Non-abelian Anomalous Hydrodynamics: a Dimensional Reduction Approach

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In collaboration with:
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Holograpic Perspectives on Chiral Transport   March 2023

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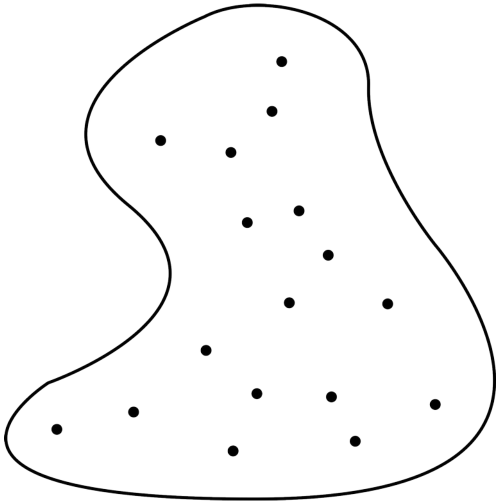
Hydrodynamics

Hydrodynamics is an effective field theory describing systems near equilibrium. Its equations express conservation laws of whatever is conserved.

Symmetries are given by Ward identities:

$$\nabla_\mu T^{\mu\nu} = \dots \qquad \nabla_\mu J^{\mu}{}_{a} = \dots$$



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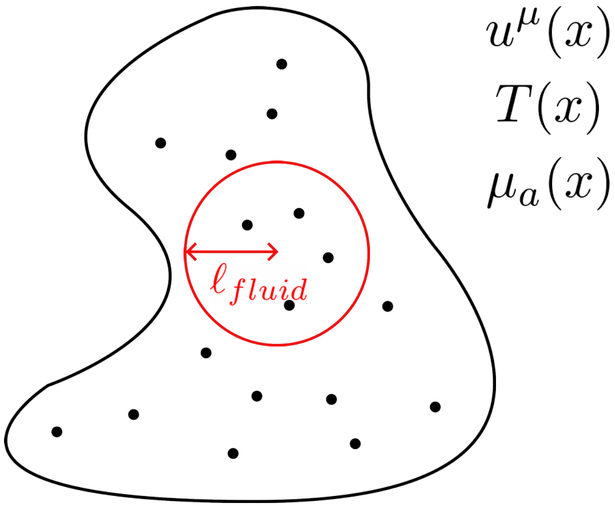

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The expansion counts in derivative order.

$$\langle T^{\mu\nu} \rangle = \mathcal{O}(\partial^0) + \mathcal{O}(\partial^1) + \ldots$$

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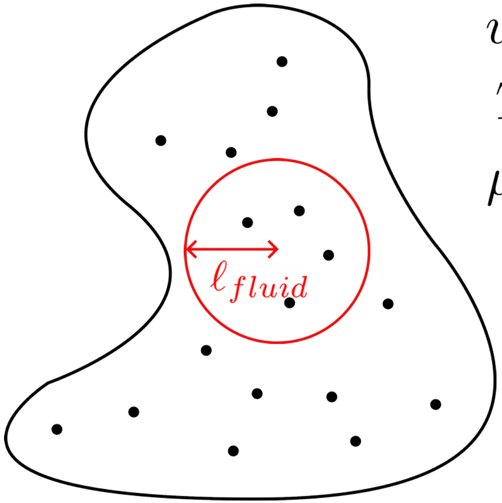


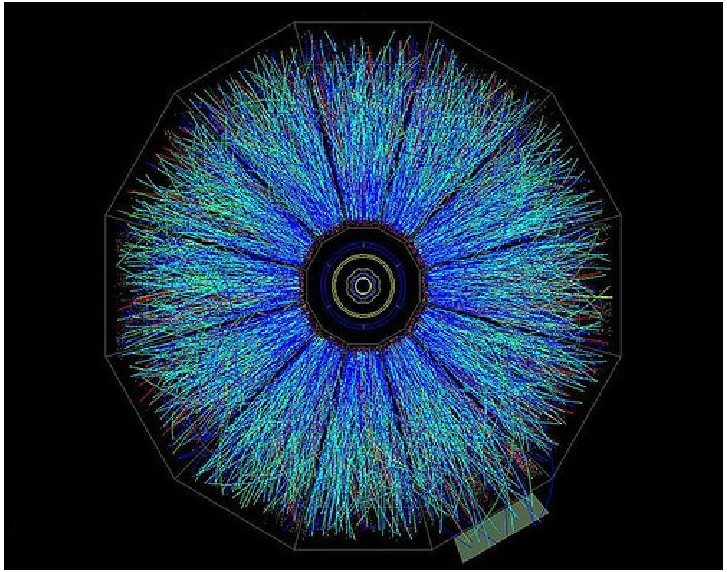
$u^\mu(x)$

$T(x)$

$\mu_a(x)$

Transport Phenomena

Hydrodynamics have been used to explain a wide variety of phenomena, for example, plasma physics or chiral transport effects.



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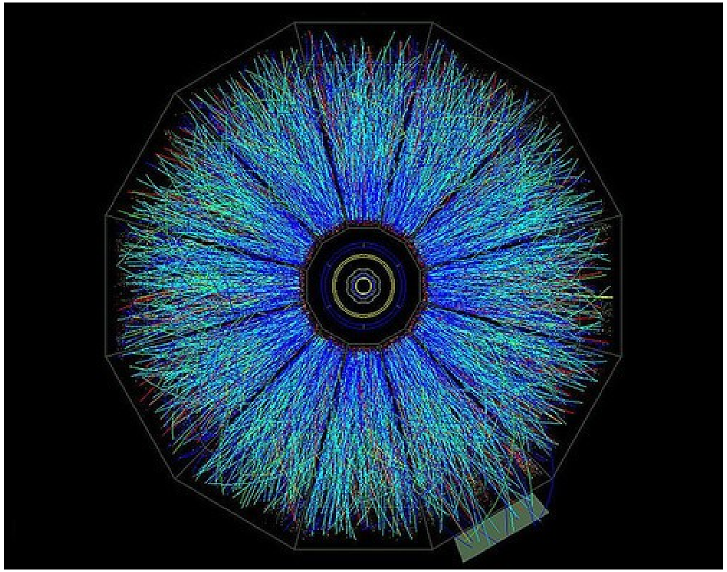
In chiral fermion theories, anomalies induce chiral transport phenomena, such as the Chiral Vortical Effect and the Chiral Magnetic Effect. Those are supposed to play important roles on physical systems like Weyl Semimetals in condensed matter and the Quark-Gluon Plasma in high energy physics.



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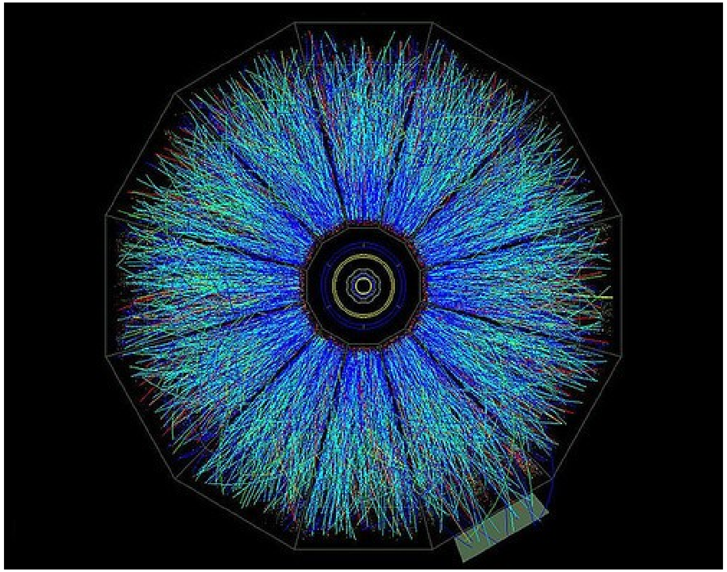

New approaches to (non-)Abelian hydrodynamics has been studied.

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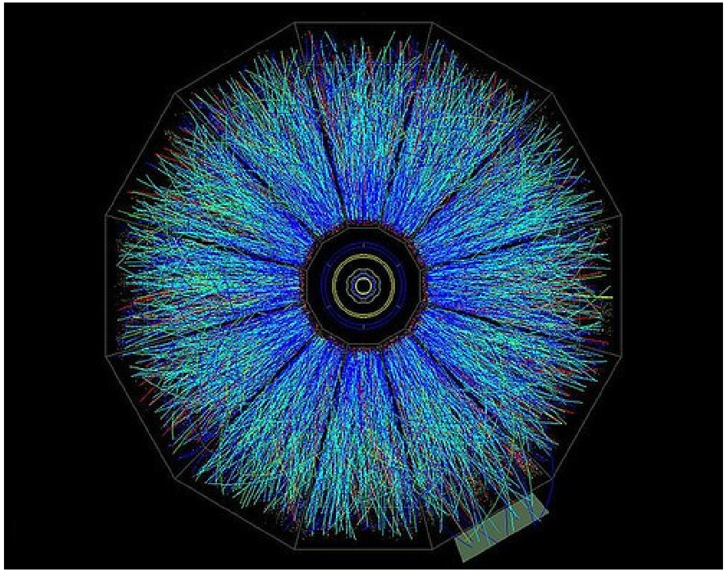

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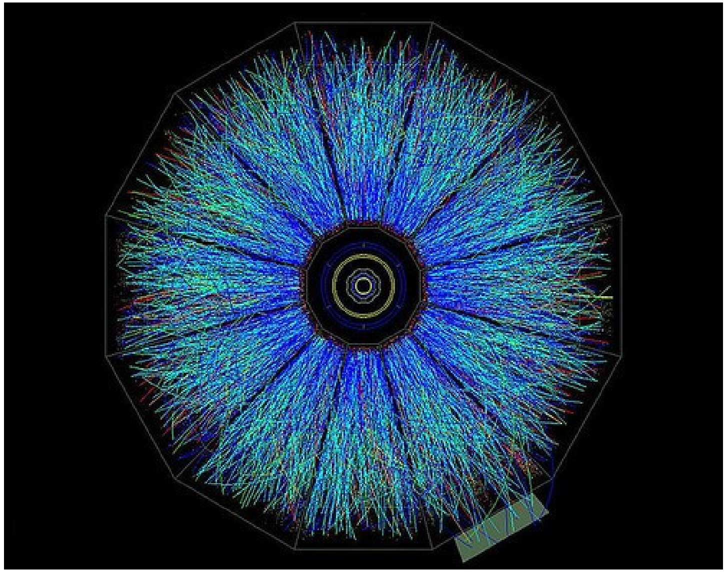

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Kaluza-Klein dimensional reduction — Di Dato, 2013; Fernández-Melgarejo, Rey, Surówka, 2017

Table of contents

Table of contents

Kaluza-Klein Dimensional Reduction

The Kaluza-Klein (KK) dimensional reduction is a procedure to obtain a d-dimensional field theory coupled to gravity and charged under an n-dimensional gauge group starting from a (D = d + n)-dimensional field theory just coupled to gravity.

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Gravity + gauge fields in d dimensions

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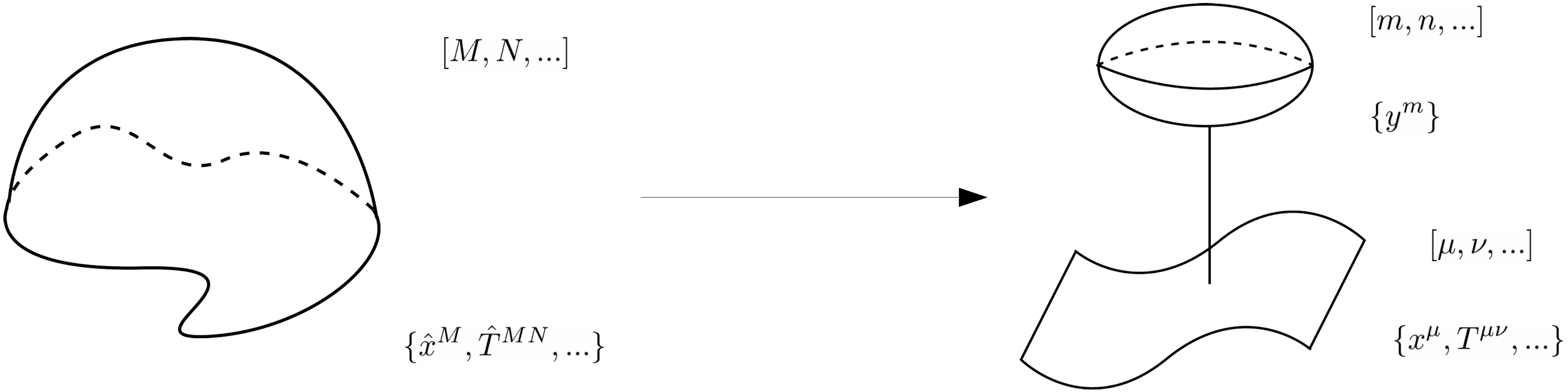

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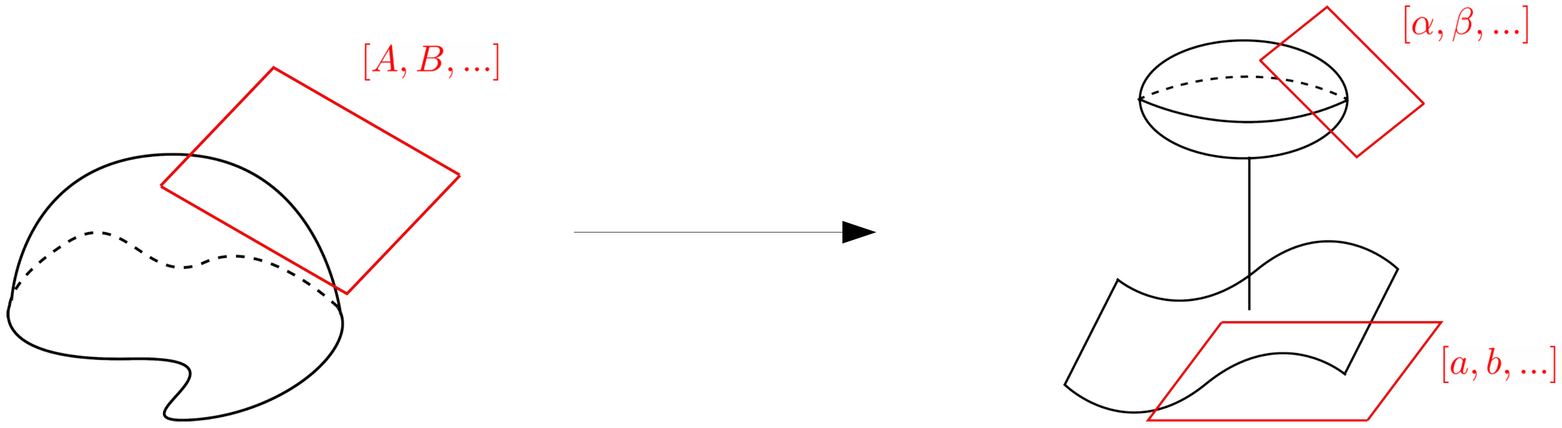
For convenience we work with the tangent space:



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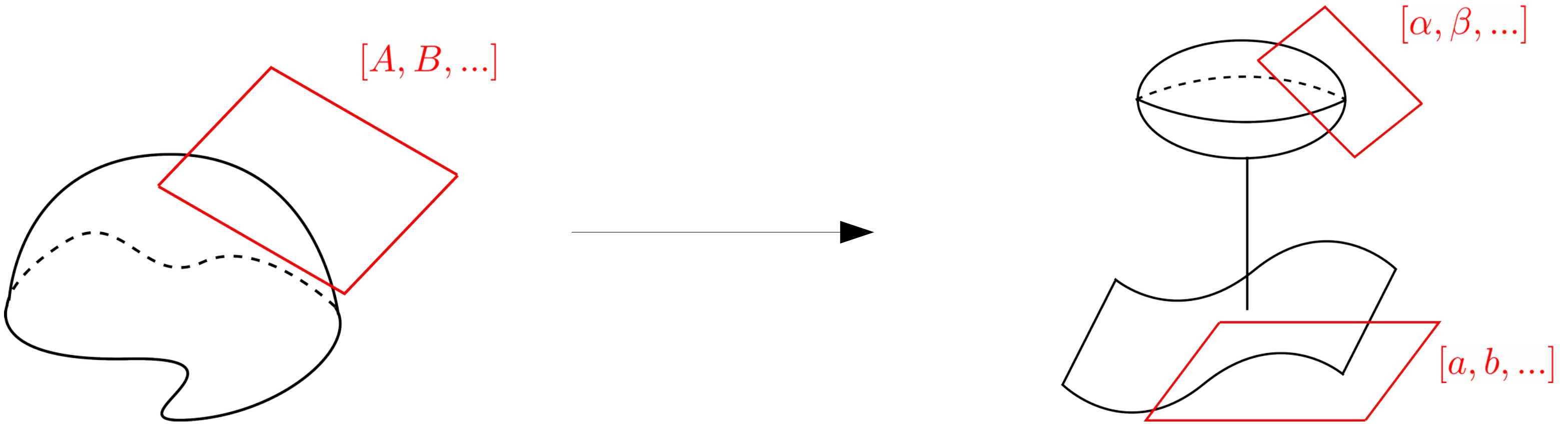

Using the vielbein formalism we can choose a parametrization directly on the vielbein. We choose the so-called triangular parametrization:

$$\hat{E}_M{}^A(\hat{x}) = \begin{pmatrix} e^{\alpha\phi}\hat{e}_\mu{}^a(\hat{x}) & e^{\beta\phi}\hat{A}^p{}_\mu(\hat{x})\hat{e}_p{}^\alpha(\hat{x}) \\ 0 & e^{\beta\phi}\hat{e}_m{}^\alpha(\hat{x}) \end{pmatrix}$$

Scherk and Schwartz, 1979

We want to preserve general coordinate invariance:

$$[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{\xi_3} \qquad {\xi_3}^M(\hat{x}) = {\xi_2}^N(\hat{x})\partial_N {\xi_1}^M(\hat{x}) - {\xi_1}^N(\hat{x})\partial_N {\xi_2}^M(\hat{x})$$

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Preserves the algebra in d-dimensions. The commutator between two internal transformations:

$${\xi_3}^p(x) = f^p{}_{mn}{\xi_1}^m(x){\xi_2}^n(x) \qquad f^p{}_{mn} = (\mathfrak{u}^{-1})^{m'}{}_m(\mathfrak{u}^{-1})^{n'}{}_n(\partial_{n'}\mathfrak{u}_{m'}{}^p - \partial_{m'}\mathfrak{u}_{n'}{}^p)$$

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$f^p{}_{mn}$ cannot depend on internal coordinates. As these objects characterize the algebra of the internal manifold, we are identifying them with the structure constants of the Lie algebra.

The Final Ansatz

To preserve general coordinate invariance, then:

$$X^m(\hat{x}) = (\mathfrak{u}^{-1})^m{}_n(y) X^n(x)$$

$$X_m(\hat{x}) = \mathfrak{u}_m{}^n(y) X_n(x)$$

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$$\hat{E}_M{}^A(\hat{x}) = \begin{pmatrix} e^{\alpha\phi}\hat{e}_\mu{}^a(\hat{x}) & e^{\beta\phi}\hat{A}^p{}_\mu(\hat{x})\hat{e}_p{}^\alpha(\hat{x}) \\ 0 & e^{\beta\phi}\hat{e}_m{}^\alpha(\hat{x}) \end{pmatrix} = \begin{pmatrix} e^{\alpha\phi}e_\mu{}^a(x) & e^{\beta\phi}A^n{}_\mu(x)\mathcal{V}_n{}^\alpha(x) \\ 0 & e^{\beta\phi}\mathfrak{u}_m{}^n(y)\mathcal{V}_n{}^\alpha(x) \end{pmatrix}$$

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Table of contents

The Hydrodynamic Variables

Following the rules to preserve isometries, we find that the hydrodynamic variables do not depend on internal coordinates ⟶ we are compactifying an specific flow pattern.

$$\hat{p}(\hat{x}) = \hat{p}(x)$$
$$\hat{\epsilon}(\hat{x}) = \hat{\epsilon}(x)$$
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The velocity must follow the constraint:

$$\hat{u}^M(\hat{x})\hat{u}^N(\hat{x})\hat{g}_{MN}(\hat{x}) = \hat{u}^A(\hat{x})\hat{u}^B(\hat{x})\eta_{AB} = -1$$

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One can choose the following parametrization:

$$\left.\begin{aligned} \hat{u}^a(x) &= u^a(x)\cosh\varphi(x), \quad u^a u^b \eta_{ab} = -1 \\ \hat{u}^\alpha(x) &= n^\alpha(x)\sinh\varphi(x), \quad n^\alpha n^\beta \delta_{\alpha\beta} = +1 \end{aligned}\right\}$$

The d-dimensional velocity field

$$u^\mu = e_a{}^\mu u^a$$

Ideal Fluid

The stress-energy tensor of the ideal fluid:

$$\hat{T}^{AB}(\hat{x}) = (\hat{\epsilon} + \hat{p})\hat{u}^A\hat{u}^B + \hat{p}\eta^{AB}$$

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With those definitions we obtain:

$$p = V\hat{p} \qquad \epsilon = V\left(\cosh^2\varphi\hat{\epsilon} + \sinh^2\varphi\hat{p}\right) \qquad \rho^m = (\epsilon + p)\mathcal{V}_\alpha{}^m n^\alpha \tanh\varphi$$

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We can obtain the other hydrodynamical variables using thermodynamics:

$$\left.\begin{aligned} \hat{\epsilon} + \hat{p} &= \hat{s}\hat{T} \\ \epsilon + p &= sT + \rho^m\mu_m \end{aligned}\right\} \longrightarrow \quad \mu_m = \tanh\varphi\mathcal{V}_m{}^\alpha n_\alpha \qquad T = \frac{\hat{T}}{\cosh\varphi}$$

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This procedure can also be used for a fluid with dissipative terms.

Di Dato, 2013

Fernández-Melgarejo, Rey, Surówka, 2017

Table of contents

Anomalous Hydrodynamics

We want to compactify an anomalous stress-energy tensor. The simplest case is to consider a non-dissipative fluid:

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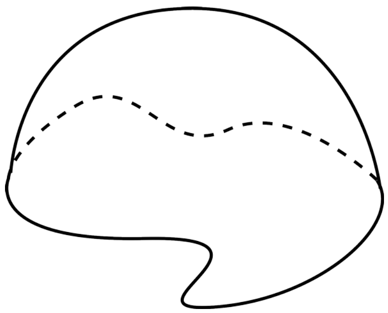
In the KK approach, we have to start with a neutral fluid. Thus, our starting point will be a dimension in which pure gravitational anomalies can be considered:

Alvarez-Gaumé and Witten, 1983

$$D = 4k + 2$$

$D = 6$



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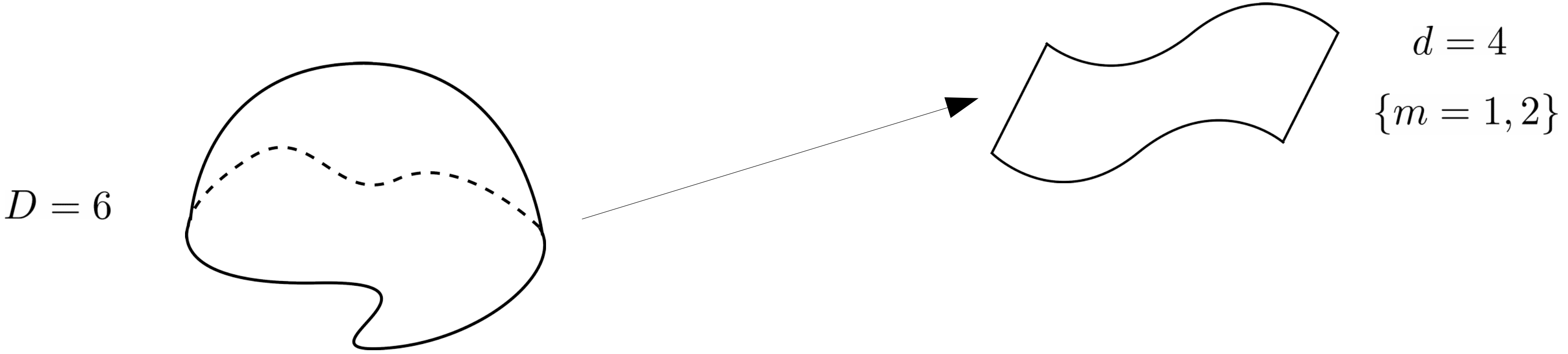

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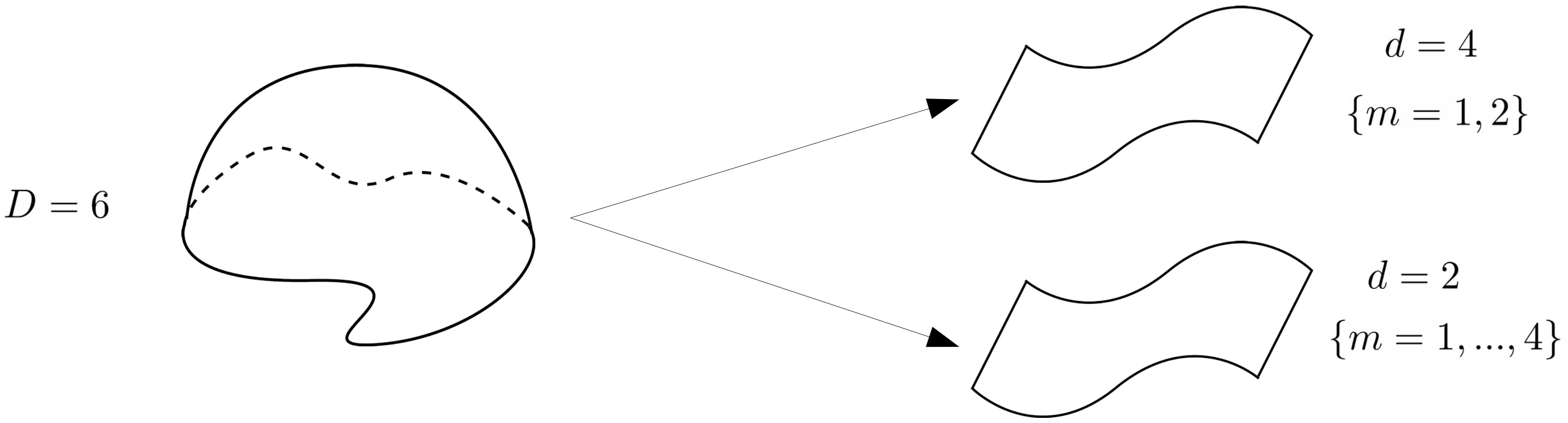

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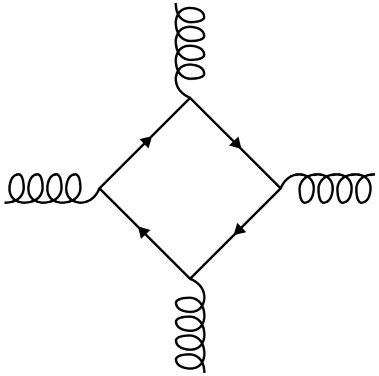
From the anomaly polynomial for pure gravitational anomalies in 6 dimensions:

$$\mathcal{P} = c_1 (\mathrm{Tr}\,\mathbf{R}^2)^2 + c_2\,\mathrm{Tr}\,\mathbf{R}^4$$



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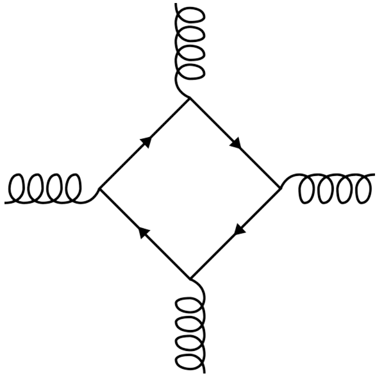

We obtain the most relevant term in derivative order:

$$\hat{T}^{AB}_{anom} = 2\hat{c}\hat{T}^4(\hat{x})\hat{u}^{(A}\epsilon^{B)CDEFG}\hat{u}_C\hat{\omega}_{DE}\hat{\omega}_{FG}$$

$$\hat{c} = (2\pi)^4(2c_1 + c_2)$$
$$\hat{\omega}_{AB} = \hat{\Pi}_{[A|}{}^{C}\hat{\nabla}_C\hat{u}_{|B]}$$
$$\hat{\Pi}_A{}^B = \hat{u}_A\hat{u}^B + \delta_A{}^B$$

We can also compute the currents for 2 dimensions:

Jensen, Loganayagam, Yarom, 2012

$$T^{ab}_{anom} = -2u^{(a}\epsilon^{b)c}u_c\left(c_A\mu_m\mu_m + 8\pi^2 c_g T^2\right)$$

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In 4D the results for chiral transport are well-known:

$$T^{ab}_{anom} = 2\left(d_{mnp}\frac{\mu_m\mu_n}{8\pi^2} + b_p\frac{T^2}{24}\right)u^{(a}B^{b)}{}_p + 2\left(d_{mnp}\frac{\mu_m\mu_n\mu_p}{6\pi^2} + b_p\mu_p\frac{T^2}{6}\right)u^{(a}\Omega^{b)}$$

Son and Surówka, 2009

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We can also compute the currents for 2 dimensions:

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Magnetic Field

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CME | CVE

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Magnetic Field | Angular velocity

From 6D to 2D

The stress-energy tensor:

$$T^{ab}_{anom} = \frac{1}{2} V \cosh^2 \varphi e^{-2\beta\phi} \hat{c} \hat{T}^4 u^{(a} \epsilon^{b)c} u_c \left(\epsilon^{mnpq} f_{mn}{}^{r} f_{pq}{}^{s}\right) \left(\mathcal{V}_r{}^{\delta} n_\delta \sinh\varphi\right) \left(\mathcal{V}_s{}^{\gamma} n_\gamma \sinh\varphi\right)$$

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From 6D to 4D

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Work in Progress: Analyzing the Coefficients

In hydrodynamics we can impose a local version of the second principle of thermodynamics:

$$\hat{\nabla}_A \hat{S}^A = 0$$

$$\hat{S}^A = \hat{s}\hat{u}^A - \frac{\hat{u}_B}{\hat{T}} \hat{T}^{AB}_{anom}$$

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This condition could allow us to identify the hydrodynamic variables

$$\nabla_a S^a = 0 \longrightarrow \begin{aligned} T &= f(\hat{T}, \varphi, \ldots) \\ \mu_m &= f({\mathcal{V}_m}^{\alpha}, \varphi, \ldots) \end{aligned}$$

Table of contents

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    - There are other ansatze that can be considered and may be able to give us a correct Abelian description: The Freund-Rubin ansatz. Freund and Rubin, 1980