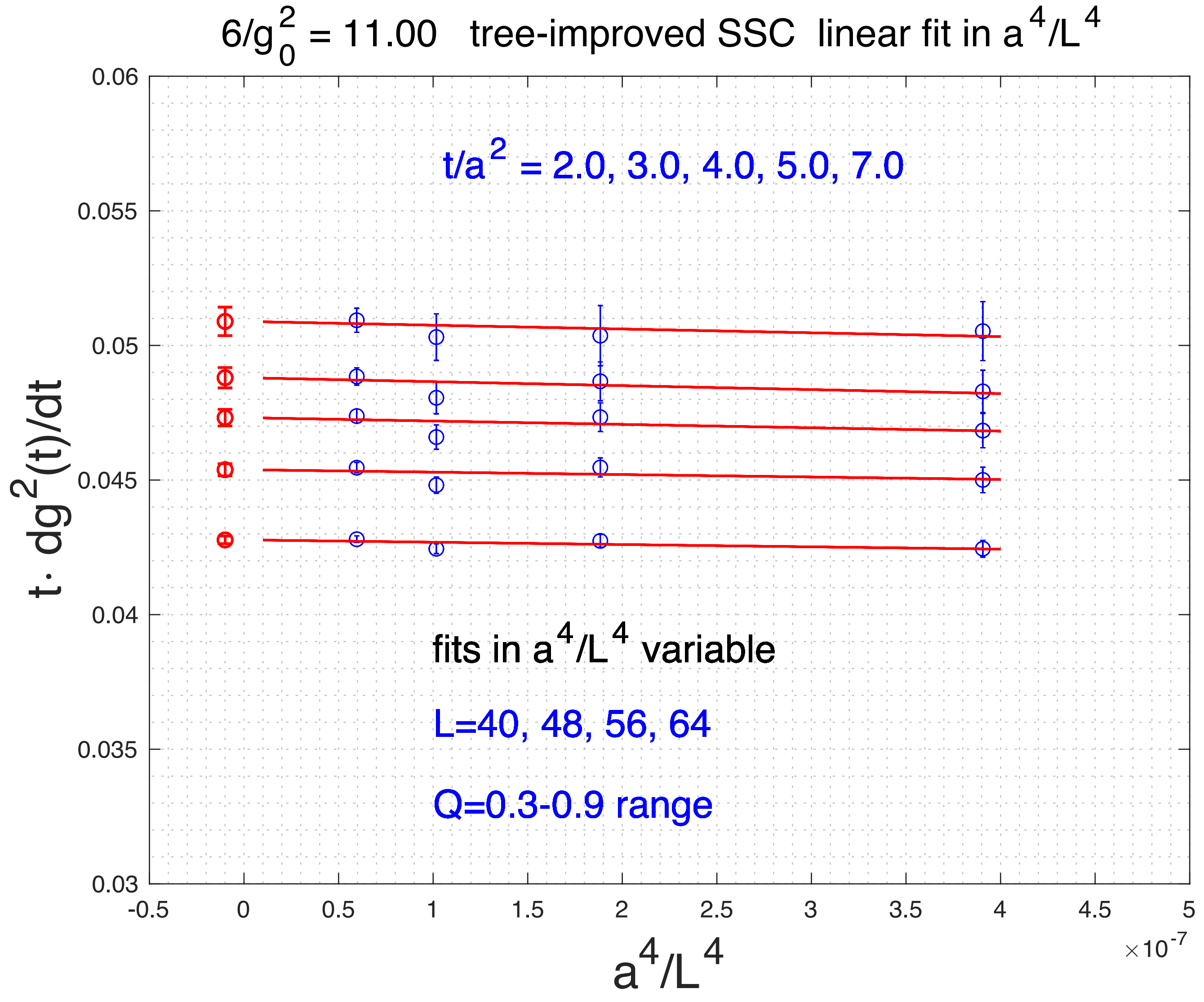
step 1 similarly at $6/g_0^2$ = 11.1 and 11.2

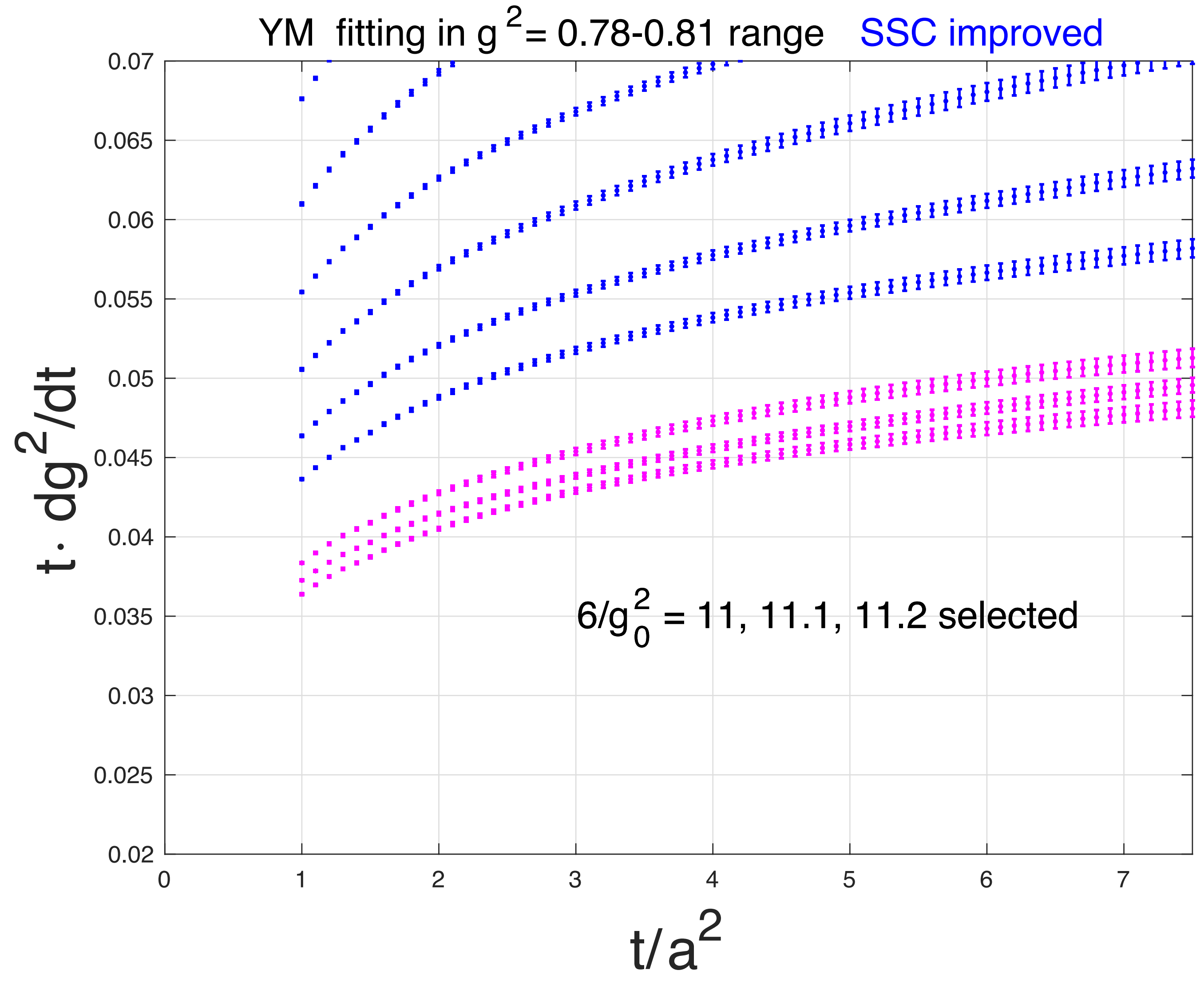
step 2 and step 3 at weak coupling

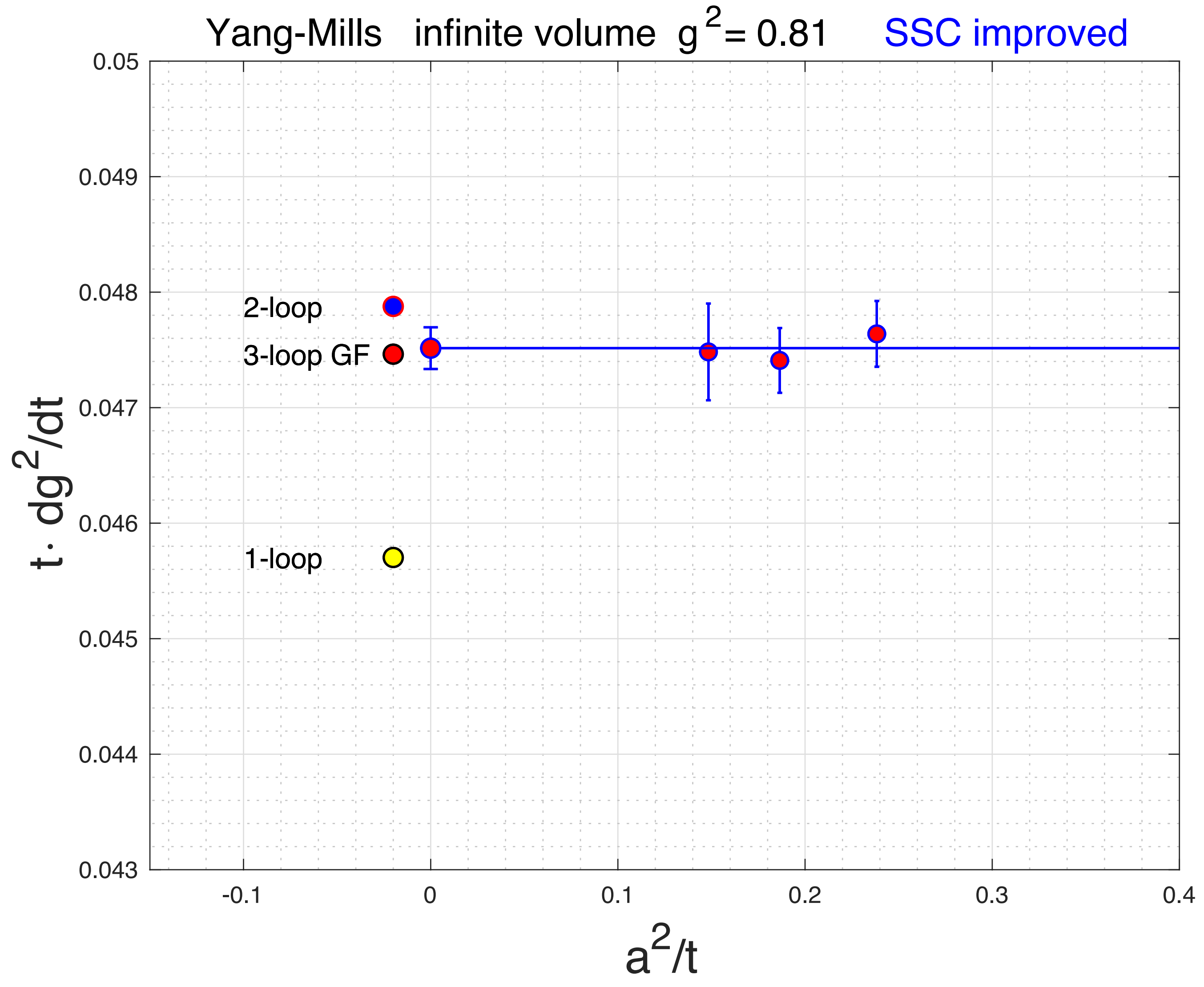




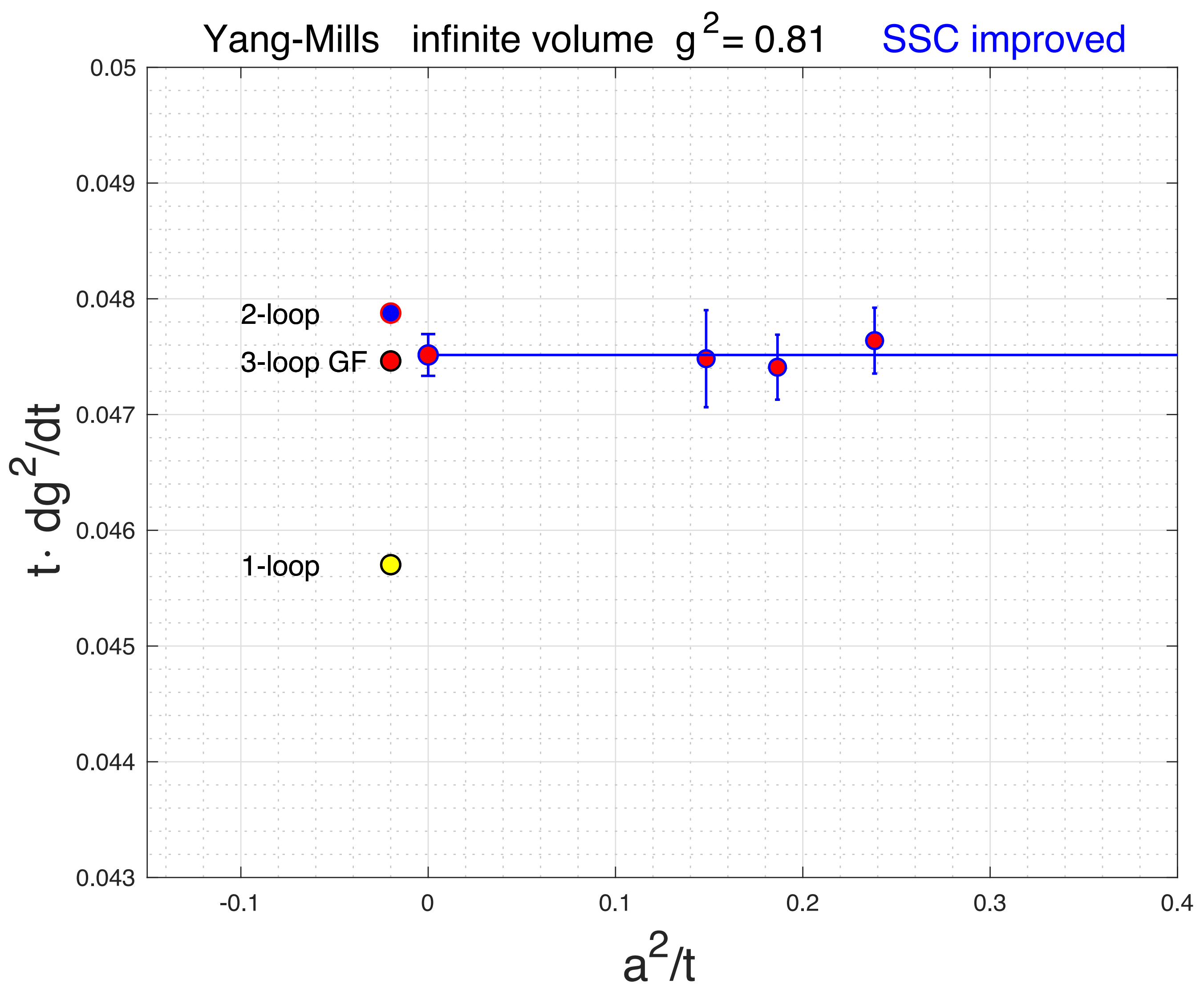


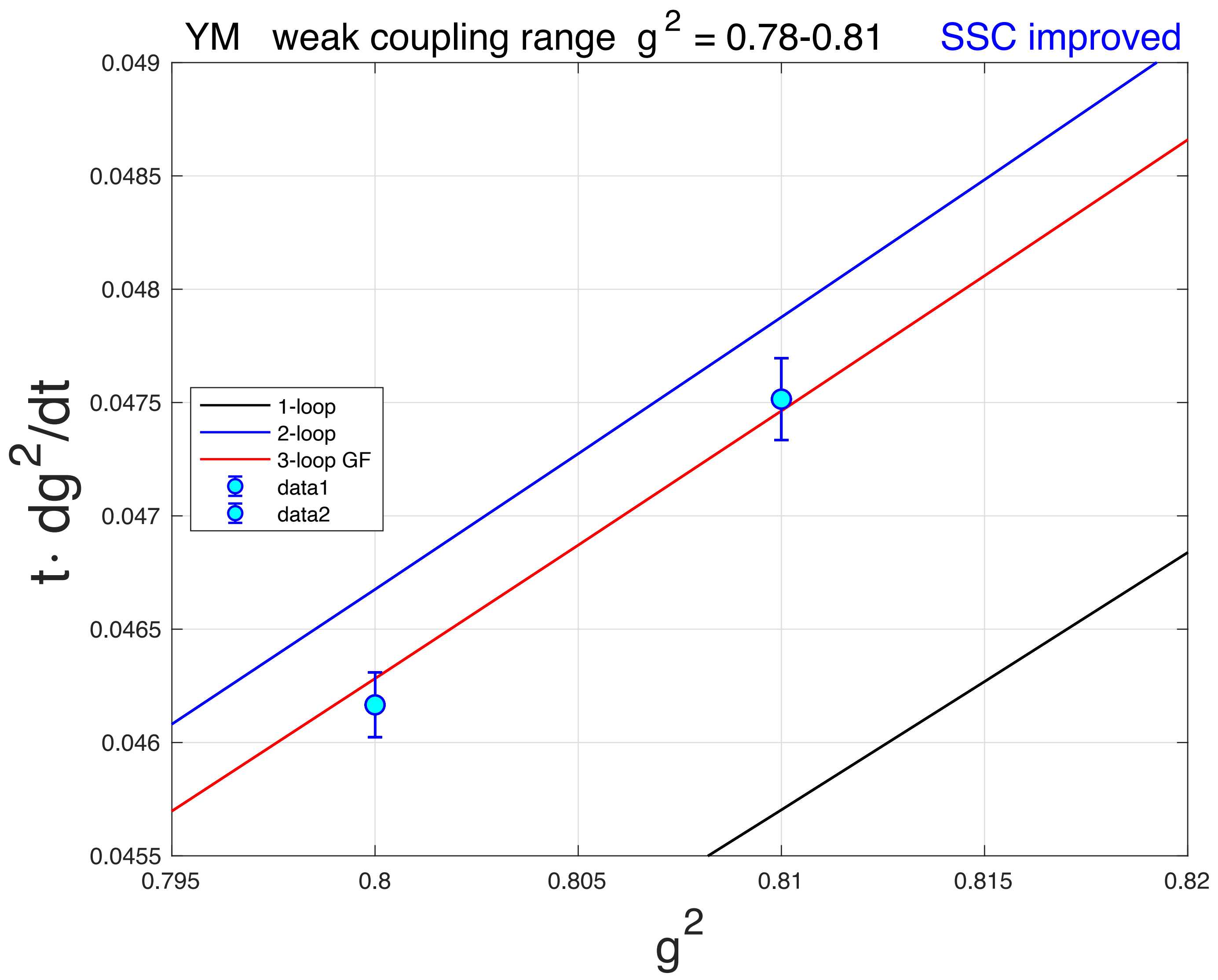


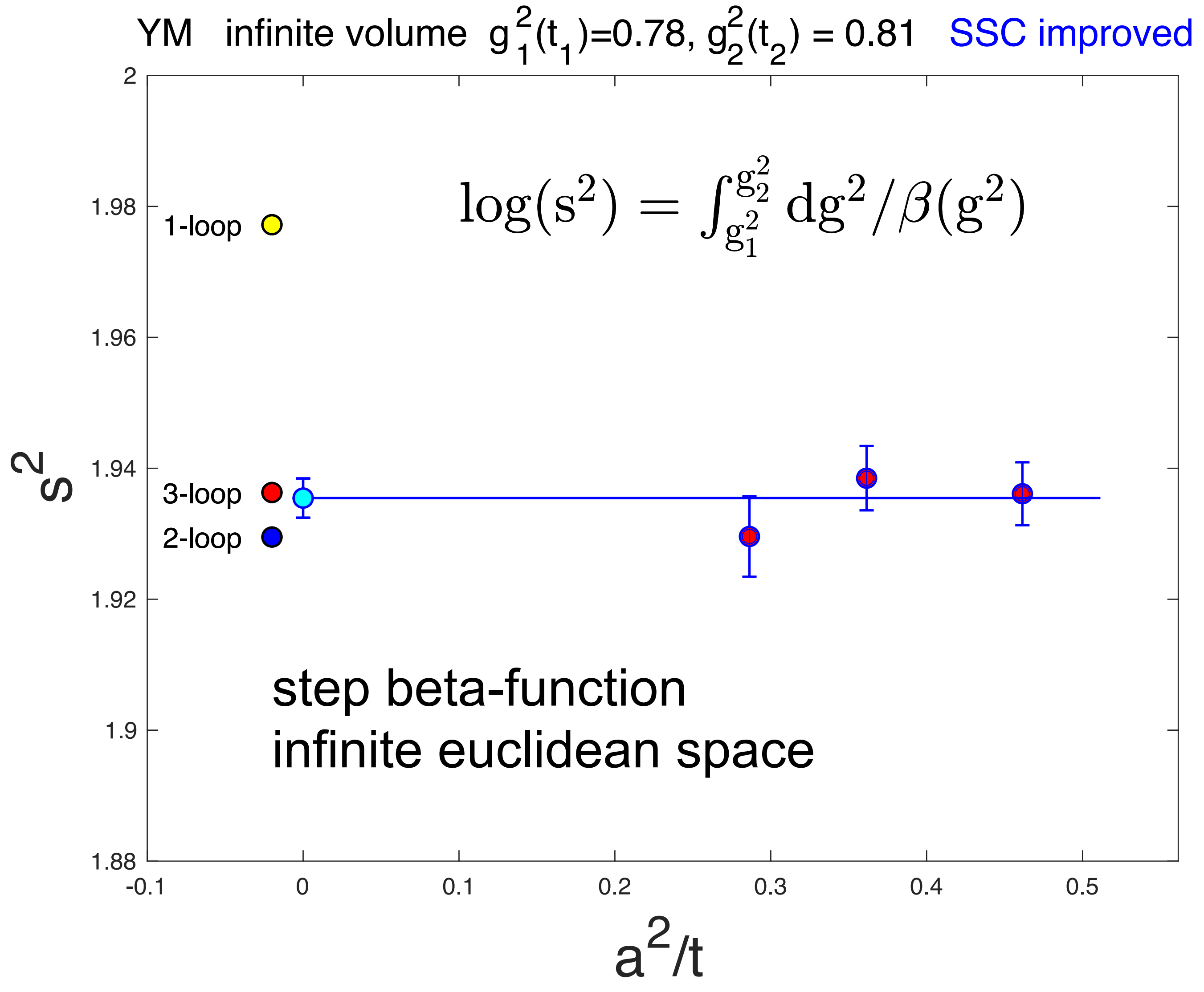


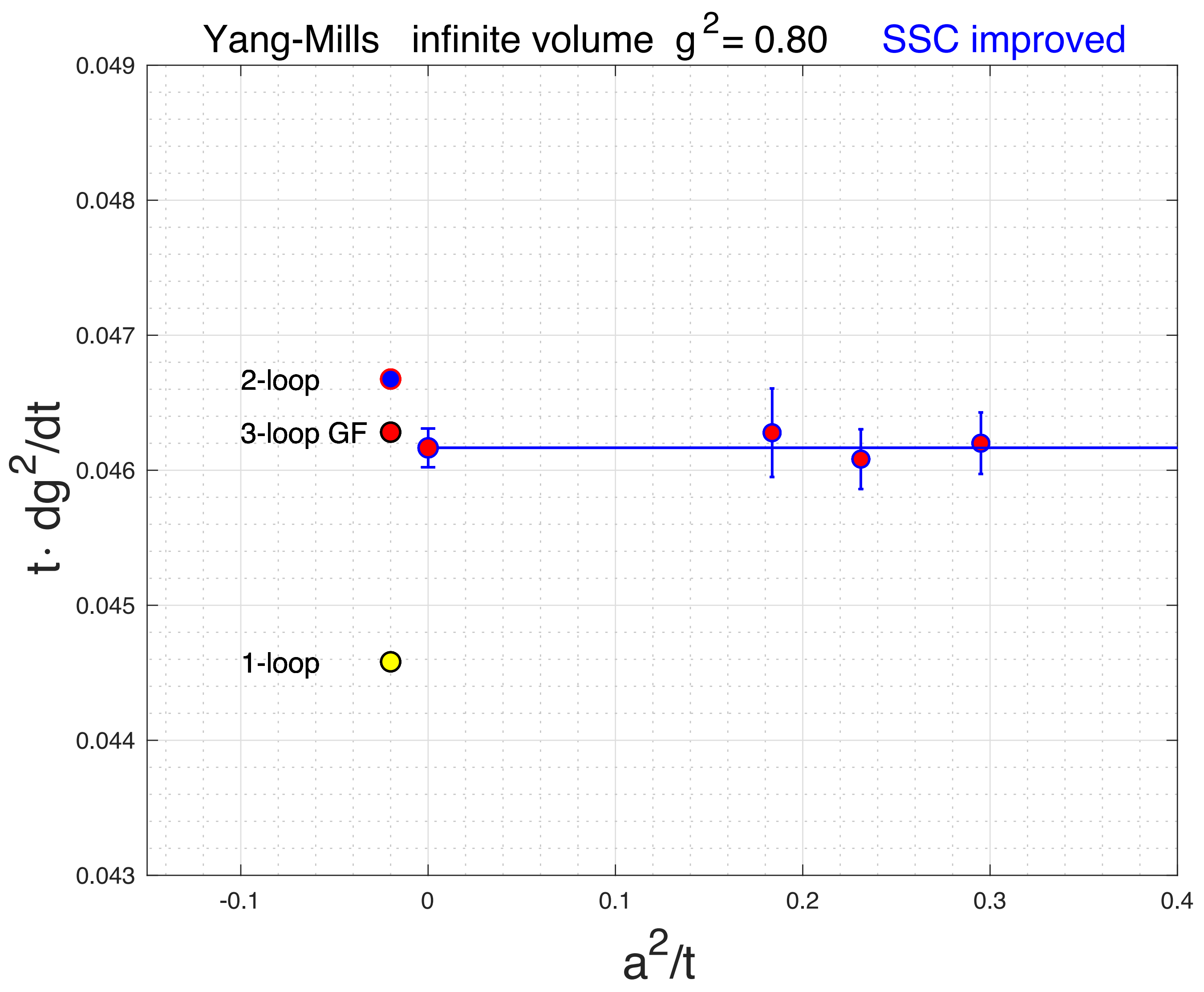


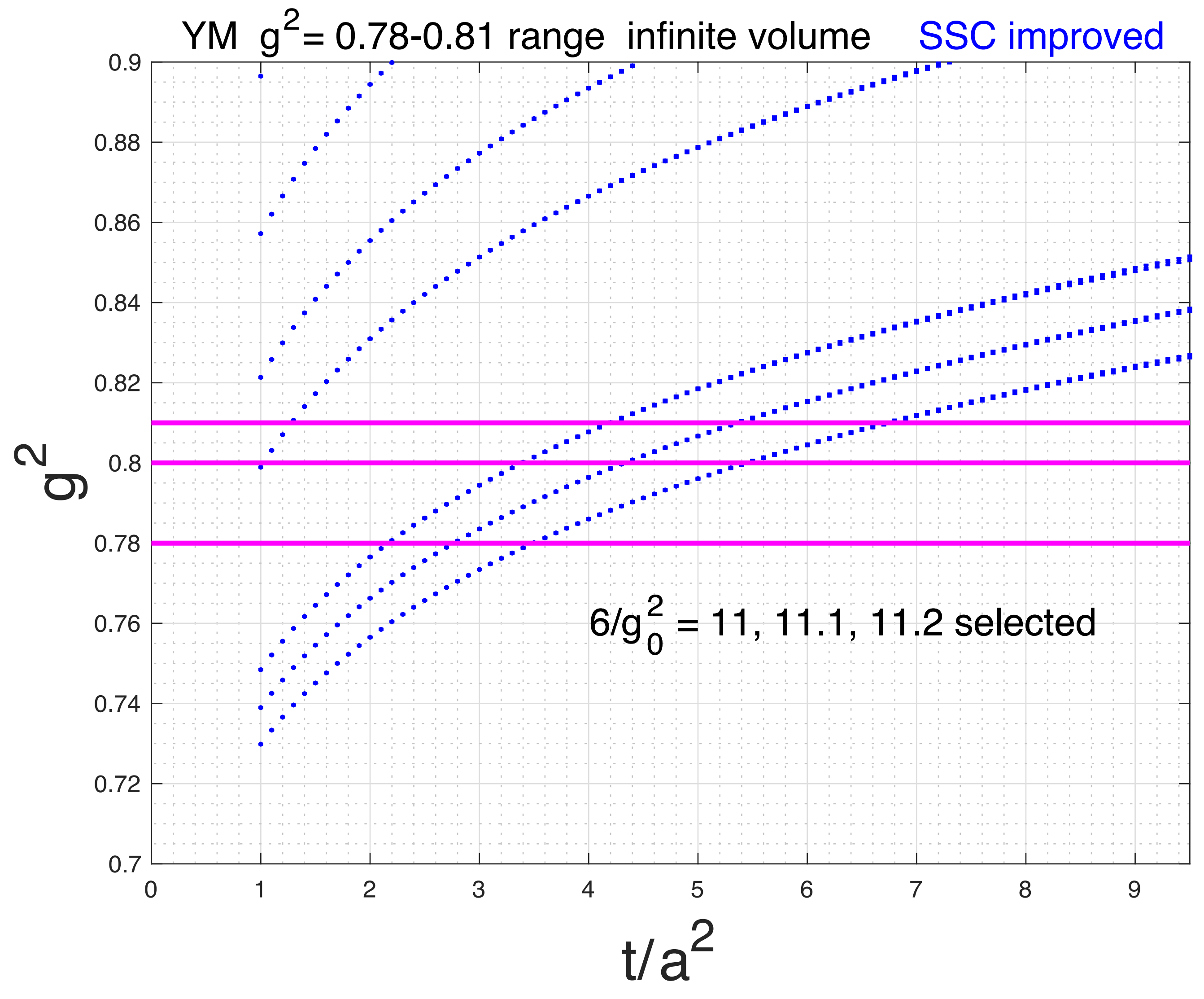
tdg^2/dt and step beta-function at weak coupling



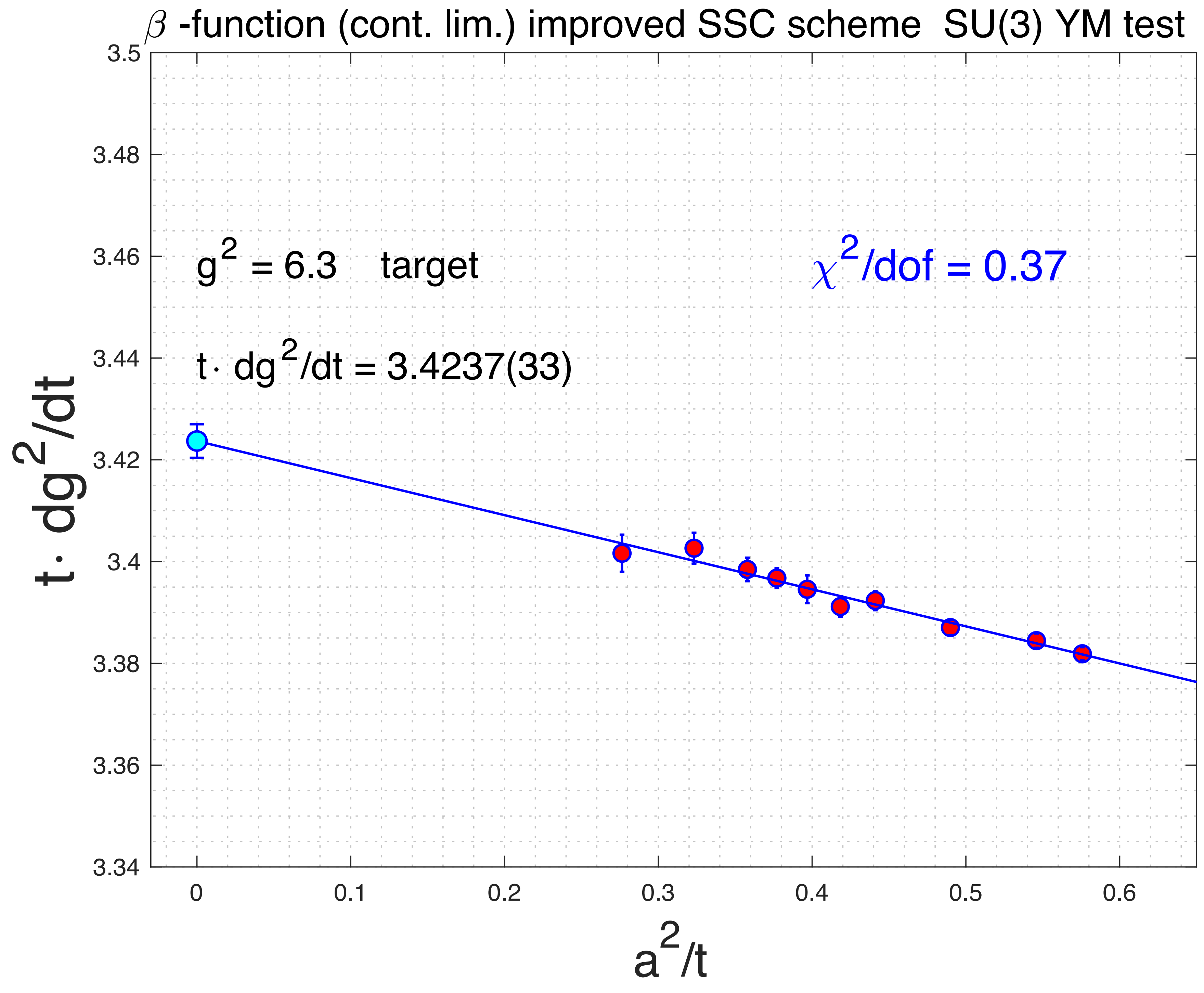


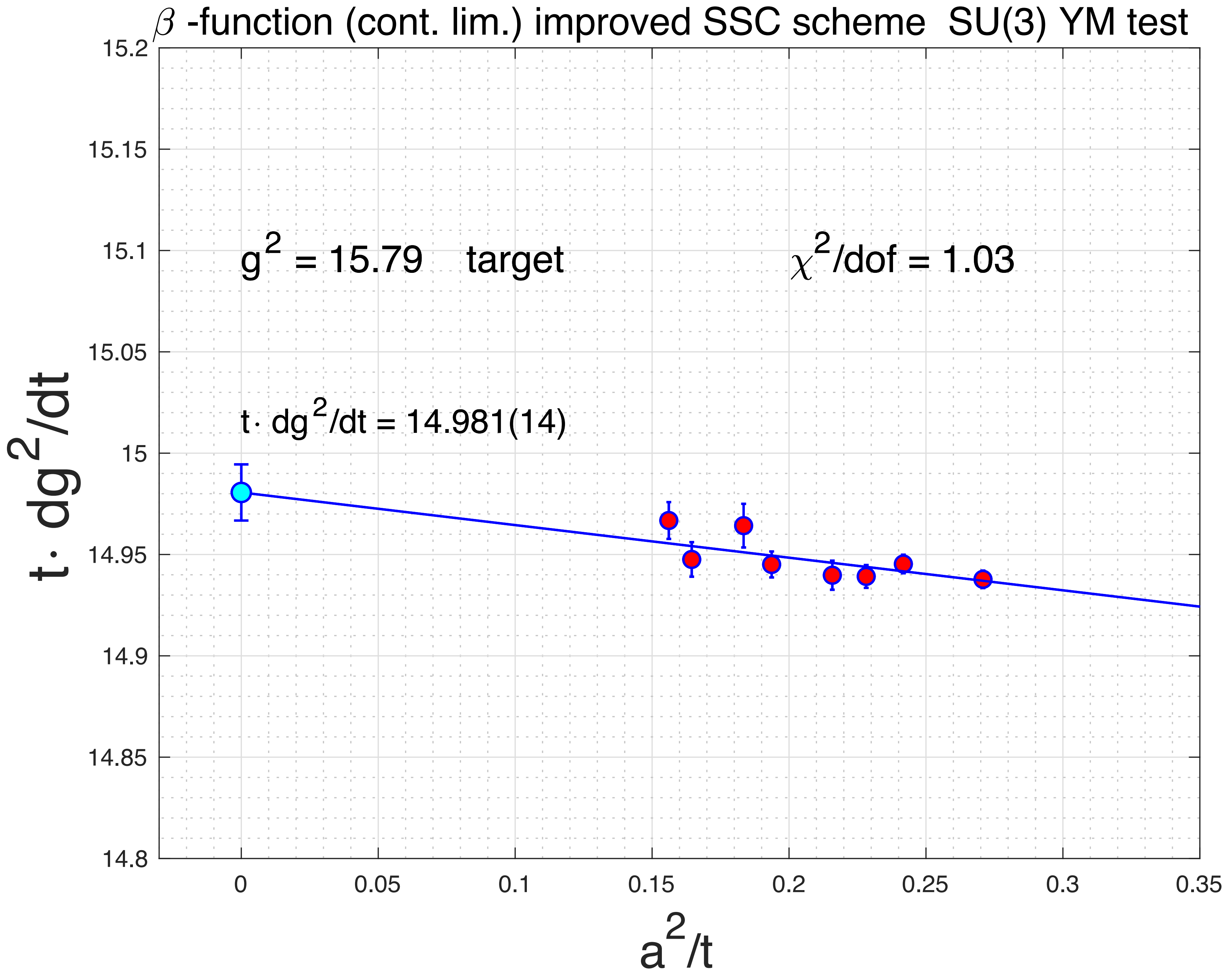






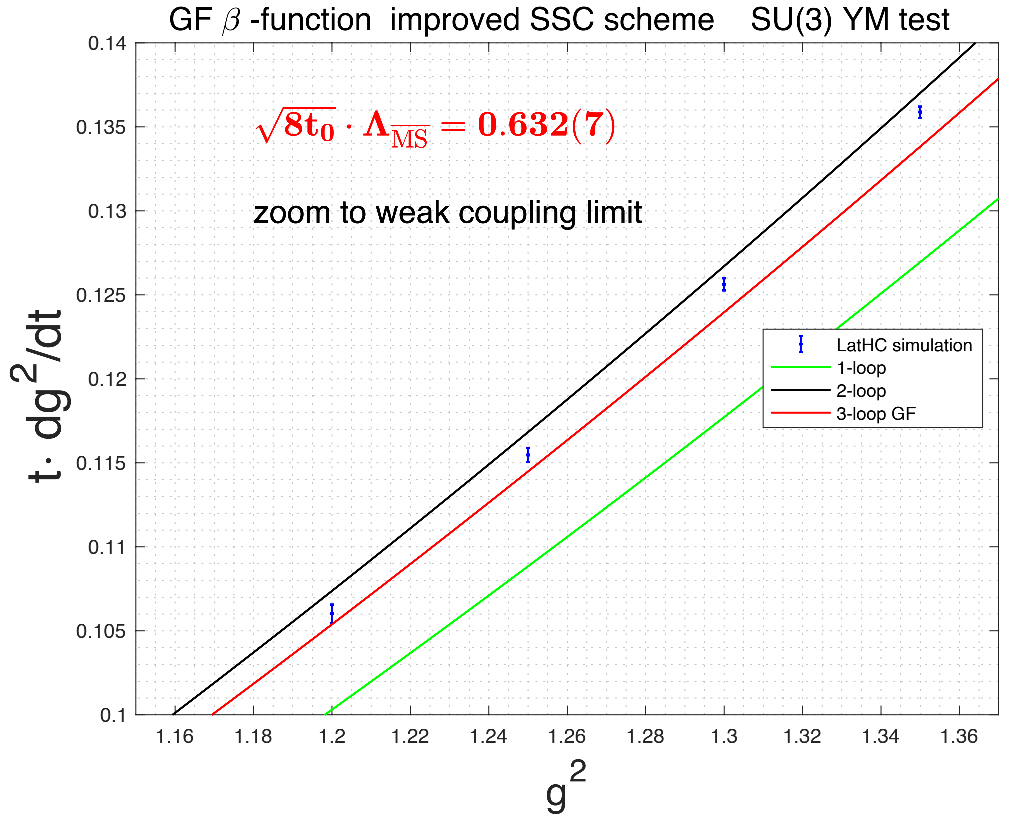
tdg^2/dt beta-function at strong coupling

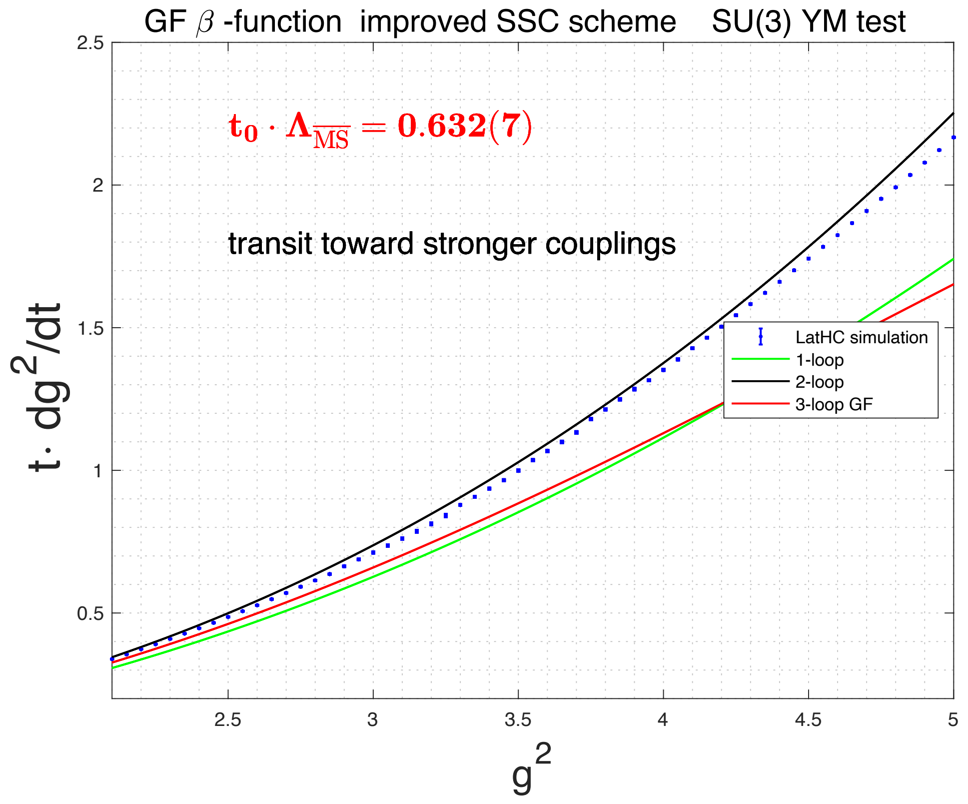


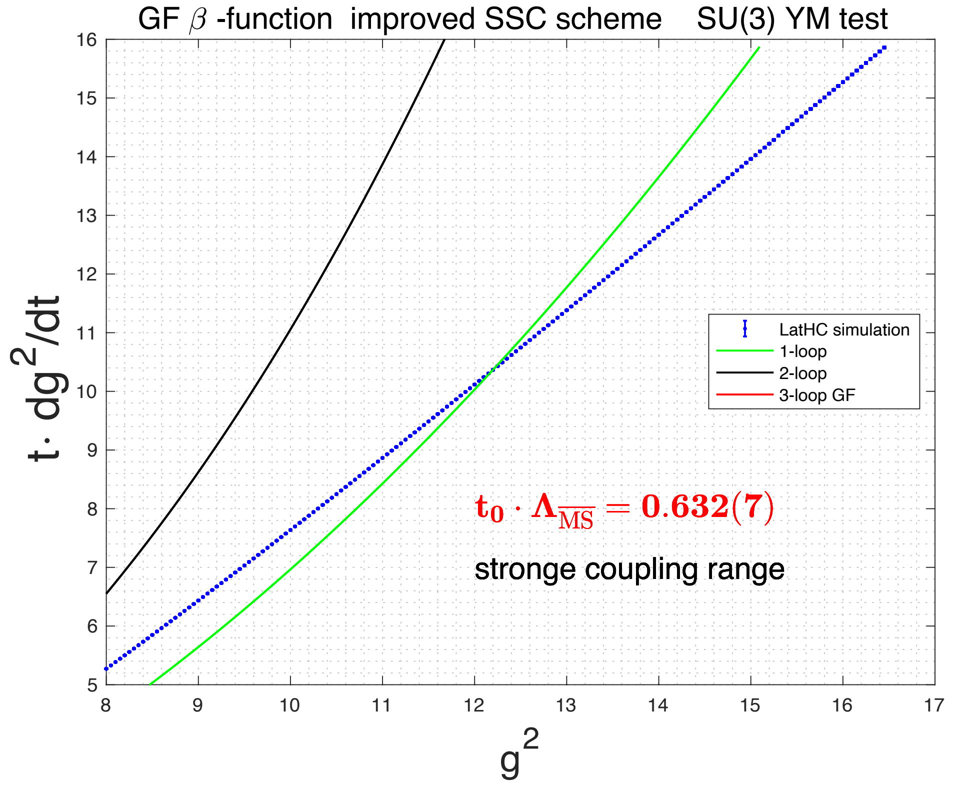


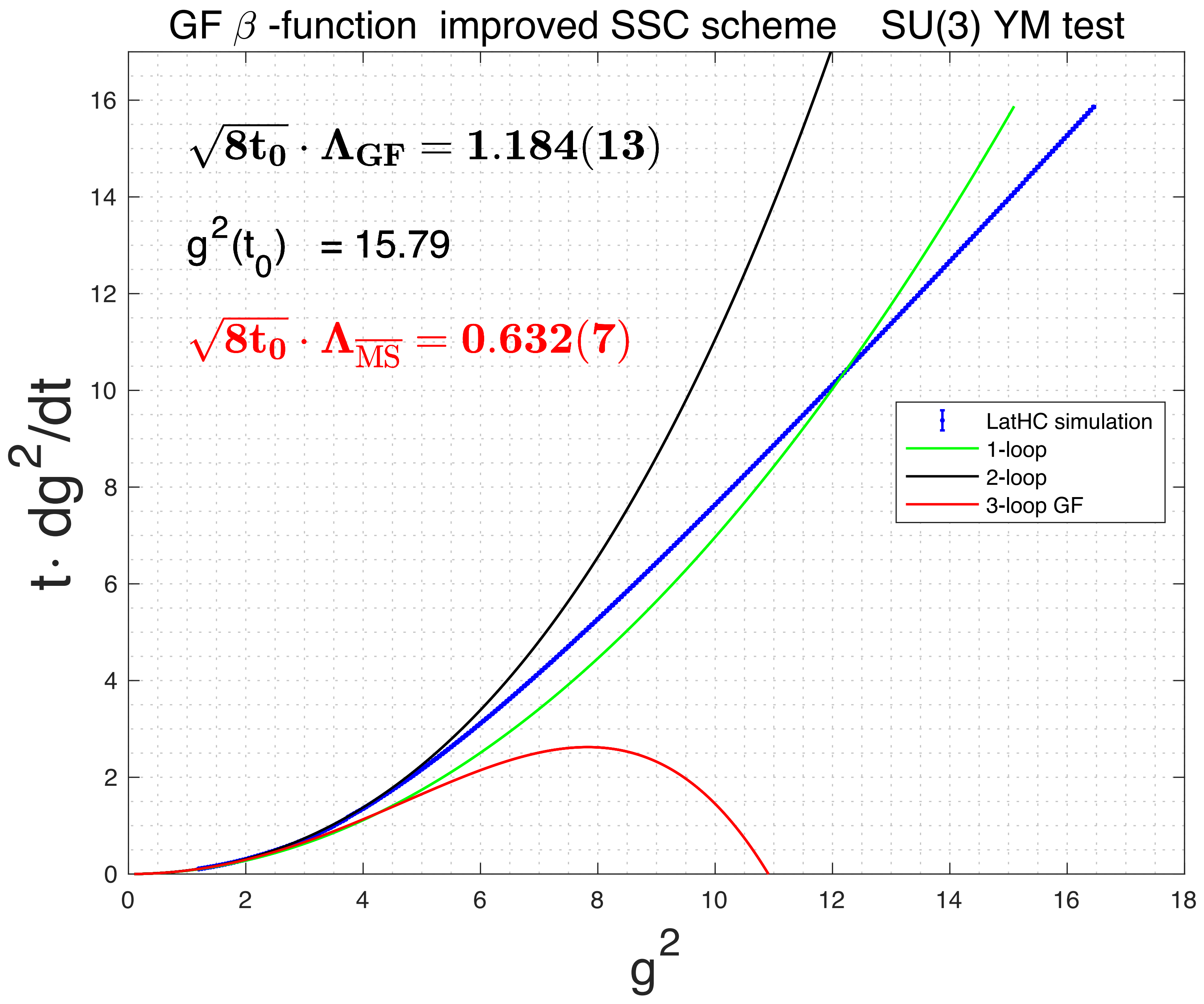
zoomed in:

2022 analysis









new 2023 analysis with extended data set is in the works

some details of the analysis:

$$t_0 \cdot \Lambda_{GF} = (b_0 \bar{g}^2)^{-b_1/2b_0^2} \cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\Big(-\int_0^{\bar{g}} dx\big[1/\beta(x) + 1/b_0x^3 - b_1/b_0^2x\big]\Big)$$

integral broken up into two parts:

- in the $g^2 = 0 - 1.2$ range the three-loop Harlander-Neumann beta-function was used
- in the $g^2 = 1.2 - g^2(t_0)$ range spline based numerical integration was used with error analysis

QCD with three massless flavors

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periodic gauge field boundary conditions (the notorious zero mode!)

smearing with 4 stout steps $\rho = 0.12$

staggered fermions, apbc, RHMCalgorithm

lattice ensembles at 22 lattice gauge couplings in $6/g_0^2$ = 4.0 - 10.5 range

several volumes at each lattice gauge coupling including L=32,36,40,48,56,64

not all L-values exist at every gauge coupling

gradient flow

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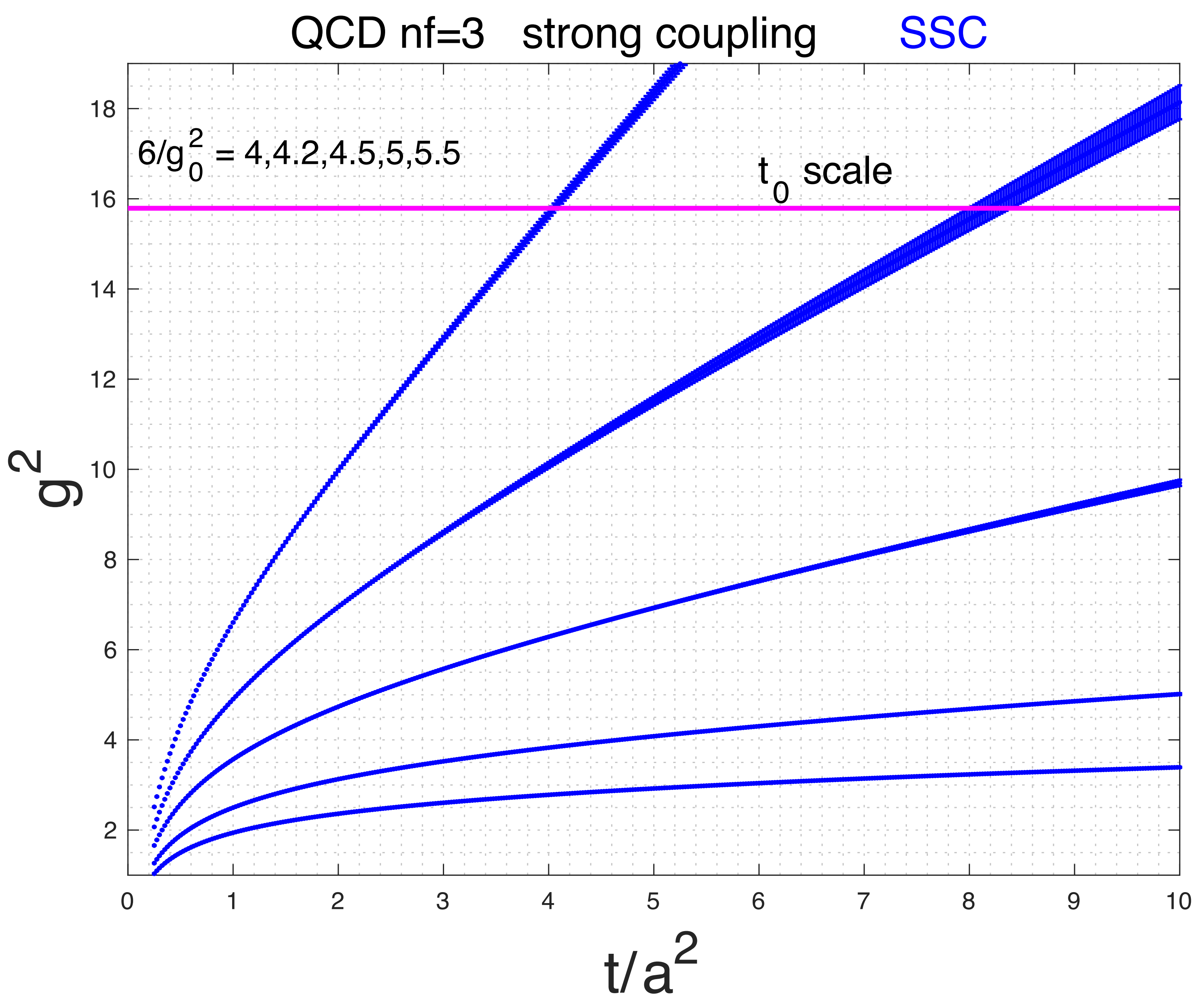
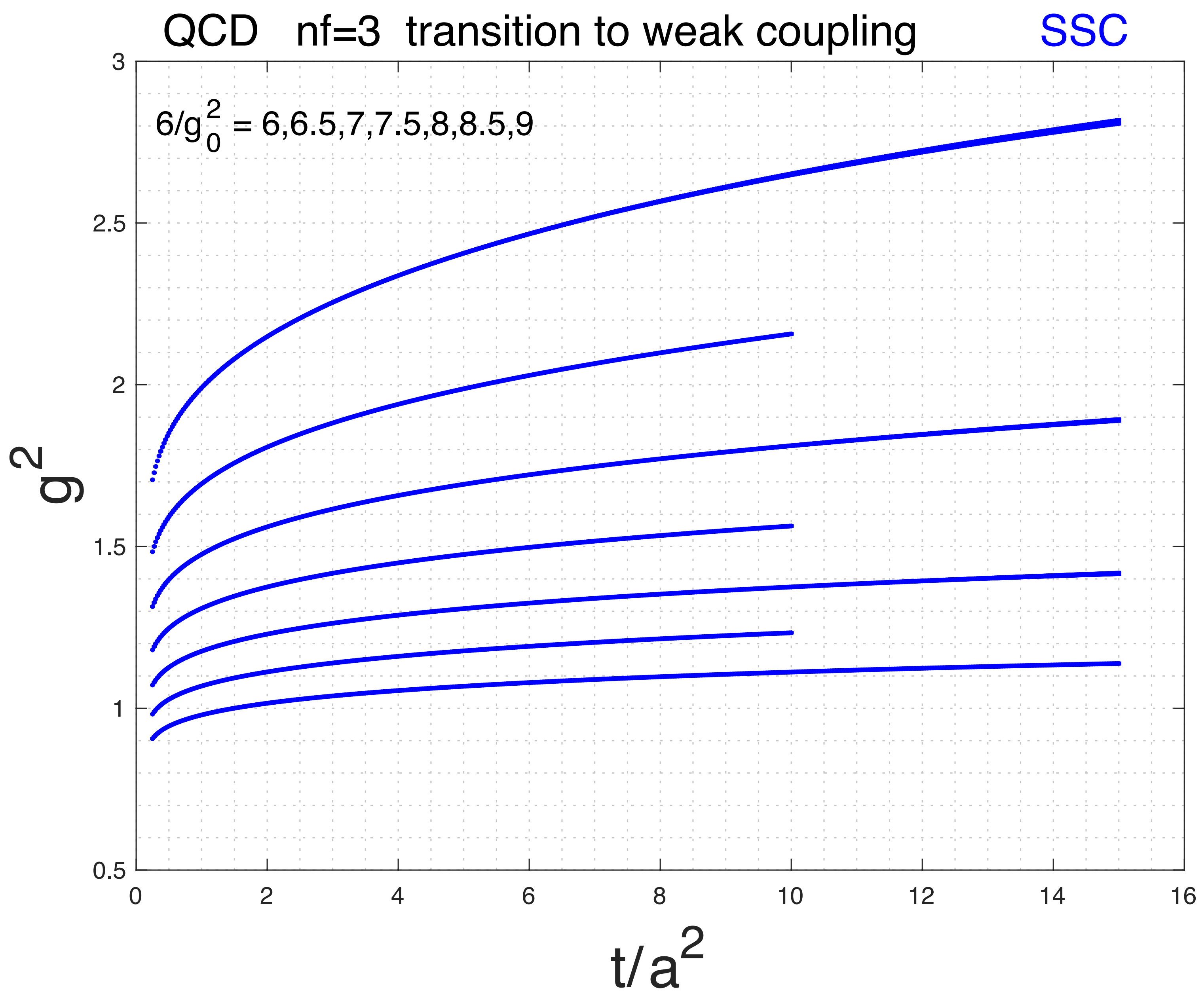
adaptive Runge-Kutta integration on the flow

strategies for getting $\beta(g^2(t)) = t dg^2/dt$ over infinite Euclidean volume will be discussed in the QCD analysis with massless fermions

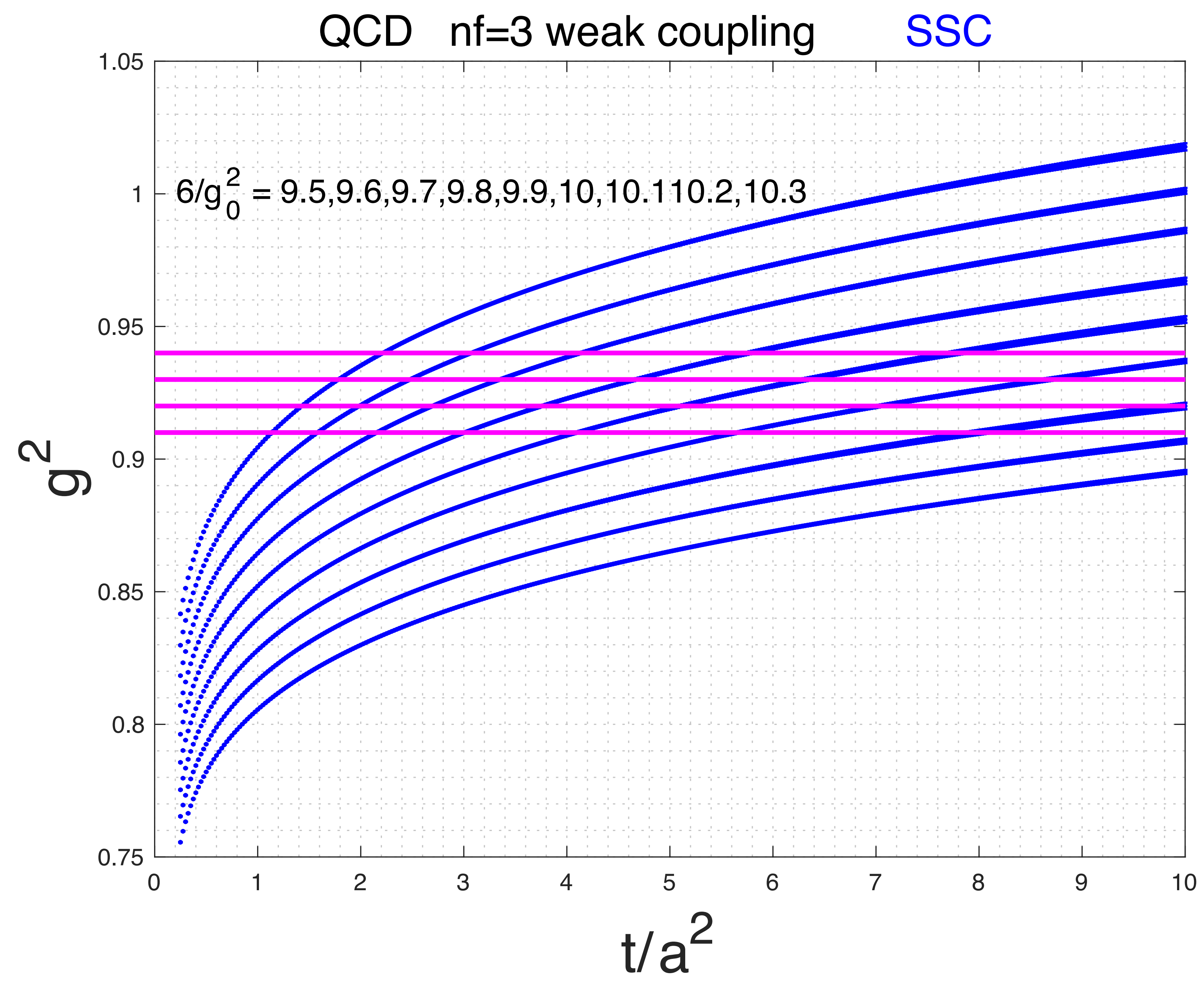
nf=3 data base (strong)



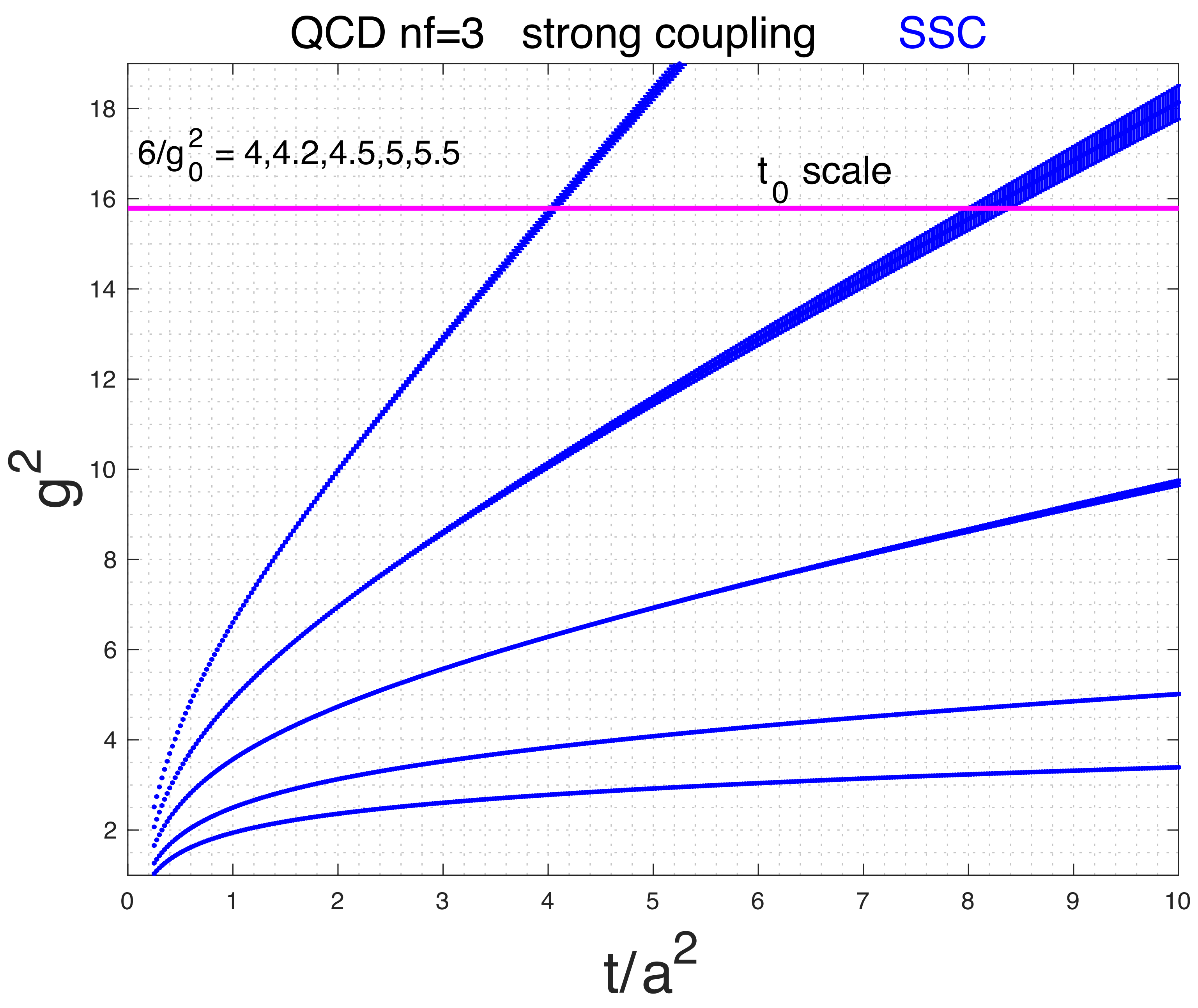
nf=3 data base (intermediate)



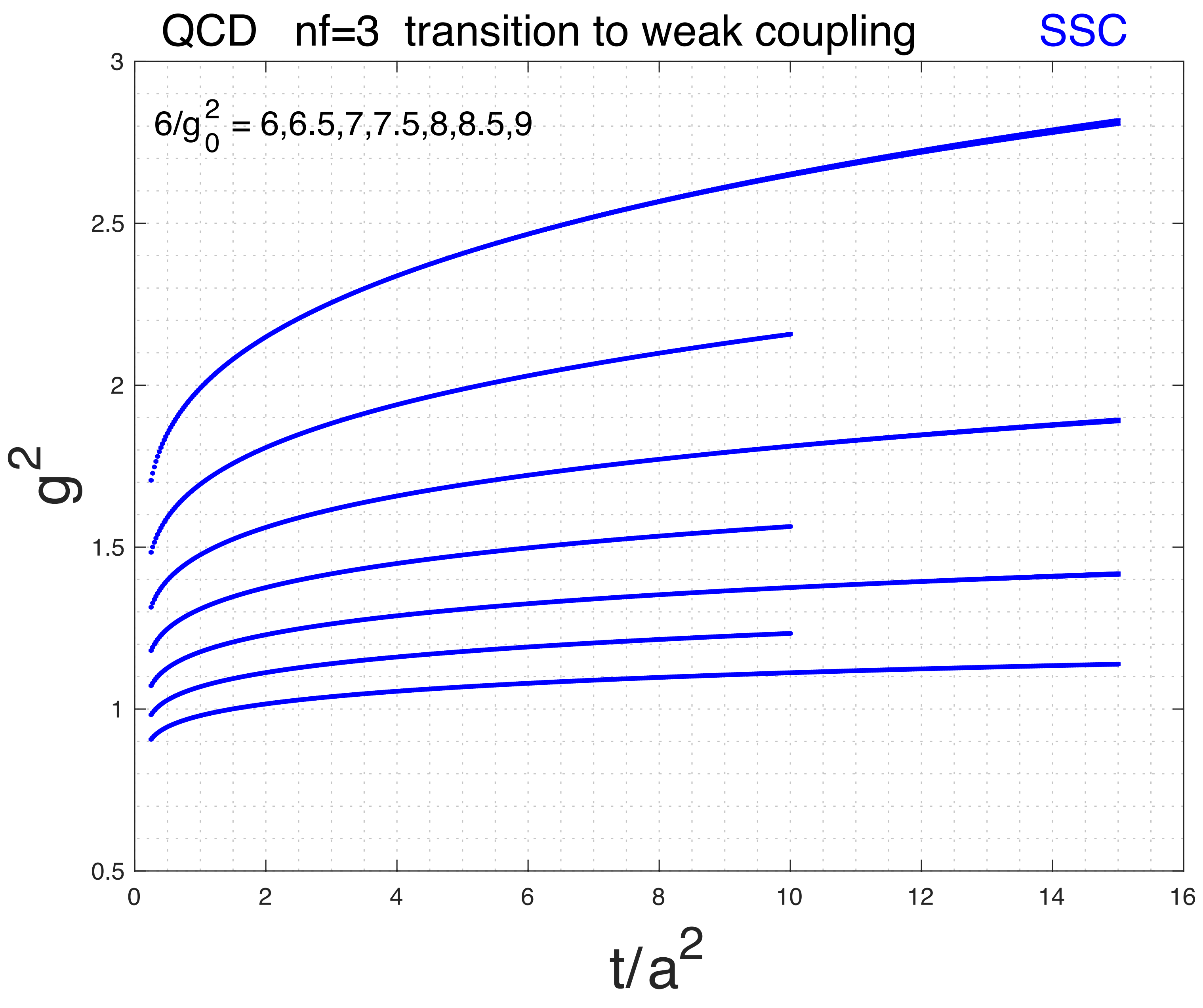
nf=3 data base (weak)



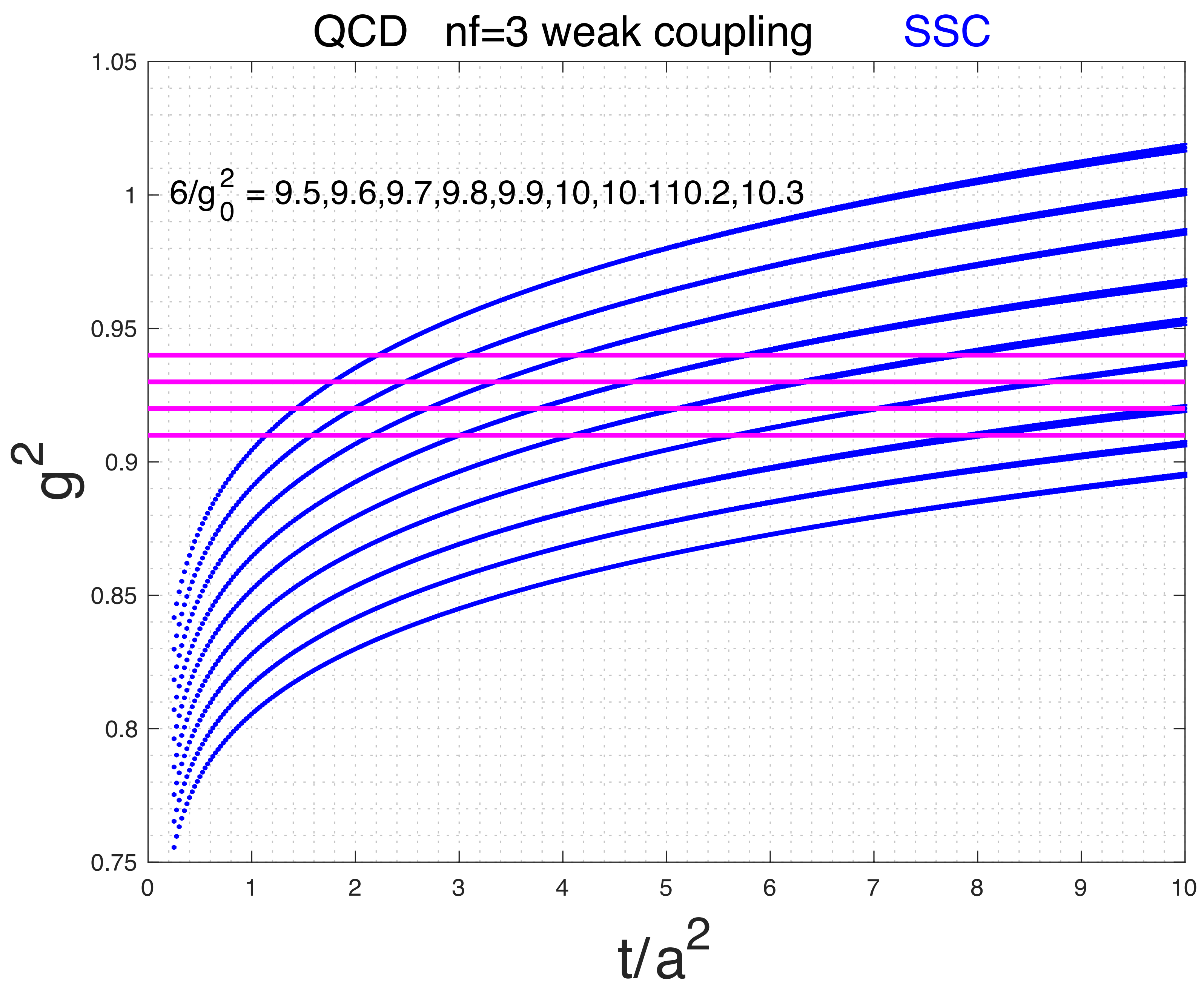
nf=3 data base (strong)

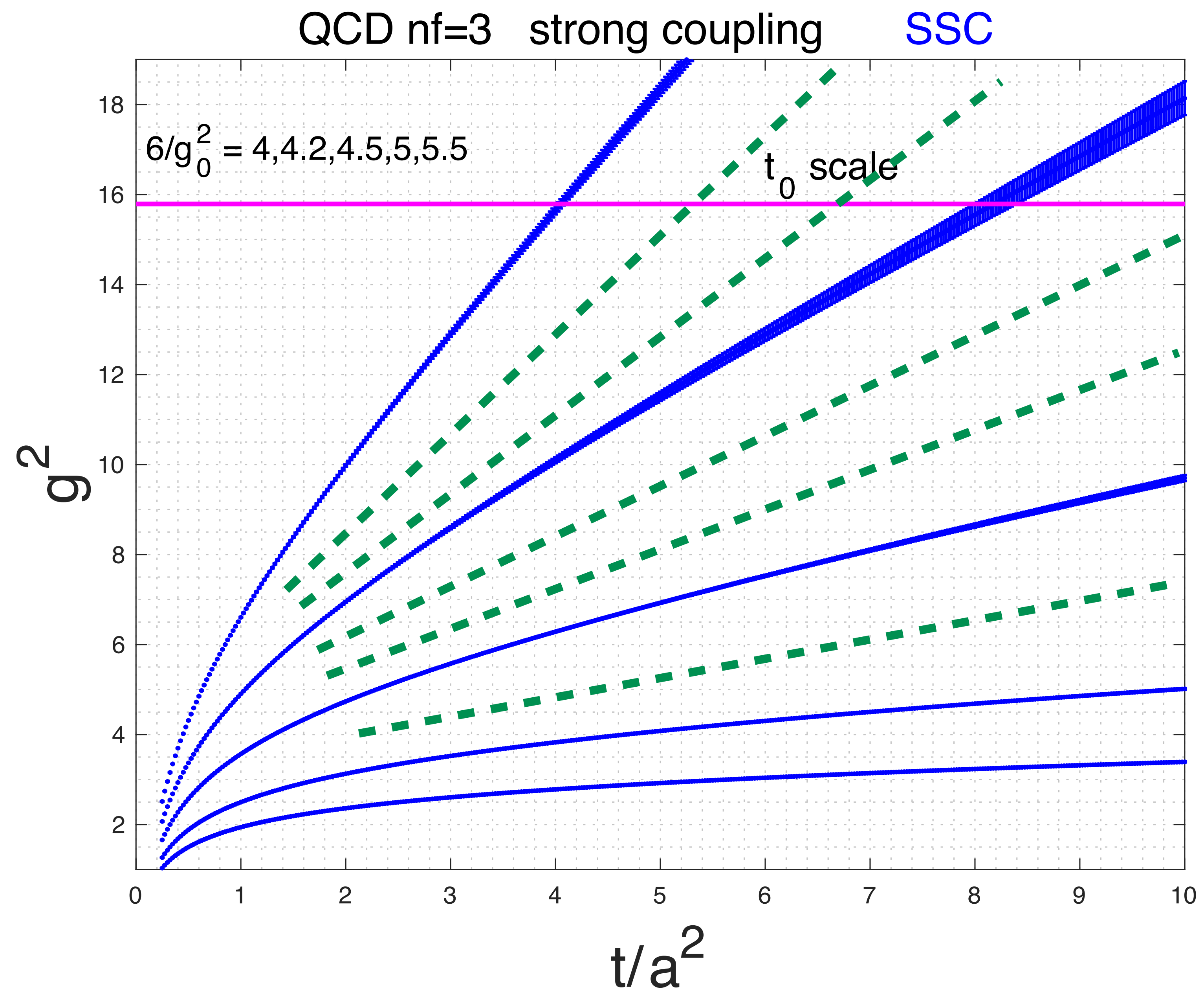


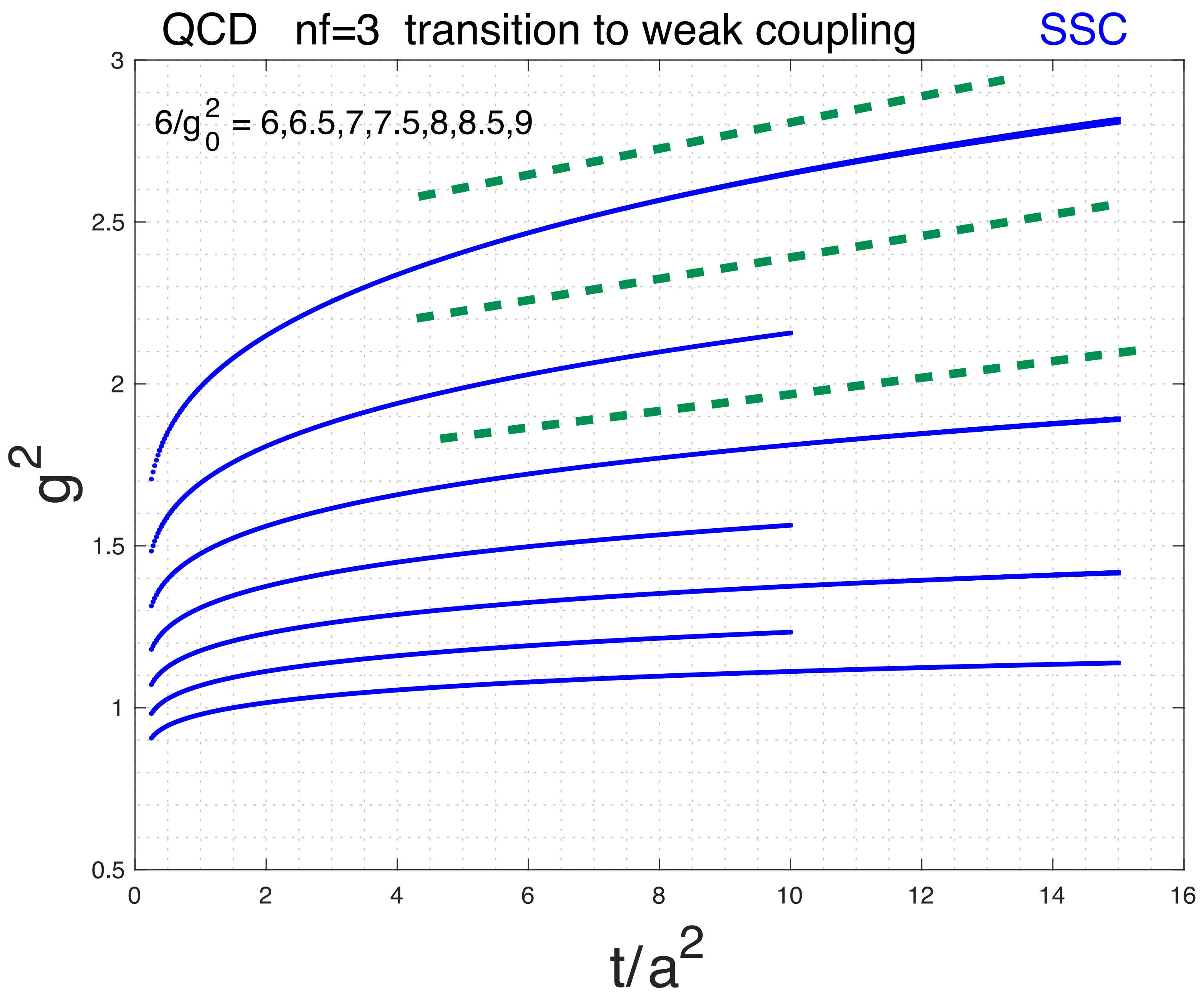
nf=3 data base (intermediate)



nf=3 data base (weak)



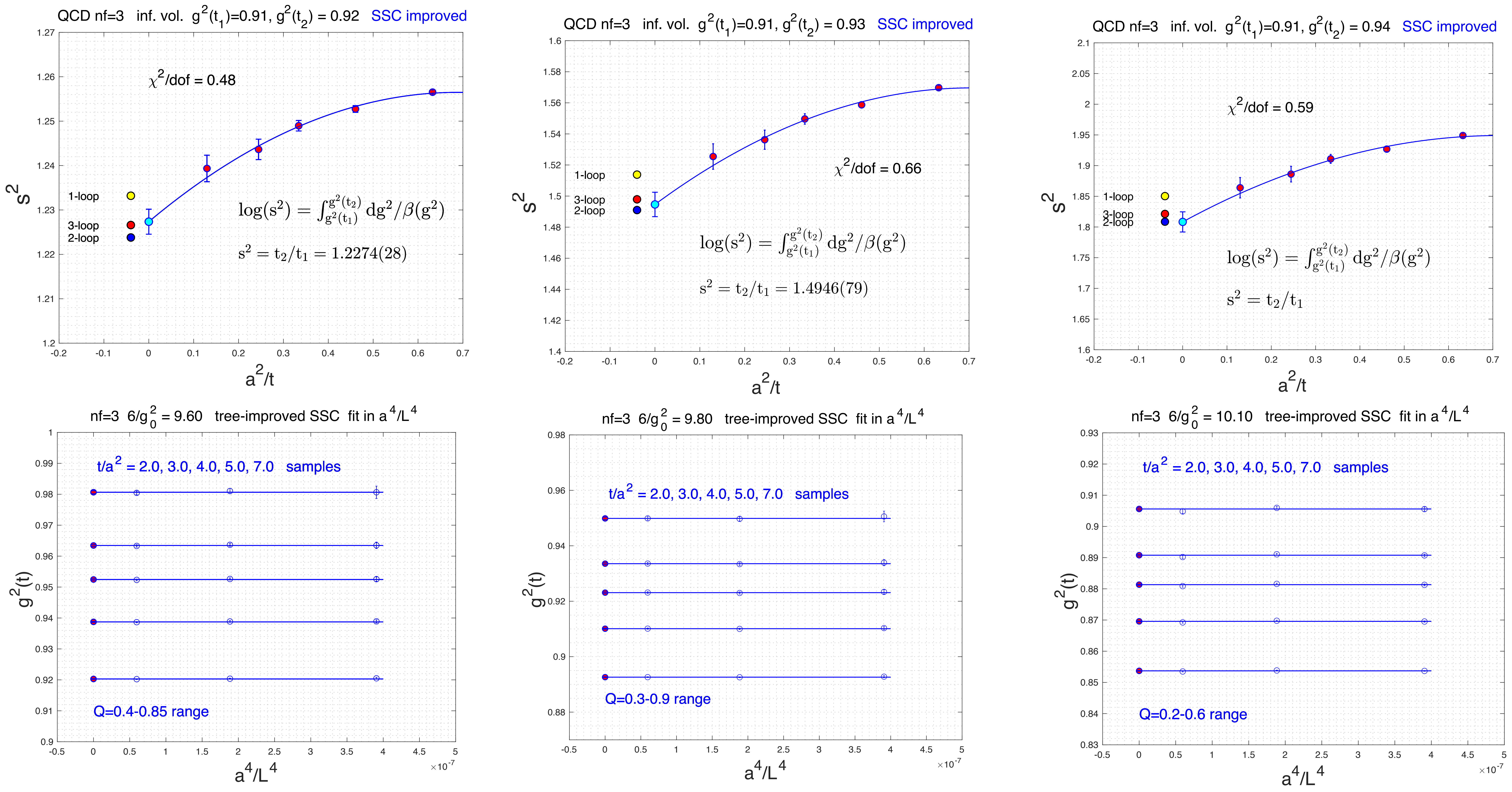




4x8 added ensembles

L=40,48,56,64

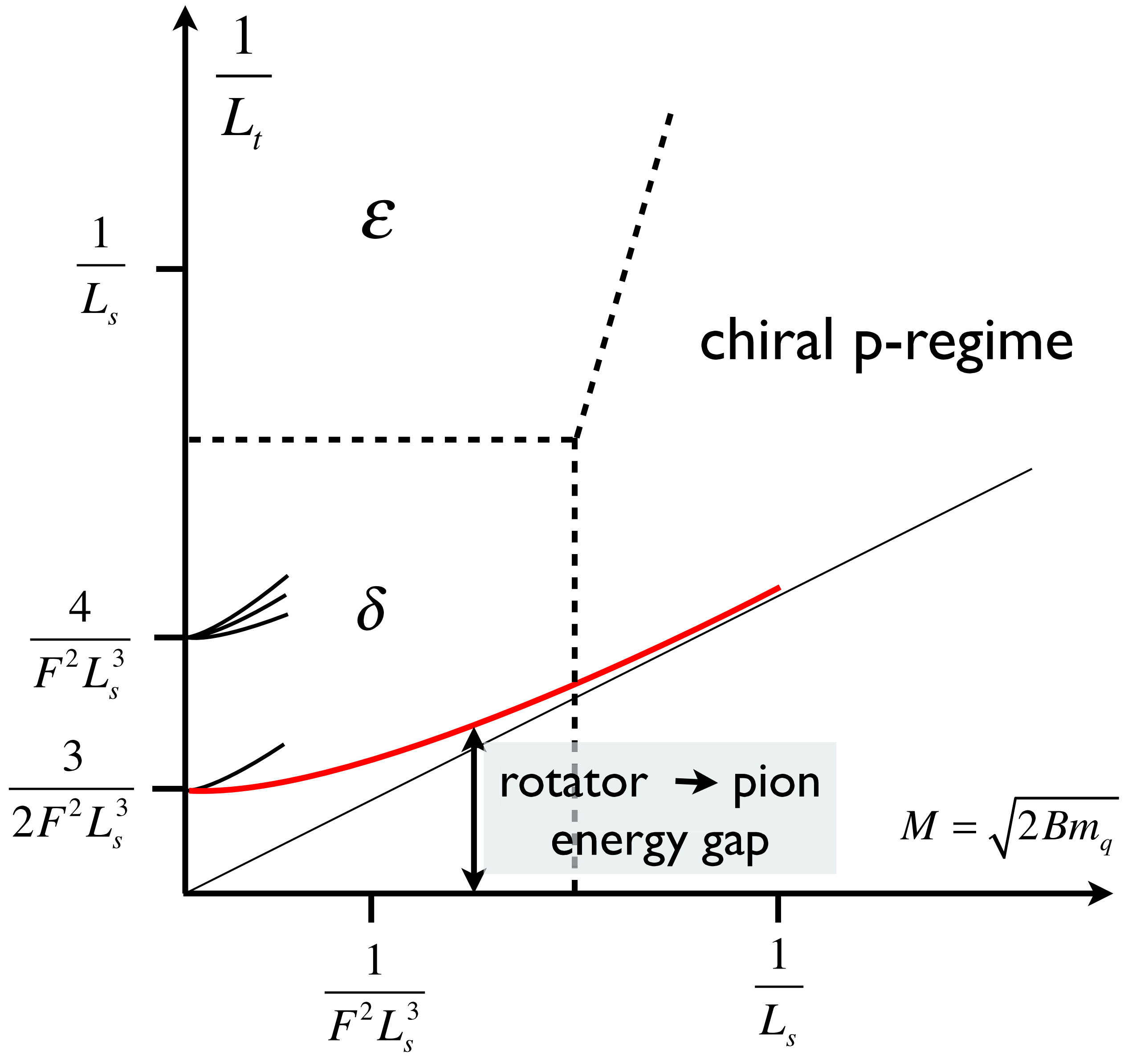
nf=3 infinite volume step beta-function at weak coupling

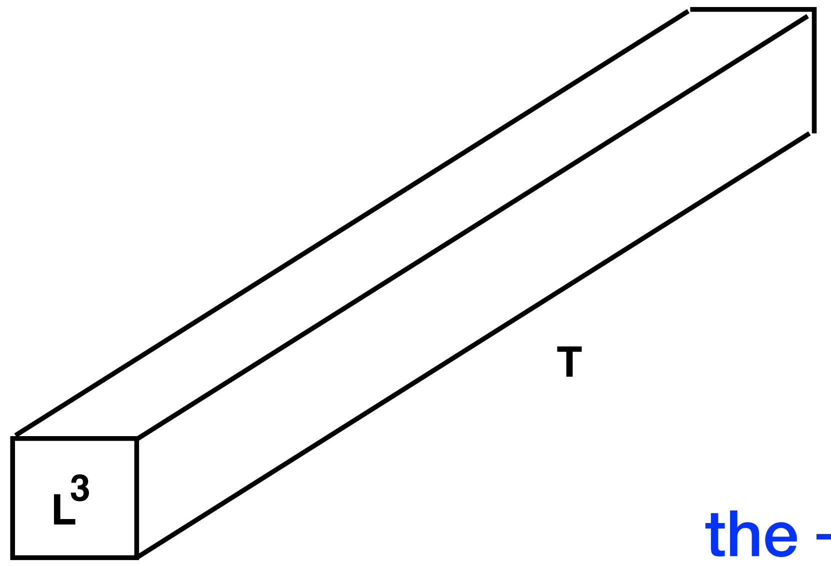


nf=3 infinite volume step beta-function at strong coupling ?

rotator geometry to reach t_0 scale and match to hadronic scale?

known results in rotator geometry





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Nearly conformal gauge theories in finite volume

Zoltan Fodor[a], Kieran Holland[b], Julius Kuti[c,*], Dániel Nógrádi[c], Chris Schroeder[c]

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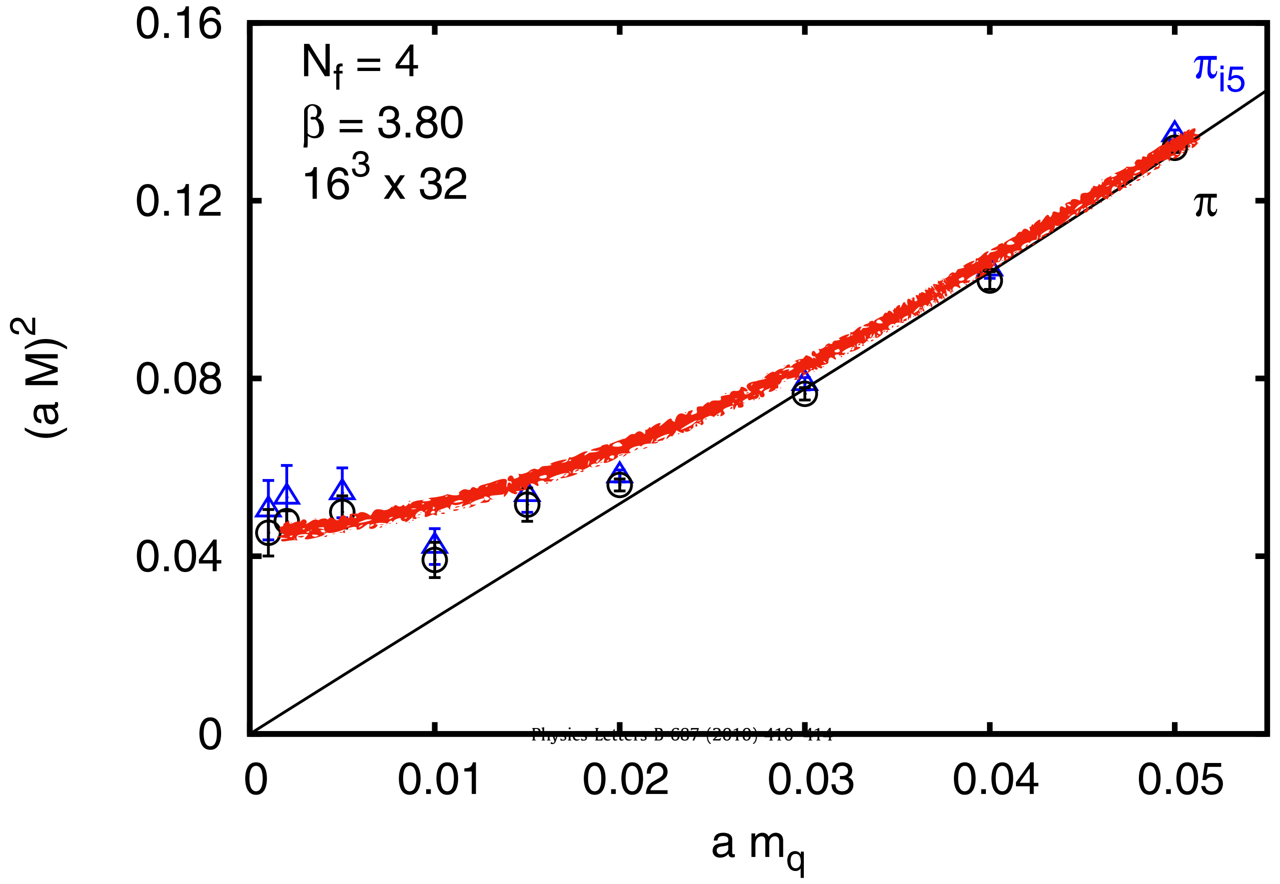
[b] Department of Physics, University of the Pacific, 3601 Pacific Ave, Stockton, CA 95211, USA
[c] Department of Physics 0319, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093, USA



massless pion approached from δ-regime

the $\frac{t_0}{F_\pi}$ ratio is set from matching the scale evolution from the beta-function to the rotor spectrum of the delta regime

Physics Letters B 687 (2010) 410–414

Pion in a box

QCDSF Collaboration

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[c] School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK
[d] NIC/DESY, 15738 Zeuthen, Germany
[e] Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
[f] Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany

massless pion approached from p-regime

$$E_l = \frac{l(l+2)}{2\Theta}, \qquad l = 0, 1, 2, \ldots,$$

Hasenfratz, Weingart

$$\Theta = F^2 L_s^3 \left[1 - \frac{2\bar{G}^*}{F^2 L_s^2} + \frac{1}{F^4 L_s^4} \left[0.088431628 + \partial_0\partial_0\bar{G}^* \frac{1}{3\pi^2} \left(\frac{1}{4}\log(\Lambda_1 L_s)^2 + \log(\Lambda_2 L_s)^2 \right) \right] \right]$$

$$\bar{G}^* = 0.2257849591\,, \qquad \partial_0\partial_0\bar{G}^* = 0.8375369106$$

$$\eta = F^2 L_s^3 M^2 \left[1 - \frac{3\bar{G}^*}{F^2 L_s^2} \right]$$

$$E_{L_s} = \frac{3}{2\Theta} \left[1 + \frac{(\Theta\eta)^2}{15} - \frac{193}{120}\frac{(\Theta\eta)^4}{15^2} \right]$$

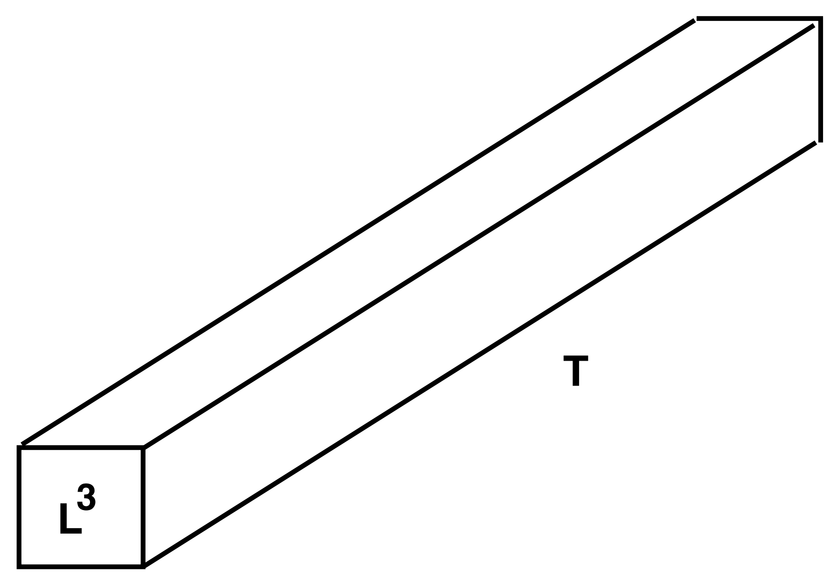
known results in rotator geometry

$$E_L = -(N^2-1)\gamma_0/2L + N(N^2-1)/8F^2L^3.$$ vacuum (Casimir) energy Leutwyler N flavors

$$(-\Delta - r^2 \,\mathrm{Re\,tr}\, U)\psi_n = e_n(r)\psi_n,$$
$$E_n = E_L + (1/2F^2L^3)e_n(F^2L^3M)$$

$$-2(N^2-1)/N.$$ $2N^2$ degeneracy rotator eigenvalues

$$(N^2-1)/NF^2L^3$$ gap



Hasenfratz, Weingart

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Ferenc Niedermayer*, Christoph Weiermann

$$\begin{aligned}\Theta = L_s^3F^2\Bigg[1 &+ \frac{1}{F^2L_s^2}0.225784959441(n-2)\\ &+ \frac{1}{F^4L_s^4}(-0.0692984943 + 0.0101978424n)\\ &- \frac{1}{F^4L_s^4}0.007071685925[(3n-10)\log M_2L_s + 2n\log M_3L_s]\\ &- \frac{g_4^{(4)}}{F^4L_s^4}[-0.55835794046(n+1)]\\ &- \frac{g_4^{(5)}}{F^4L_s^4}[0.55771822866 - 1.11639602502n] + \mathcal{O}\left(\frac{1}{F^6L_s^6}\right)\Bigg].\end{aligned}$$

some concluding remarks:

YM theory is under reasonable control at weak and strong coupling
(undiscovered hidden problems?)

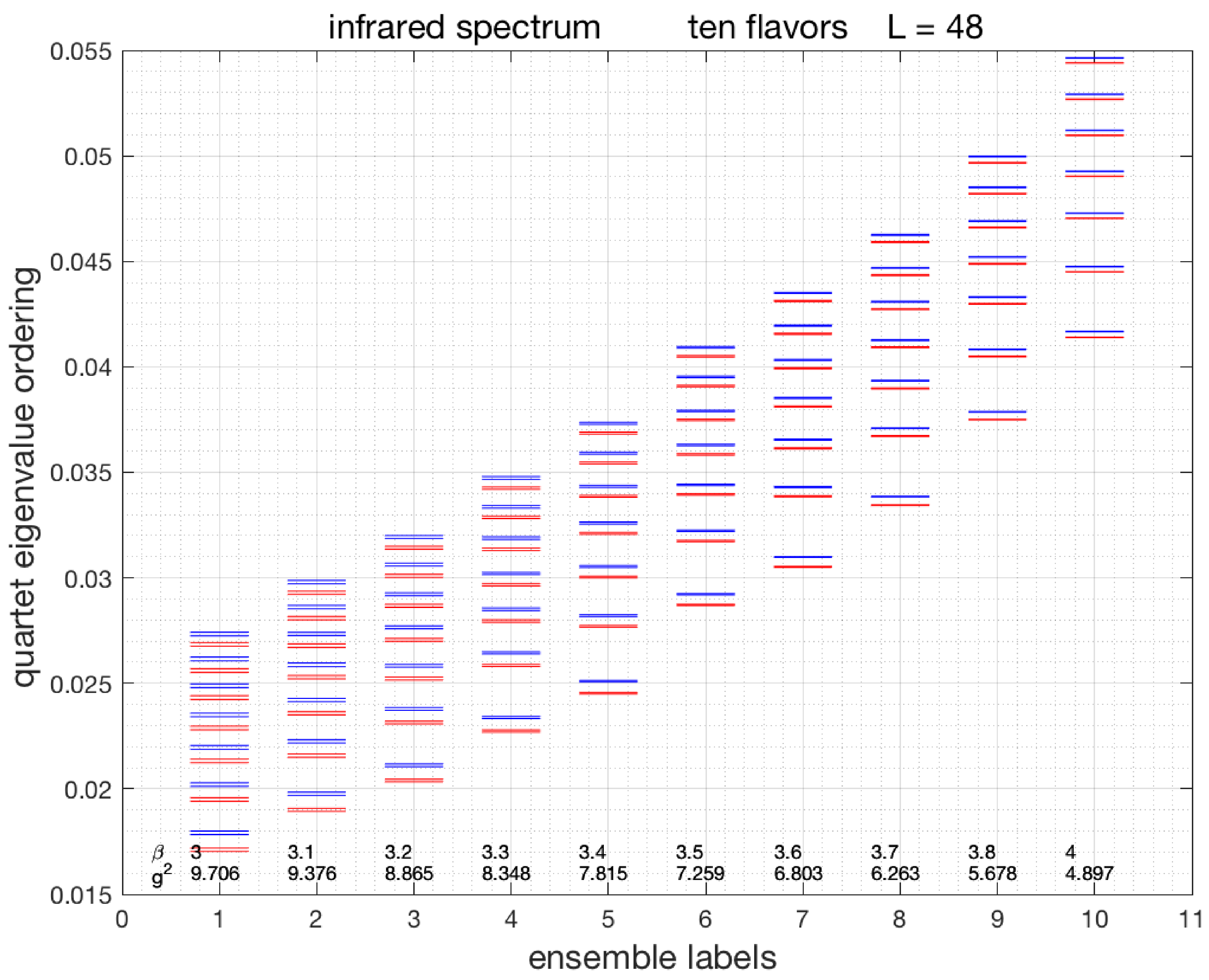
new analysis of YM based on the extended data set in the works

QCD with three massless fermions   control at weak coupling

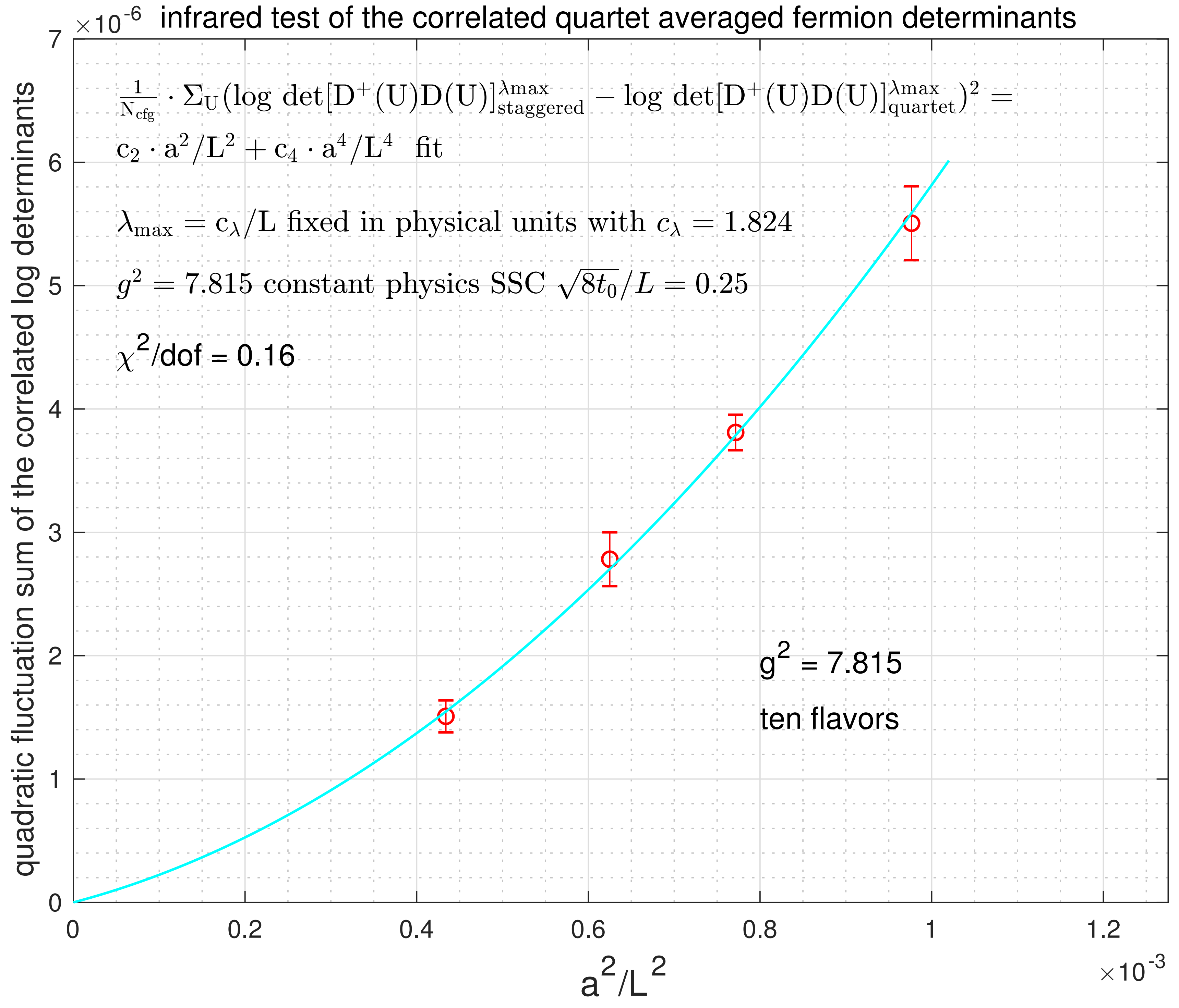
harder at strong coupling   (is frozen topology the only ~ $1/V$ effect on hypercube?)

balanced use of hypercubic and rotator geometry?

backup slides



infrared spectrum ten flavors L = 48
quartet eigenvalue ordering
ensemble labels
β 3 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 4
g² 9.706 9.376 8.865 8.348 7.815 7.259 6.803 6.263 5.678 4.897



infrared test of the correlated quartet averaged fermion determinants
$\frac{1}{N_{cfg}} \cdot \Sigma_U (\log \det[D^+(U)D(U)]^{\lambda max}_{staggered} - \log \det[D^+(U)D(U)]^{\lambda max}_{quartet})^2 =$
$c_2 \cdot a^2/L^2 + c_4 \cdot a^4/L^4$ fit
$\lambda_{max} = c_\lambda/L$ fixed in physical units with $c_\lambda = 1.824$
$g^2 = 7.815$ constant physics SSC $\sqrt{8t_0}/L = 0.25$
χ^2/dof = 0.16
g^2 = 7.815
ten flavors
quadratic fluctuation sum of the correlated log determinants
a^2/L^2