Chiral Anomaly
in
Effective Open Quantum Systems

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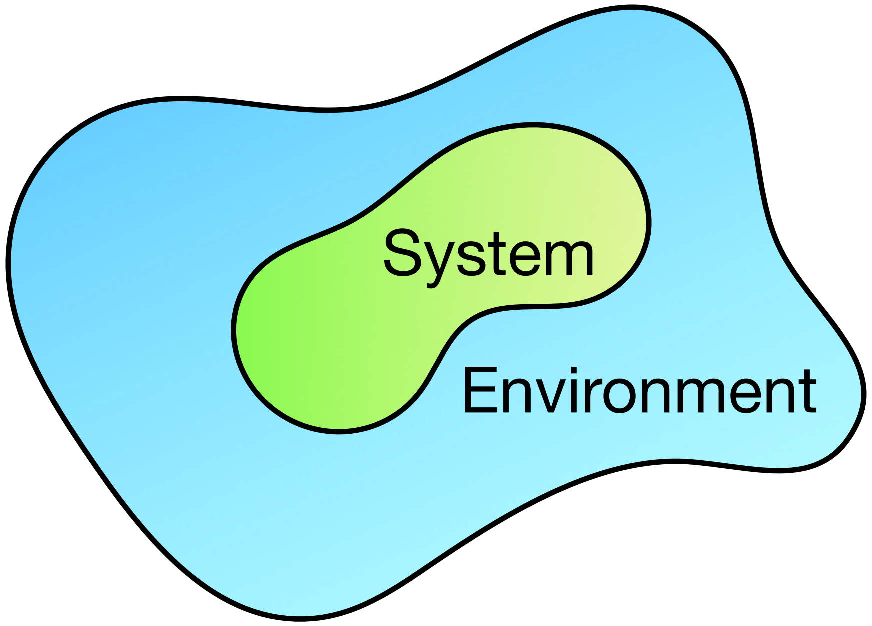
Adolfo Grushin (Institut Néel, France)

S.Sayyad, et al., Phys. Rev. Research 4, L042004 (2022)

#Open quantum systems



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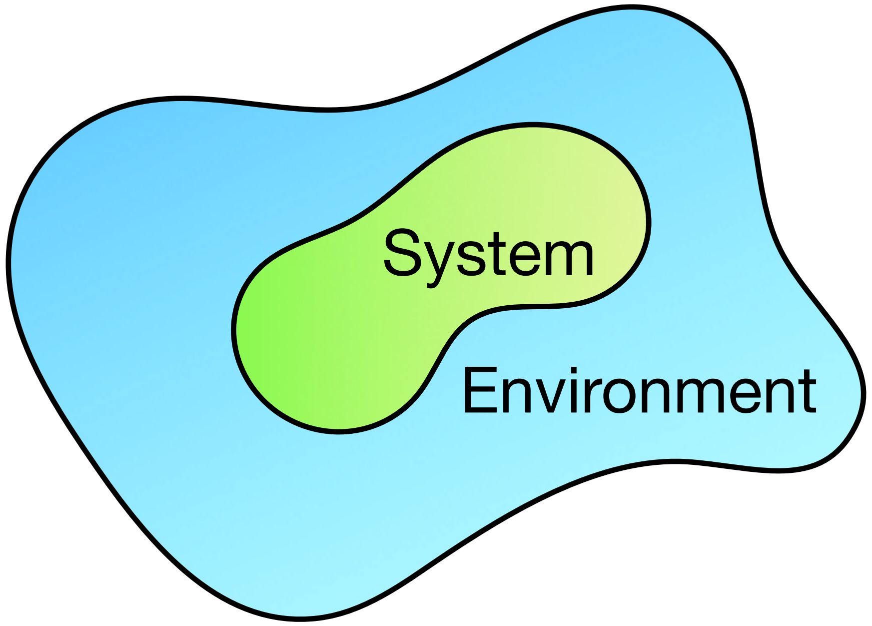

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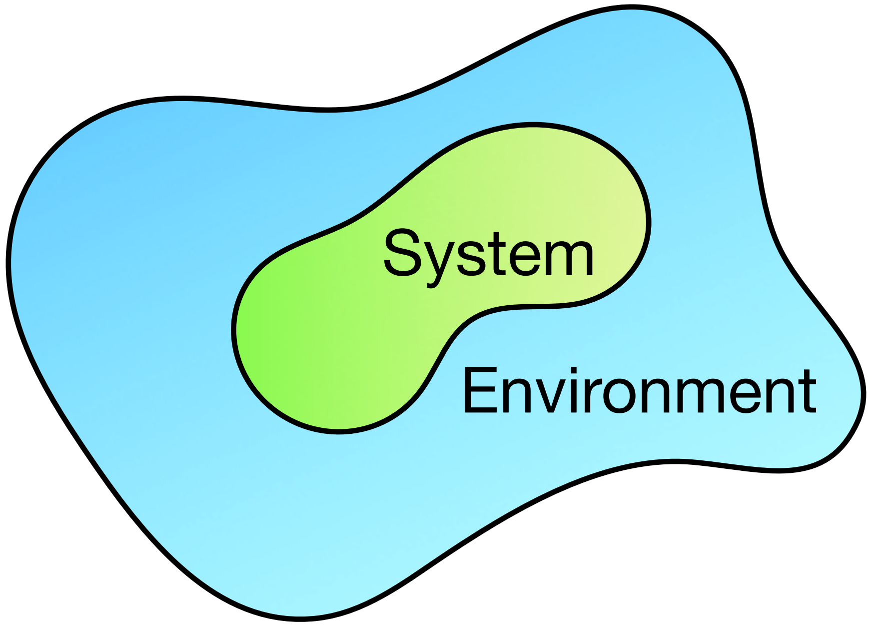

Closed quantum systems

$$\mathscr{H}_{\text{tot}} = \mathscr{H}_{\text{sys}} \otimes 1_{\text{env}} + 1_{\text{sys}} \otimes \mathscr{H}_{\text{env}} + \mathscr{H}_{\text{sys-env}}$$

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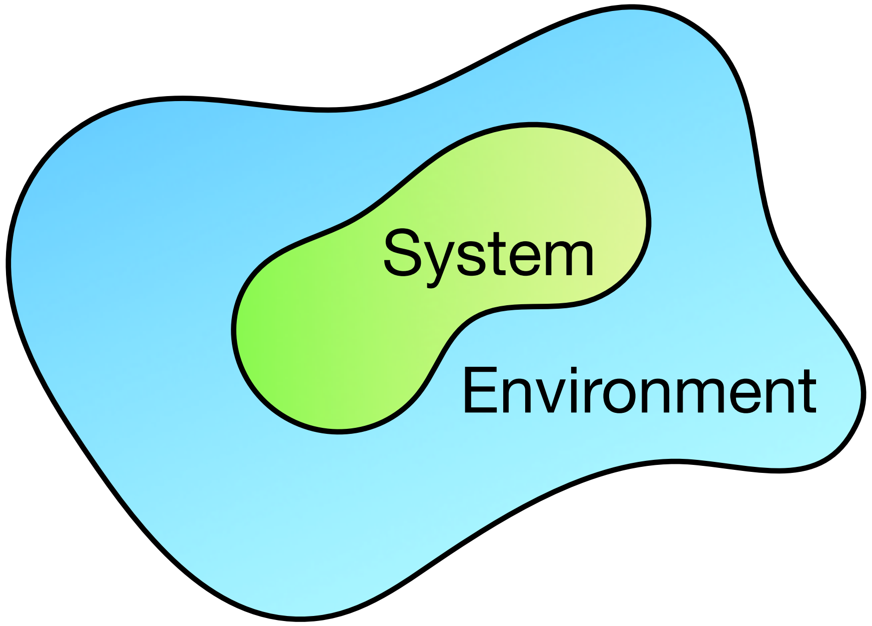

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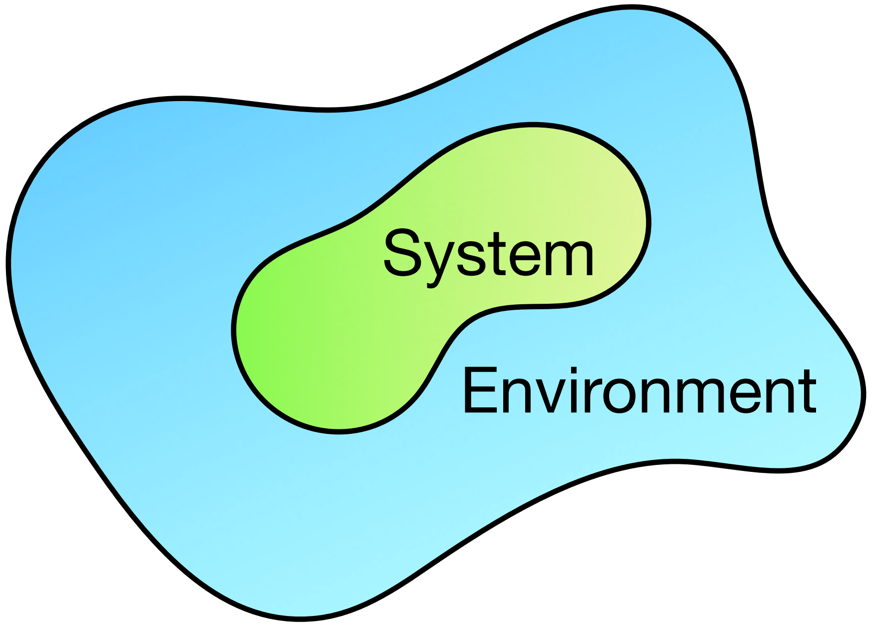

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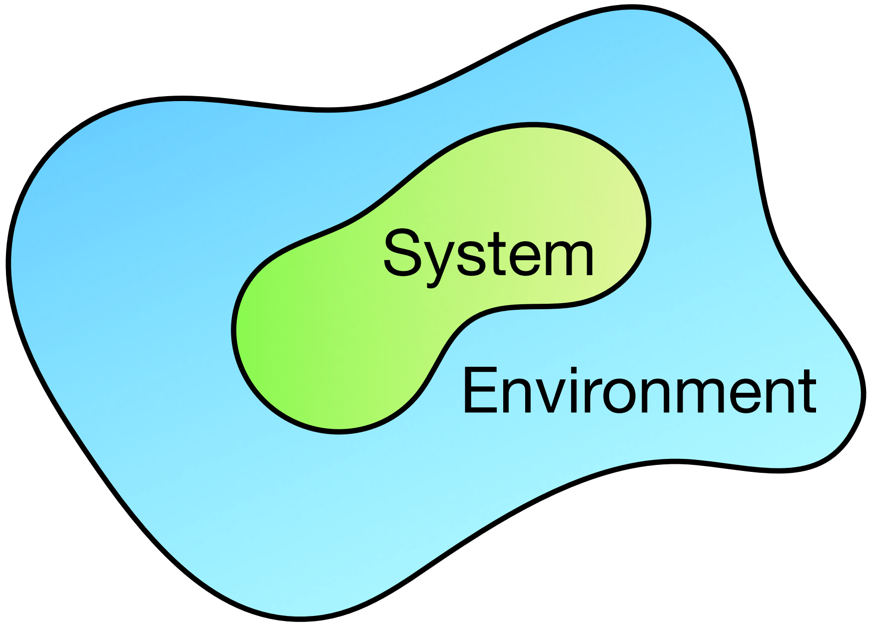

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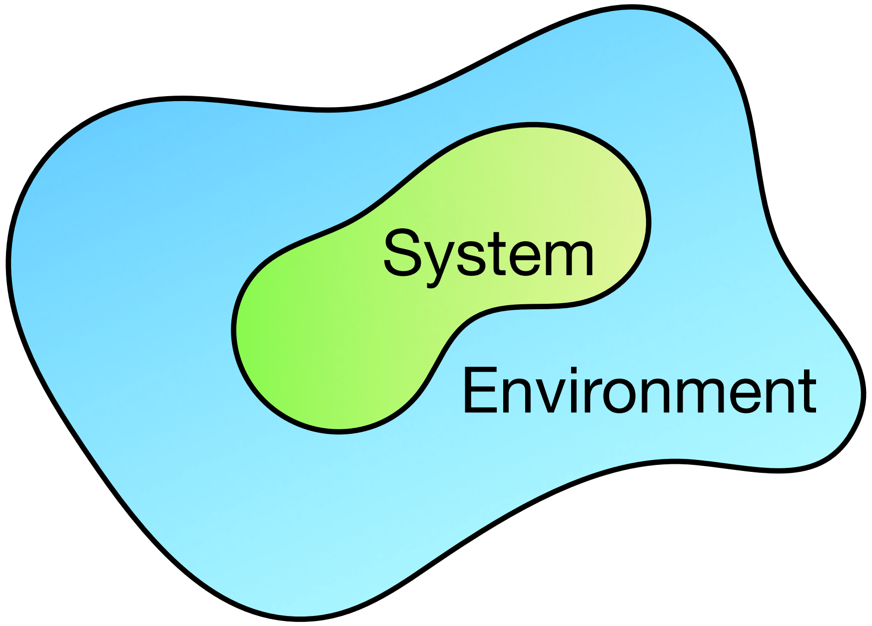

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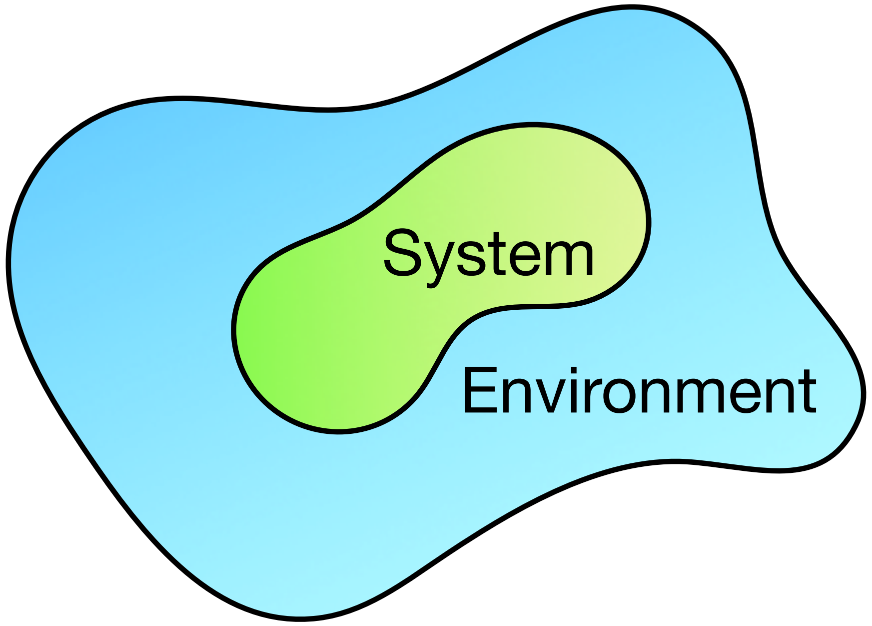

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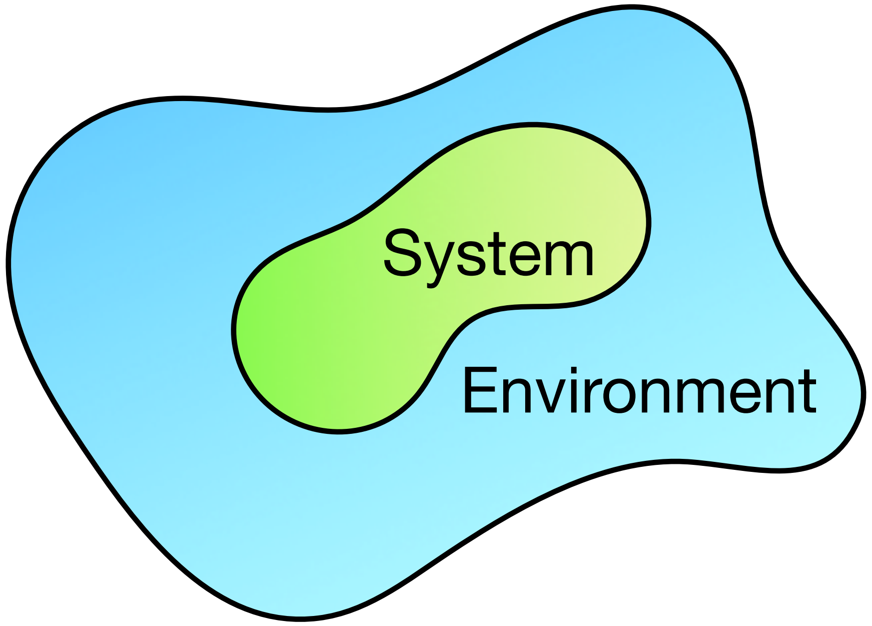

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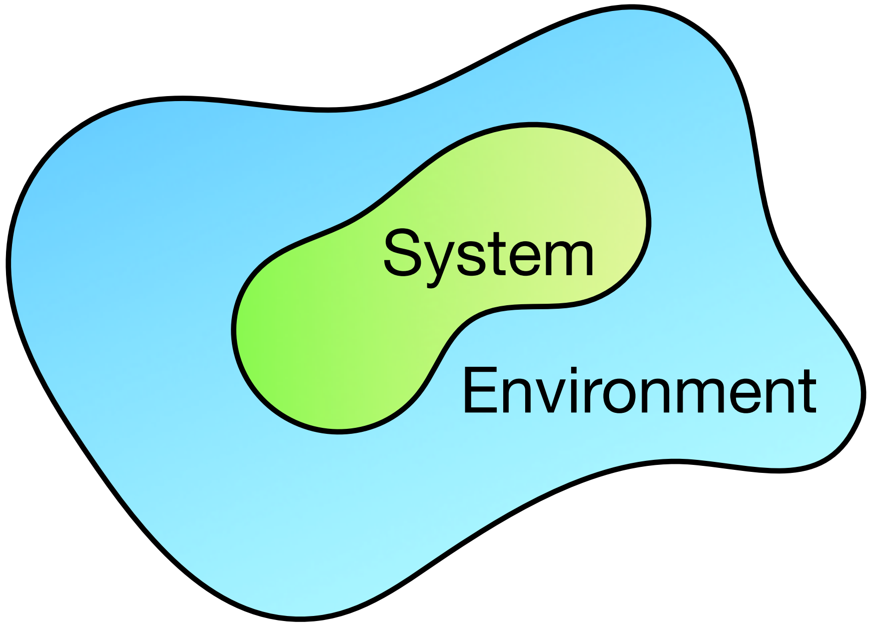

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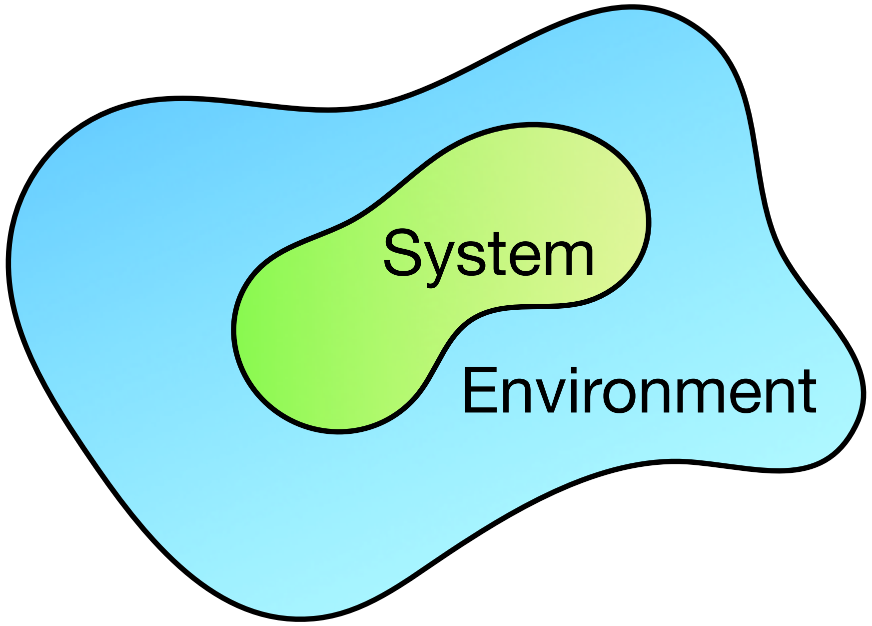

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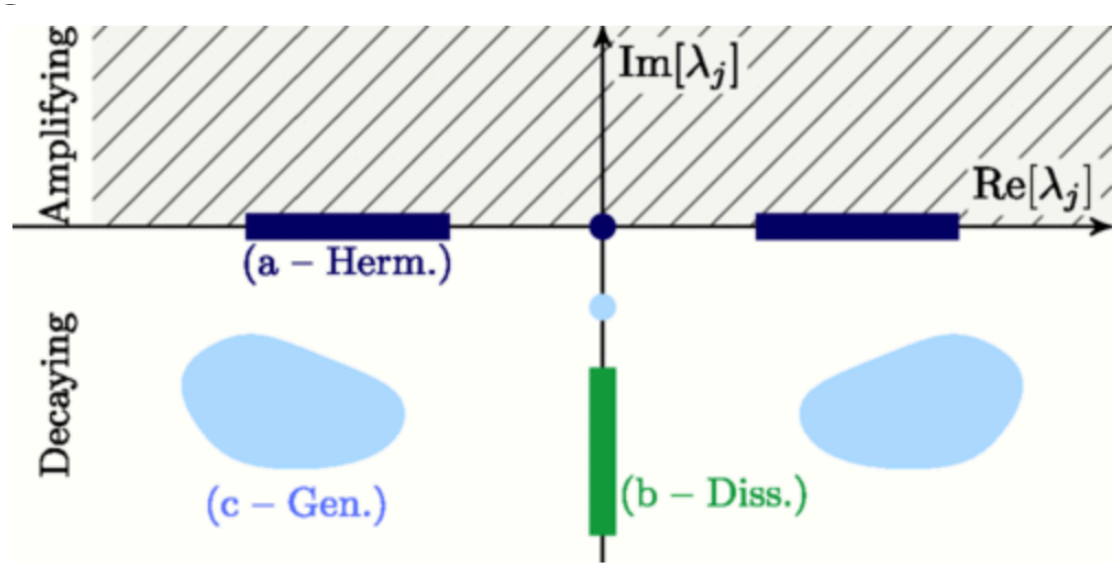

S. Lieu, et al. PRL(2020)

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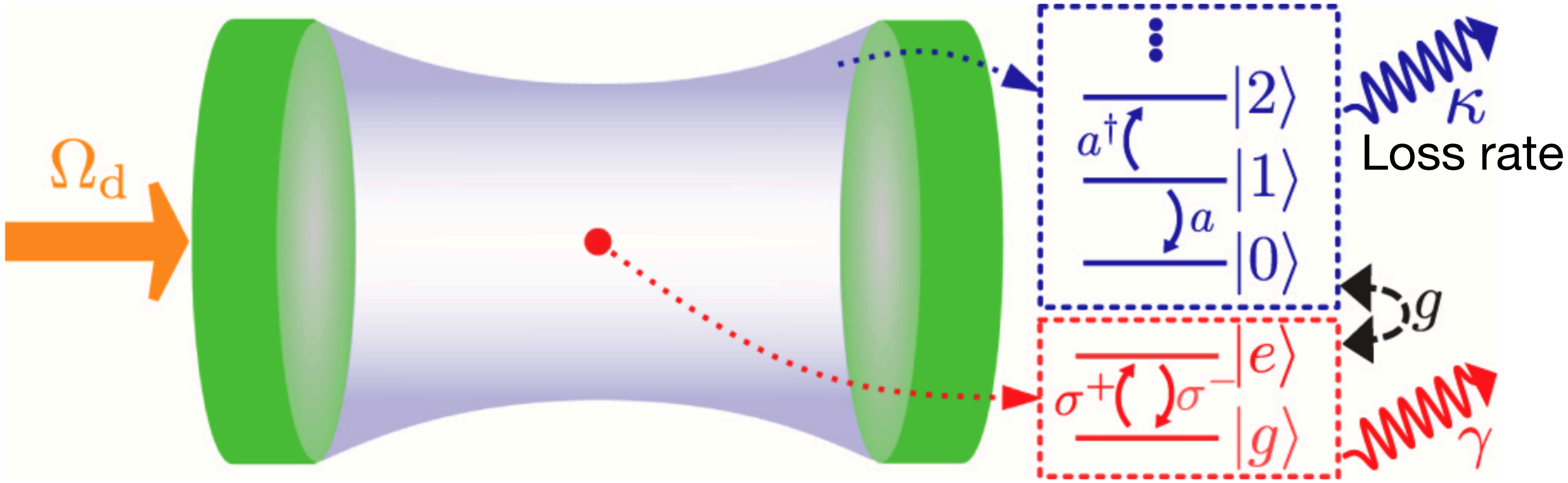

W. Salmon, j. Nanophotonics (2022)

Quantum optics

Strong light-matter interaction

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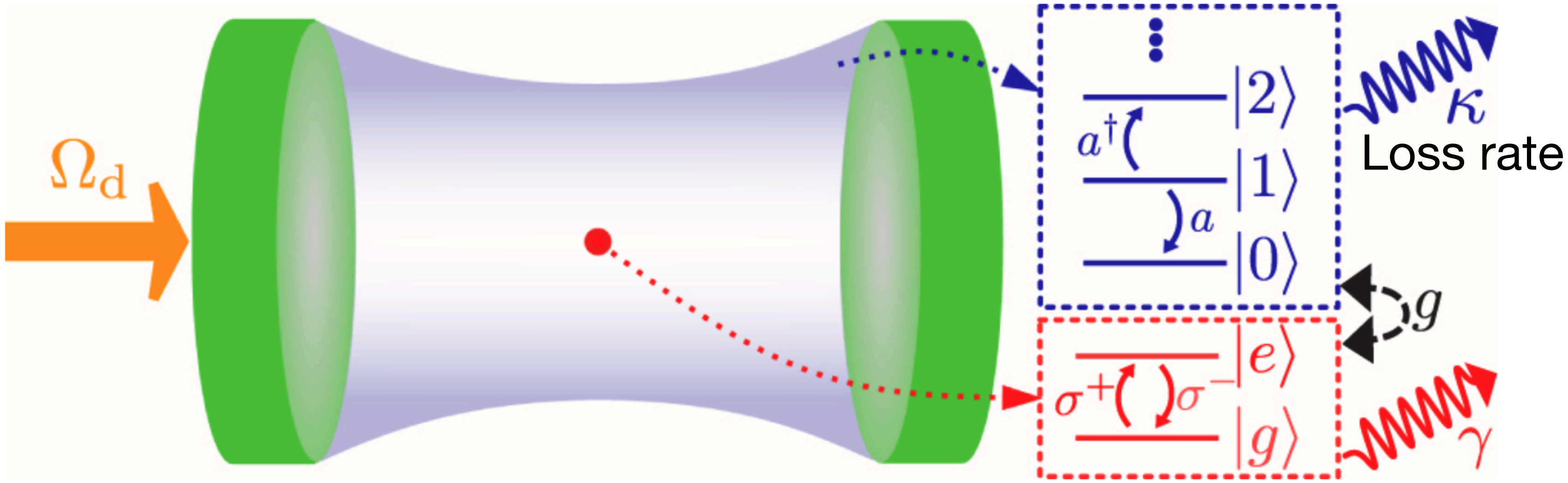

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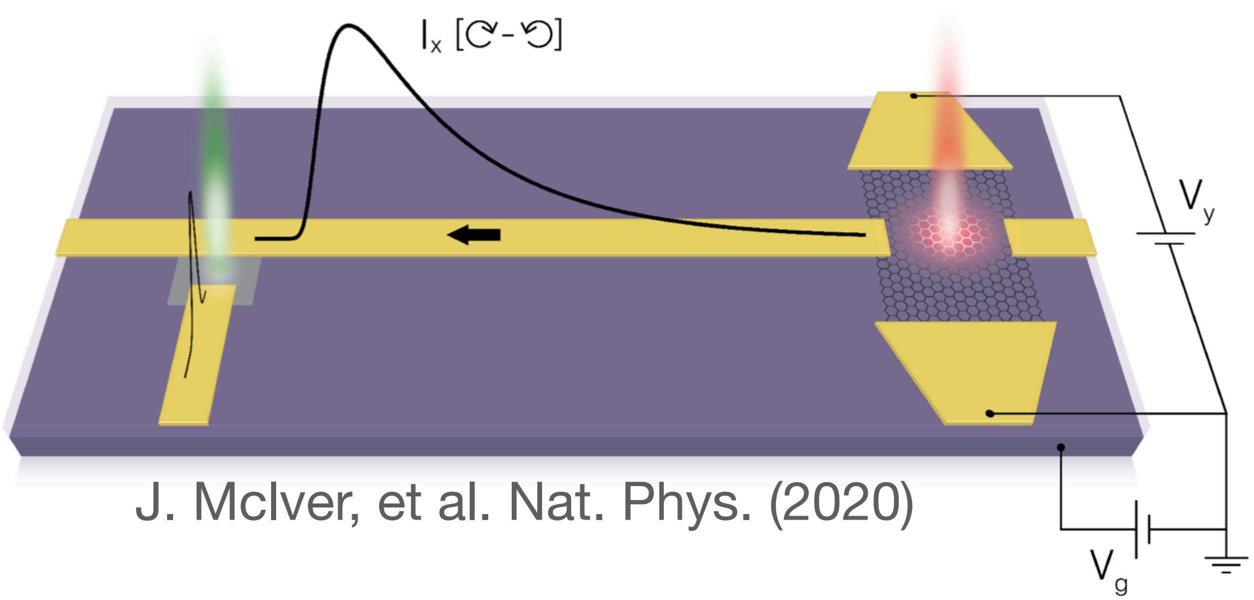

J. McIver, et al. Nat. Phys. (2020)

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Light-driven systems
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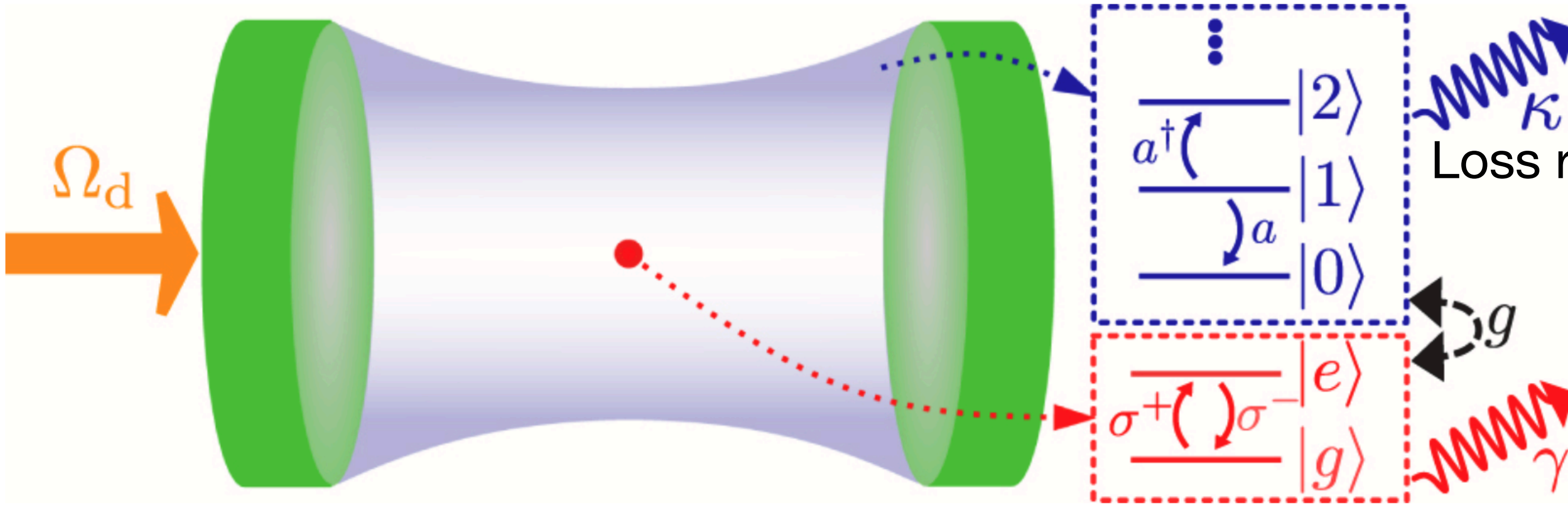

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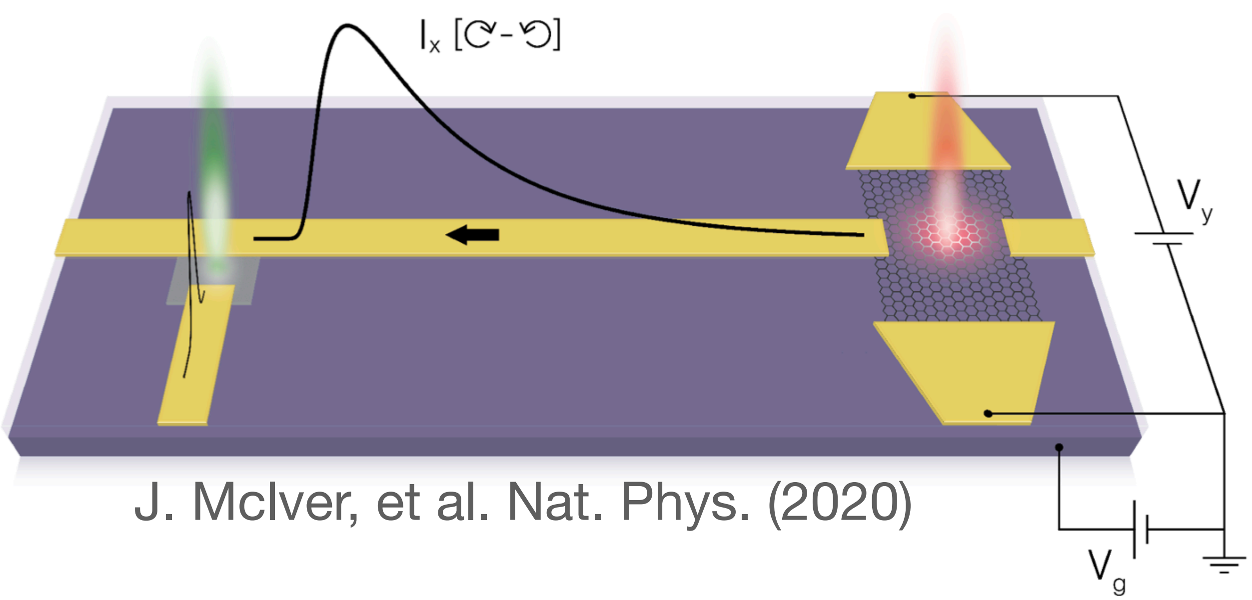

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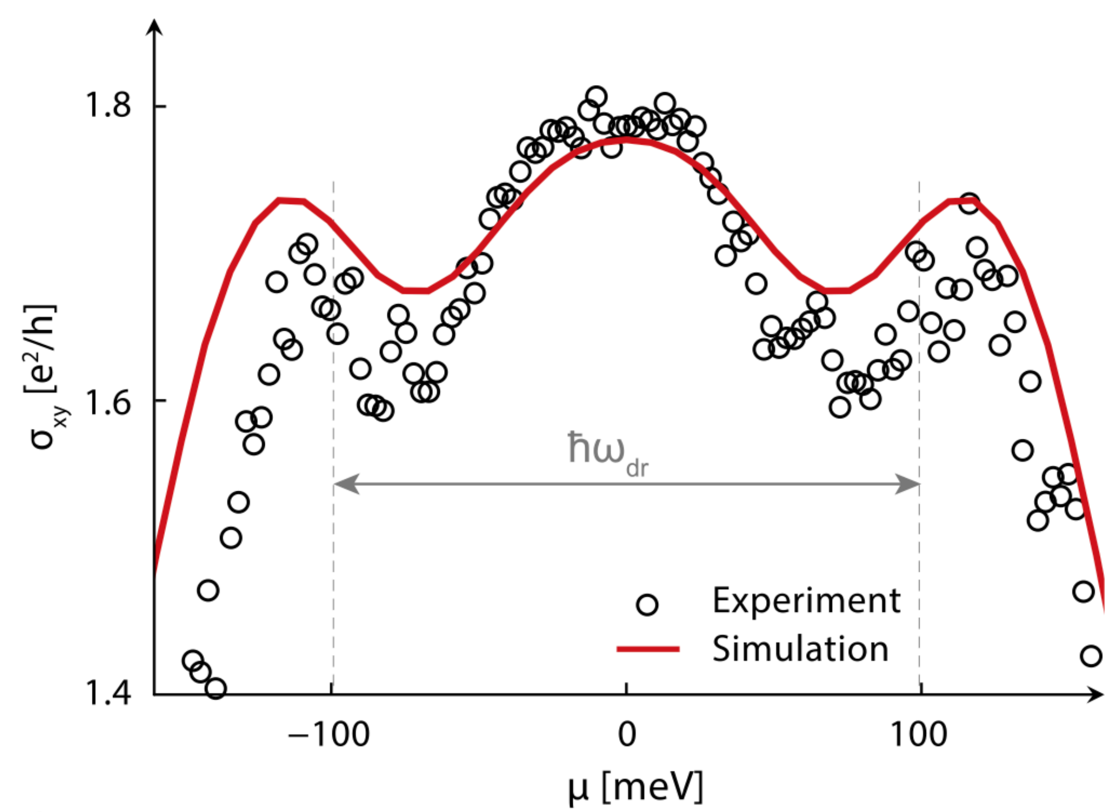

M. Nuske, et al. PRR (2020)

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#Effective description of open quantum systems

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Y. Ashida, et al, Adv. in Phys. (2020)

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Generating effective **non-Hermitian** Hamiltonians

Momentum-based effective Hamiltonian

Vectorized form of Lindbladian

Effective Hamiltonian

Third quantization

Effective recycled Hamiltonian

Dynamical matrix

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#Hermitian vs. non-Hermitian systems

	Hermitian QM	Non-Hermitian QM
Eigenvectors		
Spectrum		

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	Degenerate eigenvalues	Degenerate eigenvalues/ Coalescence of eigenvalues and eigenvectors
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#Degeneracies in non-Hermitian systems

NH degeneracies

Non-defective degeneracy

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Both eigenvalues and eigenvectors coalesce

S. Sayyad, et al, PRR (2022)

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$$\mathcal{H} = \begin{pmatrix} 0 & \alpha \\ 1 & 0 \end{pmatrix} \qquad E_{\pm} = \pm\sqrt{\alpha}$$

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Defective degeneracy at $\alpha = 0$
& self-orthogonal basis $\langle \psi_L | \psi_R \rangle = 0$

$$\mathcal{H} | \psi_{R\pm} \rangle = E_{\pm} | \psi_{R\pm} \rangle$$

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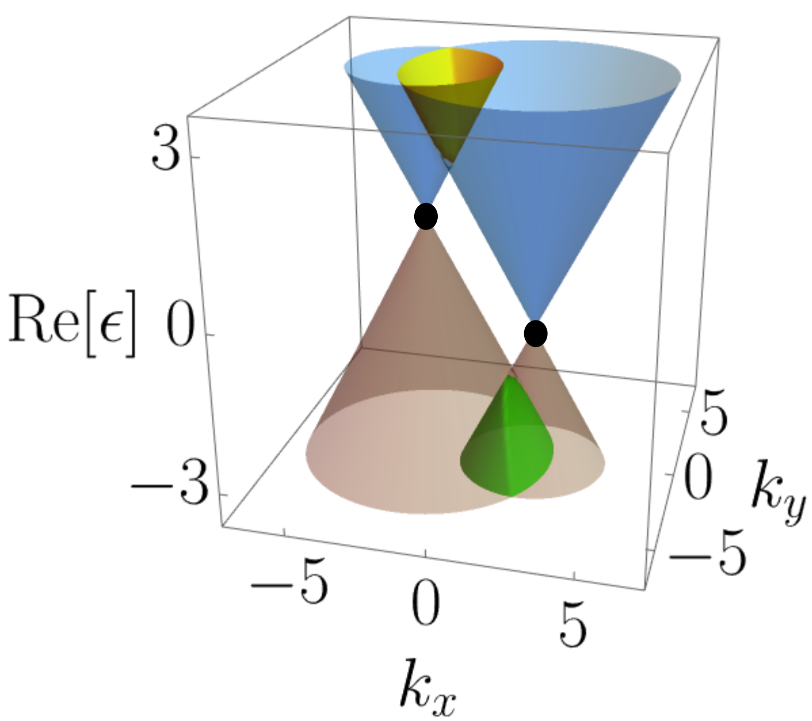
$$H_{\text{sys}} = \pm\,[b_0 + M_i^j (k_j \mp b_j)\sigma^i]$$

Real Fermi velocity: $M = \text{diag}[v_1, v_2, v_3]$



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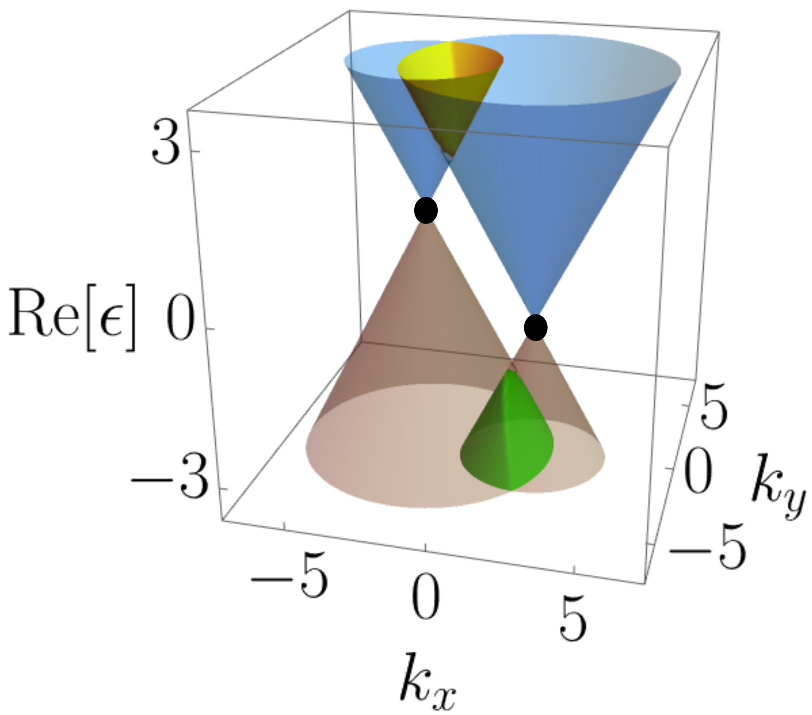

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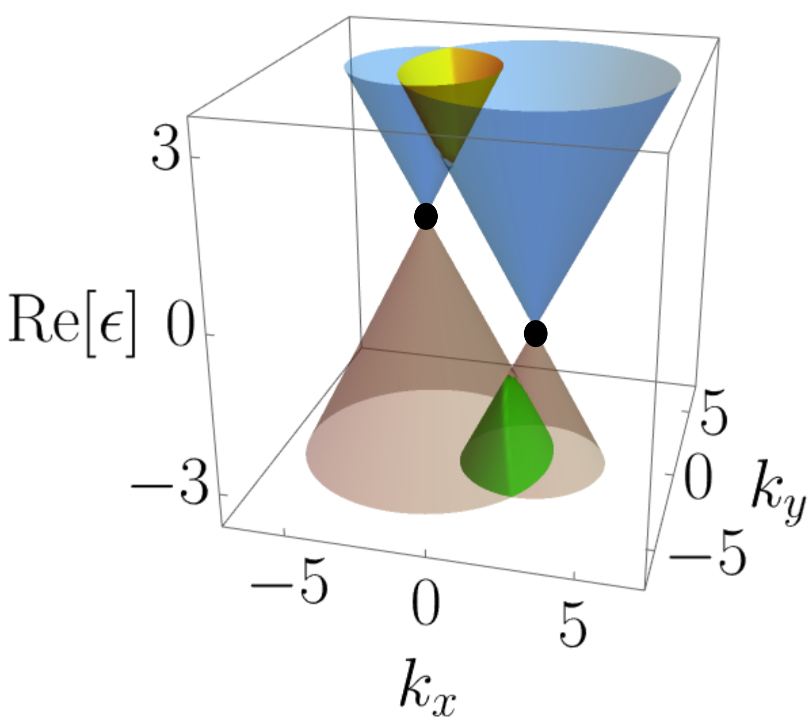

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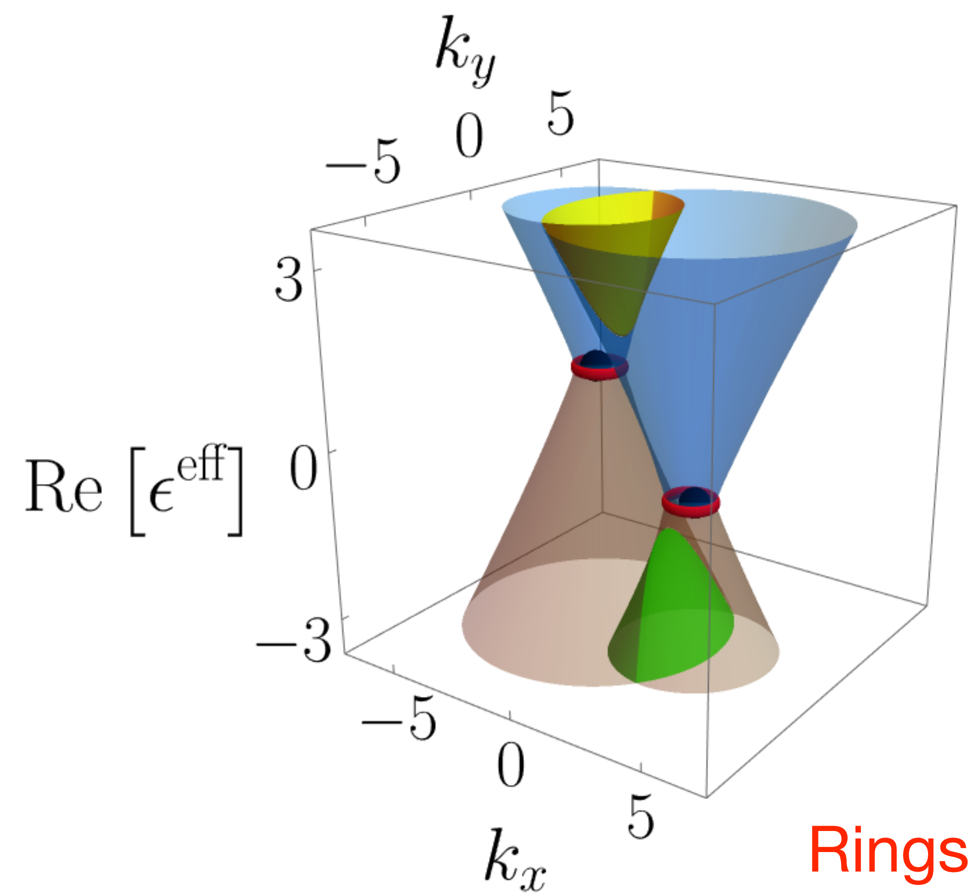

Rings:
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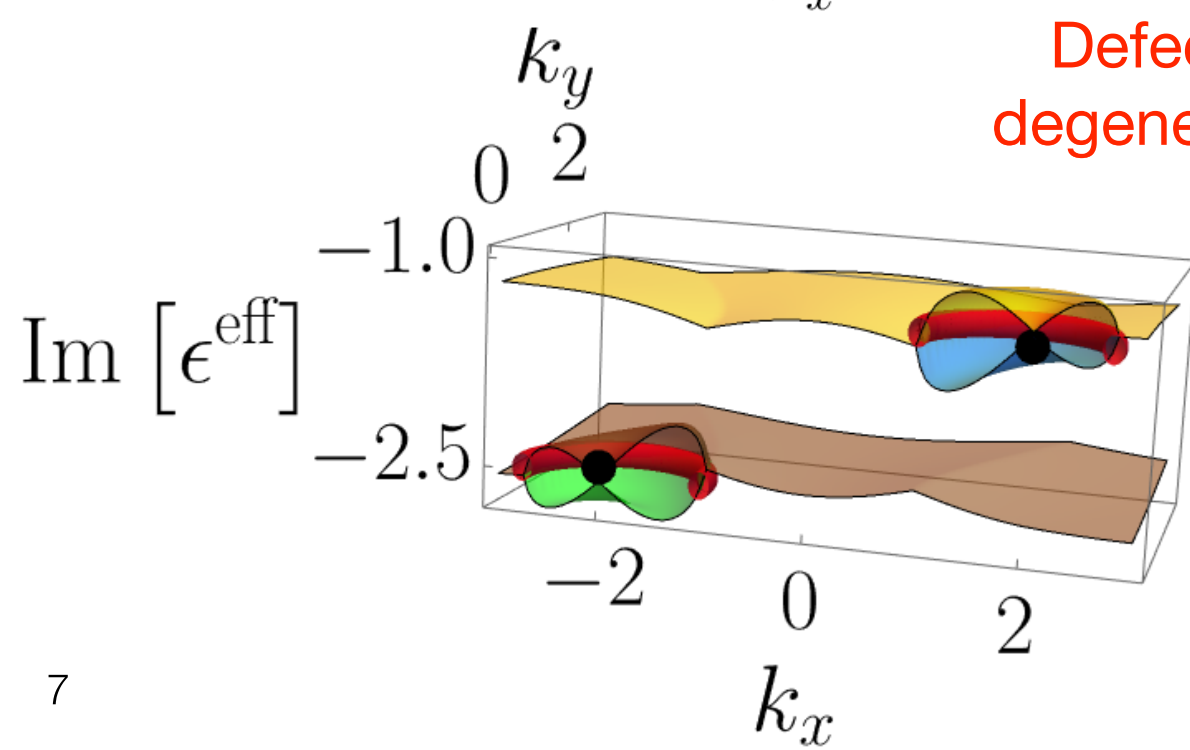

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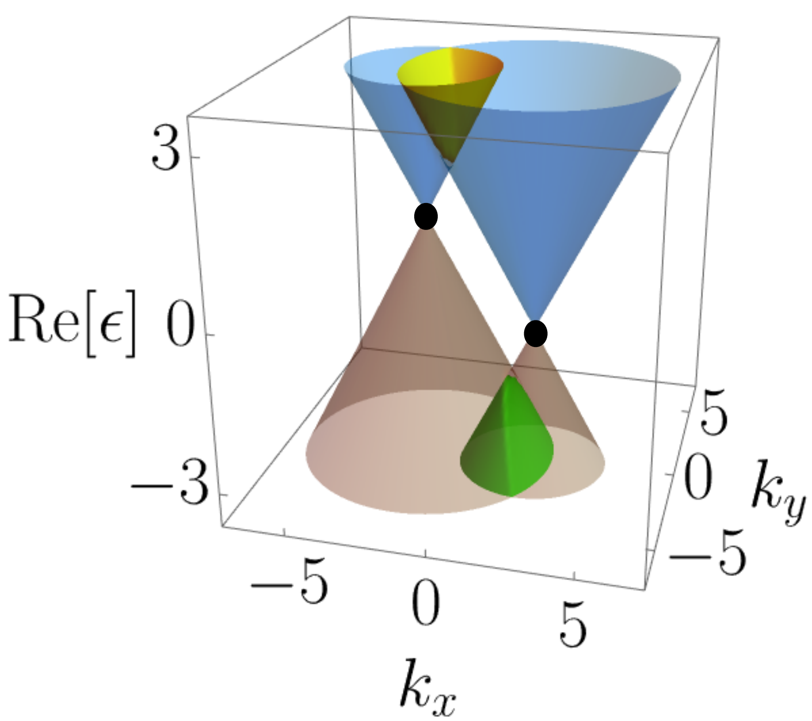

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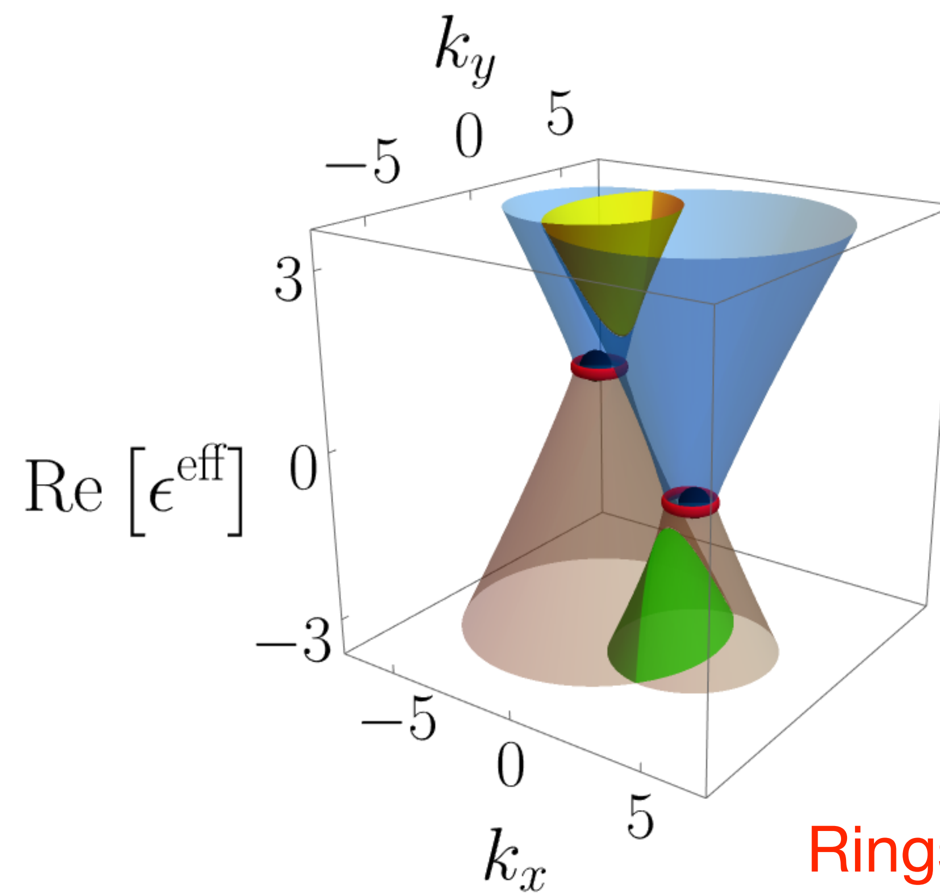

Rings:
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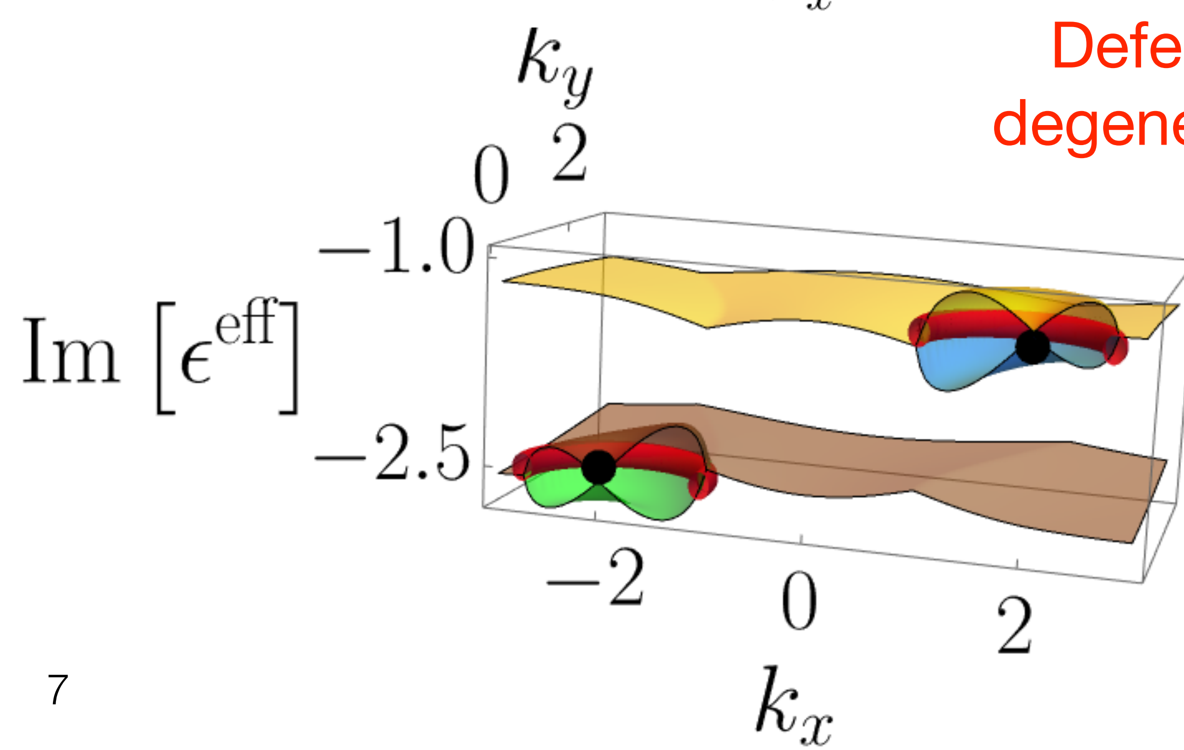
In the vicinity of
non-defective degeneracies

$$H_{\text{eff}}^{\text{low-en}} = \pm\,[ib_0 + iM_i^j(k_j \mp b_j)\sigma^i]$$

Imaginary Fermi velocity



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Conserved classical currents $\partial_\mu j^\mu = 0$ $\partial_\mu j_5^\mu = 0$

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Complex-valued

$$\mathscr{Z} \propto \int \mathscr{D}\Psi \mathscr{D}\bar{\Psi} e^{-\int \mathrm{d}^d x \left[\bar{\Psi}\gamma^\mu \left(M_\mu^{c\nu}\partial_\nu - iM_\mu^{c\nu}A_\nu^c + i\gamma_5 M_\mu^{c\nu}A_\nu^{5c}\right)\Psi\right]}$$

$$M^c = \text{diag}[1, v_1, v_2, v_3]$$

$$\mathscr{D}[A^c, A^{5c}] \neq \mathscr{D}^\dagger[A^c, A^{5c}]$$

Conserved classical currents $\partial_\mu j^\mu = 0$ $\partial_\mu j_5^\mu = 0$

How about divergence of quantum currents?

Fujikawa method + Heat-kernel regularization

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$$\mathcal{D}_{\text{herm}}^2 |\psi_n\rangle = \lambda_n^2 |\psi_n\rangle$$

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→

Non-Hermitian

$$\tilde{\mathcal{D}}\tilde{\mathcal{D}}^\dagger|\eta_n\rangle = |\lambda|_n^2|\eta_n\rangle, \quad \tilde{\mathcal{D}}^\dagger\tilde{\mathcal{D}}|\xi_n\rangle = |\lambda|_n^2|\xi_n\rangle,$$

$$\tilde{\mathcal{D}}^\dagger|\eta_n\rangle = \lambda_n^*|\xi_n\rangle, \quad \tilde{\mathcal{D}}|\xi_n\rangle = \lambda_n|\eta_n\rangle.$$

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Divergence of covariant quantum currents:

$$\tilde{d}_\mu j^{5,\mu} \propto v_1 \tilde{E}_1^\dagger + v_1^* \tilde{E}_1 \quad \text{in } d = 2,$$

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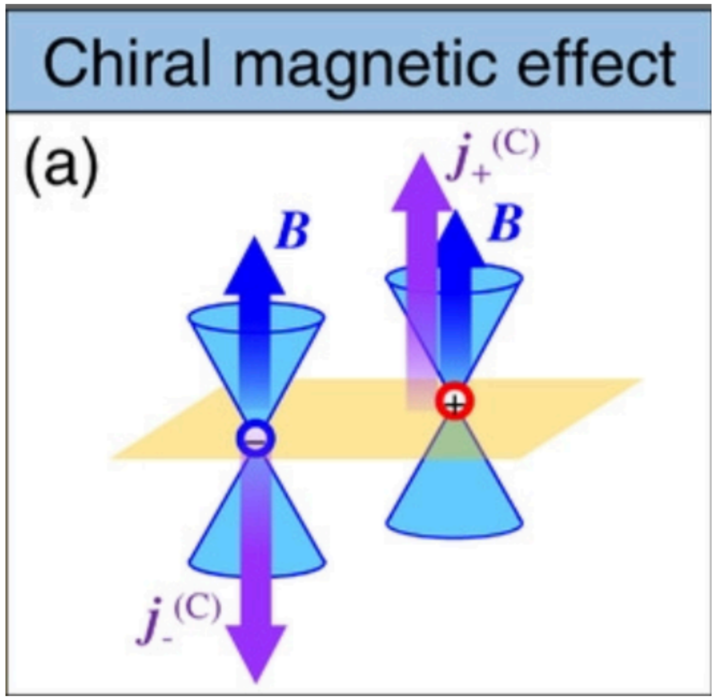
Electric field $\tilde{E}_j = e^{2i\phi_j}\partial_t A_j^c - \partial_j A_t^c$

Magnetic field $\tilde{B}_i = \varepsilon^{ijk}\tilde{B}_{jk} = \varepsilon^{ijk}(e^{2i\phi_k}\partial_j A_k^c - e^{2i\phi_j}\partial_k A_j^c)$

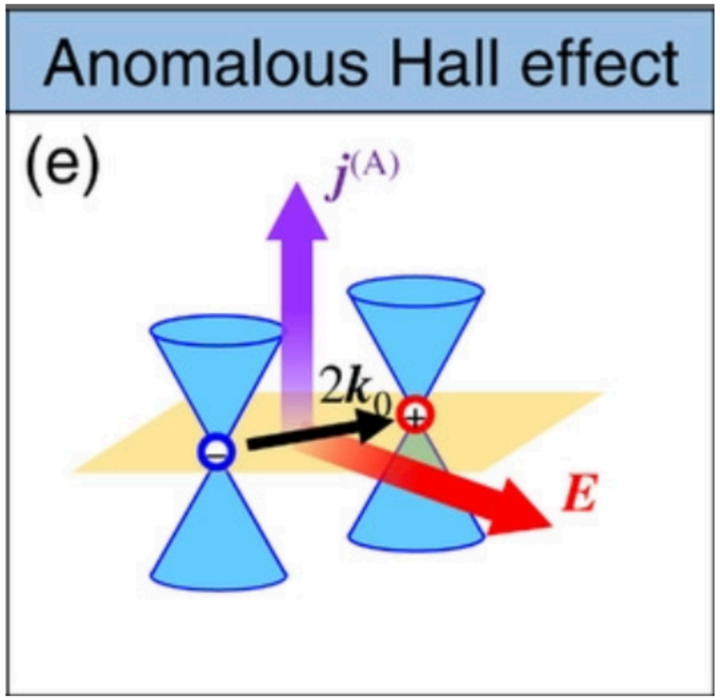
Phase of complex Fermi velocity

$$\frac{v_j}{v_j^*} = e^{i2\phi_j}$$

#Physical consequences of non-Hermitian chiral anomaly



For each Weyl node $j^{(c)} \propto B$



$j^{(A)} \propto \overrightarrow{k_0} \times E$

Y. Araki, Annalen der Physik (2020)

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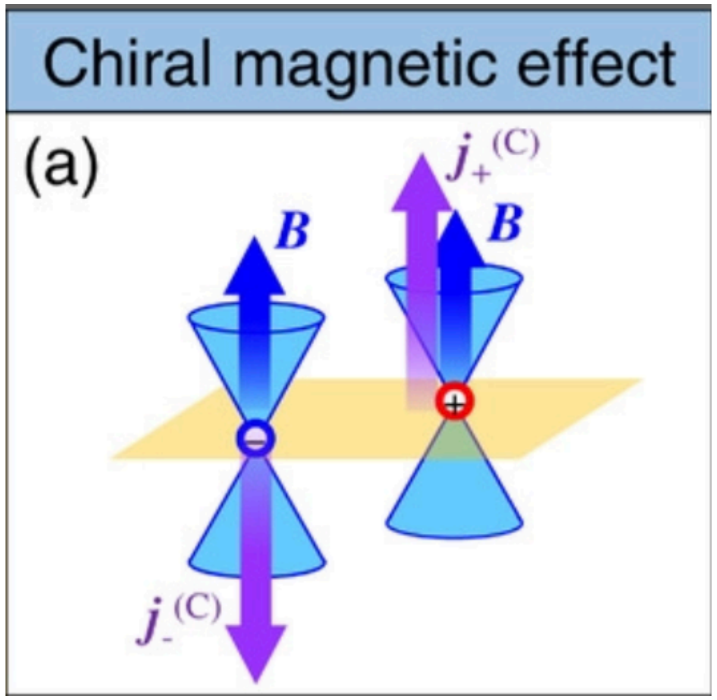

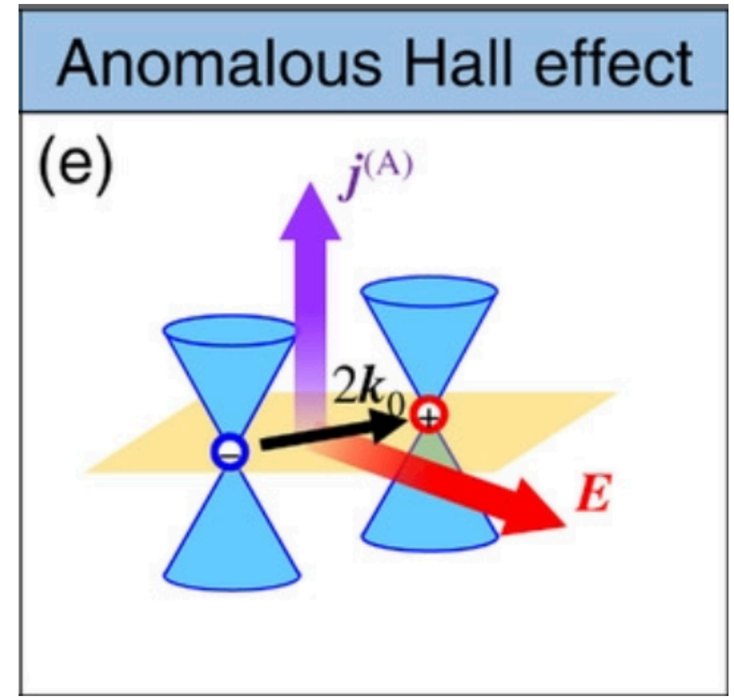

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Y. Araki, Annalen der Physik (2020)

$$M^{\alpha}_{\nu} j^{\nu} \propto \varepsilon^{0\nu\eta\xi} \mathrm{Re}[M^{\alpha *}_{\nu} M^{\iota}_{\eta} M^{\rho}_{\xi} \tilde{B}^{\dagger}_{\iota\rho}]$$

$$M^{\alpha}_{\nu} j^{\nu} \propto \varepsilon^{\mu\nu\eta\xi} \tilde{k}_{0\delta} \mathrm{Re}[M^{\alpha *}_{\nu} M^{\delta}_{\mu} M^{\rho}_{\xi} \tilde{E}^{\dagger}_{\rho}]$$

S.Sayyad, et al., PRR (2022)

#Conclusion

Effective NH Weyl semimetals

$$\tilde{d}_\mu j^{5,\mu} \propto v_1 \tilde{E}_1^\dagger + v_1^* \tilde{E}_1 \quad \text{in } d=2,$$

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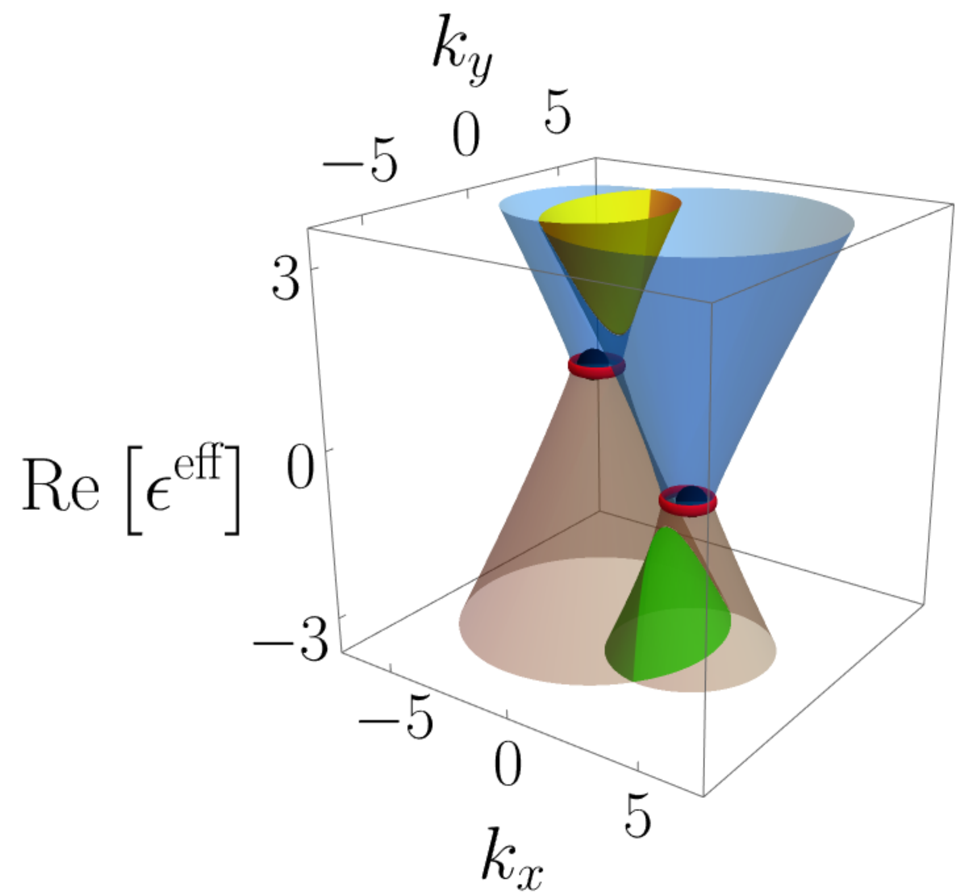

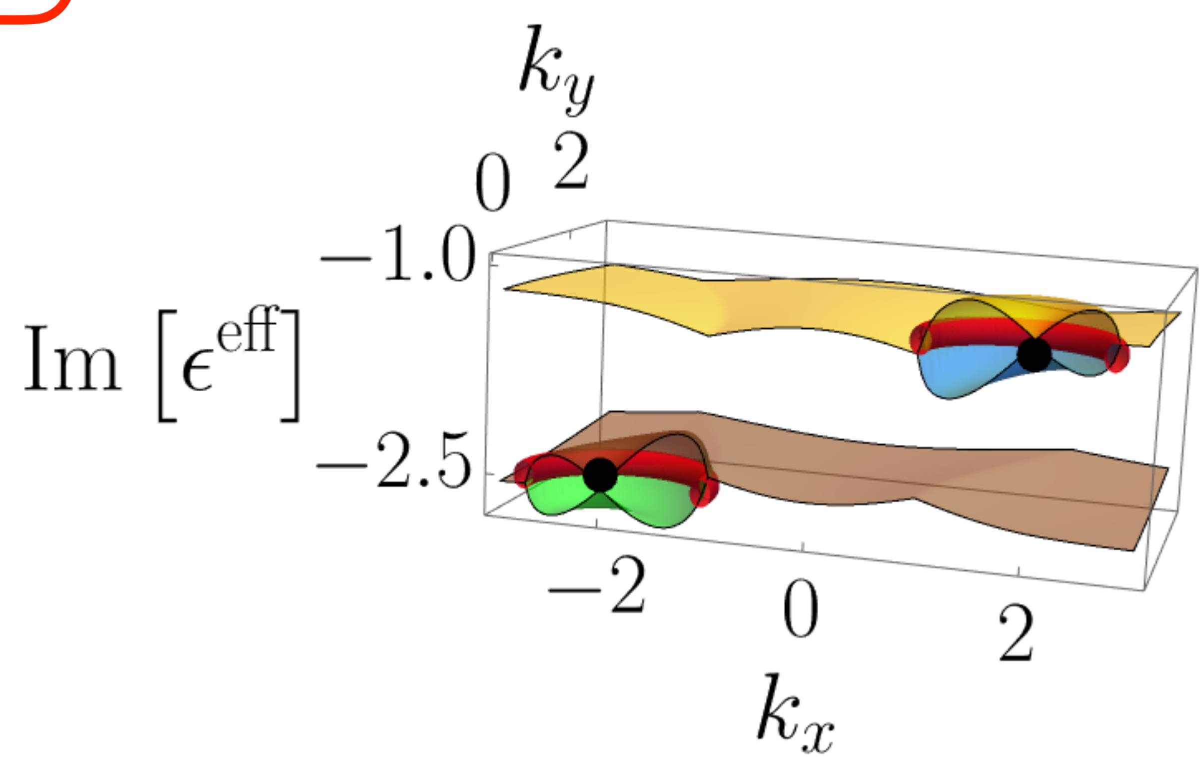

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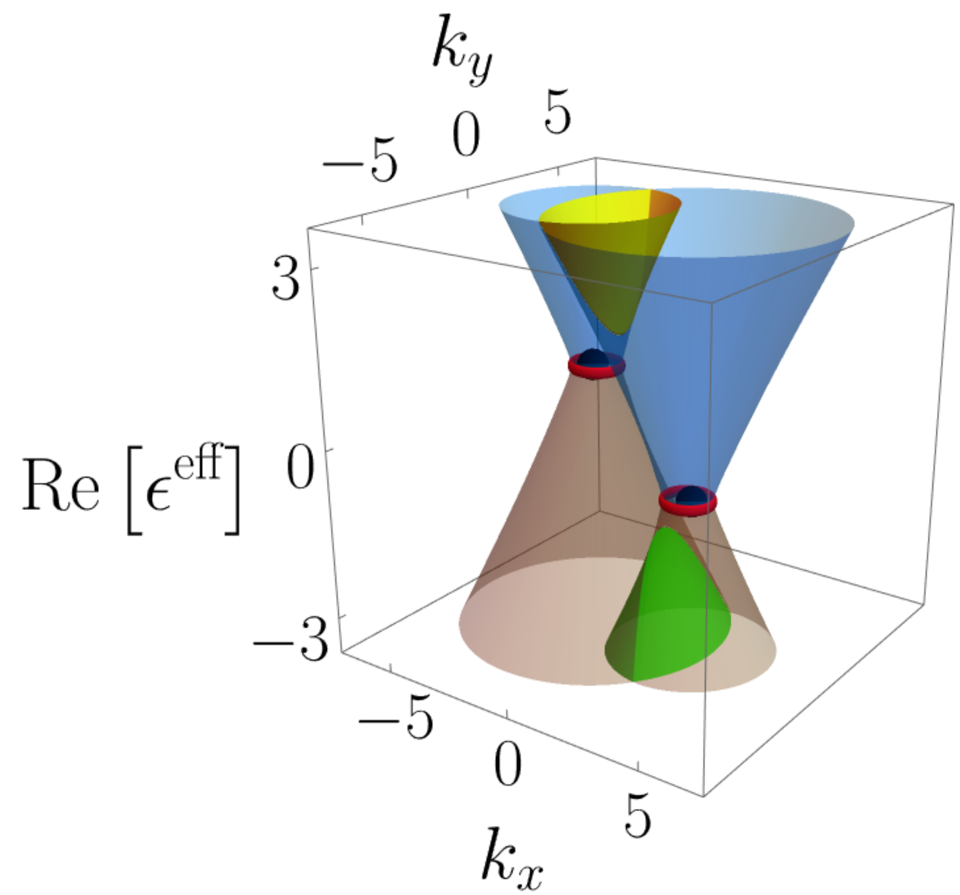
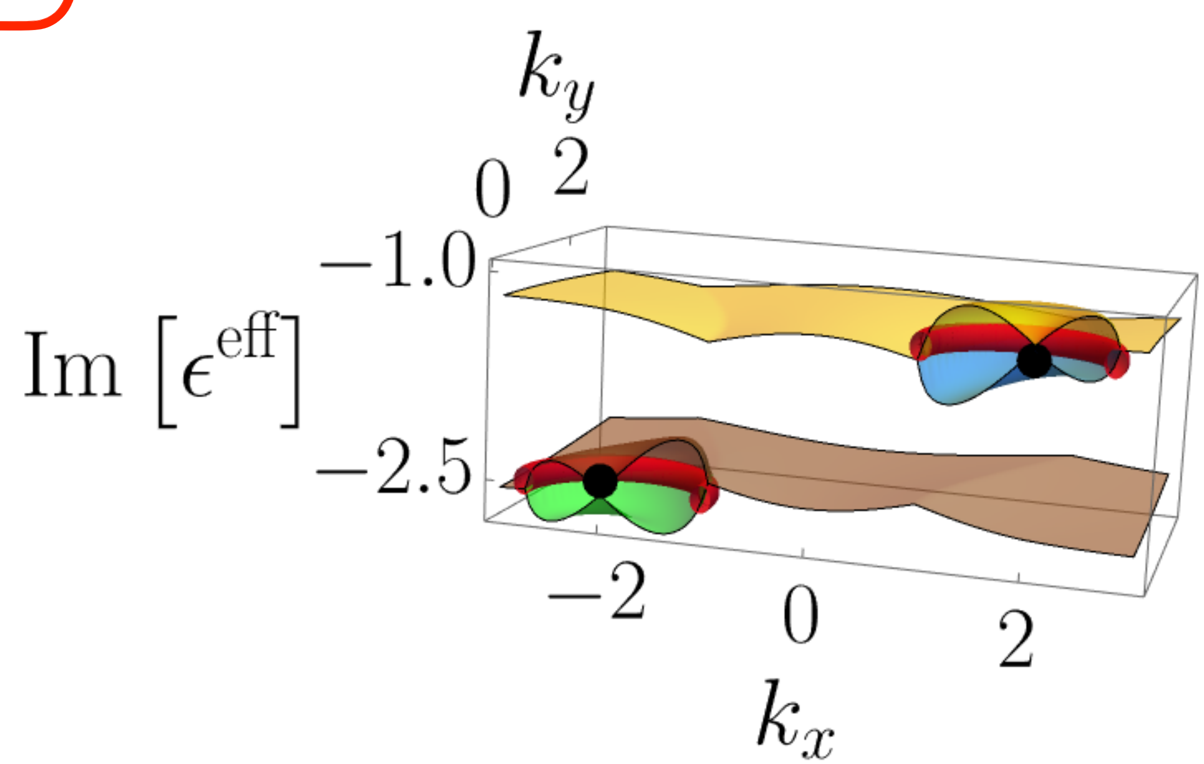
Effective NH Weyl semimetals

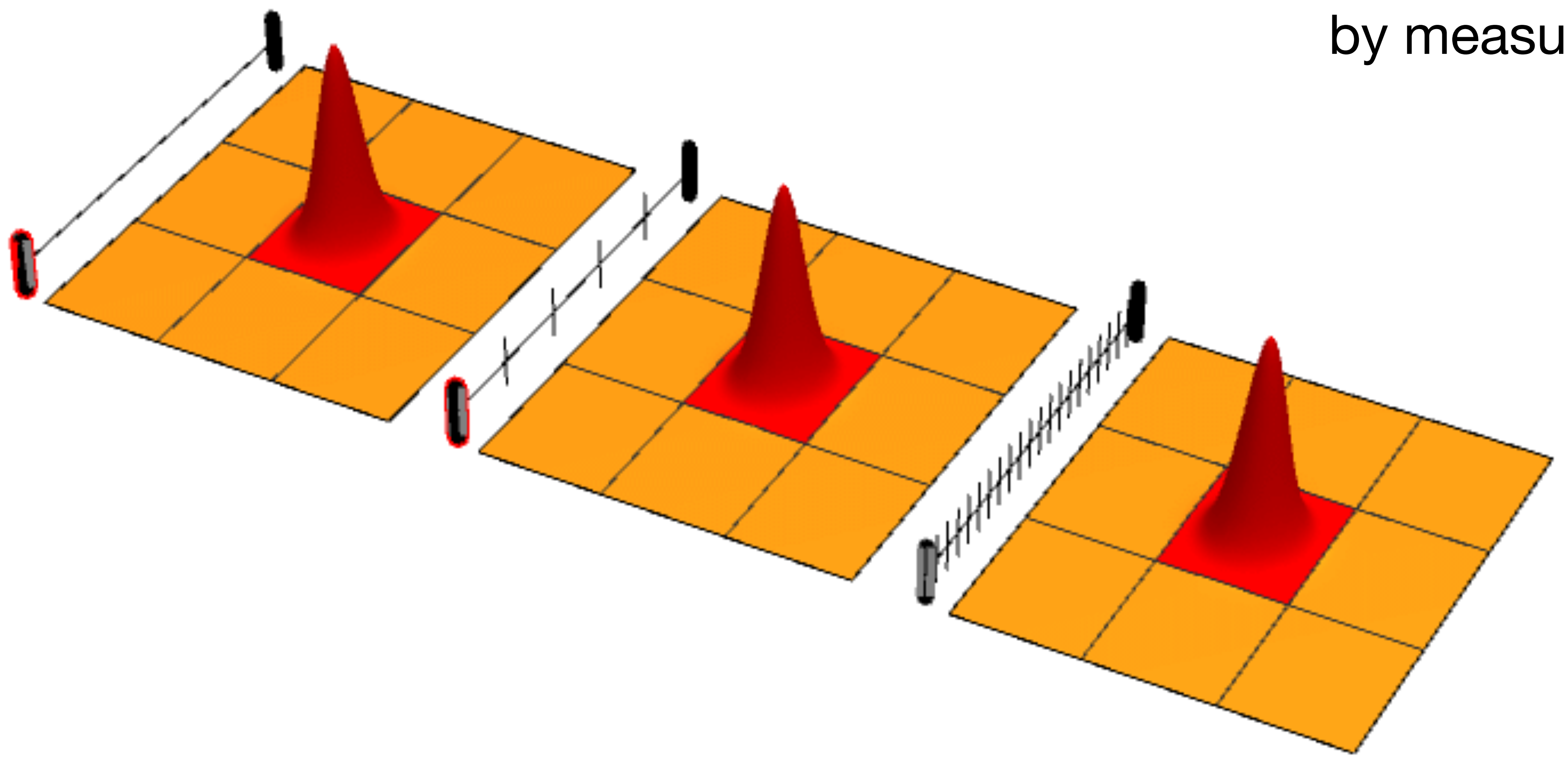




Thank you for your attention!

#Open quantum systems in transport experiments

Quantum Zeno effect
Slowing down the time-evolution
by measurements



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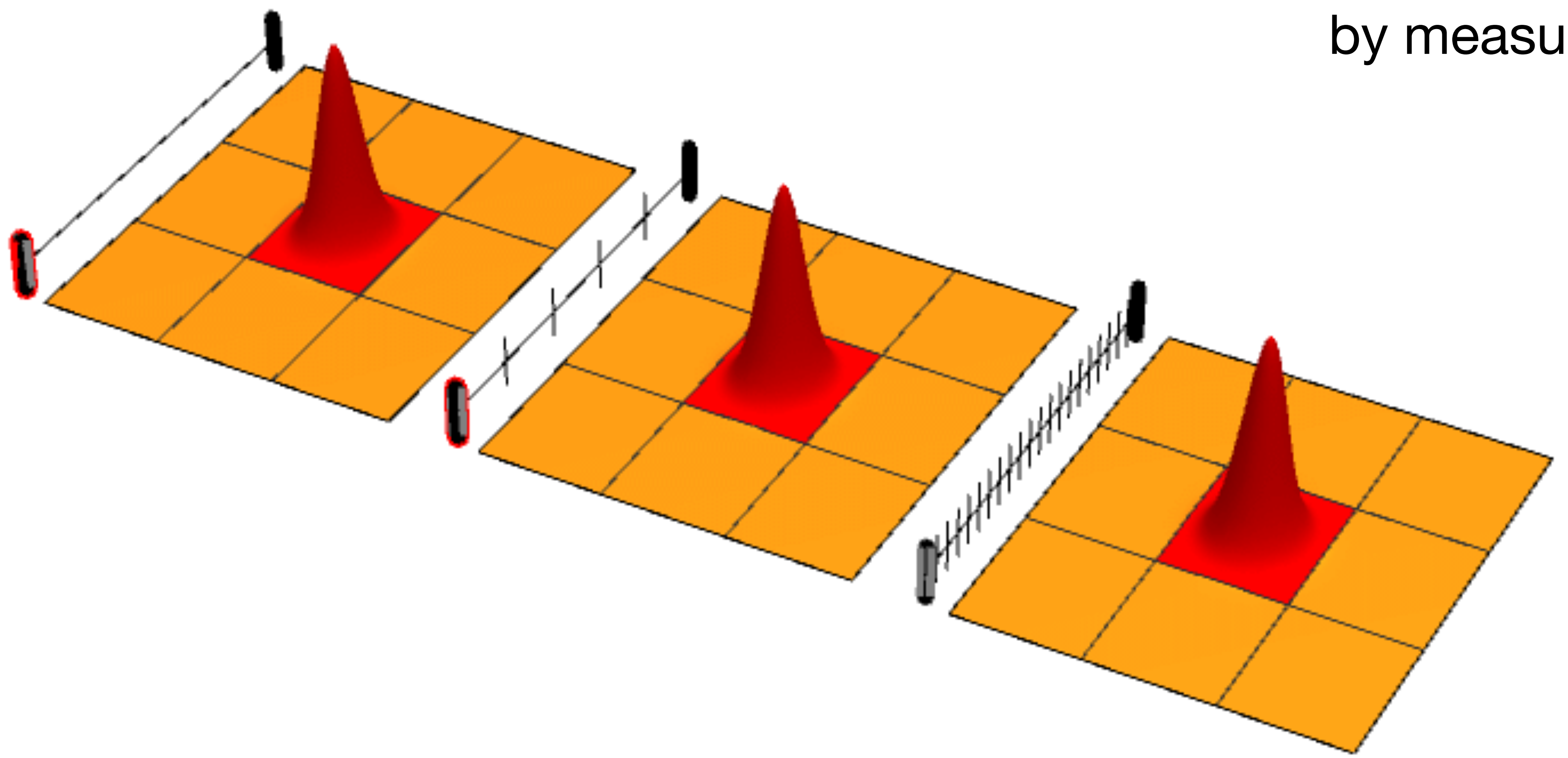

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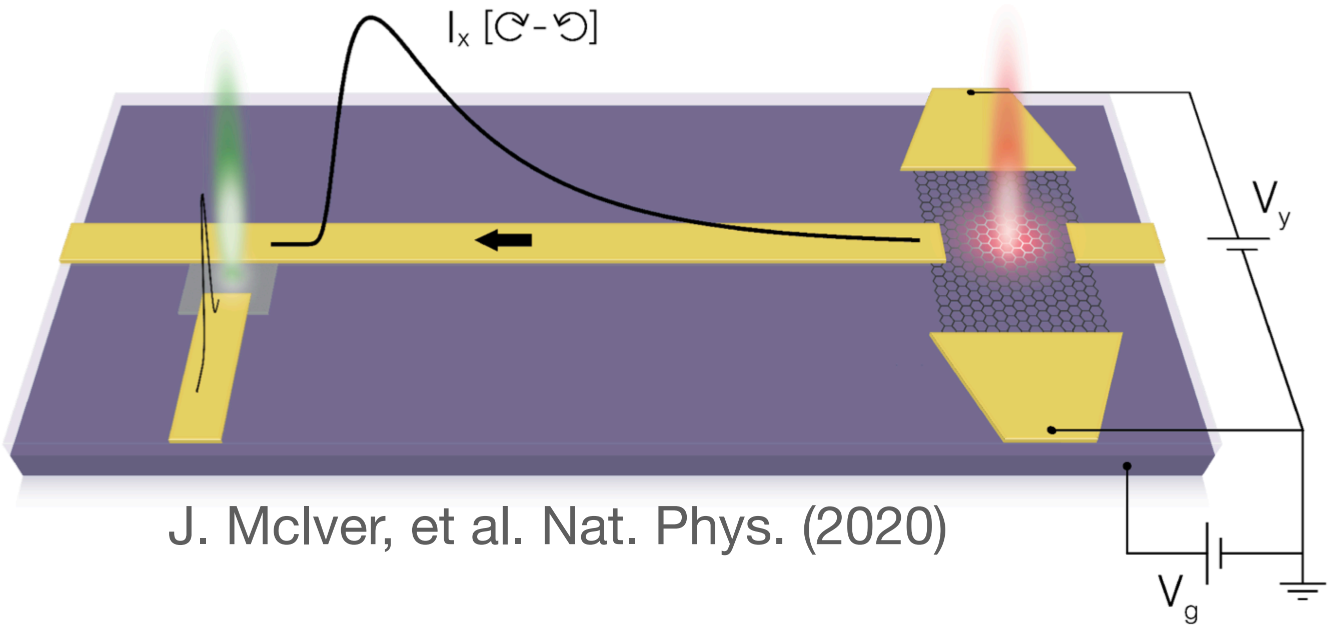
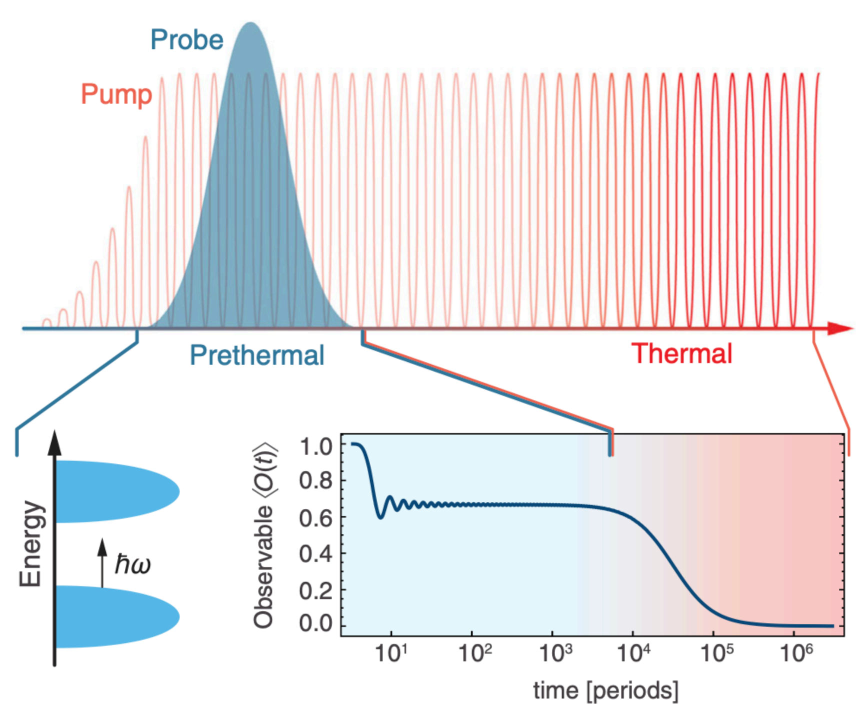
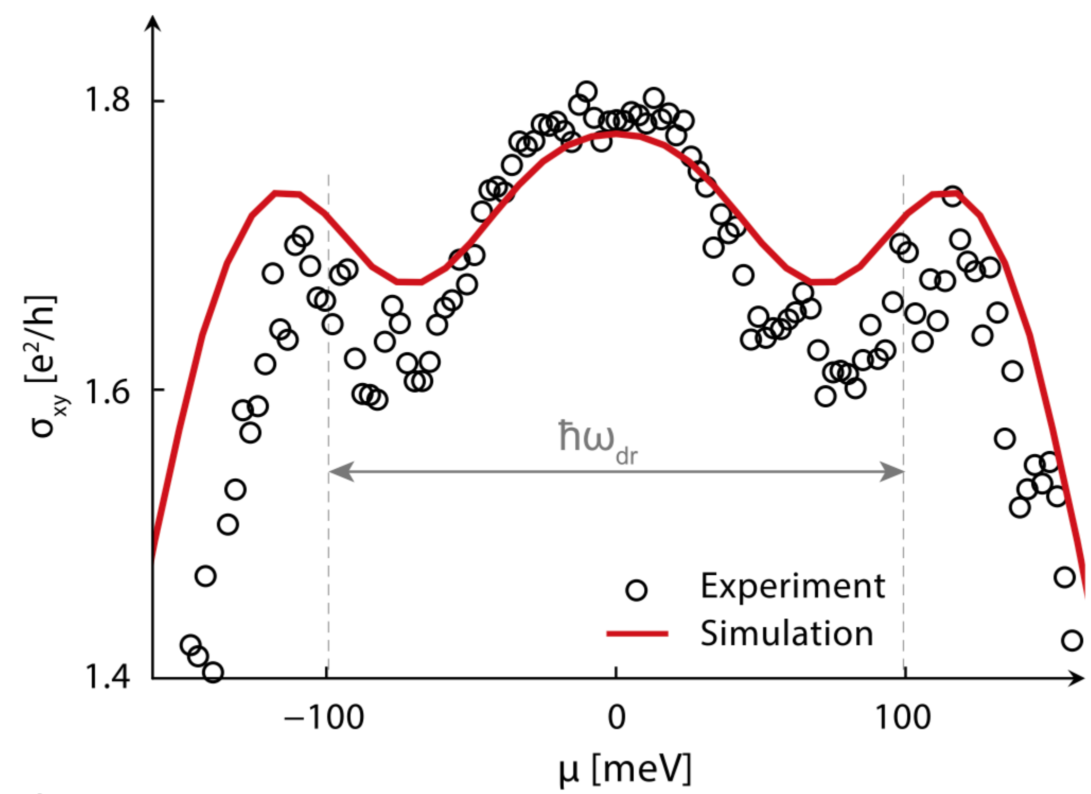


J. McIver, et al. Nat. Phys. (2020)



A. De la Torre, et al RMP (2021)



M. Nuske, et al. PRR (2020)

#Chiral anomaly in Non-Hermitian systems

Non-Hermitian effective model

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Complex-valued

$$\mathcal{Z} \propto \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi} e^{-\int \mathrm{d}^d x \left[\bar{\Psi}\gamma^\mu \left(M_\mu^{c\nu}\partial_\nu - iM_\mu^{c\nu}A_\nu^c + i\gamma_5 M_\mu^{c\nu}A_\nu^{5c}\right)\Psi\right]}$$

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Divergence of quantum currents:

$$[M_\mu^{c\nu}\partial_\nu j^\mu]_{\text{cov}} \propto \frac{1}{32\pi^2 \det[B]} \varepsilon^{\mu\nu\rho\lambda}\left(\tilde{F}_{\mu\nu}[A^{c\dagger}]\tilde{F}_{\rho\lambda}[A^{5c\dagger}] + \tilde{F}^\dagger_{\mu\nu}[A^c]\tilde{F}^\dagger_{\rho\lambda}[A^{5c}]\right)$$

$$[M_\mu^{c\nu}\partial_\nu j_5^\mu]_{\text{cov}} \propto \frac{1}{32\pi^2 \det[B]} \varepsilon^{\mu\nu\rho\lambda}\left(\tilde{F}_{\mu\nu}[A^{c\dagger}]\tilde{F}_{\rho\lambda}[A^{c\dagger}] + \tilde{F}^\dagger_{\mu\nu}[A^c]\tilde{F}^\dagger_{\rho\lambda}[A^c] + \tilde{F}_{\mu\nu}[A^{5c\dagger}]\tilde{F}_{\rho\lambda}[A^{5c\dagger}] + \tilde{F}^\dagger_{\mu\nu}[A^{5c}]\tilde{F}^\dagger_{\rho\lambda}[A^{5c}]\right)$$

Related to M^c

$$\tilde{F}_{\mu\nu}[V^\dagger] = M_\mu^{c\eta}M_\nu^{c\beta *}(\partial_\eta V_\beta^* - \partial_\beta V_\eta^*)$$

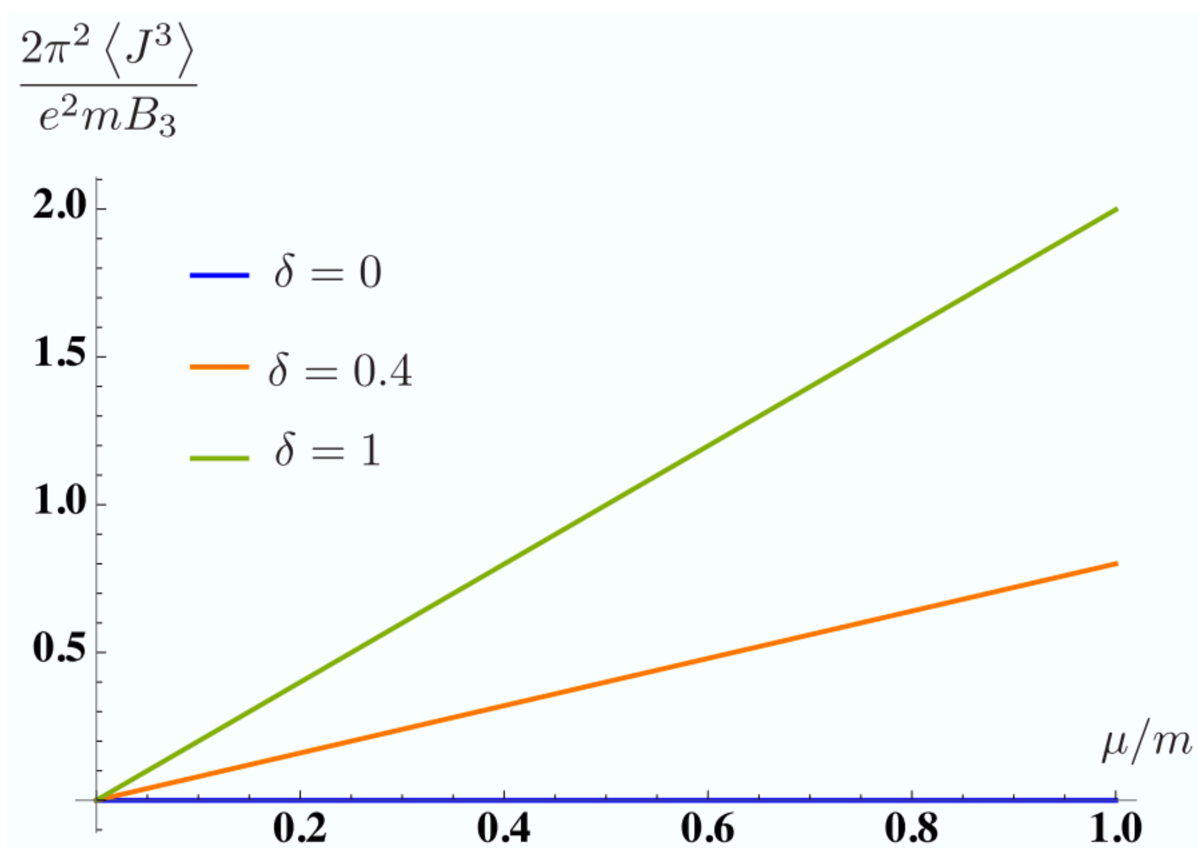
$$\tilde{F}^\dagger_{\mu\nu}[V] = M_\mu^{c\eta *}M_\nu^{c\beta}(\partial_\eta V_\beta - \partial_\beta V_\eta)$$

#Chiral anomaly in $\mathscr{PT}$-symmetric systems

$$H_{\mathscr{PT}} = \gamma_0\gamma\,.\,\mathbf{k} + m\gamma_0 + m_5\gamma_0\gamma_5 - \mu(I - \frac{m_5}{m}\gamma_5)$$

$$H_{\text{herm}} = SH_{\mathscr{PT}}S^{-1}$$

$$\left\langle J^3 \right\rangle = \frac{e^2 B_3}{2\pi^2}\frac{m_5}{m}\mu.$$

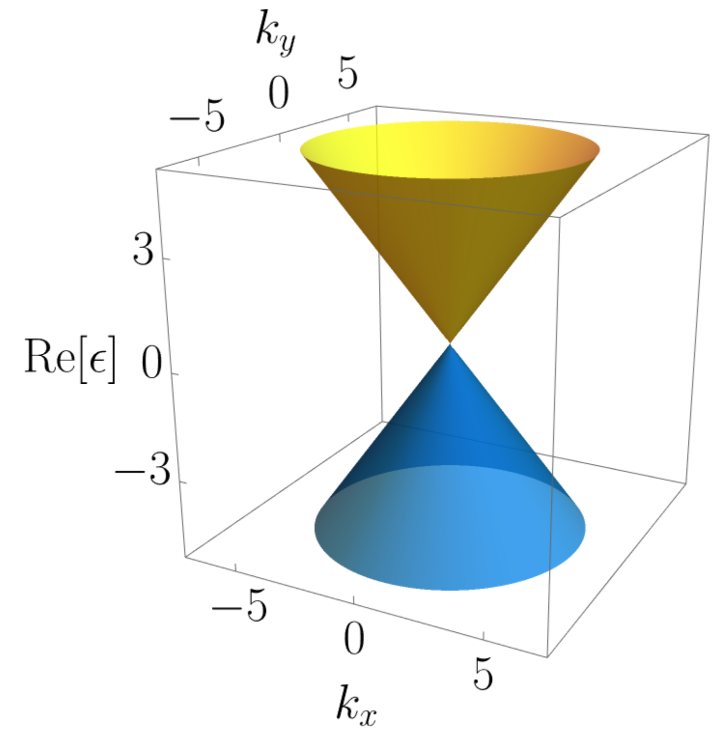


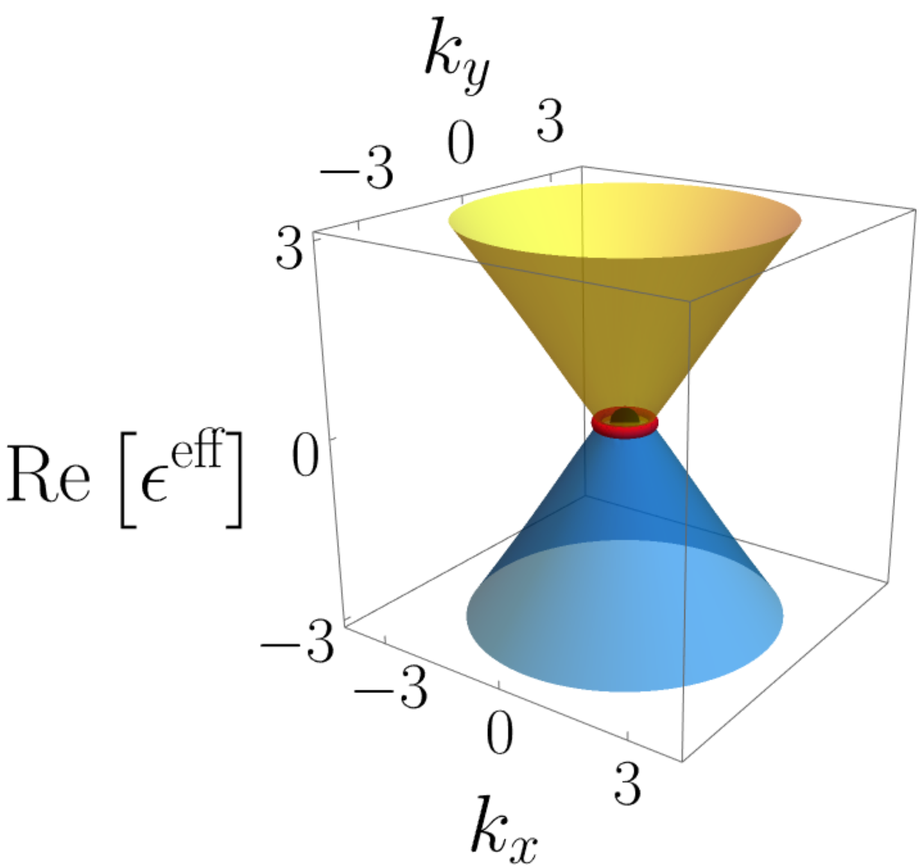
M. Chernodub, et al. Symmetry (2020)

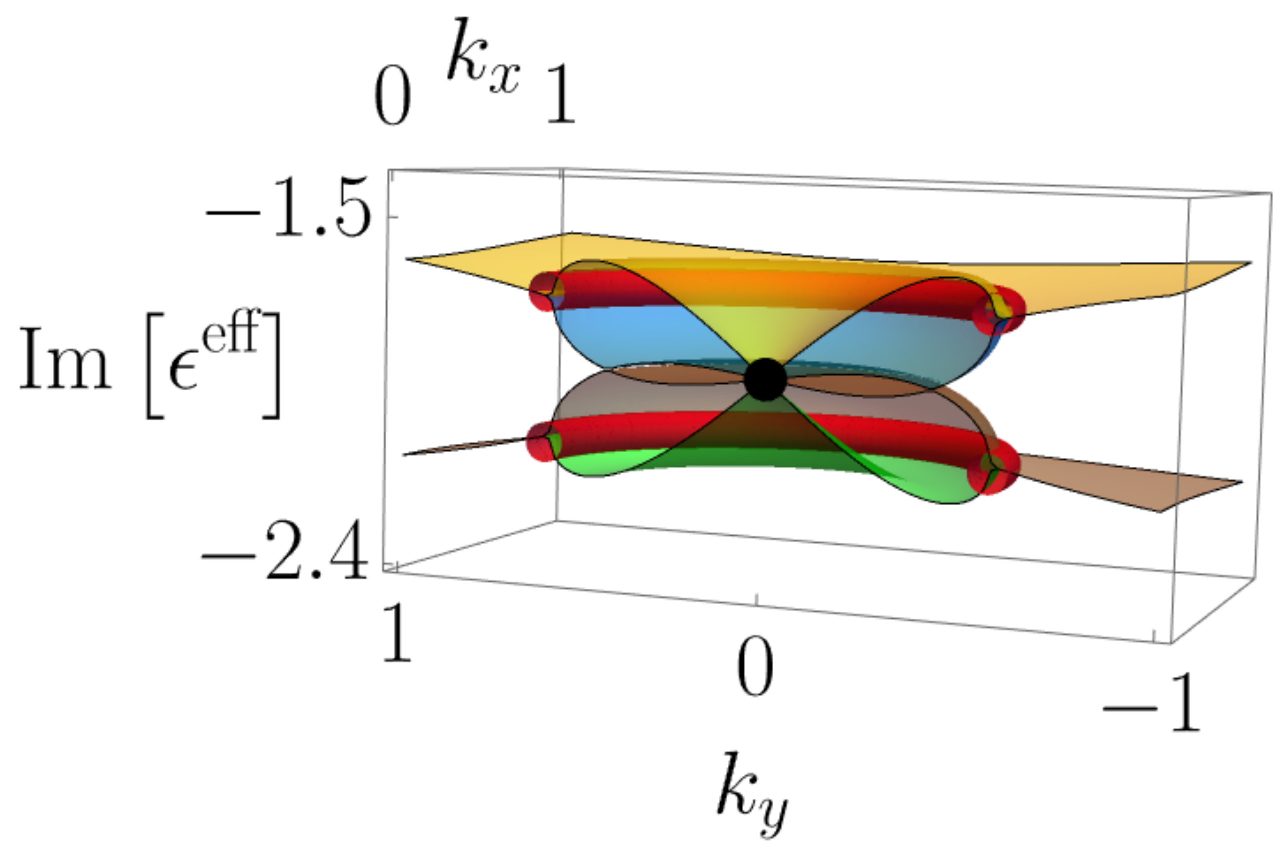
#Effective Description of Open Quantum Systems

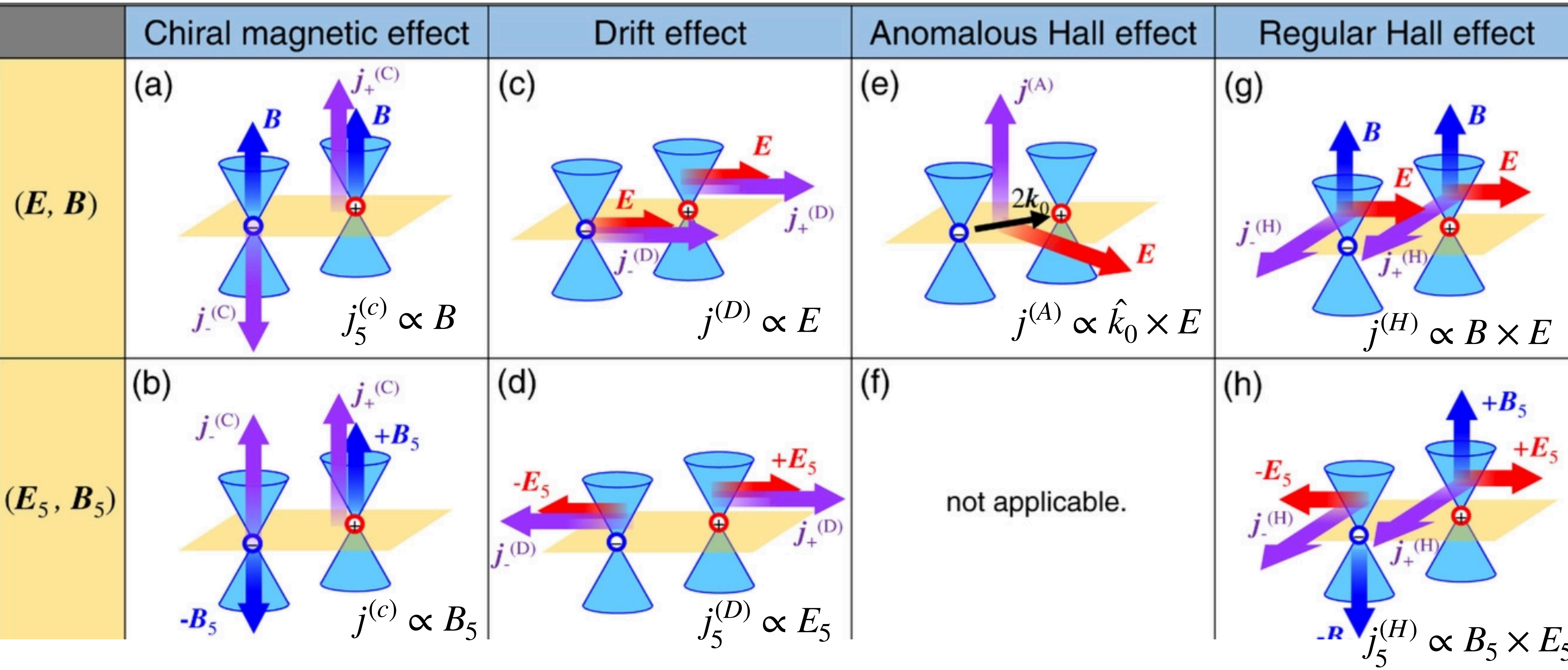
$$\dot{\rho}_{\text{sys}} = -i(H_{\text{eff}}\rho_{\text{sys}} - \rho_{\text{sys}}H_{\text{eff}}^{\dagger}) + \sum_i 2L_i\rho_{\text{sys}}L_i^{\dagger}$$

$$H_{\text{eff}} = H_{\text{sys}} - i\sum_l L_l^{\dagger}L_l$$



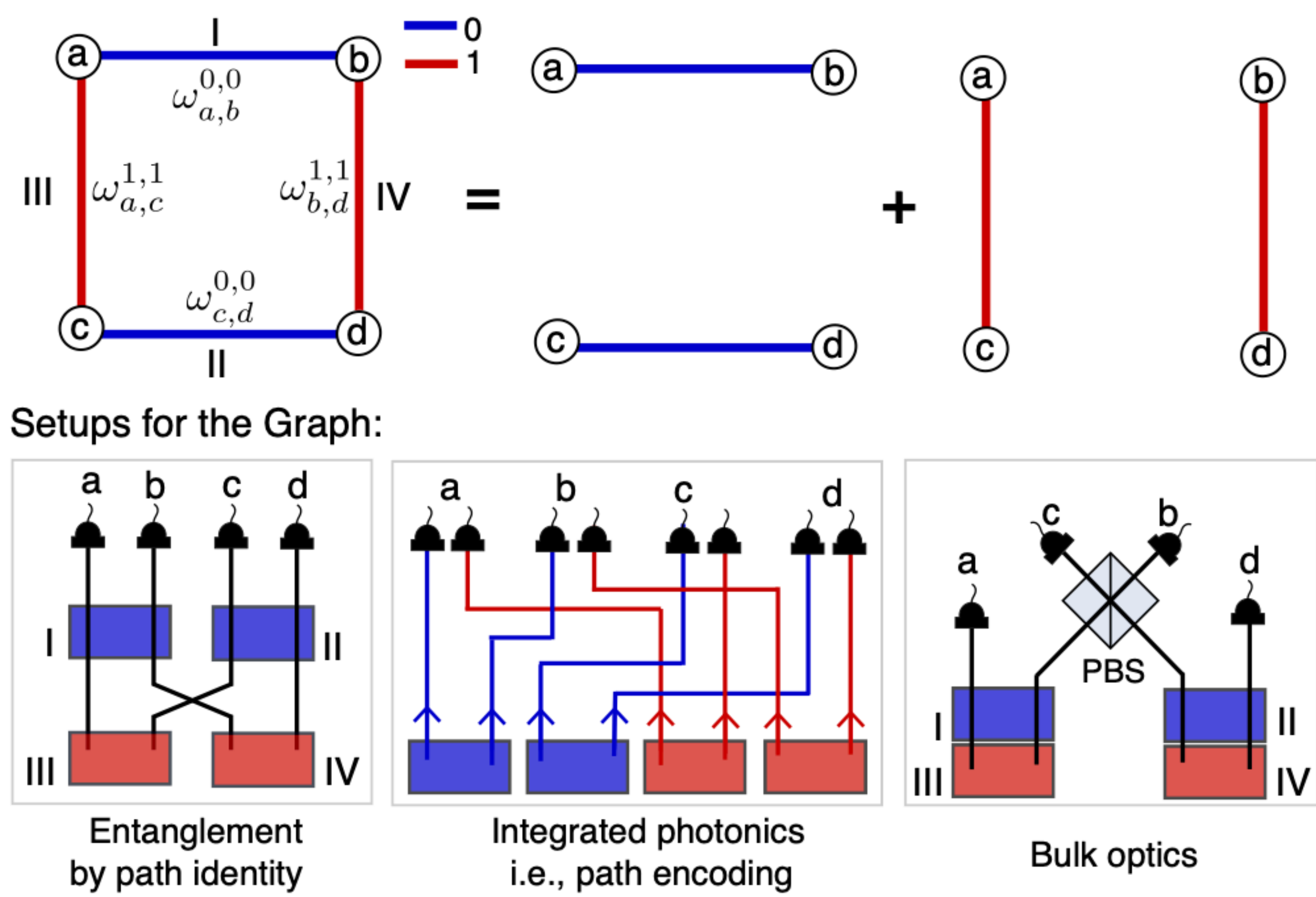






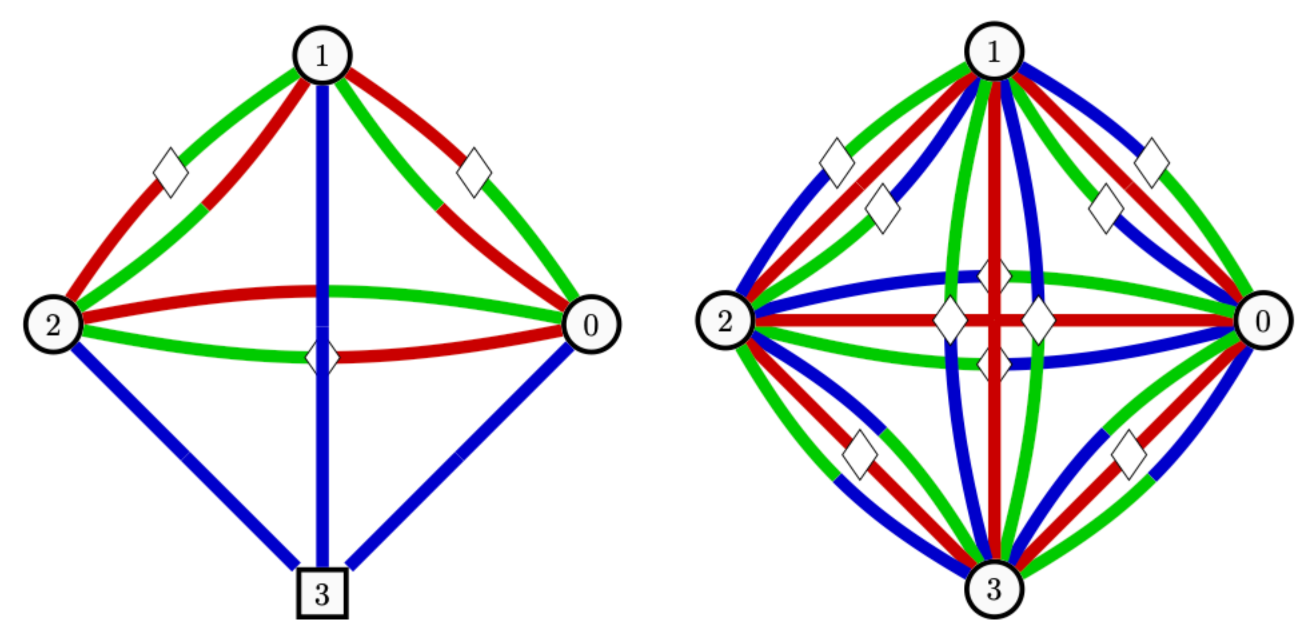
Y. Araki, Annalen der Physik (2020)

#Computer-discovery of Optical experiments



$$\left|\psi^{(3)}\right\rangle = |0,-1,+1\rangle - |-1,0,+1\rangle + |-1,+1,0\rangle - |0,+1,-1\rangle + |+1,0,-1\rangle - |+1,-1,0\rangle\,, \quad (85)$$

$$\left|\psi^{(4)}\right\rangle = 2(|-1,1,-1,1\rangle + |1,-1,1,-1\rangle) + |1,0,-1,0\rangle - |1,-1,0,0\rangle + |0,1,0,-1\rangle - |0,1,-1,0\rangle - |0,0,1,-1\rangle + |0,0,0,0\rangle - |0,0,-1,1\rangle - |0,-1,1,0\rangle + |0,-1,0,1\rangle - |-1,1,0,0\rangle - |1,0,0,-1\rangle + |-1,0,1,0\rangle - |-1,0,0,1\rangle\,. \quad (86)$$



C. Ruiz-Gonzalez, et al, arXiv:2210.09980 (2022)