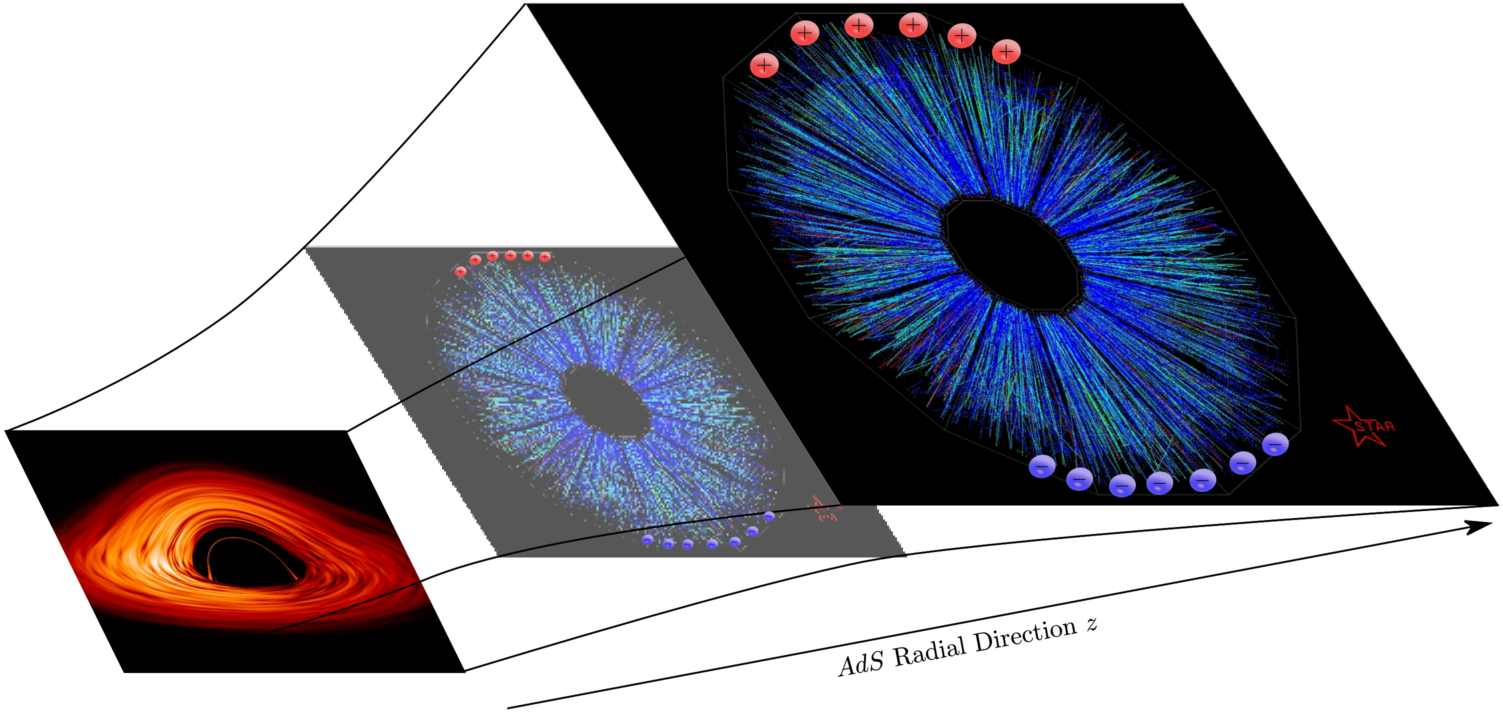
An example of convergence of hydrodynamics in strong magnetic fields



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arXiv:*Manuscript in progress*

Convergence of hydrodynamics in rapidly spinning strongly coupled plasma

[S. Hawking, C. Hunter, M. Taylor-Robinson, 1999]
[I. Papadimitriou, K. Skenderis, 2005]
[Garbiso Amano, Kaminski, 2020]
[Murata, 2009]

4+1 D asymptotically locally AdS Kerr (Myers Perry) black hole in Boyer-Lindquist coordinates

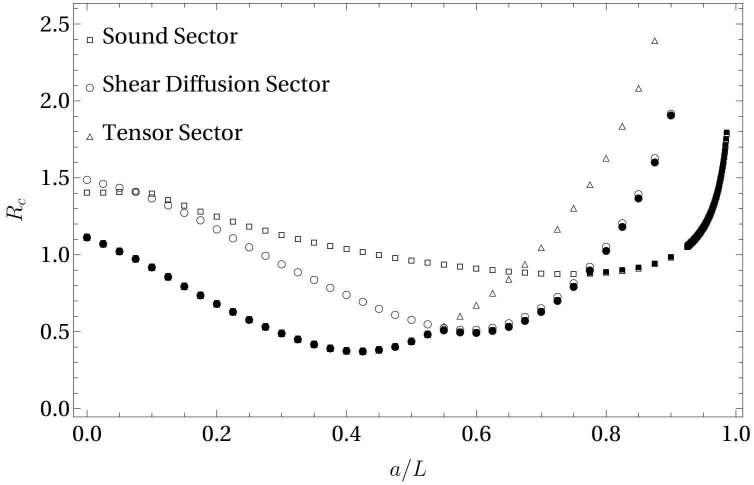
$$ds^2 = \frac{(l^2r^2+1)}{\rho^2 r^2}\left(-\frac{bd\phi\,(a^2+r^2)\sin^2(\theta)}{\Xi_a} - \frac{ad\psi\,(b^2+r^2)\cos^2(\theta)}{\Xi_b} + abdt\right)^2$$
$$+\frac{\Delta_\theta \sin^2(\theta)}{\rho^2}\left(adt - \frac{d\phi\,(a^2+r^2)}{\Xi_a}\right)^2 - \frac{\Delta_r}{\rho^2}\left(-\frac{d\phi\,(a\sin^2(\theta))}{\Xi_a} - \frac{bd\psi\cos^2(\theta)}{\Xi_b} + dt\right)^2$$
$$+\frac{\Delta_\theta}{\rho^2}\cos^2(\theta)\left(bdt - \frac{d\psi\,(b^2+r^2)}{\Xi_b}\right)^2 + \frac{d\theta^2\rho^2}{\Delta_\theta} + \frac{dr^2\rho^2}{\Delta_r}$$

$$\Delta_r = \frac{1}{r^2}(r^2+a^2)(r^2+b^2)(r^2l^2+1) - 2M$$
$$\Delta_\theta = 1 - a^2l^2\cos(\theta)^2 - b^2l^2\sin(\theta)^2$$
$$\rho^2 = r^2 + a^2\cos(\theta)^2 + b^2\sin(\theta)^2.$$
$$\Xi_a = 1 - a^2l^2$$
$$\Xi_b = 1 - b^2l^2$$

Magic

$$ds^2 = \frac{2\mu\left(\frac{a\sigma_3}{2}+dt\right)^2}{r^2} + \frac{dr^2}{G(r)} - dt^2\left(\frac{r^2}{l^2}+1\right) + \frac{1}{4}r^2\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right),$$

*Magic**



[Cartwright, Garbiso-Amano, Noroha, Speranza, PRL in review]

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Chaos and pole-skipping in a simply spinning plasma

[A. G. Amano, Blake, Cartwright, Kaminski, Thompson, 2023]

OTOC

$$D_\pm(t,\Psi) \sim 1 - cG_N\frac{\exp\left(2\pi T_\pm\left(t - \frac{L\Psi}{2v_B^\pm}\right)\right)}{4\pi|\Psi-\Omega t|}, \quad \pm\Psi \mp \Omega t > 0,$$

$$2\pi T_\pm = \frac{2r_0}{L^2\sqrt{1-a^2/L^2}}\left(1 \mp \sqrt{\frac{3}{2}}\frac{a}{L}\right),$$
$$\frac{2\pi T_\pm}{v_B^\pm} = \frac{2r_0}{L^2\sqrt{1-a^2/L^2}}\left(\sqrt{\frac{3}{2}} \mp \frac{a}{L}\right).$$

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Introduction – MHD and hydro in external fields

[Hernandez, Kovtun, 2017]

Hydrodynamics – A universal, effective field theory, of the near equilibrium collective behavior of many body (quantum) systems described by the evolution of (non)-conserved quantities

What if the underlying microscopic theory is weakly coupled to U(1) gauge fields?

Coupling between thermal and electromagnetic d.o.f's is negligible – Maxwell's equations in matter

$$\nabla \cdot D = \rho_f \qquad \nabla \cdot B = 0$$
$$\nabla \times H = J_f + \partial_t D \qquad \nabla \times E = \partial_t B$$

Coupling between thermal and electromagnetic d.o.f's is non-negligible – Hydrodynamics coupled to Maxwell in matter

If one treats, the fields as *external*, then relevant hydrodynamical variables are $u^\mu,\ T,\ \mu$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$
$$\nabla_\mu J^\mu = 0$$

If one treats, the fields as *dynamical*, then relevant hydrodynamical variables are $u^\mu,\ T,\ \mu,\ E^\mu,\ B^\mu$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J^{ext}_\lambda$$
$$\nabla_\nu \left(F^{\mu\nu} - M^{\mu\nu}\right) = J^\mu_{ext} + J^\mu_{free}$$
$$\epsilon^{\mu\nu\alpha\beta} \nabla_\nu F_{\alpha\beta} = 0$$

Magnetohydrodynamics (MHD) – The fields are dynamical, the coupling between thermal and electromagnetic fields is non-negligible and the matter is electrically conducting while the electric field is screened

$$E \sim O(\partial) \quad B \sim O(1) \qquad B/T^2 \ll 1$$

Convergence of series and plane curves – Simple Series

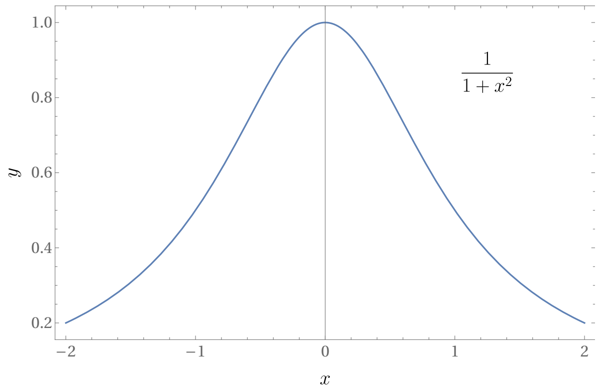
Consider the simple function $y(x) = \frac{1}{1+x^2}$

For the time being take $x \in \mathbb{R}$

It has a Taylor series expression around the origin $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

Convergence? Ratio test

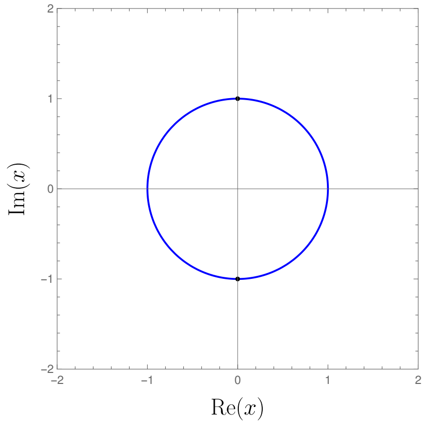
$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{(2n+2)}}{(-1)^n x^{2n}} \right| < 1 \quad \Rightarrow \quad |x| < 1$$



So what is the problem?

Back in grade school we learned that the obstruction to convergence is a singular point

Complexify x and solve for the poles $x = \pm i$



Convergence of series and plane curves – Linearized hydro

[Grozdanov, Kovtun,Starinets, Tadić, 2019]

Consider fluctuations around a homogeneous-isotropic equilibrium state,

$$T^{\mu\nu} = (T^{\mu\nu})_{eq} + \delta T^{\mu\nu}$$

Combine the EOM with constitutive relations and Fourier expand. → Linear system $L(q^2,\omega)_{\alpha\mu}\delta T^{0\mu} = 0$

Nontrivial solutions if: → $\det\left(L(q^2,\omega)_{\alpha\mu}\right) = P(q^2,\omega) = 0$

Define: *Hydrodynamic Spectral Curve* $P(q^2,\omega) = \det(L_{\alpha\mu})$

If $P(q^2,\omega)$ is analytic at $(q,\omega) = (q_0,\omega_0)$ then by the implicit function theorem

$\omega(q^2)$ can be in general expanded as a Puiseux series,

$$\omega(q^2) = \sum_{i=0}^{\infty} a_k (q^2 - q_0^2)^{i/m}$$

with radius of convergence R given by the distance to the nearest critical point q_c defined by $P(q_c^2,\omega_c) = 0, \partial_\omega P(q_c^2,\omega_c) = 0$ with $R = |q_0 - q_c|$

AdS/CFT – Background solutions

[D'Hoker, Kraus, 2009]
[D'Hoker, Kraus, 2009]
[D'Hoker, Kraus, 2010]
[Ammon, Leiber, Macedo, 2016]
[Gauntlett, Varela, 2007]
[Buchel, Liu, 2006]
[Gauntlett, Colgain, Varela, 2006]

Asymptotically AdS_5 solutions dual to a $\mathcal{N}=4$ SYM theory in the presence of an external magnetic field and uniform charge density.

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F^{\mu\nu}F_{\mu\nu}\right) - \frac{\gamma}{6}\int A\wedge F\wedge F + S_{ct}$$

We will put this term to zero for now

For $\gamma = 2/\sqrt{3}$ the theory is a consistent truncation of supergravity, and the chiral anomaly coefficient of the dual theory is then given by $C = \gamma/6$

The dual energy momentum tensor and current obey the (non)-conservation laws

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu}J_\mu$$

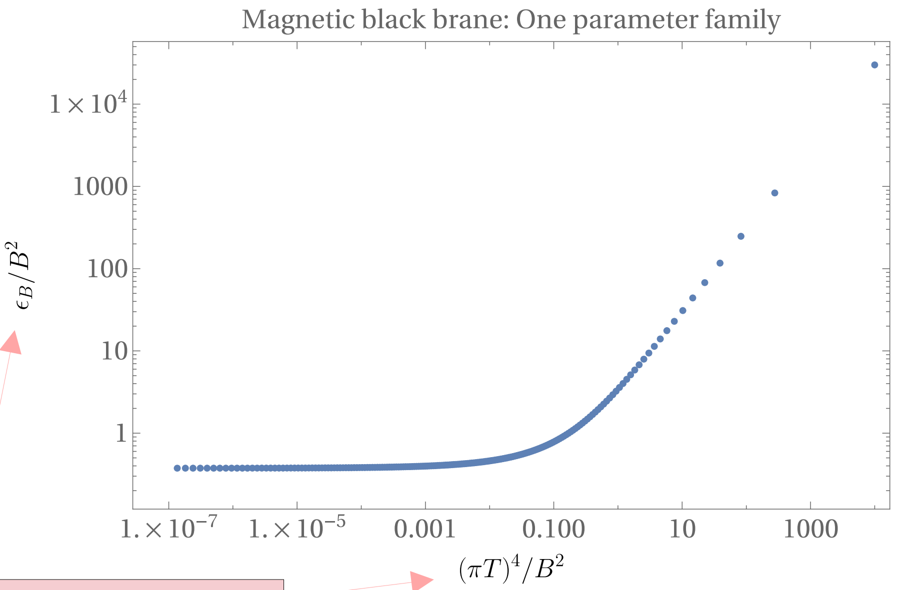
$$\nabla_\mu J^u = \frac{3}{4}C\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$$

We will take the ansatz of the metric and gauge field as the following, in Edd. Fink. coordinates

$$ds^2 = \frac{1}{z^2}\left(-U(z)d\nu^2 - 2d\nu dz + v(z)^2(dx_1^2 + dx_2^2) + w(z)^2 dx_3^2\right)$$

$$A = \frac{B}{2}(-x_2 dx_1 + x_1 dx_2)$$

Leads to a one parameter family of solutions parameterized by B/T^2



Dimensionless ratios are important!

AdS/CFT – Goal

[Kovtun, Starinets, 2005]
[Grozdanov, Kovtun,Starinets, Tadić, 2019]

Metric and gauge field perturbations of the geometry of correspond to deviations away from thermal equilibrium. i.e. hydrodynamics

1. Linearize around a background geometry

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + O(\epsilon^2)\,, \quad A_\mu = A^{(0)}_\mu + \epsilon A^{(1)}_\mu + O(\epsilon^2)$$

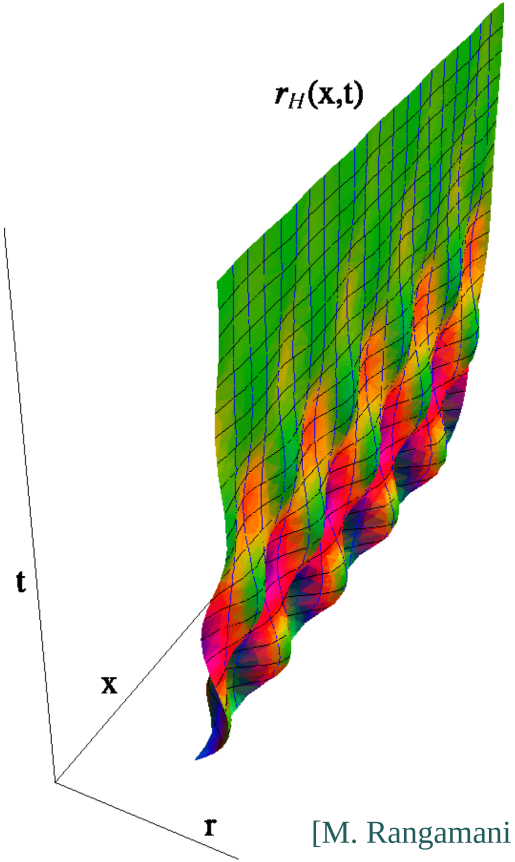
2. Solve the fluctuations equation of motion

$$0 = \frac{1}{2}\nabla_\mu\nabla_\nu g^{(1)} - \frac{1}{2}\nabla^\lambda\nabla_\lambda g^{(1)}_{\mu\nu} + \nabla^\lambda\nabla_{(\mu}g^{(1)}_{\nu)\lambda} - \frac{2\Lambda}{D-2}g^{(1)}_{\mu\nu} - s(A^{(0)}, A^{(1)})\,,$$

$$0 = 8\partial_\mu\left[\sqrt{-g^{(0)}}\left(\frac{1}{2}(g^{(0)\alpha\beta}g^{(1)}_{\beta\nu})F^{(0)\mu\nu} + F^{(1)\mu\nu} + \left(g^{(0)\mu\alpha}g^{(1)\nu\beta} + g^{(1)\mu\alpha}g^{(0)\nu\beta}\right)F^{(0)}_{\alpha\beta}\right)\right]$$
$$+ \gamma\epsilon^{\nu\alpha\beta\lambda\sigma}\left(F^{(0)}_{\alpha\beta}F^{(1)}_{\lambda\sigma} + F^{(1)}_{\alpha\beta}F^{(0)}_{\lambda\sigma}\right)$$

Subject to the boundary conditions:
1. Ingoing at the horizon (gives infalling wave)
2. Vanishing at the conformal boundary (Dirichlet)
3. Vanishing partial derivative of the frequency (Neumann-ish)

3. Extract the critical momentum



[M. Rangamani 2009]

AdS/CFT – Sectors and mode decomposition

A general fluctuation can then be represented as

$$g^{(1)}_{\mu\nu}(z,x^i) = \int \mathrm{d}^4 k e^{-ik_i x^i} g^{(1)}_{\mu\nu}(z,k^i)\,, \quad A^{(1)}_{\mu}(z,x^i) = \int \mathrm{d}^4 k e^{-ik_i x^i} A^{(1)}_{\mu\nu}(z,k^i)$$

Align the momentum with the magnetic field $\vec{k} \,||\, \vec{B}$ leaving us with a residual $SO(2)$

Spin	Fields	$\begin{cases} B \to -B \\ \gamma \to -\gamma \end{cases}$	$\begin{cases} B \to -B \\ k \to -k \end{cases}$	Hydrodynamic modes $\lim_{q\to 0} w(q) = 0$
2^+	$g^{(1)}_{x_1x_2}$, $g^{(1)}_{x_1x_1} - g^{(1)}_{x_2x_2}$	$2^+ \to 2^+$	$2^+ \to 2^+$	✗
1^+	$g^{(1)}_{\nu x_+}$, $g^{(1)}_{x_3x_+}$, $A^{(1)}_{x_+}$	$1^+ \to 1^-$	$1^+ \to 1^-$	✓
1^-	$g^{(1)}_{\nu -}$, $g^{(1)}_{x_3 -}$, $A^{(1)}_{-}$	$1^- \to 1^+$	$1^- \to 1^+$	✓
0^+	$g^{(1)}_{x_3x_3}$, $g^{(1)}_{x_1x_1} + g^{(1)}_{x_2x_2}$, $g^{(1)}_{\nu x_3}$ $g^{(1)}_{\nu\nu}$, $A^{(1)}_{\nu}$, $A^{(1)}_{x_3}$	$0^+ \to 0^+$	$0^+ \to 0^+$	✓

[Janiszewski, Kaminski, 2016]
[Ammon,Kaminski, Koirala, Leiber, Wu, 2017]

AdS/CFT – Constructing solutions

In Edd. Fink. Coordinates the fields are regular at the horizon $g_{ab}^{(1)} = (z-1)^\alpha z^\beta g_{ab}^{(1),Reg}(z)$ for $\alpha = 0$

Decompose the fluctuation equations

$$M(\mathfrak{q}^2,\mathfrak{w})\phi = \left(M_0(\mathfrak{q}^2) + M_1(\mathfrak{q}^2)\mathfrak{w} + M_2(\mathfrak{q}^2)\mathfrak{w}^2 + \cdots\right)\phi = 0$$
$$M_i = M_i^2(\mathfrak{q}^2)\frac{d^2}{dz^2} + M_0^1(\mathfrak{q}^2)\frac{d}{dz} + M_0^0(\mathfrak{q}^2)$$

ϕ -fields of a given sector
$$\mathfrak{w} = \frac{\omega}{2\pi T},\ \mathfrak{q} = \frac{q}{2\pi T}$$

To obtain QNM modes one solves $P(\mathfrak{q}^2,\mathfrak{w}) = \det(M(\mathfrak{q}^2,\mathfrak{w})) = 0$ See for instance [Jansen, 2017]

To obtain critical points one simultaneously solves $P(\mathfrak{q}^2,\mathfrak{w}) = 0\,,\ \partial_{\mathfrak{w}} P(\mathfrak{q}^2,\mathfrak{w}) = 0$

Numerical procedure:

1. Guess initial value of $\mathfrak{q}^2$ & $\mathfrak{w}$ and use a pesudo-spectral Chebyshev decomposition to represent the matrix M

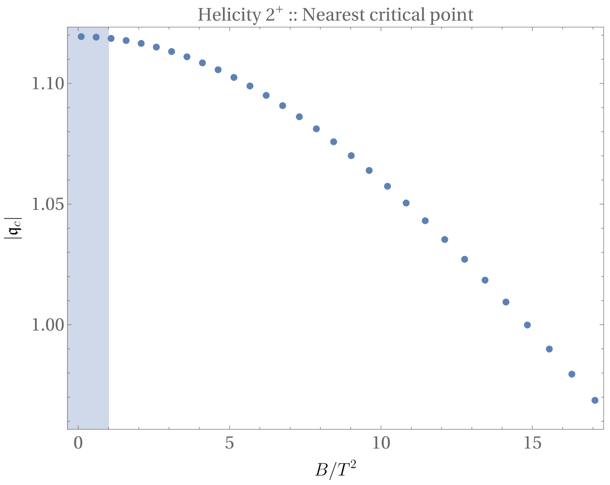
2. Use a Newton-Raphson root finder to find $\mathfrak{q}^2$ & $\mathfrak{w}$ which satisfy the two conditions

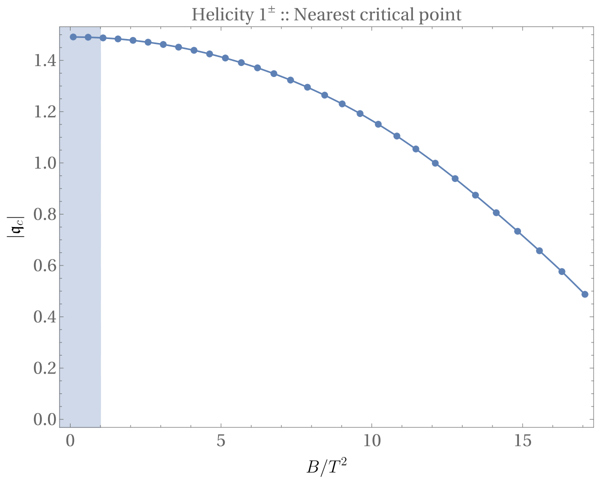
$$(\mathfrak{q}_i^2,\mathfrak{w}_i) = (\mathfrak{q}_{i-1}^2,\mathfrak{w}_{i-1}) - J^{-1}.\chi \qquad \chi = (P,\partial_{\mathfrak{w}}P)\,,\ J_{ij} = \frac{\partial \chi_i}{\partial x^j}\,,\ x = (\mathfrak{q}^2,\mathfrak{w})$$

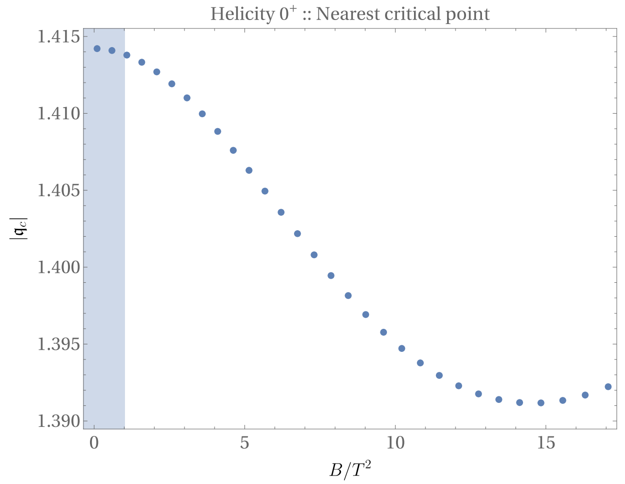
Results – N=4 SYM plasma in an external magnetic field

Preliminary

What happens near the regime of validity of hydro in external fields? $B \ll T^2$







Helicity 2

No hydro mode

-Remains smooth
-Remains roughly the same value for regime of validity of hydro approx.

Helicity 1

-Remains smooth
-Remains roughly the same value for regime of validity of hydro approx.
-Drops significantly for increasing B/T^2

Helicity 0

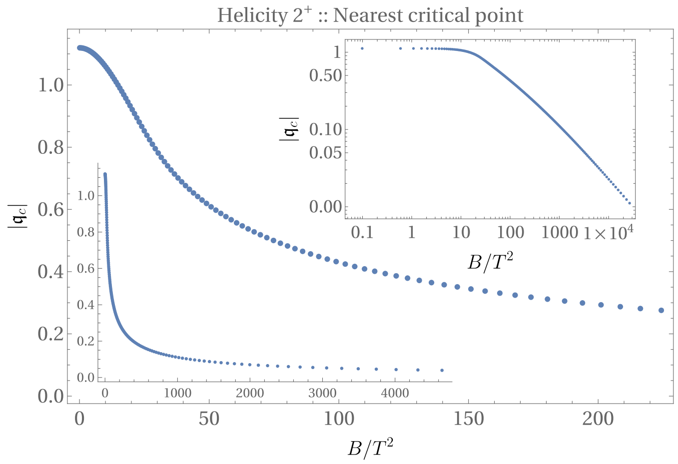
-Remains smooth
-Remains roughly the same value for regime of validity of hydro approx.

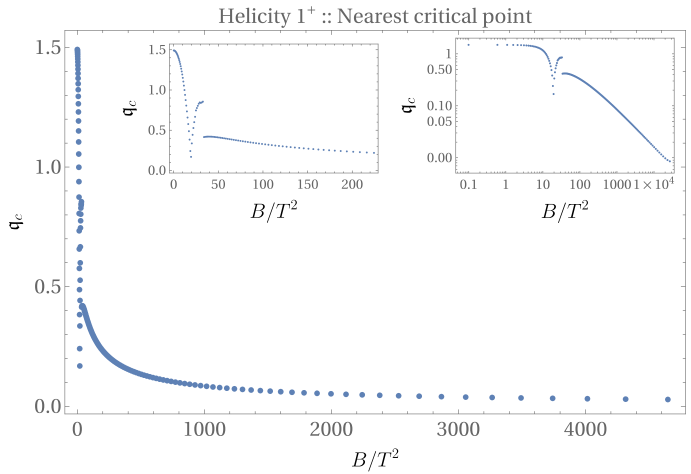
The bound on the radius of convergence remains finite and non-zero within and outside of the regime of applicability of hydro.

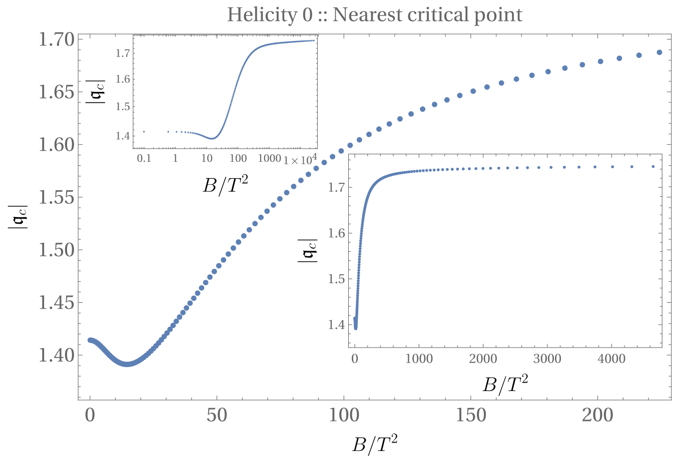
Results – N=4 SYM plasma in an external magnetic field

Preliminary

What happens far outside the regime of validity of hydro in external fields? $B \gg T^2$







Helicity 2

-Remains smooth
-Appears to asymptote to zero

Helicity 1

-Gaps
-Appears to asymptote to zero

Helicity 0

-Remains smooth
-Appears to asymptote to finite value

The bound on the radius of convergence remains finite and non-zero for substantially longer then one might expect.

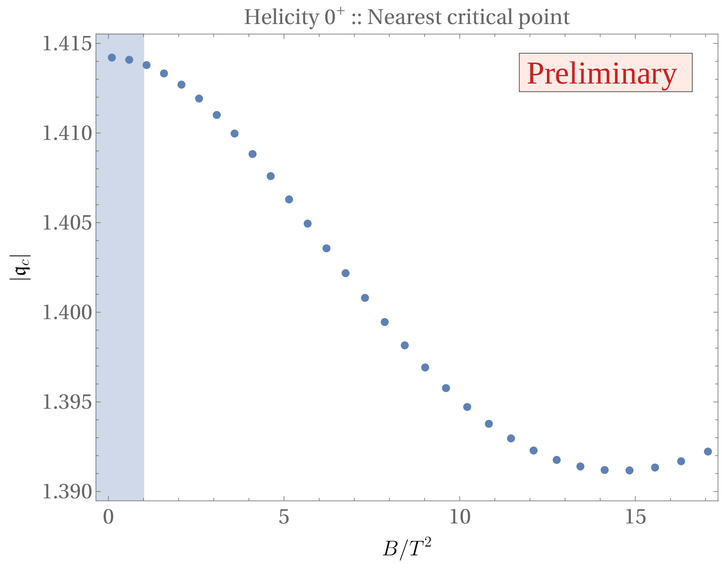
Current "observations"

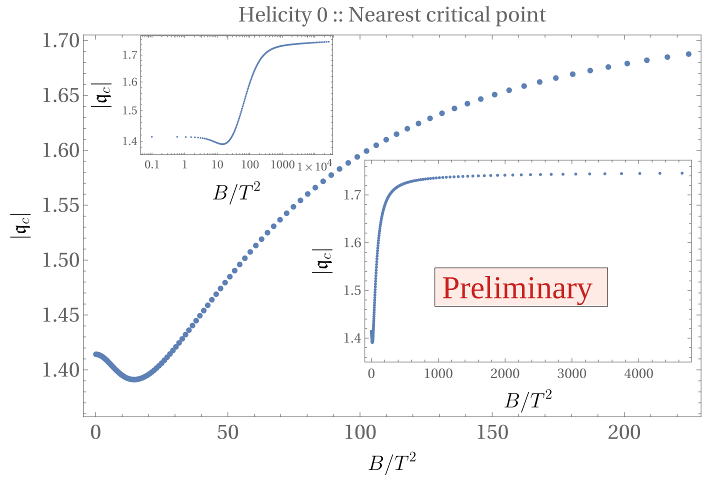
-The lack of evidence of the CME in HICs necessitates deeper theoretical control to moderate the analysis and direction of experimental searches.

-The radius of convergence of the linearized hydrodynamic gradient expansion:

- *decreases* for shear modes of SYM in strong external magnetic fields
- *decreases* for sound modes of SYM in strong external magnetic fields

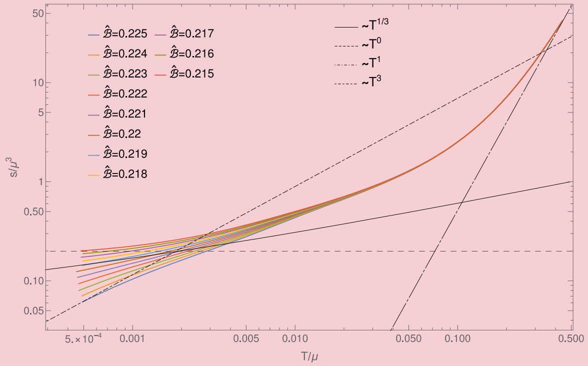
-Can this be extended to full MHD?





Stay tuned:

- Work on critical points with anomalous current is in progress.
- Full Einstein-Maxwell-Chern-Simons geometry contains a quantum critical point in its dual CFT



- Near the critical point in QCD as well as quantum critical point of EMCS in AdS, hydro should break down, these methods should provide a look at how rapidly this occurs
- *An example of convergence in hydrodynamics in strong external fields, C. Cartwright*, will appear soon.

[D'Hoker, Kraus, 2012][Hernandez, Kovtun, 2017][Grozdanov, Poovuttikul, 2019]

Results – N=4 SYM plasma in an external magnetic field

Preliminary