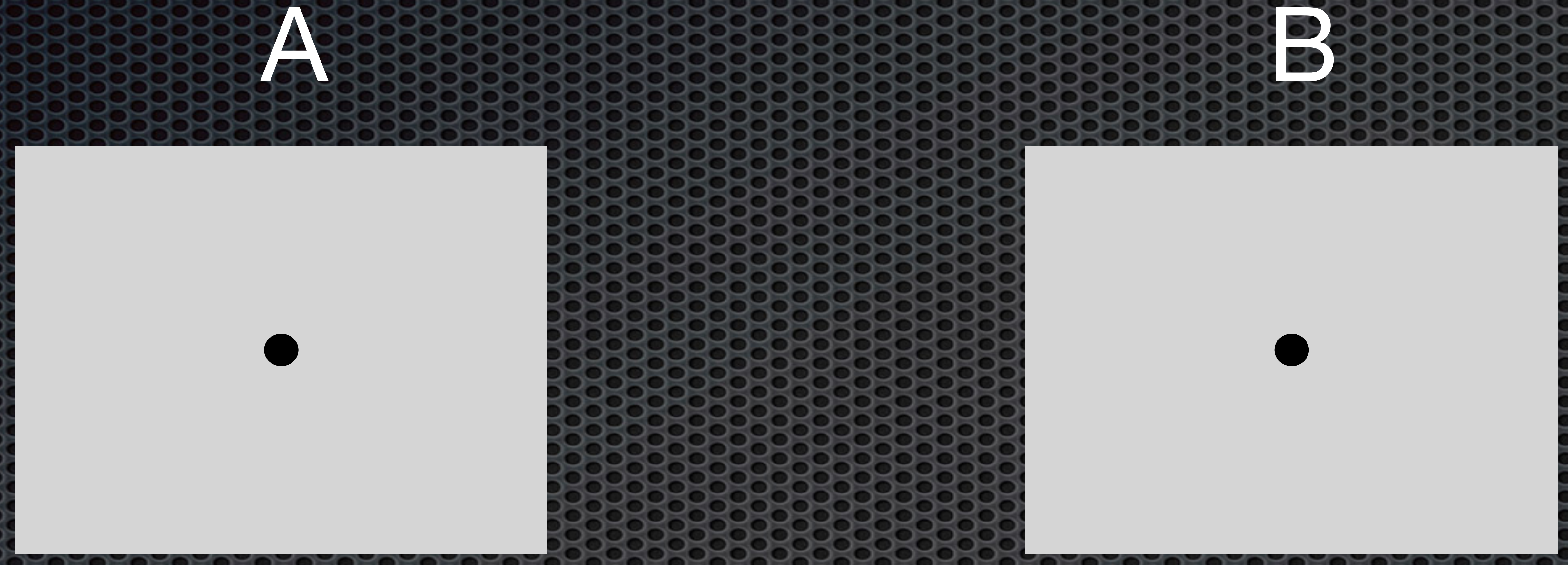
Non-Hermitian Holography

Based on 1912.06647 w/ K. Landsteiner and I. Salazar; and in progress w/ D. García

Daniel Areán, Trento, March 2023

PT-symmetric QM. 2 state Hamiltonian

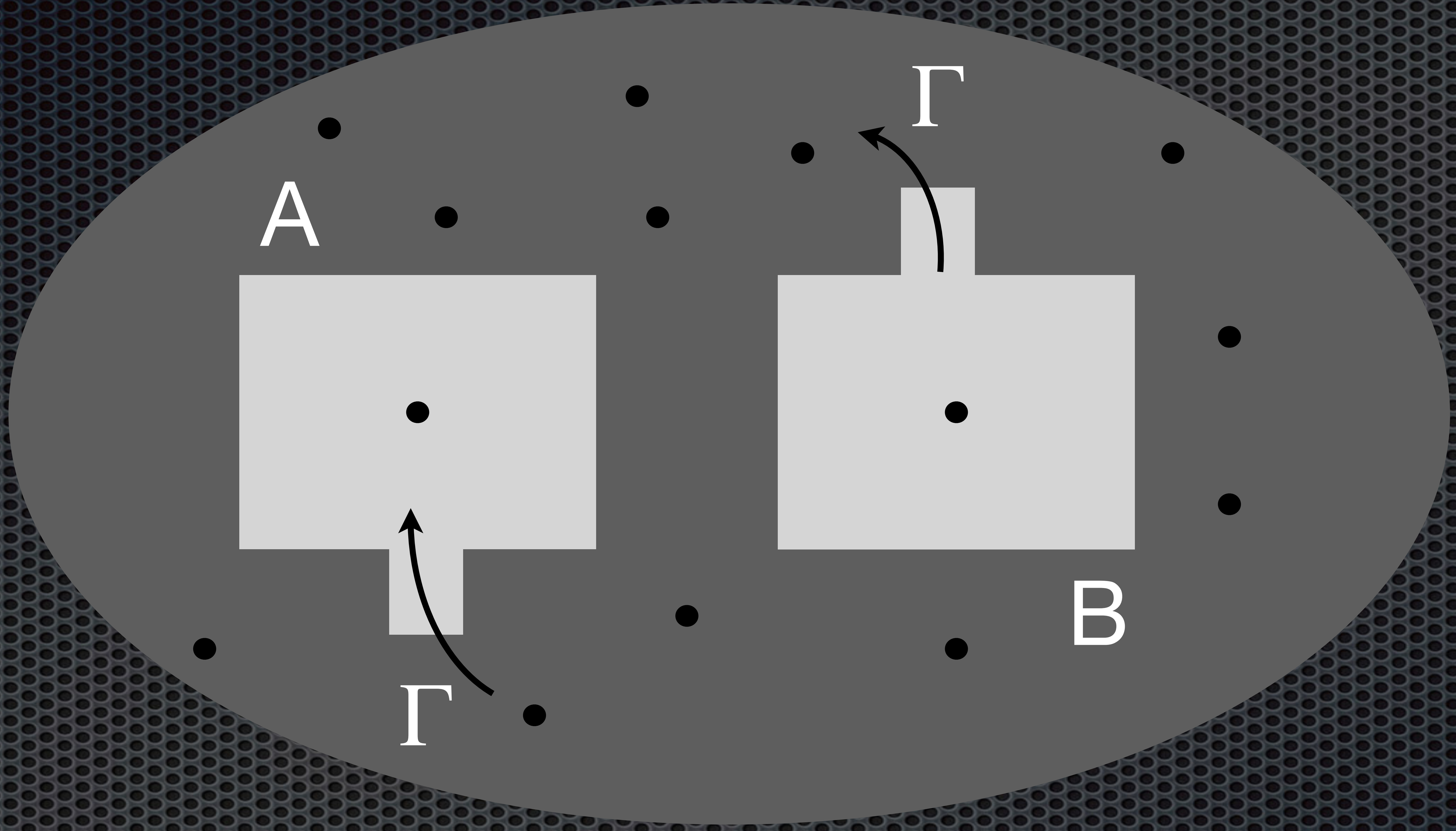
[Bender'97]



$$H = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$$

$$\psi_{A,B} = e^{-iEt}$$

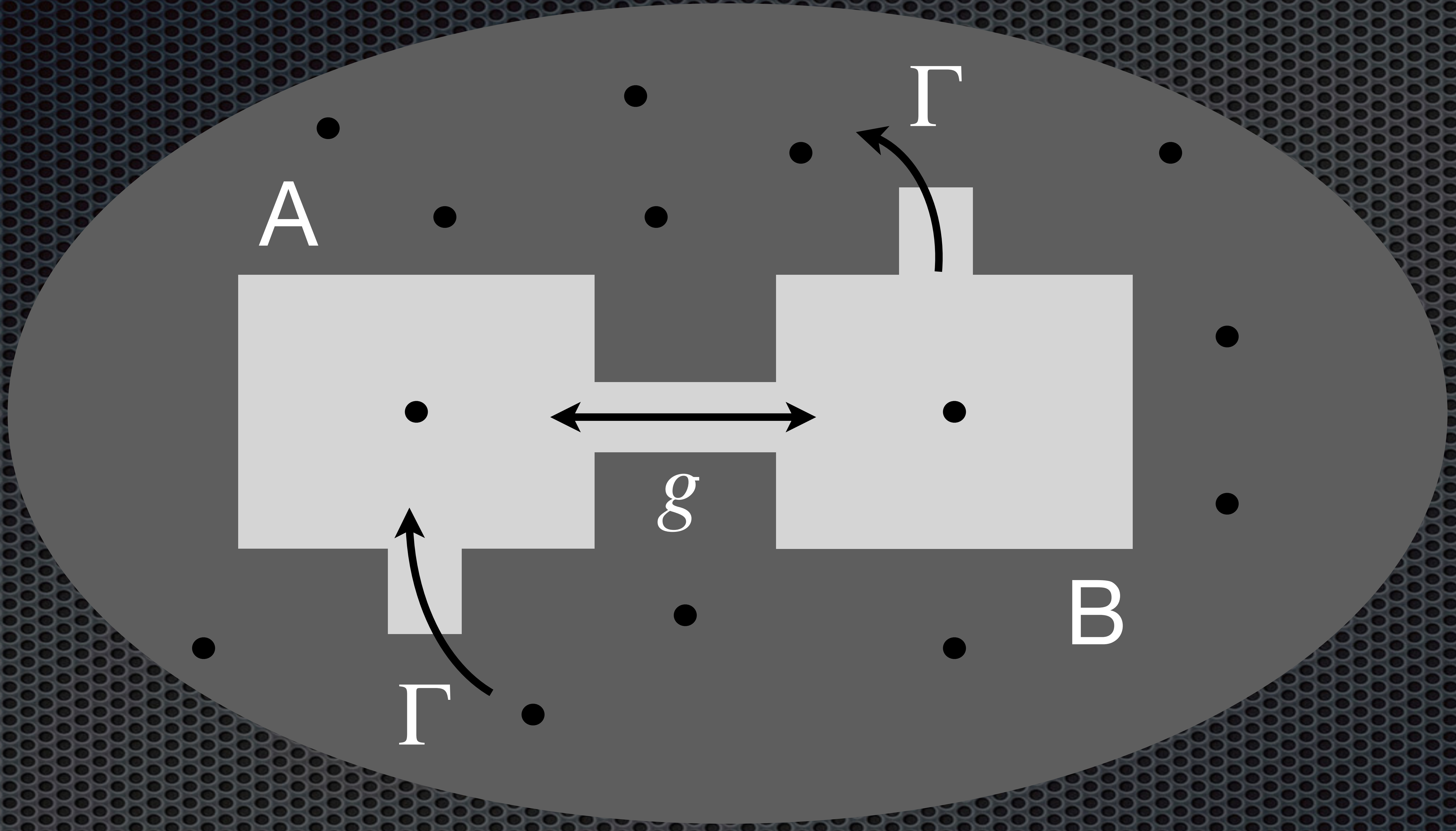
PT-symmetric QM. 2 state Hamiltonian



$$H = \begin{pmatrix} E - i\Gamma & 0 \\ 0 & E + i\Gamma \end{pmatrix}$$

$$\psi_{A,B} = e^{-(iE \pm \Gamma)\, t}$$

PT-symmetric QM. 2 state Hamiltonian



Eigenvalues

$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix}$$

$$\lambda_{\pm} = E \pm \sqrt{g^2 - \Gamma^2}$$

PT-symmetric QM

$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix}$$

$$\lambda_{\pm} = E \pm \sqrt{g^2 - \Gamma^2}$$

PT-symmetric phase: $|g| > \Gamma$

PT-broken phase: $|g| < \Gamma$

Exceptional point: $g = \Gamma$

PT-symmetric QM

$$T(H) = \begin{pmatrix} E + i\Gamma & g \\ g & E - i\Gamma \end{pmatrix} \qquad P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P\,T(H)\,P = H$$

PT-symmetric phase: $|g| > \Gamma$

PT-broken phase: $|g| < \Gamma$

PT-symmetric QM. Pseudo-hermitian H

[Mostafazadeh'02, '03, '20; Fring '22]

$$[H, PT] = 0\,, \quad PT\,|\psi\rangle = e^{i\phi}\,|\psi\rangle\,, \quad PT\,\lambda\,|\psi\rangle = \lambda^*\,|\psi\rangle$$

[➥ $(PT)^2 = 1$]

H is pseudo-Hermitian (it has real eigenvalues)

Dyson map: $\eta : H \longrightarrow h = \eta H \eta^{-1}\,, \quad h = h^\dagger$

➥ metric $\rho = \eta^\dagger \eta \;\Leftrightarrow\; \rho\, H^\dagger = H$

$$\langle \psi | \tilde{\psi} \rangle_\rho \equiv \langle \psi | \rho\, \tilde{\psi} \rangle \Rightarrow \langle \psi | H \tilde{\psi} \rangle_\rho = \langle H\psi | \tilde{\psi} \rangle_\rho$$

PT-symmetric QM. Pseudo-hermitian H

$$H_2(\vec{g}) = E\,\mathbb{I} + \vec{g}\cdot\vec{\sigma} \Longleftrightarrow H_2'(\vec{g}') = D(\overrightarrow{\alpha})^\dagger H_2(\vec{g})\,D(\overrightarrow{\alpha})$$

$SU(2)$ rotation $D(\overrightarrow{\alpha}) = e^{i\vec{\alpha}\cdot\vec{\sigma}/2}$ $\Rightarrow$ Dyson map: $\eta(\vec{\beta}) = D(-i\vec{\beta})$

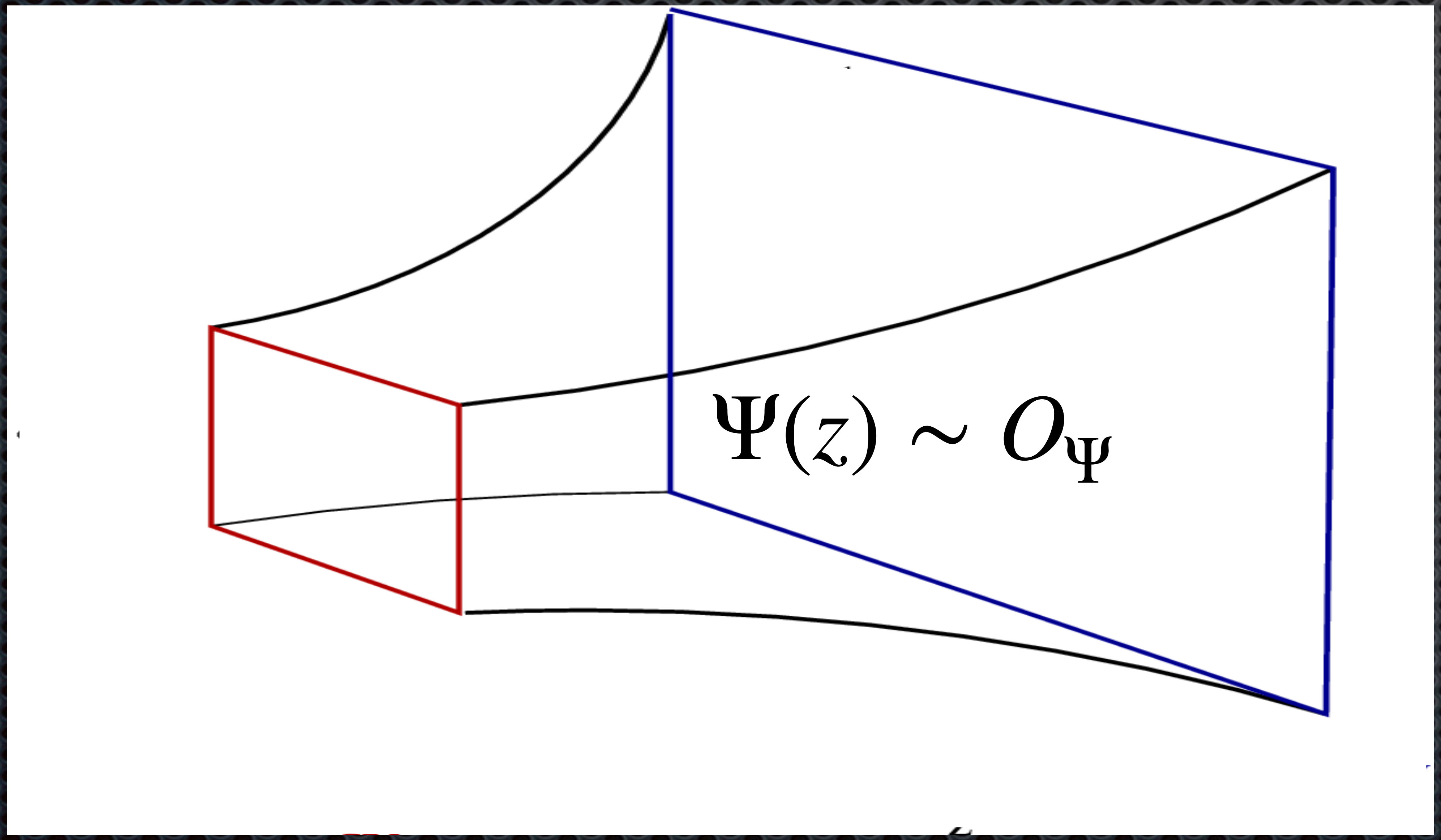
E.g. $\vec{g} = (g',0,0)\,,\ \overrightarrow{\alpha} = (0,i\beta,0) \Longrightarrow H_{\rm nH} = E + g'(\cosh\beta\sigma_1 + i\sinh\beta\sigma_3)$

Eigenvalues $\lambda_\pm = E \pm g'\sqrt{\cosh^2\beta - \sinh^2\beta}$

Exceptional point $\begin{cases} \beta \to \infty \\ g' \to e^{-\beta}\tilde{g} \end{cases} \Rightarrow$

Non-Hermitian Holography

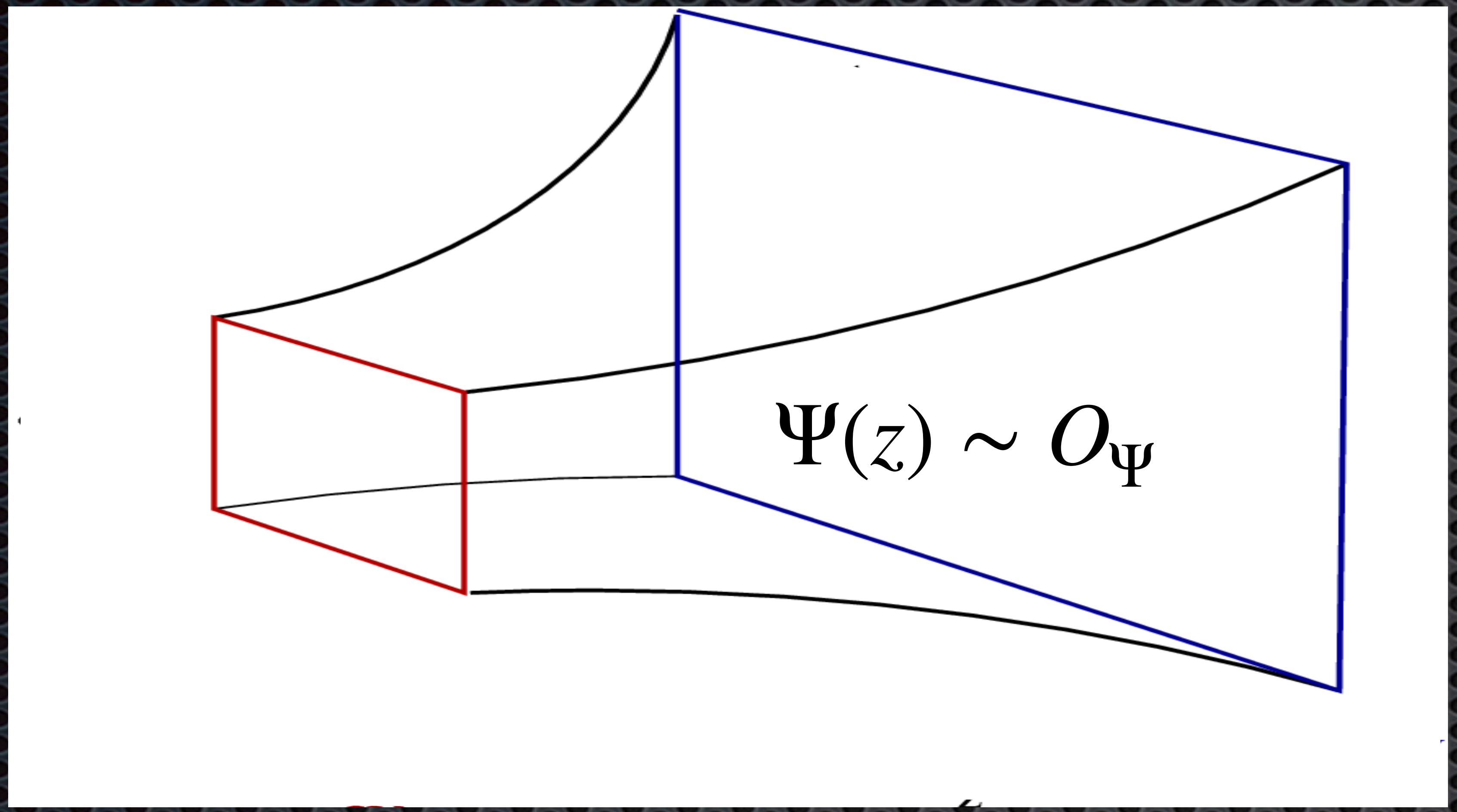
Complex scalar field Ψ in $\sim$ AdS geometry: $\Psi|_{\text{boundary}} = M$,



$$\sim \mathscr{L} = \ldots + \bar{M}\langle O\rangle + M\langle O^\dagger\rangle$$

Non-Hermitian Holography

Complex scalar field Ψ in $\sim \mathrm{AdS}$



Dyson map: $U(1)$ w/ imaginary phase

$$\Psi \sim M \to \Psi \sim M e^{-\theta}$$

$$\bar{\Psi} \sim M \to \Psi \sim M e^{\theta}$$

[Gubser&Rocha'08]

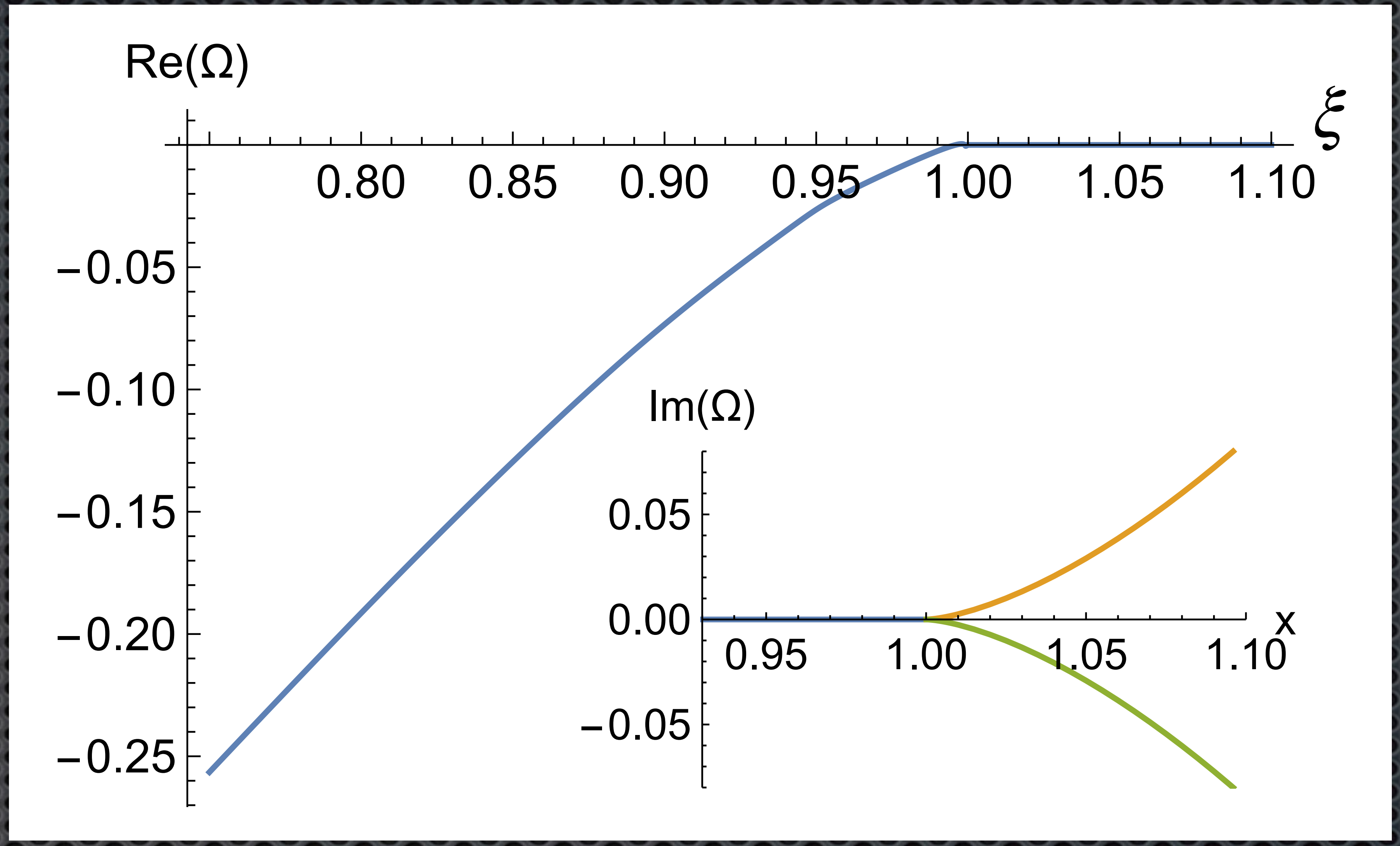
$$S = \int d^{d+1}x\sqrt{-g}\left[R - 2\Lambda + F^2 + \frac{1}{2}g^{ab}\left(D_a\Psi\,\bar{D}_b\bar{\Psi} + a \leftrightarrow b\right) + \ldots\right]$$

Non-Hermitian Holography [1912.06647]

$\Psi \sim s\,(1 - \xi)$

$\bar{\Psi} \sim s\,(1 + \xi)$

Pseudo Hermitian for $\xi < 1$



PT-symmetric phase: $\xi < 1$ $\sim$ $\xi = 0$

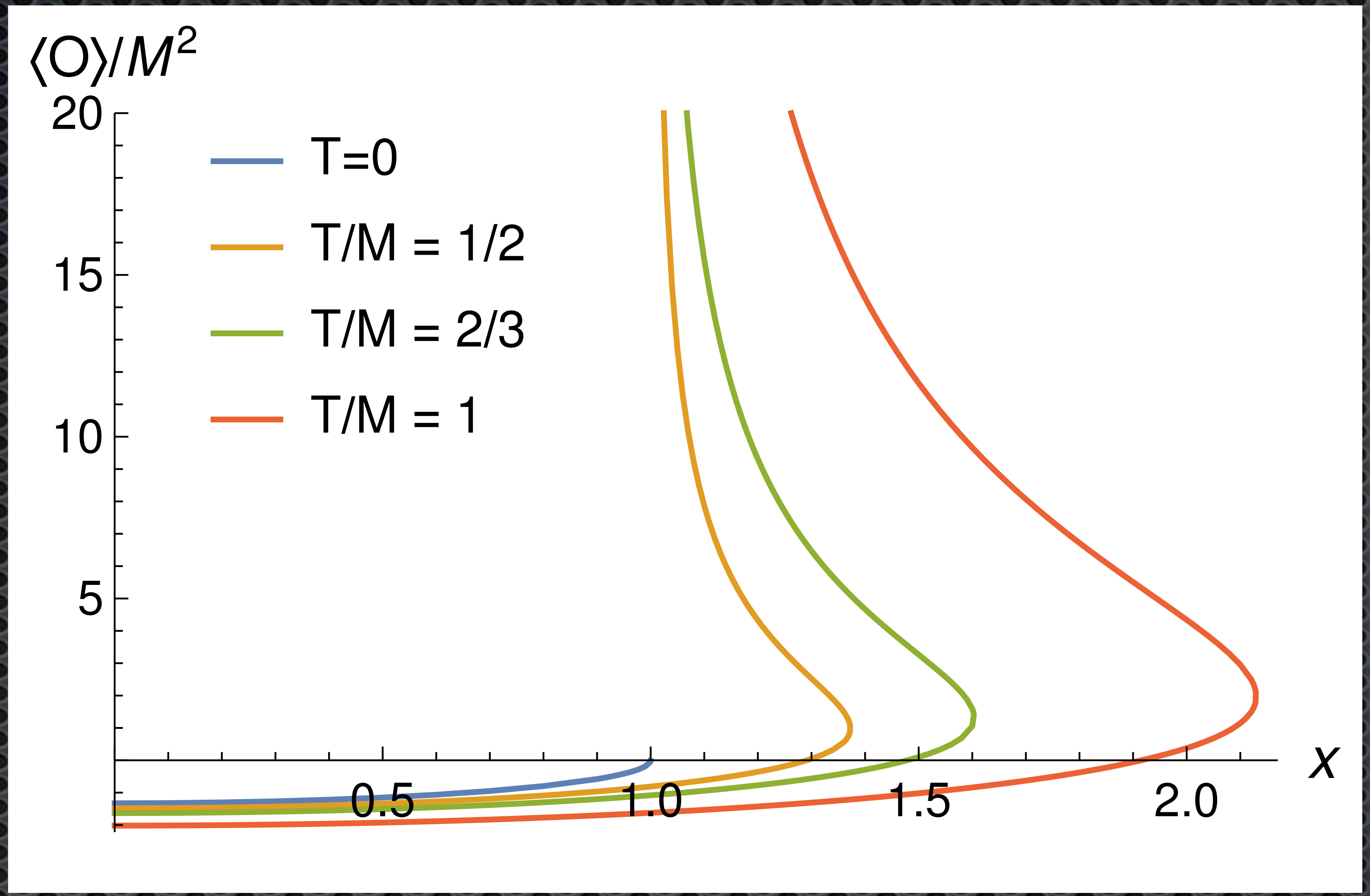
PT-broken phase: $\xi > 1 \;\Rightarrow$ complex backgrounds

Non-Hermitian Holography [1912.06647]

Finite Temperature

$$\Psi \sim s\,(1-\xi)$$

$$\bar{\Psi} \sim s\,(1+\xi)$$



Real geometries up to $x_c > 1$

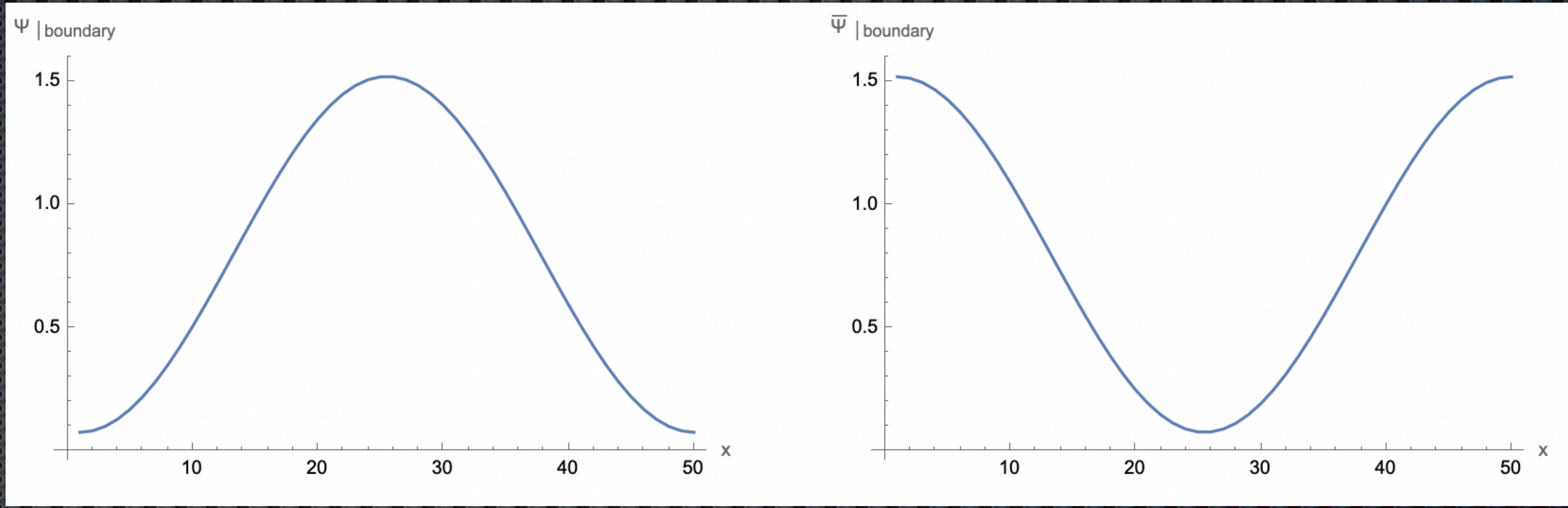
(unstable solutions)

Non-Hermitian inhomogeneous holography

Space dependent non-Hermitian deformation [Chernodub&Millington'97]

$$\Psi \sim s\,(1 - \xi(x))$$

$$\bar{\Psi} \sim s\,(1 + \xi(x))$$

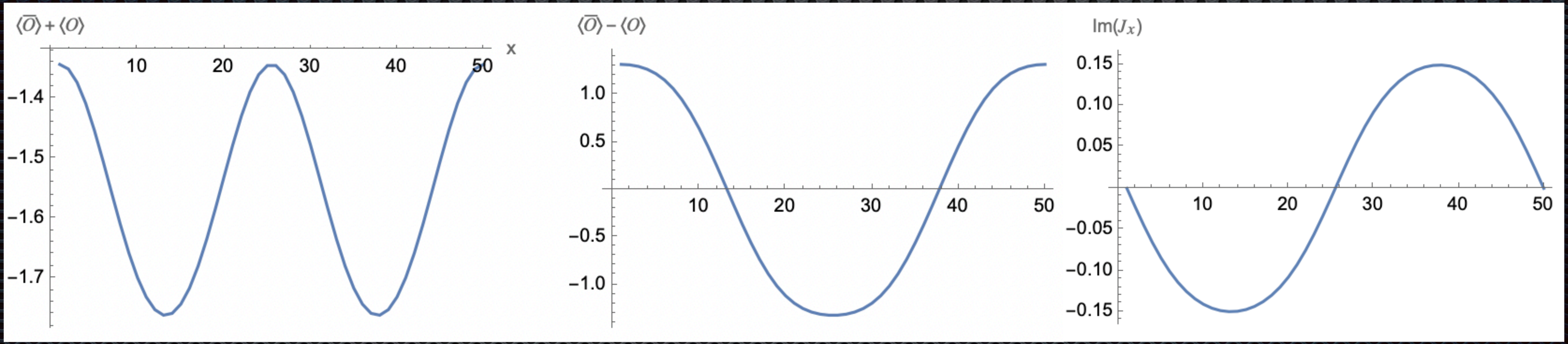


in asymptotically AdS black brane geometry $T/s \approx 0.30$

See Landsteiner&Morales-Tejera'22 for non-Hermitian quenches

Non-Hermitian inhomogeneous holography

$$\Psi \sim s\,(1-\xi(x))\,,\ \ \bar{\Psi} \sim s\,(1+\xi(x)) \quad \Longrightarrow \quad \langle \mathrm{Im}\,(J_x)\rangle \neq 0$$



$T/s \approx 0.30$

Real geometries support PT-broken solutions

Non-Hermitian inhomogeneous holography

PT-symmetric (pseudo-Hermitian) phase: gauging the Dyson map

PT-symmetric

$$\Psi \sim s\,(1 - \xi(x))$$

$$\bar{\Psi} \sim s\,(1 + \xi(x))$$

$$\sim$$

$$\Psi \sim s\sqrt{1 - \xi(x)^2}$$

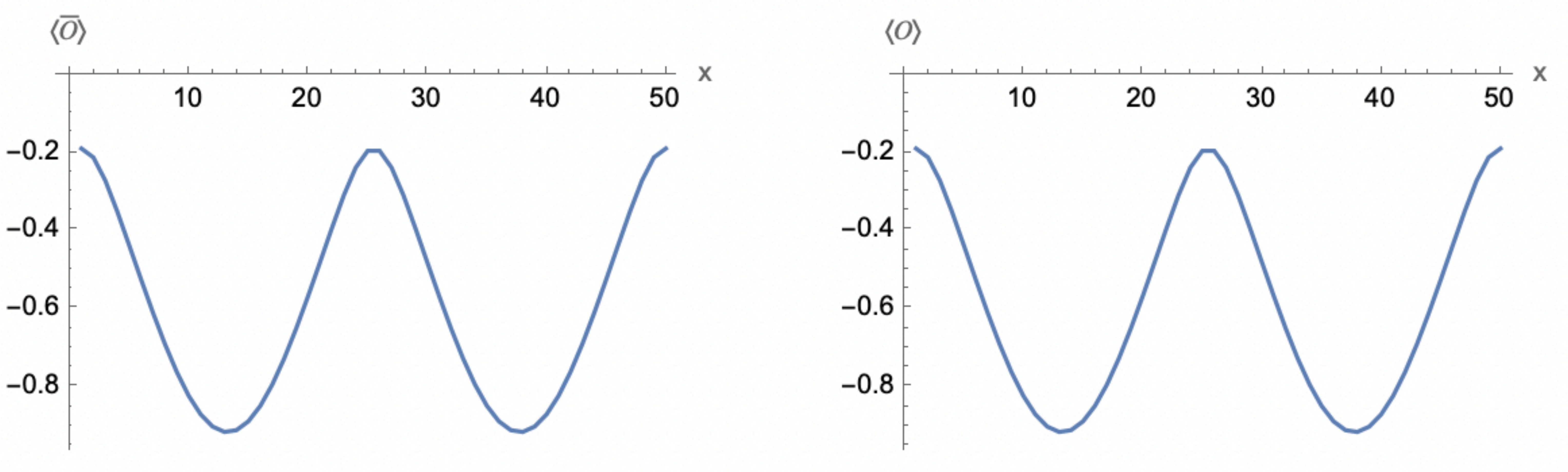
$$\bar{\Psi} \sim s\sqrt{1 - \xi(x)^2}$$

$$A_x = -\frac{i}{2}\partial_x \log\left(\frac{1 - \xi(x)}{1 + \xi(x)}\right)$$

Non-Hermitian inhomogeneous holography

PT-symmetric (pseudo-Hermitian) phase: gauging the Dyson map

$$\Psi \sim s\,(1 - \xi(x))\,,\;\; \bar{\Psi} \sim s\,(1 + \xi(x))\,,\;\; A_x = -\frac{i}{2}\partial_x \log\left(\frac{1-\xi(x)}{1+\xi(x)}\right)$$

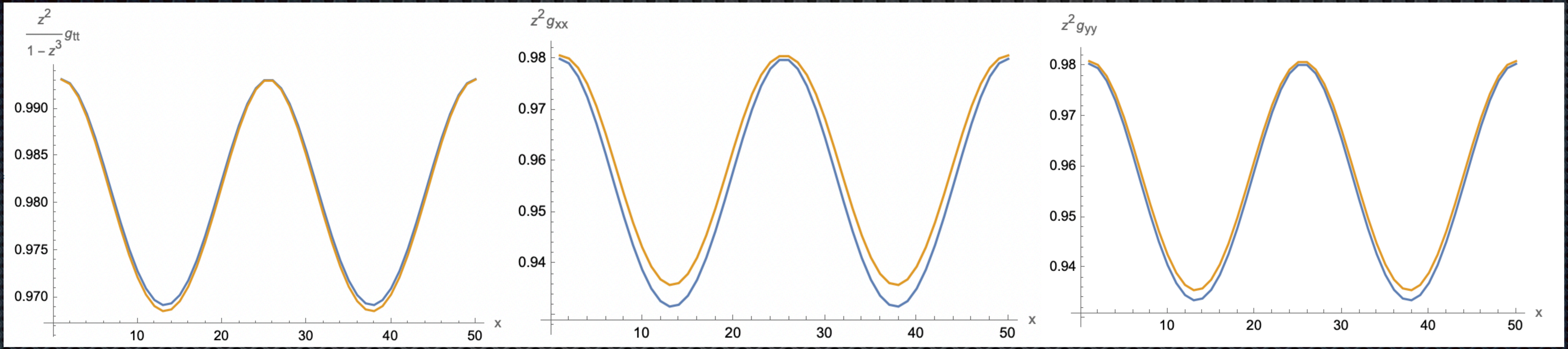


$\langle J_x \rangle = 0$

$T/s \approx 0.30$

Non-Hermitian inhomogeneous holography

PT-broken $\langle \mathrm{Im}(J_x) \rangle \neq 0$ vs PT-symmetric geometry



$T/s \approx 0.30$

Non-hermitian holography

Overview & To do

- (Minimal) Holographic model of nH PT-symmetric theories
- Exhibits PT-symmetric and PT-broken phases
- Explore parameter space: T/s, amplitude of modulation ($\exists\ \xi_{\mathrm{max}}$?)…
- Study stability of solutions (correlators?)
- H-nH-H junction
- Add charge, look for other phases
- Symmetry breaking