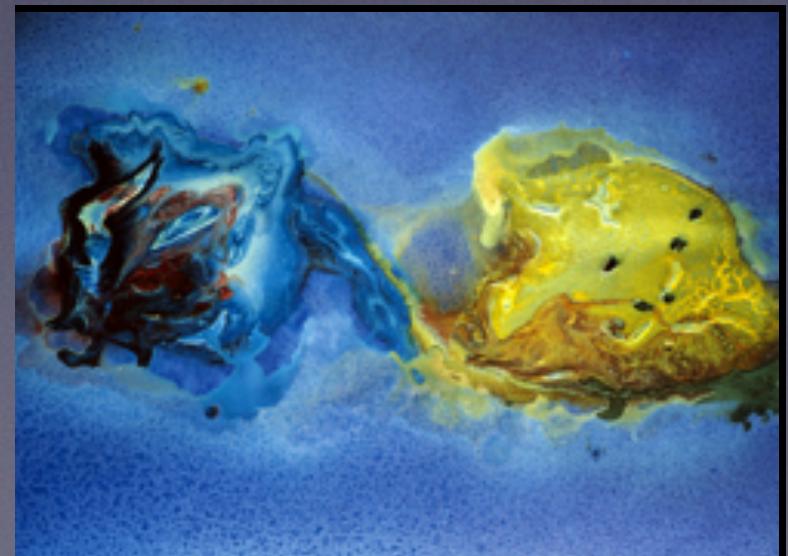


# GRADIENT FLOW and NONRELATIVISTIC EFFECTIVE FIELD THEORIES



NORA BRAMBILLA

## Plan of the talk

Gradient flow may play a crucial role to increase the predictivity of some of the contemporary Effective Field Theories (EFTs)

Multiscale systems represent an opportunity for strong interactions but a challenge for a Quantum Field Theory description: the EFT description has several advantages

Focusing on system made by two heavy quarks: their physics reach and the need for Nonrelativistic EFTs (NREFTs)

NREFTs have (or are) revolutionised(ing) our understanding of a number of problems at the frontier of particle and nuclear physics from the Exotics X Y Z states to the QCD phase diagram

Further progress depends however on our ability to calculate low energy purely glue dependent correlators nonlocal in time or in time and in space

Gradient flow may be a best tool for these calculations

General features of this gradient flow application to NREFTs and opportunities and problems considering some concrete cases

Outlook

**This talk is based on a bulk of works in NREFTs: some references are given in these slides after the outlook**

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**For more details on specific work on the gradient flow in NREFTs see at this meeting:**

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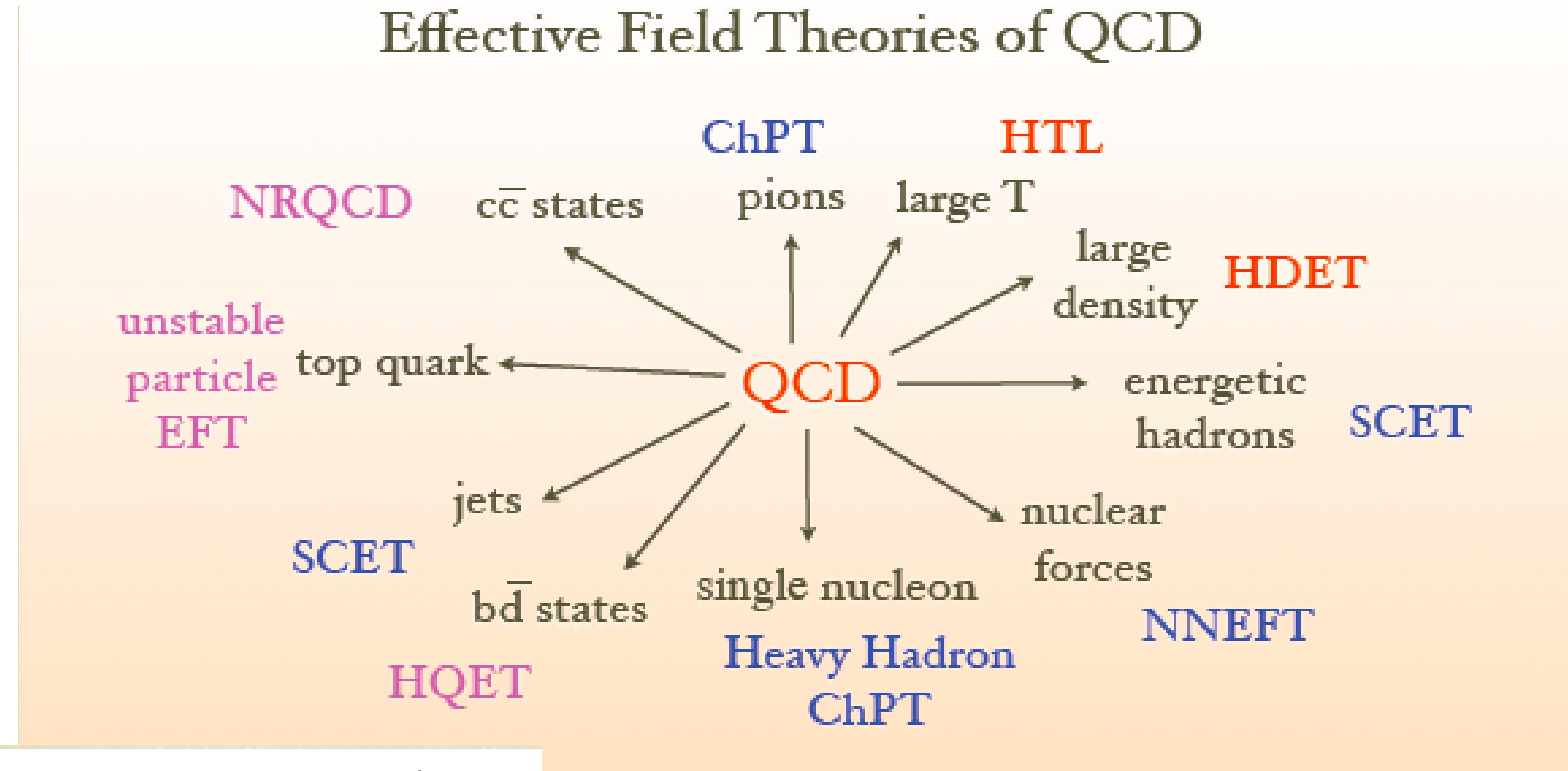
**Talk of V. Leino on gradient flow calculation of the heavy quark momentum diffusion coefficient**

**Talk of J. Mayer-Steudte on gradient flow calculation of the QCD force**

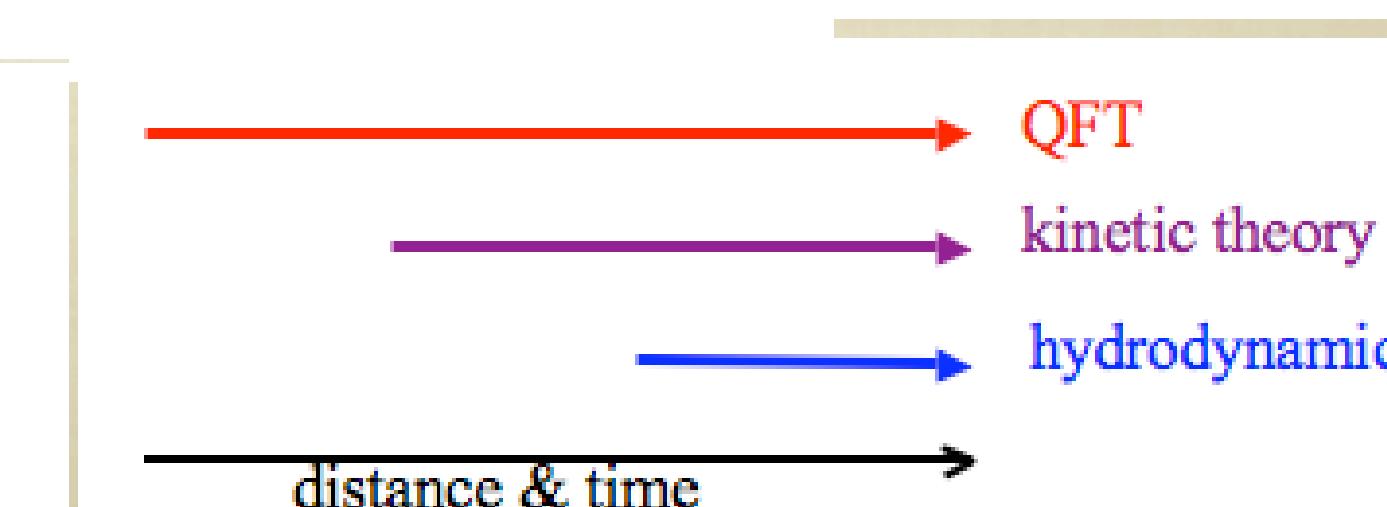
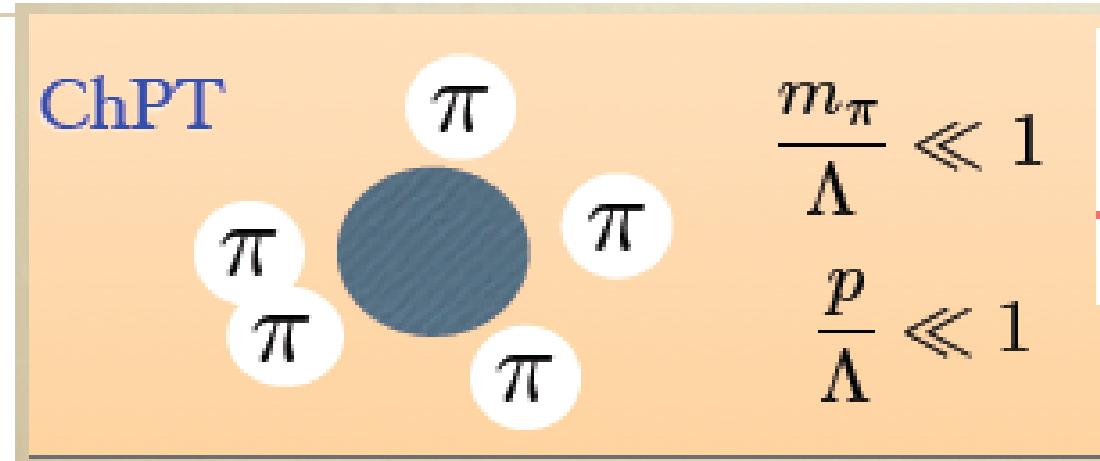
**Talk of M. Eichberg on gradient flow calculation of the QCD potentials (with relativistic corrections)**

**Talk of X.P. Wang on gradient flow calculation of the QCD force in perturbation theory**

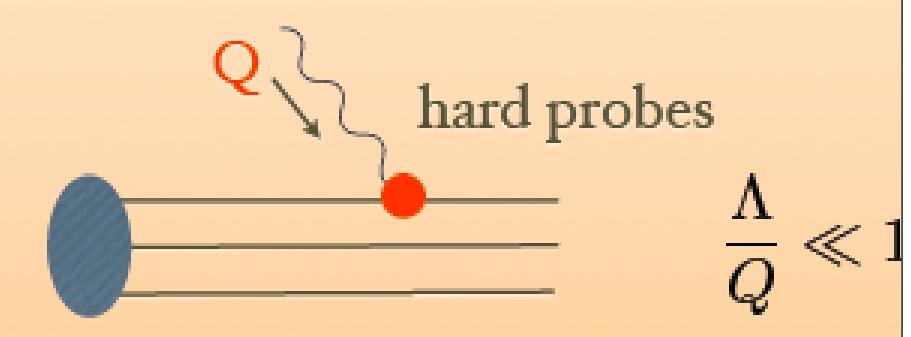
To address the research frontier of strong interactions several EFTs have been constructed: they address multiscale systems



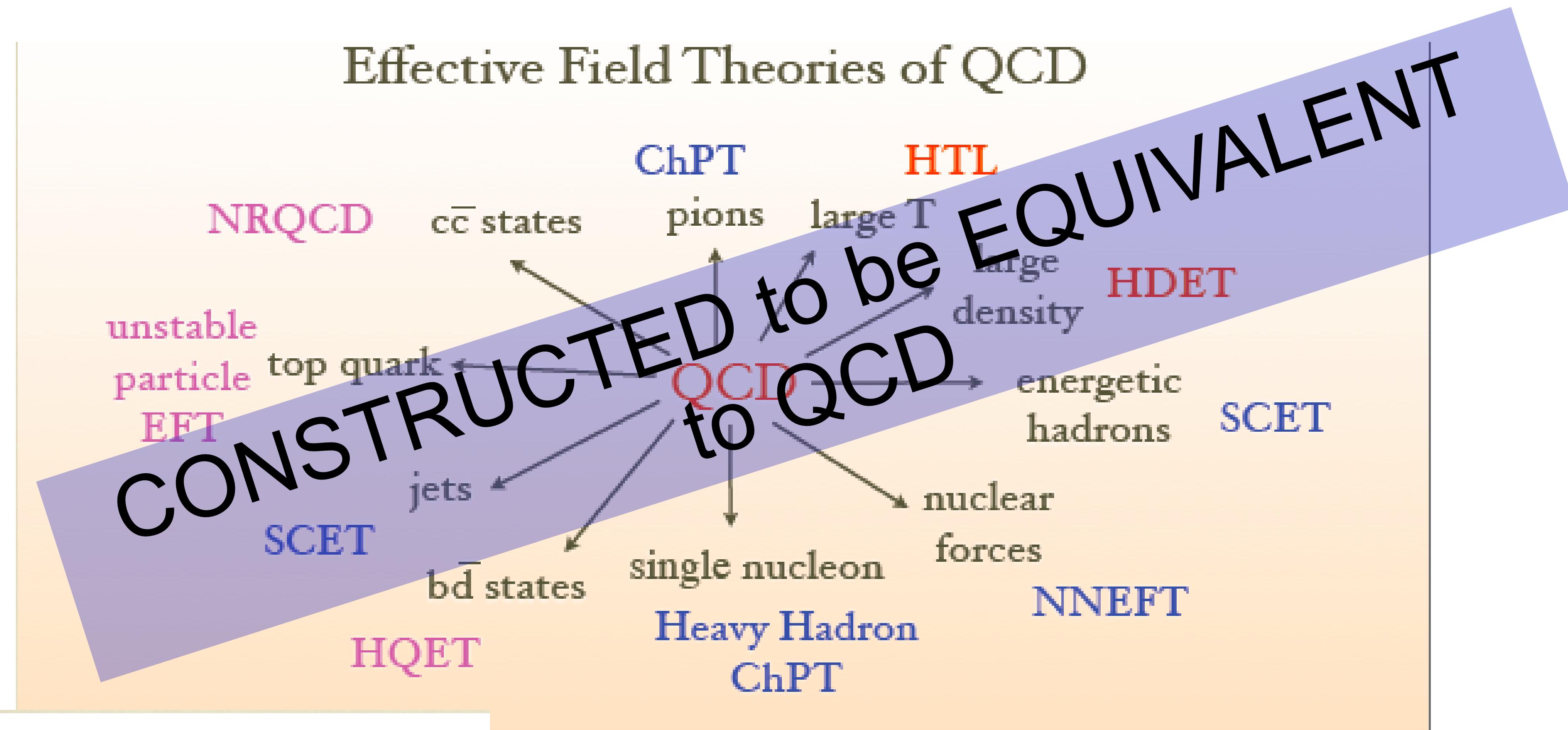
- Heavy quark effective theory (HQET):  $\frac{\lambda}{\Lambda} = \frac{\Lambda_{\text{QCD}}}{m}$



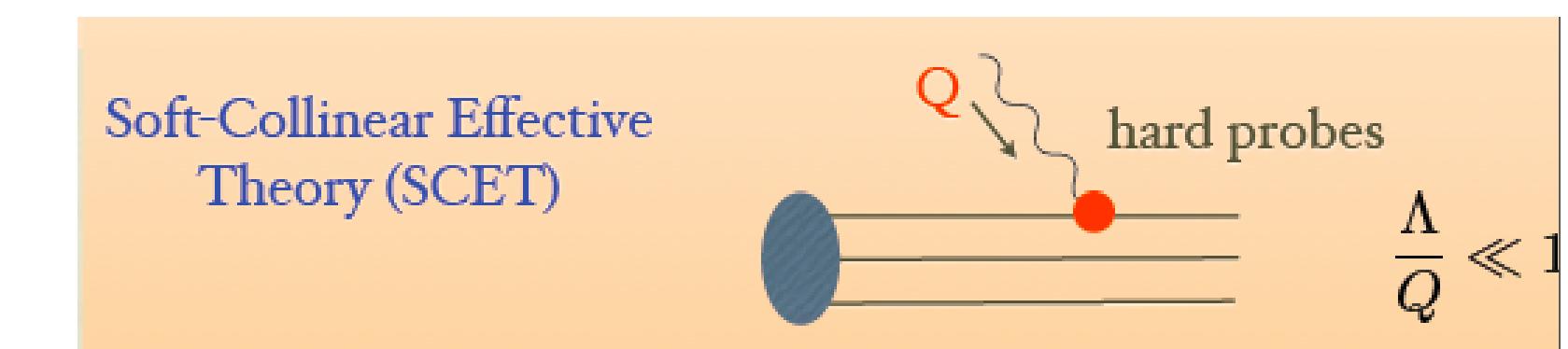
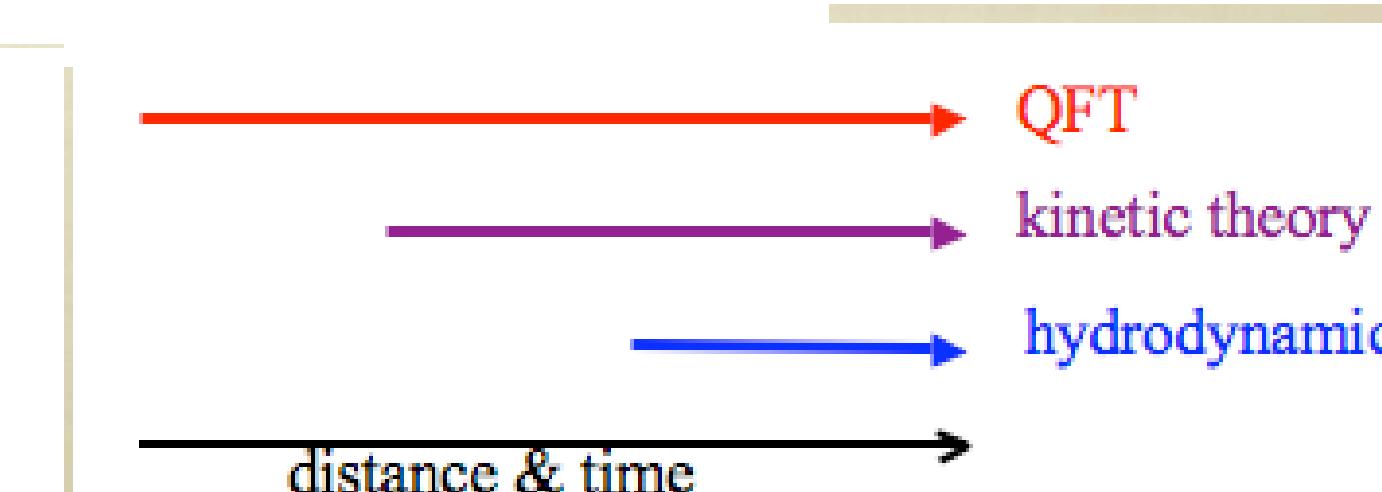
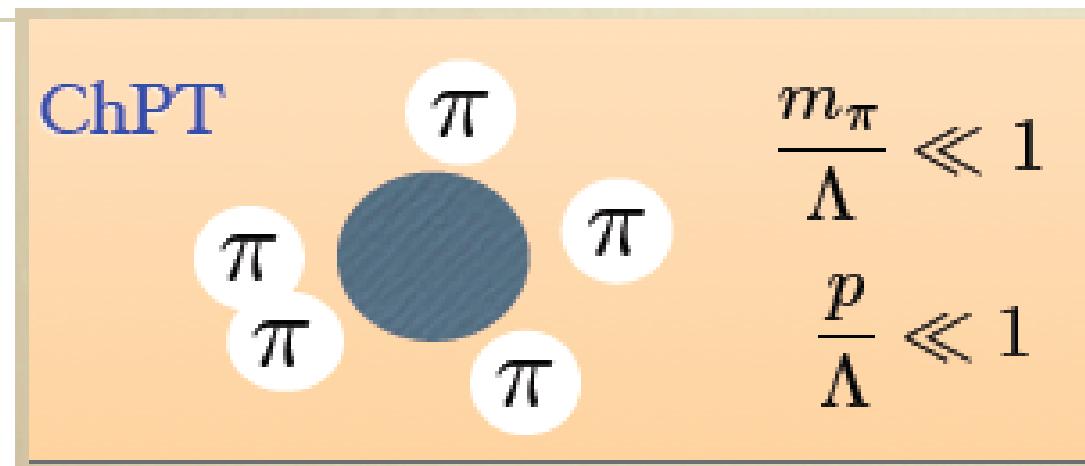
Soft-Collinear Effective Theory (SCET)



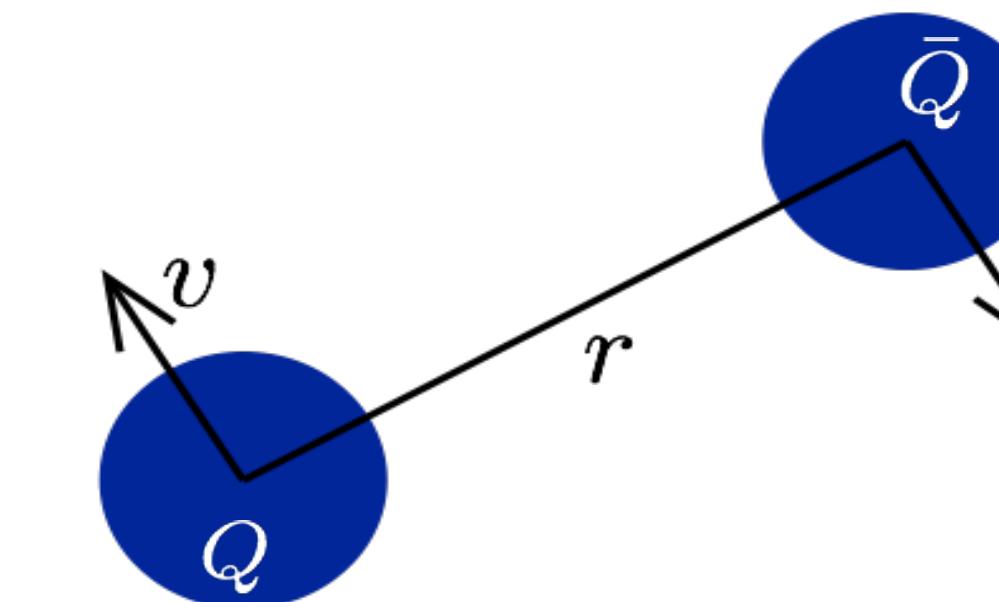
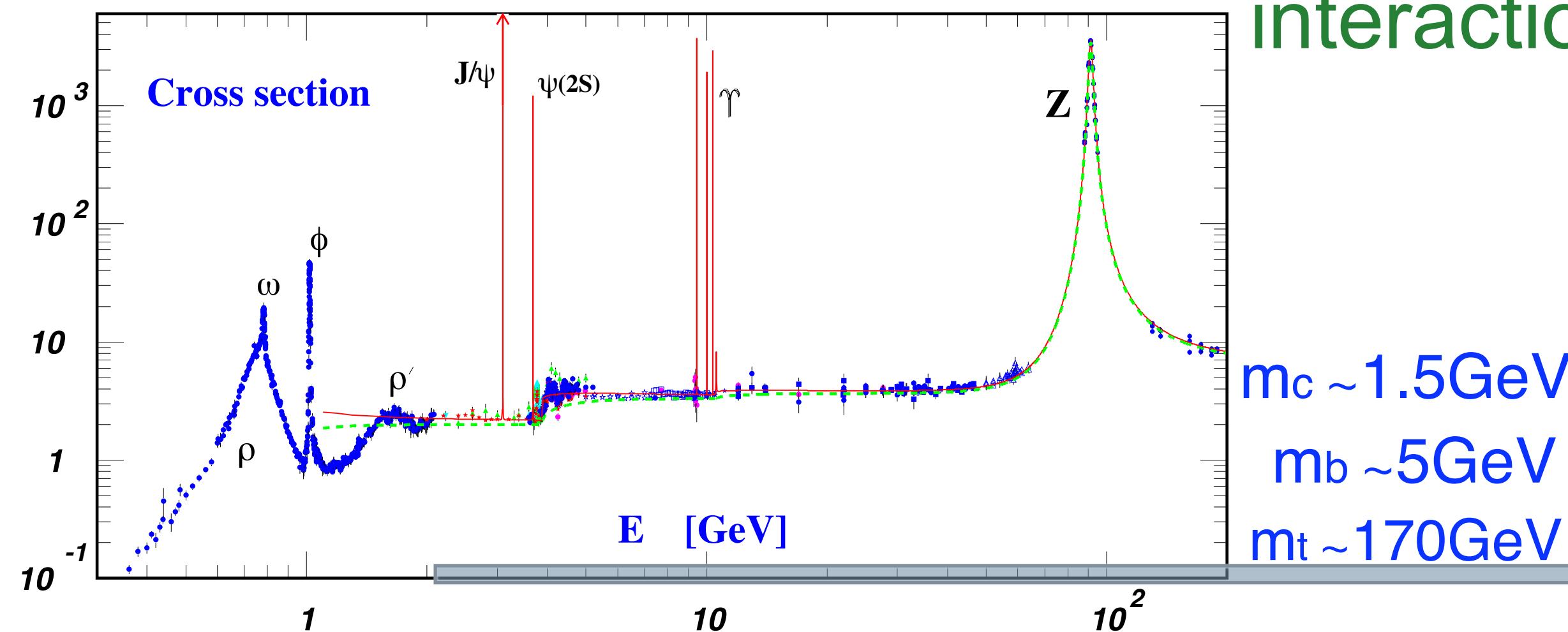
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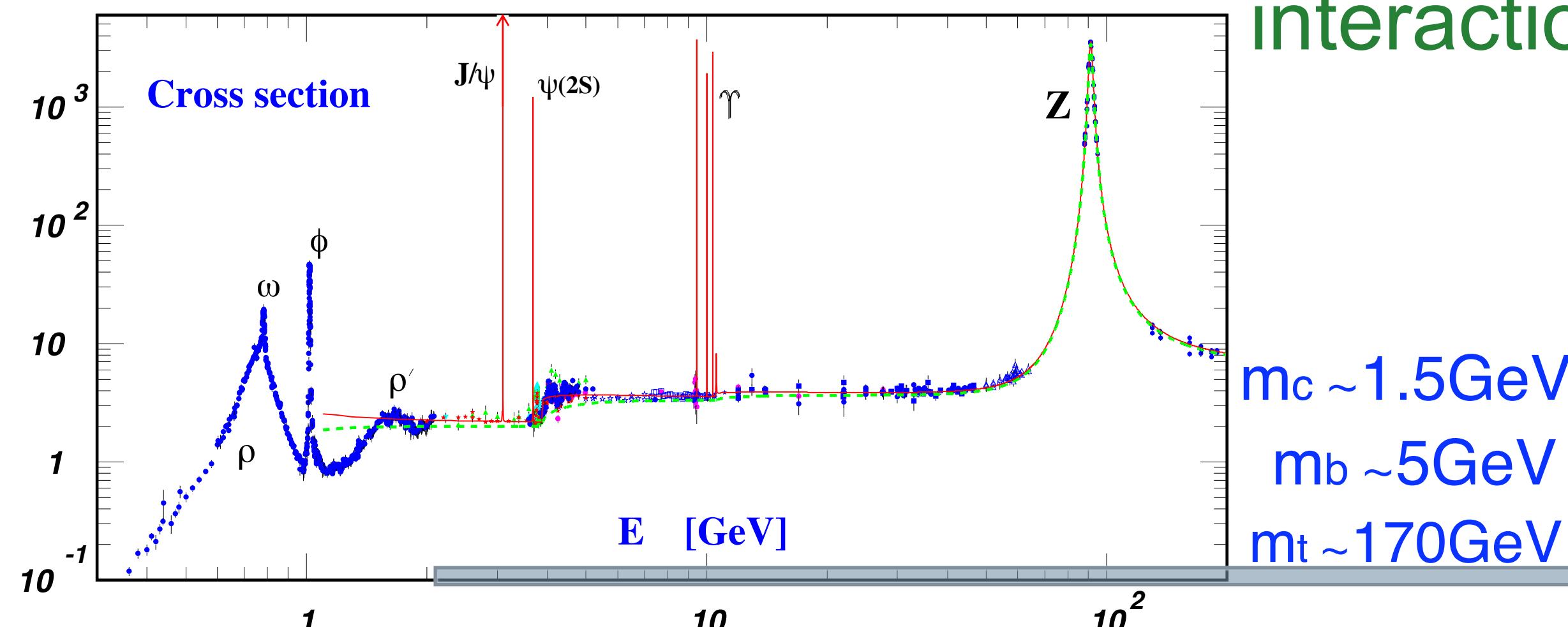


# NR bound states formed by heavy quarks offer a privileged access to strong interactions



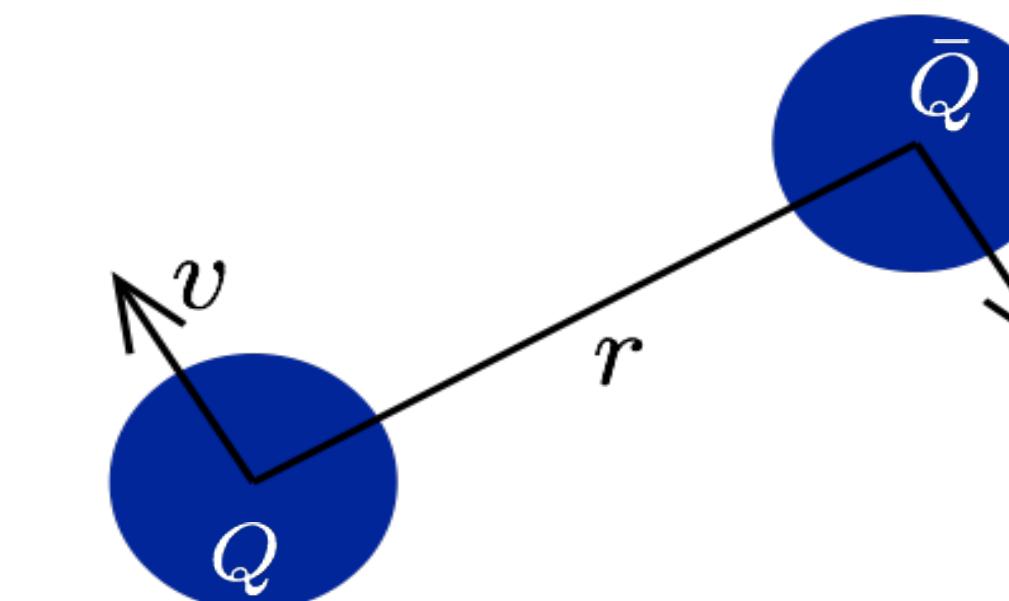
with  $Q, \bar{Q} = c, b, t$

# NR bound states formed by heavy quarks offer a privileged access to strong interactions



$$\begin{aligned} m_c &\sim 1.5 \text{ GeV} \\ m_b &\sim 5 \text{ GeV} \\ m_t &\sim 170 \text{ GeV} \end{aligned}$$

A large scale

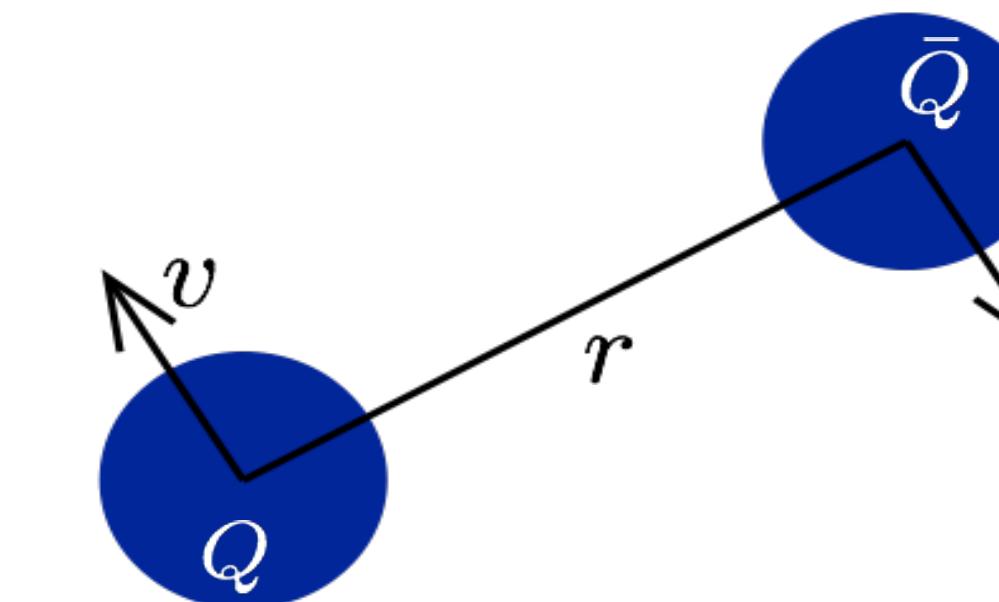
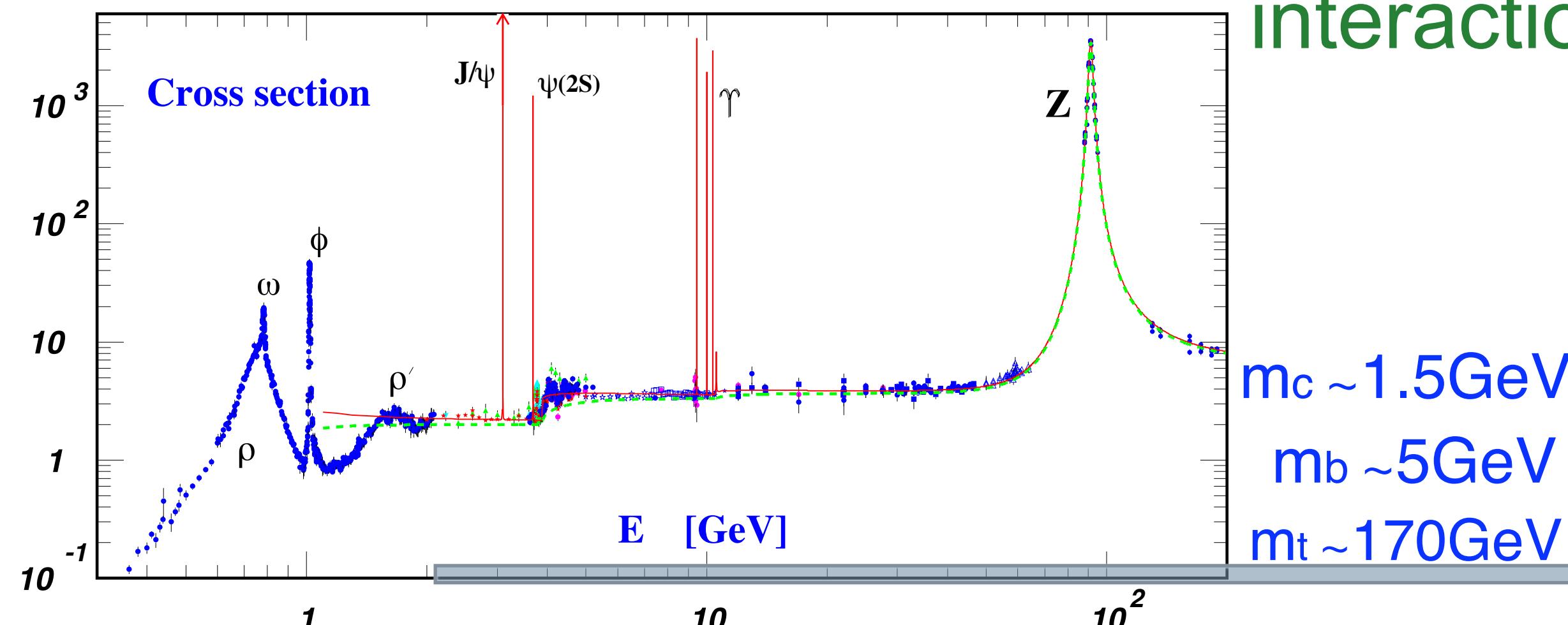


$$m_Q \gg \Lambda_{\text{QCD}}$$

with  $Q, \bar{Q} = c, b, t$

$$\alpha_s(m_Q) \ll 1$$

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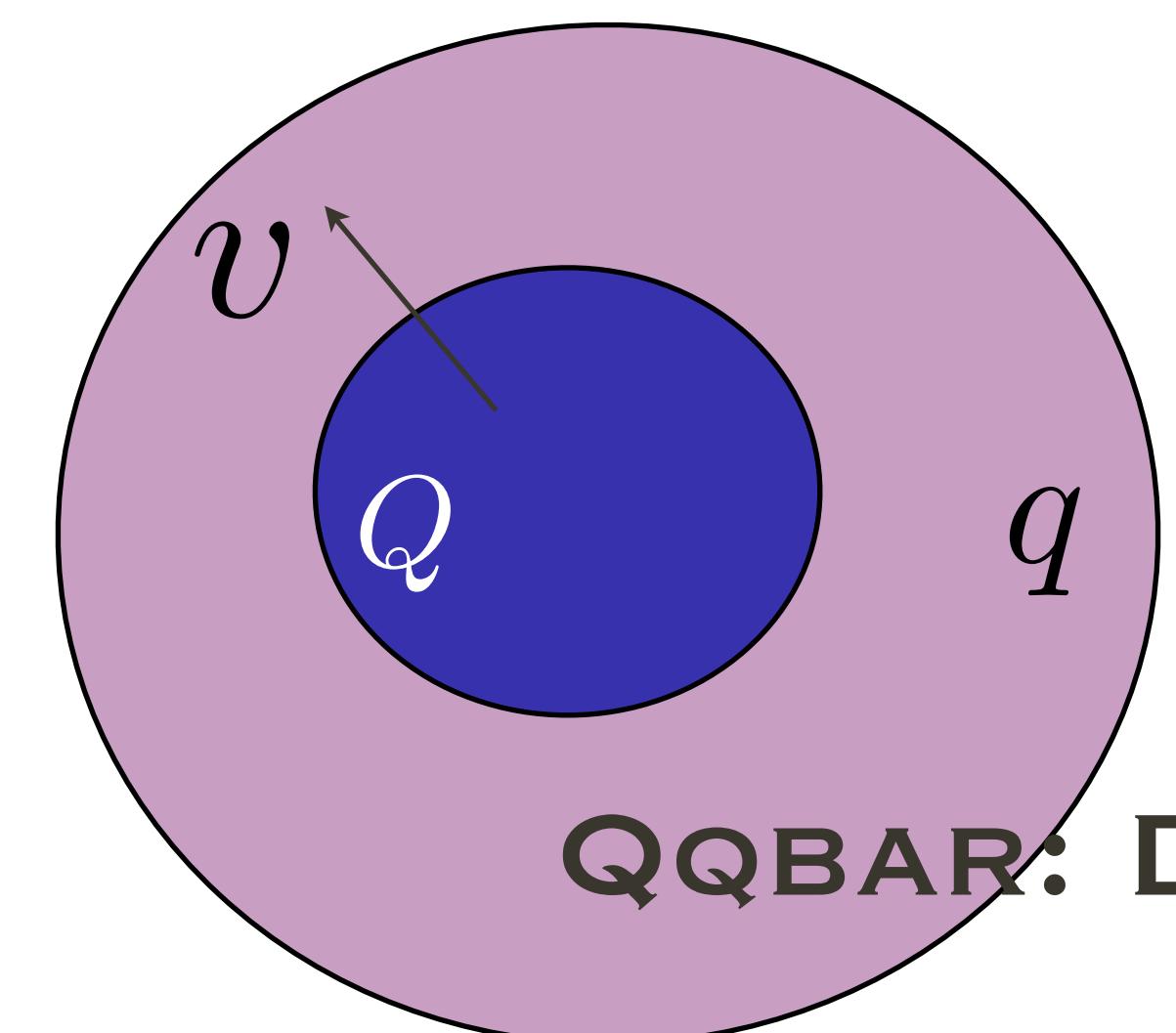
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Heavy quarkonium is very different from heavy-light hadrons

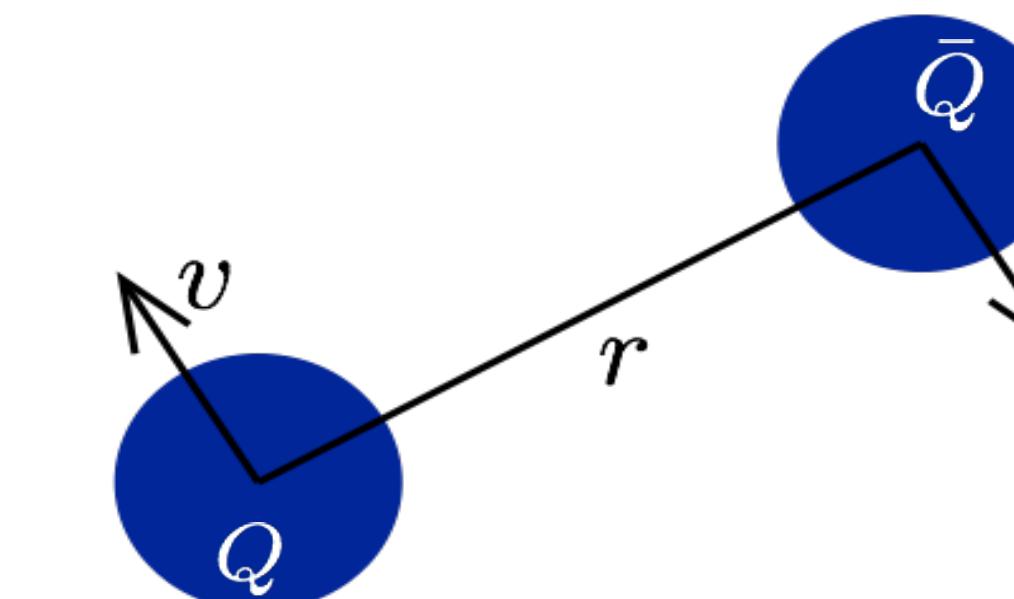
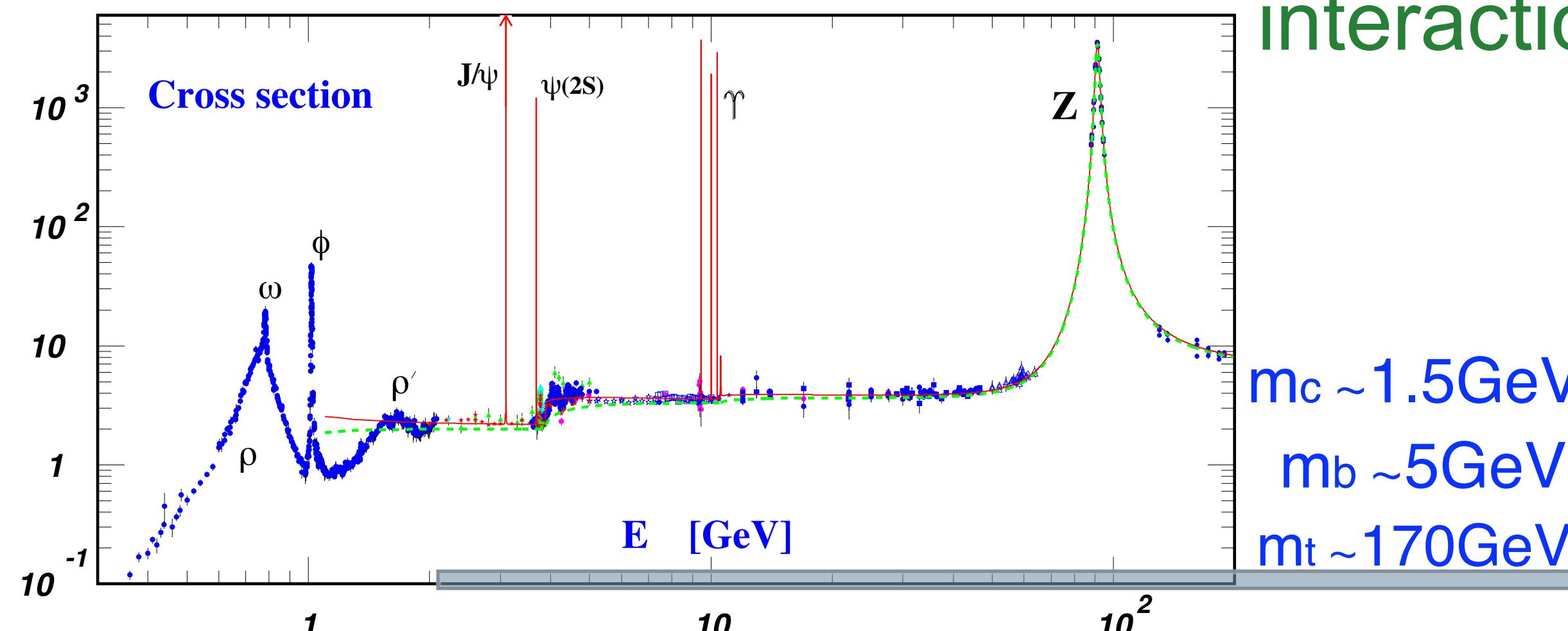


different physics from the  
heavy light meson where only  
two scales exist  $m$  and  $\Lambda_{\text{QCD}}$

for heavy-light the  
EFT is HQET  
(Heavy Quark Effective  
Theory)

QQBAR: D, B MESONS

NR bound states formed by heavy quarks offer a privileged access to strong interactions



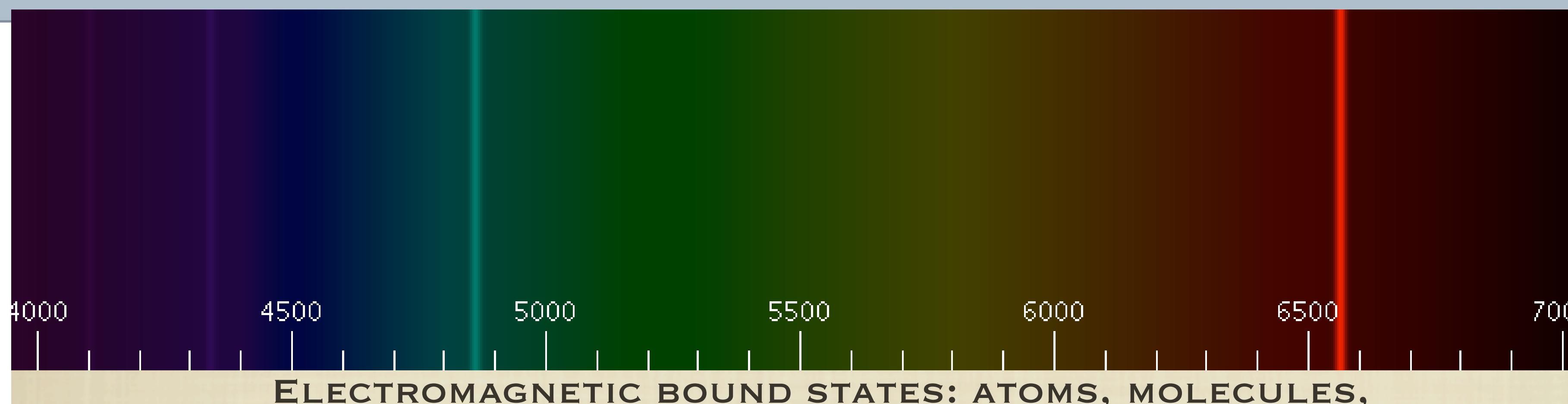
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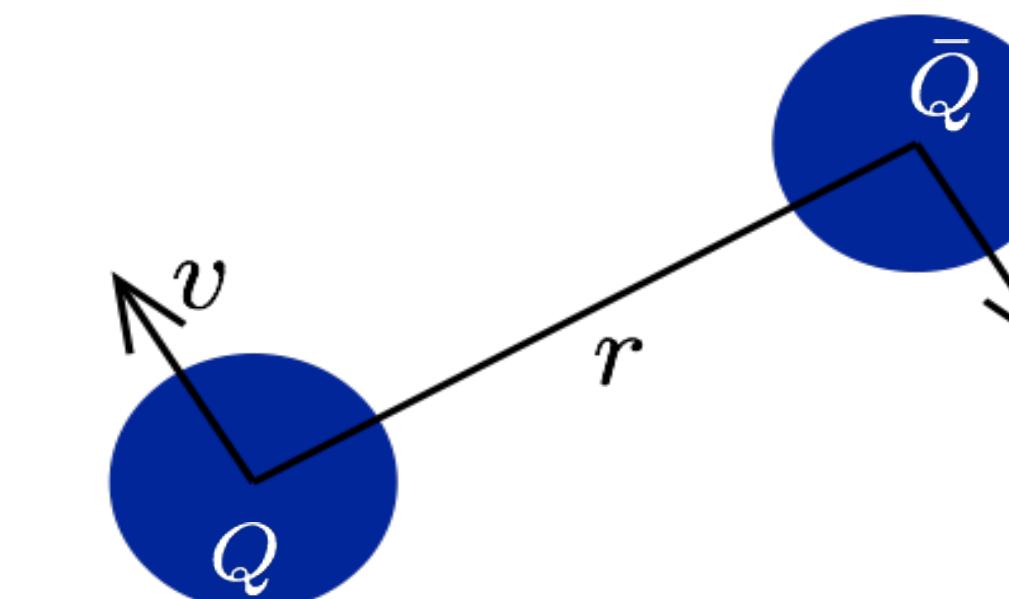
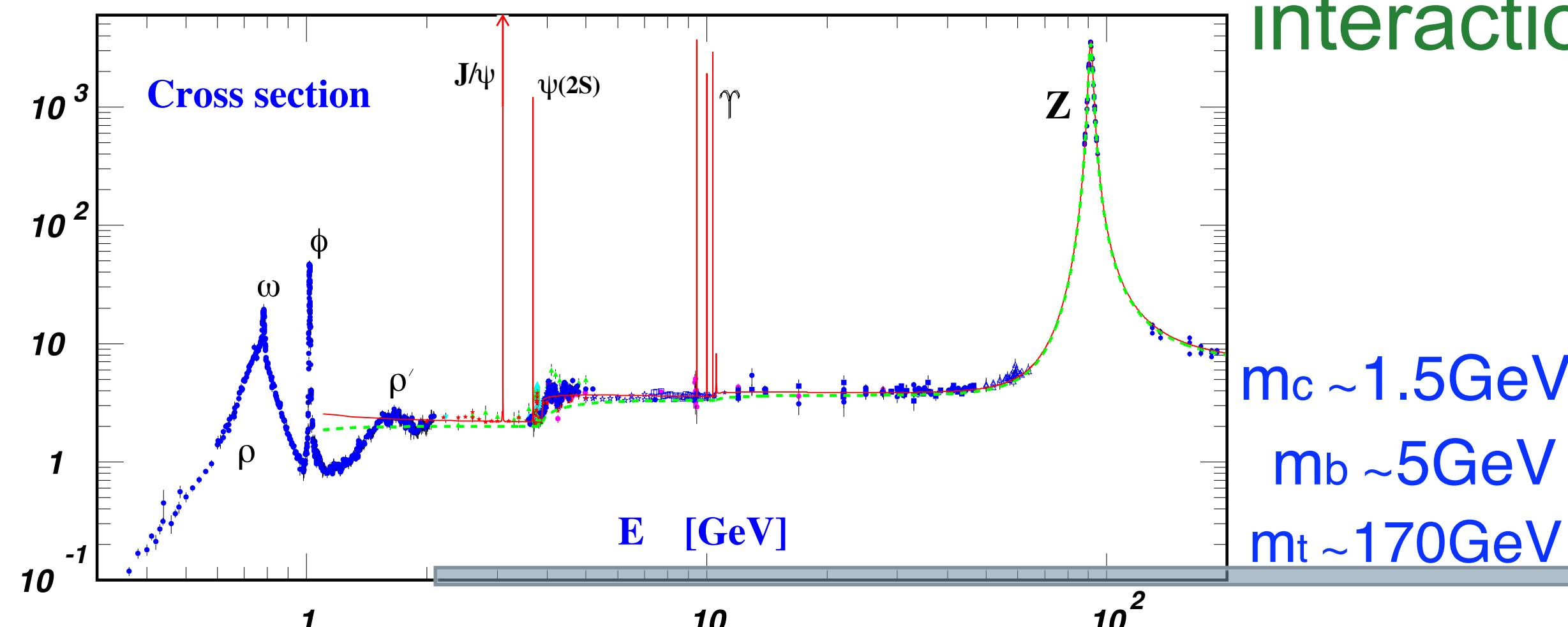
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Heavy quarkonia are nonrelativistic bound systems:  
multiscale systems



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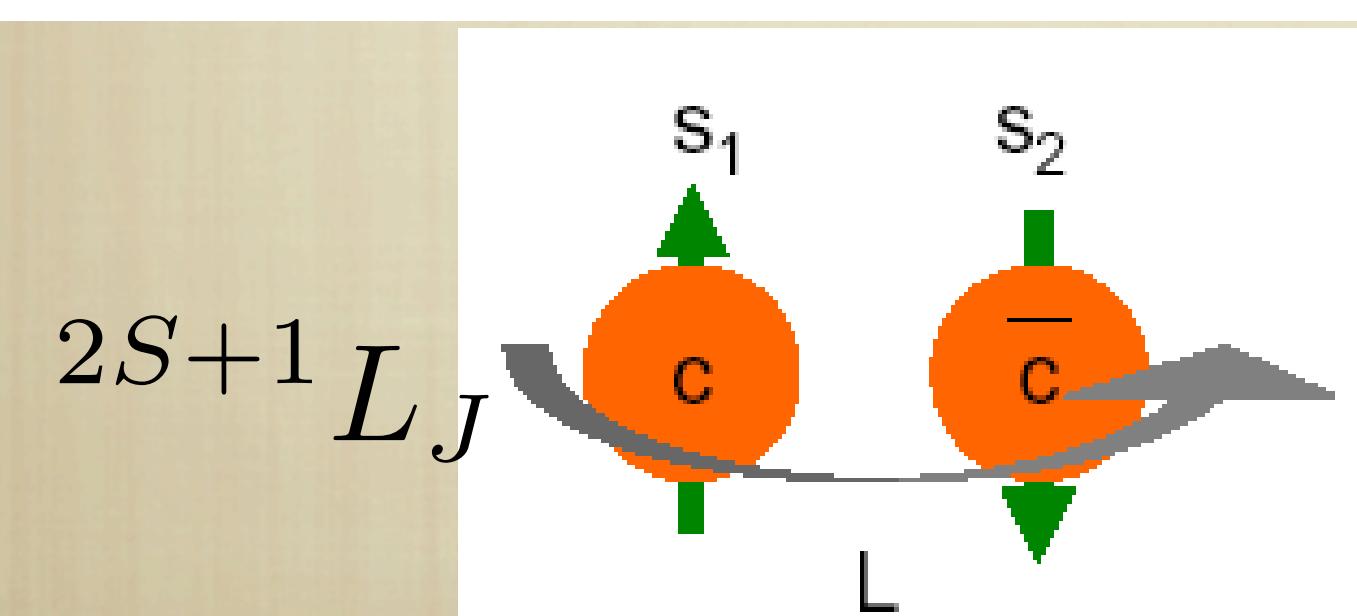
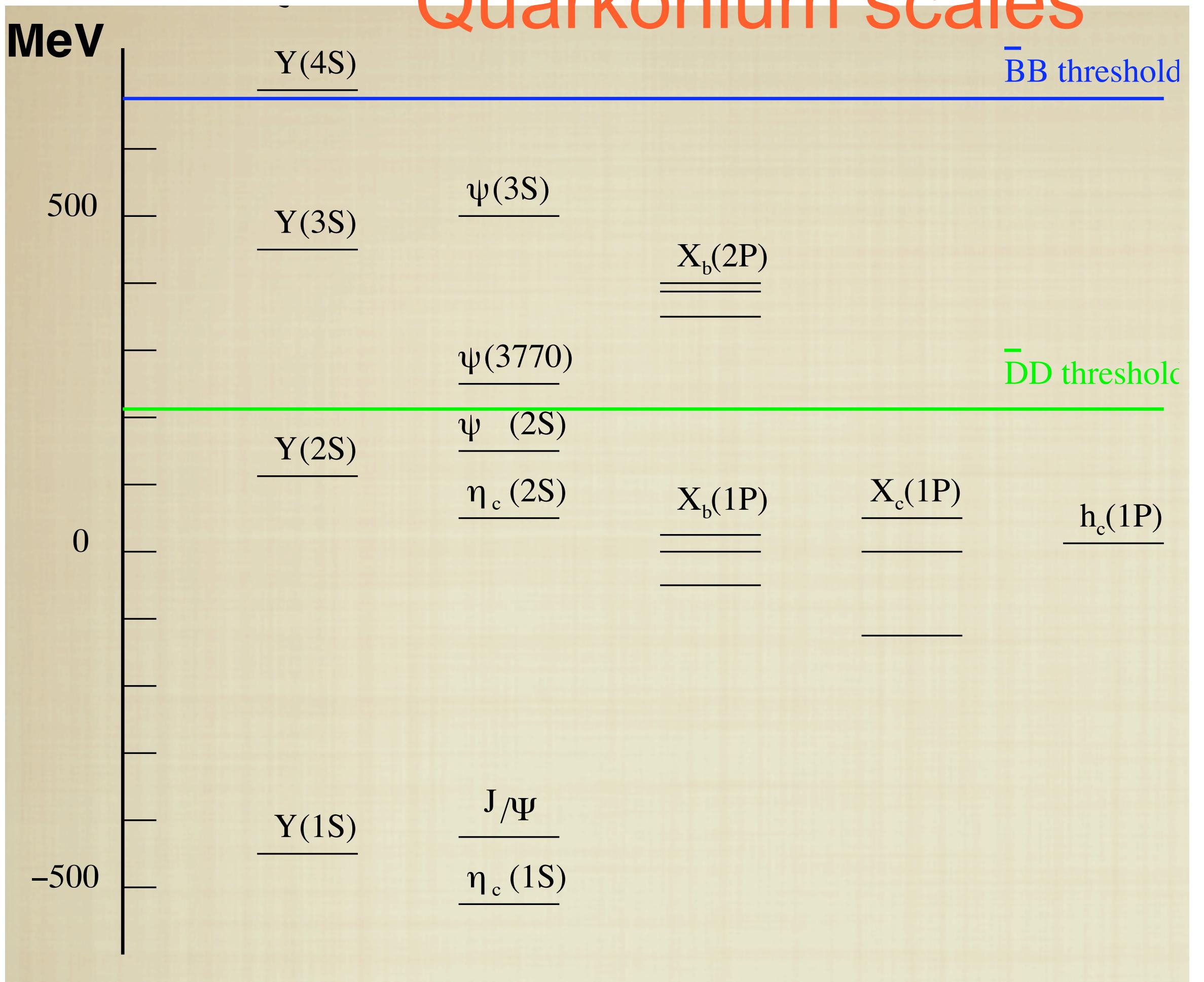
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many scales: a challenge and an opportunity



# Quarkonium scales



THE MASS SCALE IS PERTURBATIVE

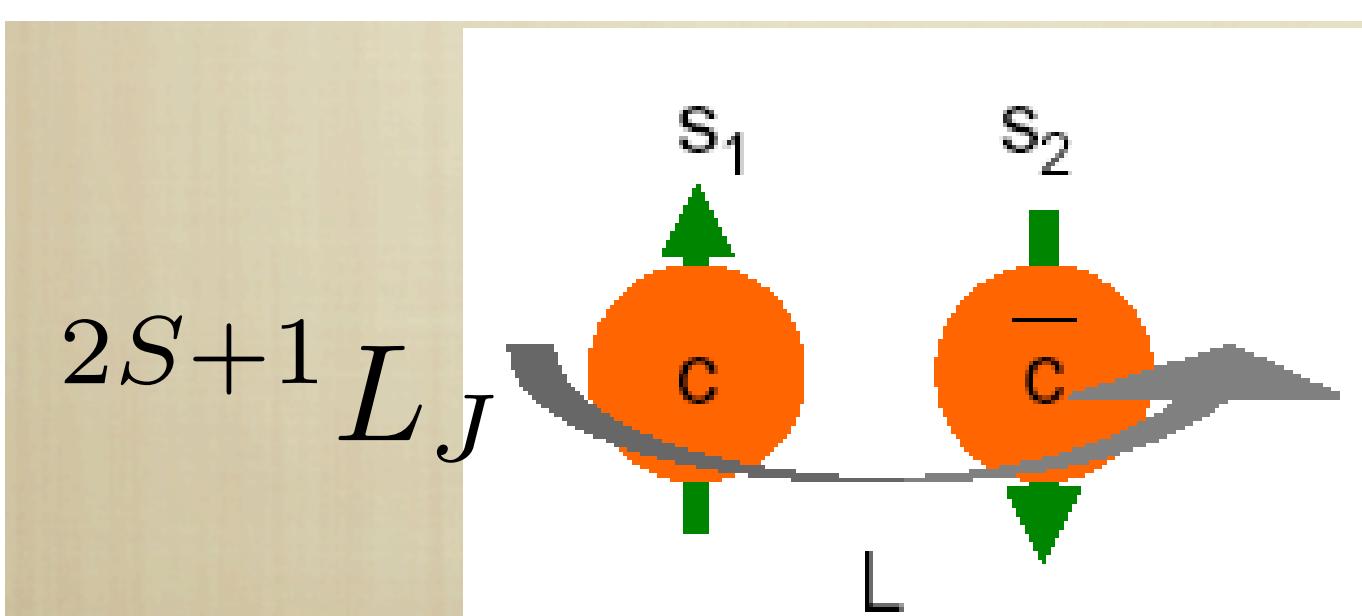
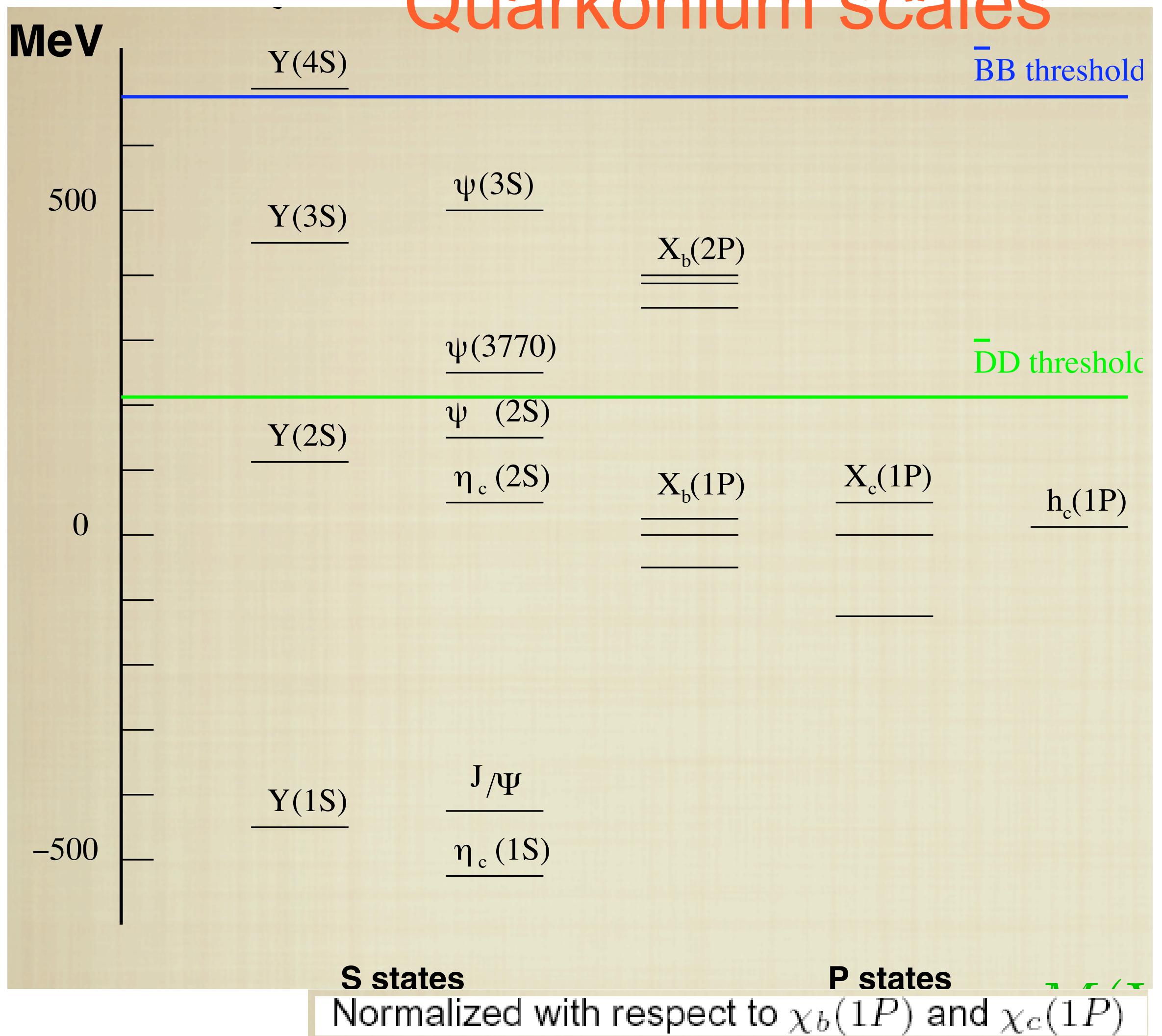
$$m_Q \gg \Lambda_{\text{QCD}}$$

$$m_b \simeq 5 \text{ GeV}; m_c \simeq 1.5 \text{ GeV}$$

$$M(Y(1S)) = 9460 \text{ GeV}$$

$$M(J/\Psi) = 3097 \text{ GeV}$$

# Quarkonium scales



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THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs}E \sim mv^4$$

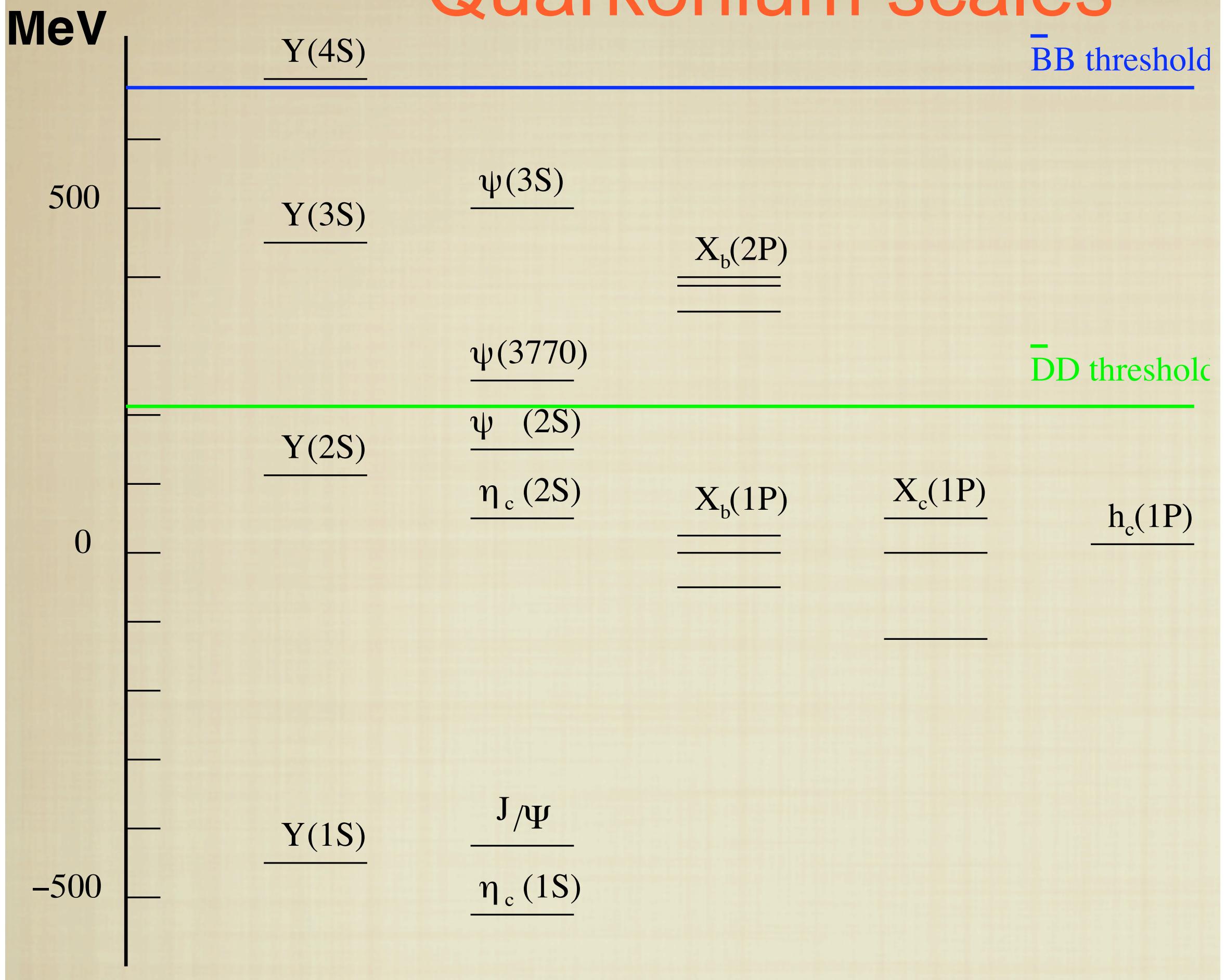
$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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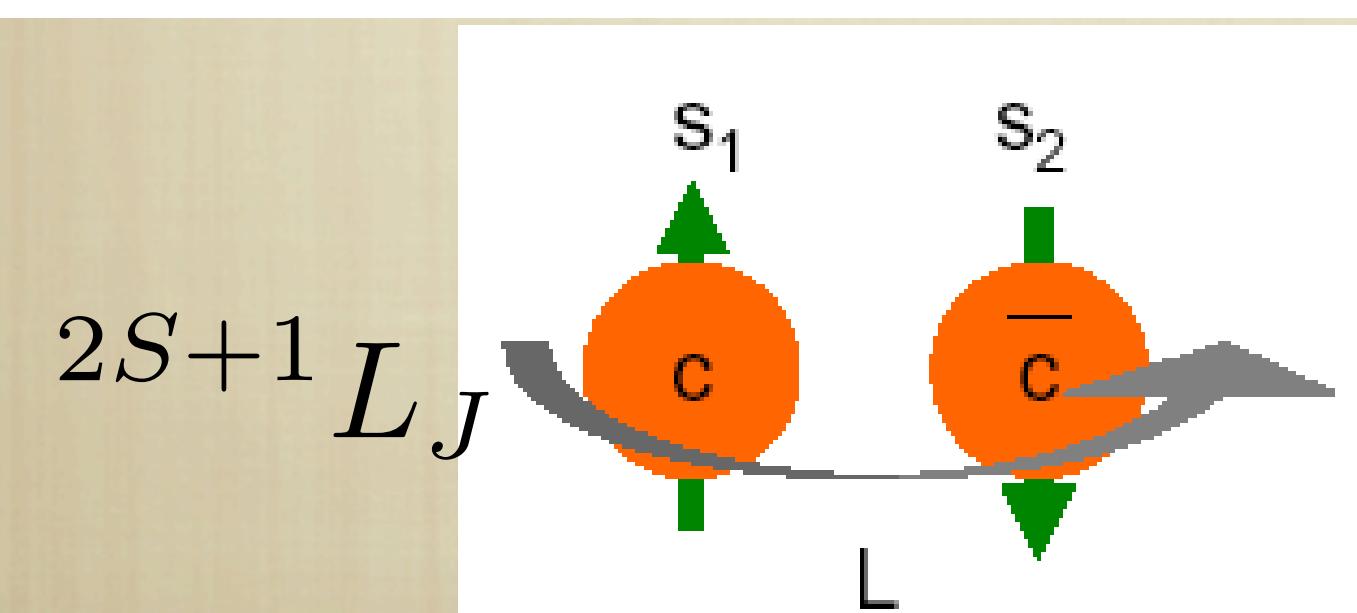
S states

Normalized with respect to  $\chi_b(1P)$  and  $\chi_c(1P)$

P states

Normalized with respect to  $\chi_b(1P)$  and  $\chi_c(1P)$

$$2S+1 L_J$$



NR BOUND STATES HAVE AT LEAST  
3 SCALES

$$m \gg mv \gg mv^2 \quad v \ll 1$$

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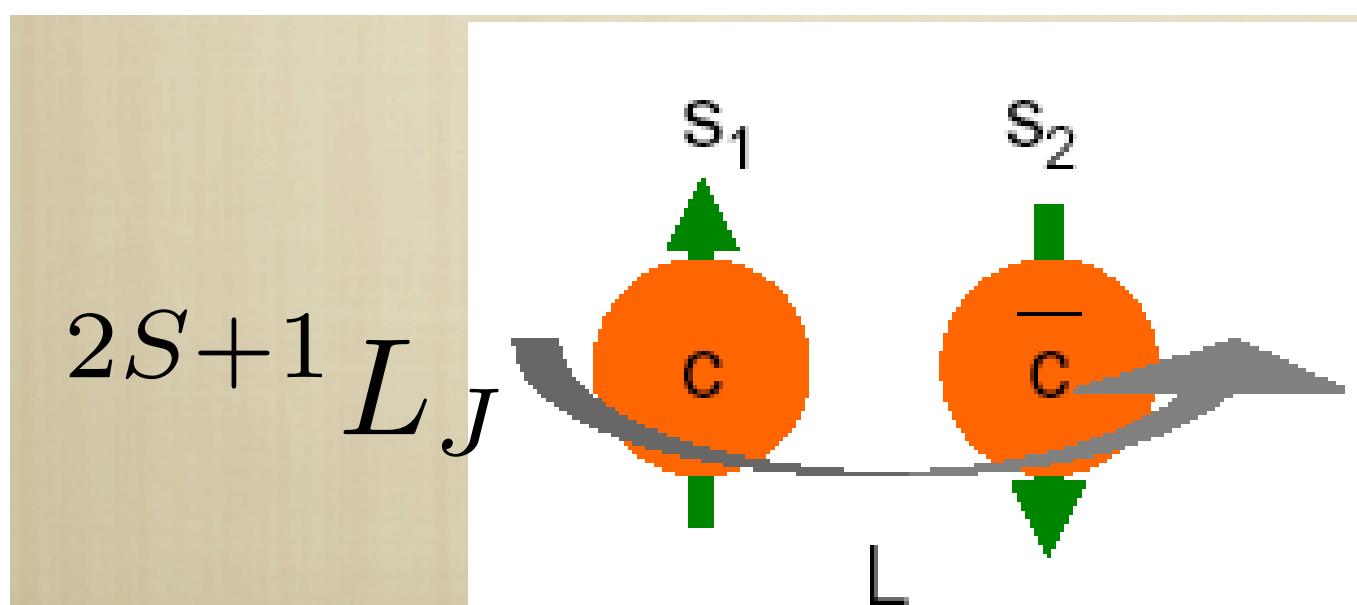
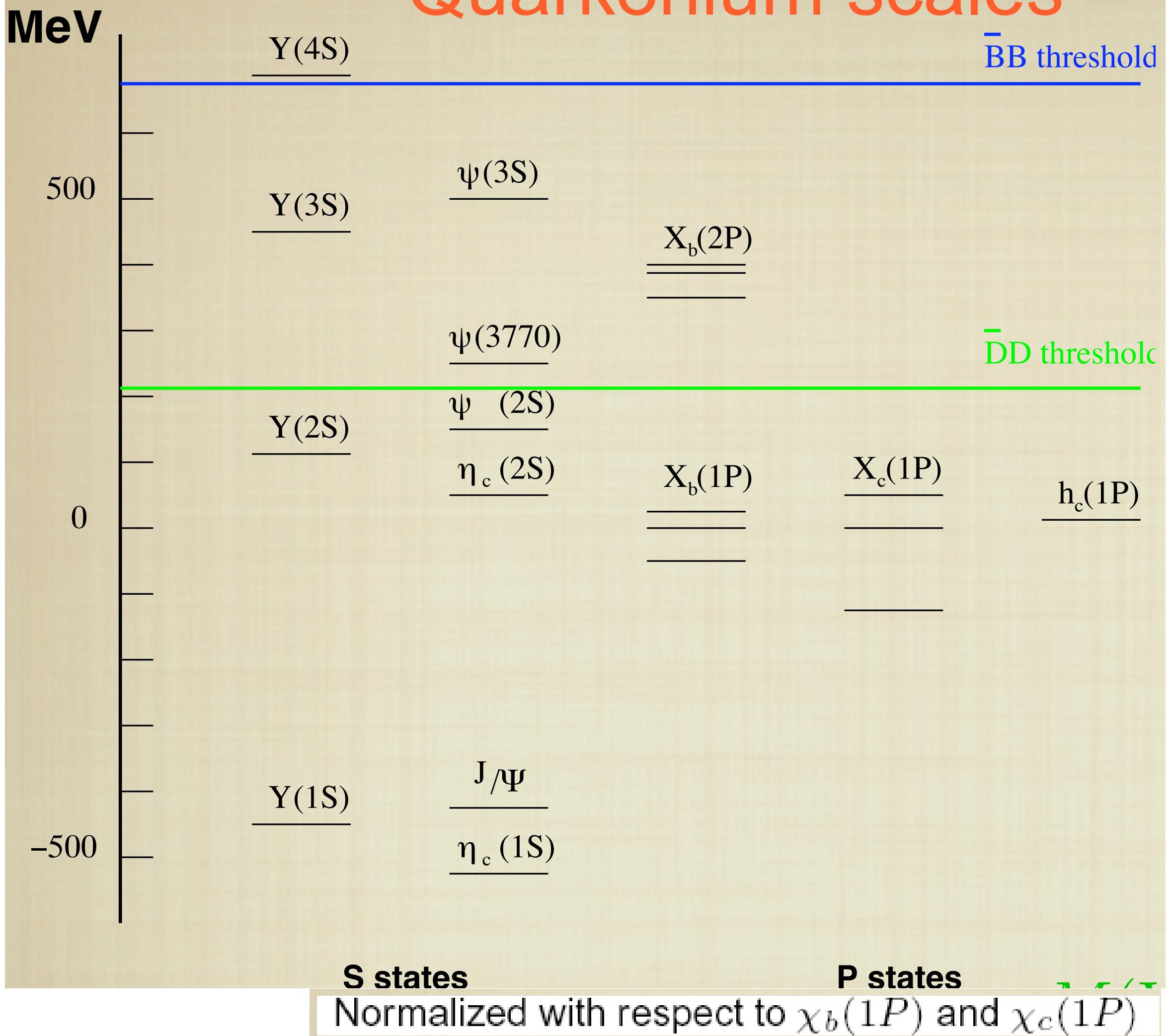
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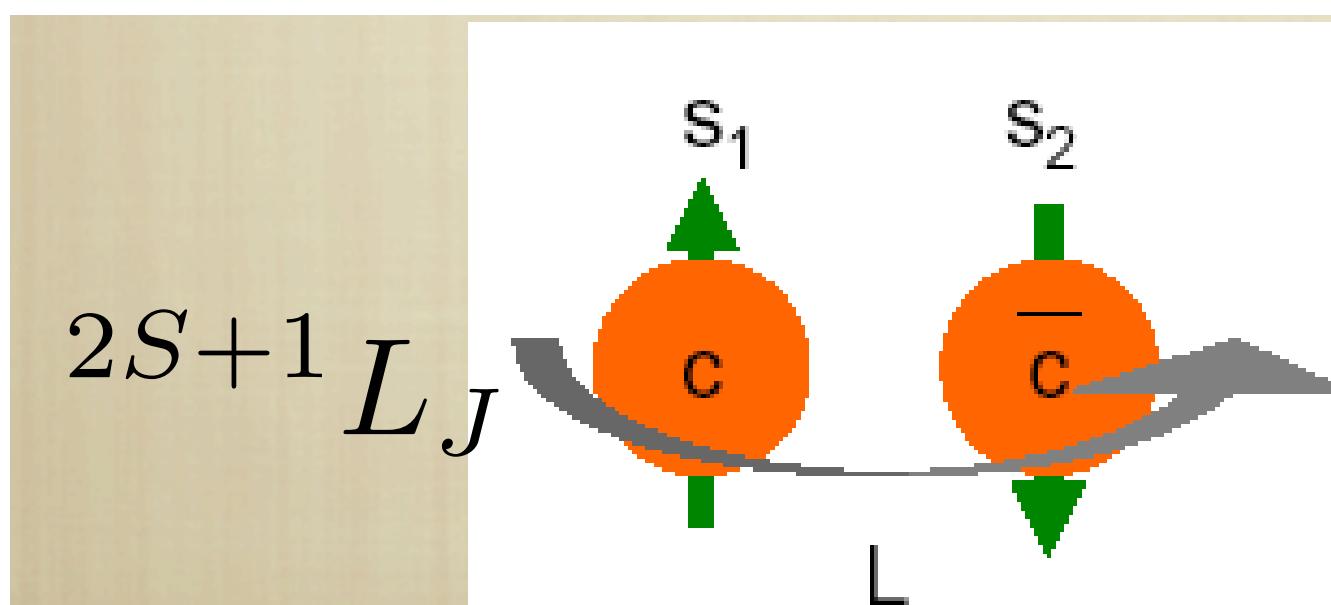
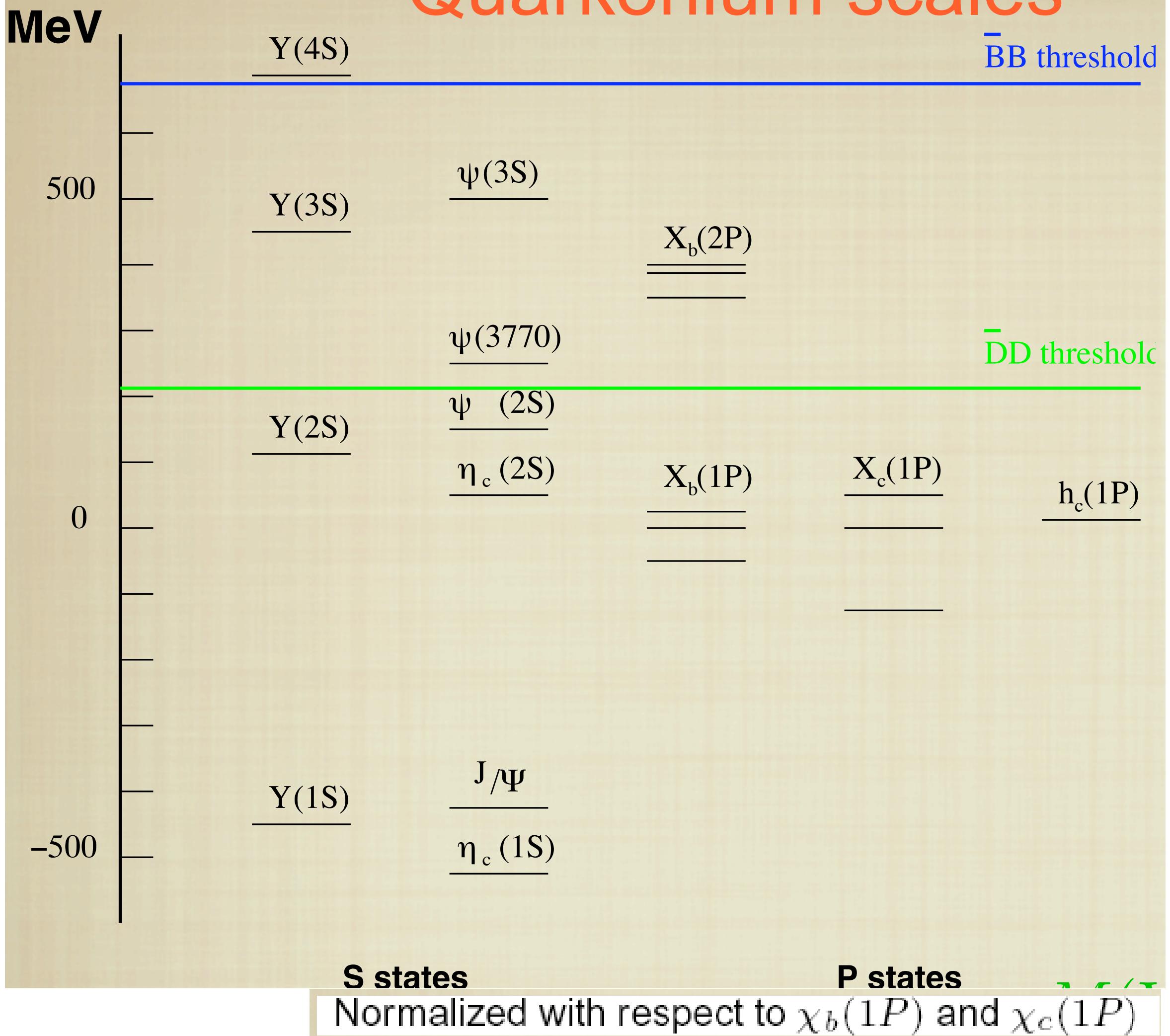
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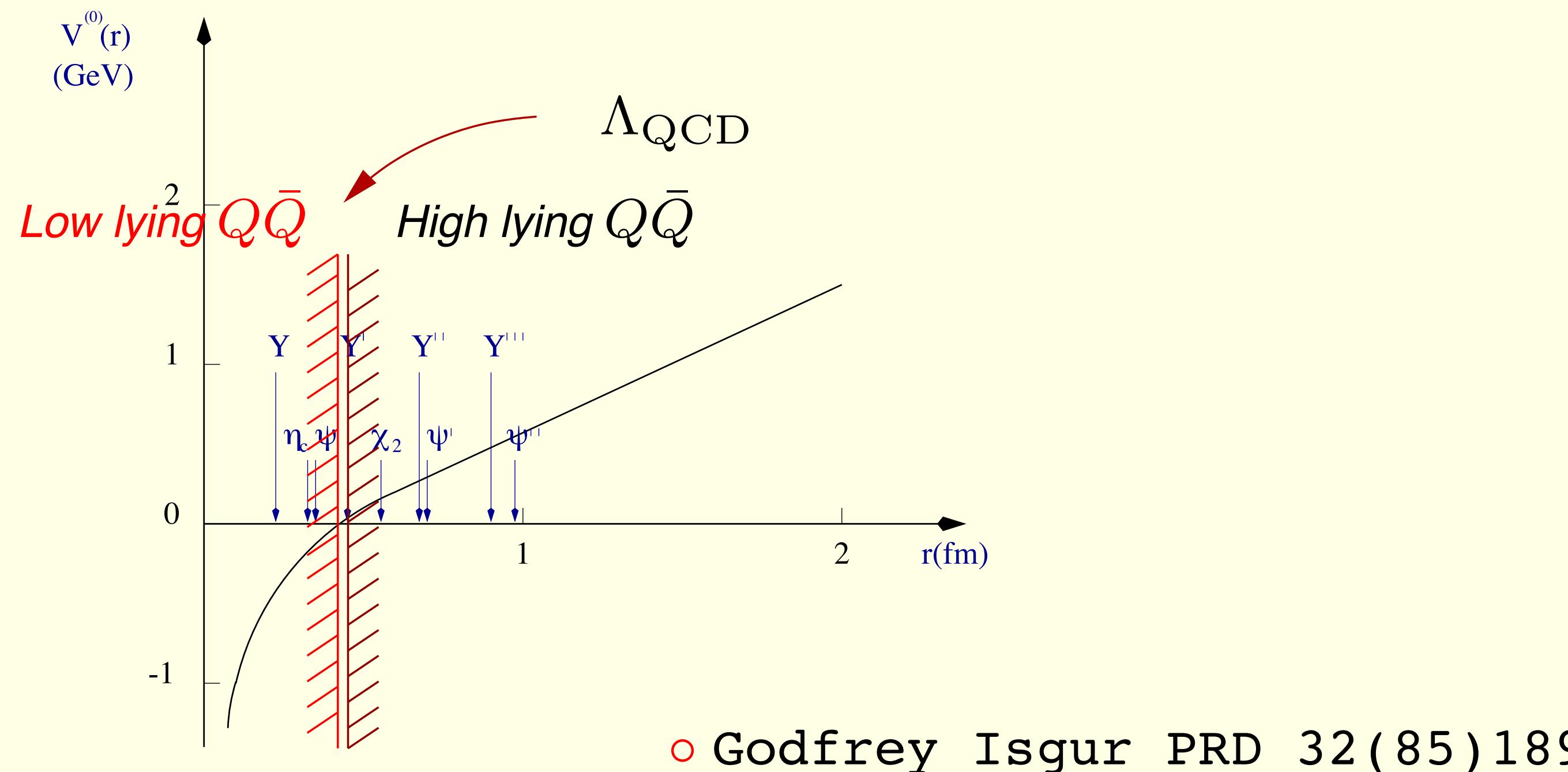
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# Quarkonium as a confinement probe

The rich structure of separated energy scales makes  $Q\bar{Q}$  an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



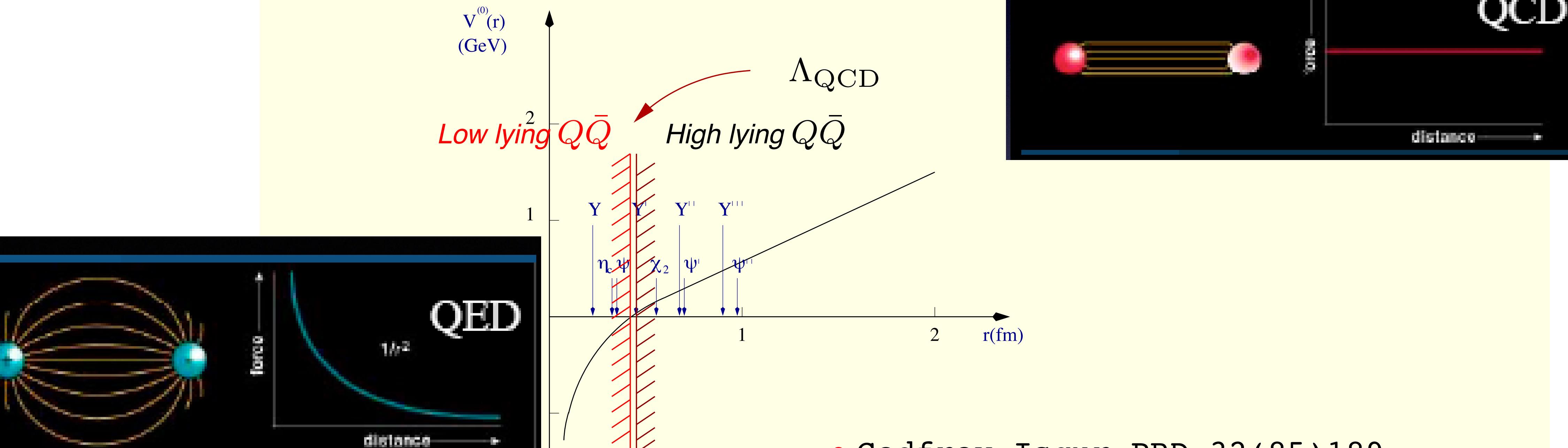
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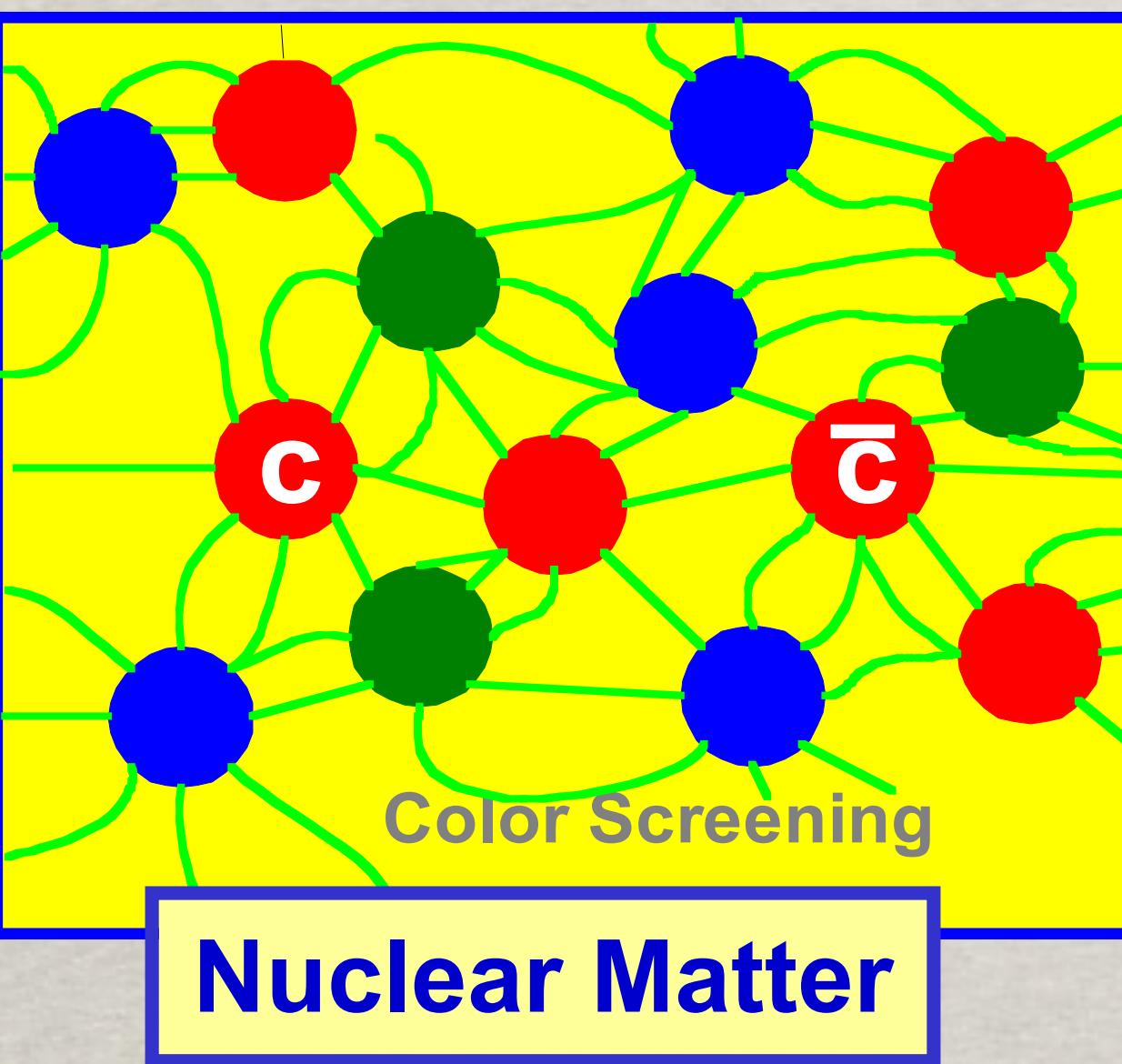
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# Quarkonium as a confinement and deconfinement probe

At finite temperature  $T$  they are sensitive to the formation of a quark gluon plasma via color screening



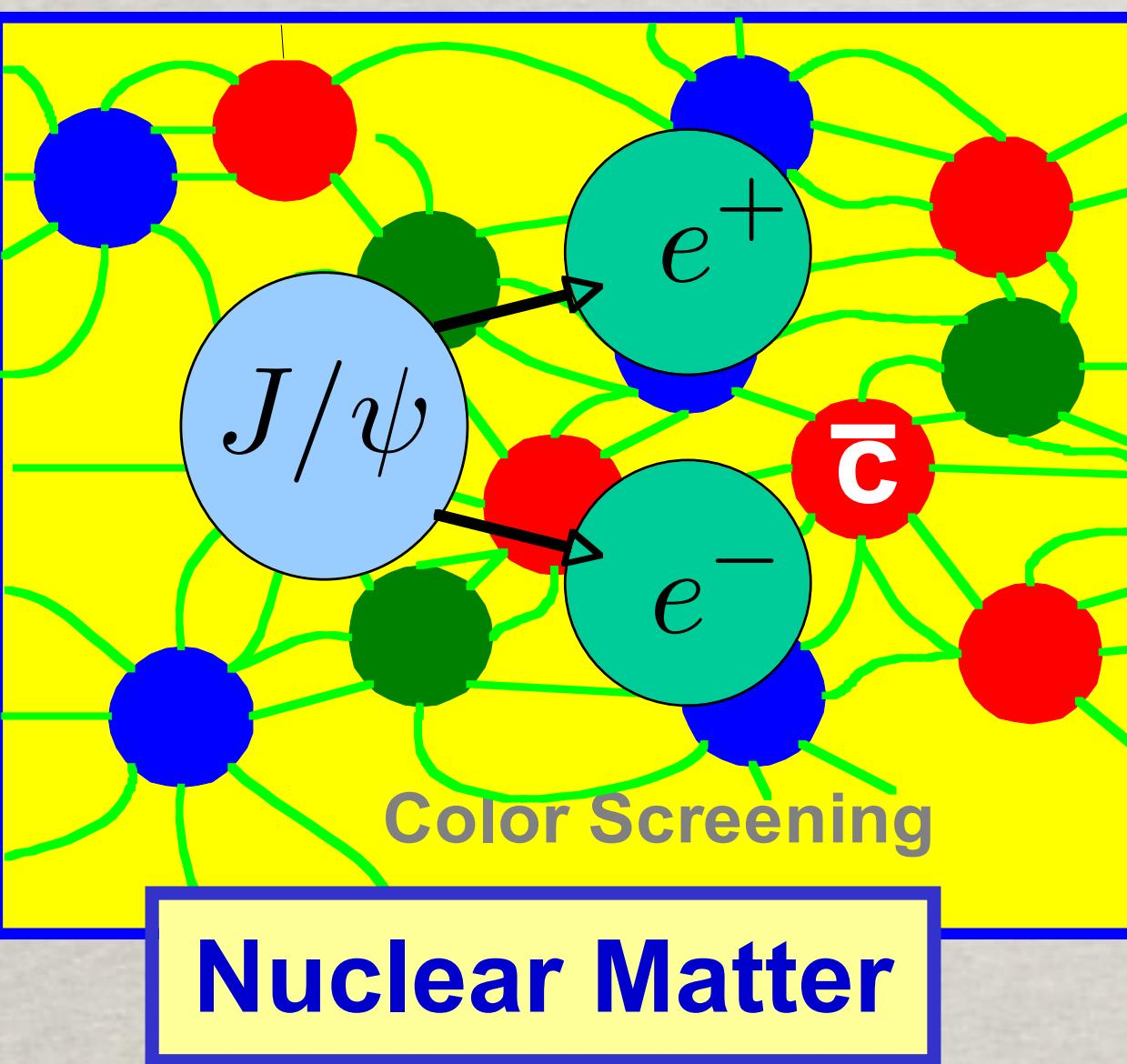
Debye charge screening  $m_D \sim gT$

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

Matsui Satz 1986

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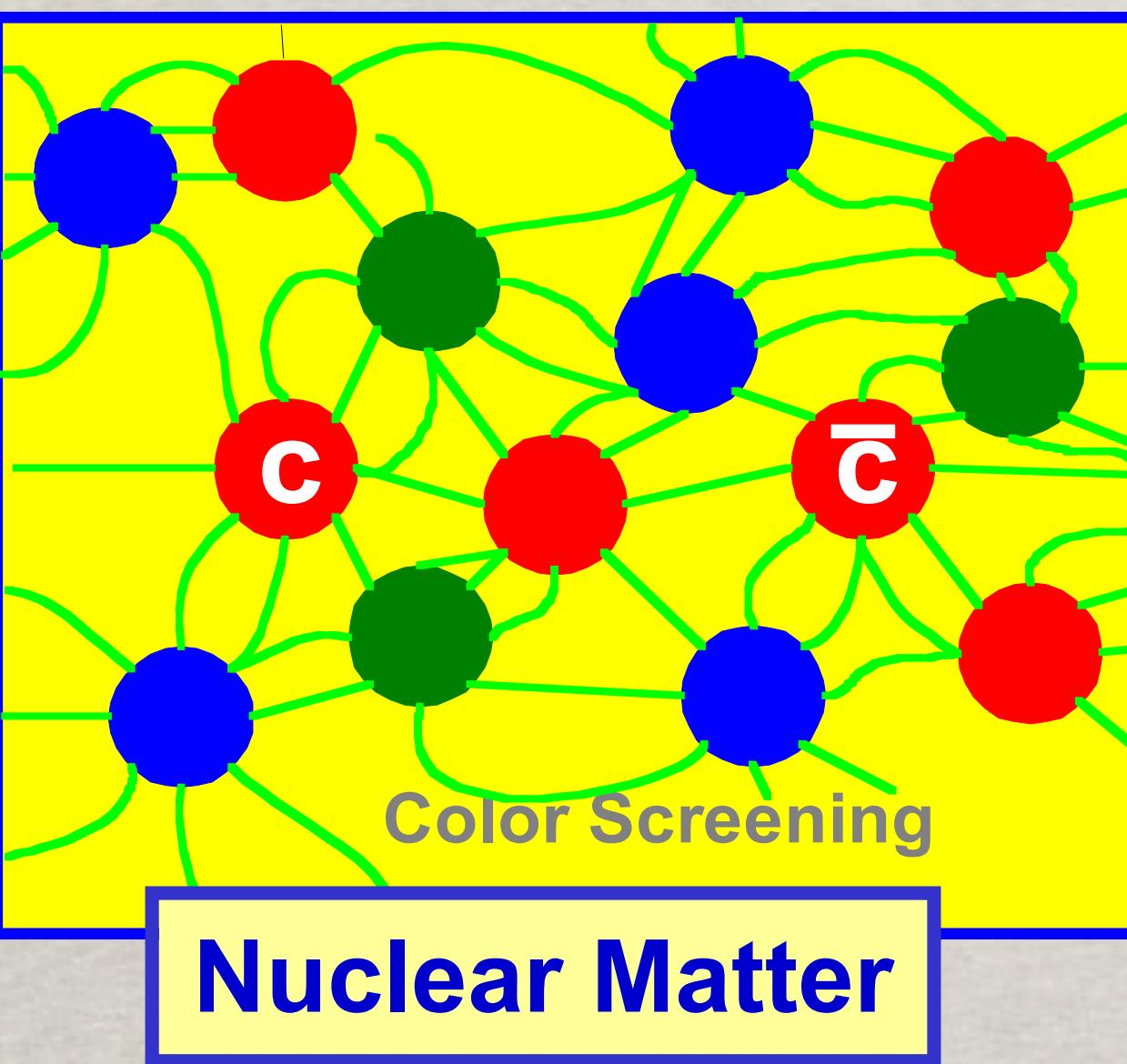
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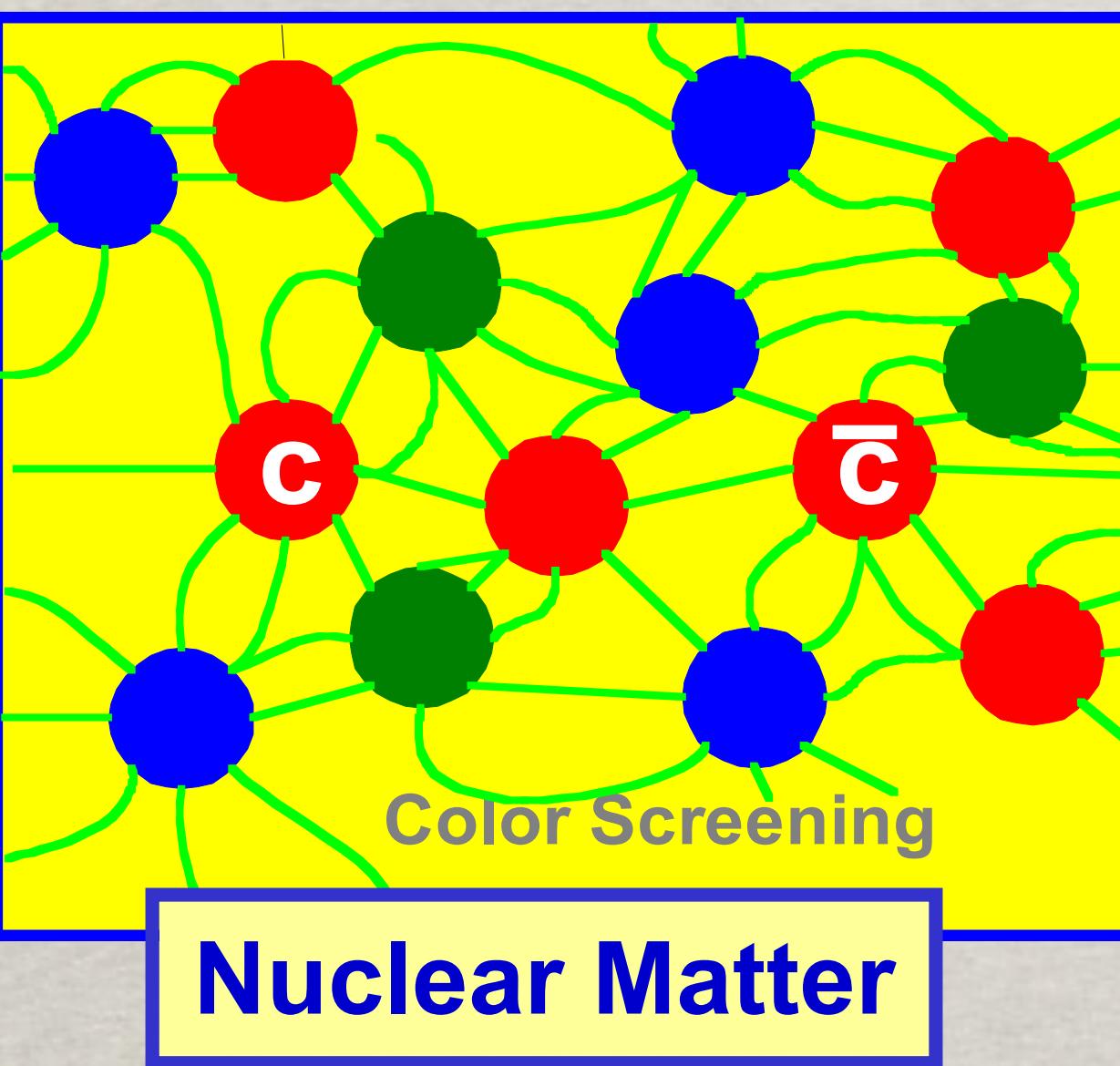
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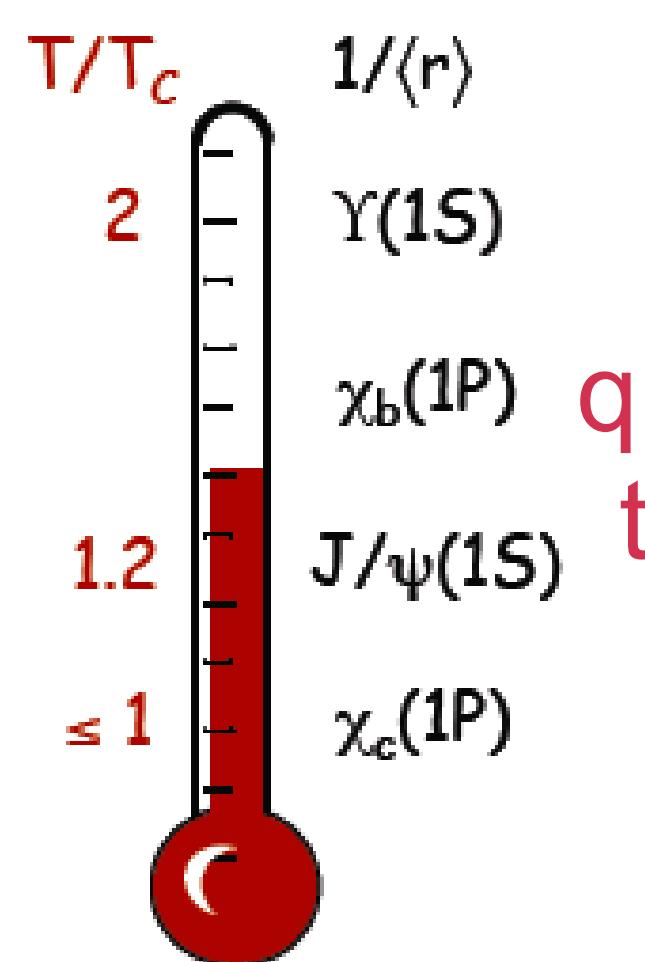
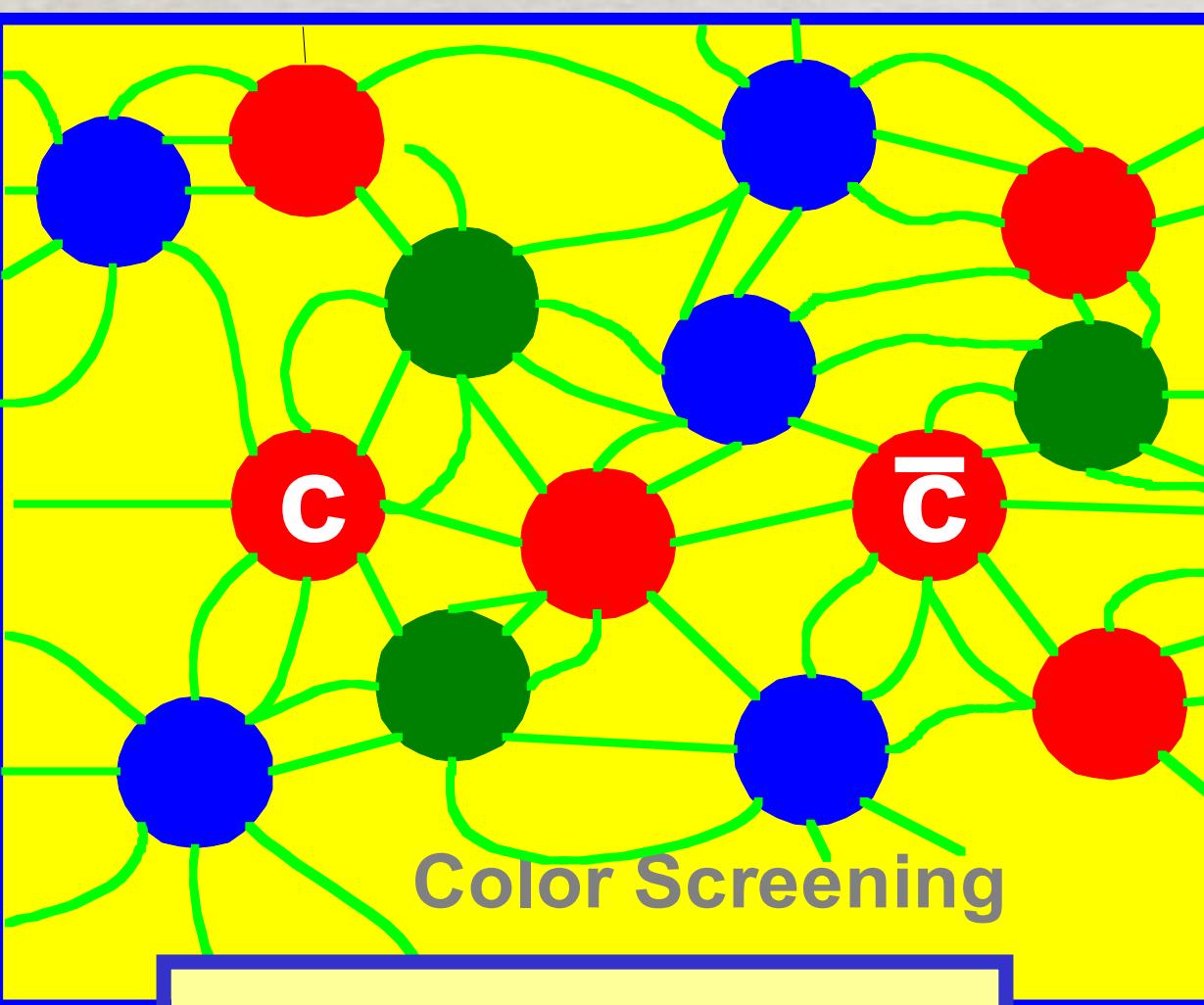
Matsui Satz 1986

$$r \sim \frac{1}{m_D} \longrightarrow$$

Bound state dissolves

# Quarkonium as a confinement and deconfinement probe

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quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

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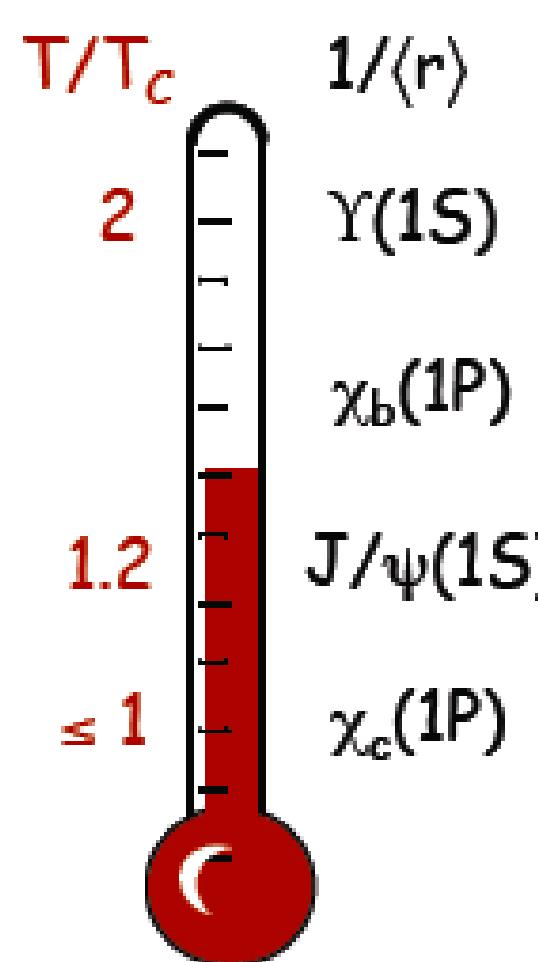
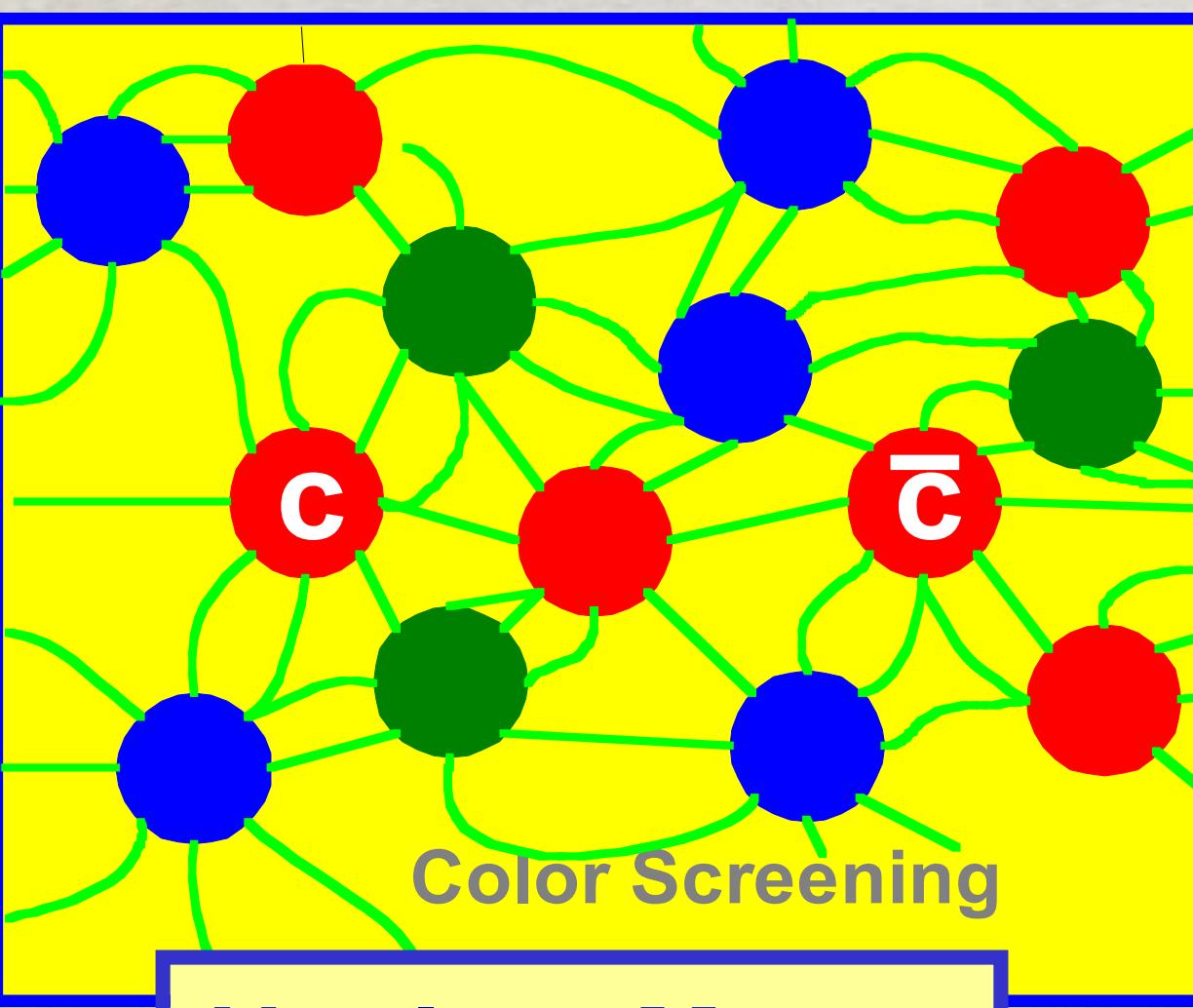
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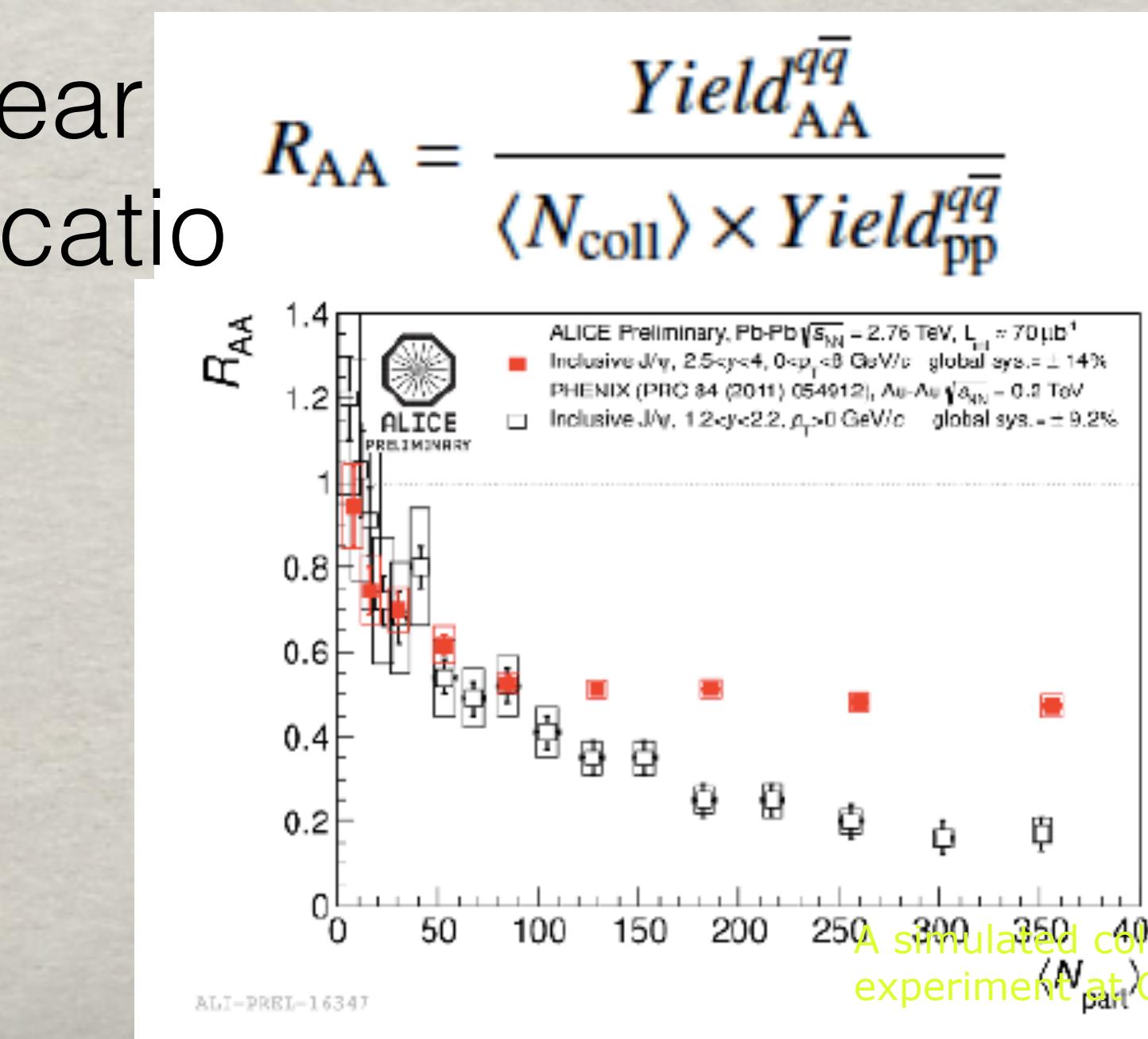
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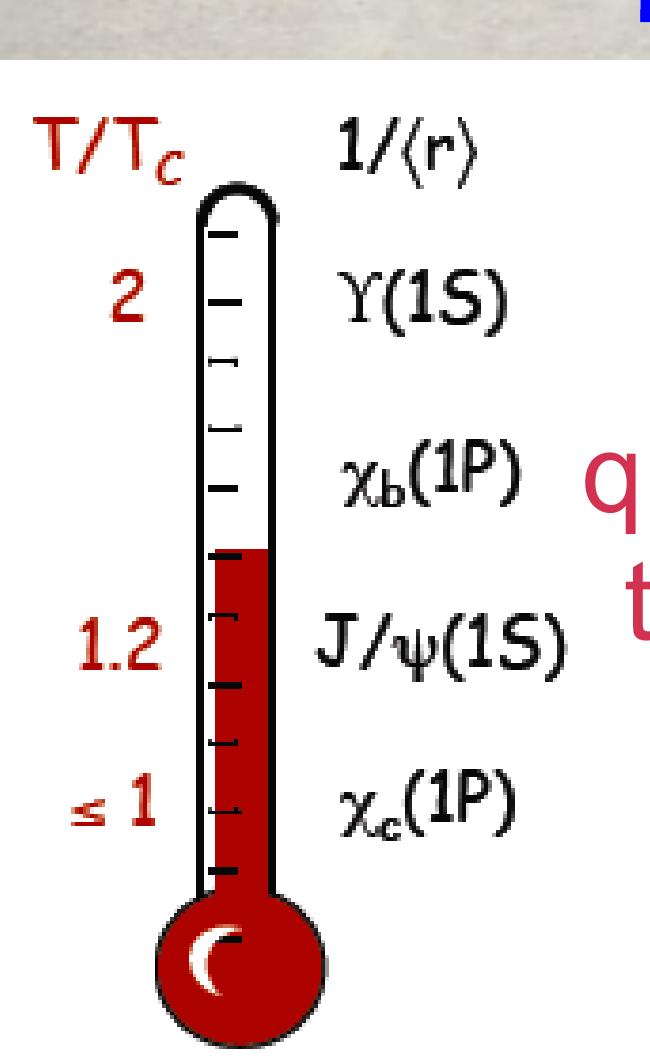
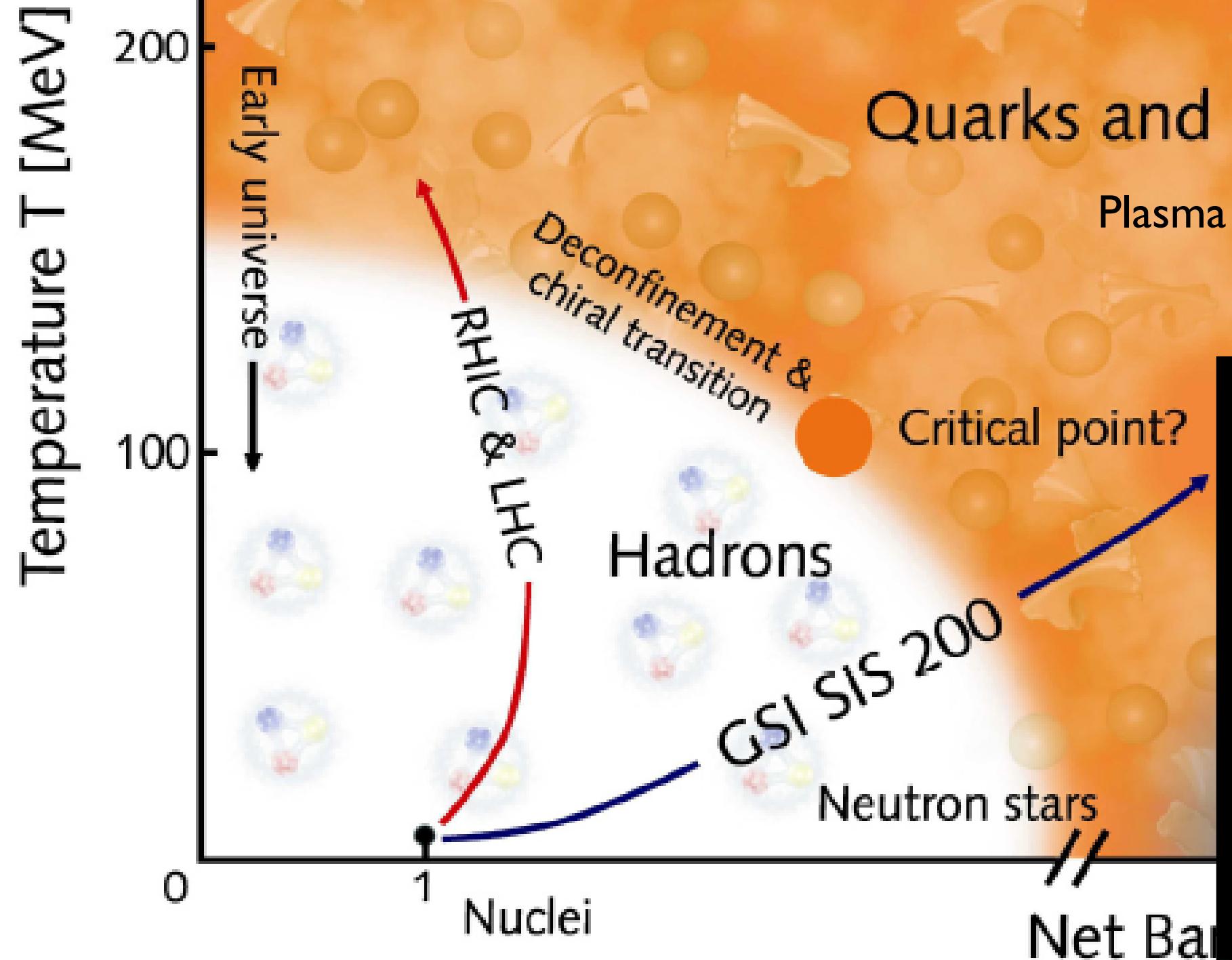
nuclear modification



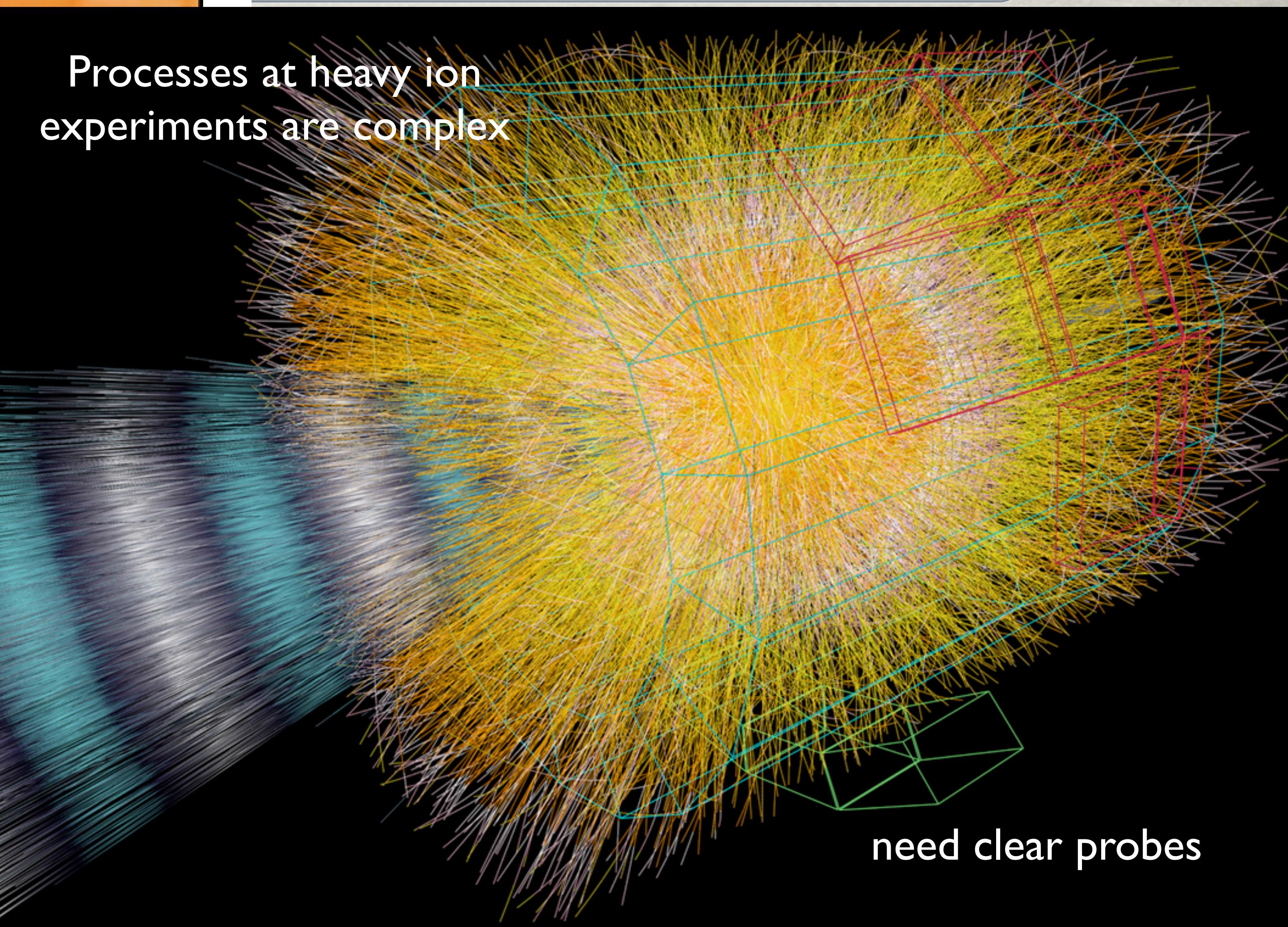
A simulated collision of lead ions, courtesy the ALICE experiment at CERN

# Deconfinement and deconfinement probe

sensitive to the formation of a quark gluon



quarkonia dissociate at different temperatures depending on their radius: they are a Quark Gluon Plasma thermometer



CLAIM:

Nonrelativistic multiscale systems are great probes of strong interactions  
and could impact on our understanding of several open problems

PROBLEM:

How to calculate properties of these states in QFT

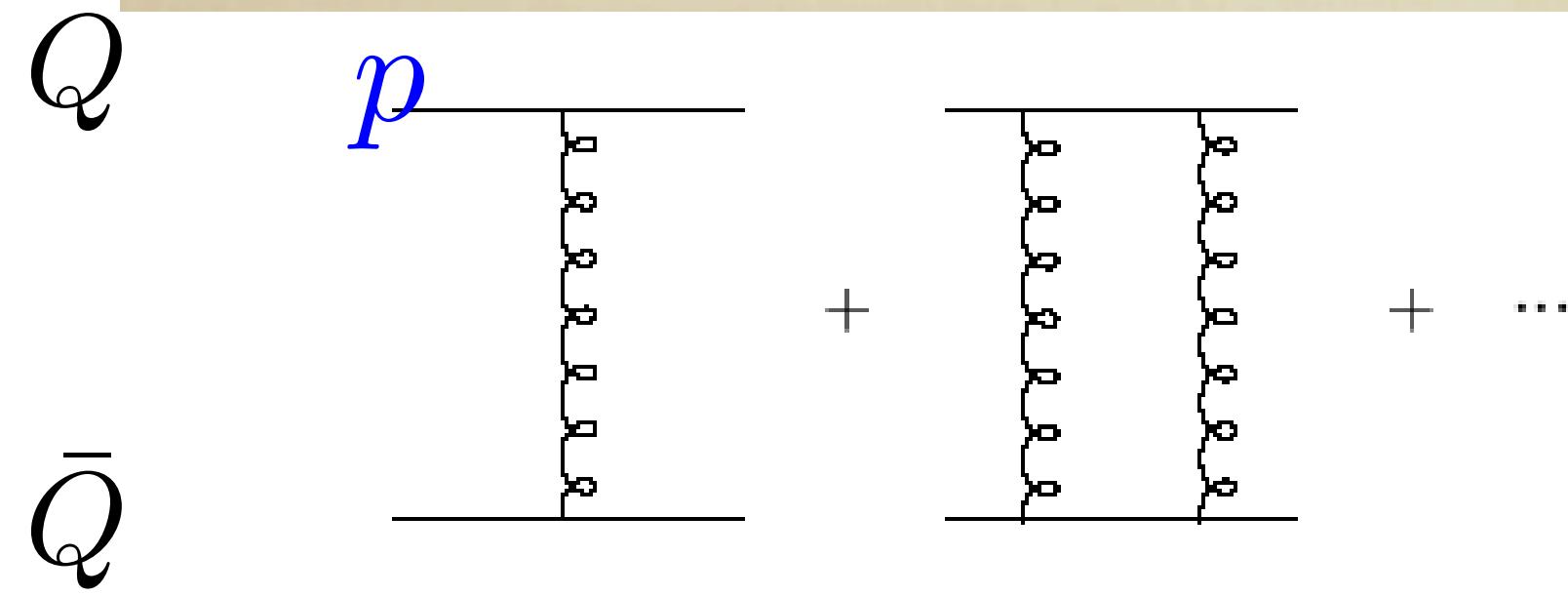
# QCD theory of Quarkonium: a very hard problem even at T=0

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Close to the bound state  $\alpha_s \sim v$

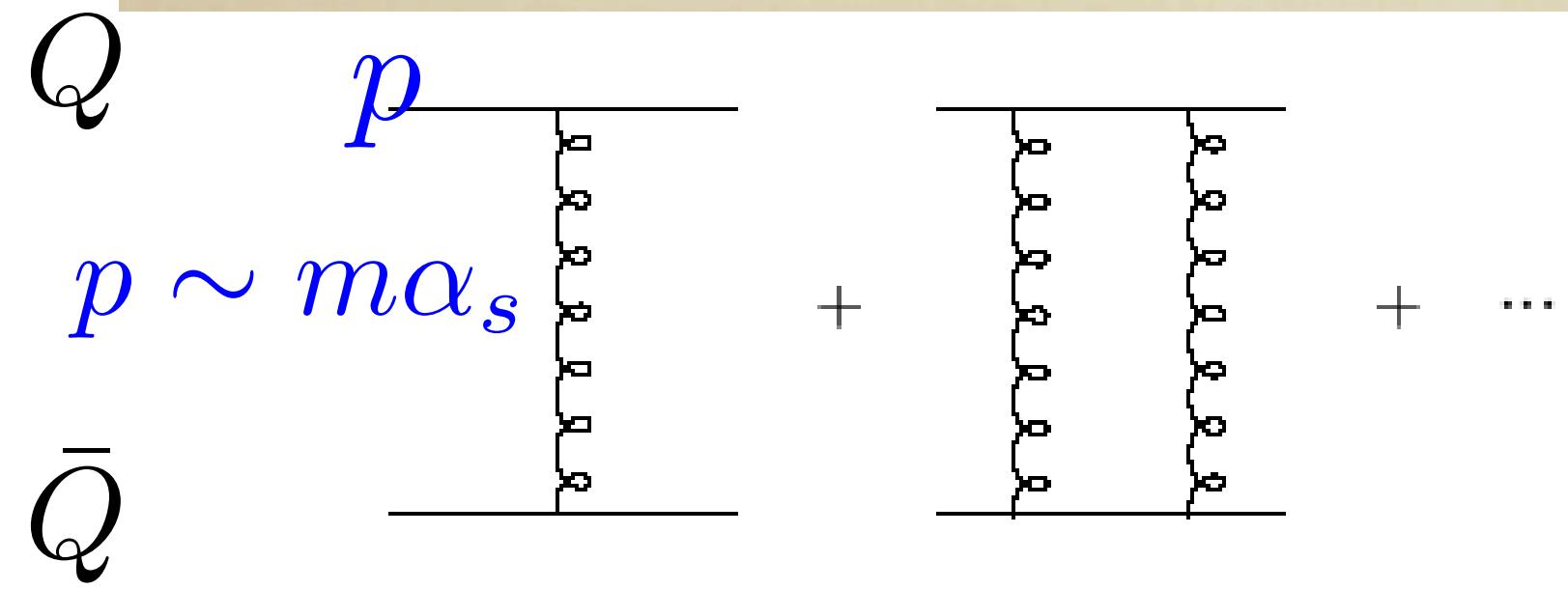
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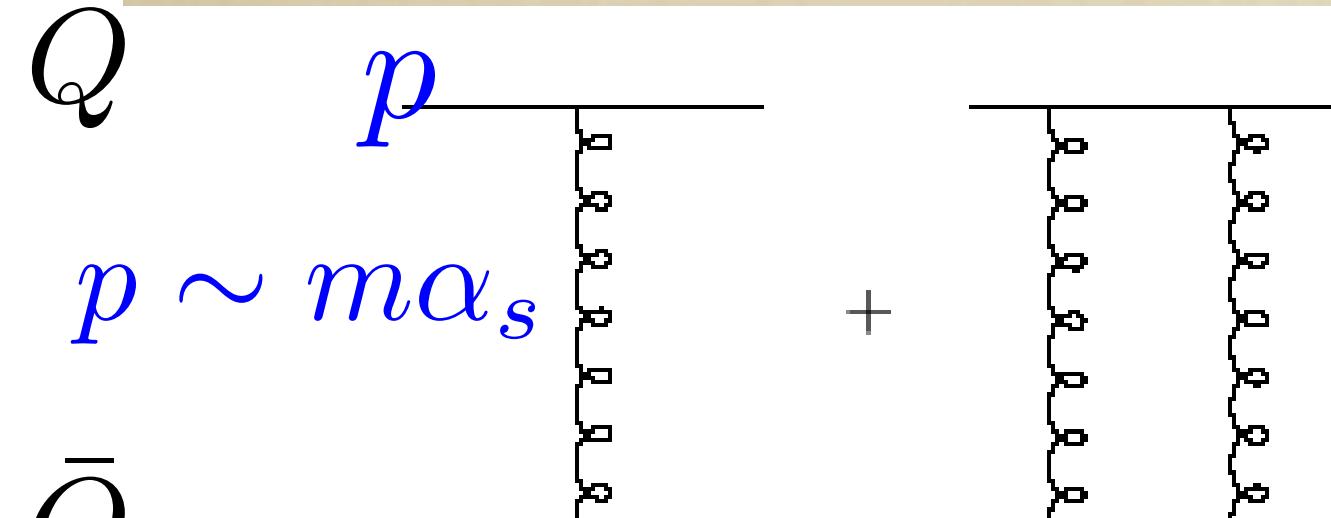


# QCD theory of Quarkonium: a very hard problem even at T=0

Close to the bound state  $\alpha_s \sim v$

$$Q \quad p \quad + \quad + \quad \dots$$

$p \sim m\alpha_s$

$$\bar{Q} \quad \frac{g^2}{p^2} \left( 1 + \frac{m\alpha_s}{p} \right)$$


The diagram shows a quark (Q) and an antiquark (bar{Q}) interacting via gluons. The quark has momentum p. The interaction is represented by a vertical gluon loop between the quark and antiquark lines. The loop is labeled with the equation  $p \sim m\alpha_s$ . The potential is given by the expression  $\frac{g^2}{p^2} \left( 1 + \frac{m\alpha_s}{p} \right)$ .

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$$\frac{g^2}{p^2} \left( 1 + \frac{m\alpha_s}{p} \right)$$
$$\sim \frac{1}{E - \left( \frac{p^2}{m} + V \right)}$$

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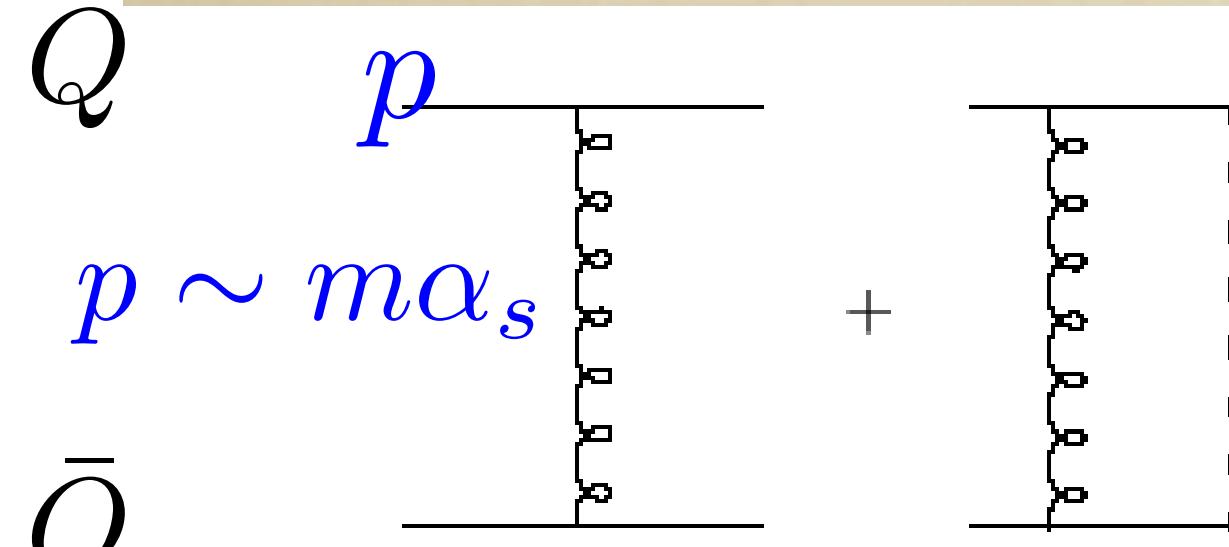
Close to the bound state  $\alpha_s \sim v$

$$Q \quad p \quad + \quad \text{Diagram} \quad + \quad \dots \quad \sim \frac{1}{E - (\frac{p^2}{m} + V)}$$
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- From  $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim m v$  and  $E = \frac{p^2}{m} + V \sim m v^2$ .

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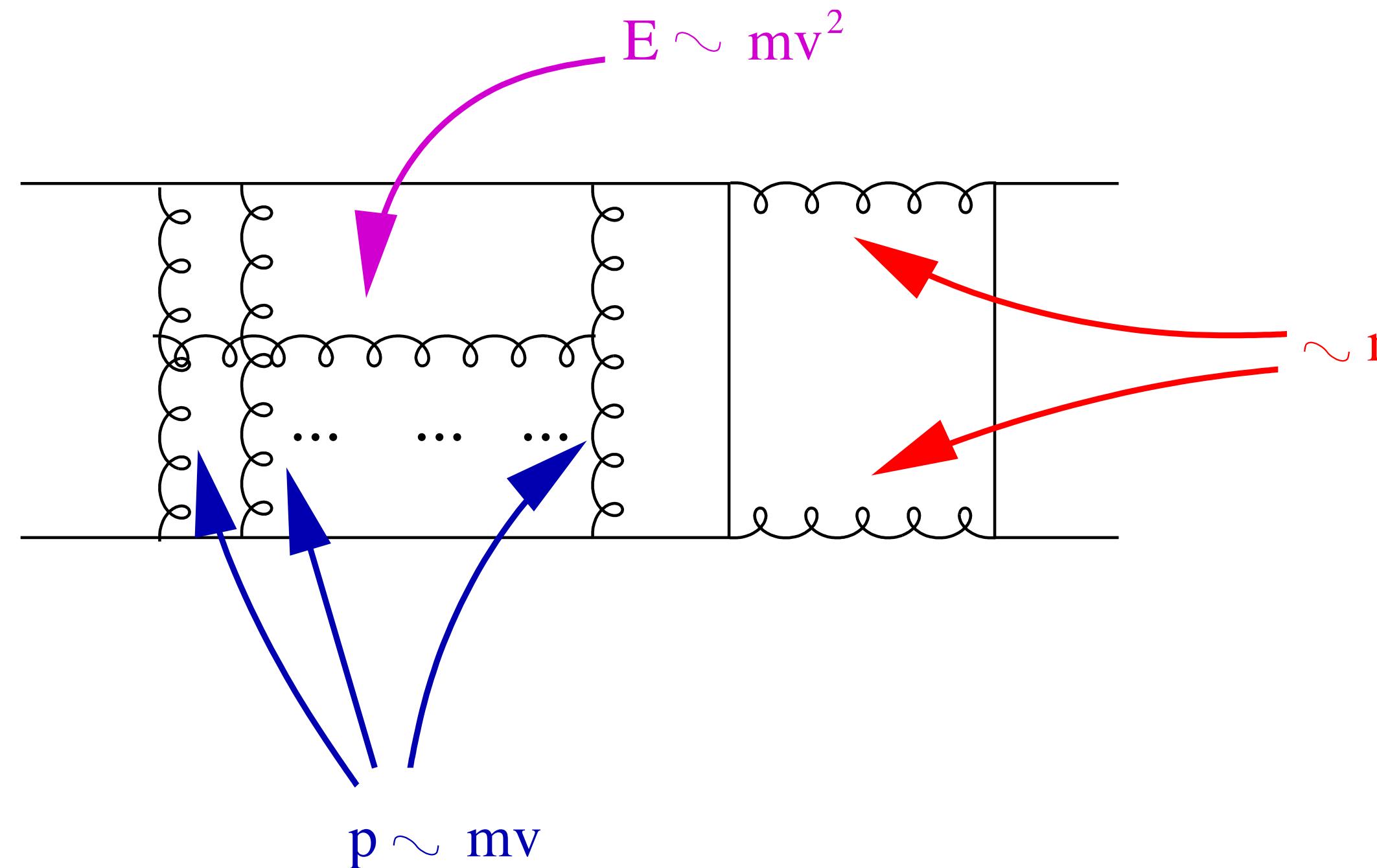
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$$+ \dots \sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

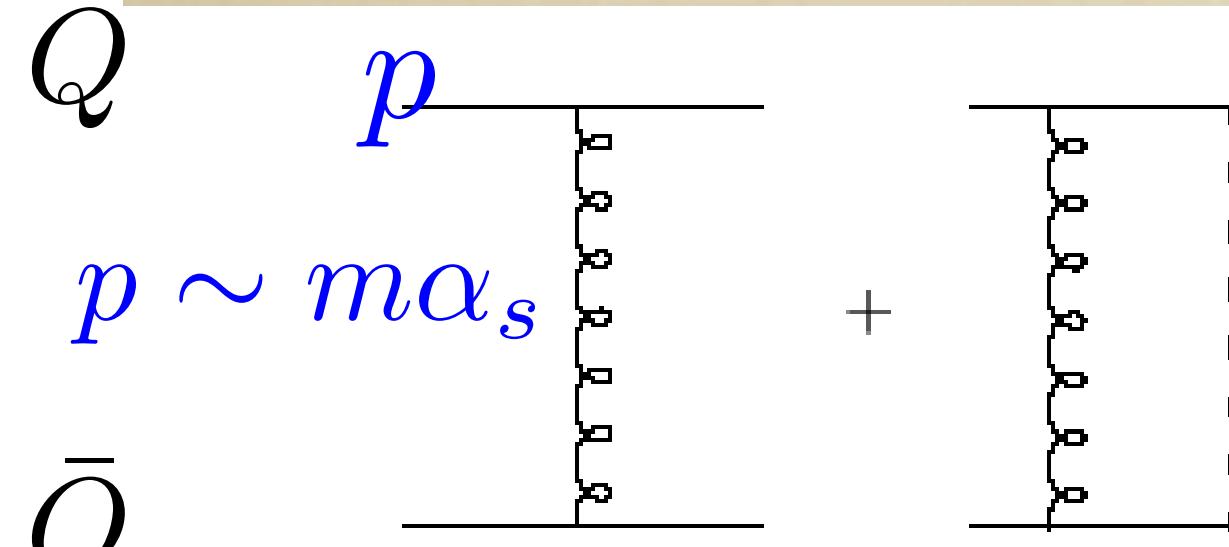
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# QCD theory of Quarkonium: a very hard problem even at T=0

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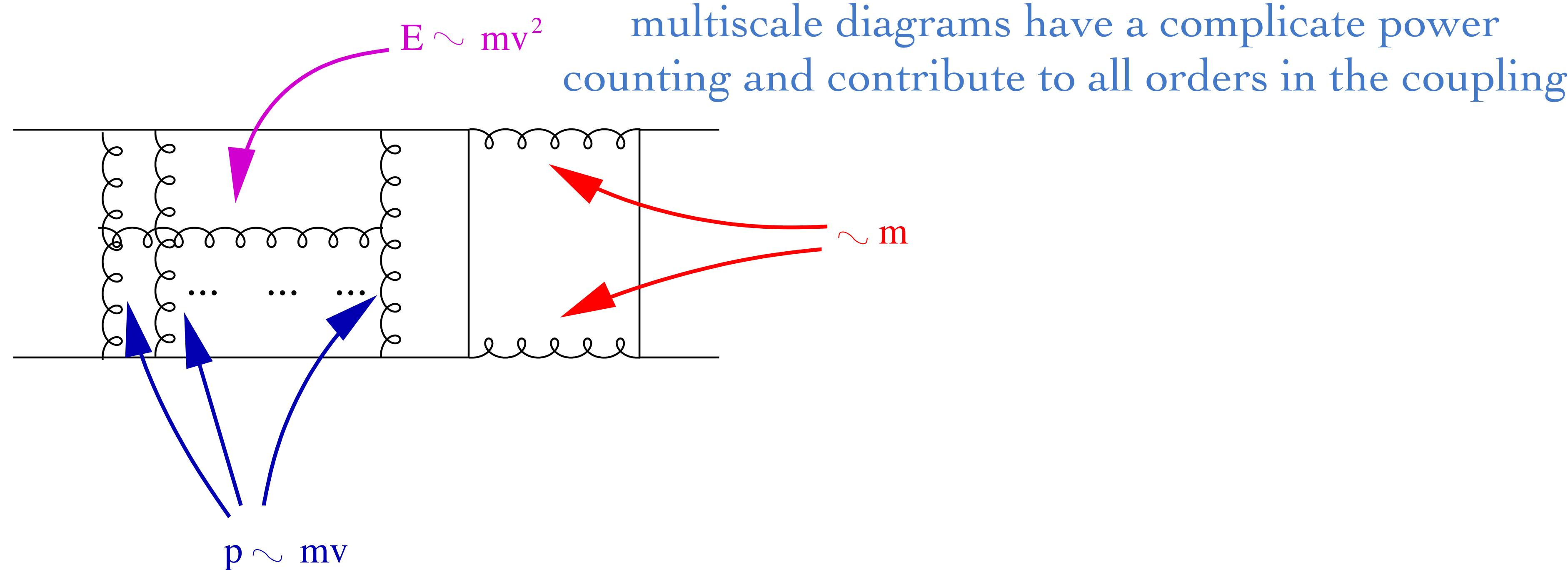
$$p \sim m\alpha_s$$

$$\bar{Q}$$

$$\frac{g^2}{p^2} \left( 1 + \frac{m\alpha_s}{p} \right)$$

$$\sim \frac{1}{E - \left( \frac{p^2}{m} + V \right)}$$

- From  $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$  and  $E = \frac{p^2}{m} + V \sim mv^2$ .



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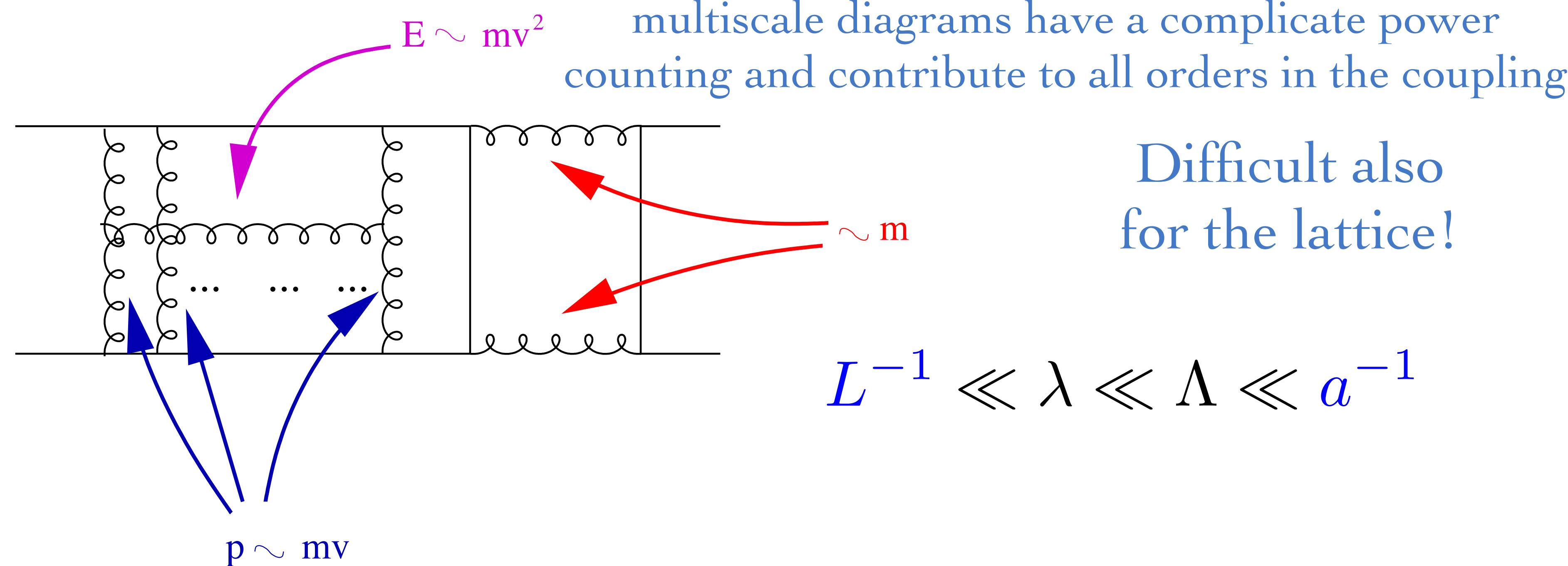
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It is cumbersome in perturbation theory even in QED

Ex.

- It shows the difficulty of the approach the fact that going from the calculation of the  $m\alpha^5$  correction in the hyperfine splitting of the positronium ground state to the  $m\alpha^6 \ln \alpha$  term took twenty-five years!
  - Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36  
Bodwin Yennie PR 43(78)267

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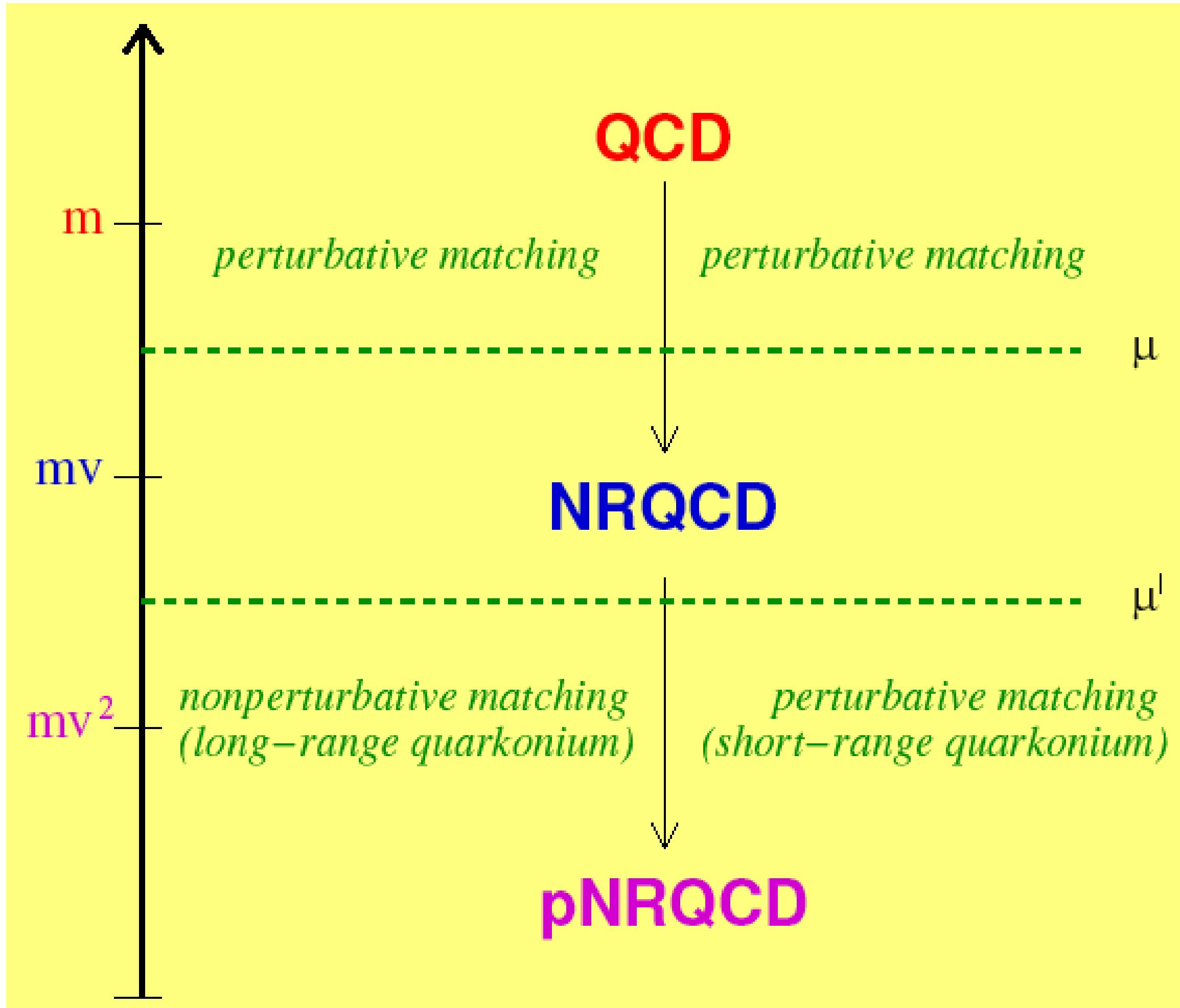
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CLAIM: NREFTS are a good tool to address this physics

# Quarkonium with NR EFT



Color degrees of freedom

$$3 \times 3 = 1 + 8$$

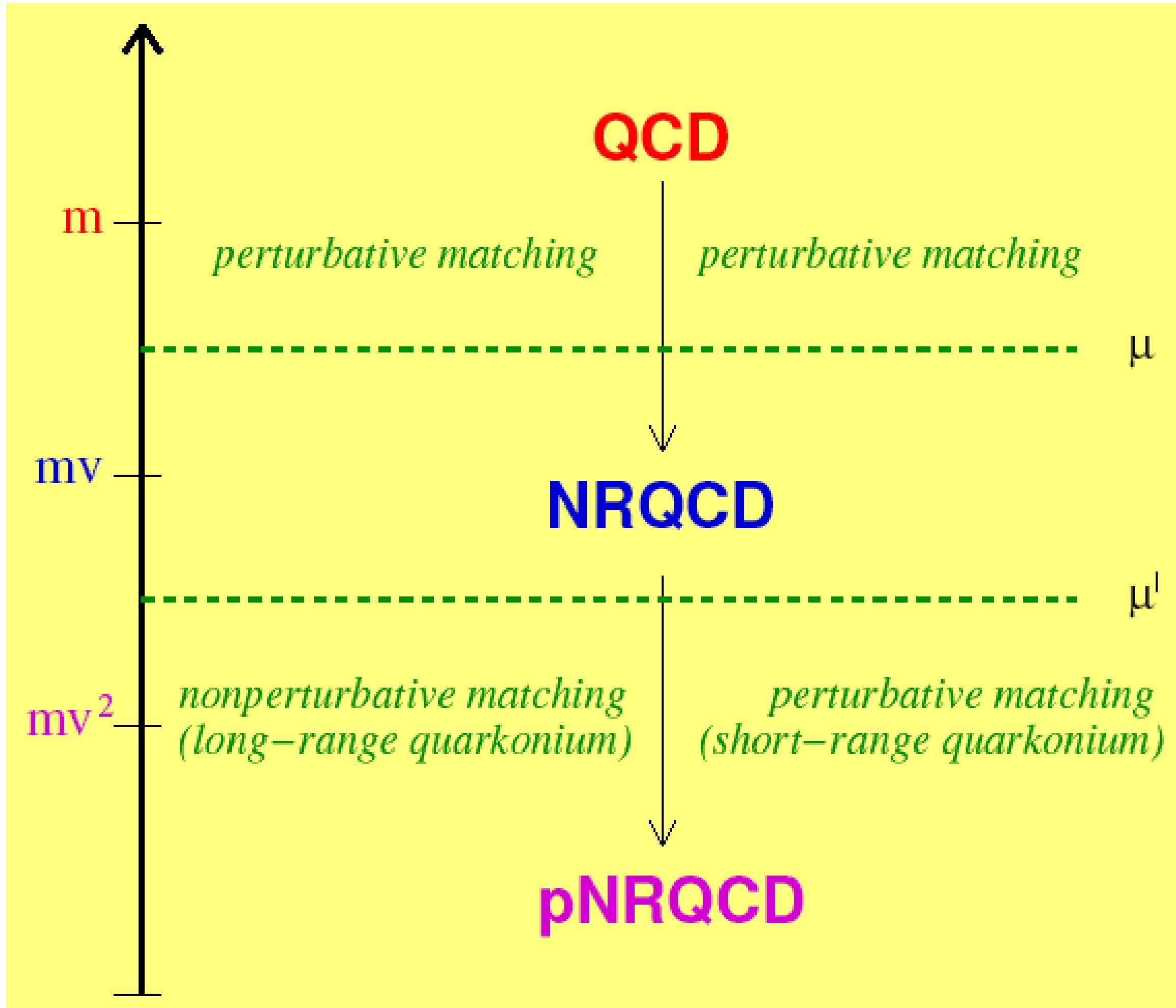
singlet and octet QQbar

Hard

Soft  
(relative  
momentum)

Ultrasoft  
(binding energy)

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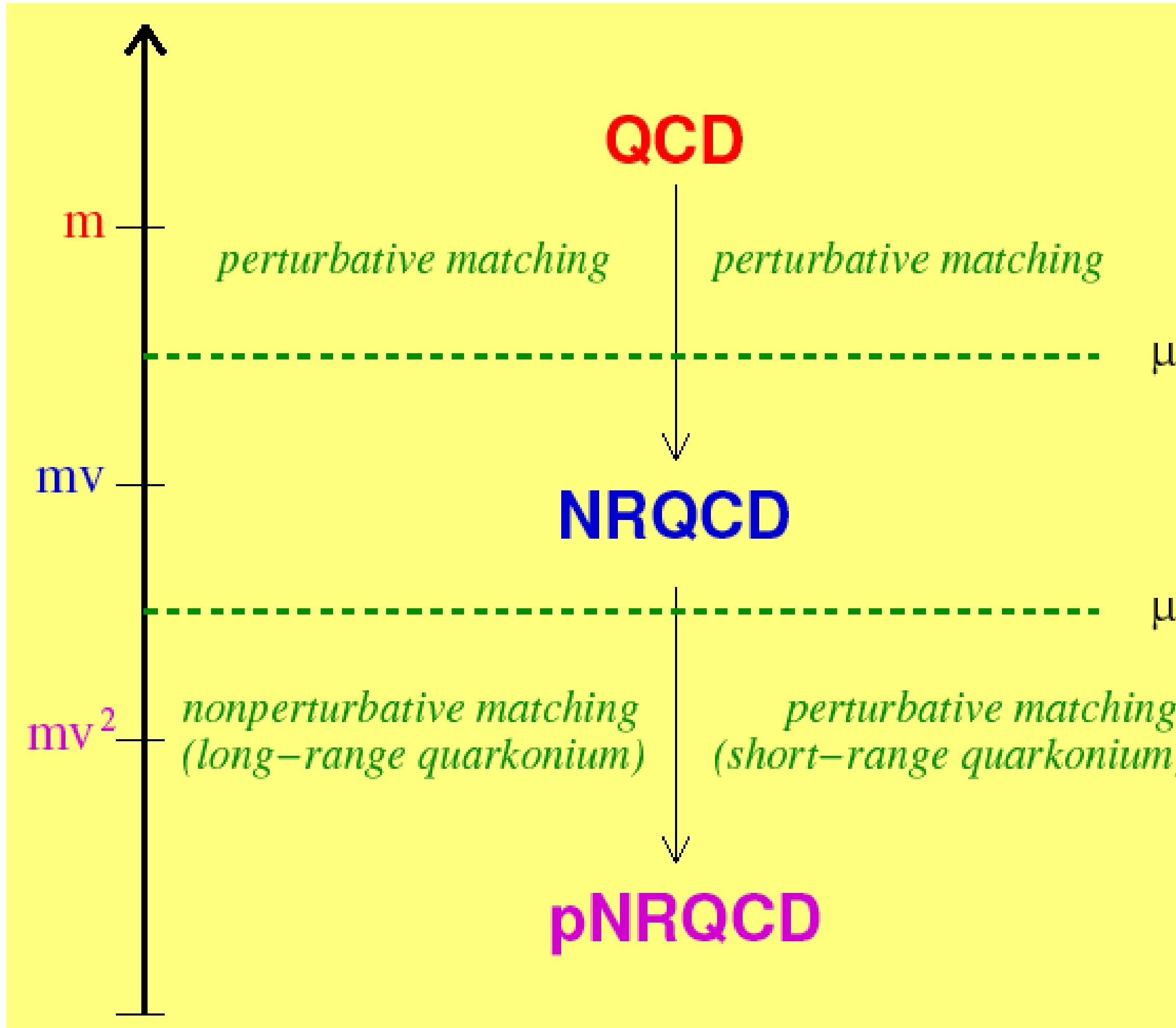
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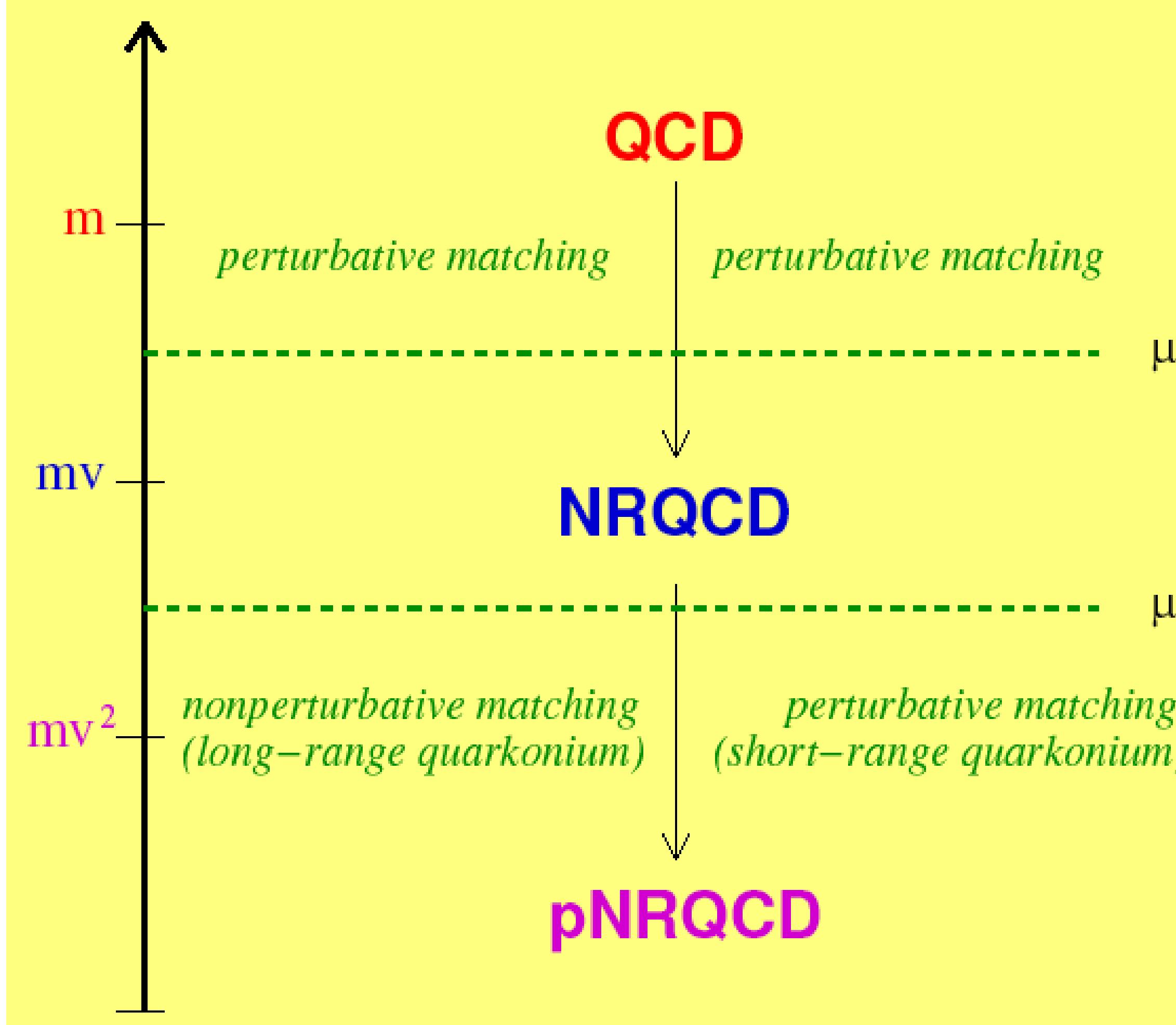
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$$\frac{E_\lambda}{E_\Lambda} = \frac{m v}{m}$$

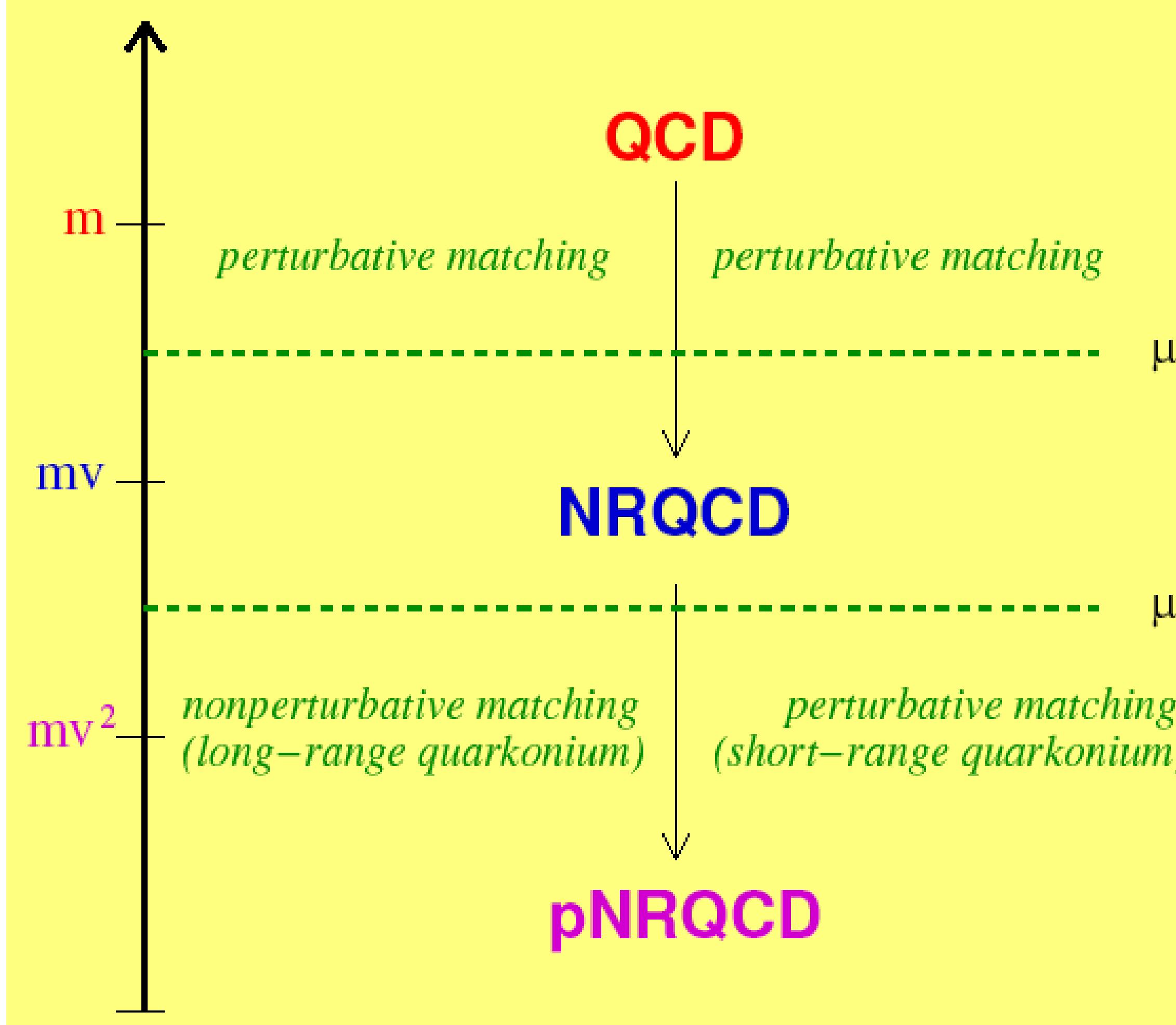
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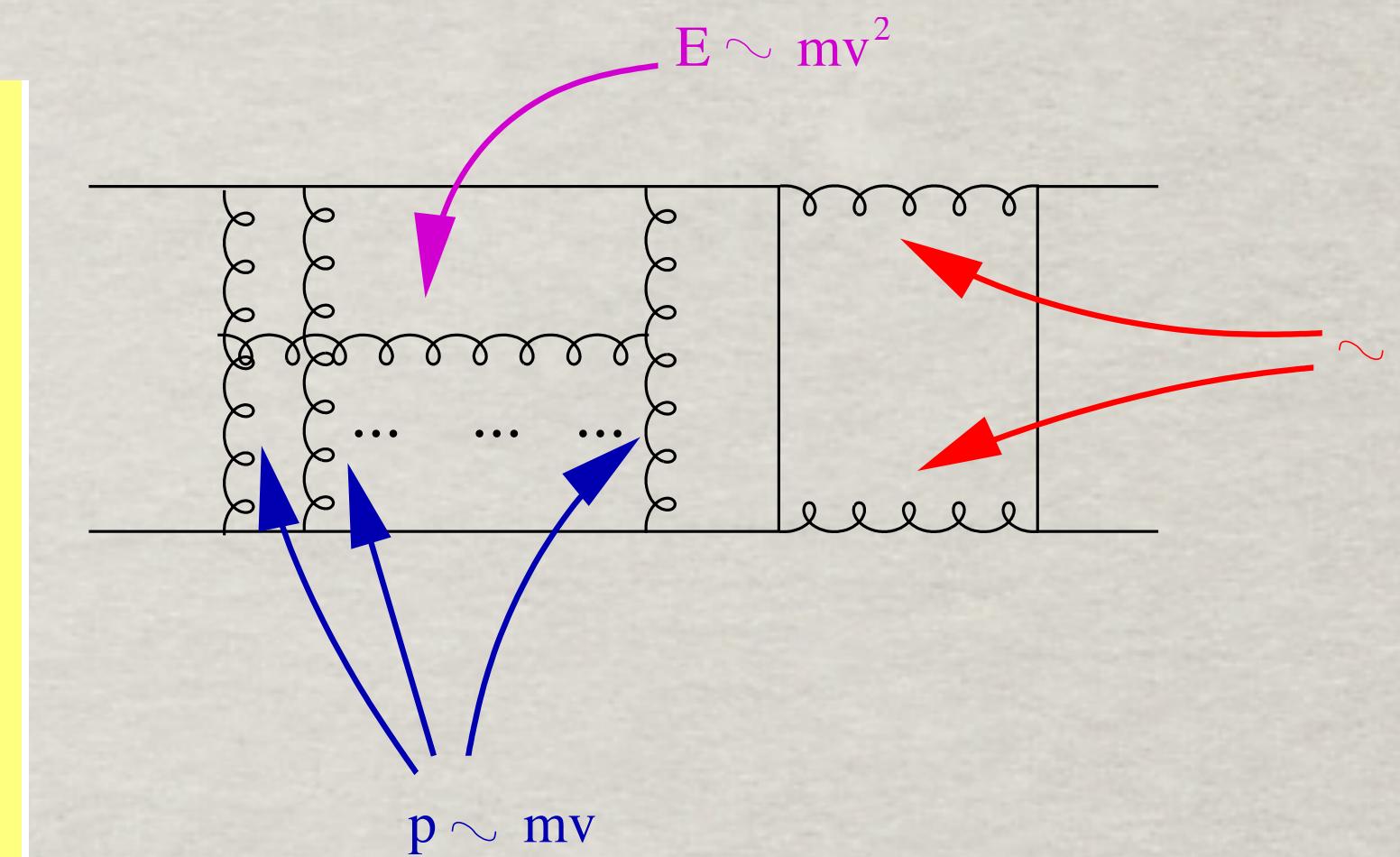
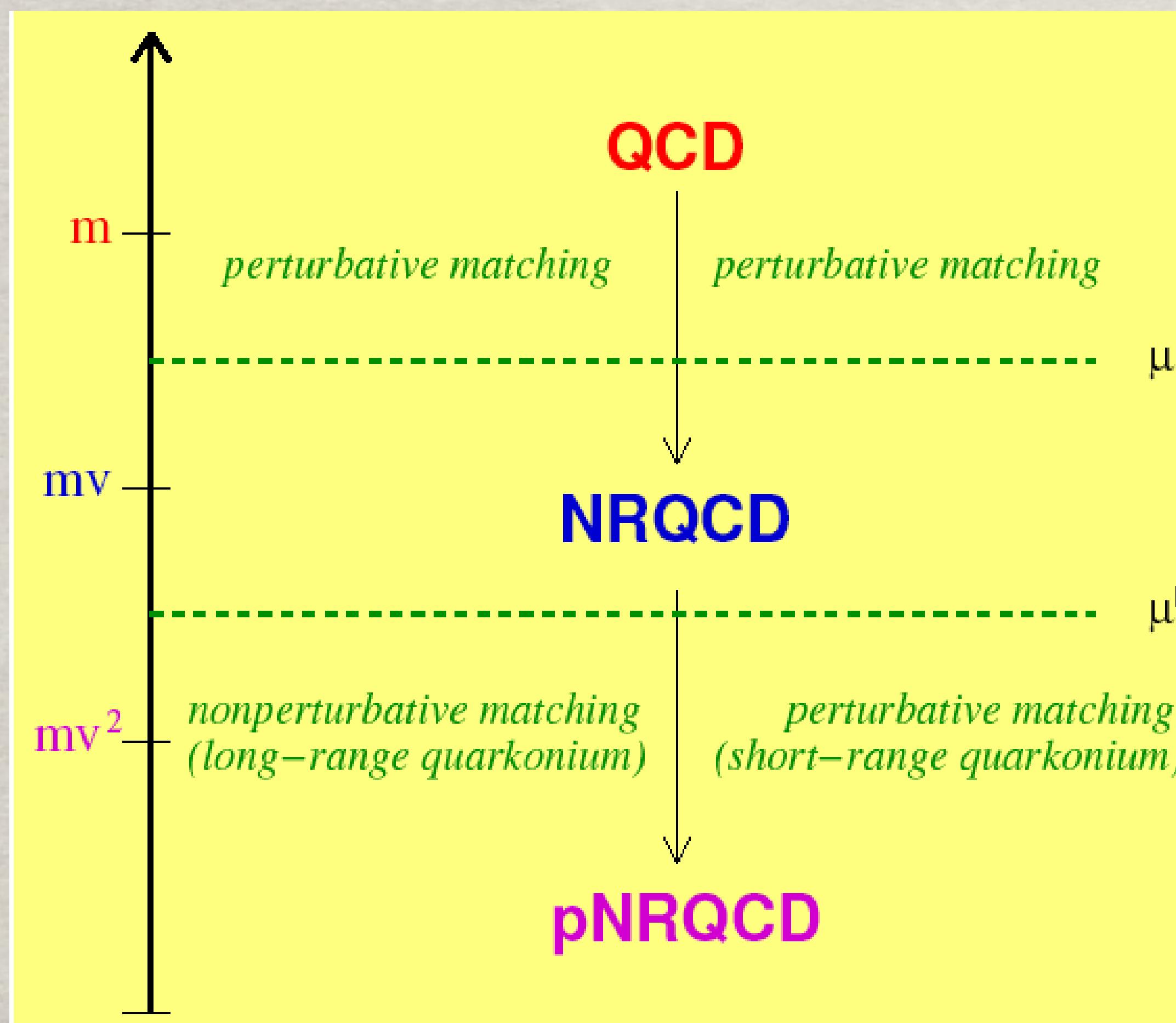
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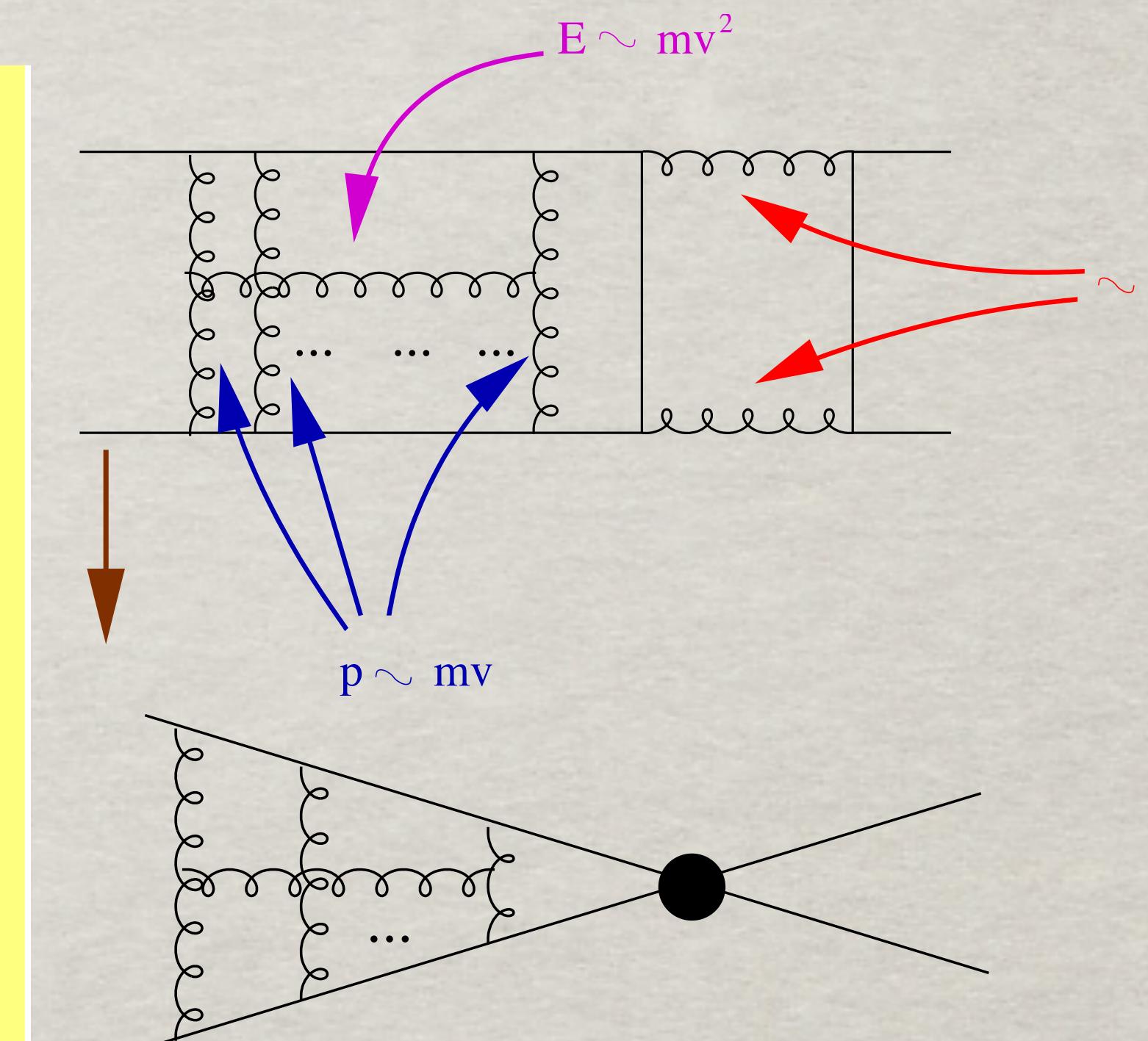
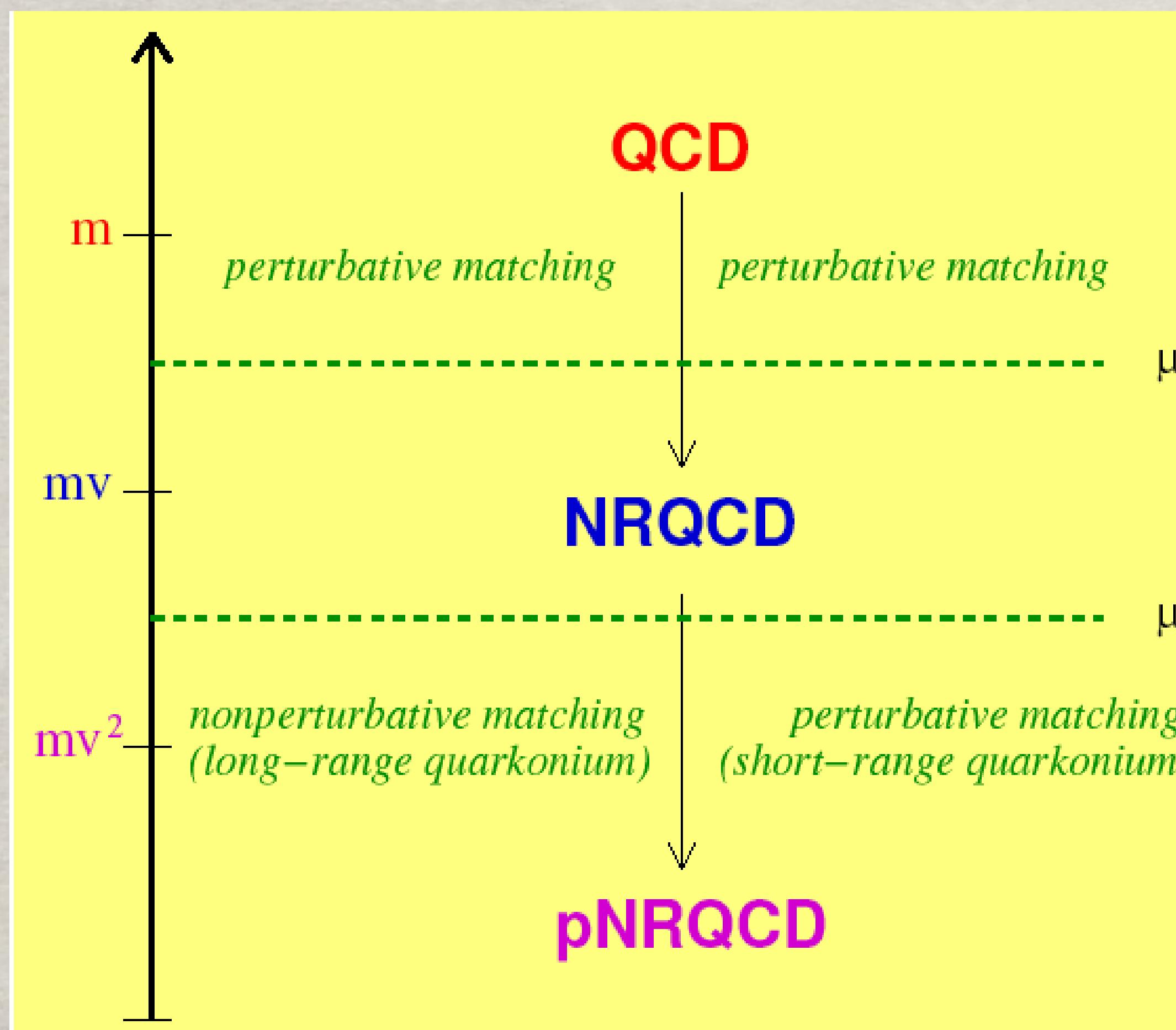
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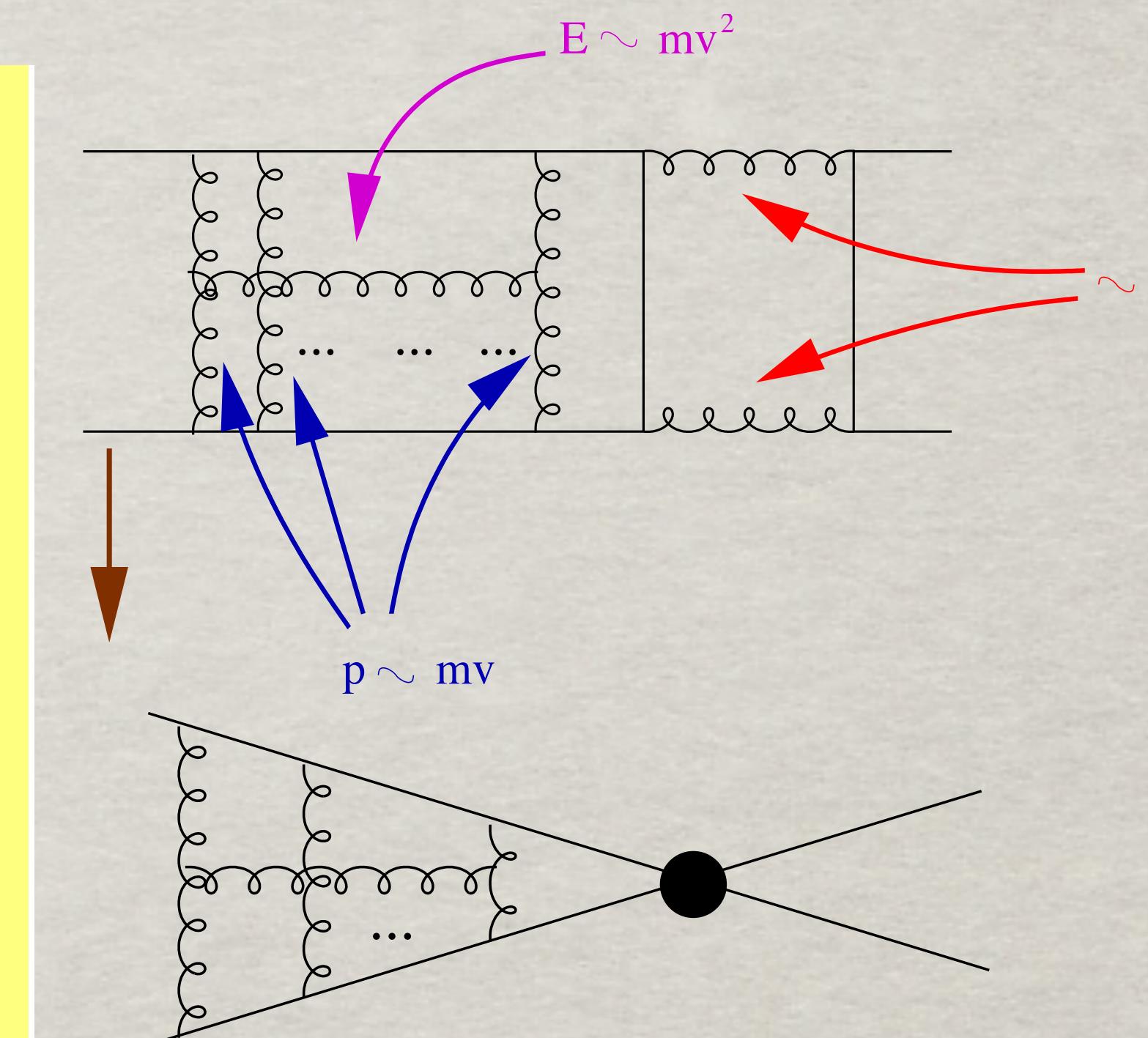
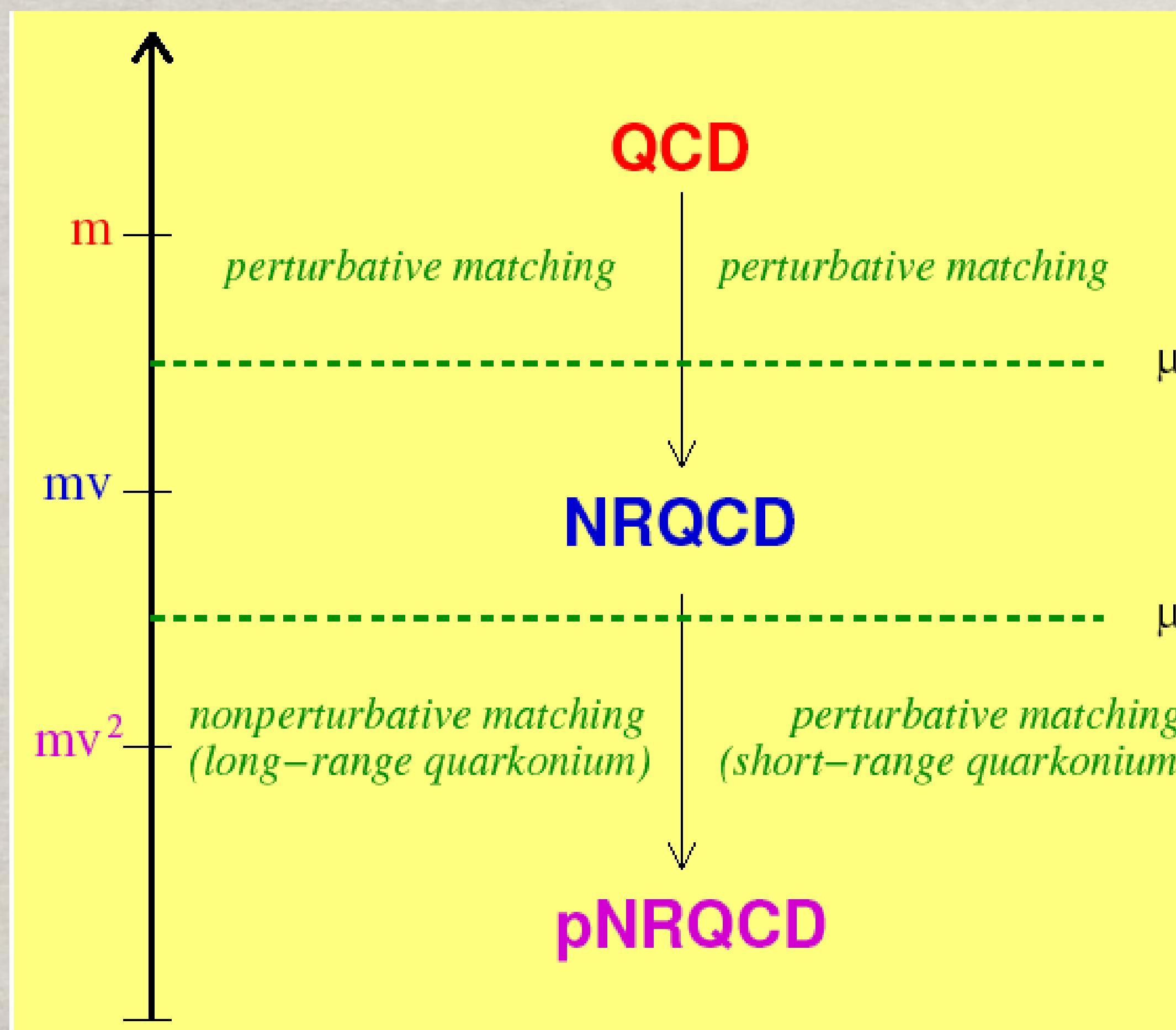
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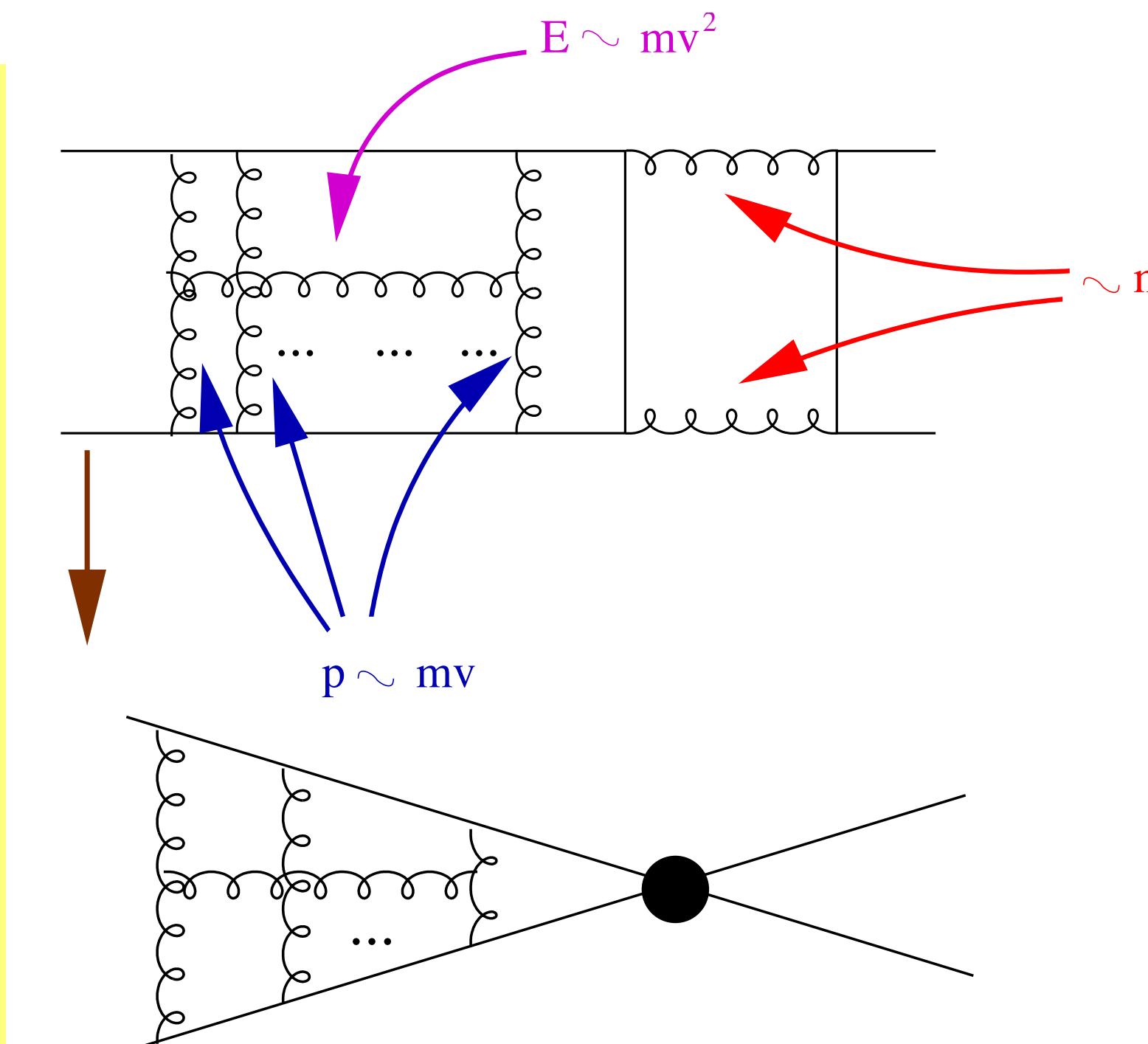
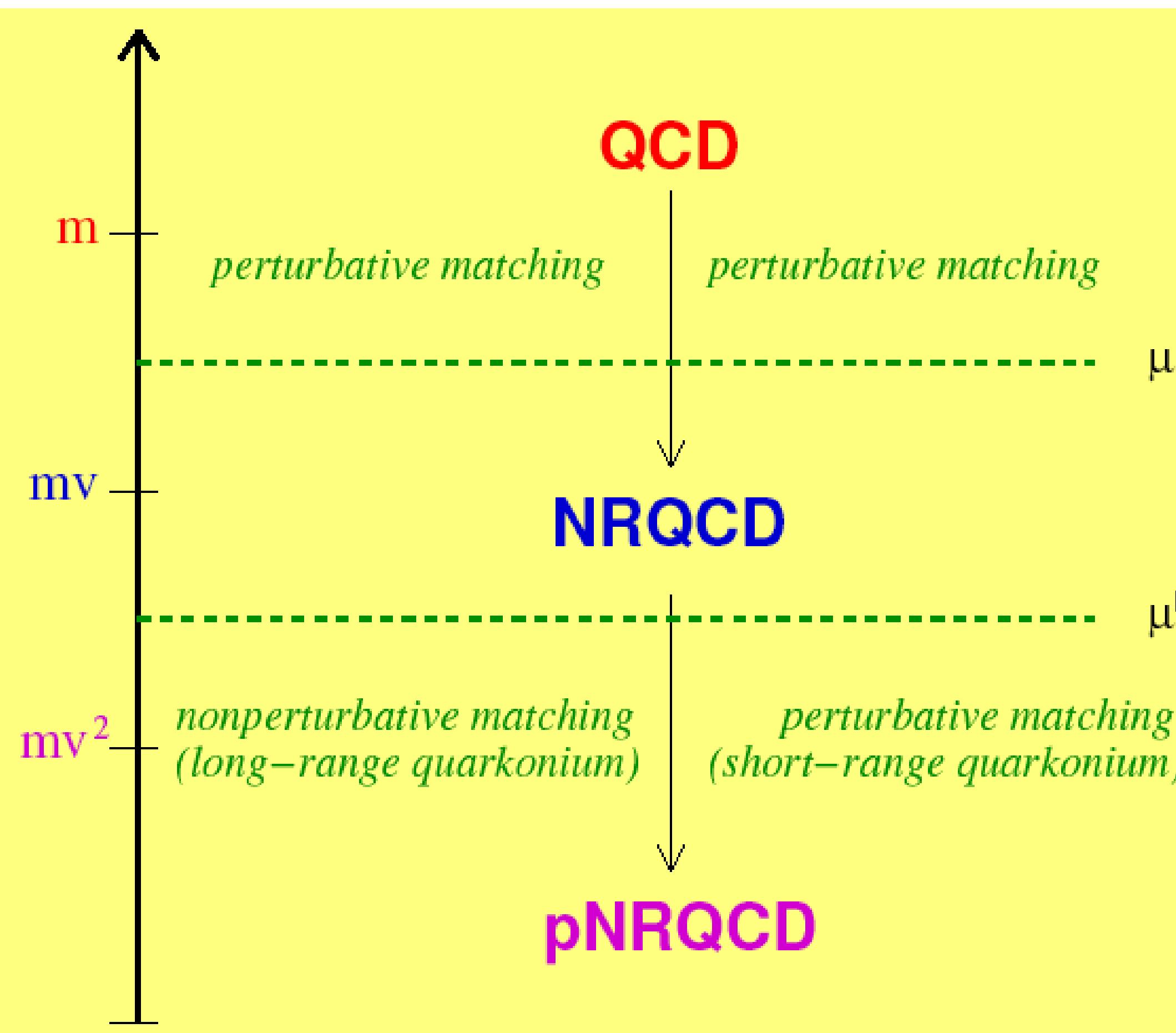


# Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

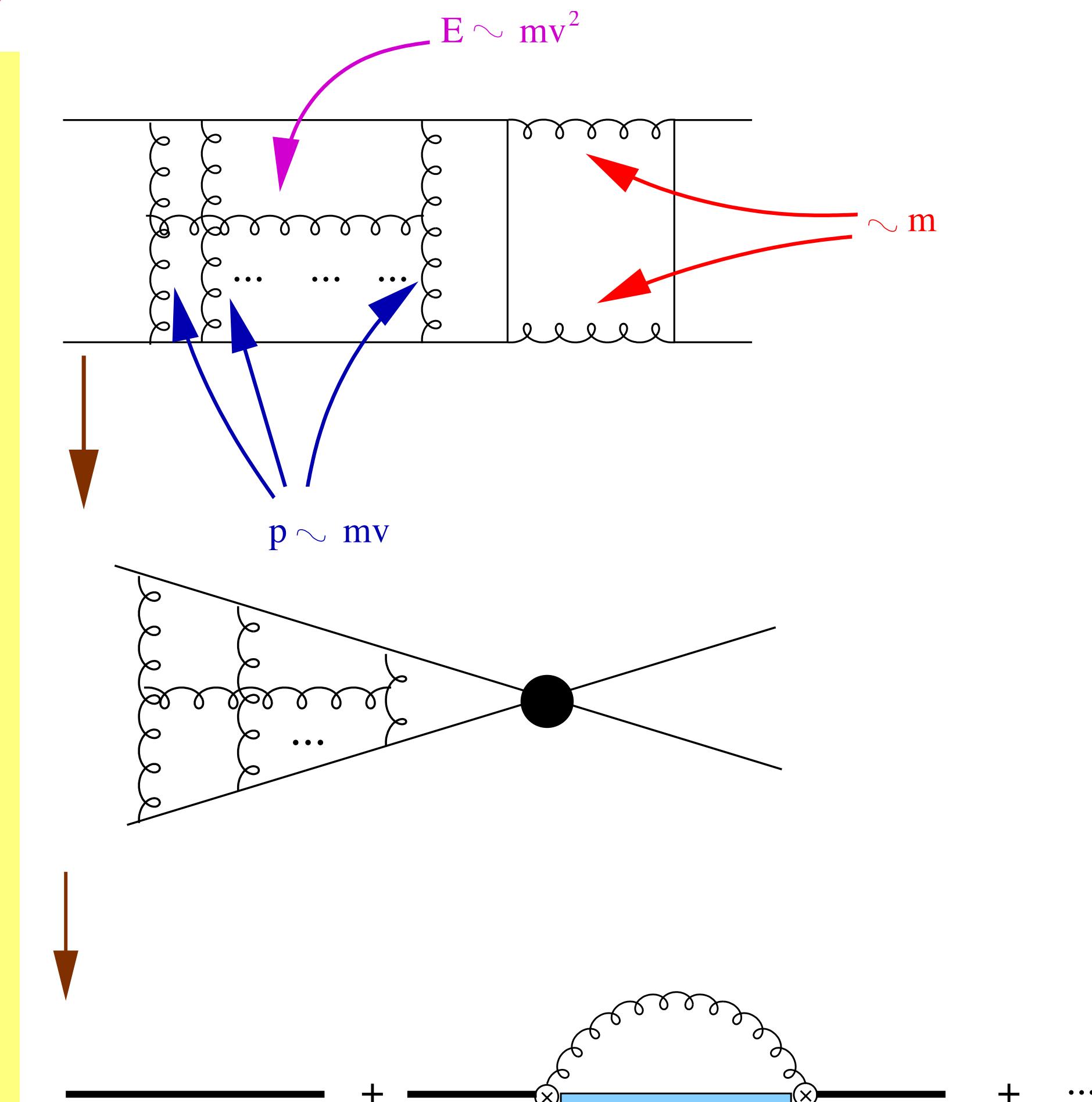
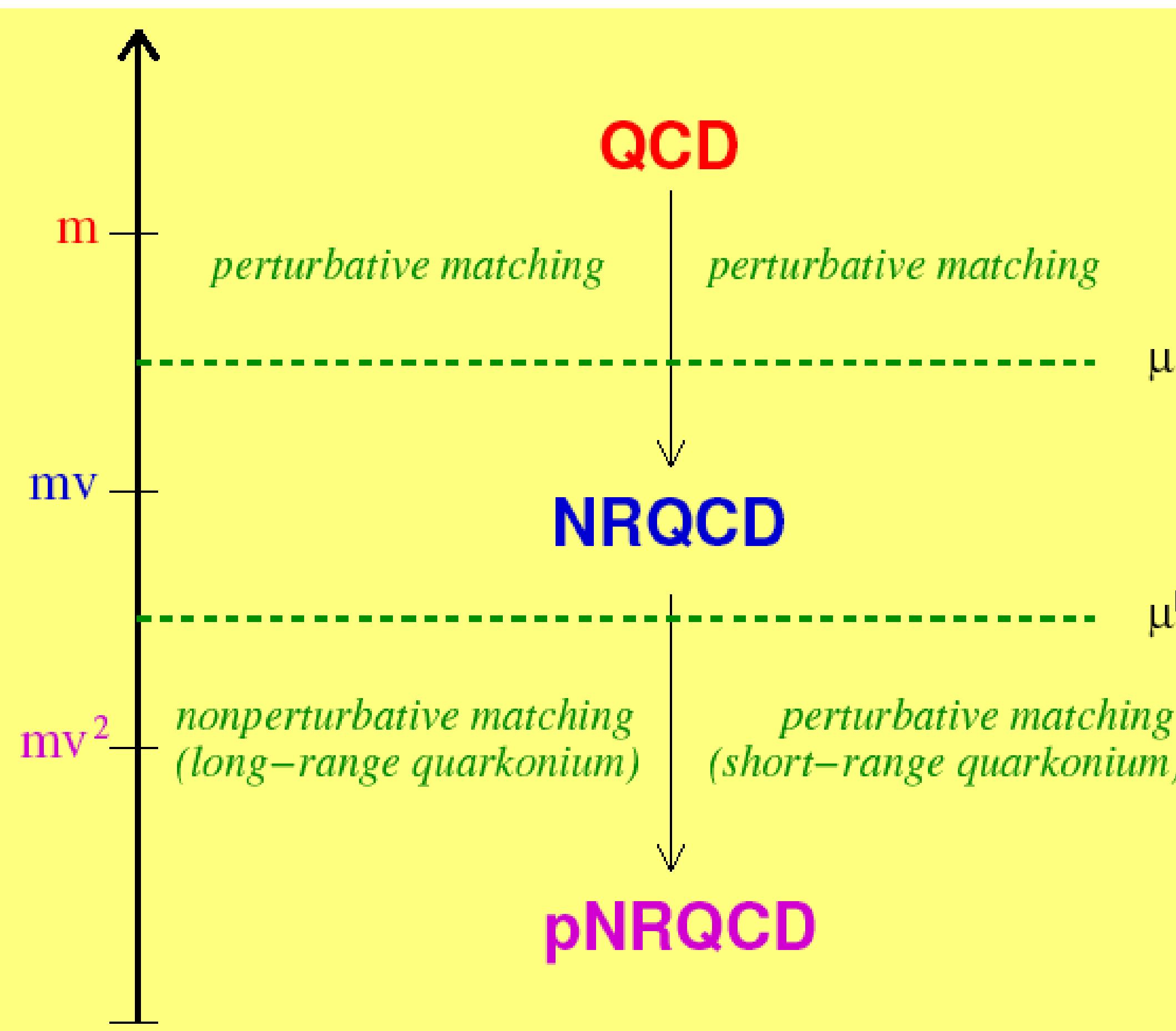


$$\mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

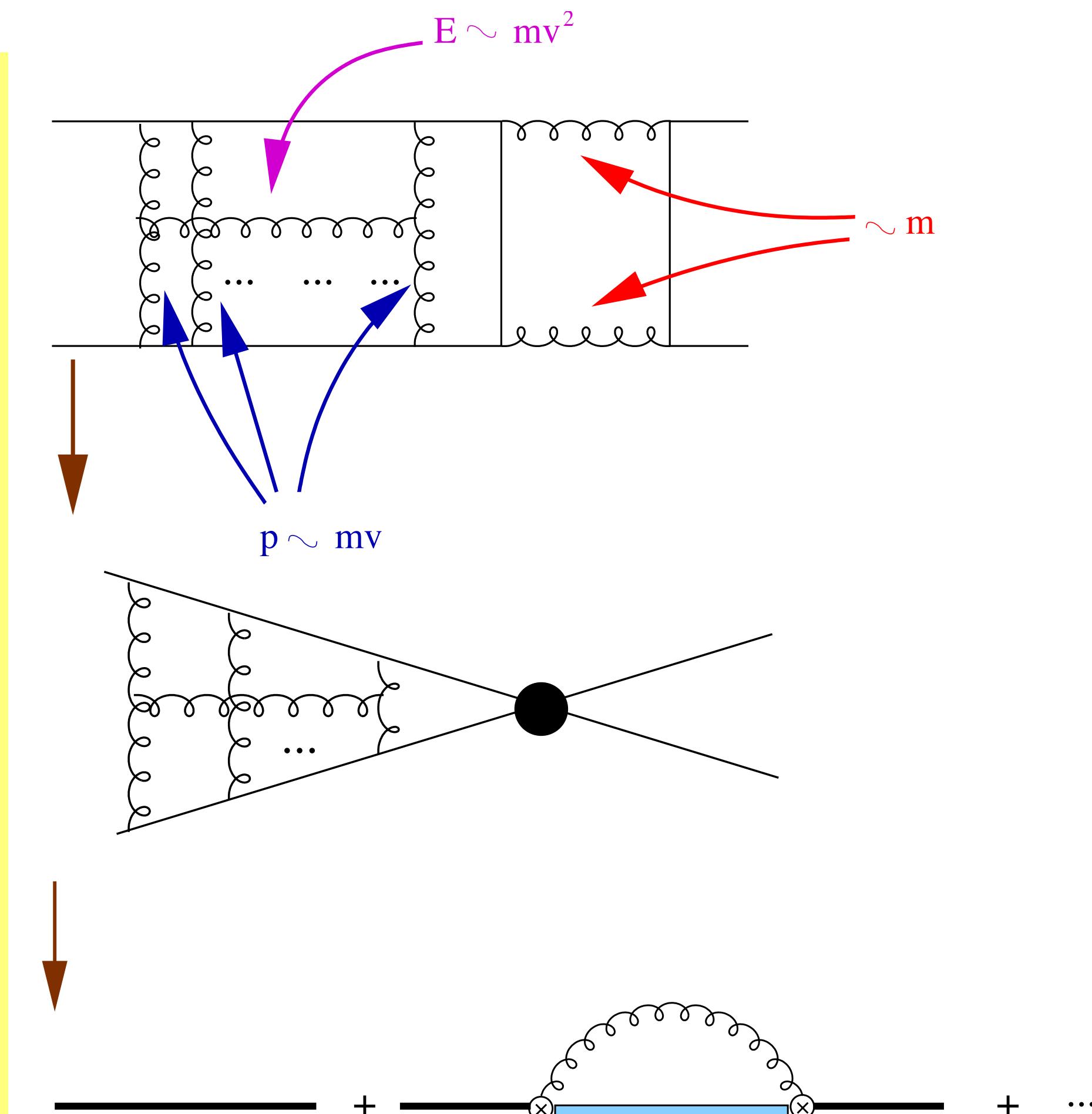
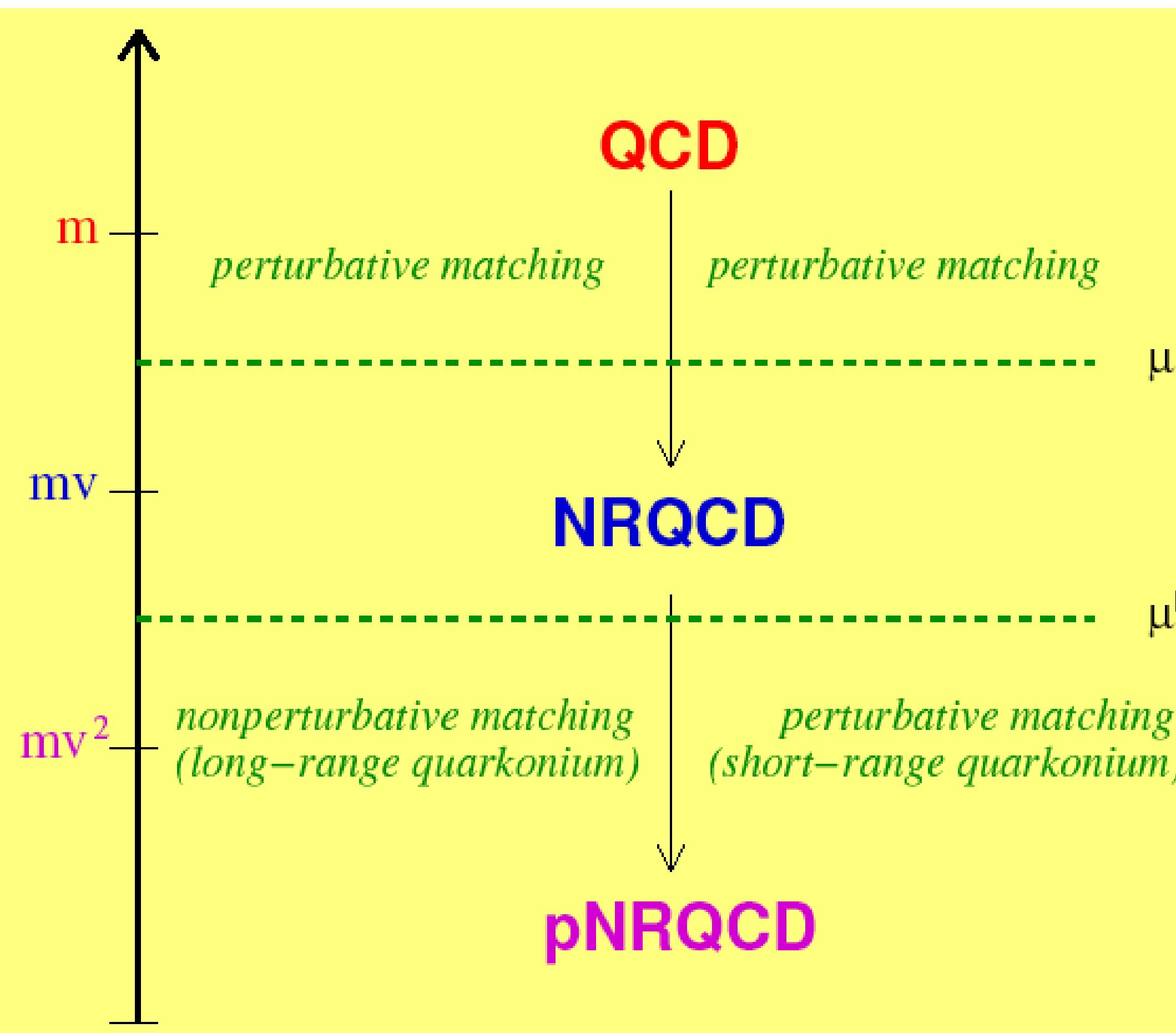
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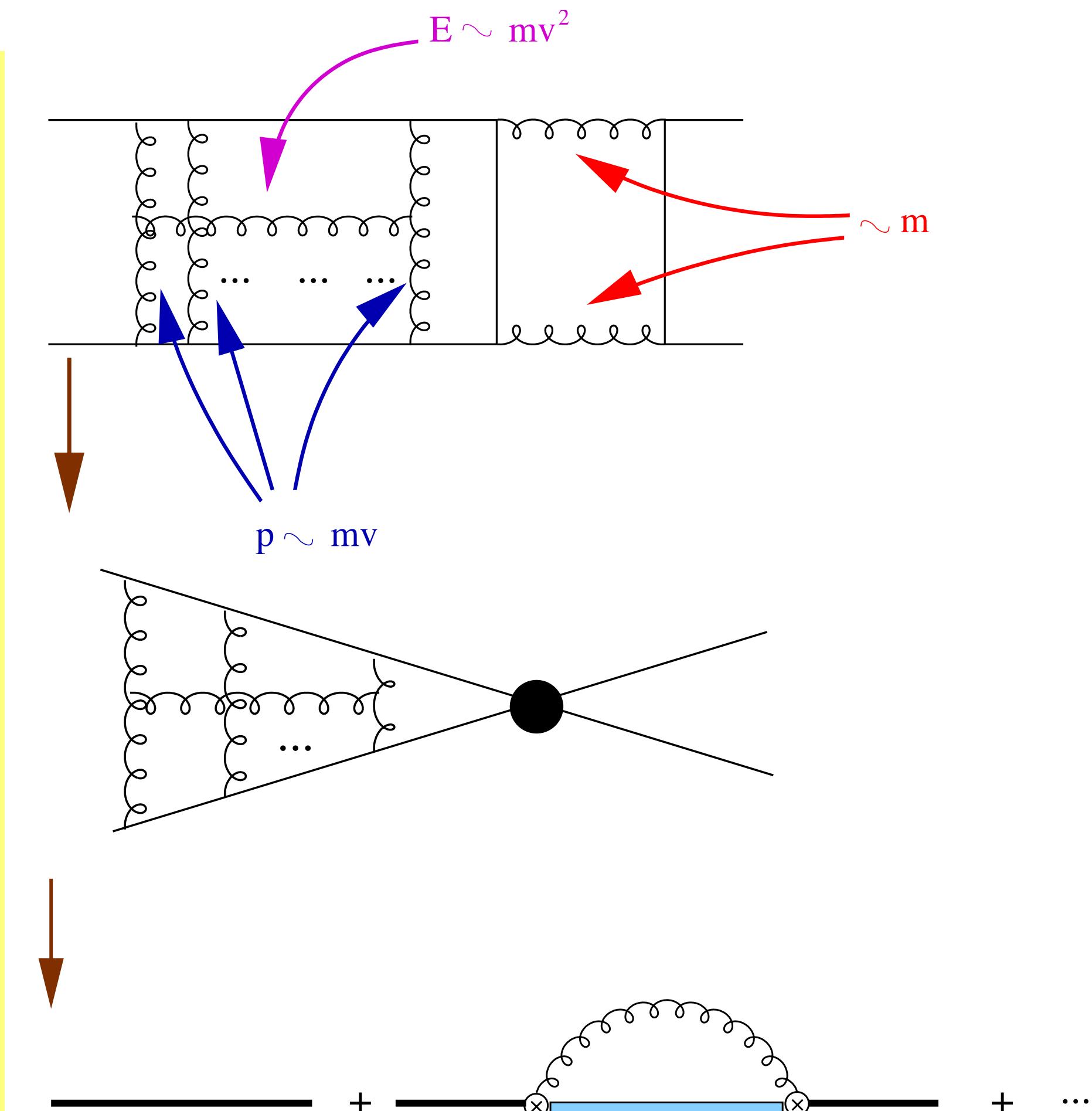
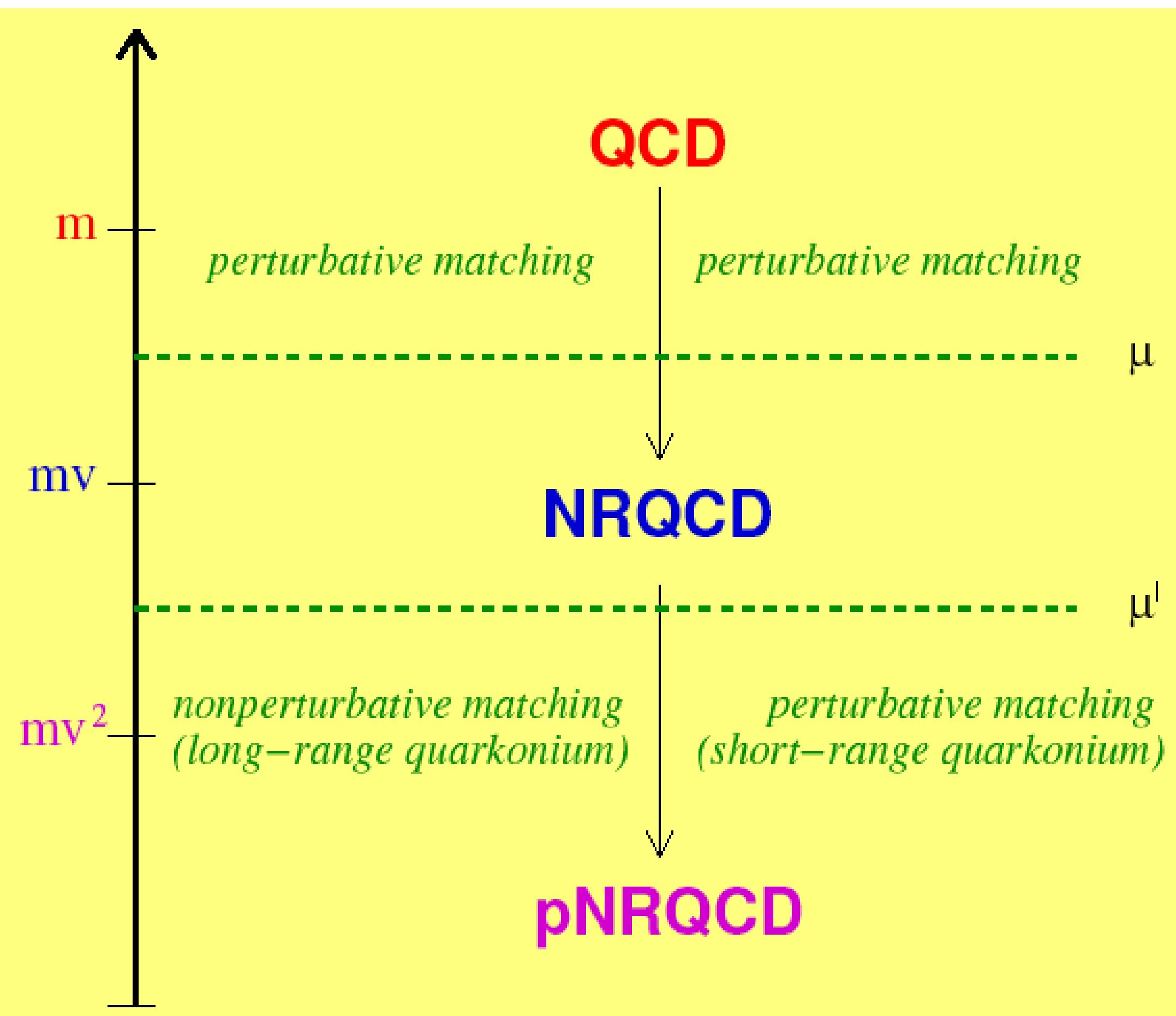


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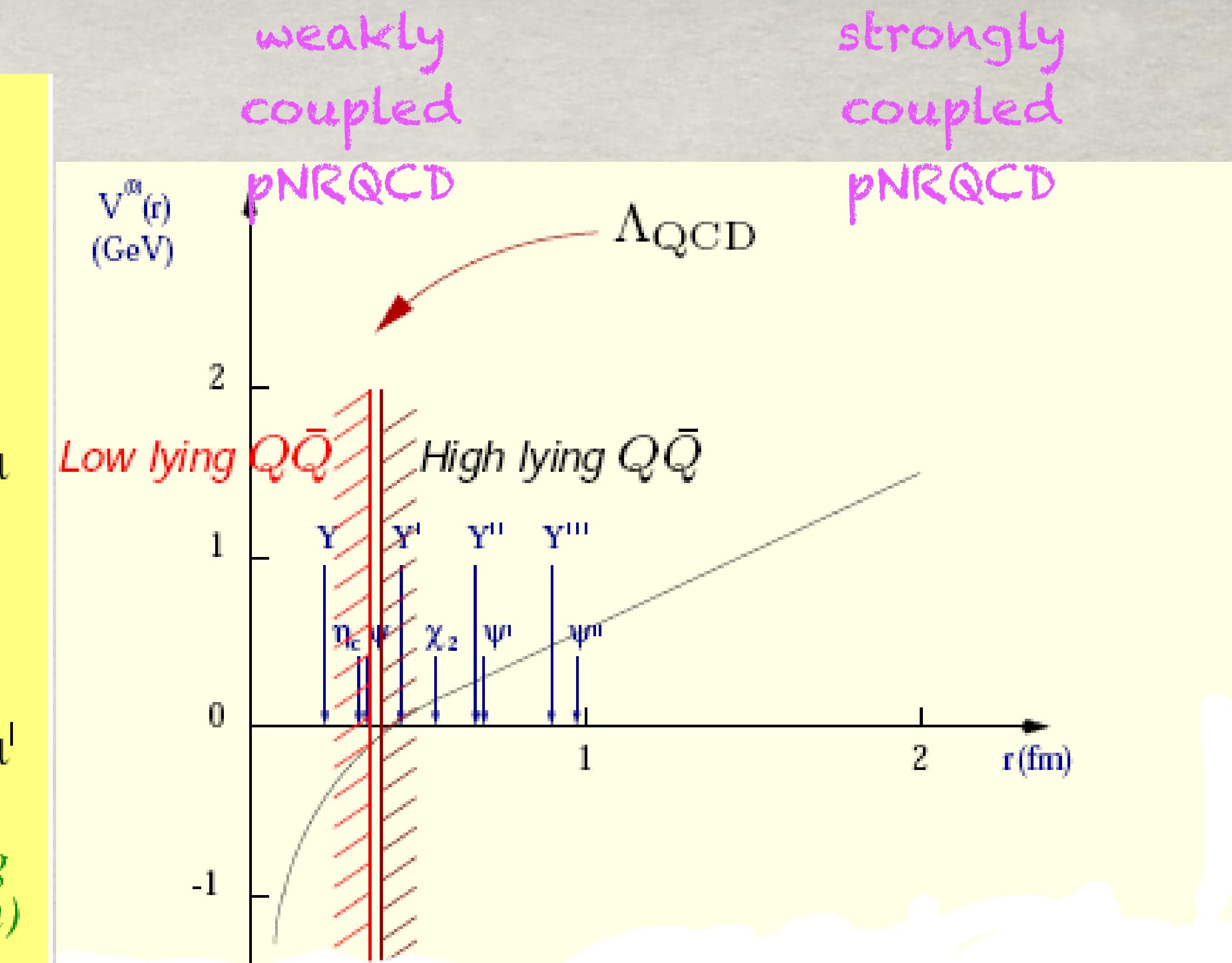
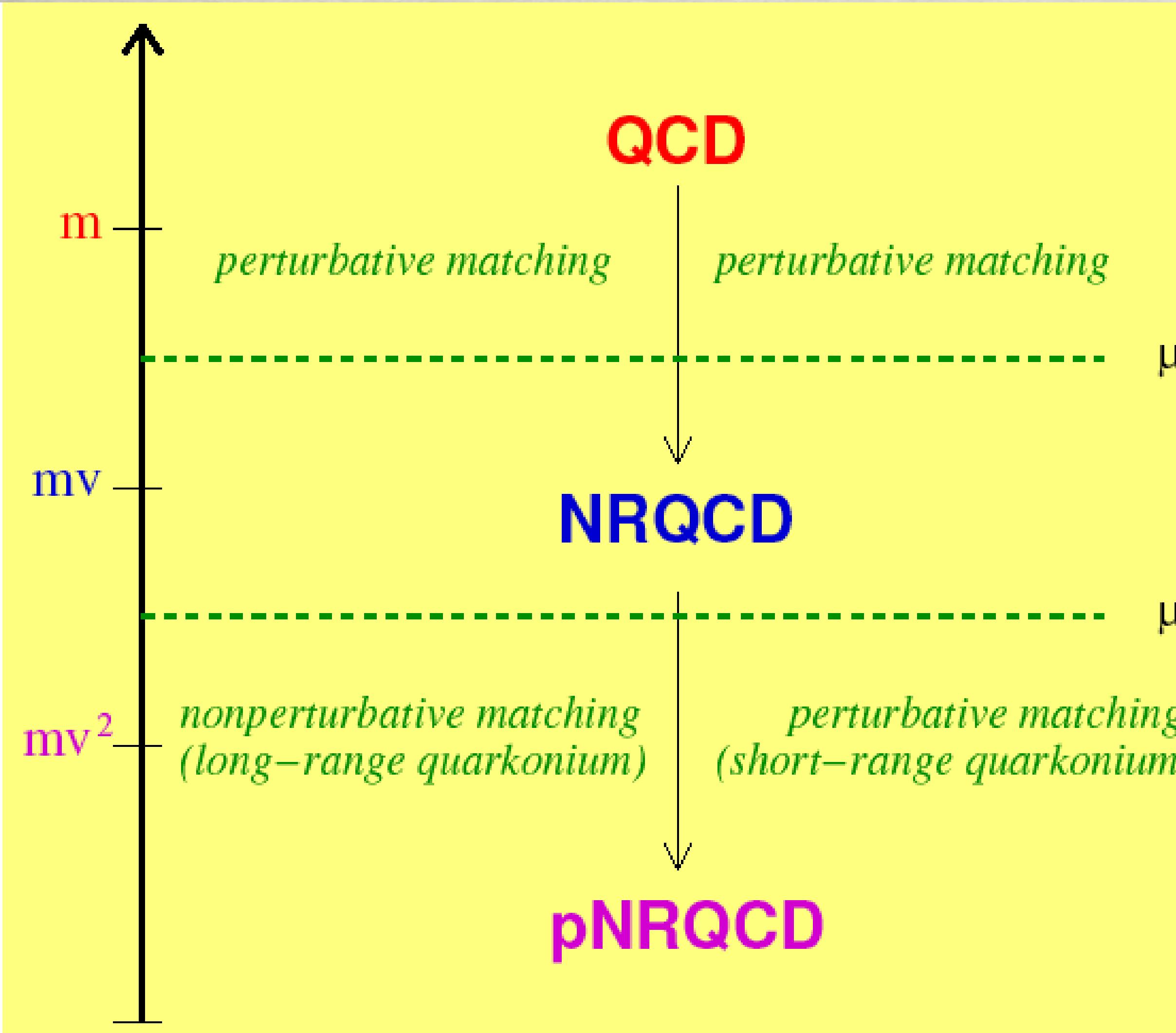
$$\mathcal{L}_{\text{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda) r^n$$

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# Quarkonium with NREFT: pNRQCD

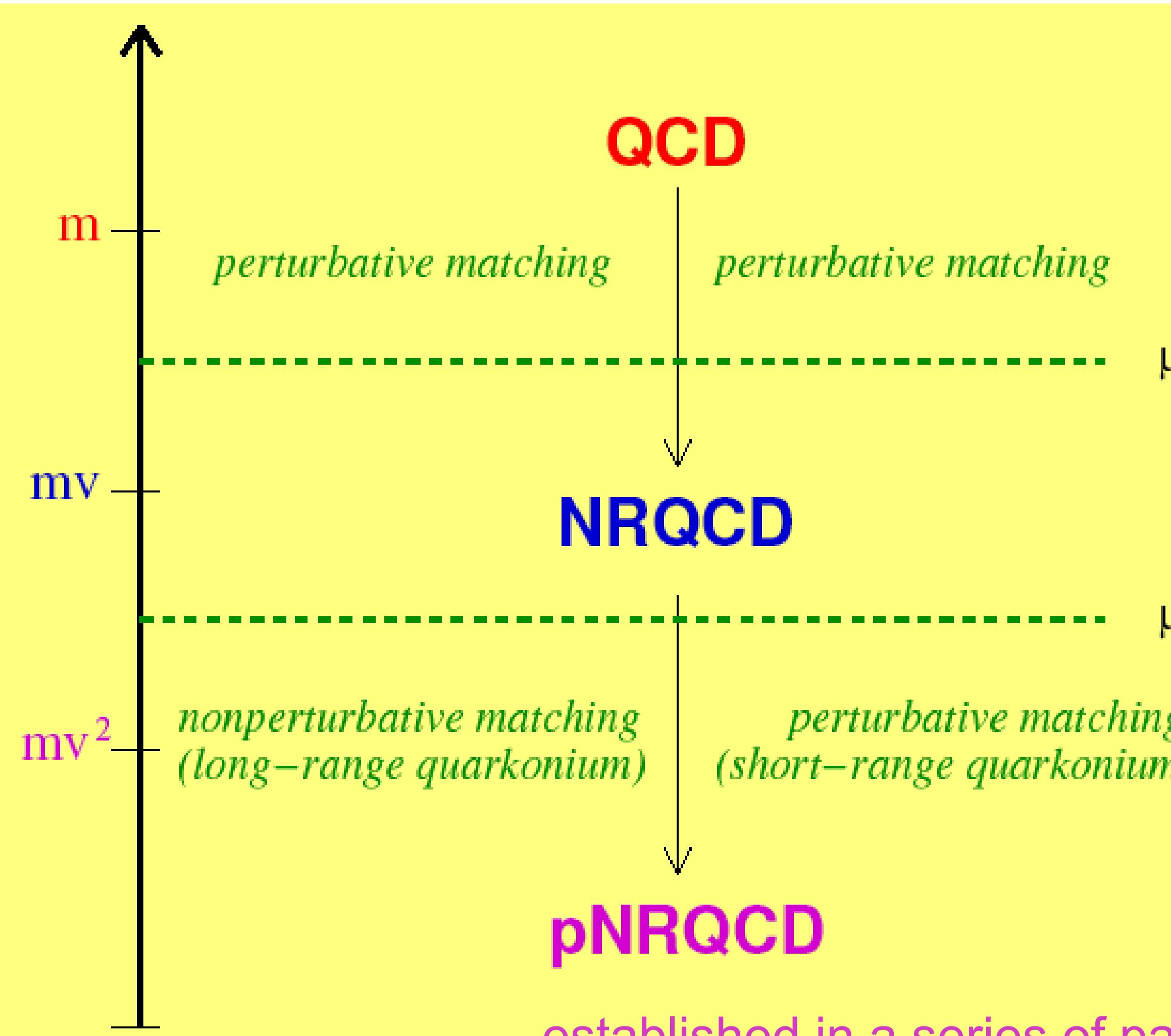


A potential picture arises at the level of pNRQCD:

- the potential is perturbative if  $mv \gg \Lambda_{\text{QCD}}$
- the potential is non-perturbative if  $mv \sim \Lambda_{\text{QCD}}$

In QCD another scale is relevant  $\Lambda_{\text{QCD}}$

# Quarkonium with NREFT



Caswell, Lepage 86,  
Lepage, Thacker 88  
Bodwin, Braaten, Lepage 95.....

Pineda, Soto 97, N.B. et al, 99,00,  
Luke Manohar 97, Luke Savage 98,  
Beneke Smirnov 98, Labelle 98  
Labelle 98, Grinstein Rothstein 98  
Kniehl, Penin 99, Griesshammer 00,  
Manohar Stewart 00, Luke et al 00,  
Hoang et al 01, 03->

established in a series of papers:

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99

N.B. Vairo, et al. 00–023

N.B., Pineda, Soto, Vairo Review of Modern Physics 77(2005) 1423

pNRQCD addresses the bound state dynamics

$$\mathcal{L}_{\text{pNREFT}} = \int d^3r \phi^\dagger \left( i\partial_0 - \frac{\mathbf{p}^2}{m} - V \right) \phi + \Delta\mathcal{L}$$

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- It is obtained by integrating out hard and soft gluons with  $p$  or  $E$  scaling like  $m, mv$ .

- The d.o.f. are  $Q\bar{Q}$  pairs (sometimes cast in color singlet  $S$  and color octet  $O$ ) and ultrasoft modes (e.g. light quarks, low-energy gluons):

$$\phi = S$$

- The Lagrangian is organized as an expansion in  $1/m$  and  $r$ .
- The form of  $\Delta\mathcal{L}$  and of the ultrasoft modes depends on the low energy dynamics.

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- The leading picture is Schrödinger eq., the potentials appear once all scales above the energy have been integrated out
- non potential effects appear as correction to the leading picture and are nonperturbative
- Any prediction of pNRQCD is a prediction of QCD at the given order of expansion
- Effects at the nonperturbative scale are carried by gauge invariant purely glue dependent correlators to be calculated on the lattice or in QCD vacuum models

# Weakly coupled pNRQCD

- If  $mv \gg \Lambda_{\text{QCD}}$ , the matching is perturbative

Non-analytic behaviour in  $r \rightarrow$  matching coefficients  $V$

Pineda Soto NP PS 64 (1998) 428

Brambilla Pineda Soto Vairo NPB 566 (2000) 275

$\mathbf{R}$  = center of mass

$\mathbf{r} = Q\bar{Q}$  distance

The gauge fields are multipole expanded:

$$A(R, r, t) = A(R, t) + \mathbf{r} \cdot \nabla A(R, t) + \dots$$

LO in  $\mathbf{r}$

NLO in  $\mathbf{r}$

$$\begin{aligned} \mathcal{L}^{\text{pNRQCD}} = & \int d^3r \text{Tr} \left\{ S^\dagger \left( i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \dots \right) S + O^\dagger \left( iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \dots \right) O + \right. \\ & + V_A (S^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger \mathbf{r} \cdot g\mathbf{E} S) + \frac{V_B}{2} (O^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger O \mathbf{r} \cdot g\mathbf{E}) \} + \dots \\ & - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \sum_{i=1}^{n_f} \bar{q}_i i \not{D} q_i \end{aligned}$$

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The matching coefficients are the Coulomb potential

$$V_S(r) = -C_F \frac{\alpha_s}{r} + \dots,$$

$$V_o(r) = \frac{1}{2N} \frac{\alpha_s}{r} + \dots,$$

$$| V_A = 1 + \mathcal{O}(\alpha_s^2), V_B = 1 + \mathcal{O}(\alpha_s^2).$$

## Feynman rules

$$= \theta(t) e^{-it(\mathbf{p}^2/m + V)}$$

$$= \theta(t) e^{-it(\mathbf{p}^2/m + V_o)} \left( e^{-i \int dt A^{\text{adj}}} \right)$$

$$= O^\dagger \mathbf{r} \cdot g\mathbf{E} S$$

$$= O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, \mathbf{O} \}$$

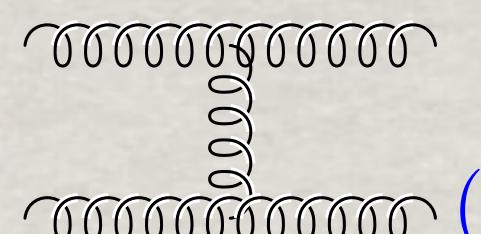
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The degrees of freedoms now are

→ nonperturbative problem, use lattice

Hitting the scale  $\Lambda_{\text{QCD}}$

$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \text{Glueball}$

$r \sim \Lambda_{QCD}^{-1}$

$(Q\bar{Q})_8 G$   
**Hybrids**  
 $h_v h_r$

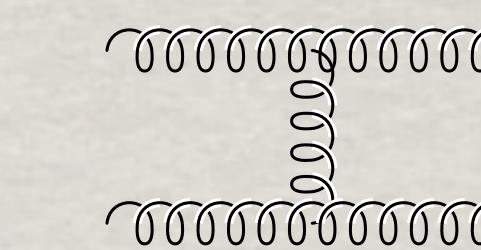
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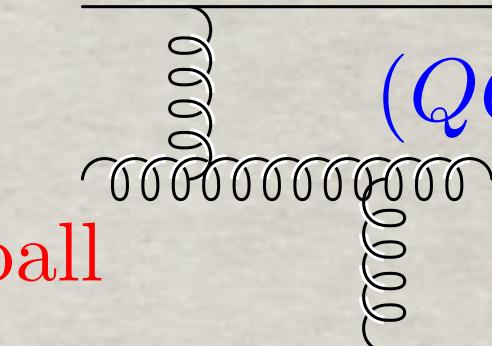
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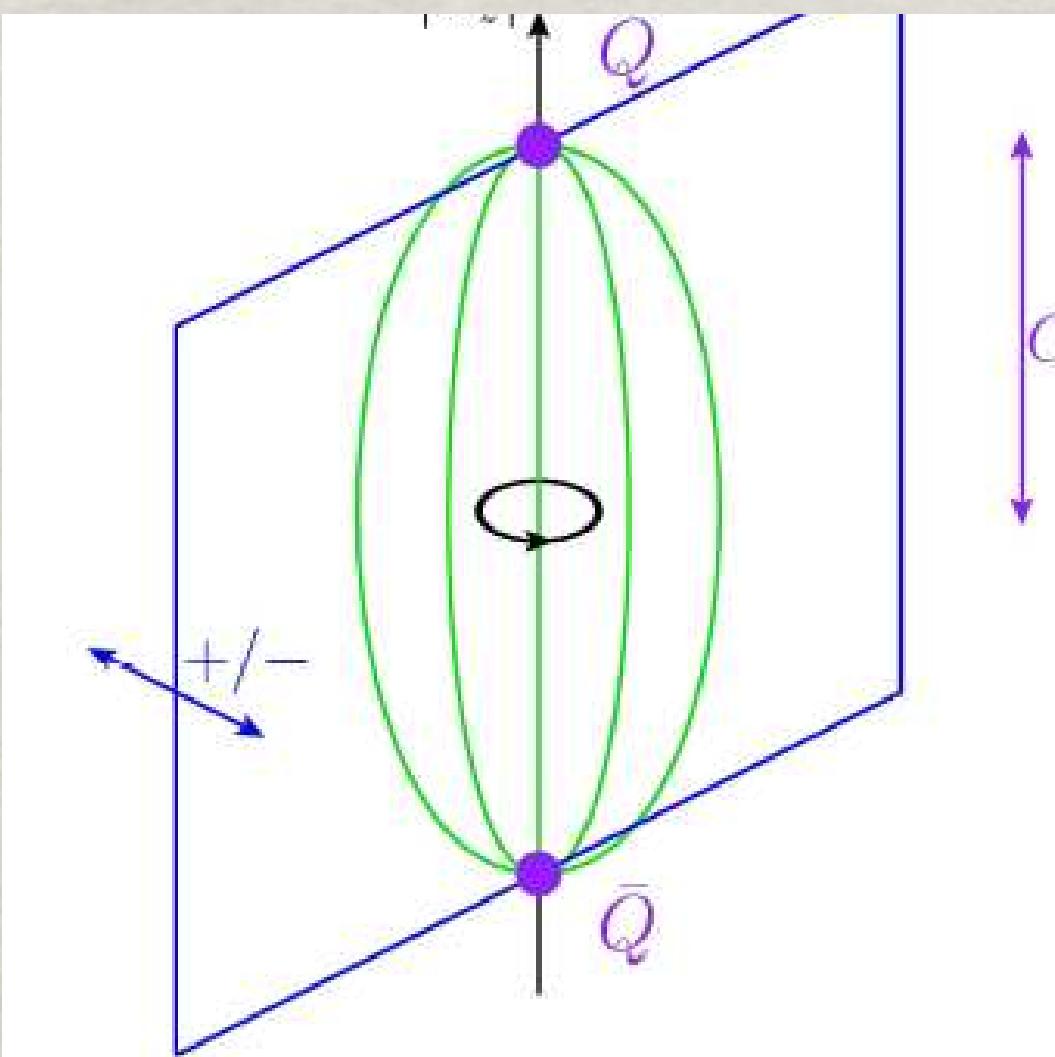
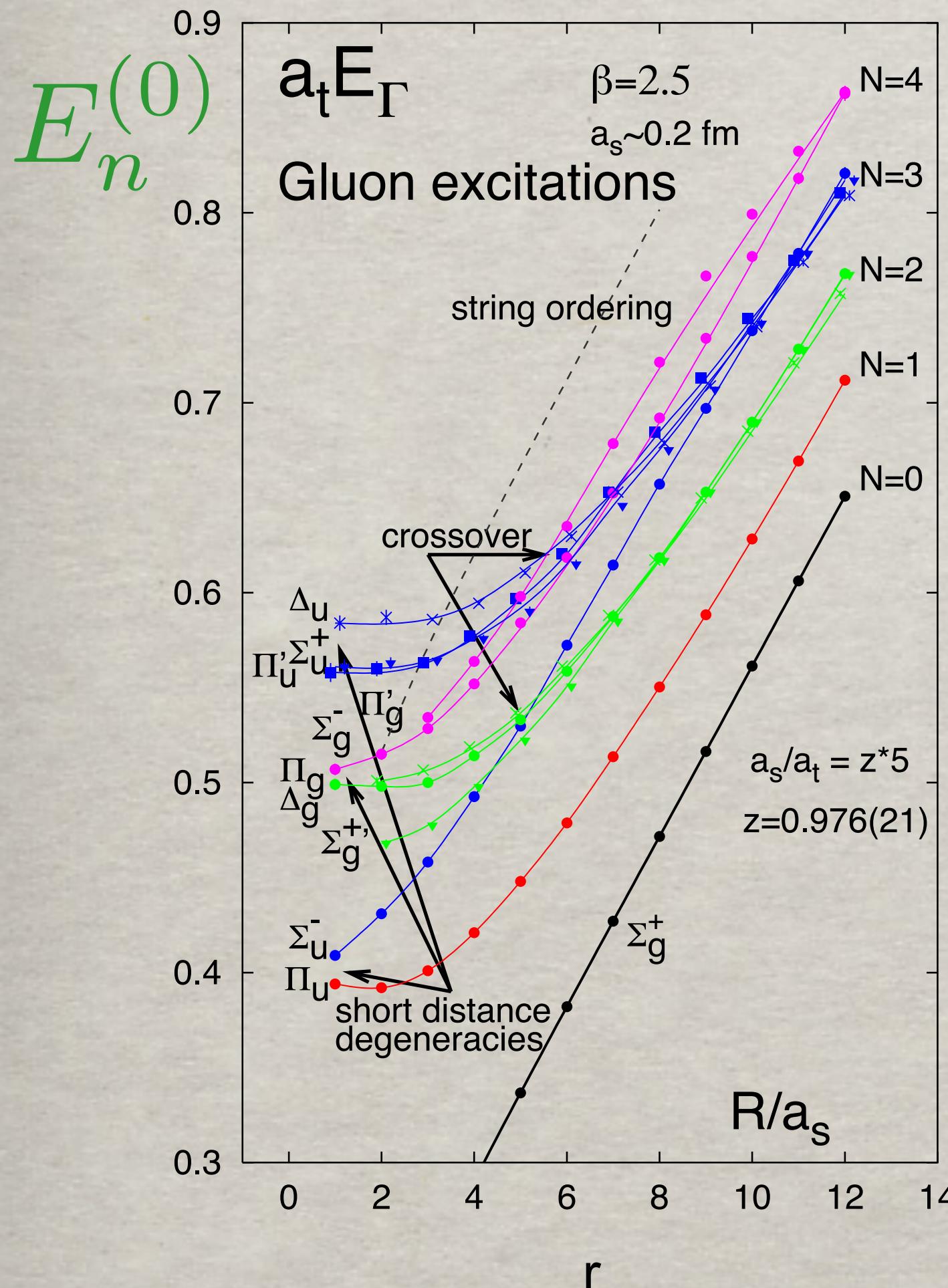
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Irreducible representations of  $D_{\infty h}$

- $\mathbf{K}$ : angular momentum of light d.o.f.  
 $\lambda = \hat{\mathbf{r}} \cdot \mathbf{K} = 0, \pm 1, \pm 2, \pm 3, \dots$   
 $\Lambda = |\lambda| = 0, 1, 2, 3, \dots$  ( $\Sigma, \Pi, \Delta, \Phi, \dots$ )
- Eigenvalue of  $CP$ :  $\eta = +1 (g), -1 (u)$
- $\sigma$ : eigenvalue of reflection about a plane containing  $\hat{\mathbf{r}}$  (only for  $\Sigma$  states)

$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1) \chi(\mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$\Lambda_\eta^\sigma \dashrightarrow n$

NRQCD states

$|0; \mathbf{x}_1 \mathbf{x}_2\rangle^- > |(Q\bar{Q})_1\rangle \rightarrow \text{Quarkonium Singlet}$  pNRQCD states

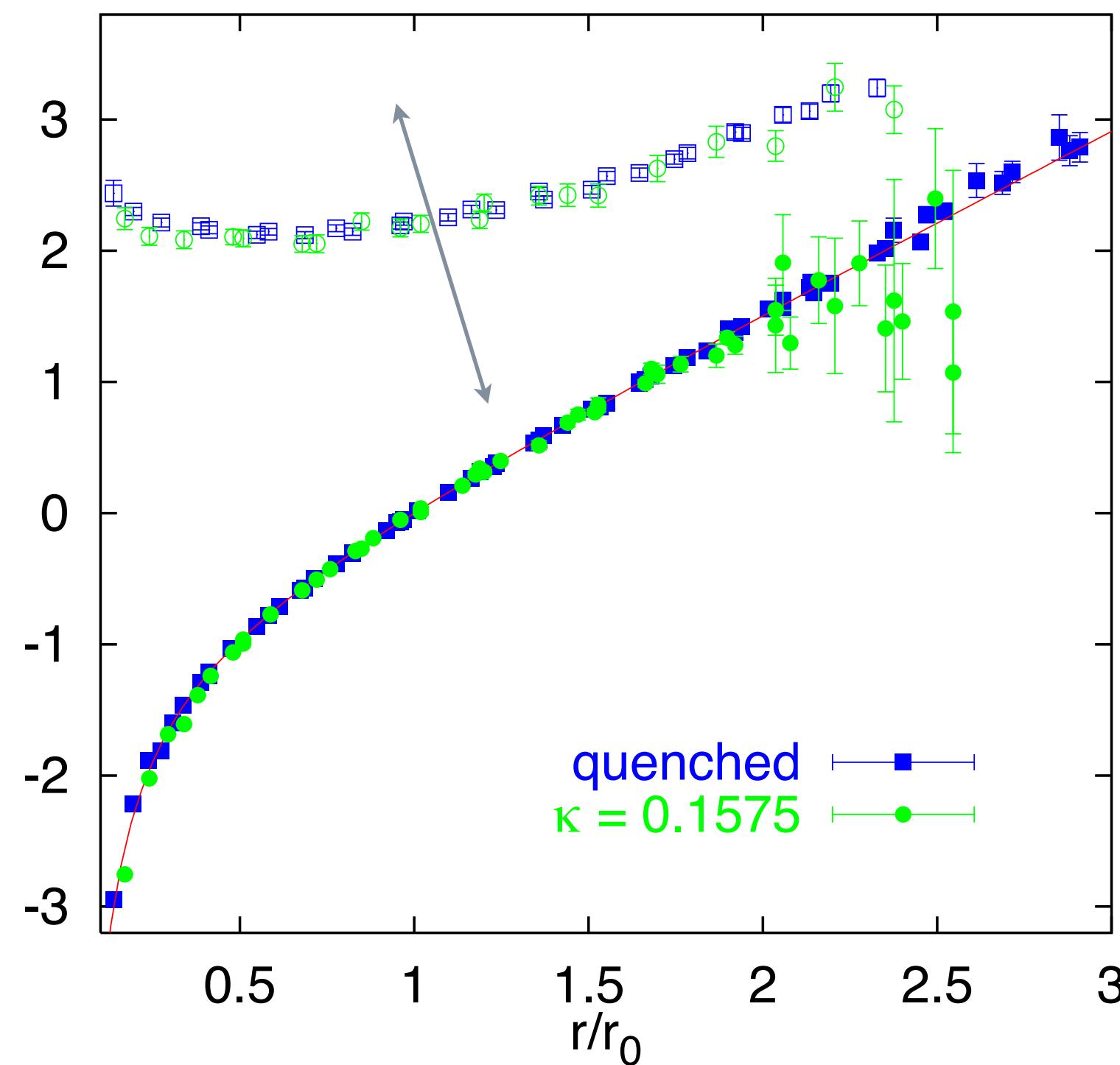
$|\underline{n} > 0; \mathbf{x}_1 \mathbf{x}_2\rangle^- > |(Q\bar{Q})g^{(n)}\rangle \rightarrow \text{Higher Gluonic Excitations}$

# Strongly coupled pNRQCD

$$r \sim \Lambda_{QCD}^{-1}$$

Bali et al. 98

- the potentials come from integrating out all scales up to  $mv^2$



- gluonic excitations develop a gap  $\Lambda_{QCD}$  and are integrated out

Brambilla Pineda Soto Vairo 00

- It can be applied spectra, decays and production

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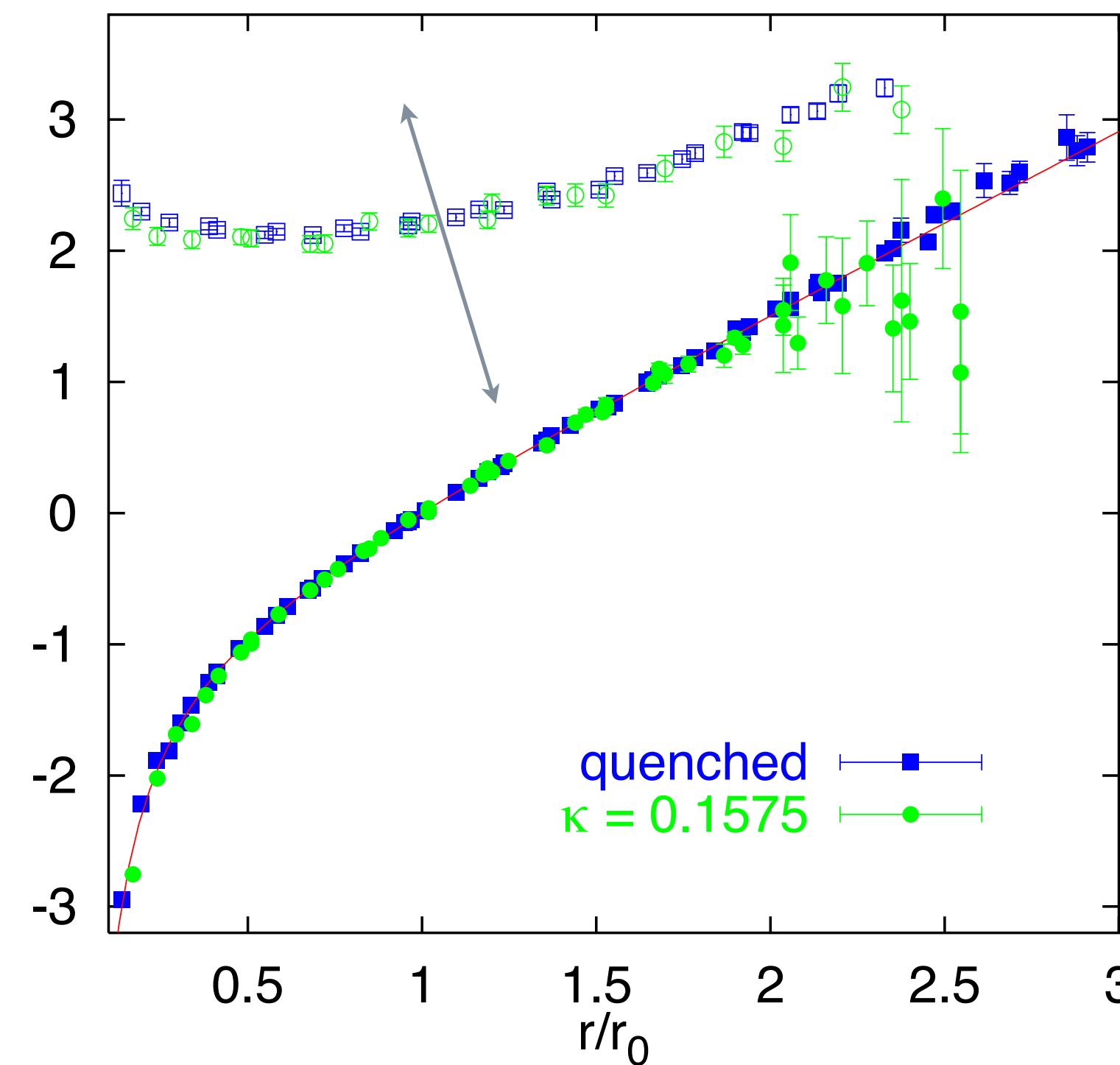
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$\Rightarrow$  The singlet quarkonium field  $S$  of energy  $mv^2$  is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).



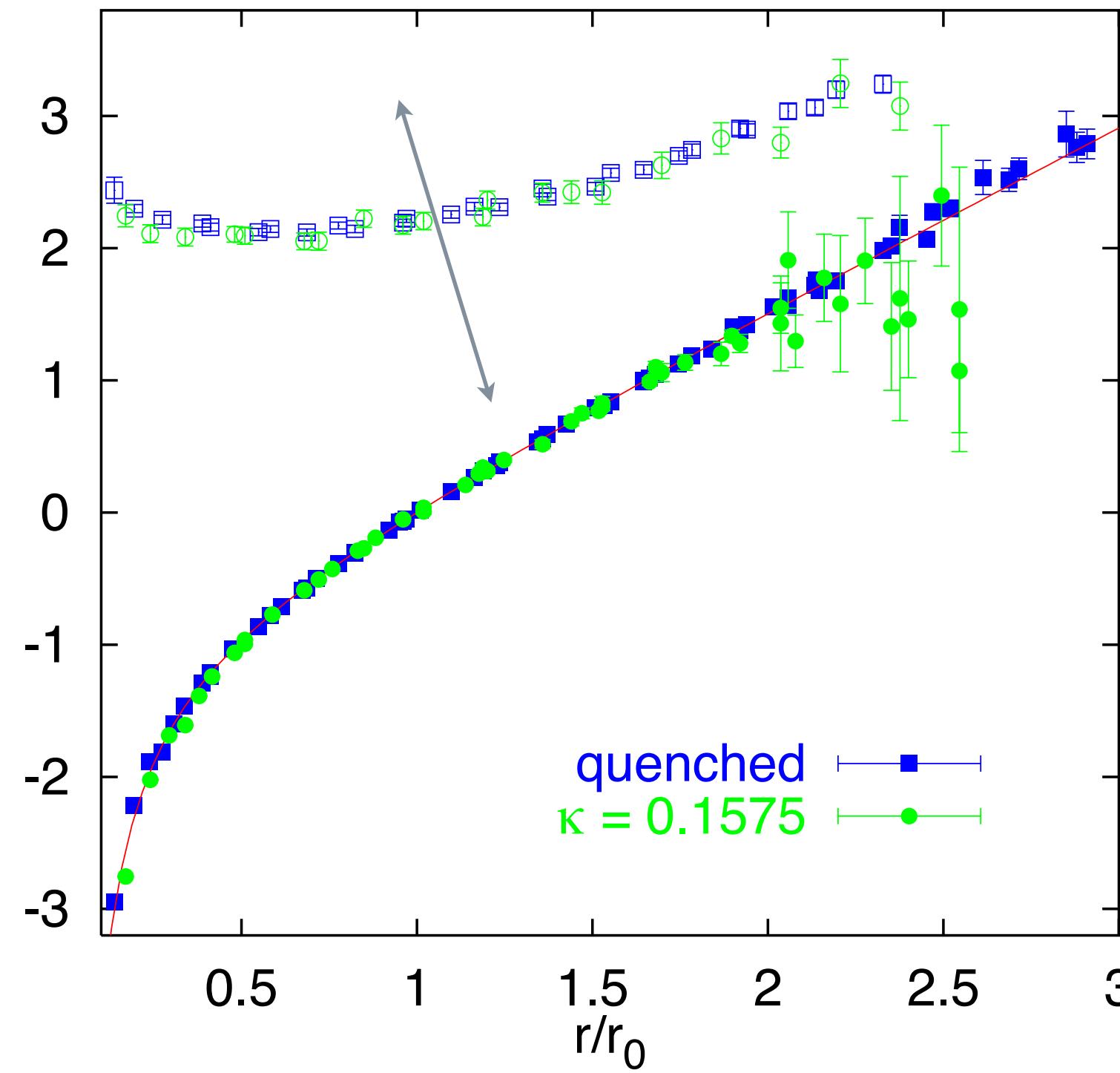
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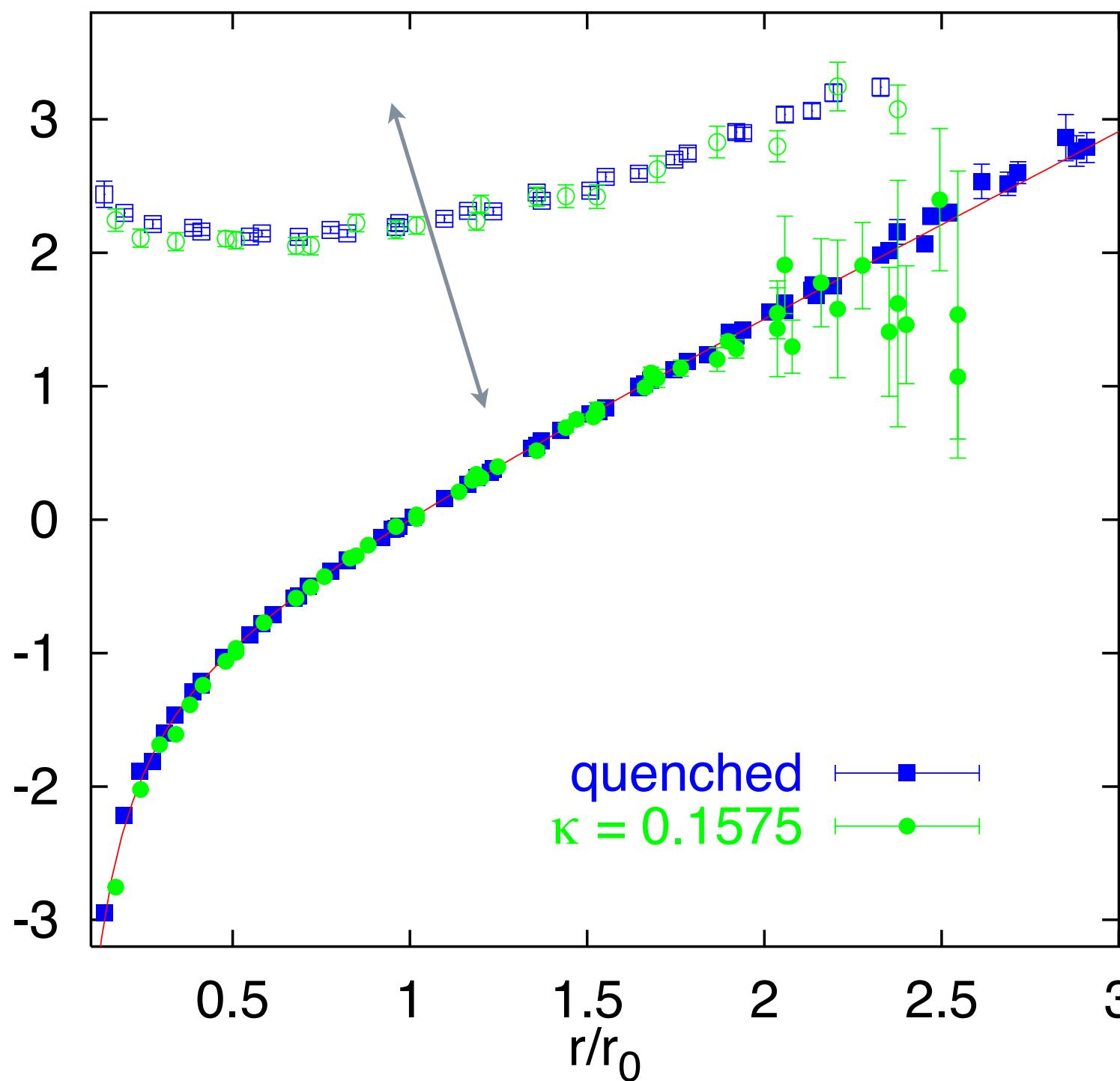
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- A pure potential description emerges from the EFT **however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters**
- The potentials  $V = \text{Re}V + \text{Im}V$  from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials  $V$  in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out
- It can be applied spectra, decays and production

# Born-Oppenheimer EFT (BOEFT): EFT for nonrelativistic pairs and light degrees of freedom

## For Exotic X Y Z states at the or above strong decay threshold

Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ( $Q\bar{Q}g$  states) or tetraquarks ( $Q\bar{Q}q\bar{q}$  states):

### Hierarchy of scales

$$\Lambda_{QCD} \gg mv^2$$

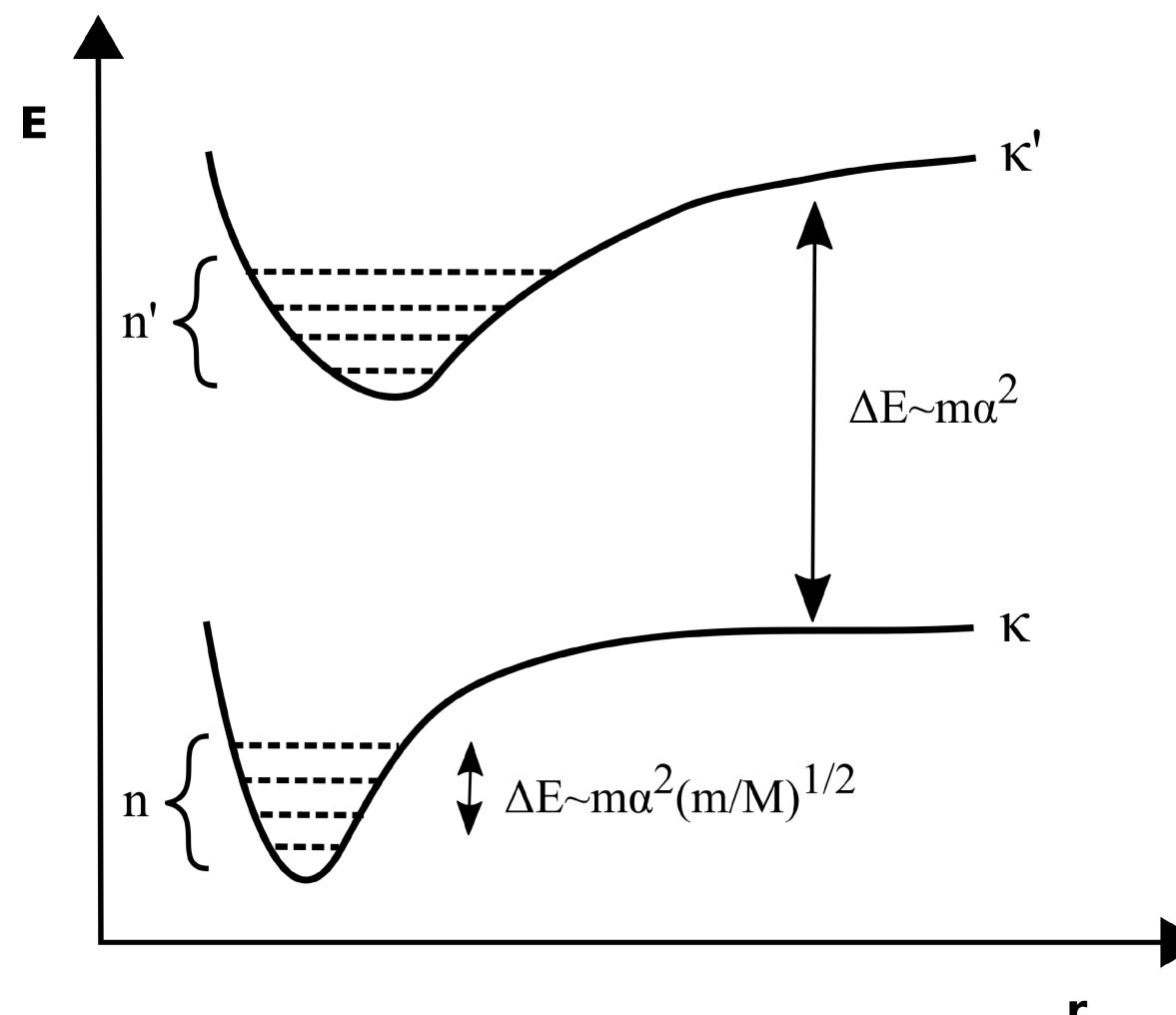
- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential  $V_\kappa$  between static sources, where  $\kappa$  labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials  $V_\kappa$  by solving the corresponding Schrödinger equation.

analogous to

$$E_{electrons} \gg E_{nuclei}$$

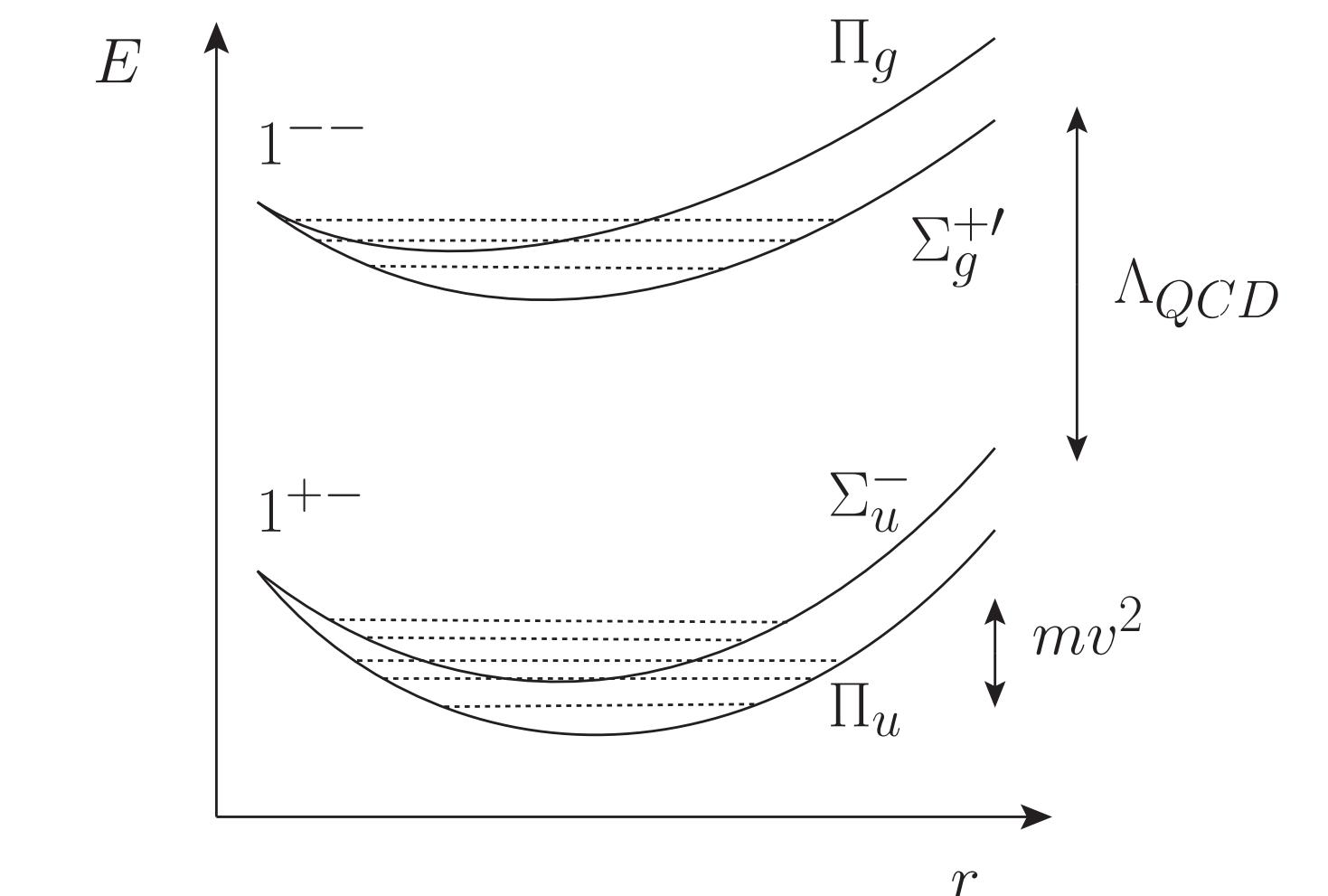
in QED

This picture goes also under the name of **Born-Oppenheimer approximation**. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called **Born–Oppenheimer EFT (BOEFT)**.

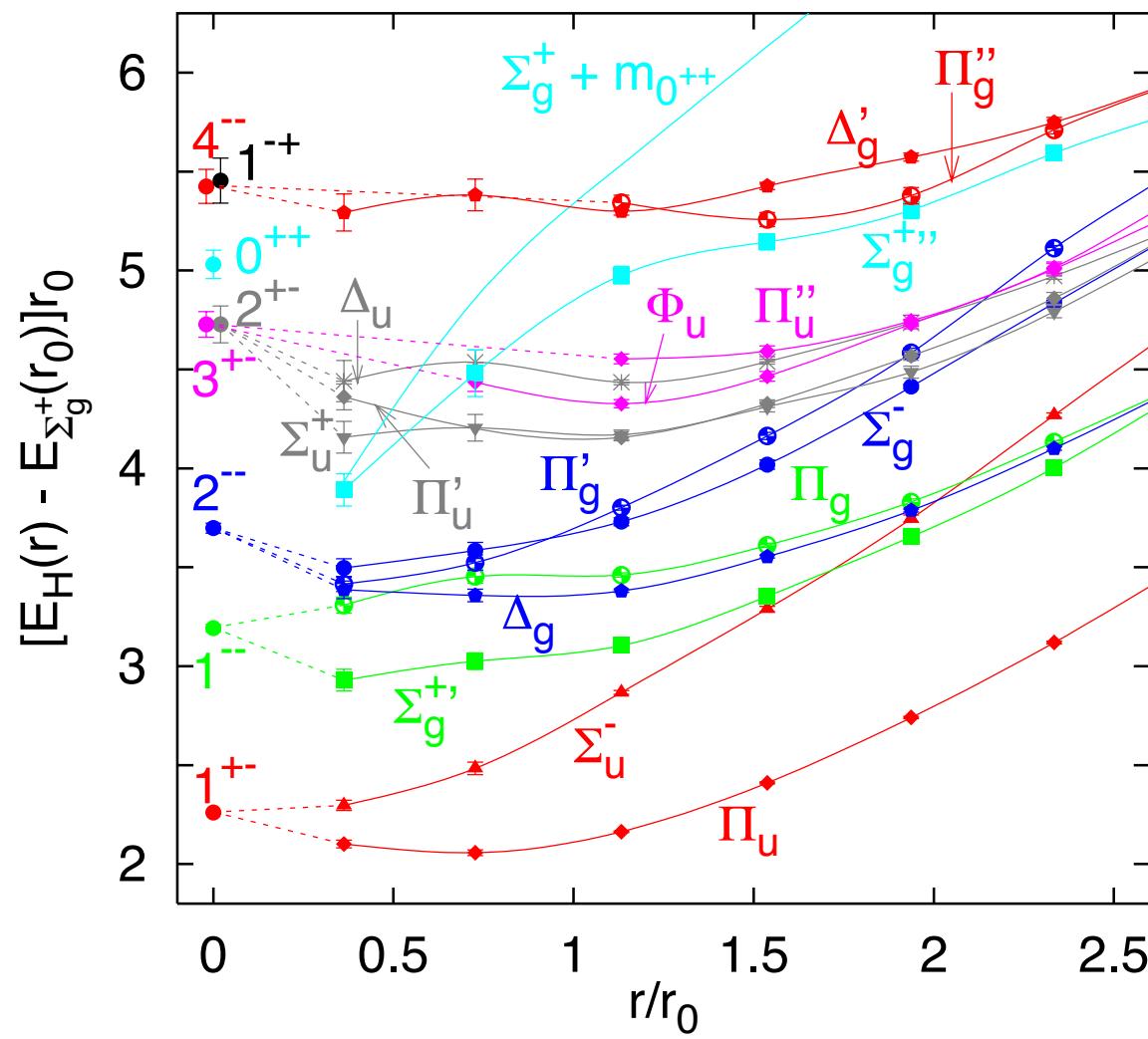


Lattice evaluation of the QCD static energies:  
Michael et al. 1983,  
Juge, Kuti, Mornigstar 1997, 1998,  
Bali Pineda 2004, Capitani, Philipsen, Reisinger,  
Riehl, Wagner 2018

BOEFT N. B.; A. Vairo, J. Tarrus,  
M. . Berwein, Wk. Lai, A. Mohapatra 2015—



# BOEFT : hybrids



In the short-range hybrids become **gluelumps**, i.e., quark-antiquark octets,  $O^a$ , in the presence of a gluonic field,  $H^a$ :  $H(R, r, t) = H^a(R, t)O^a(R, r, t)$ .

octet potential  $\rightarrow$  static energy can be written as a (multipole) expansion in  $r$ :

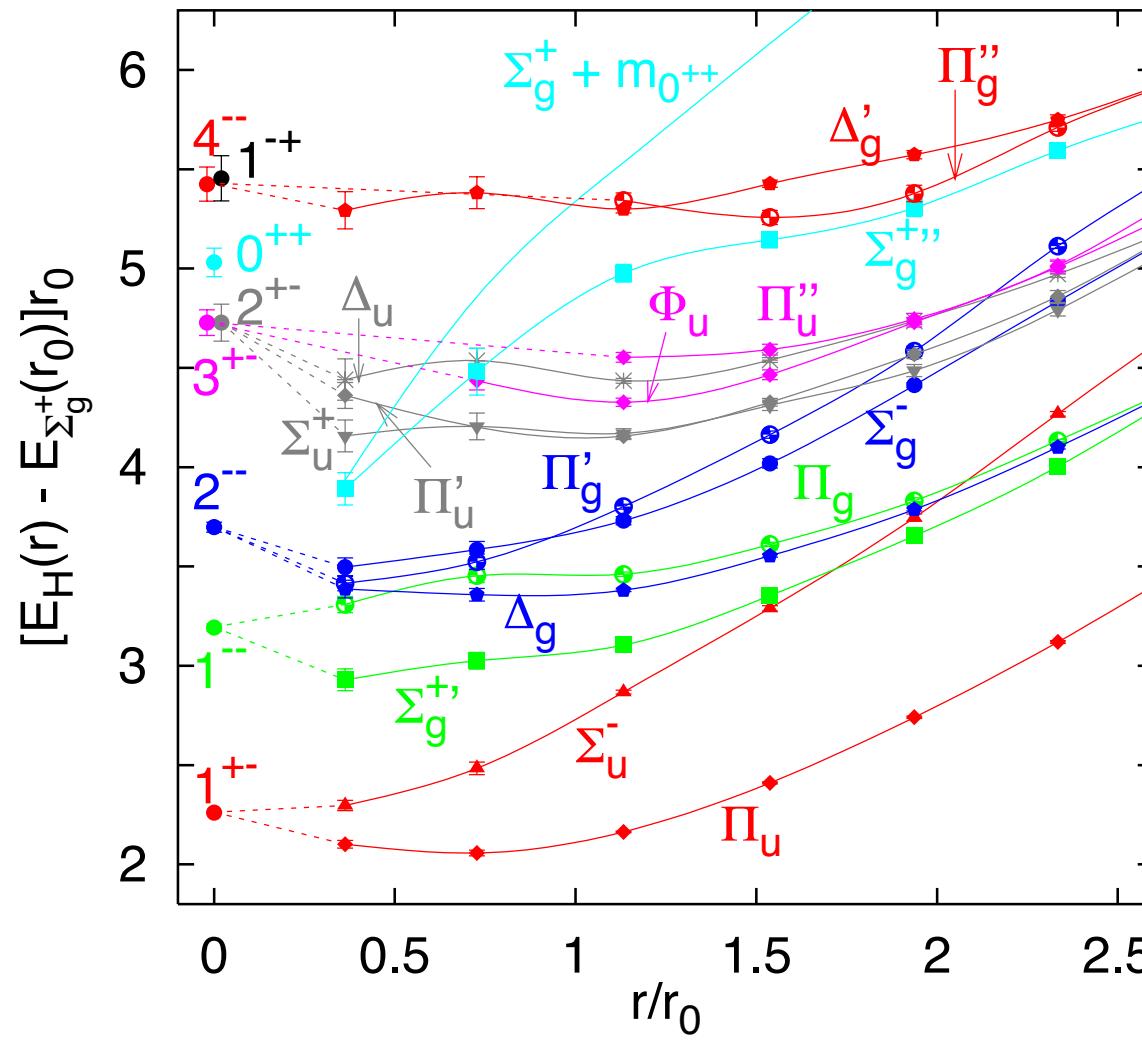
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \dots$$

$\Lambda_g$  is the **gluelump mass**:  $\Lambda_g = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle H^a(T/2) \phi_{ab}^{\text{adj}}(T/2, -T/2) H^b(-T/2) \rangle$   
calculated on the lattice

non perturbative coefficients

Foster Michael PRD 59 (1999) 094509  
Bali Pineda PRD 69 (2004) 094001  
Lewis Marsh PRD 89 (2014) 014502

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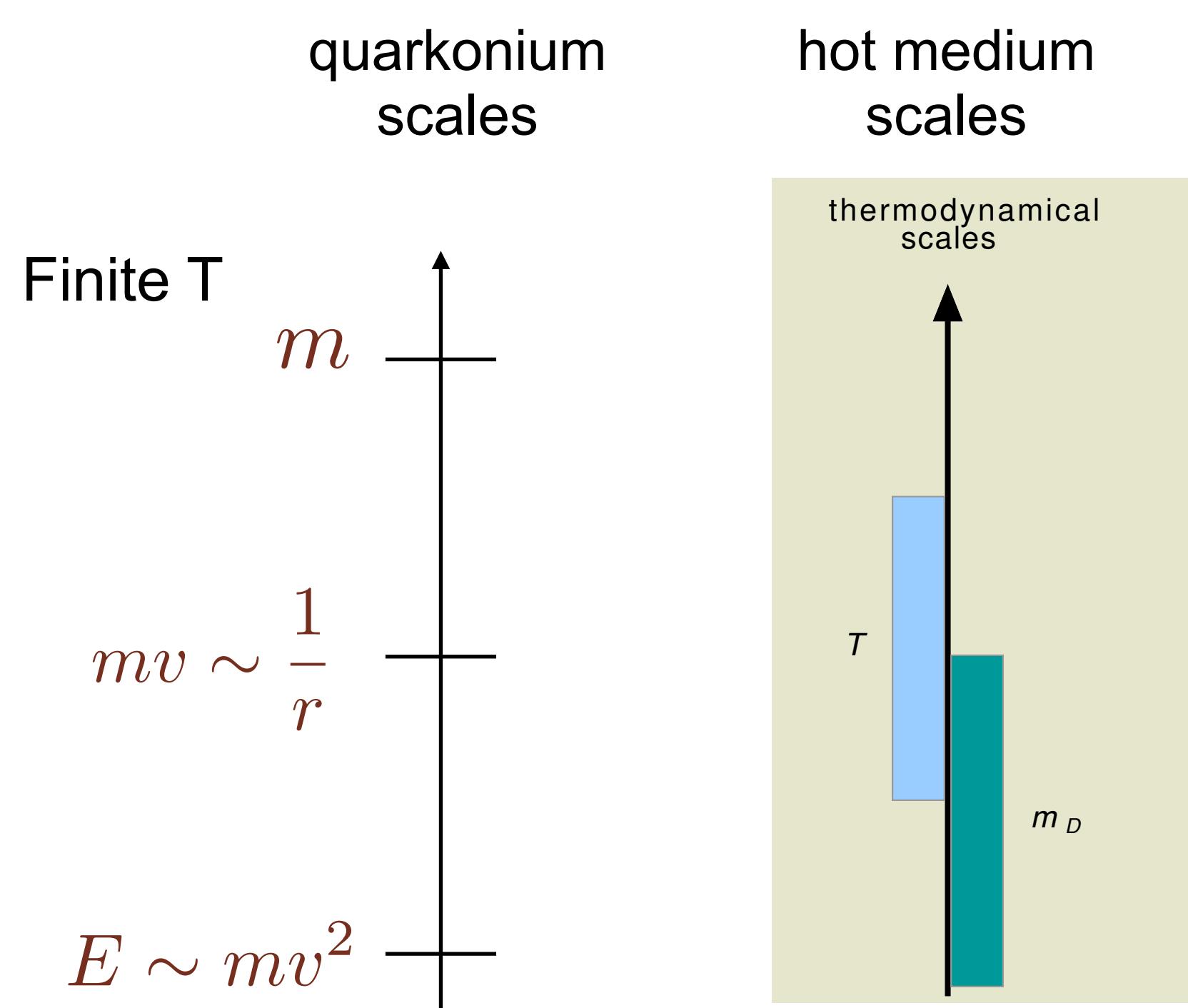
## BOEFT for $E_{\Pi_u}$ and $E_{\Sigma_u^-}$ hybrids

$$\left[ -\frac{1}{mr^2} \partial_r r^2 \partial_r + \frac{1}{mr^2} \begin{pmatrix} l(l+1) + 2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix} \right] \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix}$$

$$\left[ -\frac{1}{mr^2} \partial_r r^2 \partial_r + \frac{l(l+1)}{mr^2} + E_\Pi^{(0)} \right] \psi_{+\Pi}^{(N)} = \mathcal{E}_N \psi_{+\Pi}^{(N)}$$

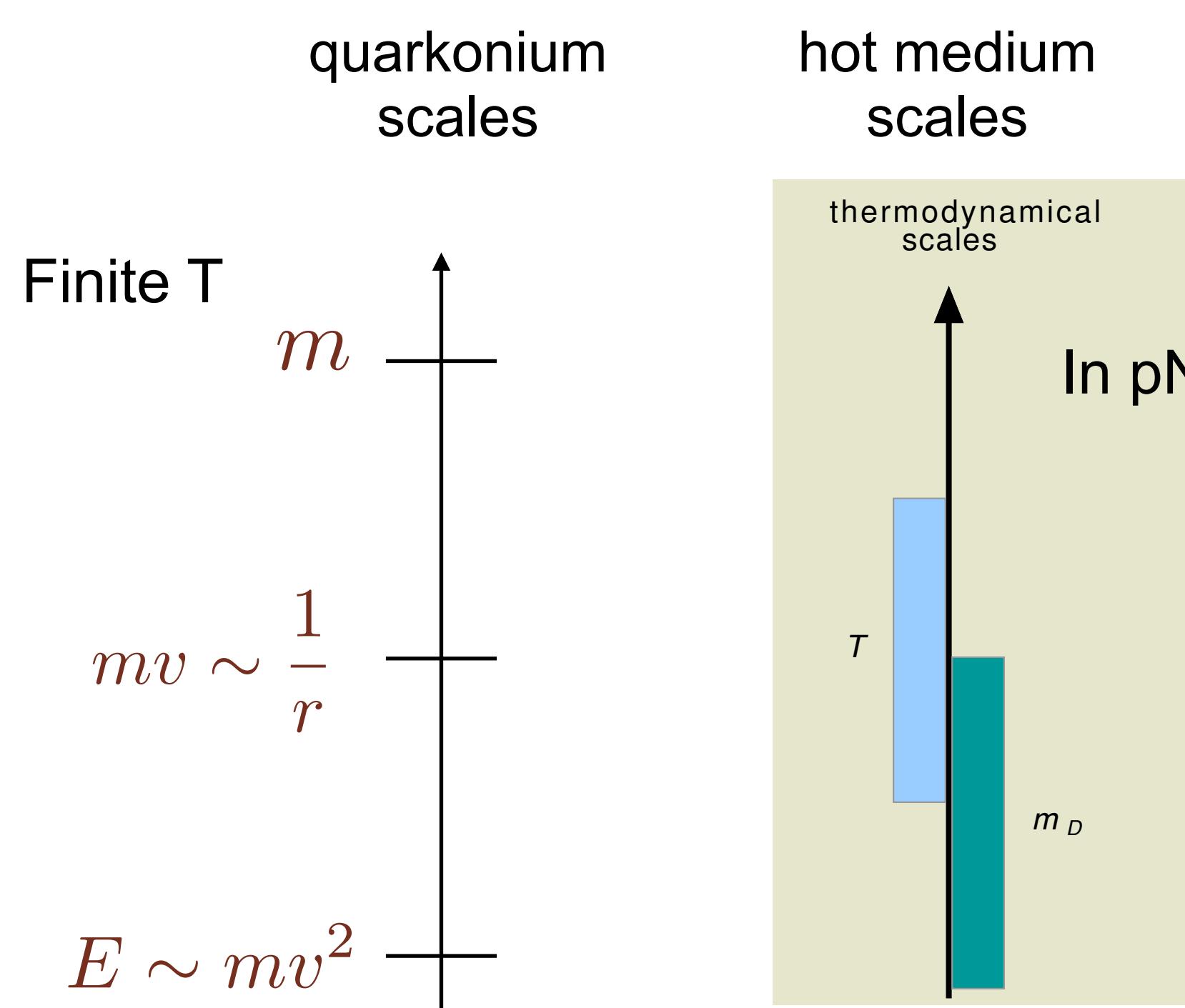
- $l(l+1)$  is the eigenvalue of angular momentum  $L^2 = (\mathbf{L}_{Q\bar{Q}} + \mathbf{L}_g)^2$
- the two solutions correspond to **opposite parity** states:  $(-1)^l$  and  $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation:  $(-1)^{l+s}$  and  $(-1)^{l+s+1}$

# pNRQCD at finite T: another scale to integrate out (using Hard Thermal Loops HTL)



For decades it was not clear what is the QQbar potential at finite T: free energies or internal energies from the lattice have been used

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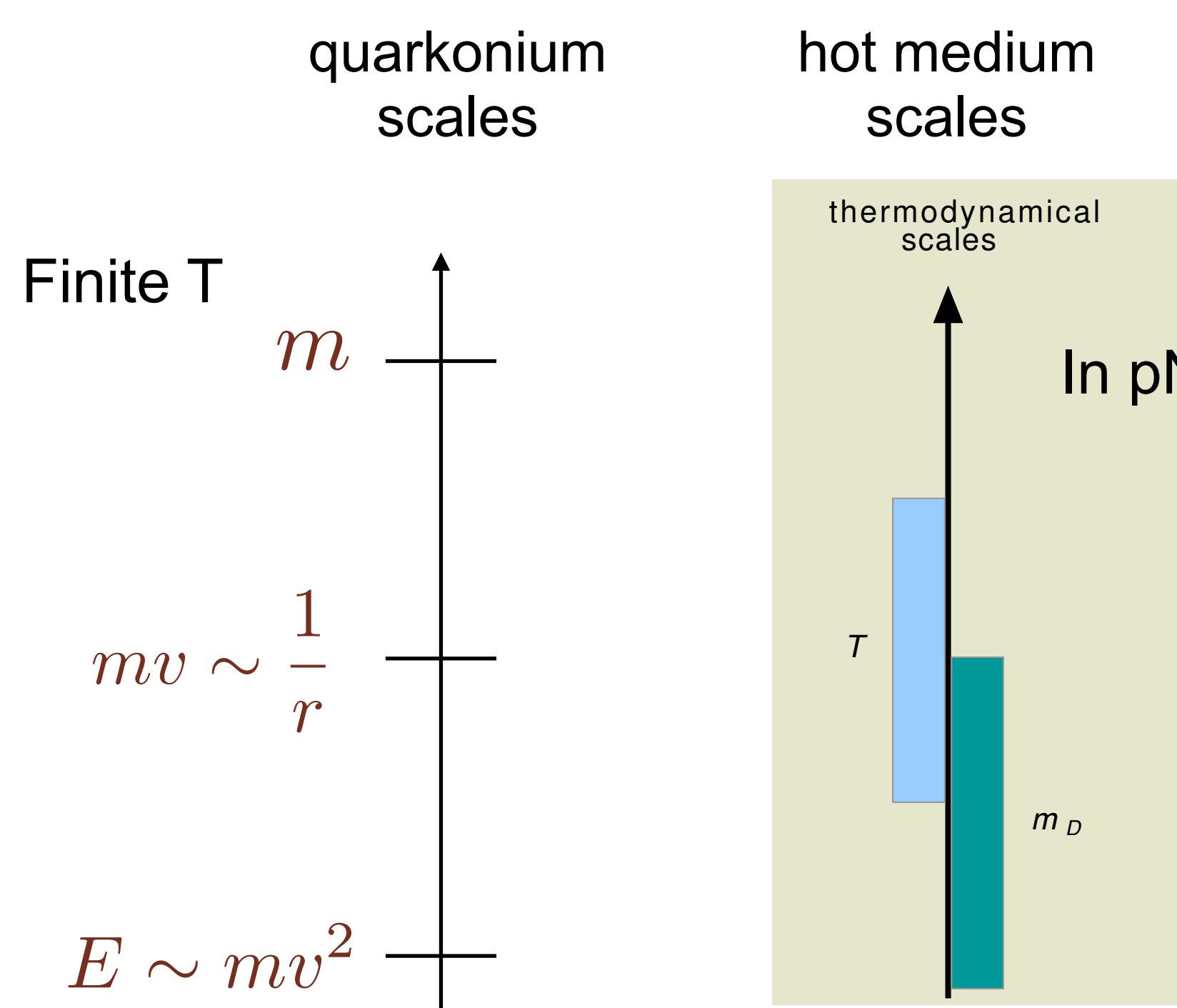
In pNRQCD to define the potential we have to integrate out all the scales bigger than E including T and  $m_d$

if T is of order E or less will give contribution to the energy and not to the potential

Notice:

The potential  $V(r,T)$  dictates through the Schroedinger equation the real time evolution of the QQbar in the medium

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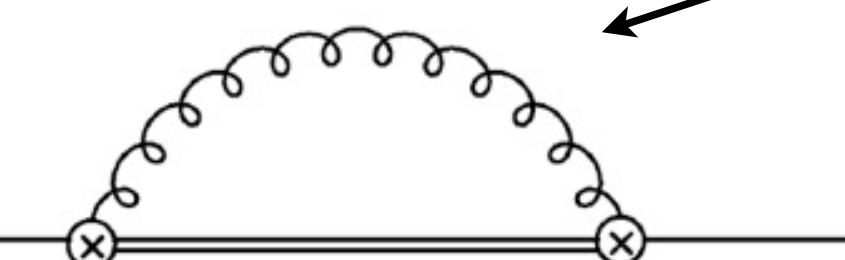
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$\text{Re}V_s(r,T)$

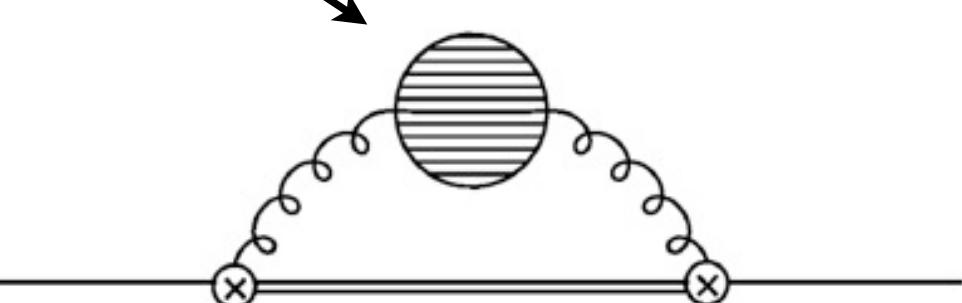
$\text{Im}V_s(r,T)$

thermal width of  $Q\bar{Q}$

Result: the static potential has a real and an Imaginary part: dissociation happens due to the imaginary part **not due to screening**



Singlet-to-octet



Landau damping

# pNRQCD at finite T and open quantum systems: nonequilibrium propagation of bottomonium in QGP

Hierarchy of scales:

strongly coupled QGP  
 $m \gg 1/r \sim m\alpha_s \gg T \sim gT \gg E$

Coulombic bound state:

quark-antiquark color singlet Hamiltonian  $h_s = \frac{\mathbf{p}^2}{m} - \frac{4}{3} \frac{\alpha_s}{r}$

quark-antiquark color octet Hamiltonian  $h_o = \frac{\mathbf{p}^2}{m} + \frac{1}{6} \frac{\alpha_s}{r}$

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## Linblad equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$

The octet potential describes an unbound quark-antiquark pair.

$C_i$  collapse or jump operators: connect different internal states

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \quad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

the QGP is characterised by two nonperturbative parameters (transport coefficients) kappa and gamma that must be calculated on the lattice

$\kappa$  is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18} \operatorname{Re} \int_{-\infty}^{+\infty} ds \langle \operatorname{T} E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

$$\gamma = \frac{g^2}{18} \operatorname{Im} \int_{-\infty}^{+\infty} ds \langle \operatorname{T} E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

**CLAIM 2: Factoring scales pays off!**

We integrate out scales in perturbation theory and we are left with low energy  
(nonperturbative) correlators only depending on the glue (flavor independent)

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pNRQCD at zero and finite T has brought **huge, model independent progress on a number of problems of contemporary interest**

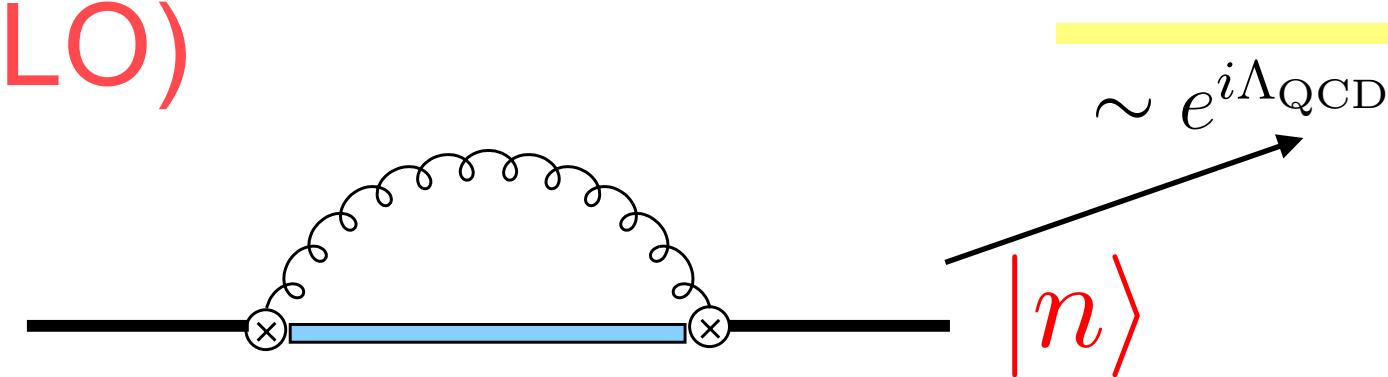
We have already seen how the EFT changed old paradigms

Now we see example of predictions on observables

# PRECISION PHYSICS (system with small radius): precise extraction of alphas and $m_b$ $m_t$ $m_c$

Energies at order  $m \alpha^5$  (NNNLO)

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$



NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler

$m\alpha_s^5 \ln \alpha_s$  Brambilla Pineda Soto Vairo 99, Kniehl Penin 99

$m\alpha_s^5$  Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

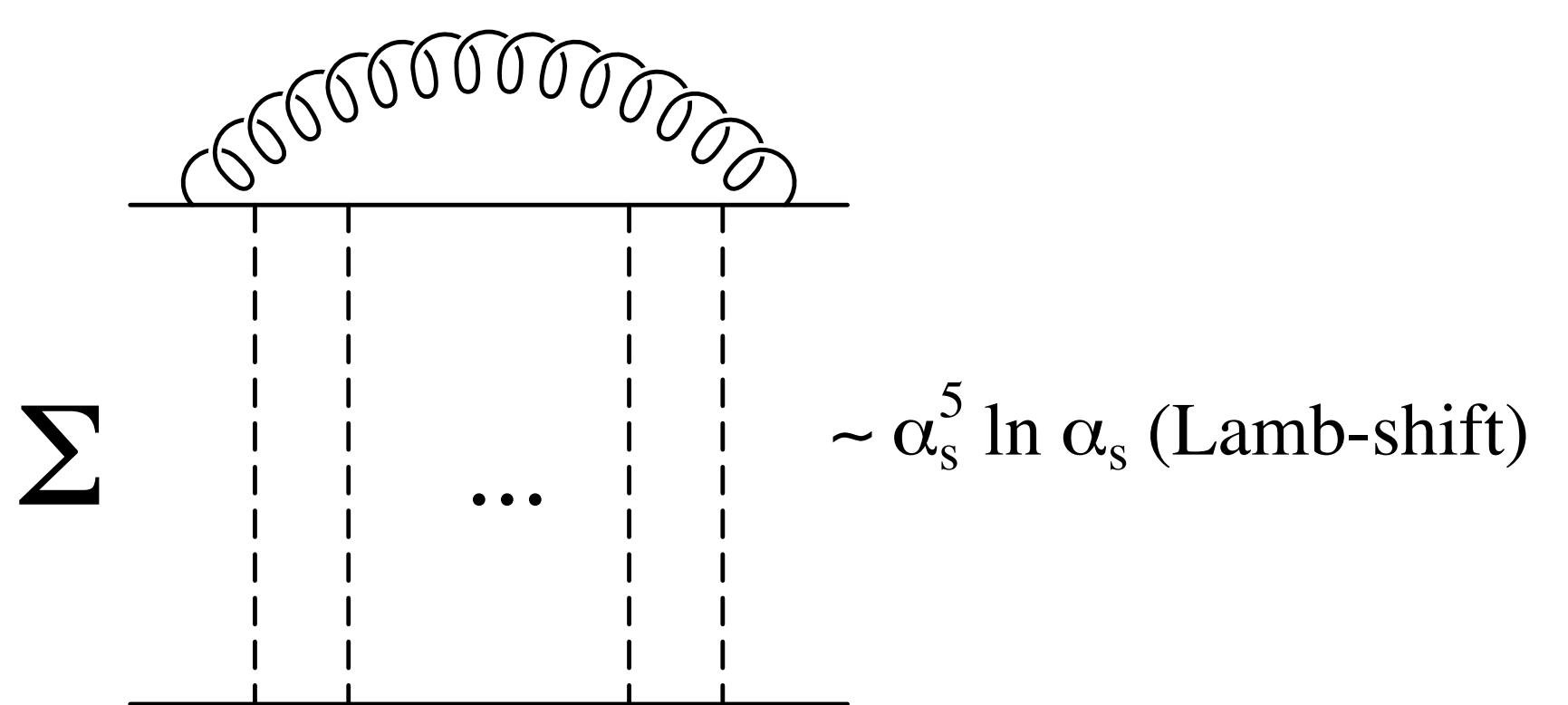
$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \langle \mathbf{E}(t) \mathbf{E}(0) \rangle(\mu)$$

$$E_n^{(0)} - H_o \gg \Lambda_{QCD} \Rightarrow \langle \mathbf{E}(t) \mathbf{E}(0) \rangle(\mu) \rightarrow \langle \mathbf{E}^2(0) \rangle$$

local condensates as predicted in a paper by Misha Voloshin in 1979

→ used to extract precise (NNNLO)

$E_n^{(0)} - H_o \sim \Lambda_{QCD}$  ⇒ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



Applications of weakly coupled pNRQCD include:

t̄tbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies,

# Confinement physics (system with large radius): spectra and transitions

$$\mathcal{L} = \text{Tr} \left\{ S^\dagger \left( i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\}$$

+  $\Delta\mathcal{L}$ (US light quarks)

static	spin dependent	velocity dependent
$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$		

$$V_{SD}^{(2)} = -\frac{r^k}{r^2} c_F \epsilon^{kij} i \int_0^\infty dt t \langle \begin{array}{c} \text{E(t)} \\ \square \\ \square \end{array} \rangle \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2) |V_{LS}^{(2)}|$$

$$-\frac{r^k}{r^2} \left( c_F \epsilon^{kij} i \int_0^\infty dt t \langle \begin{array}{c} \bullet \\ \square \\ \square \end{array} \rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) |V_{LS}^{(1)}|$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left( \langle \begin{array}{c} \bullet \\ \square \\ \square \end{array} \rangle - \frac{\delta_{ij}}{3} \langle \begin{array}{c} \bullet \\ \square \end{array} \rangle \right) \left( \mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right) |V_T|$$

$$+ \left( \frac{2}{3} c_F^2 i \int_0^\infty dt \langle \begin{array}{c} \bullet \\ \square \\ \square \end{array} \rangle - 4 \left( d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2 |V_S|$$

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2 \ln m/\mu) + \dots$ ,  $d_{sv,vv} = O(\alpha_s^2)$  from NRQCD.

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

# Confinement physics (system with large radius): spectra and transitions

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$$V^{(1)} = -\frac{1}{2} \int_0^\infty dt t \langle \begin{array}{c} \bullet \\ \square \end{array} \rangle$$

$$-\frac{r^k}{r^2} \left( c_F \epsilon^{kij} i \int_0^\infty dt t \langle \begin{array}{c} \bullet \\ \square \\ \square \end{array} \rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) |V_{LS}^{(1)}|$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left( \langle \begin{array}{c} \bullet \\ \square \\ \square \end{array} \rangle - \frac{\delta_{ij}}{3} \langle \begin{array}{c} \bullet \\ \square \end{array} \rangle \right) \left( \mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right) |V_T|$$

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$$c_F = 1 + \alpha_s / \pi (13/6 + 3/2 \ln m/\mu) + \dots, d_{sv,vv} = O(\alpha_s^2) \text{ from NRQCD.}$$

- the potentials contain the contribution of the scale  $m$  inherited from NRQCD matching coefficients —> they cancel any QM divergences, good UV behaviour
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

# Quarkonium production at colliders is studied with NRQCD

NRQCD factorization formula for quarkonium production  
valid for large  $p_T$

cross section

short distance coefficients  
partonic hard scattering cross section  
convoluted with parton distribution

$$\sigma(H) = \sum_n F_n \langle 0 | \mathcal{O}_n^H | 0 \rangle.$$

The LDMEs depend on the scale  $m_V$ ,  $m_V^2$  and  $\lambda_{\text{QCD}}$   
long distance matrix elements  
(LDME)  
give the probability of a  $q\bar{q}$  pair with certain quantum number to evolve into a final quarkonium  $H$   
they are vacuum expectation values of four fermion operators and contain color singlet and color octet contribution

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Bodwin Braaten Lepage 1995, proved at NNLO JW Qiu, Sterman et al

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One problem is the proliferation of LDMEs: nonperturbative objects **that cannot be evaluated on the lattice** and should be extracted from the data, they depend on the considered quarkonium: different groups extract different LDMEs values

# Quarkonium production at colliders with pNRQCD factorisation at lower energy

The LDMEs can be factorized in terms of wave functions and universal nonperturbative gauge invariant correlators

This reduces by half the number of nonperturbative unknowns that can be extracted from the data unambiguously

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$$pp \rightarrow J/\psi + X, \quad pp \rightarrow \psi(2S) + X \quad \text{and} \quad pp \rightarrow \Upsilon(nS) + X$$

## NRQCD

$$\begin{aligned}\sigma_{V+X} = & \hat{\sigma}_{3S_1^{[1]}} \langle \mathcal{O}^V ({}^3S_1^{[1]}) \rangle + \hat{\sigma}_{3S_1^{[8]}} \langle \mathcal{O}^V ({}^3S_1^{[8]}) \rangle \\ & + \hat{\sigma}_{1S_0^{[8]}} \langle \mathcal{O}^V ({}^1S_0^{[8]}) \rangle + \hat{\sigma}_{3P_J^{[8]}} \langle \mathcal{O}^V ({}^3P_0^{[8]}) \rangle\end{aligned}$$

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**pNRQCD**

$$\begin{aligned} \langle \mathcal{O}^V(^3S_1^{[8]}) \rangle &= \frac{1}{2N_c m^2} \frac{3|R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{10;10}(\mu) \\ \langle \mathcal{O}^V(^1S_0^{[8]}) \rangle &= \frac{1}{6N_c m^2} \frac{3|R_V^{(0)}(0)|^2}{4\pi} c_F^2(\mu) \mathcal{B}_{00}(\mu) \\ \langle \mathcal{O}^V(^3P_0^{[8]}) \rangle &= \frac{1}{18N_c} \frac{3|R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{00} \end{aligned}$$

$$\begin{aligned} \mathcal{E}_{10;10} &= d^{a'bc'} d^{e'xy'} \int_0^\infty dt_1 t_1 \int_{t_1}^\infty dt_2 \langle \Omega | \Phi_\ell^{\dagger ad}(0) \Phi_0^{a'a\dagger}(0; t_1) g E^{b,i}(t_1) \Phi_0^{cc'\dagger}(t_1; t_2) g E^{c,i}(t_2) \\ &\times \int_0^\infty dt'_1 t'_1 \int_{t'_1}^\infty dt'_2 g E^{y,j}(t'_2) \Phi_0^{yy'}(t'_1; t'_2) g E^{x,j}(t'_1) \Phi_0^{e'e}(0; t'_1) \Phi_\ell^{de}(0) | \Omega \rangle \end{aligned}$$

$$\mathcal{E}_{00} = \int_0^\infty dt \int_0^\infty dt' \langle \Omega | \Phi_\ell^{\dagger ab}(0) \Phi_0^{\dagger ad}(0; t) g E^{d,i}(t) g E^{e,i}(t') \Phi_0^{ec}(0; t') \Phi_\ell^{bc}(0) | \Omega \rangle$$

$$\mathcal{B}_{00} = \int_0^\infty dt \int_0^\infty dt' \langle \Omega | \Phi_\ell^{\dagger ab}(0) \Phi_0^{\dagger ad}(0; t) g B^{d,i}(t) g B^{e,i}(t') \Phi_0^{ec}(0; t') \Phi_\ell^{bc}(0) | \Omega \rangle$$

# Quarkonium production at colliders with pNRQCD: predictions

- Universality of the gluonic correlators leads to the prediction for *cross section ratios*, independently of the correlators

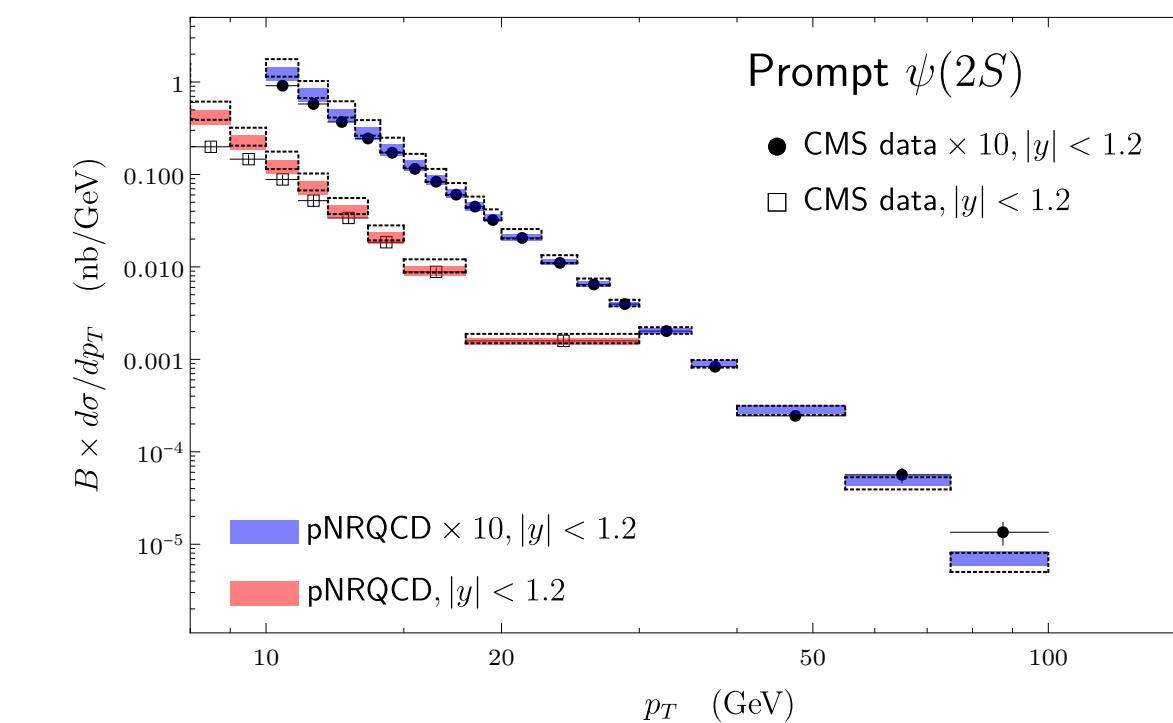
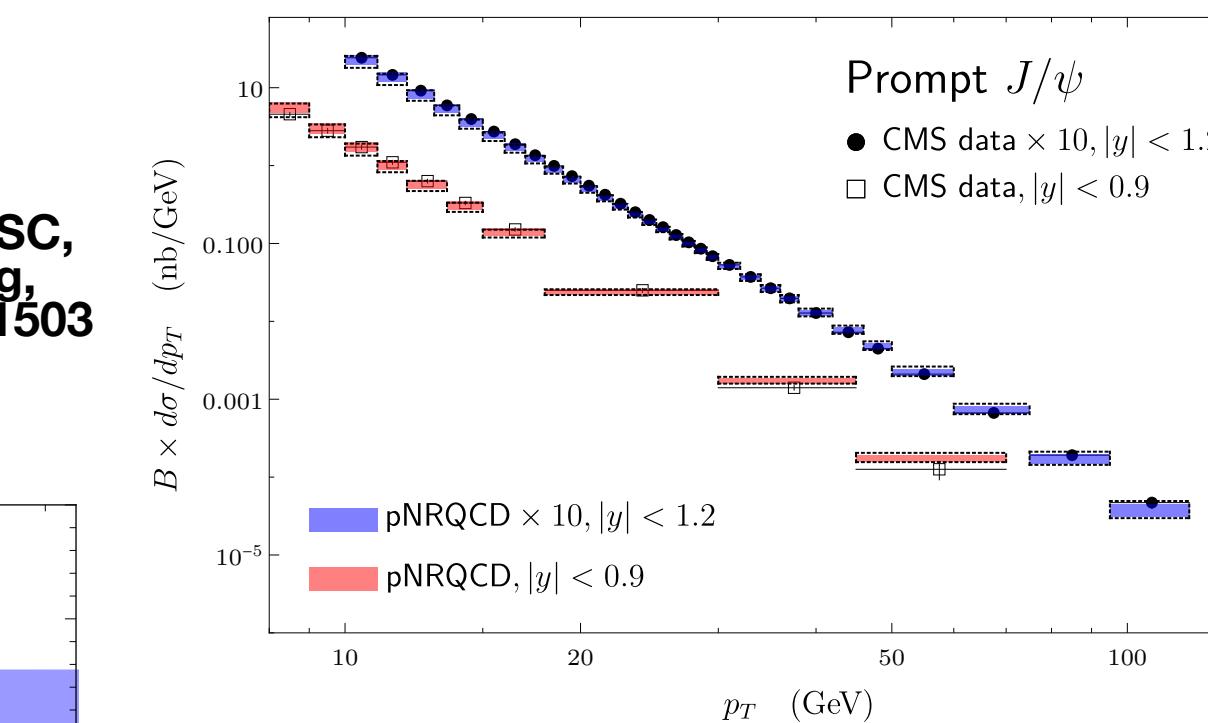
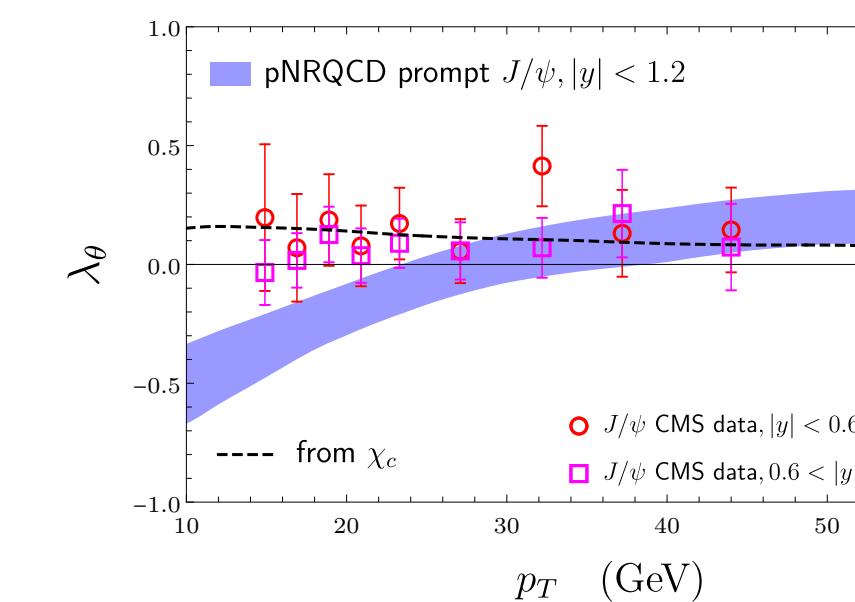
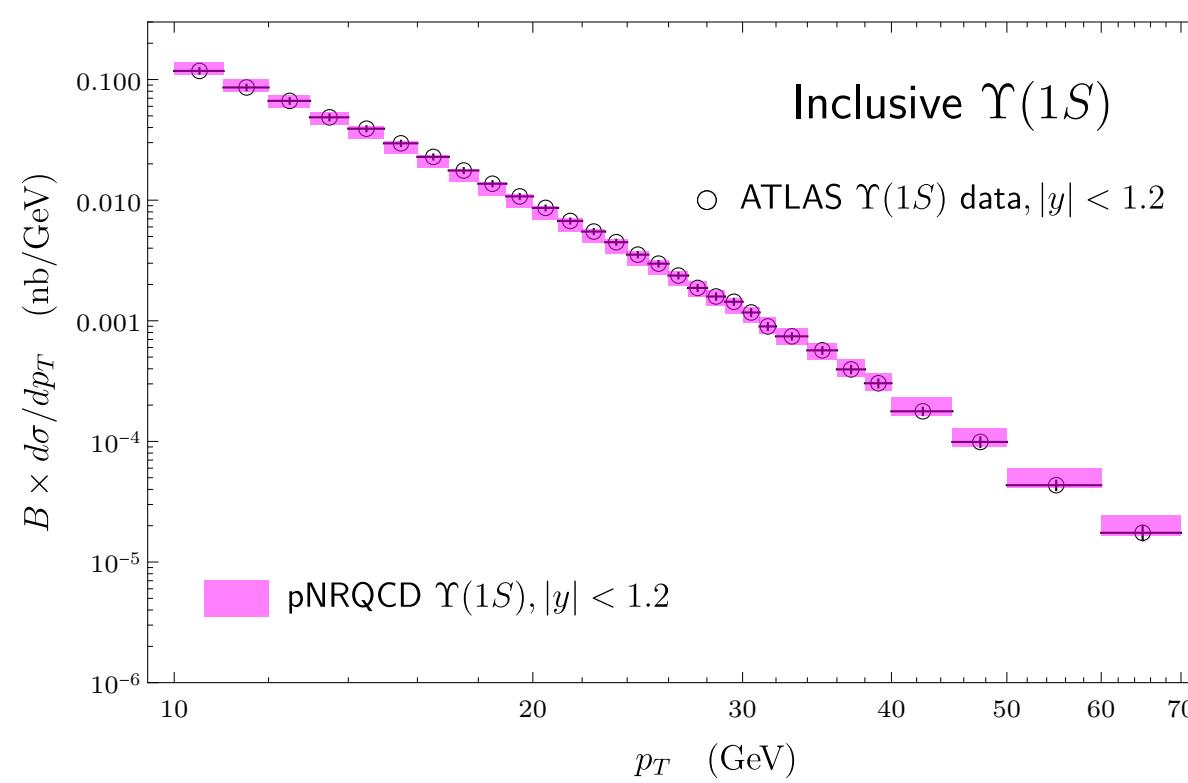
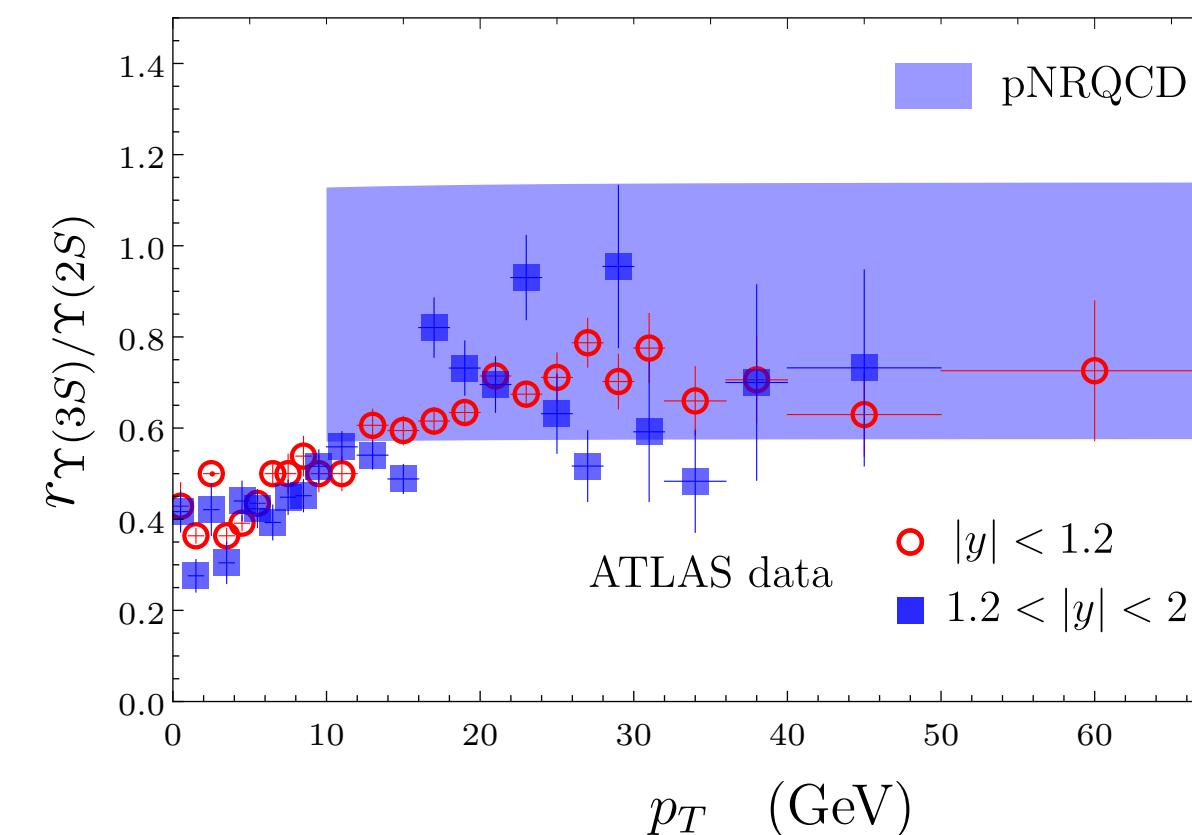
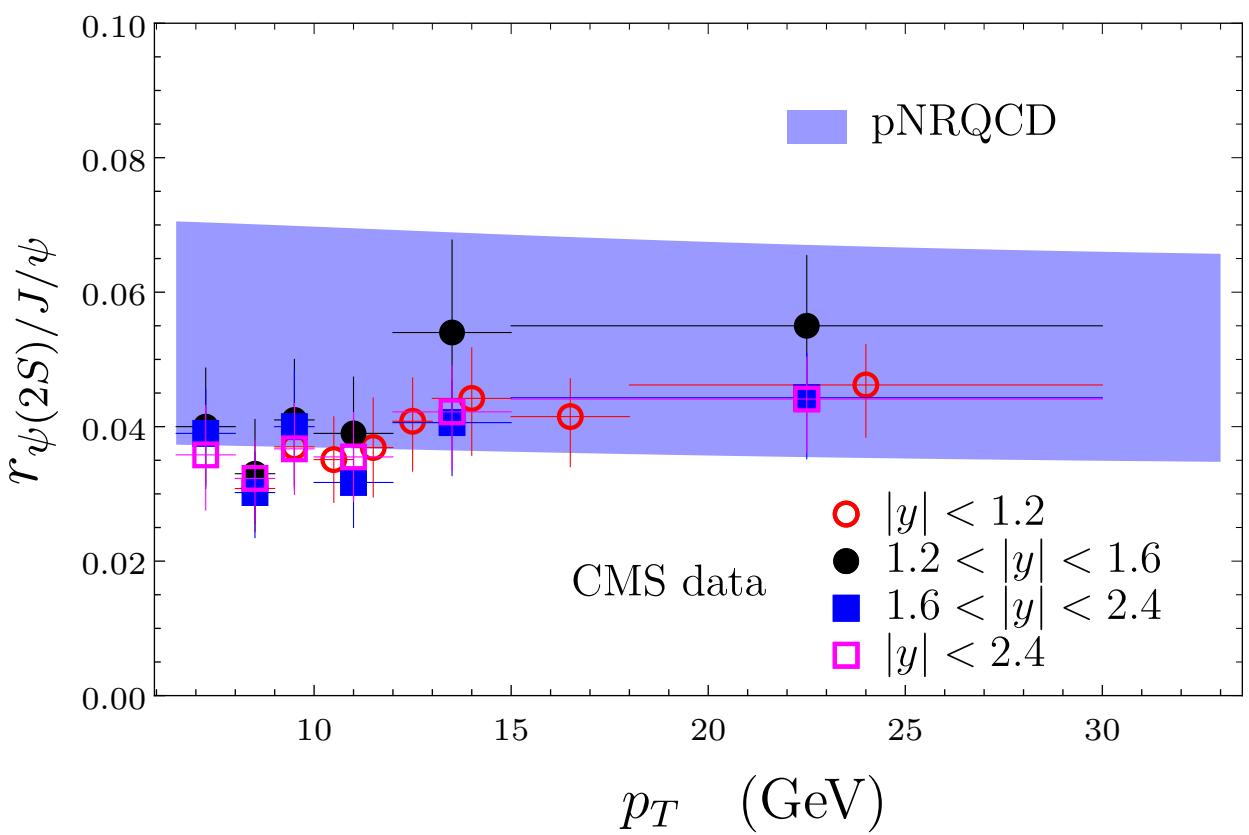
$$\frac{\sigma_{\psi(2S)}^{\text{direct}}}{\sigma_{J/\psi}^{\text{direct}}} = \frac{|R_{\psi(2S)}^{(0)}(0)|^2}{|R_{J/\psi}^{(0)}(0)|^2}$$

$$\frac{\sigma_{\Upsilon(3S)}^{\text{direct}}}{\sigma_{\Upsilon(2S)}^{\text{direct}}} = \frac{|R_{\Upsilon(3S)}^{(0)}(0)|^2}{|R_{\Upsilon(2S)}^{(0)}(0)|^2}$$

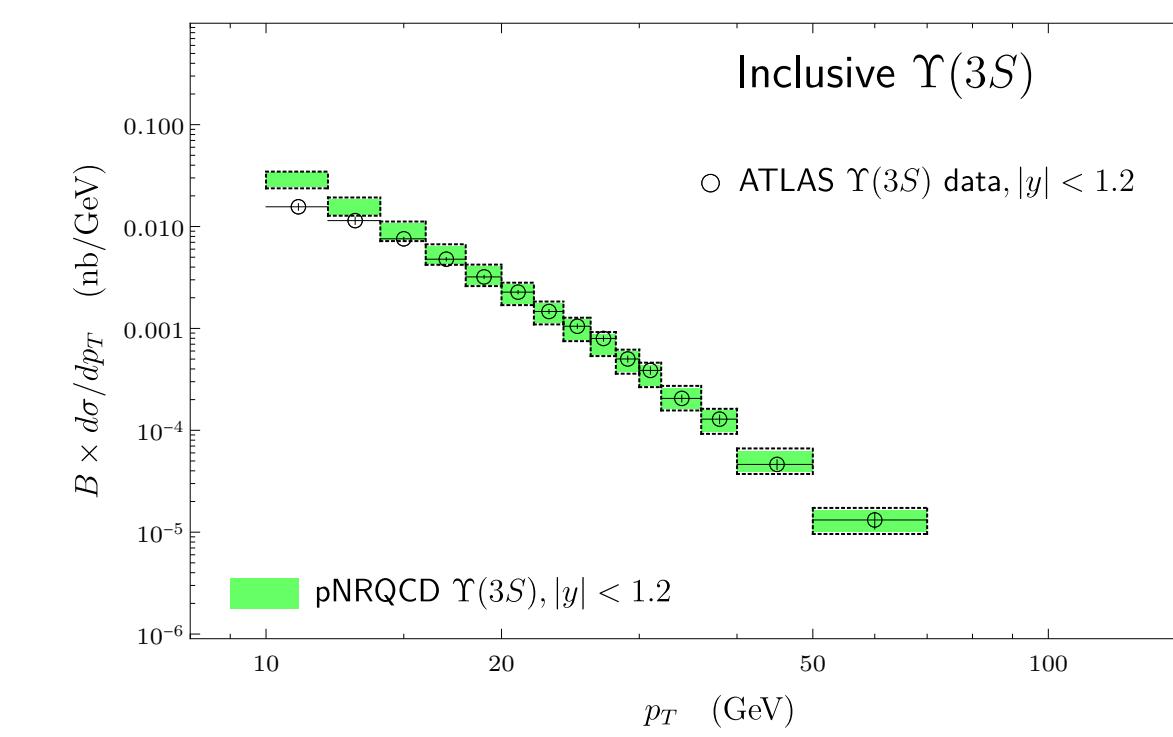
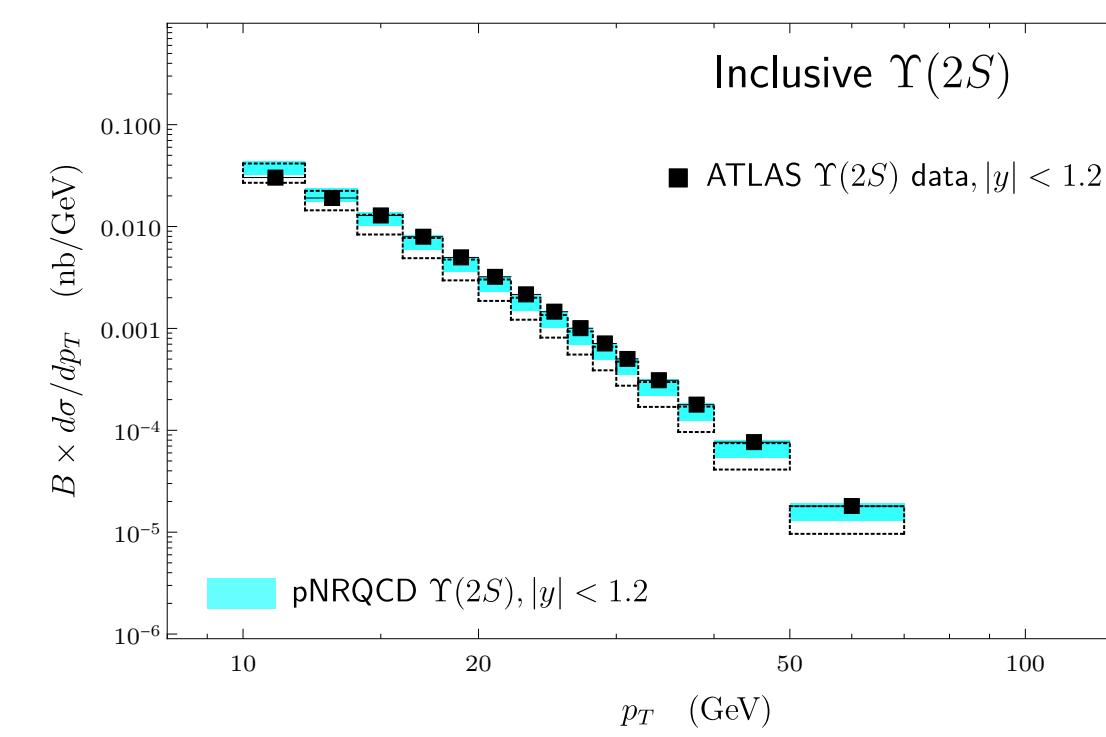
Brambilla, HSC,  
Vairo, Wang,  
PRD105, L111503  
(2022)

- Compared to experiment, including feeddown effects:

$$r_{A/B} = (\text{Br}_{A \rightarrow \mu^+ \mu^-} \sigma_A) / (\text{Br}_{B \rightarrow \mu^+ \mu^-} \sigma_B)$$



**Figure 9.** Production cross section of prompt  $J/\psi$  and  $\psi(2S)$  at the LHC center of mass energy  $\sqrt{s} = 7$  TeV compared to CMS data [69, 72];  $B$  is the dimuon branching fraction. Results from the LDMEs given in table 3 are shown as dotted outlined bands.



# Quarkonium inclusive decays

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## NRQCD at order $m v^5$

$$\begin{aligned}\Gamma(H \rightarrow \text{LH}) &= -2 \operatorname{Im} \langle H | \mathcal{H} | H \rangle \\ &= \sum_n \frac{2 \operatorname{Im} f^{(n)}}{m^{d_n - 4}} \langle H | \psi^\dagger K^{(n)} \chi \chi^\dagger K'^{(n)} \psi | H \rangle\end{aligned}$$

$$\Gamma(\chi_J \rightarrow \text{LH}) = 9 \operatorname{Im} f_1 \frac{|R'(0)|^2}{\pi m^4} + \frac{2 \operatorname{Im} f_8}{m^2} \langle \chi | O_8(^1S_0) | \chi \rangle$$

\* Octet and singlet contribute to the same order.  
 $\Rightarrow$  The IR divergences of  $\operatorname{Im} f_1$  are absorbed into the non-perturbative operator  $\langle \chi | O_8(^1S_0) | \chi \rangle$ .

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$$\Gamma(\chi_J \rightarrow \text{LH}) = \frac{|R'(0)|^2}{\pi m^4} \left[ 9 \text{ Im } f_1 + \frac{\text{Im } f_8}{9} \mathcal{E} \right]$$

The quarkonium state dependence factorizes.

$$\mathcal{E} \equiv \int_0^\infty dt t^3 \langle \text{Tr}(g \mathbf{E}(t) g \mathbf{E}(0)) \rangle$$

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$$\mathcal{E}(1 \text{ GeV}) = 5.3_{-2.2}^{+3.5} \text{ [exp]}$$

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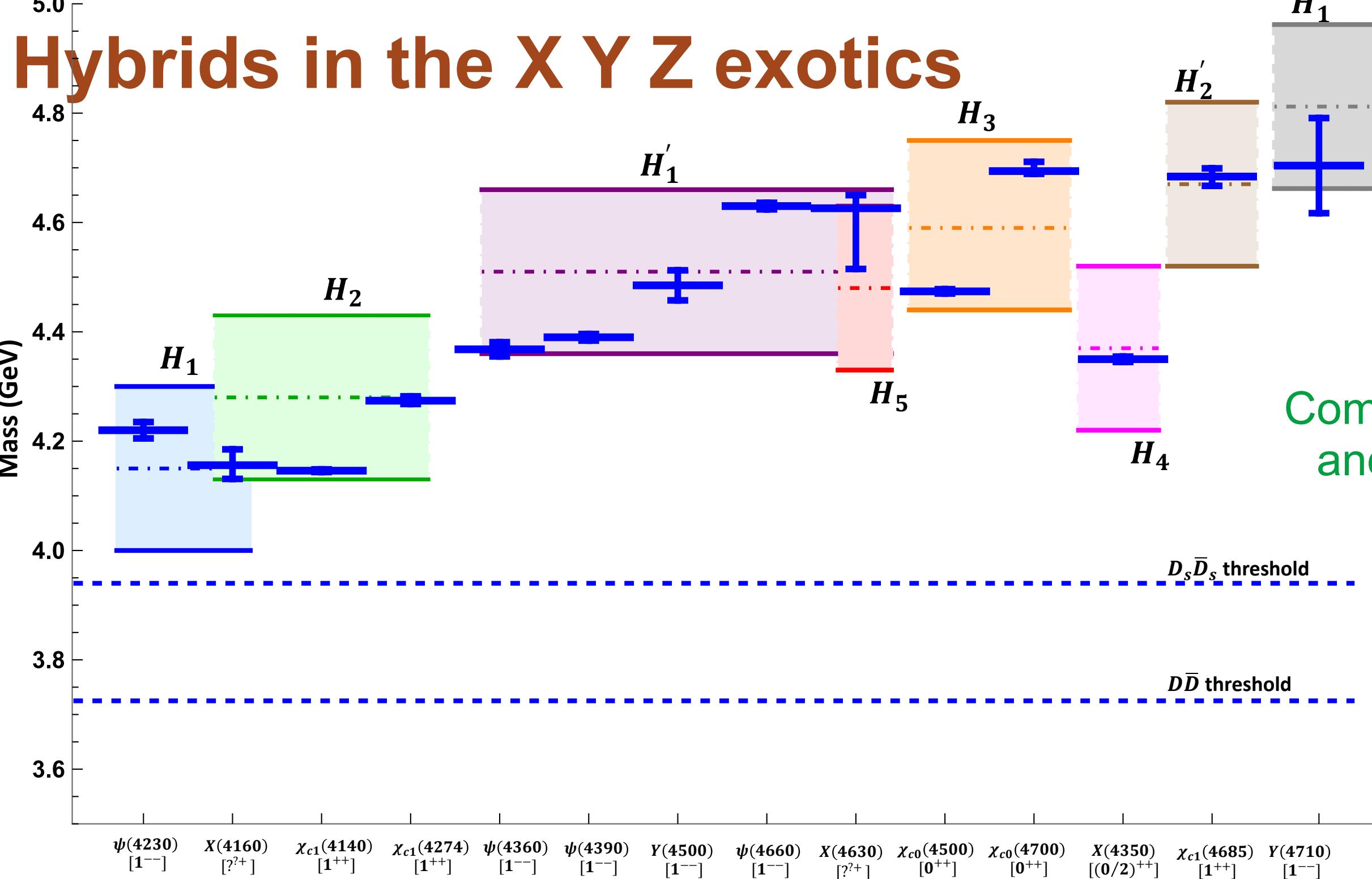
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Similar nonperturbative only glue dependent objects appear for all inclusive decays : they could be calculated on the lattice

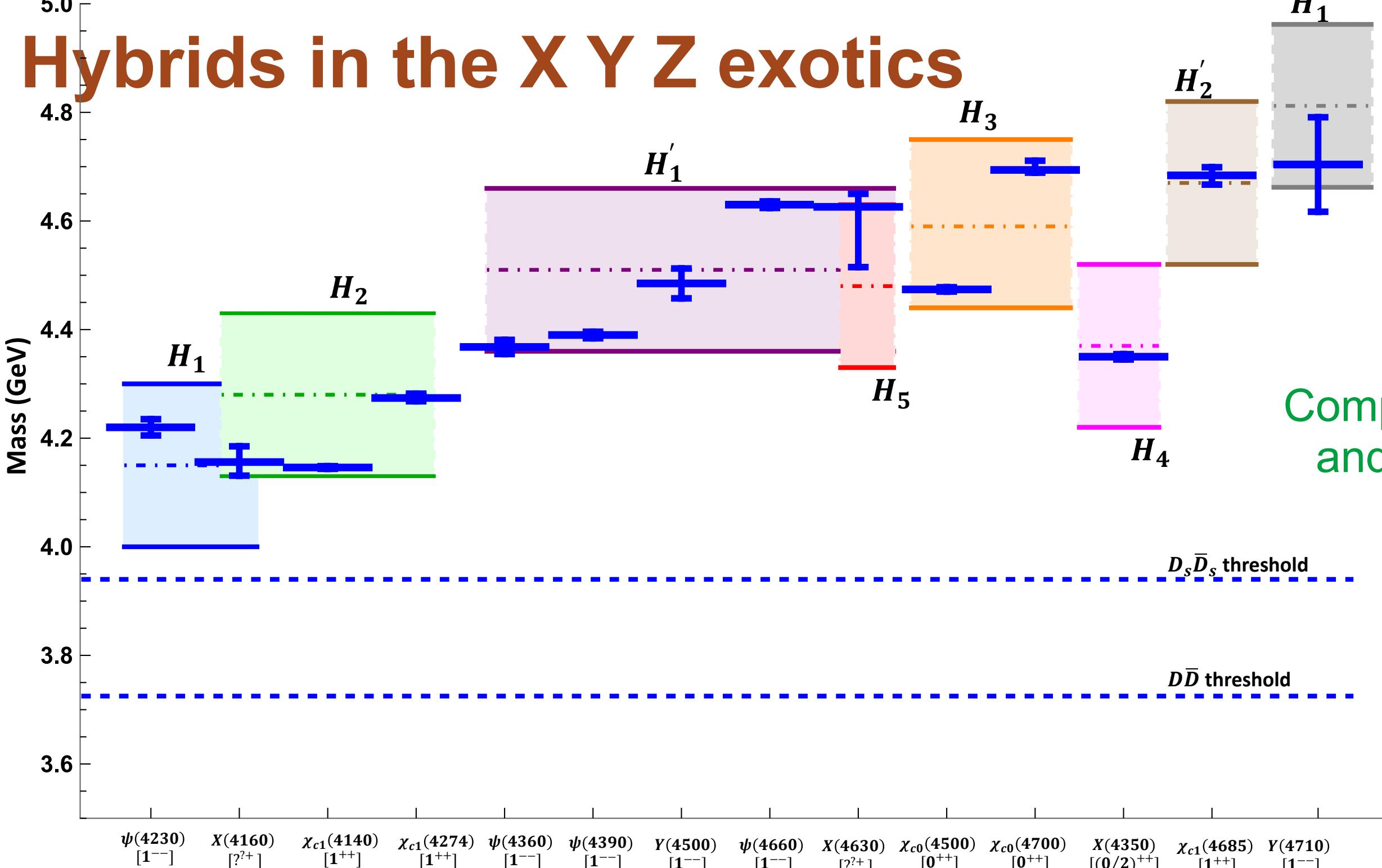


Multiplet	$T$	$J^{PC}(S = 0)$	$J^{PC}(S = 1)$	$E_\Gamma$
$H_1$	1	1 <sup>--</sup>	(0, 1, 2) <sup>-+</sup>	$E_{\Sigma_u^-}, E_{\Pi_u}$
$H_2$	1	1 <sup>++</sup>	(0, 1, 2) <sup>+-</sup>	$E_{\Pi_u}$
$H_3$	0	0 <sup>++</sup>	1 <sup>+-</sup>	$E_{\Sigma_u^-}$
$H_4$	2	2 <sup>++</sup>	(1, 2, 3) <sup>+-</sup>	$E_{\Sigma_u^-}, E_{\Pi_u}$

Comparison between the predicted hybrids multiplets and the measured exotic Isoscalar neutral mesons

The band depends on the error on the gluelump mass

# Hybrids in the X Y Z exotics

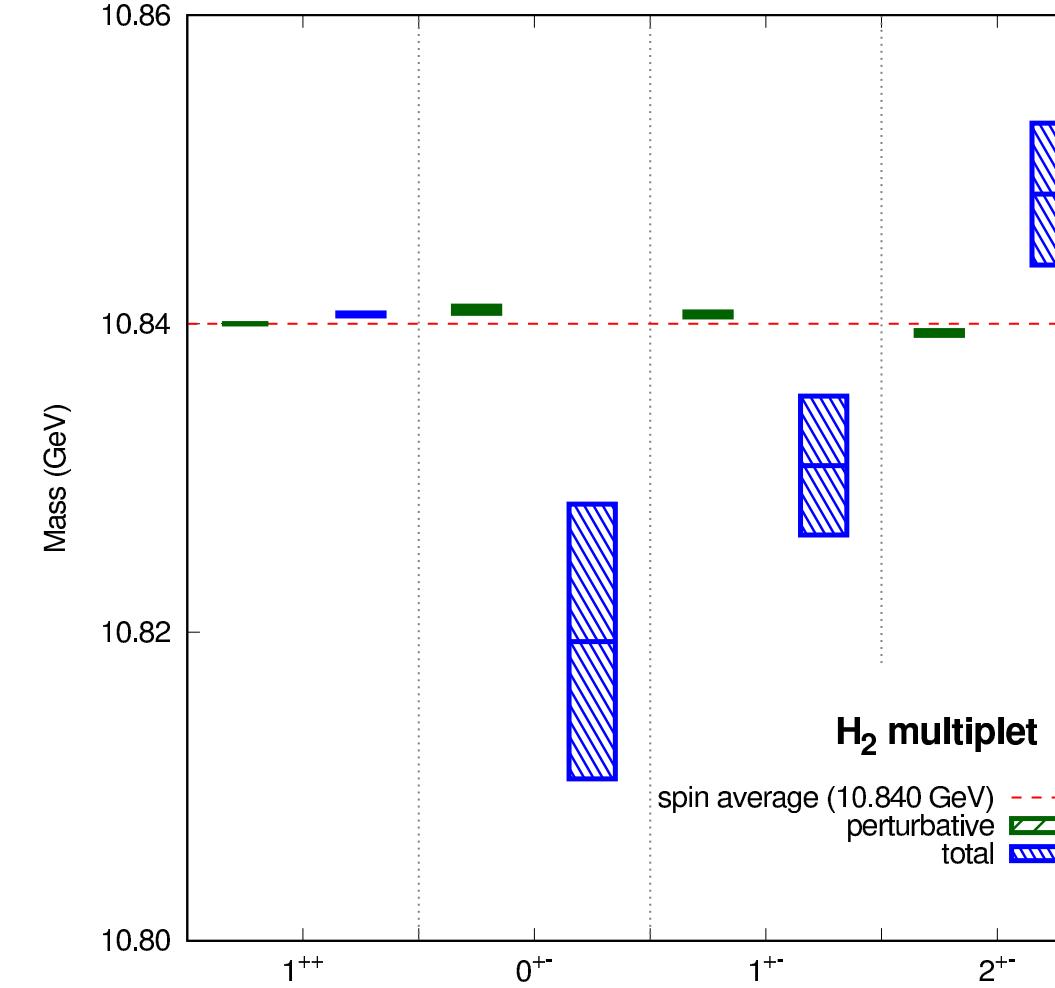


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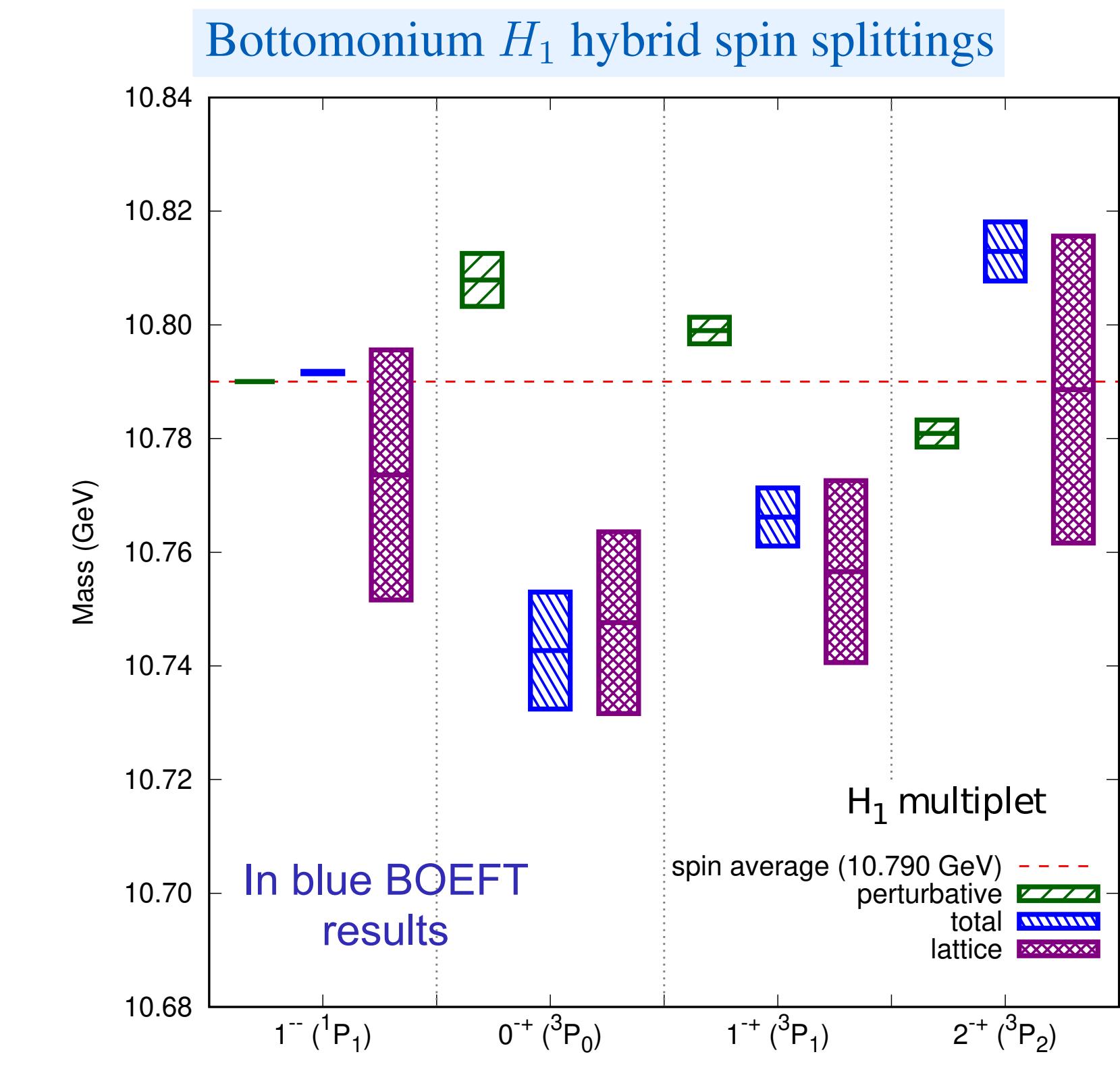
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## Spin splittings in BOEFT



Appears at order  $1/m$  we can fix the nonperturbative unknowns from a charmonium direct lattice hybrid calculation  
Then we can predict bottomonium splitting —>



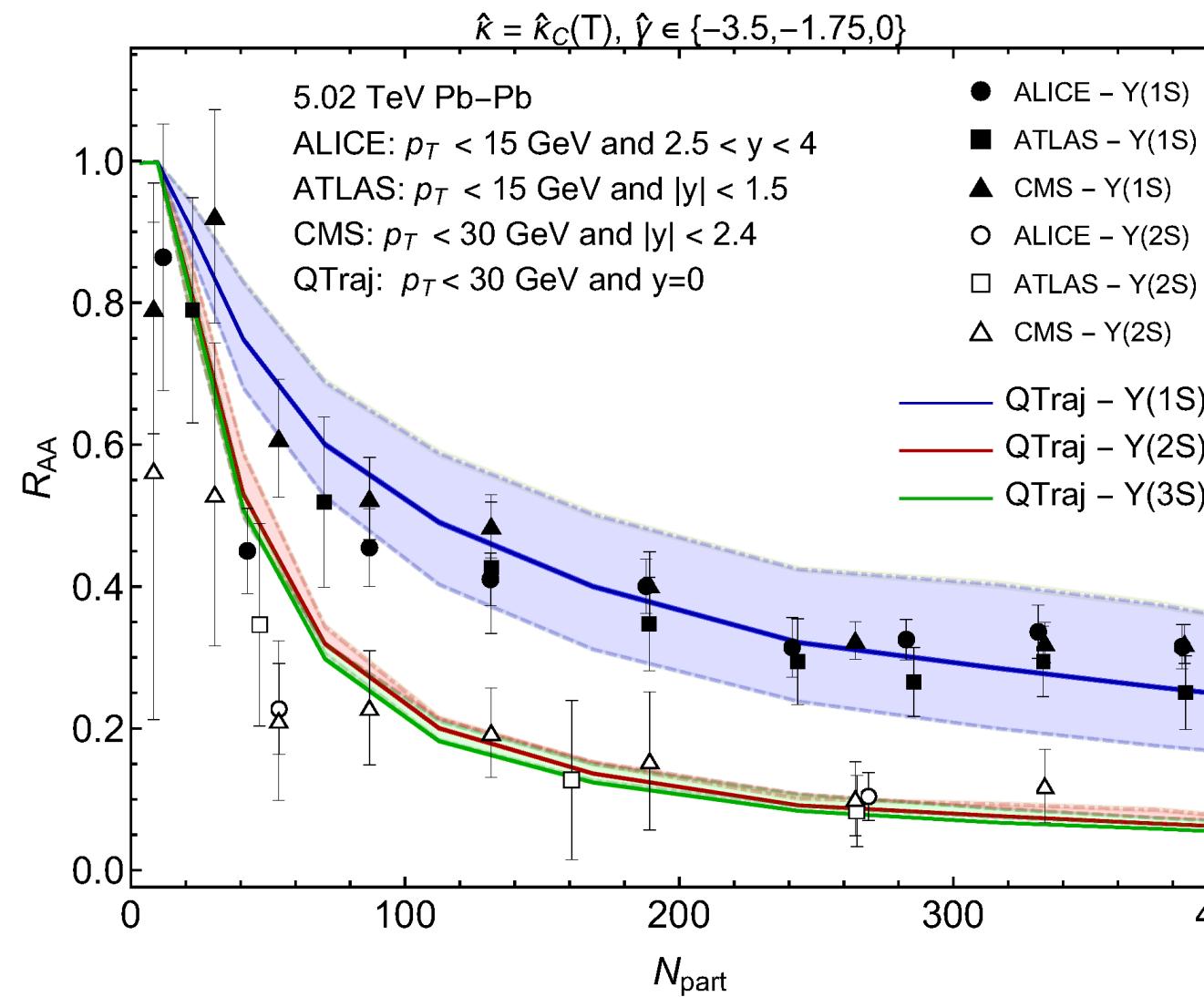
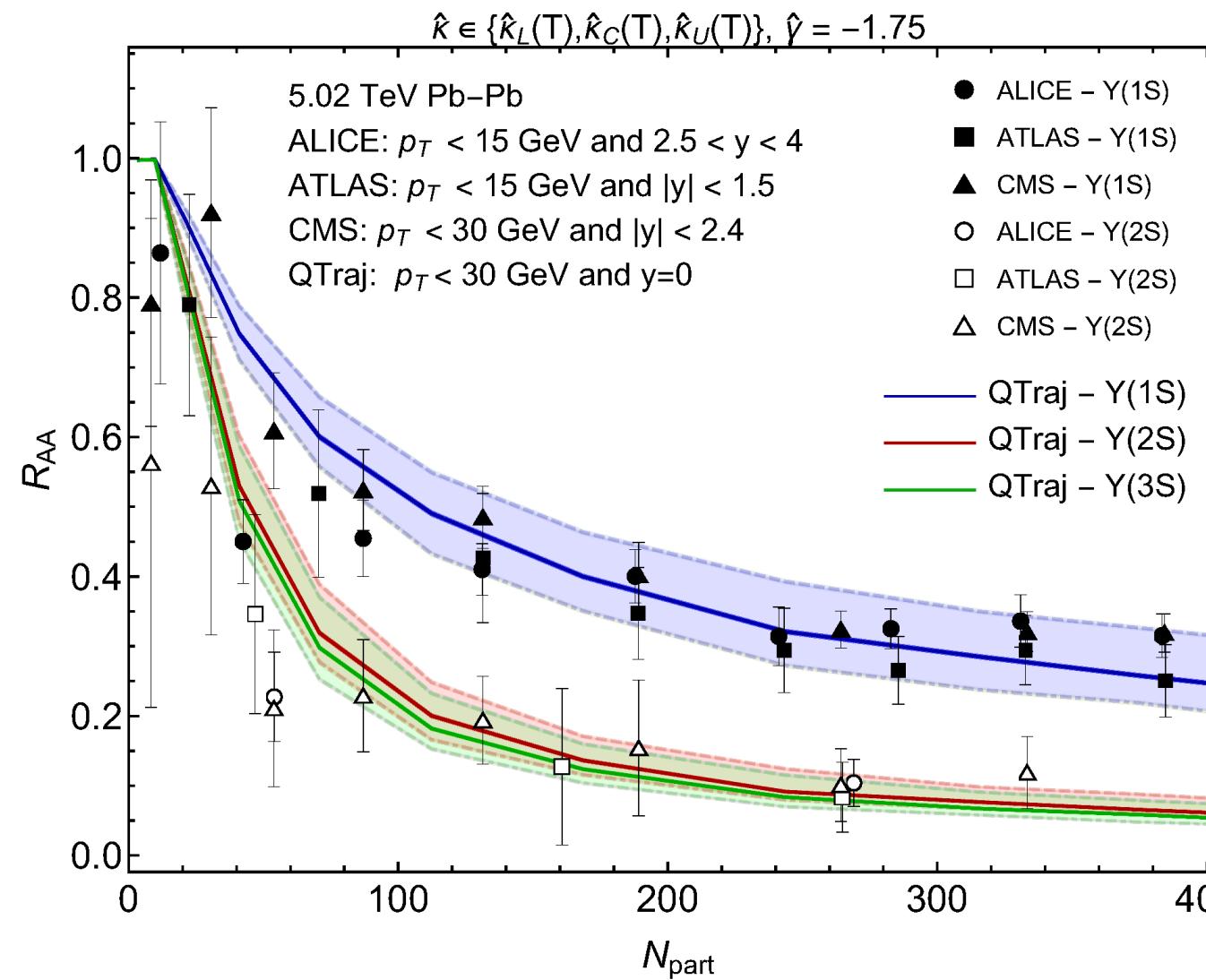
N. B. A. Mohapatra, WK Lai, J. Tarrus;  
A. Vairo, 2212.09187,  
1908.11699, 1510.04299

○ Ryan et al arXiv:2008.02656 [2+1 flavors,  $m_\pi = 400$  MeV]

# Bottomonium Nuclear Modification factor

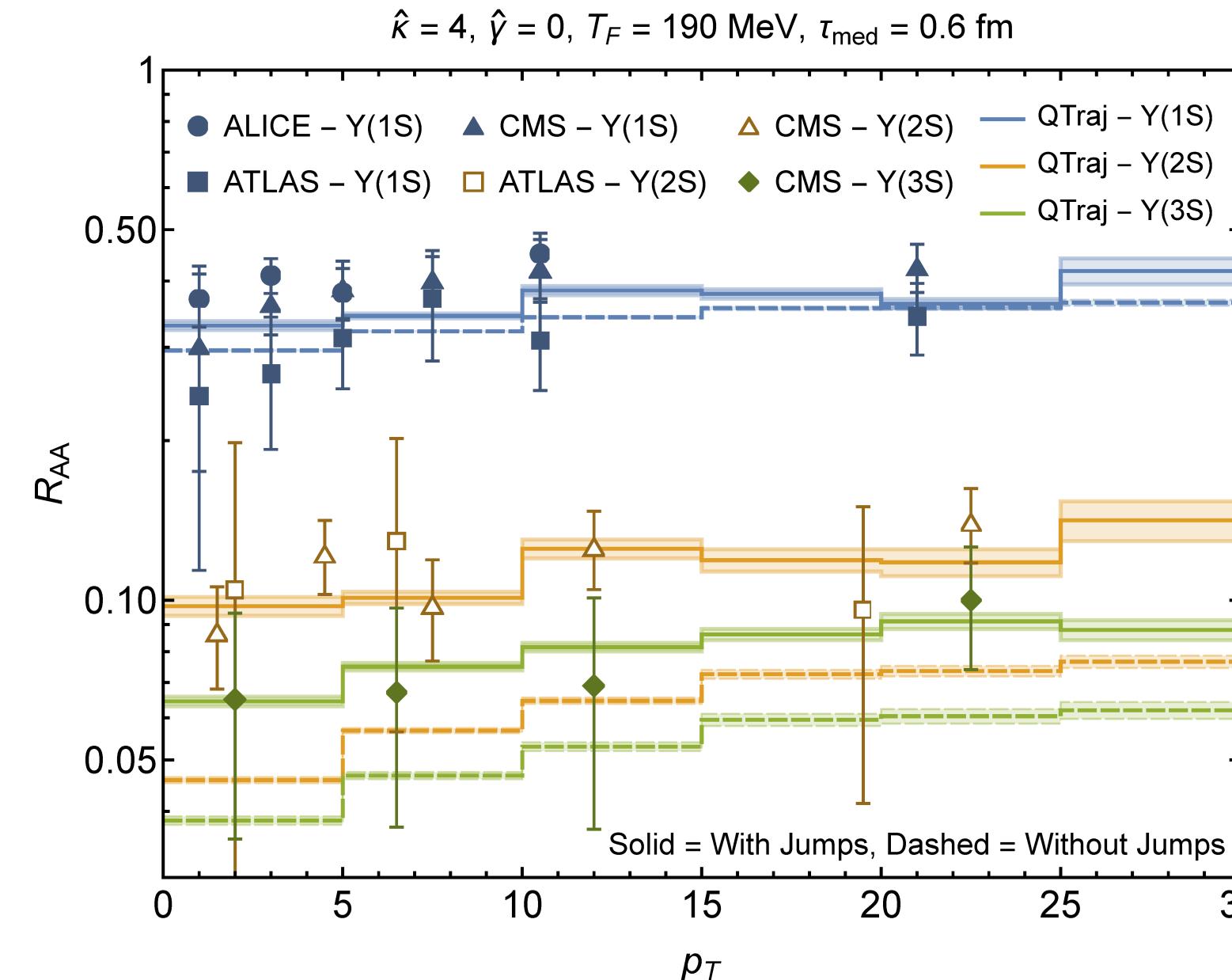
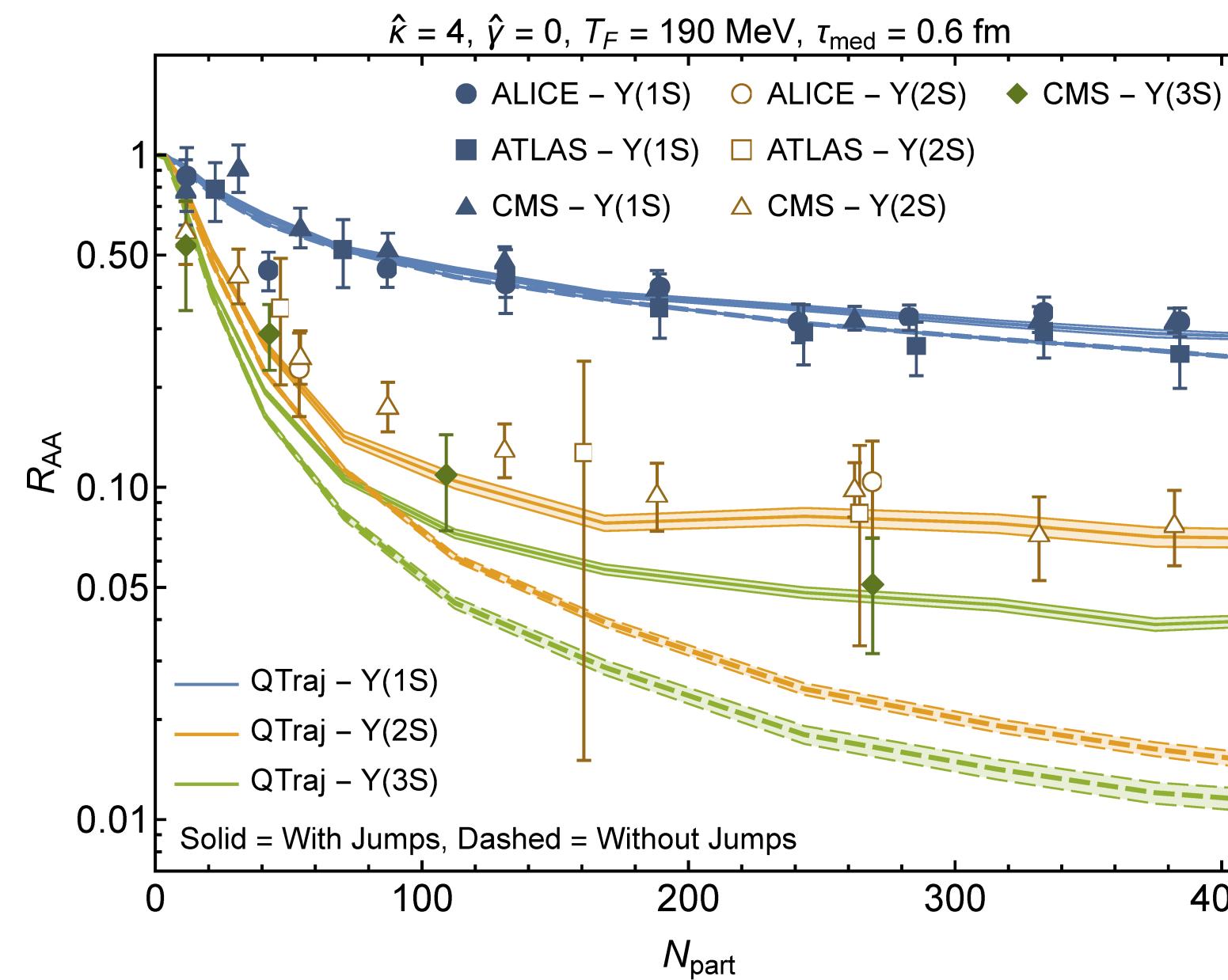
We compute

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$



and compare with LHC data

Bands are from the dependence in kappa and gamma parameters



Here with kappa and gamma  
Fixed in unquenched (undirect) lattice data

Best theory description: show the  
Quantum recombination effects are  
necessary to describe the data

N.B., M. Escobedo, M. Strickland,  
A. Vairo P. VanderGriend et al

arXiv:2302.11826

arXiv:2107.06222

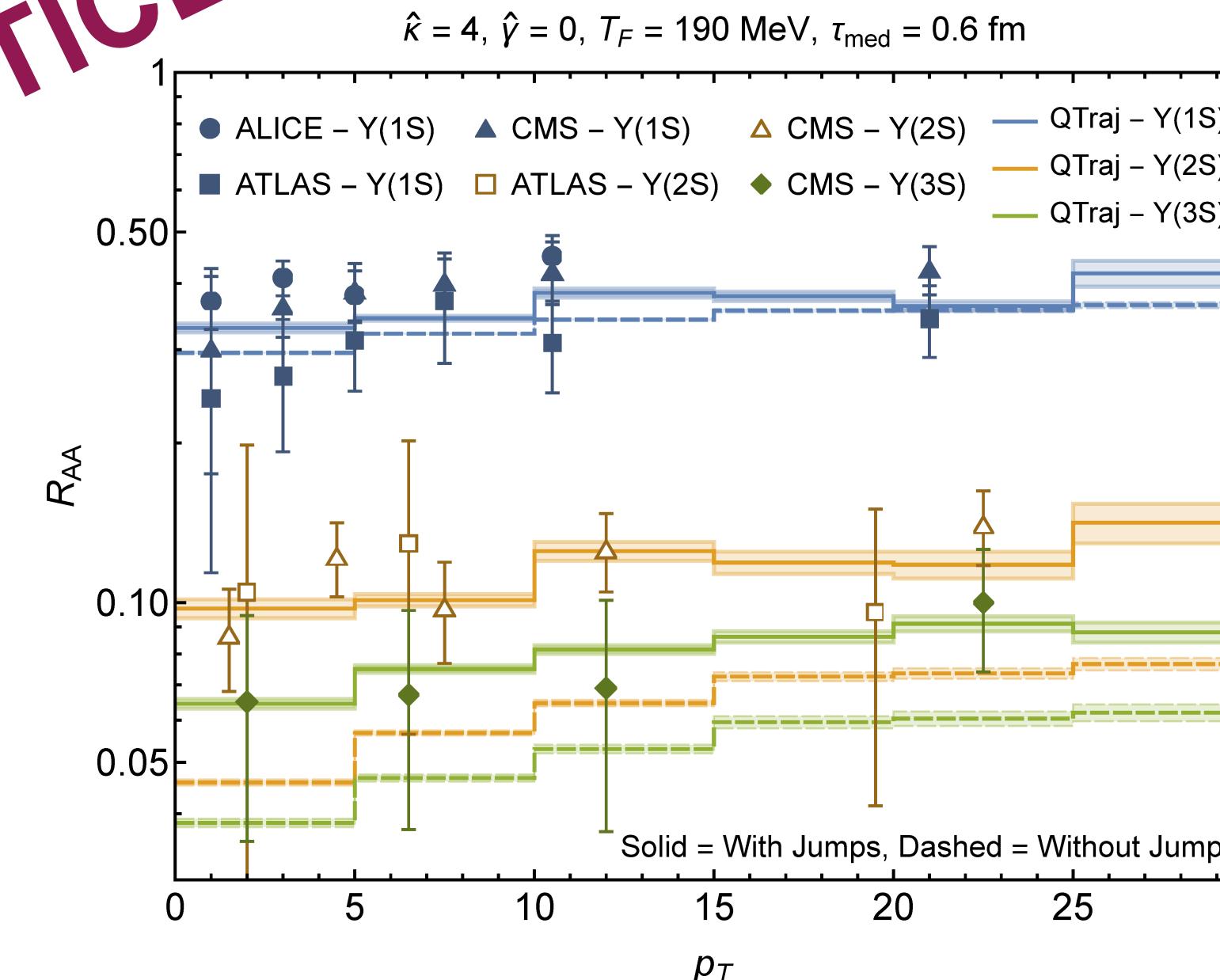
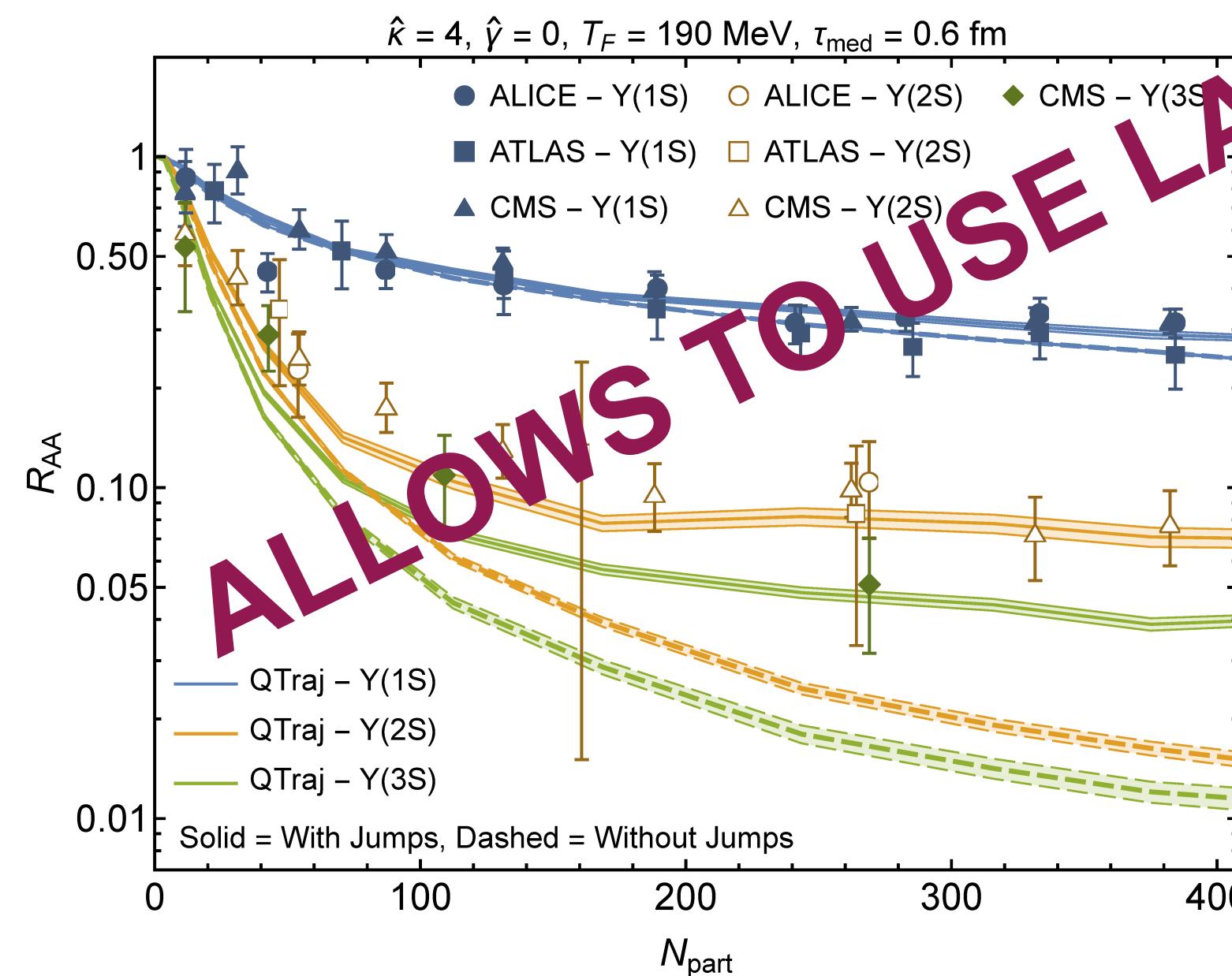
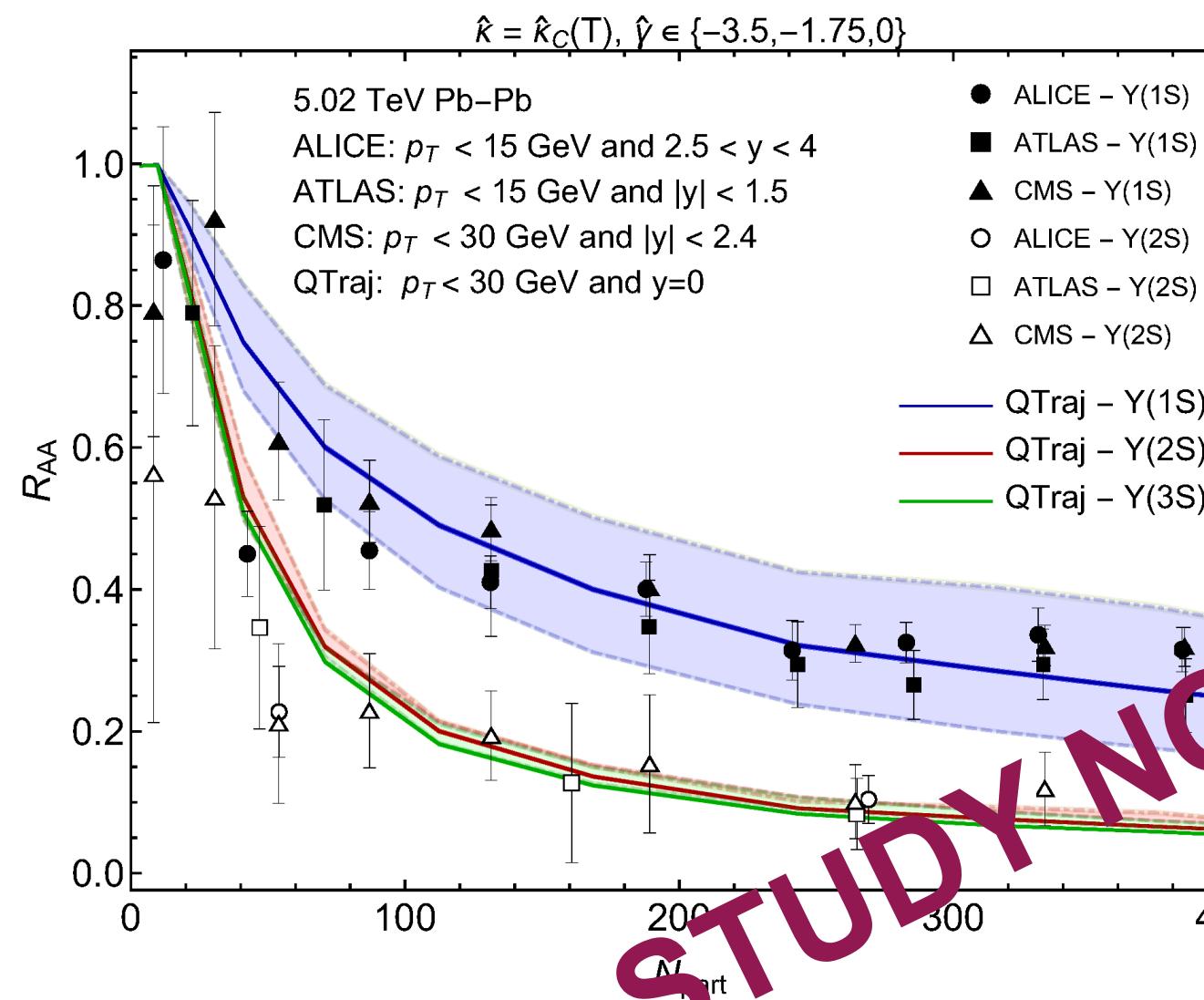
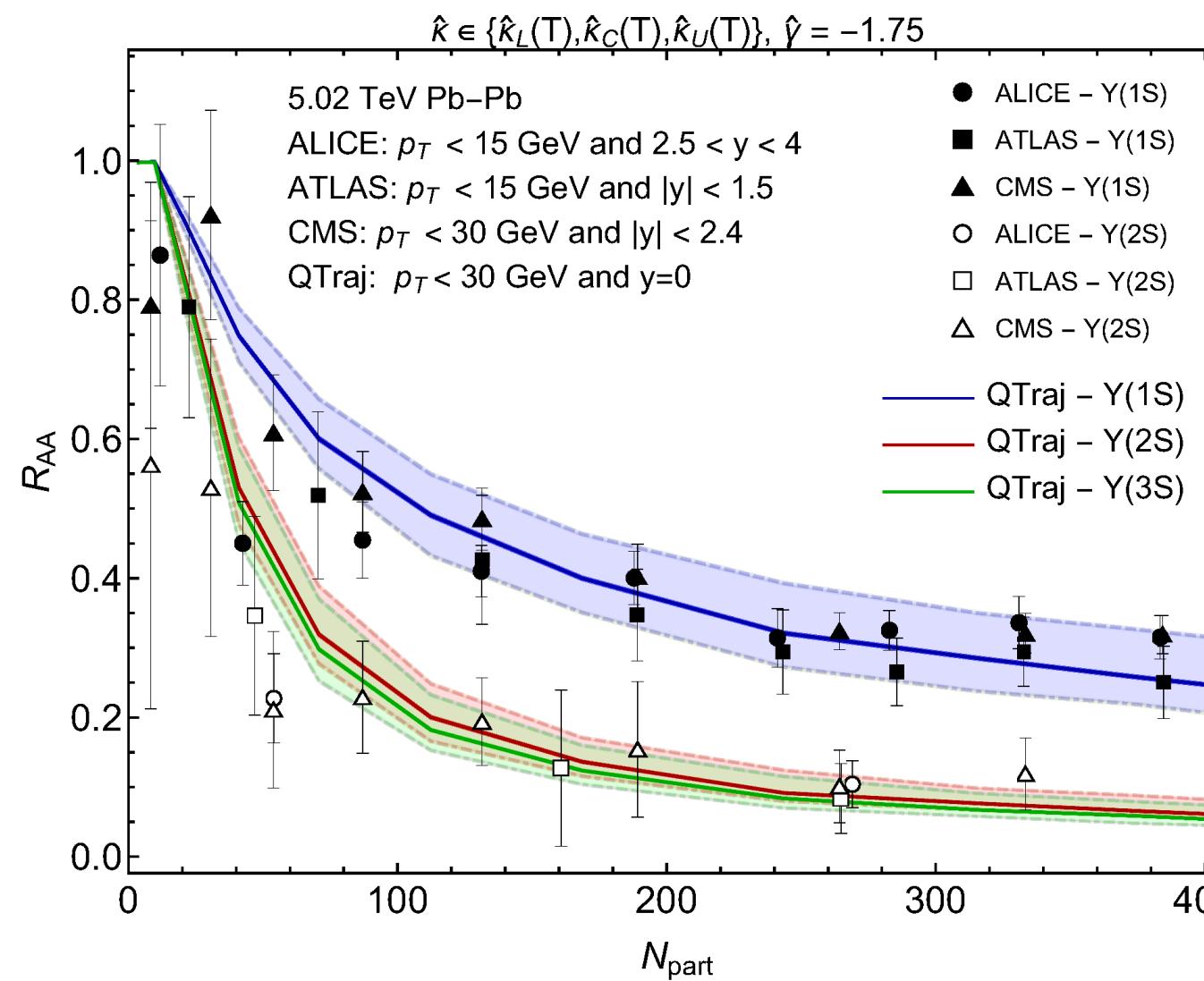
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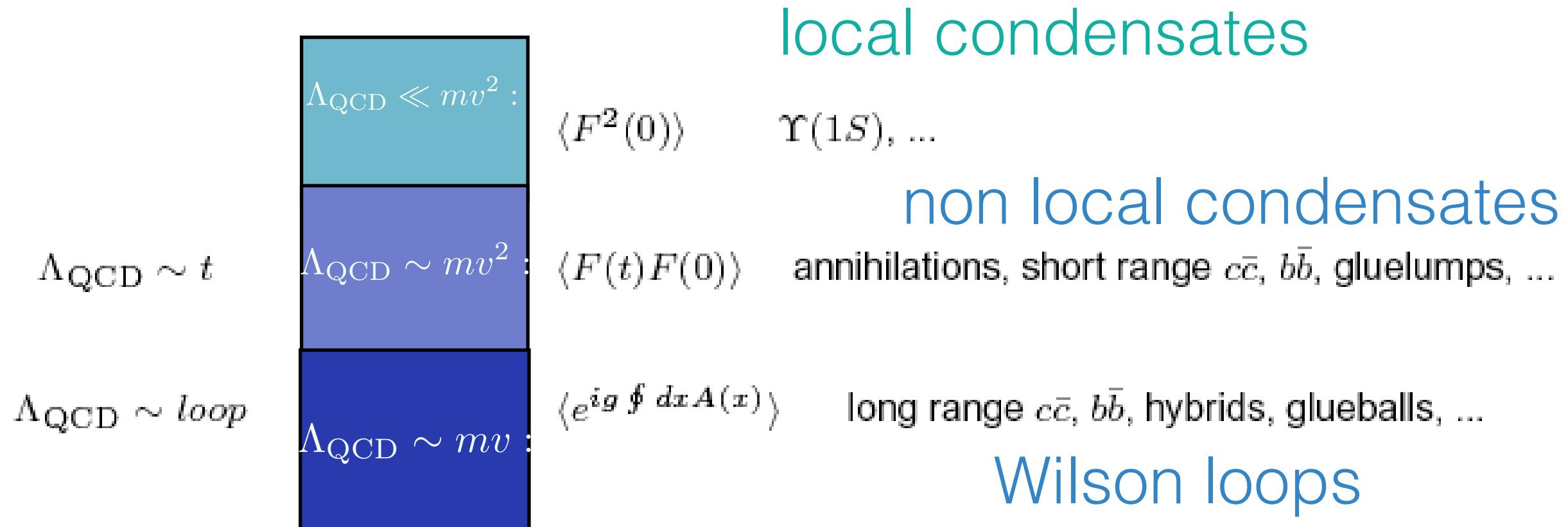
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ALLOWS TO USE LATTICE to STUDY NONEQUILIBRIUM PHENOMENA

Low energy (nonperturbative) factorized effects depend on the size of the physical system

The EFT factorizes the low energy nonperturbative part.

Depending on the physical system:

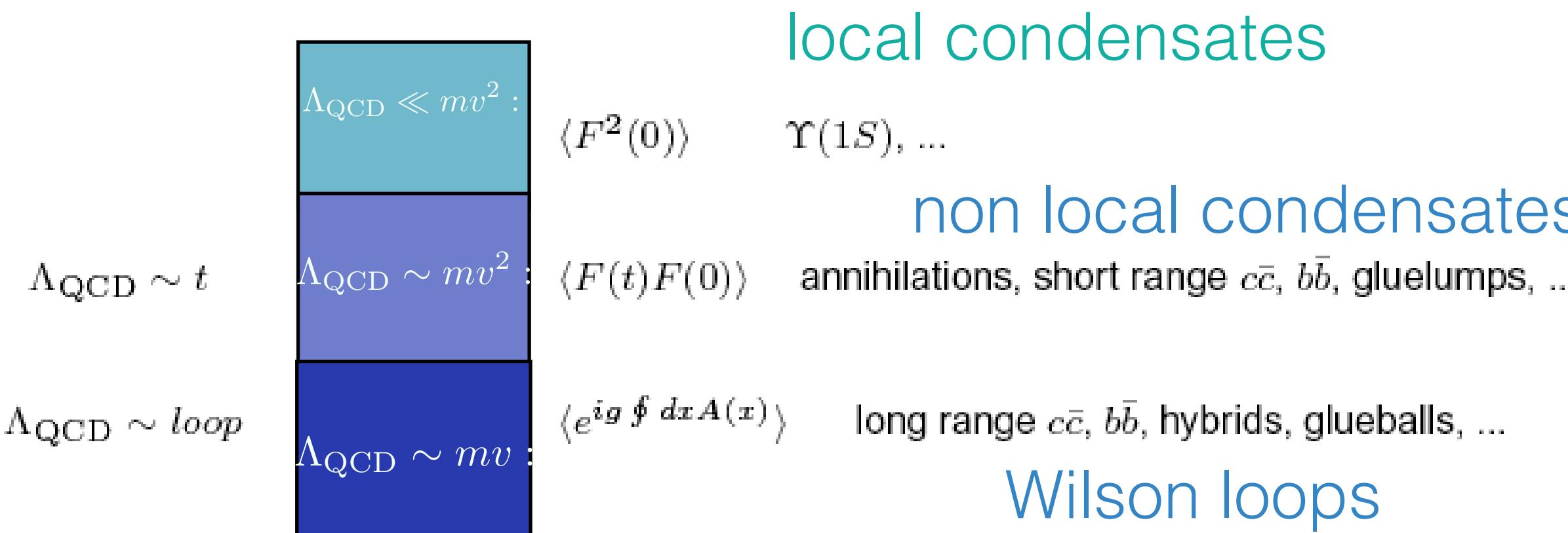


*The more extended the physical object, the more we probe the non-perturbative vacuum.*

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GAIN:

Inside the EFT: Model independent predictions, power counting

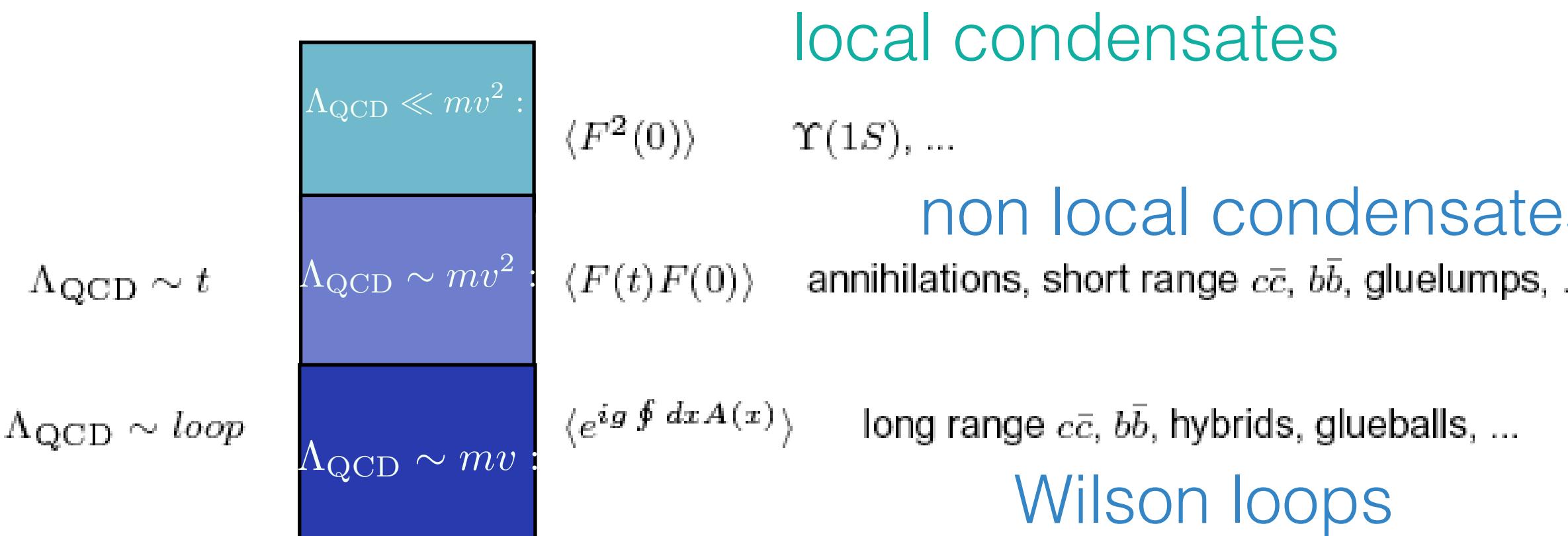
Lattice Calculation of only few nonperturbative objects, universal and depending only on the glue—> at variance with the state dependent calculation of each single observable with the full dynamics!

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### CHALLENGE:

Need techniques to reduce noise and improve convergence to continuum for calculation of chromoelectric and chromomagnetic fields—> Gradient flow

Avoid change of scheme between continuum and lattice (cutoff) regularization> Gradient flow (composite operators renormalisation in cutoff scheme is painful)

**CLAIM 3:** The Bottleneck on the NREFTs predictivity  
is now set by the low energy nonperturbative correlators  
that have to be calculated on the lattice

Let us see a list of them

# Correlators needed from the lattice for the NREFTs at zero and finite T

We need to calculate several gauge invariant correlators with chromoelectric and chromomagnetic fields:

- **spectra** (large states) : generalized (i.e. with E and B insertions) static Wilson loops
  - **for decays**, spectra (small states): correlators of E and B fields non local in time
  - **for X Y Z states**: static energies for hybrids and tetraquarks, i.e. generalized Wilson loops, time nonlocal correlators of several E and B field (for the spin stricture), three point functions of a magnetic field between singlet and hybrid (mixing between quarkonium and hybrids), gluelump masses
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BUT.....

**CLAIM 4:** These object are better  
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Why?

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Why?

Let us start from the simplest object: the force, a physical object

# Example of the QCD force

## The force as a Wilson loop with a chromoelectric field

A direct computation of the force that avoids interpolating the static energy and taking numerically the derivative is possible from the expression of a rectangular Wilson loop,  $W_{r \times T}$ , with a chromoelectric field insertion on a quark line:

$$F(r) = \frac{d}{dr} E_0(r) = \lim_{T \rightarrow \infty} -i \frac{\langle \text{Tr}\{\text{P } W_{r \times T} \hat{\mathbf{r}} \cdot g\mathbf{E}(\mathbf{r}, t^*)\} \rangle}{\langle \text{Tr}\{\text{P } W_{r \times T}\} \rangle}$$

insertion of a single  $\mathbf{E}$   
into a static Wilson loop

physical object

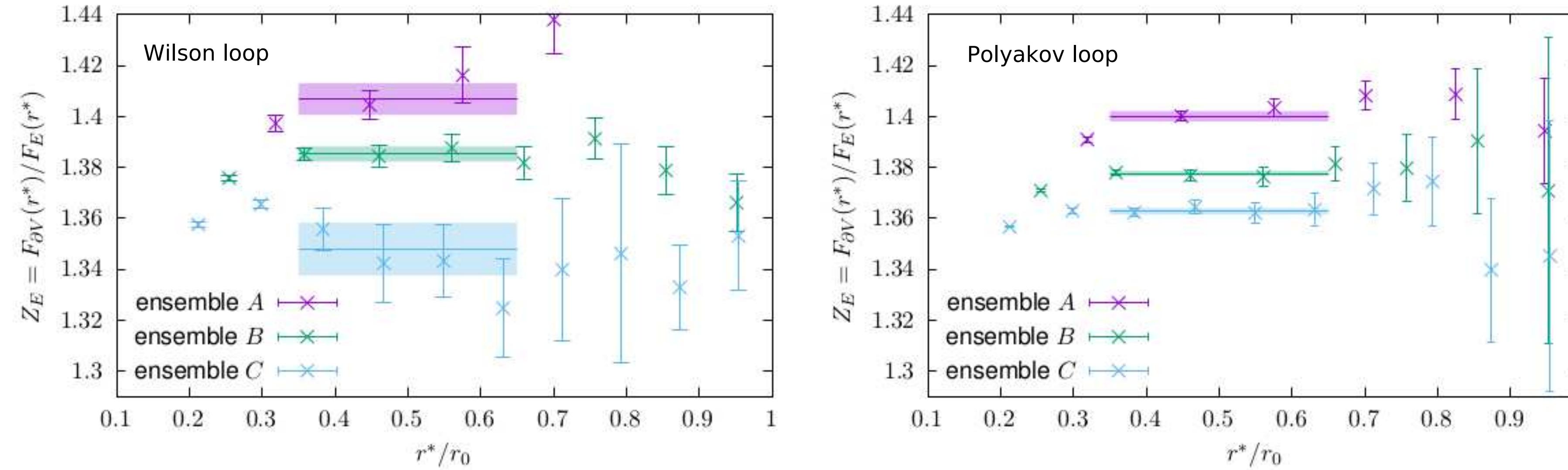
An equivalent expression can be written using a Polyakov loop instead of a Wilson loop.

At fixed  $t^*$  for  $T \rightarrow \infty$ , the rhs is independent of  $t^*$ .

The force is mass renormalon free and finite after charge renormalization.

- Brambilla Pineda Soto Vairo PRD 63 (2001) 014023  
Vairo MPLA 31 (2016) 34, 1630039

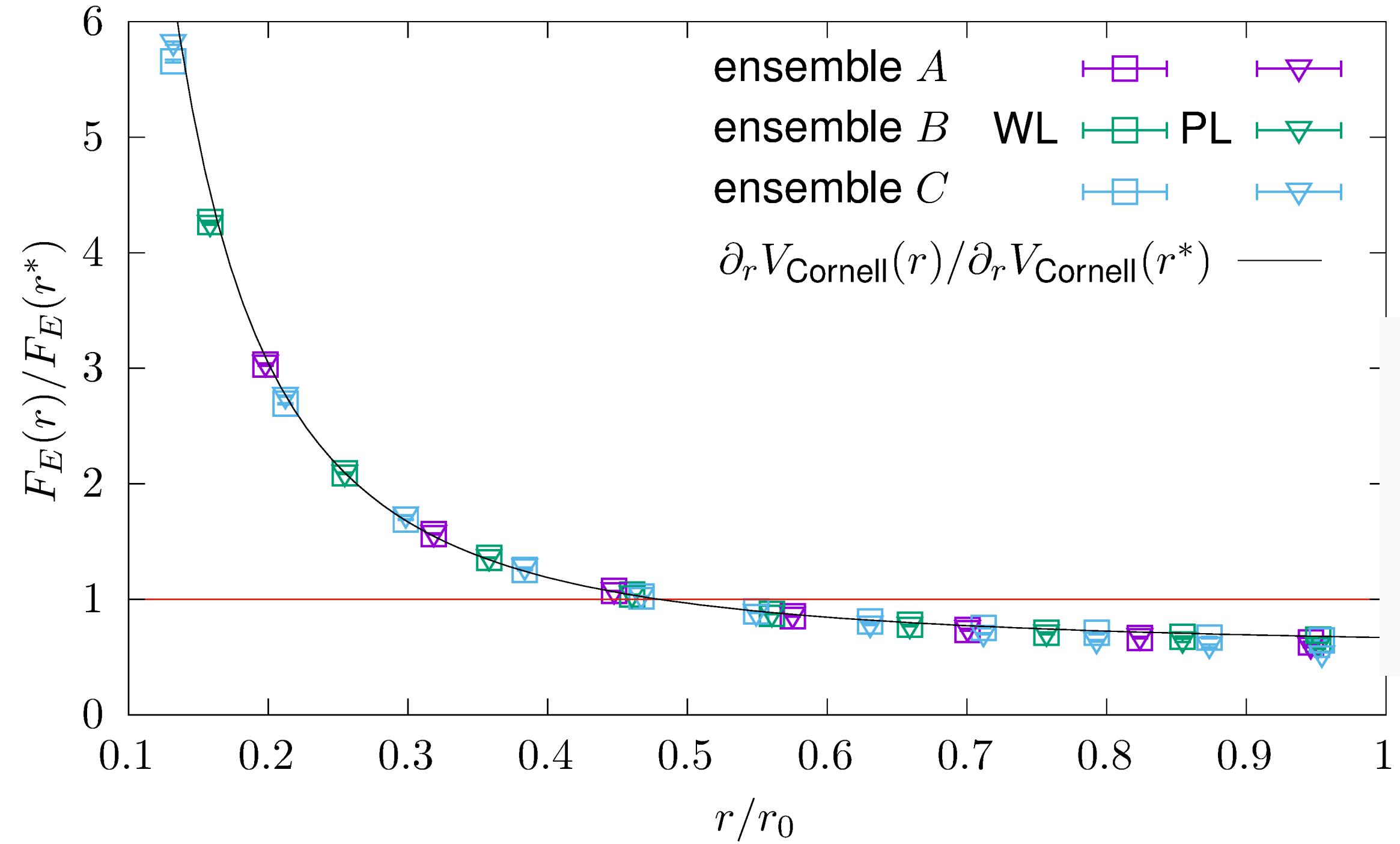
# Renormalization constant $Z_E$



The convergence of the direct force towards the continuum, i.e. the derivative of the static potential, is slow. The ratio of the two determinations is an  $r$  independent constant  $Z_E$  that may be computed once forever at some fixed (arbitrary) distance  $r^*$  ( $r_0 \approx 0.5$  fm).

ensemble	$a$ in fm	$Z_E$ from Wilson loops	$Z_E$ from Polyakov loops
A	0.060	1.4068(63)	1.4001(20)
B	0.048	1.3853(30)	1.3776(10)
C	0.040	1.348(11)	1.3628(13)

## Direct force vs lattice data



- Remove  $Z_E$  by dividing with measurement at  $r^* = 0.48r_0$
- Proof of concept:
  - Both derivative of potential and direct force agree
  - Both Wilson loop and Polyakov loops agree

Once normalized by  $Z_E$  the direct force agrees well with the Cornell parameterization based on quenched lattice data of the QCD static energy.

We have chosen  $r^* = 0.48 r_0 \approx 0.24 \text{ fm}$ .

**CLAIM5 : perform calculation in gradient flow**

**Features and strategy of calculation in gradient flow**

## Low energy correlators in gradient flow

## Physical quantities

The strategy is to calculate them in MSbar in gradient flow in continuum to guide the continuum limit of the lattice gradient flow calculation of the correlator

- We calculated the Free energy (one Q) 2+1 from Polyakov loop with GF in 1804.10600

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$$|X_n\rangle = \chi(\mathbf{x}_2)\phi(\mathbf{x}_2, \mathbf{R})T^a H^a(\mathbf{R})\phi(\mathbf{R}, \mathbf{x}_1)\psi^\dagger(\mathbf{x}_1)|\text{vac}\rangle$$

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- We extensively calculated the QQbar static energy not in GF (only smearing)

It serves to us to extract alphas —> it is needed at very short QQbar distance Could GF reach short distances?  
then we would need a 3 loop GF continuum calculation

In 2+1+1 It serves to us to quantify charm mass effects

## Low energy correlators in gradient flow

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- Hybrids static energies are calculated but not yet in GF
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## Low energy correlators in gradient flow

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- Static energies
- Hybrids static energies are calculated but not yet in GF
- Tetraquark static energies are calculated only in some cases not yet in GF
- Physical quantities like the force and the heavy quark momentum diffusion coefficient needs calculations at short distance in gradient flow to guide the continuum limit extrapolation ->
- Force (lattice Mayer Steudte, continuum GF perturbative talk Wang) and the heavy quark momentum diffusion coefficient (lattice talk Leino, one loop GF calculation (partial) Eller, Moore)

# Low energy correlators in gradient flow

Correlators appearing as part of physical quantities

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**GF will simplify the problem enhancing the signal and allowing to avoid a change of scheme**

# OUTLOOK

- There is a large number of interesting physical problems that can be better attacked with NREFTs and need gradient flow calculation of low energy correlators
- In most cases even a quenched calculation would be interesting and has never been done
- General strategies to optimise the perturbative/nonperturbative interface should still be developed
- Studies of confinement physics may be done
- Is there any better use of ML 4Lattice in case of correlators with only one scale?
  - What I discussed here is relevant for hadron structure studies, i.e. the physics of EIC (Electron Ion Collider) as similar low energy correlators mostly depending of gluon fields appear in PDF, TMDs
  - Could one directly define the NREFT in the gradient flow?
  - Systematic applications of gradient flow to calculate low energy correlators in NREFTs may bring a lot of progress in some of the most interesting contemporary problems in particle/nuclear

# Material for discussion/references

N. Brambilla, A. Pineda J. Soto, A. Vairo  
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· **Effective field theories for heavy quarkonium**  
Nora Brambilla, Antonio Pineda, Joan Soto, Antonio Vairo  
*Rev.Mod.Phys.* 77 (2005) 1423  
e-Print: [hep-ph/0410047](#)

**Inclusive production of J/psi psi(2s) Y states in pNRQCD**  
N. Brambilla, H. S. Chung, A. Vairo, X.P. Wang  
*JHEP* in press 2210.17345

## pNRQCD Foundation

## Gradient Flow Perturbative calculations

Force: [2111.07811](#)

## Quarkonium Production

## Gradient Flow Lattice calculations

· **Quarkonium Hybrids with Nonrelativistic Effective Field Theories**  
Matthias Berwein , Nora Brambilla, Jaume Tarrús Castellà, Antonio Vairo  
*Phys.Rev. D*92 (2015) no.11, 114019  
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## BOEFT

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**QCD spin effects in the heavy hybrid potentials and spectra**  
Nora Brambilla, Wai Kin Lai, J. Segovia, J. Tarrus  
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**Quarkonium suppression in heavy ion collisions:  
an open quantum system approach**  
N. Brambilla, M. Escobedo, J. Soto, A. Vairo  
*PRD* 96 (2017) 3, 034021 [1612.07248](#)

**Regeneration of bottomonia in  
an open quantum systems approach**  
N. Brambilla, M. Escobedo, M Strickland et al [2302.11826](#)

## Non-equilibrium Quarkonium evolution in QGP

Heavy quark momentum diffusion coefficient:  
[2206.02861](#)

Backup

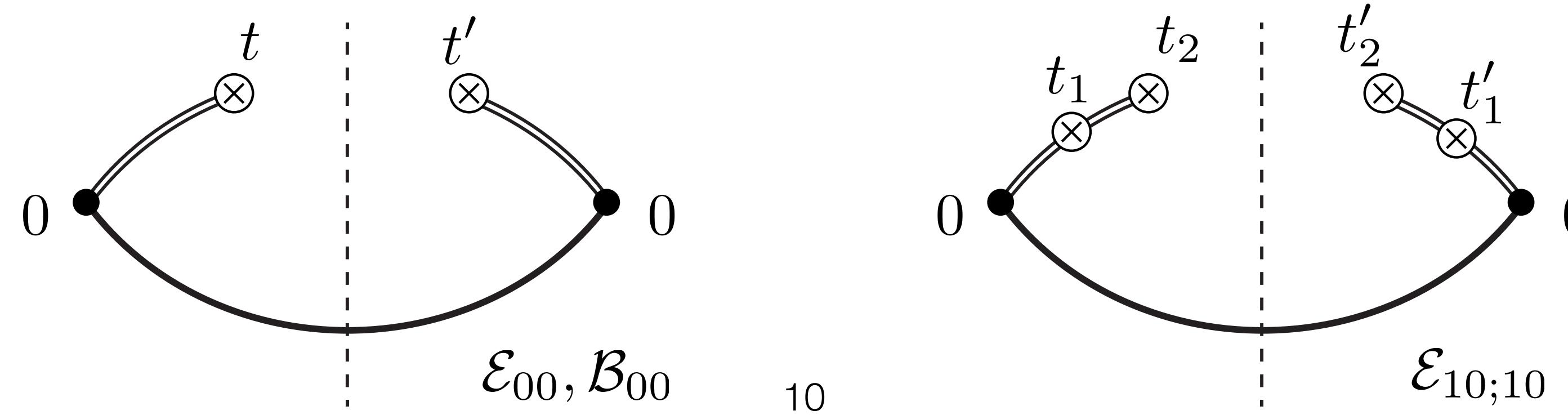
- Definitions of gluonic correlators:

$$\begin{aligned}\mathcal{E}_{10;10} &= d^{a'b c'} d^{e' x y'} \int_0^\infty dt_1 t_1 \int_{t_1}^\infty dt_2 \langle \Omega | \Phi_\ell^{\dagger ad}(0) \Phi_0^{a' a\dagger}(0; t_1) g E^{b,i}(t_1) \Phi_0^{c c'\dagger}(t_1; t_2) g E^{c,i}(t_2) \\ &\quad \times \int_0^\infty dt'_1 t'_1 \int_{t'_1}^\infty dt'_2 g E^{y,j}(t'_2) \Phi_0^{y y'}(t'_1; t'_2) g E^{x,j}(t'_1) \Phi_0^{e' e}(0; t'_1) \Phi_\ell^{d e}(0) | \Omega \rangle\end{aligned}$$

$$\mathcal{E}_{00} = \int_0^\infty dt \int_0^\infty dt' \langle \Omega | \Phi_\ell^{\dagger ab}(0) \Phi_0^{\dagger ad}(0; t) g E^{d,i}(t) g E^{e,i}(t') \Phi_0^{e c}(0; t') \Phi_\ell^{b c}(0) | \Omega \rangle$$

$$\mathcal{B}_{00} = \int_0^\infty dt \int_0^\infty dt' \langle \Omega | \Phi_\ell^{\dagger ab}(0) \Phi_0^{\dagger ad}(0; t) g B^{d,i}(t) g B^{e,i}(t') \Phi_0^{e c}(0; t') \Phi_\ell^{b c}(0) | \Omega \rangle$$

- Configurations of Wilson lines and field strength insertions:



# Decay in the Singlet Model

$$|H\rangle = \int \frac{d^3k}{(2\pi)^3} \Phi_{uv}^{ij}(\mathbf{k}) |Q(\mathbf{k})^{iu} Q(-\mathbf{k})^{jv}\rangle \quad \Phi_{uv}^{ij}(\mathbf{x}) \sim (\dots) R(x)$$

$$\Gamma(\chi_0 \rightarrow \text{LH}) = \sum_X \Gamma(\chi_0 \rightarrow X) \simeq \Gamma(\chi_0 \rightarrow gg)$$

$$\Gamma(\chi_0 \rightarrow \text{LH}) \simeq \langle H | 2 \operatorname{Im} \left[ \text{Diagram} \right] | H \rangle$$

- tree level

$$\left| \text{Diagram} \right|^2 = 2 \operatorname{Im} \left( \text{Diagram} \right) \xrightarrow[q \sim mv \ll m]{\frac{4\pi\alpha_s^2}{3} \frac{\mathbf{S} \cdot \nabla \delta(\mathbf{r}) \mathbf{S} \cdot \nabla}{m^4}}$$

$$\Gamma = \langle H | \dots | H \rangle = 9 \left( \frac{C_F \pi}{2} \alpha_s^2 \right) \frac{\left| R'(0) \right|^2}{\pi m^4}$$

- one loop

$$2 \operatorname{Im} \left( \text{Diagram} + \dots \right) \longrightarrow \left( (\dots) \alpha_s^2 + (\dots) \alpha_s^3 \ln \frac{\mu}{m} \right) \frac{\mathbf{S} \cdot \nabla \delta(\mathbf{r}) \mathbf{S} \cdot \nabla}{m^4}$$

Barbieri et al. 76, 79, 80, 81

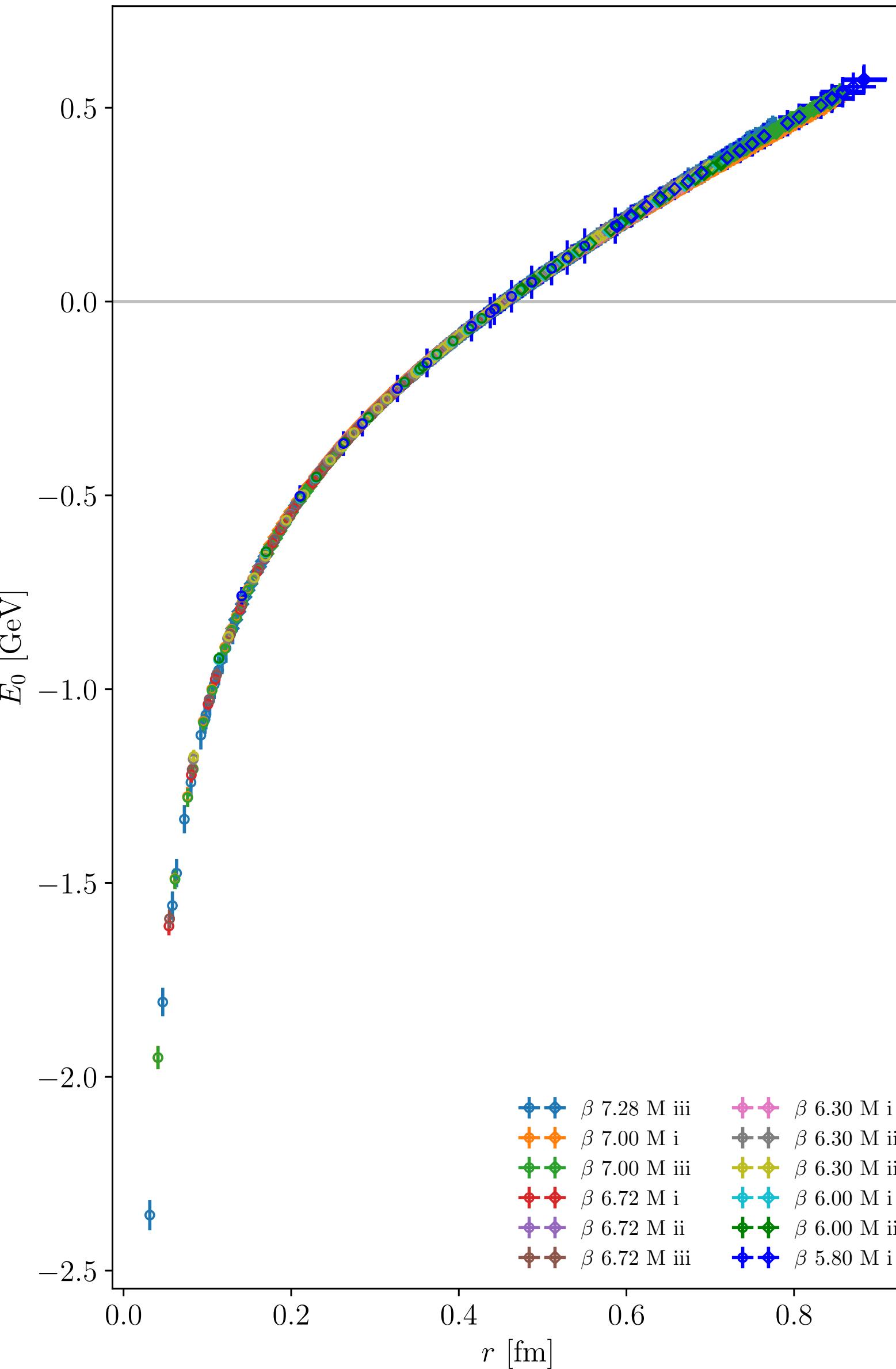


Figure 1. Results for the static energy in physical units from the calculations described in this paper. The data are from twelve ensembles of varying lattice spacing (keyed by  $\beta$ ) and three choices of light quark mass (denoted “M i”, “M ii”, “M iii”). Lattice units are eliminated via  $r_0/a$ , and the unphysical constant is eliminated by setting  $E_0(r_0) = 0$ . See Sec. IV C for details.