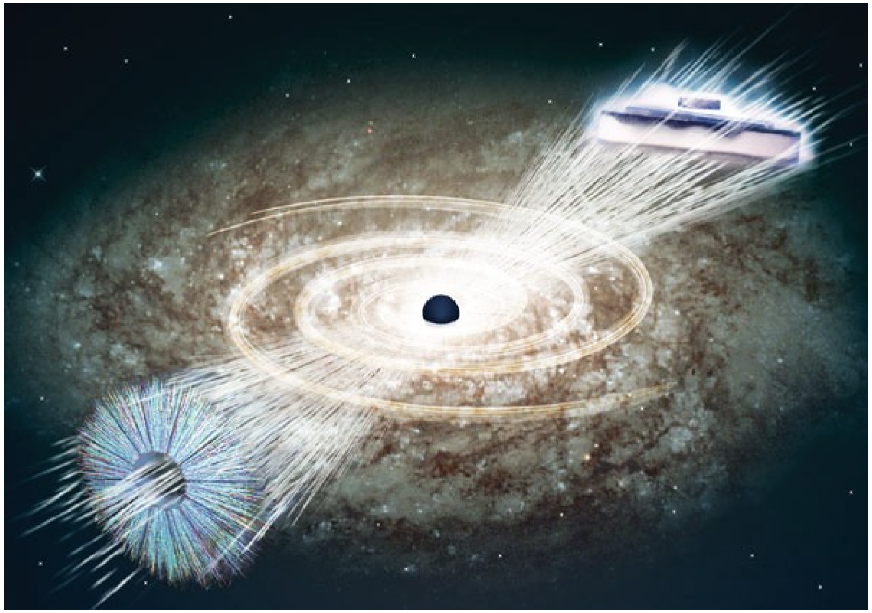
Pseudo-spontaneous symmetry breaking in hydrodynamics and holography



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March 16th 2023

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Based on work with

Daniel Areán, Matteo Baggioli, Seán Gray and Sebastian Grieninger

Pseudo-spontaneous U(1) Symmetry Breaking
in Hydrodynamics and Holography

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What is hydrodynamics?

- EFT of long-lived, long-wavelength excitations
- Equations of motion are conservation laws

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

- Recent interest in weakly broken symmetries:

– pseudo-spontaneous breaking of spatial translations

see e.g. [Amoretti, Areán, Goutéraux, Musso, '16], [Ammon, Baggioli, Jiménez-Alba, '19], [Baggioli and Grieninger, '19], [Donos, Martin, Pantelidou, Ziogas '19] ….

potentially interesting for physics of strange metals and their charge-density fluctuations

[Delacretaz, Gouteraux, Hartnoll, Karlsson, '16], [Amoretti, Arean, Gouteraux, Musso, '18]

– pseudo-spontaneous breaking of U(1) superfluid *In this talk!*

[MA, Areán, Baggioli, Gray, Grieninger, '21], see also [Donos, Kailidis, Pantelidou, '21] [Delacretaz, Gouteraux, Ziogas, '21], [Armas, Jain, Lier, '21]

Why is holography useful?

- Holography geometrises the hydrodynamic (derivative) expansion

  Fluid/Gravity correspondence

- Thermodynamic and hydrodynamic (transport) coefficients can be determined

  Are those non-zero?

- Universal relations may be detected

  If you are lucky

  Find a field-theoretic argument ...

PART I

Hydrodynamics for U(1) pseudo-spontaneous symmetry breaking

Hydrodynamics for a U(1) superfluid

for simplicity: “probe limit”

- Goldstone boson φ with gauge invariant combination:

$$\xi_\mu \equiv \partial_\mu \varphi - A_\mu$$

- Equations: $\partial_\mu \langle J^\mu \rangle = 0$ and $u^\mu \xi_\mu = -\mu$

- Constitutive and Josephson relation to first order in derivatives:

$$\langle J^\mu \rangle = \rho_t u^\mu + \rho_s n^\mu - \sigma_0 T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) + \mathcal{O}(\partial^2)$$

$$u^\mu \partial_\mu \left(\Delta^\rho_\nu \xi_\rho \right) = \Delta^\rho_\nu \left[-\partial_\rho \mu + \zeta_3 \, \partial_\rho \partial_\mu \rho_s n^\mu \right] + \mathcal{O}(\partial^2)$$

with superfluid bulk viscosity $\zeta_3 = \lim_{\omega \to 0} \omega \, \mathrm{Im} G^{\mathrm{R}}_{\varphi\varphi}(\omega, 0)$

and superfluid velocity defined by $\mu \, n^\mu \equiv \Delta^{\mu\nu} \xi_\nu$ $\Delta_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}$

Hydrodynamics for a U(1) superfluid (II)

- Dispersion relation (of fourth sound)

$$\omega(k) = \pm v_s\, k - \frac{i}{2}\, \Gamma_s\, k^2 + \ldots$$

where

$$v_s^2 = \frac{\rho_s}{\mu(\partial\rho_t/\partial\mu)} \qquad\qquad \Gamma_s = \frac{\sigma_0}{(\partial\rho_t/\partial\mu)} + \zeta_3\,\frac{\rho_s}{\mu}$$

Pseudo-spontaneous hydrodynamics

for simplicity: "probe limit"

- Goldstone boson acquires a mass, shift symmetry is broken

- Conservation law: $\partial_\mu \langle J^\mu \rangle = \Gamma\, u_\mu \langle J^\mu \rangle + m\, \varphi$

  with charge relaxation rate Γ and m related to mass of pseudo-Goldstone boson

  Assumption: Γ and m are tiny

- Modified Josephson relation to first order in derivatives:

$$(u^\mu \partial_\mu + \Omega)\ (\Delta_\nu^\rho \xi_\rho) = \Delta_\nu^\rho \left[-\partial_\rho \mu + \zeta_3\, \partial_\rho \partial_\mu \rho_s n^\mu\right] + \mathcal{O}(\partial^2)$$

  with phase relaxation rate Ω

Pseudo-spontaneous hydrodynamics (II)

for simplicity: "probe limit"

- Dispersion relation $\omega(k) = \alpha_\pm - iD_\pm k^2 + \ldots$

where $\alpha_\pm = -\frac{i}{2}(\Gamma + \Omega) \pm \sqrt{\omega_0^2 - \frac{(\Gamma - \Omega)^2}{4}}$ $\qquad \omega_0^2 \equiv \frac{m}{(\partial \rho_t / \partial \mu)}$

$$D_\pm = \frac{1}{2}\frac{\sigma_0}{(\partial \rho_t/\partial \mu)} + \zeta_3 \frac{\rho_s}{\mu} \pm \frac{i}{2}\frac{\zeta_3 \rho_s (\partial \rho_t/\partial \mu)(\Gamma - \Omega) + 2\rho_s - \sigma_0 \mu (\Gamma - \Omega)}{\mu\sqrt{4m(\partial \rho_t/\partial \mu) - (\partial \rho_t/\partial \mu)^2 (\Gamma - \Omega)^2}}$$

- 'sum relation' $D_+ + D_- = \Gamma_s$

also holds when taking temperature and fluid velocity fluctuations into account.

Pseudo-spontaneous hydrodynamics (III)

for simplicity: "probe limit", $\Gamma = 0$

- Static Correlators:

$$G_{\varphi\varphi}(0,k) = \frac{\chi_{\xi\xi}}{k^2 + \mathfrak{m}^2} \qquad \mathfrak{m}^2 = \chi_{\xi\xi}\, m\,, \qquad \chi_{\xi\xi} = \frac{\mu}{\rho_s}$$

'Gell-Mann-Oakes-Renner' relation

- Kubo-Formulae for Ω and ζ_3

$$\Omega = \lim_{\omega\to 0} \frac{m}{\omega \chi_{\rho\rho}^2} \operatorname{Im} G^{\mathrm{R}}_{J^t J^t}(\omega, 0)$$

$$\lim_{\omega\to 0} \frac{1}{\omega} \lim_{k\to 0} \frac{\partial^2}{\partial k^2} \operatorname{Im} G^{\mathrm{R}}_{J^t J^t}(\omega, k) = -2 \frac{\chi_{\rho\rho}^2}{m^2 \mu} \left(\zeta_3 m \rho_s - \rho_s \Omega - \mu \sigma_0 \Omega^2\right)$$

- AC-conductivity

$$\sigma(\omega) = \lim_{k\to 0} \frac{i}{\omega} \left[G^{\mathrm{R}}_{J^x J^x}(\omega, k) - G^{\mathrm{R}}_{J^x J^x}(0, k)\right] = \sigma_0 .$$

PART II

Holographic models for pseudo-spontaneous symmetry breaking

Holographic model: s-wave superfluid

[Hartnoll, Herzog, Horowitz, '08]

- Action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\psi|^2 - M^2 |\psi|^2 \right]$$

with background metric

$$\mathrm{d}s^2 = \frac{1}{z^2} \left[-u(z)\,\mathrm{d}t^2 - 2\,\mathrm{d}t\,\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2 \right] \qquad u(z) = 1 - z^3$$

and $M^2 = -2$. Hence

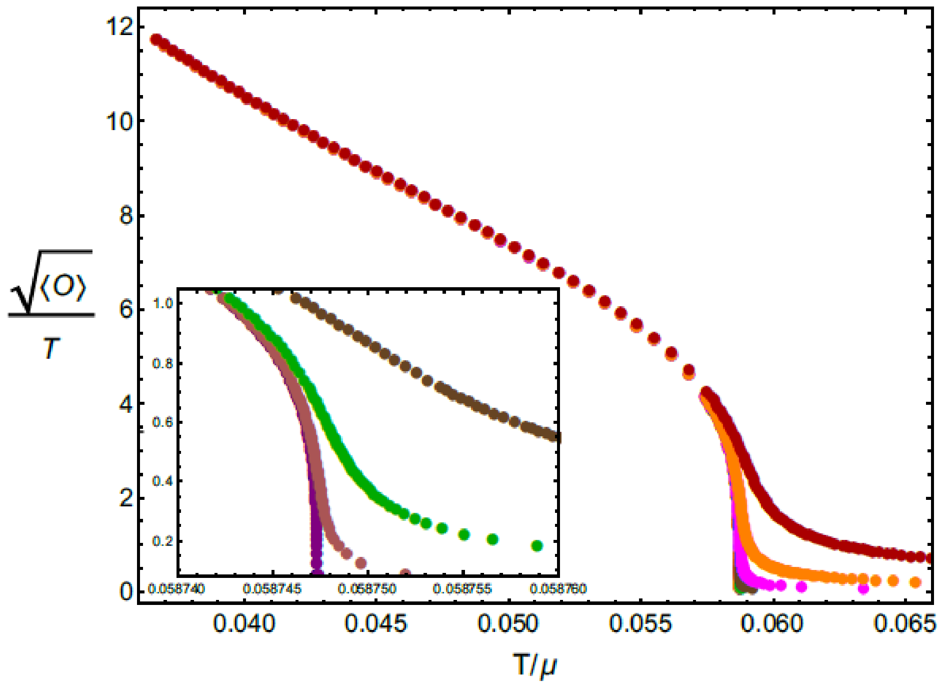
$$\psi(z) = \psi^{(l)}\, z + \psi^{(s)}\, z^2 + \mathcal{O}(z^3)$$

- **Model I:** tiny source for the scalar field $\psi^{(l)} = \lambda$

Identification of parameter: $m = q\,\lambda\,\langle \mathcal{O} \rangle_{\mathrm{eq}}$ $\qquad \Gamma = 0$

Holographic model: s-wave superfluid

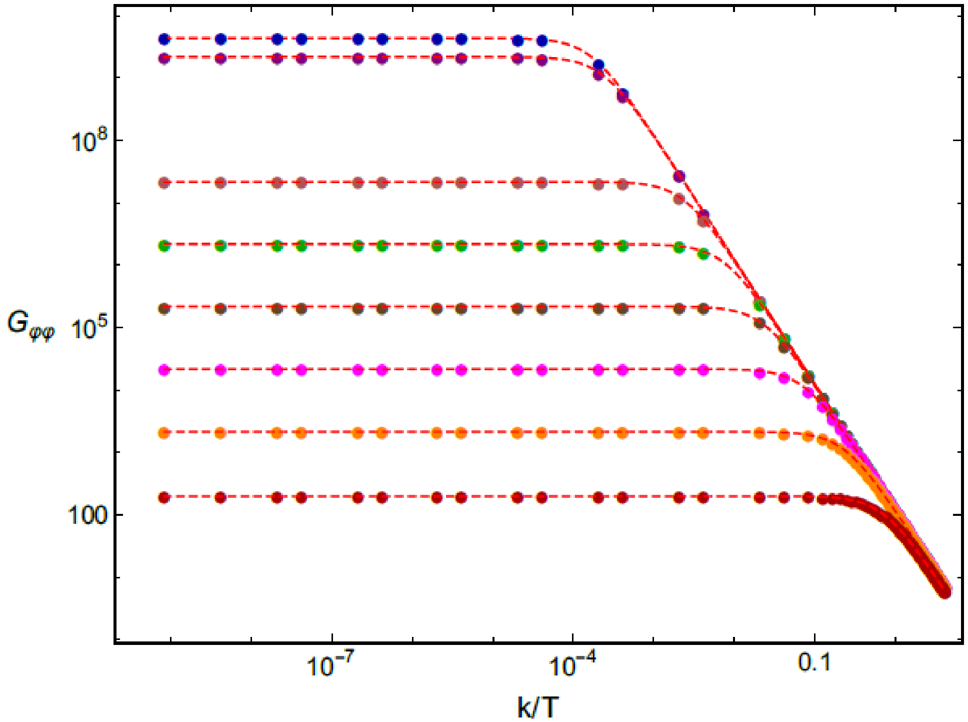
- Results:



Values for λ/T: blue $5 \cdot 10^{-10}$, purple 10^{-9}, light brown 10^{-7}, green 10^{-6}, dark brown 10^{-5} magenta 10^{-4}, orange 10^{-3}, red 10^{-2}.

Holographic model: s-wave superfluid

- Results:



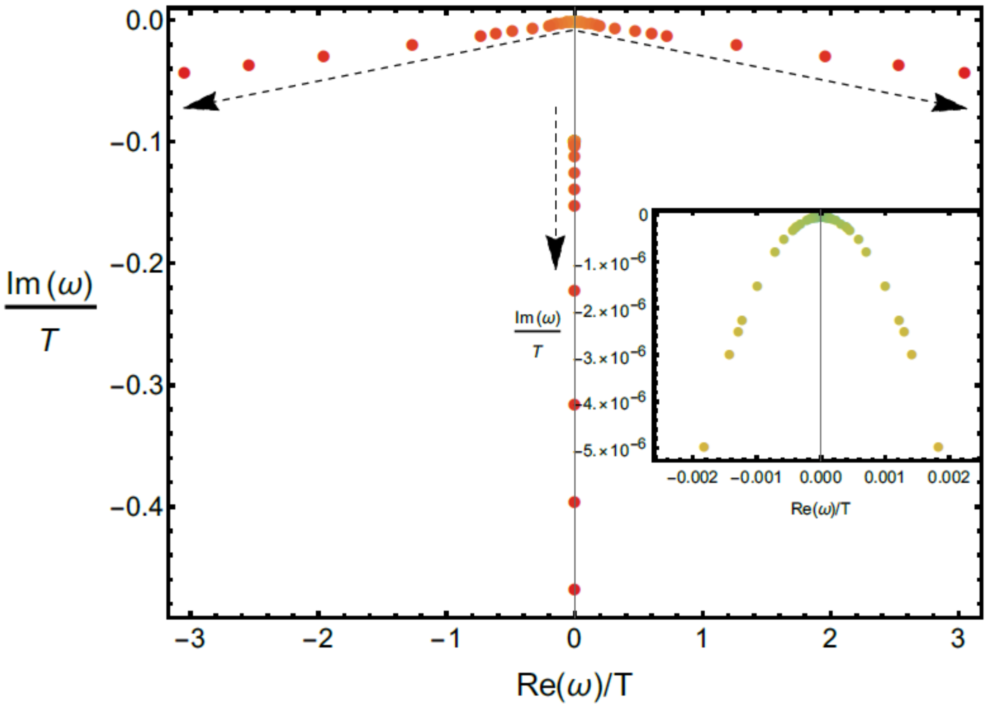
$T/\mu = 0.0575$

Values for λ/T: blue $5 \cdot 10^{-10}$, purple 10^{-9}, light brown 10^{-7}, green 10^{-6}, dark brown 10^{-5} magenta 10^{-4}, orange 10^{-3}, red 10^{-2}.

Holographic model: s-wave superfluid

- Results: Quasi-normal Modes at zero momentum

$$\omega(k=0) = -\frac{i}{2}(\Gamma+\Omega) \pm \sqrt{\frac{m}{(\partial\rho_t/\partial\mu)} - \frac{(\Gamma-\Omega)^2}{4}}$$



Values: $\lambda/T \in [10^{-16}, 0.1]$

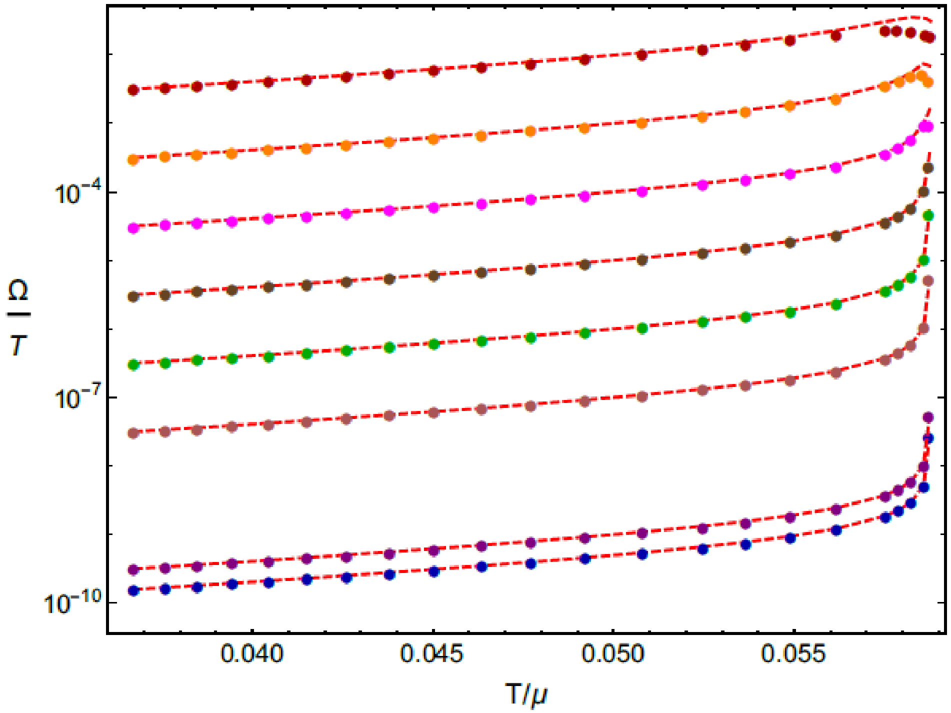
$T/\mu = 0.0582$

Conclusion:

$\Omega \neq 0$ but $\Gamma = 0$

Holographic model: s-wave superfluid

- Results:



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Holographic model: s-wave superfluid

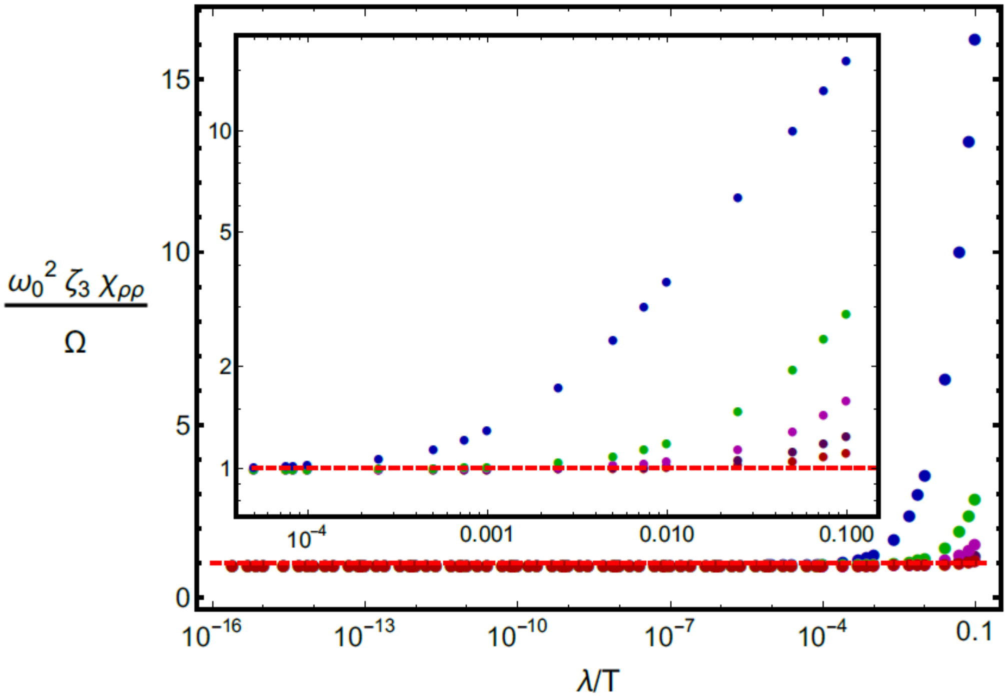
- Results: for small λ/T

$$\Omega = m\,\zeta_3$$

Universal relationship?

see also

[Delacretaz, Gouteraux, Ziogas, '21],
[Armas, Jain, Lier, '21]
[Grossi, Soloviev, Teaney, Yan, '20]



- Electrical conductivity:

finite DC conductivity despite no Drude-like behaviour

Holographic model: s-wave superfluid

- Interpretation of results:

> pseudo-Goldstone bosons
> *as well as*
> (induced) effective phase relaxation

- We considered a *second model* based on massive gauge fields
  - – induces effectively charge relaxation
  - – breaks U(1) symmetry *but not* shift symmetry

Summary & Outlook

- ‘Hydrodynamics’ for pseudo-spontaneous breaking of U(1) symmetry

  → ‘sum relation’

- Two holographic models with distinct symmetry breaking behaviour

  →’universal relation’

- Outlook: how general is the ‘universal relation’?

  – type II (pseudo-)Goldstone modes?

  – light dilatons arising from the pseudo-spontaneous breaking of scale invariance

Thank you very much for your attention!

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