Approach to Criticality in Holographic Plasmas

Matti Järvinen

asia pacific center for theoretical physics

POSTECH
POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Holographic Perpectives on Chiral Tranport
ECT* – Trento – 16 March 2023

[Umut Gürsoy, MJ, Giuseppe Policastro, Natale Zinnato
arXiv:2112.04296 (JHEP)]
[Panagiotis Betzios, Umut Gürsoy, MJ, Giuseppe Policastro
arXiv:1807.01718; arXiv:1708.02252 (PRD)]

NRF
National Research Foundation of Korea

Motivation

Critical phenomena are ubiquituos in nature. Important cases:

- Behavior near the (conjectured) critical point of the (μ, T) phase diagram in QCD
- Criticality in the Ising model
- Quantum criticality in connection to high temperature superconductors

Criticality in the gauge/gravity duality – various approaches ...

- I will study a simple example of criticality (nonconformal), where many things can be solved analytically
- This example has connections to QCD and to spin models

Critical solutions in Einstein-dilaton gravity

$$S \propto \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3} (\partial\phi)^2 + V(\phi) \right]$$

Study a class of potentials with $V(\phi) \sim e^{\alpha\phi}$ as $\phi \to \infty$ (which will be the IR limit)

- A critical value $\alpha_c = 4/3$ arises
- For $\alpha > \alpha_c$ ($\alpha < \alpha_c$) confinement (deconfinement)
  [Gürsoy, Kiritsis 0707.1324; Gürsoy, Kiritsis, Nitti 0707.1349]
- Dynamics similar to the Yang-Mills theory is obtained for $V(\phi) \sim e^{\alpha_c\phi}\sqrt{\phi}$ (Improved holographic QCD)
- At $\alpha = \alpha_c$ higher order phase transitions also possible
  [Gürsoy 1007.0500]
- Connection to spin models at criticality (the XY model)
  [Gürsoy 1007.4854]

Critical solutions in Einstein-dilaton gravity

$$S \propto \int d^5x \sqrt{-\det g}\left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right]$$

Simplest case: take exactly exponential $V(\phi) = e^{\alpha\phi}$

- Geometry can be solved exactly (also at finite T, i.e., black holes)
- Most of the fluctuations can be solved in the critical limit $\alpha \to \alpha_c$

[Betzios, Gürsoy, MJ, Policastro 1807.01718; 1708.02252]

Remarks to which I will return later:

- The results can be generalized to RG flows (e.g. from AdS_5 in the UV) ending to this geometry in the IR quite easily
- The same story works for charged backgrounds (Einstein-Maxwell-dilaton)

Connection to large d

$$S \propto \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3} (\partial\phi)^2 + e^{\alpha\phi} \right]$$

A simple way to understand the analytic solutions: generalized dimensional reduction [Goutéraux, Smolic, Smolic, Skenderis, Taylor 1110.2320]

- For generic α (with $\alpha < \alpha_c$) the geometry is a dimensional reduction of a $d+1$ -dimensional AdS black hole
- Explicit relation:

$$d = \frac{4 - \alpha^2/\alpha_c^2}{1 - \alpha^2/\alpha_c^2}$$

- The critical limit $\alpha \to \alpha_c$ maps to the large d limit $d \to \infty$
- For $\alpha = \alpha_c$ the solution is the linear dilaton background – the case $d = \infty$
- As $d \to \infty$, the horizon regime of the black hole becomes thin, width $\sim 1/d$ – membrane picture
[E.g. Emparan, Suzuki, Tanabe 1302.6382]
- Fluctuations can be solved by combining the (analytic) solutions near the horizon and elsewhere

Solutions

In the charged case

[Gürsoy, MJ, Policastro, Zinnato 2112.04296]

$$S \propto \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3} (\partial\phi)^2 + V_0 e^{\alpha\phi} + \frac{1}{4} e^{-\alpha\phi} F^2 \right]$$

The geometry is the dimensional reduction of a $d+1$ -dimensional Reissner-Nordström black hole

$$ds^2 = r^{-\frac{2}{3}(d-1)} \left[f(r)^{-1} dr^2 - f(r) dt^2 + \delta_{ij} dx^i dx^j \right]$$

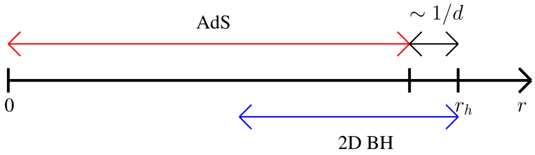
$$f(r) = 1 - \left(\frac{r}{r_h}\right)^d + \frac{dQ^2}{d-2} \left[\left(\frac{r}{r_h}\right)^{2d-2} - \left(\frac{r}{r_h}\right)^d \right]$$

$$\phi = \frac{1}{2} \frac{\alpha}{\alpha_c} (d-1) \log r$$

Near the horizon, this reduces to the well studied charged 2D linear dilaton black hole (times $\mathbb{R}^3$)

[Mandal, Sengupta, Wadia; Elitzur, Forge, Rabinovici; Witten; ...]

Solving the fluctuations



1. Far away from the horizon: empty AdS modes – Bessel functions
2. Near the horizon: known results for the 2D linear dilaton black hole [Elitzur, Giveon, Kutasov, Rabinovici hep-th/0204189]
   - Fluctuations given in terms of hypergeometric functions
   - Result characterized in terms of the reflection amplitude for ingoing waves

$$\mathcal{R} = -\left(1-Q^2\right)^{-i\widetilde{S}} \frac{\Gamma\left(1+i\widetilde{S}\right)\Gamma\left(\frac{1}{2}\left(1-i\varpi-i\widetilde{S}\right)\right)\Gamma\left(\frac{1}{2}\left(1-\frac{1+Q^2}{1-Q^2}i\varpi-i\widetilde{S}\right)\right)}{\Gamma\left(1-i\widetilde{S}\right)\Gamma\left(\frac{1}{2}\left(1-i\varpi+i\widetilde{S}\right)\right)\Gamma\left(\frac{1}{2}\left(1-\frac{1+Q^2}{1-Q^2}i\varpi+i\widetilde{S}\right)\right)}$$

where $\widetilde{S} = \sqrt{\varpi^2 - q^2 - 1}\,, \quad \varpi = \omega/(2\pi T)\,, \quad q = k/(2\pi T)$

At large d, the two descriptions overlap!

Solving the fluctuations

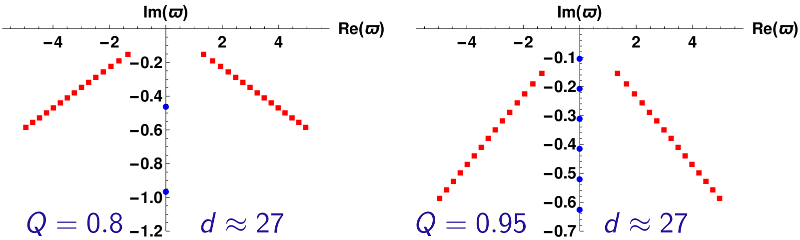
Putting together the results we obtain, e.g.

$$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0)\rangle =$$

$$\frac{2\pi\, d^d r_h^{-d}}{\Gamma\left(\frac{d}{2}\right)\Gamma\left(1+\frac{d}{2}\right)} \left(\frac{(\varpi^2 - q^2)}{16}\right)^{\frac{d}{2}} \left[i + \left(\frac{1+i\widetilde{S}}{1-i\widetilde{S}}\right)^{\frac{d}{2}} \frac{e^{-id\widetilde{S}}}{\mathcal{R}} \right]^{-1}$$

- Valid up to corrections suppressed by $1/d$
- Depends on T only through r_h^{-d} and the rescaling of ω and k
- Charge dependence only in the reflection amplitude
- Captures all nonhydrodynamic quasi normal modes (QNMs) of the gravity sector
- In the presence of hydrodynamic modes and for current-current correlators $\mathcal{R}$ not known analytically
- Hydrodynamic modes can of course be analyzed at small ω and k using standard fluid/gravity techniques

Classes of QNMs



1. Hydrodynamic modes – not captured – localized near the horizon
2. Imaginary modes of the 2D black hole also localized near the horizon – only present at finite charge in the full result

(2*. Additional imaginary modes of the 2D black hole, always present, not physical, arise due to a technical issue at certain ω)

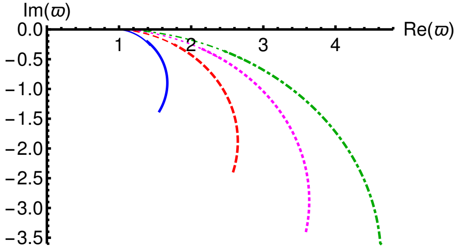
[Bertoldi, Hoyos 0903.3431]

3. Complex modes sensitive to both regions of the geometry

Approach to extremality/criticality

In the extremal limit $Q \to 1$, the imaginary modes become dense

- Breaking of hydrodynamics in this case has been studied in the literature [E.g., Arean, Davison, Goutéraux, Suzuki 2011.12301; see also talk of Jesús Cruz Rojas]

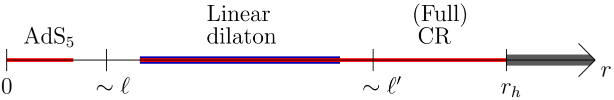


In the critical limit $\alpha \to \alpha_c$ or $d \to \infty$ the complex modes become dense and approach the real axis but remain gapped

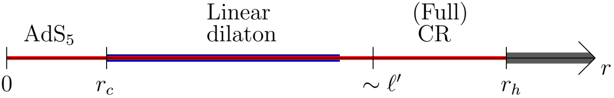
- Lifetimes of modes $\tau \sim d \Rightarrow$ slow thermalization!
- Complex modes slower than the hydro modes for $q \gtrsim 1/\sqrt{d}$
- Correlators singular as $d \to \infty$, branch cut formation unclear

Complete RG flows

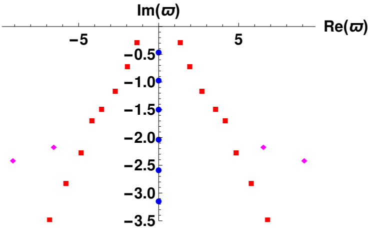
Consider RG flow from AdS_5 to the large-d "CR" geometry
Black hole solutions have the structure (with $\ell' \sim d\,\ell$)



Simplest approximation: glue the CR geometry directly to AdS_5 $\Rightarrow$



- Analytic correlators still found (but messy)
- Main results unchanged
- Nontrivial (mild) temperature dependence of the QNMs
- An additional "near boundary" set of QNMs appears
- Regularizes the large d limit of the correlators

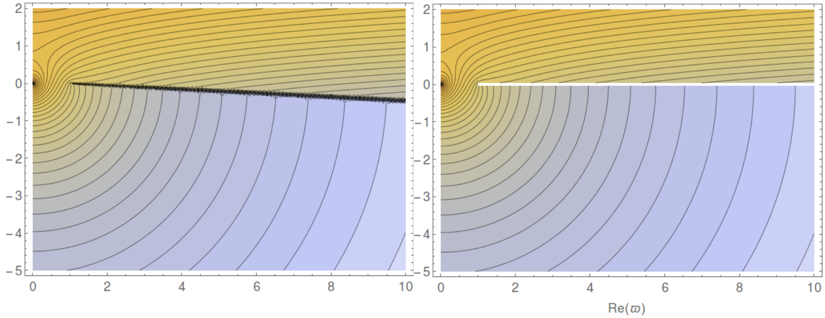


Correlators at $d = \infty$

$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle$ at

$r_h/r_c = 20$
Discrete modes

$r_h \to \infty$
Branch cut



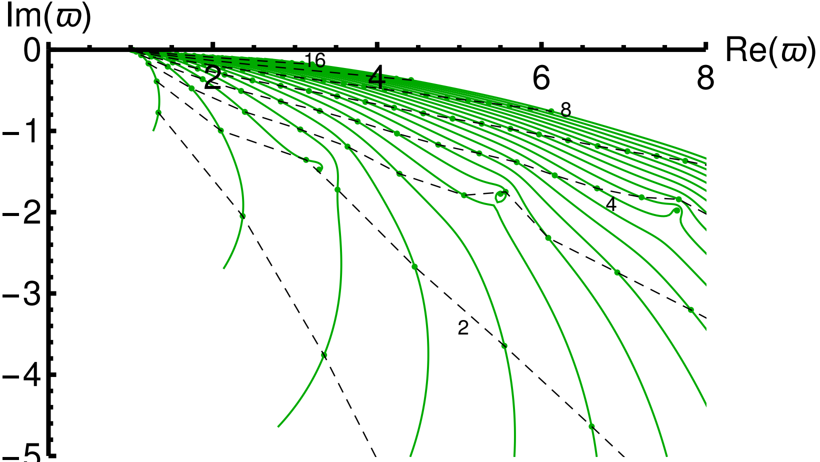
- The branch cut is replaced by a discrete set of modes, but is recovered in the limit of small black hole

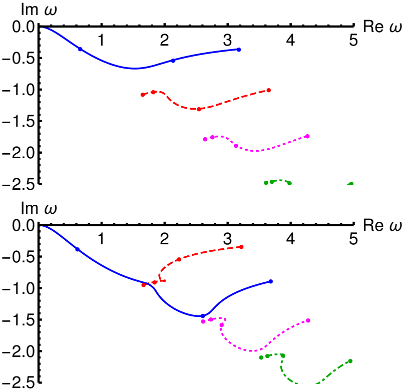
Summary

We studied the QNMs of a critical non-conformal plasma

- Analytic results for the nonhydrodynamic modes
- A setting where nonconformal behavior drastically affects the dynamics
- Infinitely many gapped long lived modes in the critical limit, forming a branch cut
- Time dependence described approximately in terms of hydrodynamics + branch cuts

Thank you!


Im(ϖ)
Re(ϖ)
0
−1
−2
−3
−4
−5
2
4
6
8
16
8
4
2


Im ω
Re ω
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
1
2
3
4
5

- Idea: explore effects due to nonconformality in quark-gluon plasma via holography
- Several studies using various approaches recently [Janik, Plewa, Soltanpanahi, Spalinski, Buchel, Heller, Myers, Ishii, Kiritsis, Rosen, Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Olivan, Sopuerta, Triana, Zilho, . . .]
- Our approach: A simple way to deviate from conformality – holographic dual of Einstein-Dilaton gravity with exponential potential (+modifications) [Chamblin, Reall]
  - Much of the analytic control of the AdS_5 solution remains in this case, but still nontrivial!

Thermalization of quark-gluon plasma

- AdS/CFT predicts that the thermalization time $\tau \sim 1/T$
- In QCD, an additional energy scale $\Lambda_{\rm QCD} \Rightarrow$ what happens to thermalization time?
- I will discuss examples where dynamics is drastically modified with respect to the conformal plasma

Connection to large D

- The same critical value as in improved holographic QCD (but approaching from the "wrong" side: no confinement)
- At critical value, higher order transition can be obtained [Gürsoy 1007.0500]
- At exactly critical value, the solution is the linear dilaton background
- Connection to spin models at criticality [Gürsoy 1007.4854]
- Corresponds to the $D \to \infty$ limit of dimensional reduction $\Rightarrow$ drastic simplifications expected

Instead of α we parametrize the solutions using

$$d = \frac{4 - \alpha^2/\alpha_c^2}{1 - \alpha^2/\alpha_c^2}$$

which diverges in the critical limit

Backgrounds at criticality

Take a (sufficiently regular) dilaton potential with

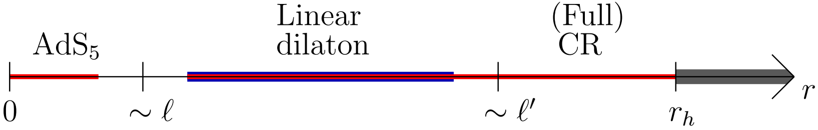
1. A minimum at some $\phi = \phi_0$
2. Asymptotics $V(\phi) \sim e^{\alpha\phi}$ with $0 < \alpha_c - \alpha \ll 1$

admitting a flow from $\phi = \phi_0$ (UV, AdS_5) to $\phi = \infty$ (IR, Chamblin-Reall background)

In conformal coordinates

$$ds^2 = e^{2A(r)} \left(f(r)^{-1} dr^2 - f(r) dt^2 - dx^2\right)$$

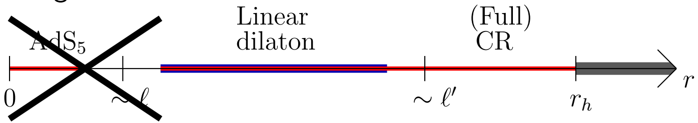
the structure of the BH geometry is as follows:



- Here $\ell' \sim d\ell \to \infty$ as $\alpha \to \alpha_c$ and $r_h \sim d\ell$ also
- Improved holographic QCD models have similar structure!

Fluctuations

Let us neglect the UV structure for a moment



and just use $V(\phi) = V_0 e^{\alpha\phi}$. Solution (Chamblin-Reall, CR):

$$ds^2 = e^{2A_0}\hat{r}^{-\frac{2}{3}(d-1)}\left[-2\ell' d\hat{r}dv - f(\hat{r})dv^2 + \delta_{ij}dx^i dx^j\right]$$

$$f(\hat{r}) = 1 - \left(\frac{\hat{r}}{\hat{r}_h}\right)^d \, , \quad \phi = \frac{1}{2}\frac{\alpha}{\alpha_c}(d-1)\log\hat{r} \, , \quad \hat{r} = 1 + \frac{r}{\ell'}$$

- QNMs at low enough T and $|\omega|$ unaffected
- As d grows fluctuations can be solved in two parts
  1. At finite r far from the horizon at any d (Bessel functions)
  2. Near the horizon, at fixed $(\hat{r}/\hat{r}_h)^d$, (hypergeometric functions) [Dijkgraaf,Verlinde^2]
- Solutions overlap at large d and small $(\hat{r}/\hat{r}_h)^d$! Full analytic control over (a subset of) fluctuations
- Without UV completion, the critical limit $d \to \infty$ is singular

Correlators

We can solve (up to corrections $\sim 1/d$)

1. All correlators at zero momentum (all of them identical)
2. Correlators of $T_{\perp\perp}$ and the scalar at any momentum

that is, we obtain (only) nonhydrodynamic QNMs

In terms of rescaled frequency and momentum

$$\varpi = \frac{\omega}{2\pi T}\,, \qquad q = \frac{k}{2\pi T}\,, \qquad \widetilde{S} = \sqrt{\varpi^2 - q^2 - 1}$$

we find

$$\langle T_{\perp\perp}(\varpi, q)\, T_{\perp\perp}(0)\rangle =$$

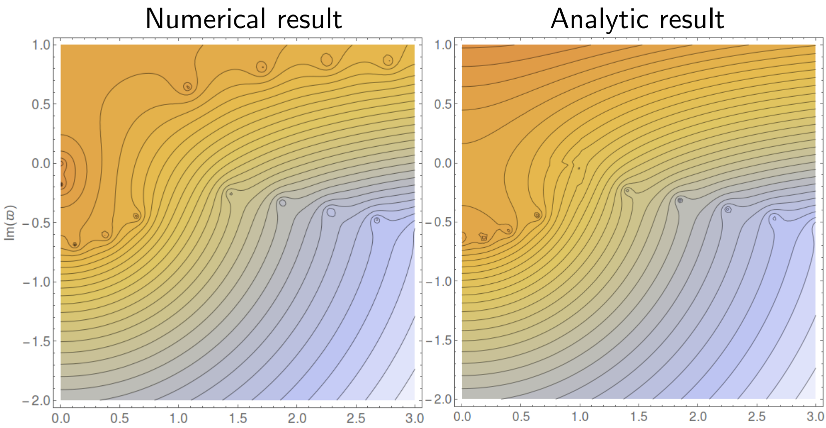
$$\frac{2\pi\, d^d\, \hat{r}_h^{-d}}{\Gamma\left(\frac{d}{2}\right)\Gamma\left(1+\frac{d}{2}\right)} \left(\frac{\left(\varpi^2 - q^2\right)}{16}\right)^{\frac{d}{2}} \left[i + \left(\frac{1+i\widetilde{S}}{1-i\widetilde{S}}\right)^{\frac{d}{2}} \frac{e^{-id\widetilde{S}}}{\mathcal{R}}\right]^{-1} + \cdots$$

where the reflection amplitude is

$$\mathcal{R}(\varpi, q) = -\frac{\Gamma\left(1+i\widetilde{S}\right)\,\Gamma\left(\frac{1}{2}\left(1 - i\varpi - i\widetilde{S}\right)\right)^2}{\Gamma\left(1-i\widetilde{S}\right)\,\Gamma\left(\frac{1}{2}\left(1 - i\varpi + i\widetilde{S}\right)\right)^2}$$

Comparison to numerics

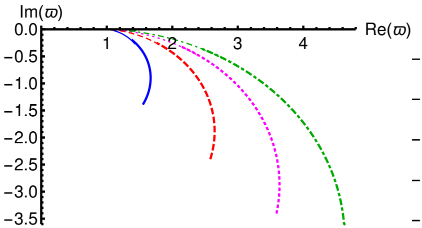
Check of correlators (log of absolute value) at $d \simeq 17$, $q = 0$



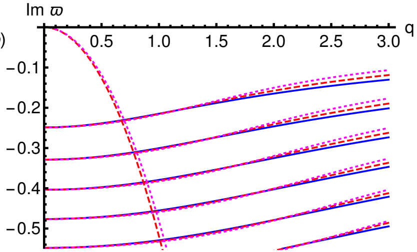
Note: result only depends on T through $\varpi = \omega/2\pi T$ (and trivial normalization factor)

Quasi Normal Modes

Evolution of nonhydro modes from $d = 4$ (conformal) to $d = \infty$



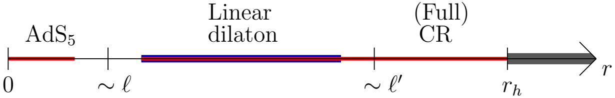
All modes for $d \simeq 17$ as a function of q



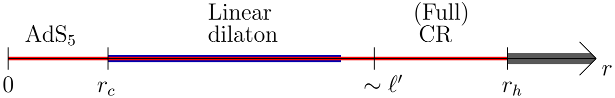
- Nonhydro QNMs accumulate on the real axis, $\varpi > \sqrt{1+q^2}$
- A branch cut on the real axis?
  - Connection to kinetic theory?
- Nonhydro modes dominate late time behavior for $q \gtrsim 1/\sqrt{d}$
  - Early breakdown of hydro?
  - Infinitely many modes but gapped, $|\varpi| > 1$

Gluing

Putting back the UV structure...



Simplest approximation: glue the CR geometry directly to AdS$_5$ $\Rightarrow$



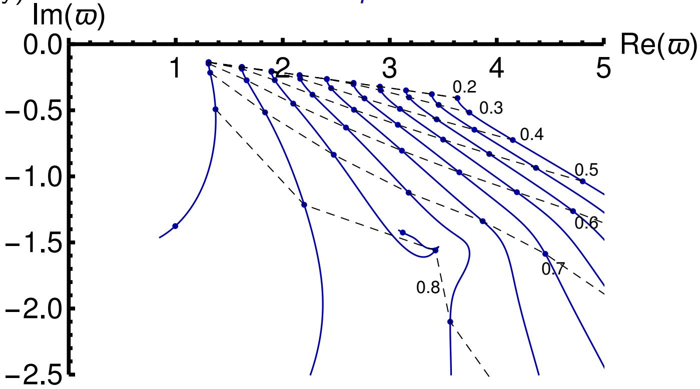
- Fluctuations can still be treated analytically if blackening factor negligible at the joint: $r_h \gg r_c$
- Results expected to be qualitatively similar to a generic background

What do we gain?

- Analytic T dependence of QNMs
- Critical limit $d \to \infty$ now regular

Results after gluing

Nonhydro QNM evolution with T extracted from the analytic (but messy) correlator for $d \simeq 27$ and $q = 0$



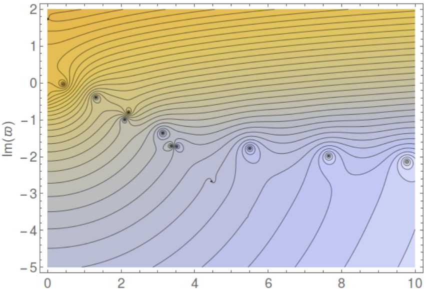
- As T decreases, QNMs move closer to real line (units: T of linear dilaton bg)
- Evolution stops at the locations determined by the CR geometry

Critical limit – branch cut

Two sets of modes:

- "AdS modes", roughly independent of T
- "CR modes", T dependent

Limit $d \to \infty$ regular after UV completion



Taking also $r_h \to \infty$,

$$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle = -\frac{81 i \pi \hat{\mu}^4}{512 r_c^4} \frac{\hat{\mu} H_1^{(1)}\left(\frac{3\hat{\mu}}{2}\right) - (i\widetilde{S}+1) H_2^{(1)}\left(\frac{3\hat{\mu}}{2}\right)}{\hat{\mu} J_1\left(\frac{3\hat{\mu}}{2}\right) - (i\widetilde{S}+1) J_2\left(\frac{3\hat{\mu}}{2}\right)}$$

with $\hat{\mu} = \sqrt{\varpi^2 - q^2}$, $\widetilde{S} = \sqrt{\varpi^2 - q^2 - 1}$

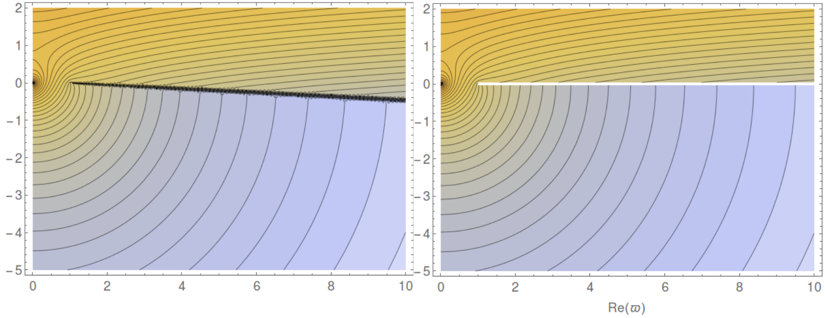
- Branch cut due to $\widetilde{S}$ running from $\varpi = \sqrt{1+q^2}$ to $\varpi = \infty$

Limit of small black holes:

$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle$ at

$r_h/r_c = 20$
Discrete modes

$r_h \to \infty$
Branch cut



Conclusions – Part II

We studied the QNMs of a non-conformal plasma (mostly analytically)

- "Large" deviation from CFT near a critical point
- Our results should be contrasted with other studies where broken scale dependence has mild effects on the QNMs
  [e.g. Janik et al; Mateos et al,. . .]
- Infinitely many gapped long lived modes in the critical limit, forming a branch cut – relations to/applications in
  - Weak coupling physics, kinetic theory?
  - Continuous phase transition with divergent correlation length? Relevant in quark-gluon plasma?
    [Gürsoy]
- How does the (gapped) branch cut affect hydrodynamics?