The Strong CP Problem revisited using the Gradient Flow

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY

With

Yoshifumi Nakamura

RIKEN Center for Computational Science, Kobe

arXiv:2106.11369

arXiv:2212.05485

Outline

Introduction

One of the most intriguing unsolved problems in particle physics is the strong CP problem. QCD allows for a CP-violating term S_θ, called the θ term, in the action,

$$S = S_0 + S_\theta$$

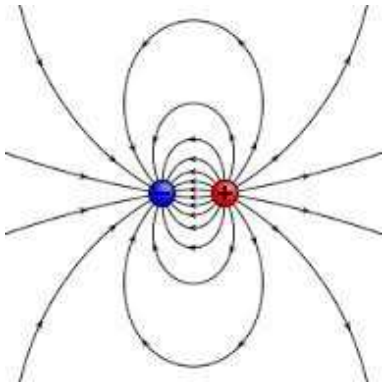
In Euclidean space-time it reads

$$S_\theta = i\,\theta\,Q\,, \quad Q = \frac{1}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\int d^4x\ \mathrm{Tr}\,[F_{\mu\nu}F_{\rho\sigma}]\ \in\ \mathbb{Z}$$

where θ is the *bare* vacuum angle

A finite value of θ would result in an electric dipole moment $d_n \propto \theta$ of the neutron. To date the most sensitive measurements of d_n are compatible with zero. The current upper bound is $|d_n| < 1.8 \times 10^{-13} e\,\mathrm{fm}$, indicating that θ is anomalously small. Why should a parameter not forbidden by symmetry be essentially zero? This puzzle is referred to as the strong CP problem

The electric dipole moment d_n is a measure of (permanent) separation of positive and negative charge in the neutron. According to the upper bound on d_n, the separation would have to be less than 10^{-13} fm. If a finite value for d_n is finally found, it would be hard to believe that it can be attributed to QCD, and to the topological properties of the theory in particular



A popular view is that the solution of the strong CP problem lies beyond QCD and the Standard Model. Indeed, the first instinct in such a situation is to propose a new symmetry that suppresses CP-violating terms in the strong interactions. Peccei and Quinn concocted such a symmetry in 1977, at the expense of introducing a hitherto undetected particle, the axion. It is widely believed that the Peccei-Quinn axion leaves the designated properties of QCD, such as confinement, chiral symmetry breaking and the chiral anomaly, unscathed. This is, however, not the case $\Longrightarrow$

In view of the topological nature of the problem, it is obvious to look for a solution within the QCD. A solution within QCD would mean that QCD does not exist as a viable physical theory unless $\theta = 0\,[\mathrm{mod}\,2\pi]$, or that QCD has an infrared (IR) fixed point at which the vacuum angle renormalizes to $\theta = 0$, or both. As a guideline it is helpful to have some model understanding how the QCD vacuum reacts to the θ term

Repercussions of the Peccei-Quinn axion on QCD

Generic action

$$S = \int d^4x \left[\frac{1}{2}\left(\partial_\mu \phi\right)^2 + i\frac{\phi}{f}\, P + L_{\rm int}(\partial_\mu\phi, \Psi)\right] + S_{\rm QCD}\,, \quad P = \frac{1}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,{\rm Tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]$$

Writing

$$\int \mathcal{D}\phi \int \mathcal{D}A_\mu = \int d\bar{\phi} \int_{\delta(\sum\phi)} \mathcal{D}\phi \int \mathcal{D}A_\mu\,, \quad \bar{\phi} = \frac{1}{V}\sum \phi$$

and integrating out ϕ, taking into account that $P = \partial_\mu \Omega^{(0)}_\mu$, $\Omega^{(0)}_\mu = \partial_\nu \Omega^{(1)}_{\mu\nu}, \cdots$, we are left with the path integral restricted to trivial topology Jackiw; Laursen, GS, Wiese

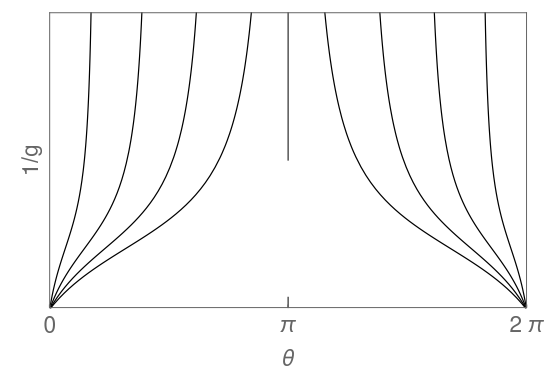
$$\int \mathcal{D}\phi \int_{\underline{Q=0}} \mathcal{D}A_\mu \exp\left\{-\int d^4x \left[\frac{1}{2}\left(\partial_\mu\phi\right)^2 + L_{\rm int}(\partial_\mu\phi, \Psi)\right] - S_{\rm QCD}\right\}$$

on finite and infinite volumes, and patch-wise

We conclude that the Peccei-Quinn model is not compatible with QCD Leutwyler, Smilga

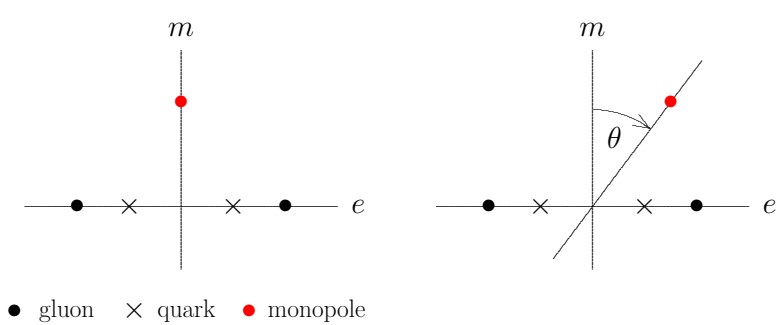
One source of information are field theories that share the main characteristics of QCD, but lend themselves to (semi-)analytic investigations

1. A prominent example is the CP^{N-1} model in two dimensions at large N, which reflects the fundamental features of the quantum Hall effect. The Hall conductivity σ_{xy} has a precise parallel in the vacuum angle θ, $\theta/2\pi \sim \sigma_{xy}$, while the linear conductivity is represented by the inverse coupling, $N/g \sim \sigma_{xx}$. The flow to quantization, $\theta = 0$, appears to be a generic feature of the instanton vacuum as well



Pruisken, Levine, Libby; Knizhnik, Morozov

2. 't Hooft has argued that the degrees of freedom responsible for confinement are color-magnetic monopoles. This is unfolded by fixing to the maximally abelian gauge that leaves the Cartan subgroup U(1) × U(1) ⊂ SU(3) unbroken. Monopoles arise from singularities of the gauge condition. Quarks and gluons have color-electric charges with respect to the U(1) subgroups. For θ different from zero the monopoles acquire a color-electric charge $q = \theta/2\pi$ Witten



Due to the joint presence of gluons and monopoles a rich phase structure is expected to emerge as a function of θ. For $\theta > 0$ it is expected that the color fields of quarks and gluons are screened by forming bound states with the monopoles. The Debye screening length of a particle of charge q immersed in the conducting vacuum is given by $\lambda_D = \sqrt{E_F/\rho\, q^2}$ (E_F: Fermi energy, ρ: monopole density) which leads to $\lambda_D = \sqrt{4\pi^2 E_F/\rho\, \theta^2}$. This suggests that θ is restricted to zero in the confining phase of the theory

3. It is known from the case of the massive Schwinger model as well that a θ term may change the phase of the system. Callan, Dashen and Gross have claimed that a similar phenomenon will occur in QCD. Their statement is that the color fields produced by quarks and gluons will be screened by instantons for $|\theta| > 0$

This is a multi-scale problem, which involves the passage from the short-distance weakly coupled regime to the long-distance strongly coupled confinement regime. The framework for dealing with physical problems involving different energy scales is the multi-scale renormalization group (RG) flow. Exact RG transformations are very difficult to implement numerically. The gradient flow provides a powerful alternative for scale setting, with no need for costly ensemble matching. It can be regarded as a particular, infinitesimal realization of the coarse-graining step of momentum space RG transformations à la Wilson, Polchinski and Wetterich, which leaves the long-distance physics unchanged

Lüscher

Makino, Morikawa, Suzuki

Carosso, Hasenfratz, Neil

Gradient Flow

The gradient flow evolves the gauge field along the gradient of the action. The flow of SU(3) gauge fields is defined by

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

where D_μ is the covariant derivative and $B_\mu(t=0,x) = A_\mu(x)$ is the lattice gauge field. The renormalization scale is set by $\mu = 1/\sqrt{8t}$, where $\sqrt{8t}$ is the smoothing range over which B_μ is averaged. The expectation value $\langle E(t)\rangle$ of the energy density

$$E(t,x) = \frac{1}{4}\, G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$$

defines a renormalized coupling

$$g^2_{GF}(\mu) = \frac{16\pi^2}{3}\, t^2 \langle E(t)\rangle \,\big|_{t=1/8\mu^2}\,, \qquad \Lambda_{GF} = \exp\left\{\frac{2\pi}{11} t_1\right\} \Lambda_{\overline{MS}}$$

in the gradient flow scheme

Lüscher; Harlander

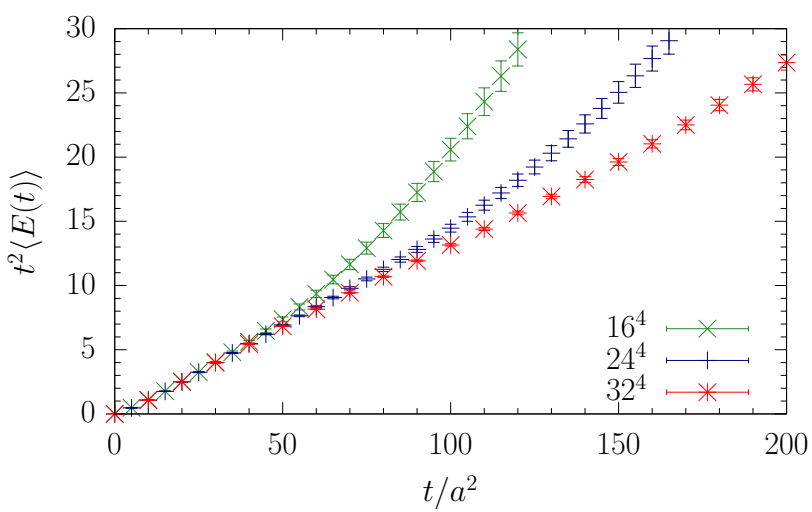
For a start we may restrict our investigations to the Yang-Mills (YM) theory. If the strong CP problem is resolved in the YM theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \left(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\; U_{\mu\nu}(x)\right)$$

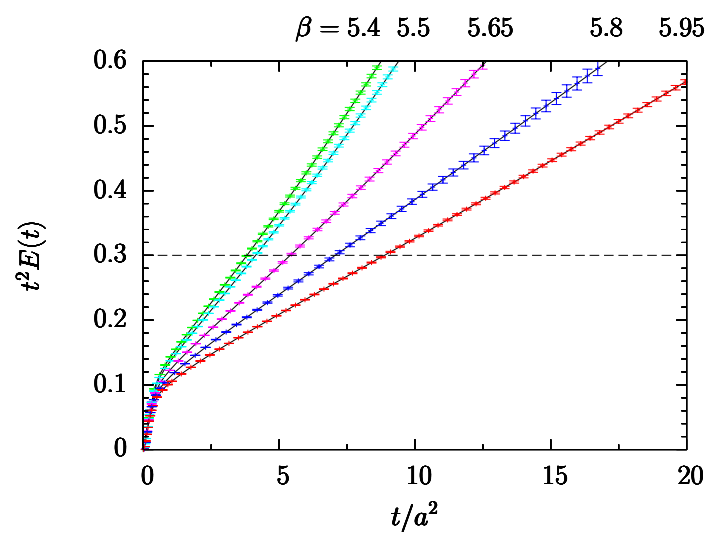
	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

We consider bulk quantities only



$N_f = 2 + 1$



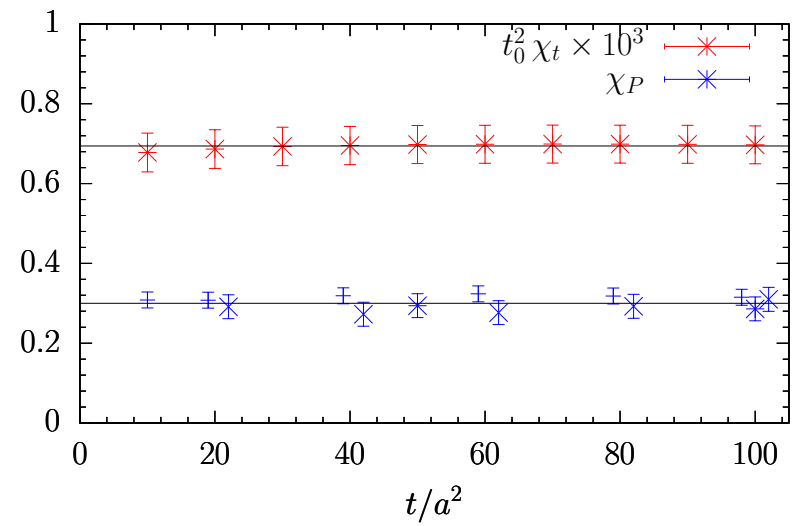
Constant physics?

Topological susceptibility

$$\chi_t = \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{V}$$

Renormalized Polyakov susceptibility

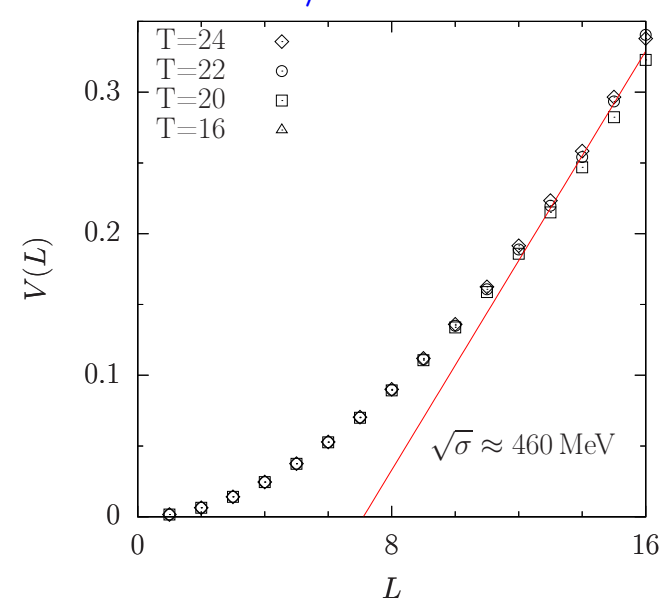
$$\chi_P = \frac{\langle |P|^2 \rangle - \langle |P| \rangle^2}{\langle |P| \rangle^2}, \quad P = \frac{1}{V_3} \sum_{\mathbf{x}} P(\mathbf{x})$$



$t/a^2 = 30$

Static potential

$$W(L,T) \simeq \exp\{-V(L)\,T\}$$

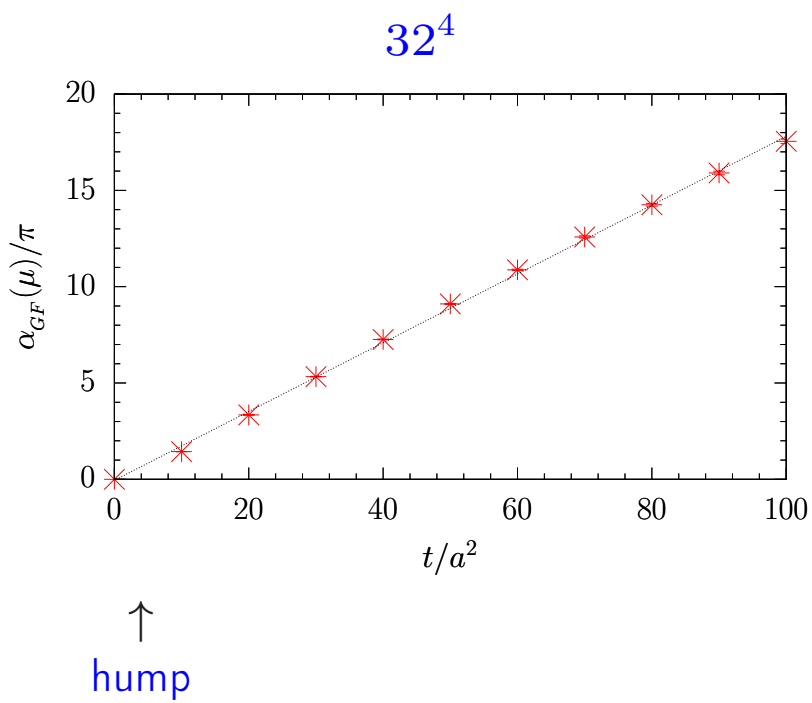


Mass gap (see later)

Running Coupling and Confinement

Confinement is intimately connected with the IR behavior ($\mu \to 0$) of the running coupling $\alpha_{GF}(\mu)$

Pure YM theory is of particular interest in this respect

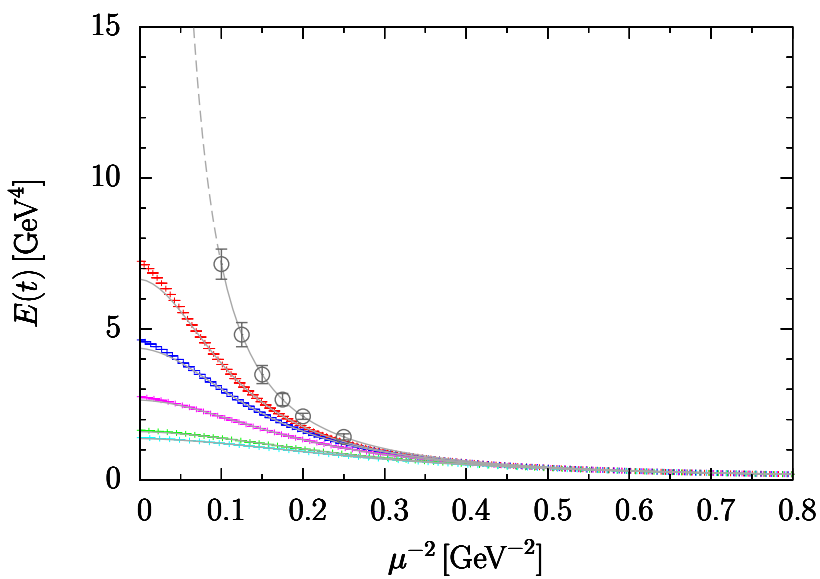


$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF})$$

$$\underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0\,\alpha_{GF})^{-\frac{b_1}{2\,b_0^2}}\; e^{-\left\{\frac{1}{8\pi b_0\,\alpha_{GF}}+\int_0^{\alpha_{GF}} d\alpha\,\frac{1}{\beta_{GF}(\alpha)}+\frac{1}{8\pi b_0\,\alpha^2}-+\frac{b_1}{2\,b_0^2\,\alpha}\right\}}$$

Matching the two-loop expression



$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

cf. arXiv:2303.00704

To make contact with phenomenology, it is desirable to transform the GF coupling α_{GF} to a common scheme. A preferred scheme in the YM theory is the V scheme: $V(q) = -4\pi C_F \alpha_V(\mu)/q^2$

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

IR behavior universal

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3} \int d^3\mathbf{q}\; e^{i\,\mathbf{qr}}\, \frac{4}{3}\, \frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

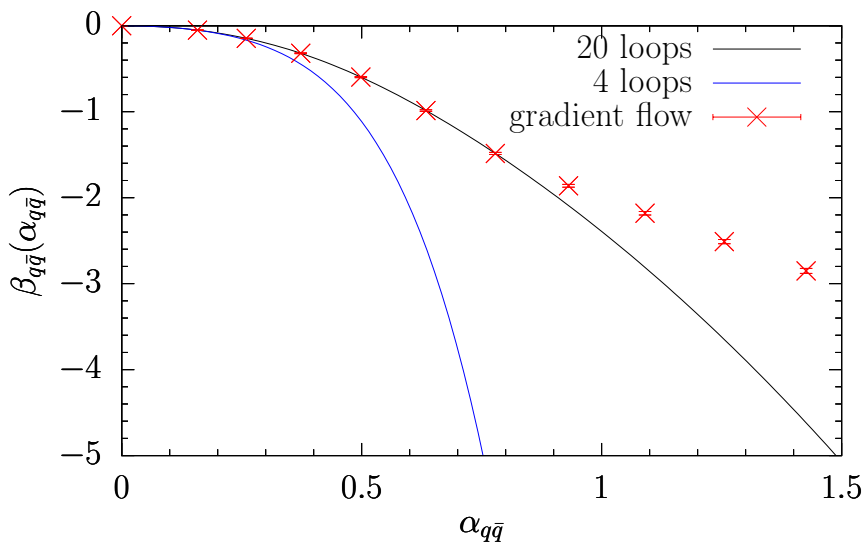
$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

It is interesting to compare the nonperturbative GF beta function with the perturbative beta function known up to twenty loops



$$\frac{\Lambda_{q\bar{q}}}{\Lambda_V} = 0.655$$

20 loops arXiv:1309.4311

4 loops arXiv:1012.3037

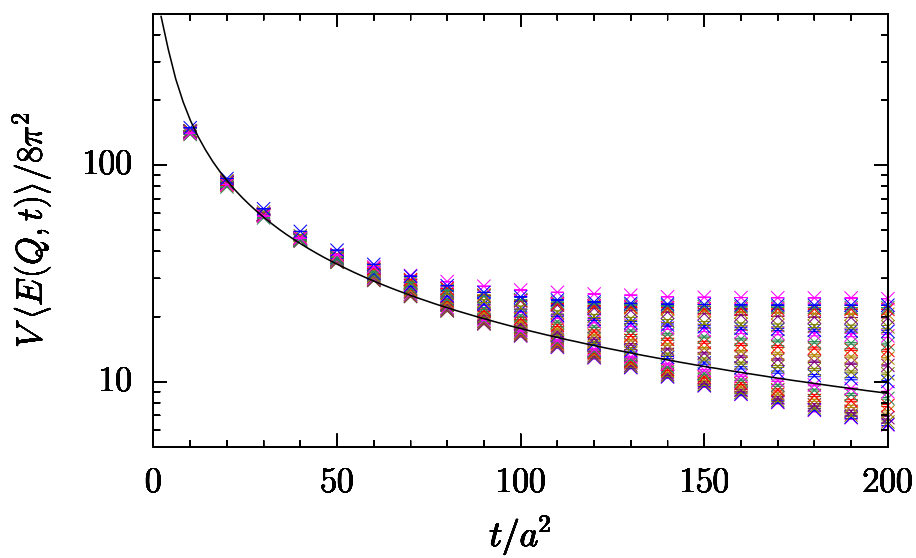
In general

$$\alpha_S \xrightarrow[\mu \to 0]{} \begin{cases} 0 \\ \infty \end{cases} \quad \text{possible}$$

As was to be expected, the perturbative beta function gradually approaches the nonperturbative beta function with increasing order

Adding the θ Term

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased



$$\mathcal{P}(\theta) = \int \mathcal{D}A_\mu \, e^{-S+i\theta Q}$$

$$= \sum_Q e^{i\theta Q} \int_Q \mathcal{D}A_\mu \, e^{-S}$$

$$Z(\theta) = \frac{\mathcal{P}(\theta)}{\mathcal{P}(0)} = \sum_Q e^{i\theta Q} \, P(Q)$$

DIG: $V\langle E(Q,t)\rangle/8\pi^2 \simeq |Q|$, while the ensemble average $\langle E(t)\rangle$ is finite and vanishes like $1/t$

$$Q = \int d^4x \, \partial_\mu \Omega_\mu^{(0)} \, , \; \partial_t \Omega_\mu^{(0)} = (1/8\pi^2) D_\rho G_{\nu\rho} \tilde{G}_{\mu\nu}$$

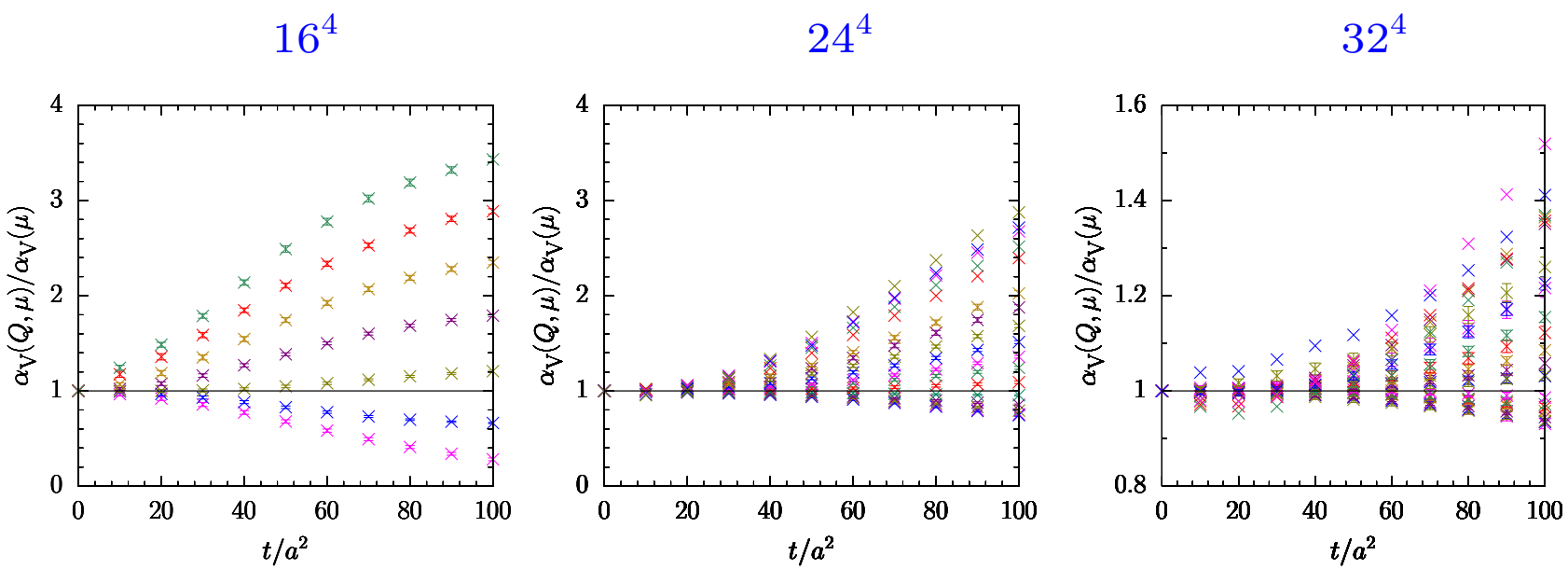
gauge invariant

$$\boxed{\Rightarrow \; \partial_t Q = 0}$$

Running coupling

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top



Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

The transformation of $\alpha_V(Q,\mu)$ from the 'Q vacua' to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q} P(Q)\, \alpha_V(Q,\mu)\,, \quad Z(\theta) = \sum_Q e^{i\,\theta\,Q} P(Q)$$

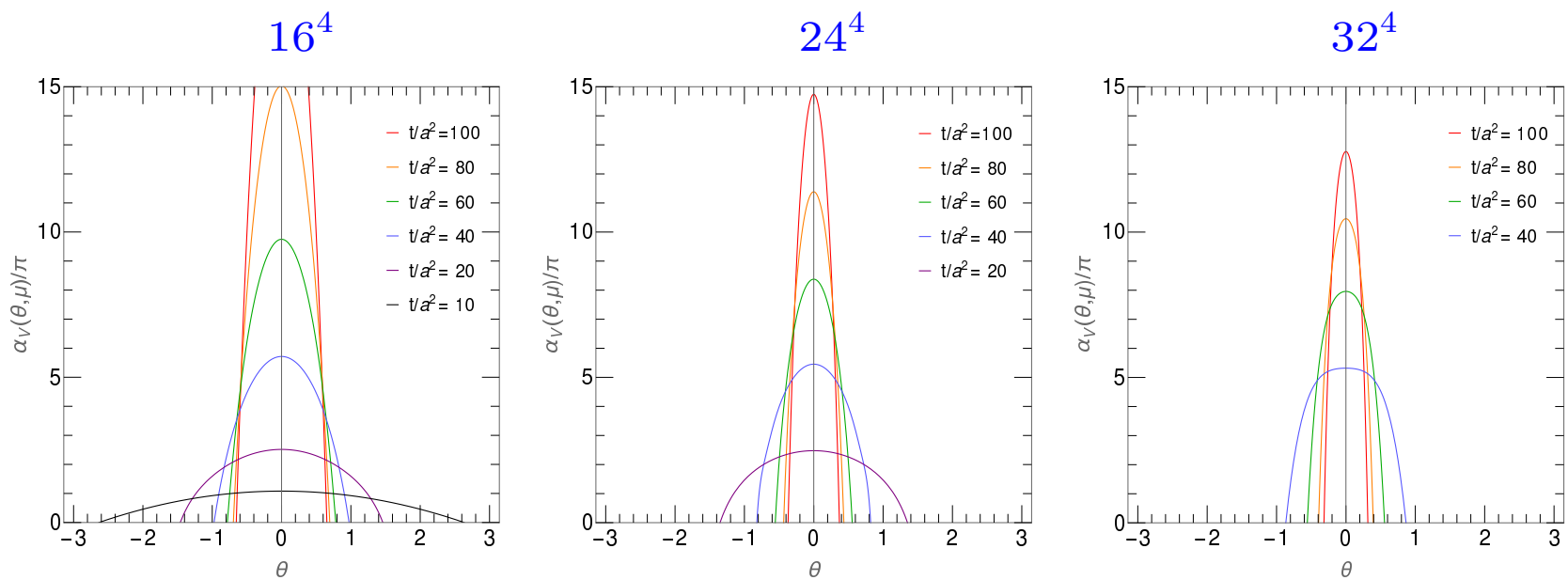
weighted by the charge density $P(Q)$, i.e. the probability of finding a configuration with charge Q

A few remarks are in order

- Here the parameter θ is the bare vacuum angle that labels the superselection sectors. It is the parameter that appears in the (lattice) action and determines the topological properties of the vacuum

- $P(Q)$ is determined by the real part of the action, $S_{\rm QCD}$, which increases proportionally to $|Q|$ and suppresses configurations which hold a large number of (anti-)instantons. It thus becomes increasingly difficult to determine $P(Q)$ precisely for large values of $|Q|$. This circumstance is completely independent of whether we simulate at $\theta = 0$ or any other value $|\theta| > 0$. This is to say, the situation would not improve if we could simulate the complex action

- As we shall see, we need to know the Fourier sum for small values of $|\theta|$ only, which is rather insensitive to fluctuations at large values of $|Q|$



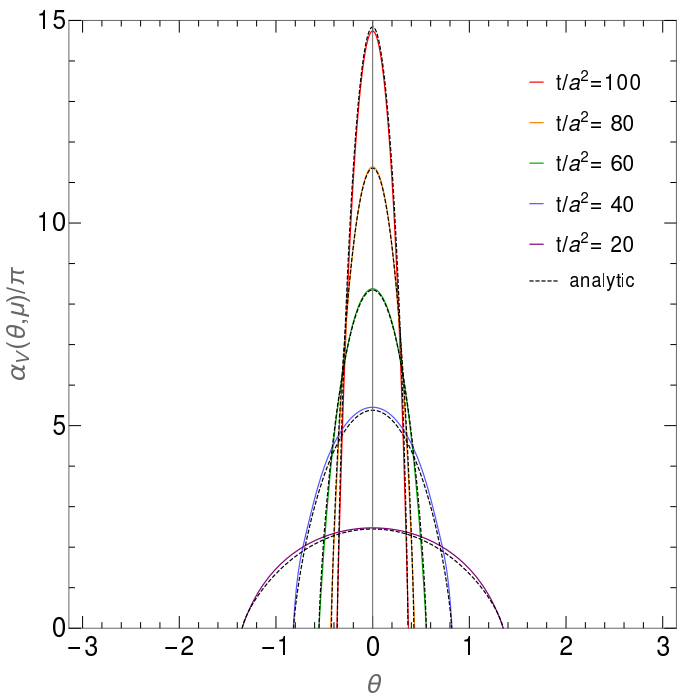
At a first glance

- The color charge gets totally screened for $|\theta| > 0$ in the infrared limit, while it becomes gradually independent of θ as we approach the perturbative regime

- The flow to confinement is constricted to the inner part of the envelope of the curves, that is to stay within QCD

Analytically

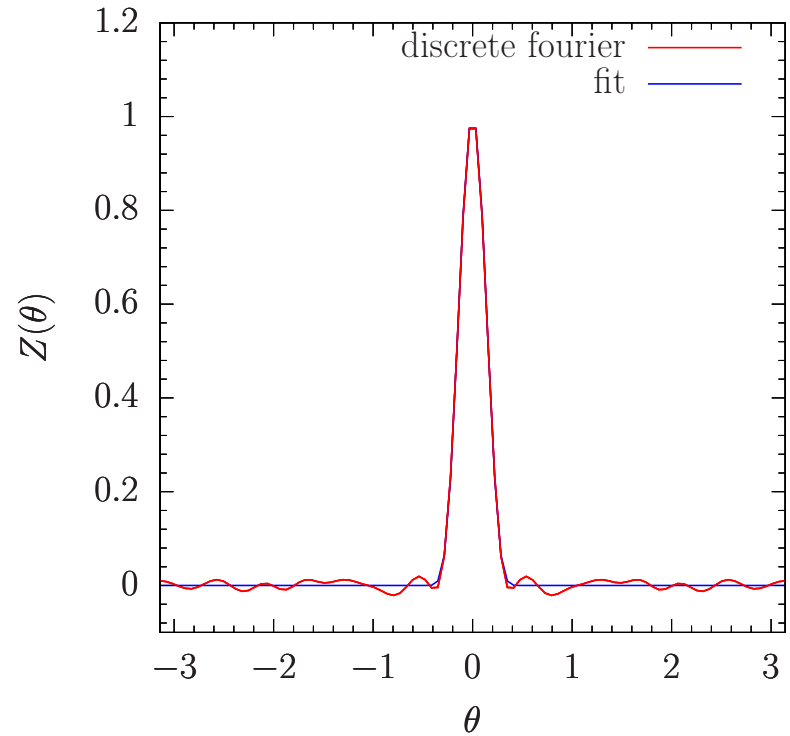
$$\alpha_V(\theta,\mu) = \alpha_V(\mu)[1 - \alpha_V(\mu)(D/\lambda)\theta^2]^\lambda$$



$$D \approx 0.12\,, \quad \lambda \approx 0.75$$

which leads to the screening length of the color charge $\lambda_c \approx 0.5/\theta\,[\mathrm{fm}] \sim \lambda_D$

Error



$$\nu \approx |Q|_{\max}$$

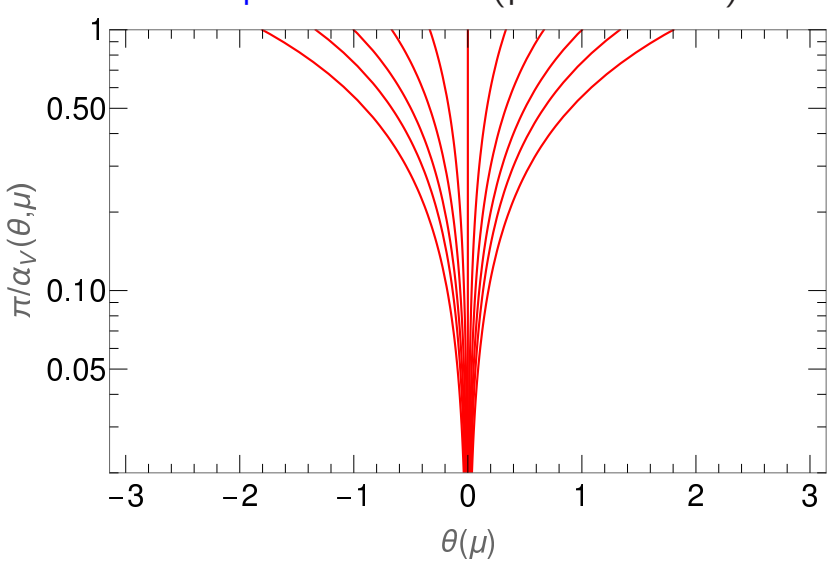
Low-pass filter gives practically the same result

From the analytic expression of $\alpha_V(\theta,\mu)$ we derive coupled RG equations, which for larger values of t decouple and take the form

$$\frac{\partial(\pi/\alpha_V)}{\partial \ln t} \simeq -\frac{\pi}{\alpha_V} + \pi D\,\theta^2\,, \quad \frac{\partial\theta}{\partial \ln t} \simeq -\frac{1}{2}\,\theta$$

Solution (full) for various initial values of $\theta(\mu)$:

Independent of θ (perturbative)



IR fixed point (?)

- Within QCD the θ parameter gets *renormalized*, $\theta = \theta(\mu)$, whereby $\theta(\mu)$ enters the effective Lagrangian
- If there is no phase transition, CP should be conserved along every trajectory
- Bar any loops, properties can be directly read off from fixed point values (universality class)

Literature:

Reuter arXiv:hep-th/9604124

derives

$$\theta(\mu = 0) = 0 \quad \text{for} \quad \alpha_s(\mu = 0) = \infty$$

based on an exact RG evolution equation à la Wetterich

Knizhnik and Morozov JETP Lett. 39 (1984) 240

derive

$$\frac{\partial(1/g^2)}{\partial \ln \mu} = C + \bar{D} \cos\theta$$
$$\frac{\partial\theta}{\partial \ln \mu} = 8\pi^2 \bar{D} \sin\theta$$

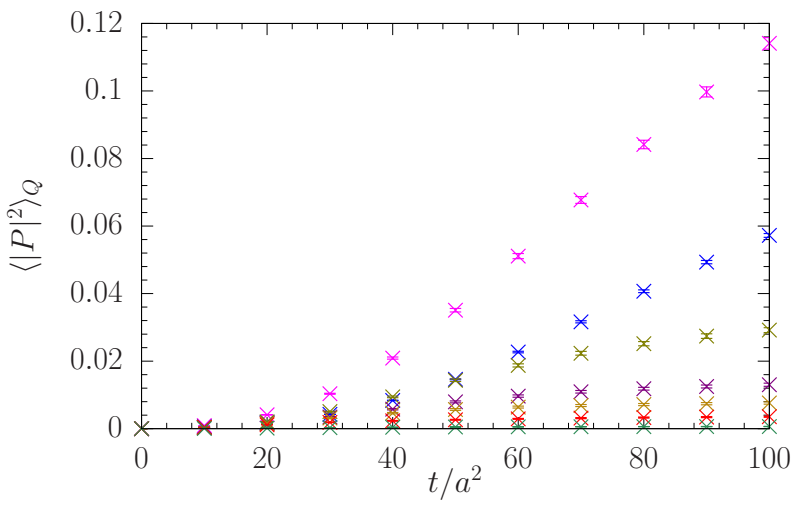
from the instanton density in an external field à la SVZ. This result is in remarkable agreement with our result for $\bar{D} \approx 1/32\pi^2$ (corresponds to $D = 1/8$)

QHE (see Introduction) arXiv:cond-mat/0101003

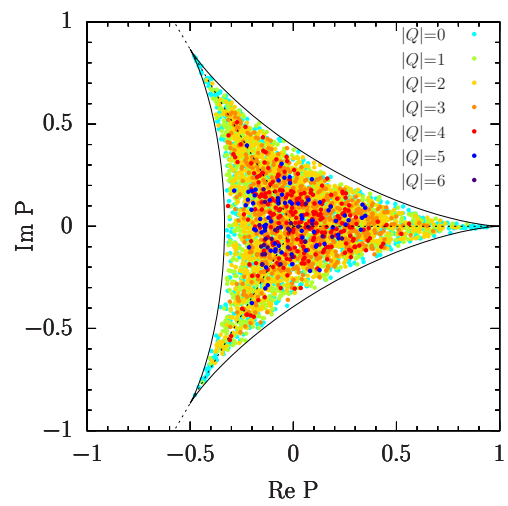
Polyakov loop

The Polyakov loop P describes the propagation of a single static quark travelling around the periodic lattice

16^4



$t/a^2 = 60$



From $Q = 0$ (top) to 6 (bottom)

$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

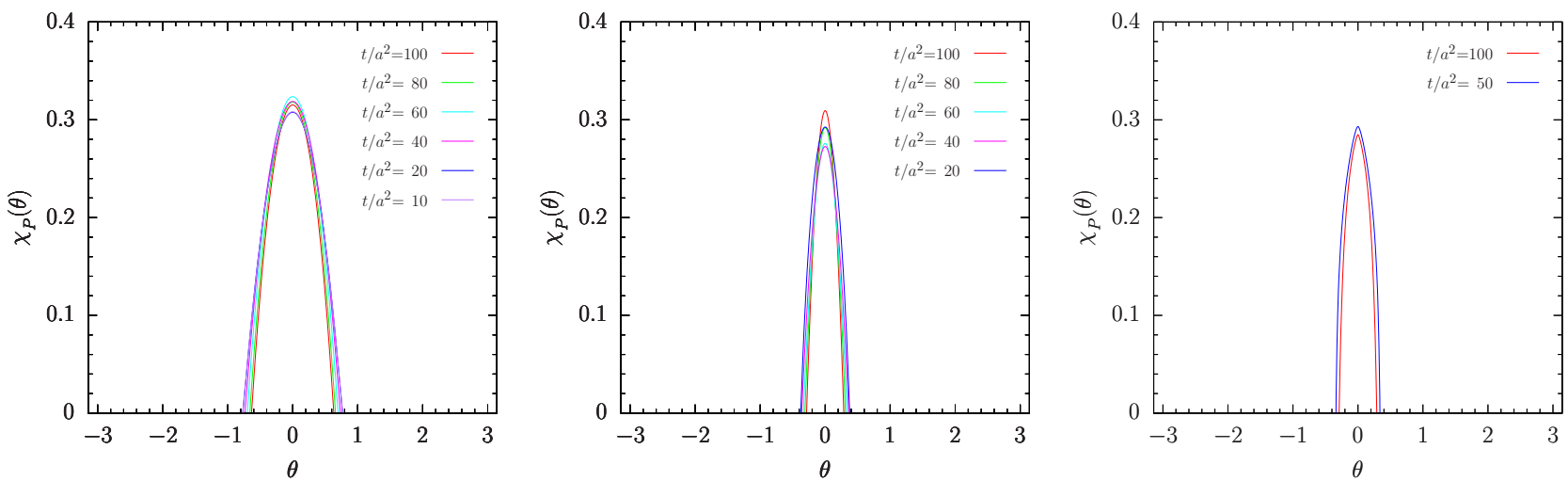
$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The connected part of $\langle |P|^2 \rangle_\theta$ is described by the normalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$

16^4 24^4 32^4



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The normalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

Mass gap

$$\langle E^2 \rangle = \frac{1}{T} \sum_t \langle E(0) E(t) \rangle$$

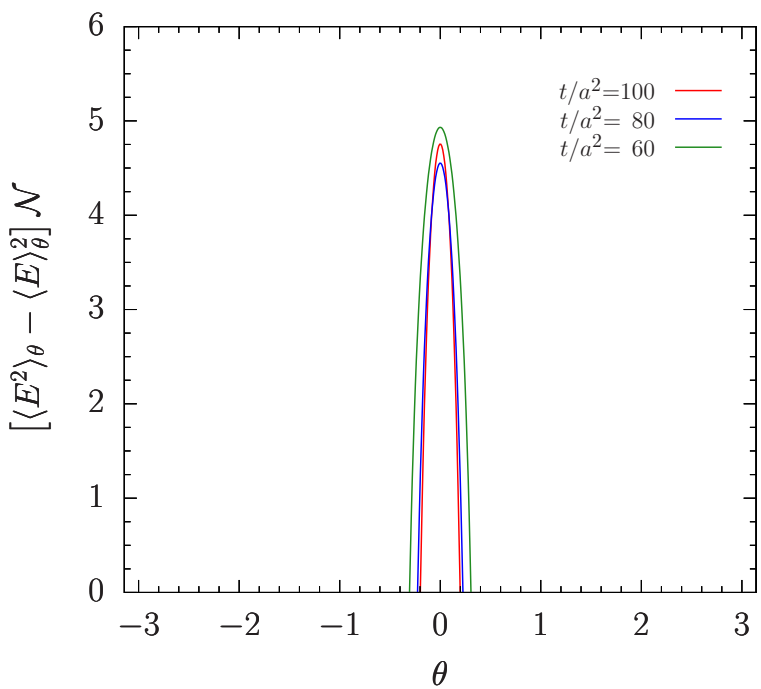
$$E(t) = \frac{1}{V_3} \sum_{\vec{x}} E(\vec{x}, t)$$

$$[\langle E^2 \rangle - \langle E \rangle^2] \mathcal{N} = \sum_{n>0,t} \frac{1}{2m_n} |\langle 0|E|n\rangle|^2 \, e^{-m_n t}$$

$$\simeq \frac{1}{m_{0^{++}}^2} |\langle 0|E|0^{++}\rangle|^2 \propto \xi^2$$

Correlation length

24^4



$\langle E^2 \rangle_\theta - \langle E \rangle_\theta^2$ Independent of flow time t

$\xi \simeq 0$ for $|\theta| \gtrsim 0$

No mass gap

Conclusions

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. As far as we can see it leaves the long-distance physics unchanged and qualifies for a RG transformation

★ A key point is that the path integral splits into disconnected topological sectors for $t \gtrsim 0$, which is expected to occur at ever smaller flow times with decreasing lattice spacing. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The novel result is that within QCD the θ angle gets renormalized and flows to $\theta = 0$ in the infrared limit. Thus CP is conserved by the strong interactions

★ In an external field though with, or equivalent to, $|\theta| > 0$, the color charge is screened, leading perhaps to the 'oblique' phases advocated by 't Hooft