Anomaly-induced transport regime in Weyl semimetals

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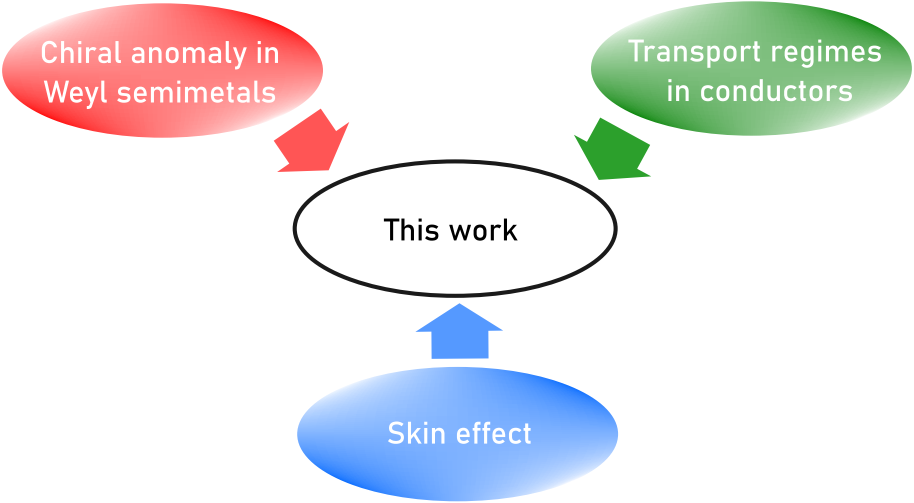

Chiral anomaly in Weyl semimetals
Transport regimes in conductors
This work
Skin effect

Chiral anomaly in Weyl semimetals

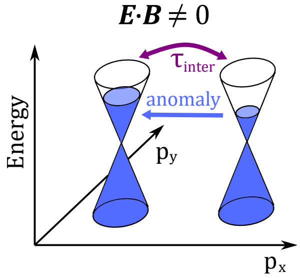


Figure: Low-energy spectrum under the effect of the anomaly.

Low-energy Hamiltonian in the vicinity of a node characterised by chirality $s = \pm 1$:

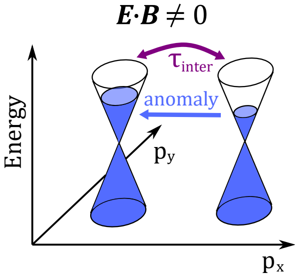
$$H = sv\sigma \cdot \mathbf{p}.$$

Chiral anomaly induces current

$$\mathbf{J}_{\mathrm{CME}} \propto \tau_{\mathrm{inter}} \left(\mathbf{E} \cdot \mathbf{B}\right) \mathbf{B},$$

where τ_{inter} is the relaxation time for the axial charge $\rho_5 = \rho_{\mathrm{L}} - \rho_{\mathrm{R}}$.

Chiral anomaly in Weyl semimetals



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$\Rightarrow$ **Negative longitudinal magnetoresistance**: standard diagnostic of chiral anomaly.

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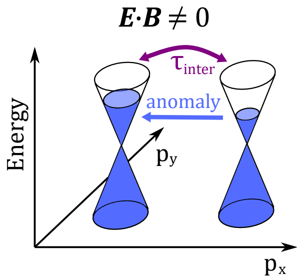

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What about transport at finite frequency? Does the presence of the chiral anomaly lead to new signature effects?

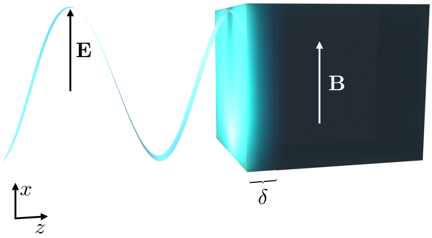
Skin effect

From Maxwell's equations:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\partial_t^2 \mathbf{E} = \mu \partial_t \mathbf{J}.$$

The relation between $\mathbf{J}$ and $\mathbf{E}$ is in general nonlocal. In the Fourier space

$$\mathbf{J}(q,\omega) = \sigma_{||}(q,\omega)\mathbf{E}(q,\omega).$$



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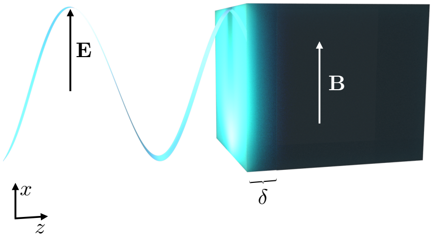

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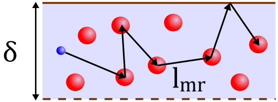
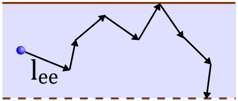
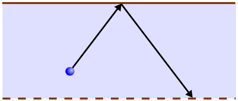
Skin depth $\delta(\omega)$ plays the role of a frequency-dependent system size!

Transport regimes: regular conductors

Normally, conductance is proportional to the effective relaxation time for momentum τ_{eff}. What is τ_{eff}?

Diffusive	$l_{\text{mr}} \ll \delta$ $\tau_{\text{eff}} \sim \tau_{\text{mr}}$	
Hydrodynamic	$l_{\text{ee}} \ll \delta \ll l_{\text{mr}}$ $\tau_{\text{eff}} \sim \delta^2/\tau_{\text{ee}} v^2$	
Ballistic	$\delta \ll l_{\text{ee}}, l_{\text{mr}}$ $\tau_{\text{eff}} \sim \delta/v$	

To relate to conductivity: $\sigma(q,\omega) \propto \tau_{\text{eff}}$ with $\delta \to q^{-1}$.

Calulations for a Weyl semimetal

Boltzmann equations for the distribution functions $f^{(s)}$, where $s = \pm 1$ denotes chirality:

$$\partial_t f^{(s)} + \dot{\mathbf{x}} \cdot \partial_{\mathbf{x}} f^{(s)} + \dot{\mathbf{p}} \cdot \partial_{\mathbf{p}} f^{(s)} = C^{(s)} \left[f^{(s)}, f^{(-s)} \right].$$

Equations of motion modified by the presence of the Berry curvature $\mathbf{\Omega}^{(s)}$:

$$\dot{\mathbf{x}} = \mathbf{v} + \dot{\mathbf{p}} \times \mathbf{\Omega}^{(s)}, \qquad \dot{\mathbf{p}} = e\mathbf{E} + e\dot{\mathbf{x}} \times \mathbf{B}.$$

We want the collision term $C^{(s)} \left[f^{(s)}, f^{(-s)} \right]$ to take into account three scattering mechanisms:

1. **Internode scattering** with time τ_{inter},
2. **Momentum-relaxing scattering** with time τ_{mr},
3. **Electron-electron scattering** with time τ_{ee}.

Methods. Collision integral

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  1. **Internode scattering** with time τ_{inter},
  2. **Momentum-relaxing scattering** with time τ_{mr},
  3. **Electron-electron scattering** with time τ_{ee}.
- The distribution function $\delta f^{(s)} \equiv f^{(s)} - f_0$ can be expanded in a suitable basis $|K_l^{m(s)}\rangle$ corresponding to different modes. For example, in an isotropic system, these are spherical harmonics $Y_l^m(\mathbf{p})$.
- We then define the collision operator

$$\hat{C}^{(s)} = -\frac{1}{\tau_{\text{inter}}} P_0^{(s)} - \frac{1}{\tau_{\text{mr}}} P_1^{(s)} - \left(\frac{1}{\tau_{\text{mr}}} + \frac{1}{\tau_{\text{ee}}}\right) P_{\text{higher}}^{(s)},$$

where

$$P_0^{(s)} = |K_0^{0(s)}\rangle\langle K_0^{0(s)}| - |K_0^{0(s)}\rangle\langle K_0^{0(-s)}|,$$
$$P_1^{(s)} = \sum_{M=-1,0,1} |K_1^{M(s)}\rangle\langle K_1^{M(s)}|, \qquad P_{\text{higher}}^{(s)} = 1 - |K_0^{0(s)}\rangle\langle K_0^{0(s)}| - P_1^{(s)}.$$

Results: anomaly-induced nonlocal regime

In addition to the aforementioned regimes, the presence of the new timescale $\tau_{\rm inter}$ allows us to distinguish a new regime. When $\tau_{\rm mr}^2 \ll (qv)^{-2} \ll \tau_{\rm mr}\tau_{\rm inter}$:

$$\sigma(q) \propto \tau_{\rm mr} + 9\alpha^2 \frac{(qv)^{-2}}{\tau_{\rm mr}},$$

where q is the wavevector and $\alpha = e\hbar|\mathbf{B}|v^2/2\mu^2$ is a dimensionless parameter proportional to the magnetic field.

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$\Rightarrow$ **In the new regime, a local classical conductivity and a nonlocal anomalous conductivity in the presence of the magnetic field.**

Anomaly-induced nonlocal regime: mechanism

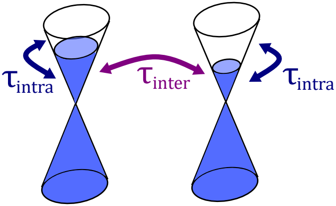

Figure: Impurity scattering between different nodes on the timescale $\tau_{\rm inter}$.

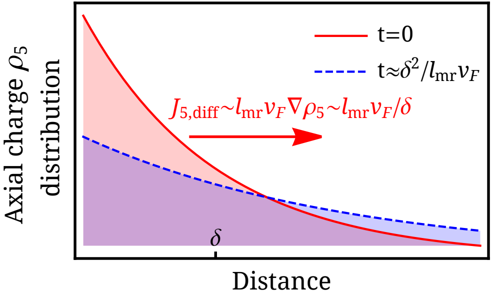

Figure: For a nonhomogeneous distribution, diffusion of the axial charge on the timescale $\delta^2/l_{\rm mr}v_{\rm F} \approx \delta^2/\tau_{\rm mr}v_{\rm F}^2$.

If $\delta \ll \sqrt{\tau_{\rm inter}\tau_{\rm mr}}v_{\rm F}$, the second relaxation mechanism dominates.

Electric field in the *AIN* regime[1]

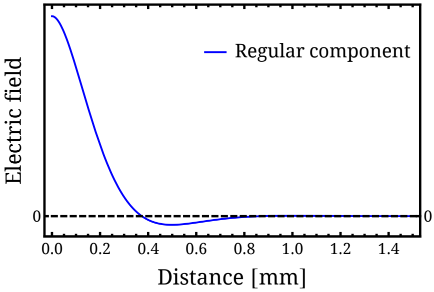

Figure: Magnetic field off.

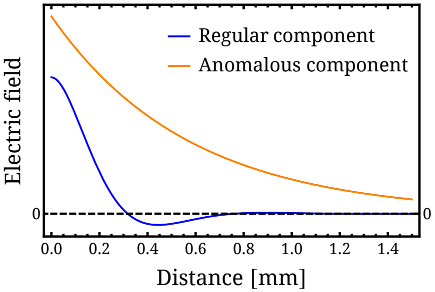

Figure: Magnetic field on.

Parameters: $\tau_{\text{inter}} = 10^{-7}$s, $\tau_{\text{intra}} = 10^{-9}$s, $v = 1.4 \times 10^{7}$ cm/s, $\omega = 10^{6}$ Hz, plasma frequency $\omega_P = 10^{13}$ Hz.

[1] See also: P. O. Sukhachov and L. I. Glazman, Phys. Rev. Lett. **128** (2022)

Surface impedance in the *AIN* regime

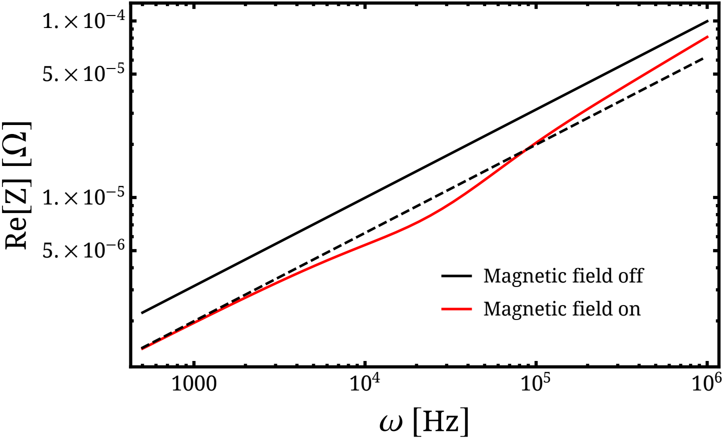

Figure: Scaling of the real part of the surface impedance $Z \equiv \mathbf{E}(z=0)/\int_0^\infty dz \mathbf{J}(z)$ with the driving frequency.

Summary

- Skin effect can be used to probe different transport regimes in conductors.
- The presence of the chiral anomaly in Weyl semimetals leads to a new nonlocal transport regime, whose presence could be detected experimentally.
- We constructed a collision integral that takes into account 3 distinct collision processes using an original procedure.

The End