Fractons Hydrodynamics

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~~Holographic~~ perspectives on ~~chiral~~ transport
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Outline

- Very Brief introduction to Fractons
- Fractons hydrodynamics
- Discussions

What is a fracton? Is an excitation with reduced mobility.

Spin liquids

Elasticity

Quantum Hall

Carroll Theories

Fractons

Galileons

...

Haah code

X-cube model

Pretko, et al

Son, et al

Gromov

Vijay, Haah, Fu

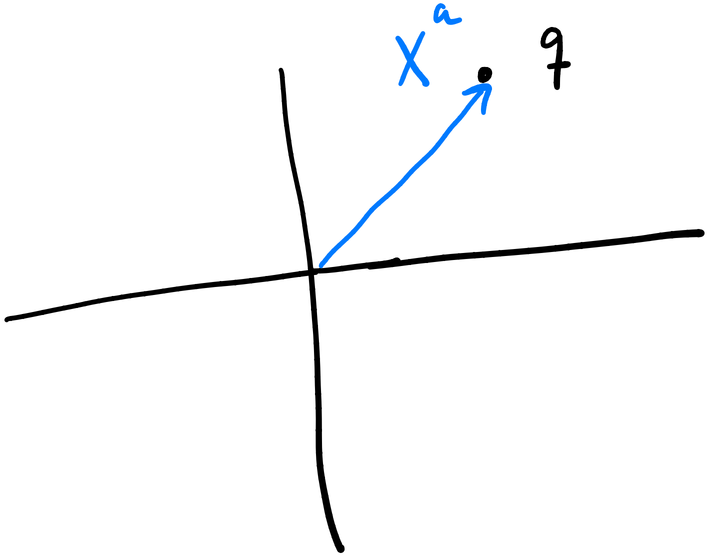
...

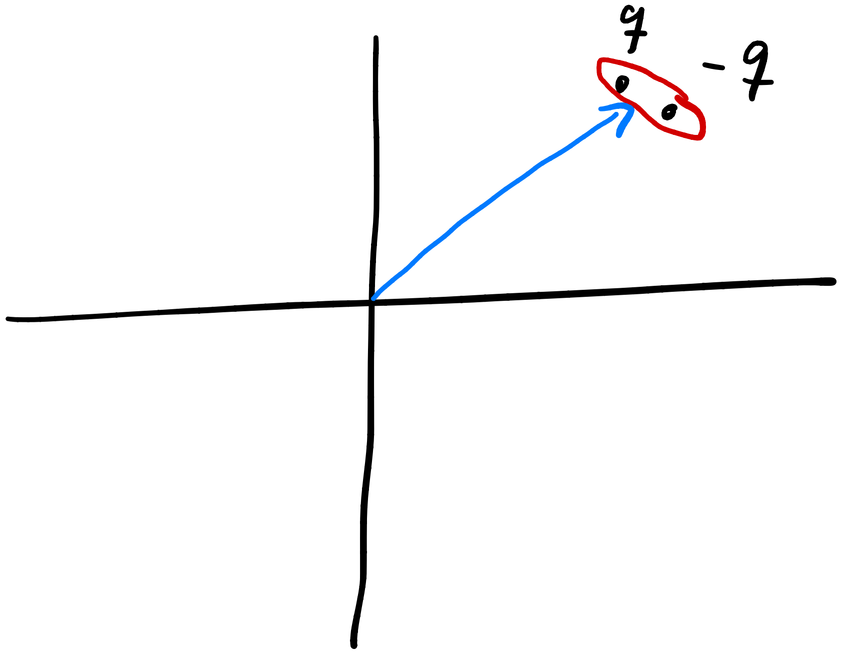
Gapless phases

Are constructed with a scalar (o vector) charge and certain amount of its moments.

Simple case:

U(1) charge + Dipole moment





Q

$$Q = \int d^D x \, \bar{\rho}(x)$$

$$\dot{Q} = 0 \Rightarrow \boxed{\partial_t \bar{\rho} + \partial_a J^a = 0}$$

$$D^a = \int d^D x \left(x^a \bar{\rho}(x) + \bar{\rho}^a(x) \right)$$

$$\dot{D}_a = 0 \Rightarrow \boxed{\partial_t \bar{\rho}^a(x) + \partial_b \bar{K}^{ba} = J^a}$$

this conservation equations can be improved to

$$\boxed{\begin{array}{l} \partial_t \rho + \partial_a \partial_b J^{ab} = 0 \\ \rho^a = x^a \rho \; ; \; K^{ab} = x^a \partial_c J^{cb} - J^{ab} \end{array}}$$

MDMA

Translations on $Q \Rightarrow \delta_\xi Q = 0$ $\Rightarrow$
on $D_a \Rightarrow \delta_\xi Q_a = \xi Q$

$$[P_a, D_a] = \delta_{ab} Q$$

Introduction of rotations extends the algebra as

$$[L, P] \sim P \quad ; \quad [L, D_a] \sim D_a \; ; \quad [L, L] \sim L$$

<u>Non-Abelian space symmetry group!</u>

Galileons 1905.05190
Quantum Hall 2103.09826

Carroll algebra Levy-Leblond 1965 ; . . .
Warped Lifschitz 1909.01157

Field theory realizations

$$\Phi$$

$$\delta\Phi = \alpha + \beta_a x^a$$

$$S = \int d^d x\, \mathcal{L}(\dot{\Phi}, \partial^2\Phi, \partial\dot{\Phi})$$

$$\Psi$$

$$\delta\Psi = i(\alpha + \beta_a x^a)\Psi$$

$$S = \int d^d x \Big[|\partial_t \psi|^2 - V(|\psi|^2) - c_1(\nabla|\psi|^2)^2 - c_2\big[\psi^{*2}(\psi\nabla^2\psi - \nabla\psi\cdot\nabla\psi) + h.c.\big] - c_3|\psi\partial_i\partial_j\psi - \partial_i\psi\partial_j\psi|^2 \Big]$$

Hydrodynamics

* Effective finite temperature dissipative low energy theory
* d.o.f are conserved charges
* e.o.m. Local conservation equations
* Local thermal equilibrium
* Currents expanded in derivatives of hydro variables with phenomenological coefficients (constrained by symmetries!)

Conserved charges : $\mathcal{E}, n, P_i$

E.O.M

$$\dot{\mathcal{E}} + \partial_i j^i_{\mathcal{E}} = 0$$

$$\dot{n} + \partial_i \partial_j J_{ij} = 0$$

$$\dot{P}_i + \partial_j T_{ji} = 0$$

Symmetry constraints

Rot. invariance $\Rightarrow$ $T_{ij} = T_{ji}$

$$[D_i, P_j] = \delta_{ij} Q \quad \Rightarrow$$

$$\delta_\beta P_i = -n\beta_i \quad ; \quad \delta_\beta T_{ij} = \partial_k J_{ij} \beta_k - \partial_k J_{kj} \beta_i - \partial_k J_{ki} \beta_j$$

see also Glorioso et. al. 2301.02680

Symmetry constraints & thermodynamics

Usually $\mathcal{E}(s, n, P_i)$ however $\delta_\beta \mathcal{E} = 0 \Rightarrow$ energy density cannot depend on P_i.

Notice $\delta_\beta \left(\frac{P_i}{n}\right) = \beta_i \Rightarrow \delta_\beta\, \partial_i \left(\frac{P_j}{n}\right) = 0$

transforms as a Goldstone

therefore we assume different values for $V_{ij} \equiv \partial_i \left(\frac{P_j}{n}\right)$ label different thermodynamic states (analogy superfluid)

$$d\mathcal{E} = T\,dS + \mu\,dn + F_{ij}\,dV_{ij}$$

Constitutive relations and derivative expansion.

Thermodynamic variables ϵ, n, s, v_{ij} are $\mathcal{O}(\nabla) \sim 0$. Therefore $\mathcal{O}(P_i) \sim -1$

the equations are truncated as follow

$$\dot{\varepsilon} = -\partial_i \dot{J}_\varepsilon + \mathcal{O}(\nabla)^{2n+3}$$

$$\dot{n} = -\partial_i \partial_j J^{ij} + \mathcal{O}(\nabla)^{2n+3}$$

$$\dot{P}_i = -\partial_j T^{ji} + \mathcal{O}(\nabla)^{2n+2}$$

Here we understand the currents as polynomials of the densities and their derivatives

Constitutive relations at <u>zeroth order</u>

$$j_\varepsilon^i = (\epsilon + P)V^i - F_{ij}\partial_t\left(\frac{P_j}{n}\right) + \alpha\,\partial_i \frac{1}{T}$$

$$J^{ij} = -F_{ij}$$

$$T^{ij} = P\delta_{ij} + V_i P_j + V_j P_i + \partial_k F_{ij}\frac{P_k}{n} + F_{ij}V_{kk}$$

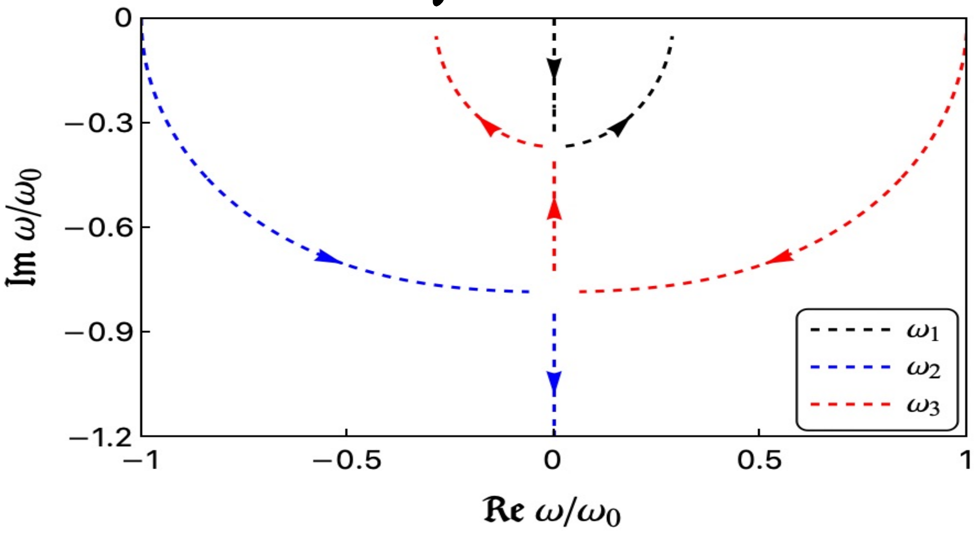
$$s^i = sV_i + \frac{\alpha}{T}\partial_i \frac{1}{T}$$

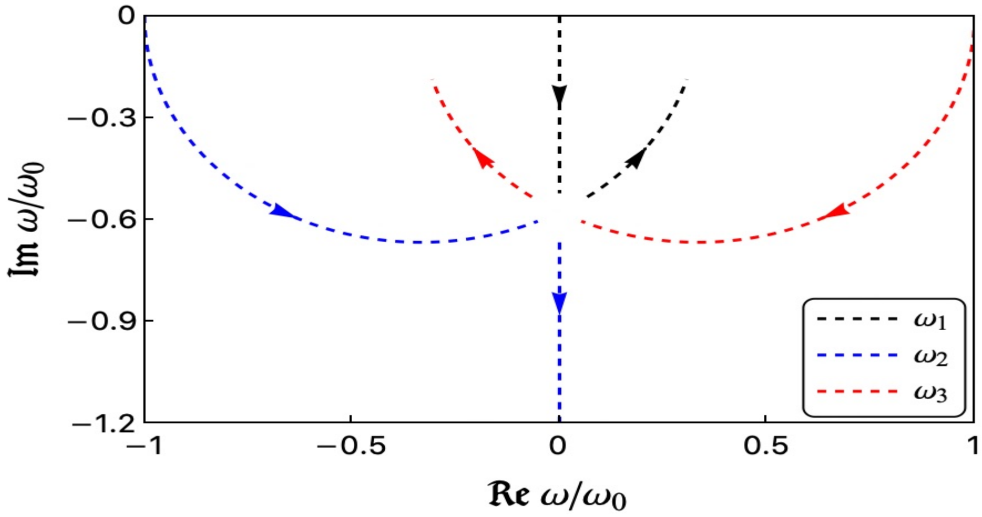
$$\boxed{\begin{array}{l}\text{Effective hydro velocity} \\ V_i = -\dfrac{\partial_j F_{ji}}{n}\end{array}}$$

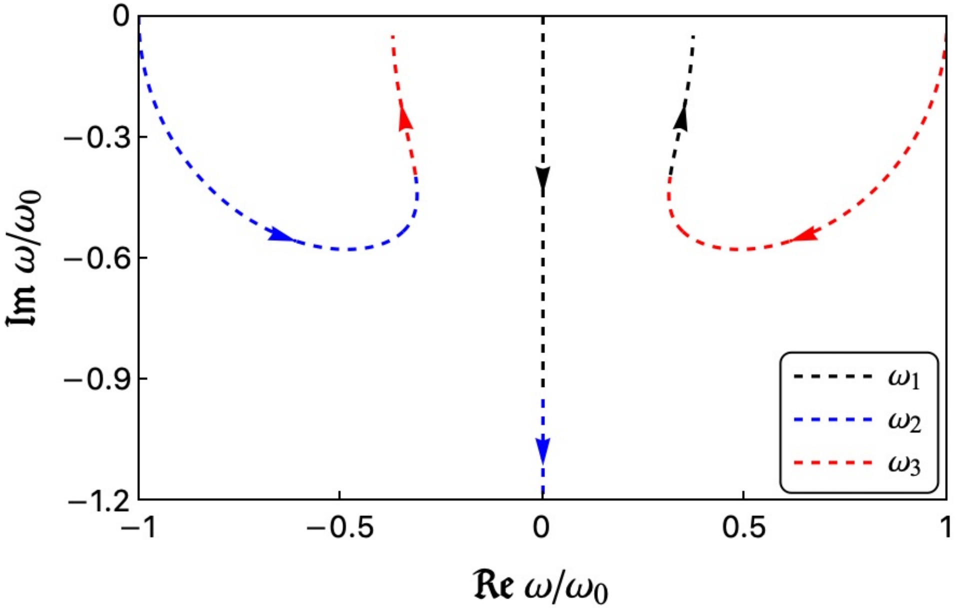
The condition

$$\dot{s} + \partial_i s^i \geq 0 \Rightarrow \alpha \geq 0$$

Longitudinal modes around $q_i = 0$







a_1/a_2 fixed

$$\left(\frac{\omega}{\omega_0}\right)^3 + i a_2 \left(\frac{\omega}{\omega_0}\right)^2 - \frac{\omega}{\omega_0} - i a_1 = 0$$

$\omega_0 \sim K^2$; $a_1 \sim \alpha$

$a_2 \sim \alpha$

$\frac{a_1}{a_2}$ depends on thermodynamic parameters only

On the other hand the shear (transverse) mode does not disperse

$$\omega = \mathcal{O}(k^4)$$

If we introduce the first order corrections we find 12 transport coefficients. the shear mode becomes

$$\omega \sim i\eta k^4$$ ← subdiffusive.

analogue to shear viscosity

the longitudinal modes receive also k^4 corrections

Can we learn anything about gravity?

Elasticity (2+1D)	Geometry	Fractons
smooth deformations: Phonons ⟷	"Gravitons" ⟷	Symmetric gauge fields
Singular configurations: Dislocations, Disclinations ⟷	Background torsion, Curvature ⟷	Fractons!