Sphalerons & topological domains at strong coupling

Holographic Perspectives on Chiral Transport, Trento (Trient)

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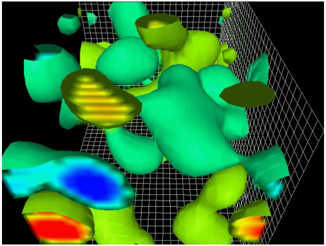
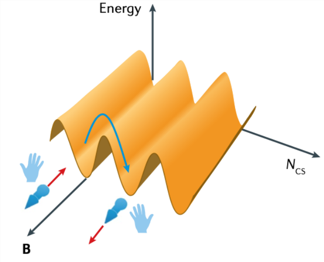
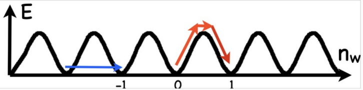
Motivation



The Chiral Magnetic Effect

[Kharzeev, Jinfeng Liao;'21]





- Axial anomaly (QED): $\partial_\mu J_5^\mu = C\,\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$
- CME current $\vec{J} = 8C\,\mu_5\,\vec{B}$

In Heavy-ion collisions

1. μ_5 is generated dynamically and not put in by hand in form of a chemical potential
2. μ_5 (and n_5) is not a conserved quantity since axial symmetry is broken explicitly by quark masses and gluonic effects

Summary

[Voloshin;'04] [Fukushima, Kharzeev, Warringa;'09]

- At high T and at weak coupling: topological fluctuations in QCD matter are enhanced due to the real-time sphalerons
- In B field fluctuations of topological charge can be directly observed $\Rightarrow$ separation of electric charge along B due to the spatial variation of the topological charge distribution + CME due to the time dependence of the topological charge density

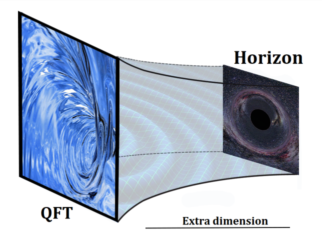
$$\Delta N_{\mathrm{CS}} = \int \mathrm{d}^4 x \, q(x^\mu), \qquad \partial_\mu J_5^\mu = -2q(x^\mu) \sim \mathrm{tr}(G \wedge G)$$

$G_{\mu\nu}$ is gluon field strength, q topological charge density

- Experimental observable directly linked to fluctuations of electric current

$$\cos(\Delta\phi_\alpha + \Delta\phi_\beta) \propto \frac{\alpha\beta}{N_\alpha N_\beta}\left(J_\perp^2 - J_\parallel^2\right).$$

Methodology



Holographic Stückelberg Model

[Klebanov, Ouyang, Witten; '02], [Anastasopoulos, Bianchi, Dudas, Kiritsi; '06], [Gursoy, Jansen; '14], [Jimenez-Alba, Landsteiner, Melgar; '14]

Gravitational Action [Jimenez-Alba, Landsteiner, Melgar; '14]

$$S = \frac{1}{2\,\kappa_5^2}\int_{\mathcal{M}} d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}F_{(5)}^2 + \frac{m_s^2}{2}(A_m - \partial_m\theta)^2 \right.$$
$$\left. + \frac{\alpha}{3}\epsilon^{mnklp}(A_m - \partial_m\theta)\left(3F_{nk}F_{lp} + F_{nk}^{(5)}F_{lp}^{(5)}\right)\right] + S_{bdy} + S_{ct}$$

with $F = \mathrm{d}V,\ F^{(5)} = \mathrm{d}A$, and $\dim[\langle J_5^\mu\rangle] = 3 + \Delta(m_s)$

Ward identities

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_5^\mu \sim m_s\, c_1\left(\mathrm{tr}G\wedge G + F^{(5)}\wedge F^{(5)}\right) + c_2\,\alpha\left(3F\wedge F + F^{(5)}\wedge F^{(5)}\right)$$

Two contributions: non-abelian anomaly + abelian QED anomaly

Setup

Background: Magnetic brane

$$\mathrm{d}s^2 = \frac{1}{u^2}\left(-f(u)\,\mathrm{d}t^2 - 2\mathrm{d}t\mathrm{d}u + v(u)^2\mathrm{d}x^2 + v(u)^2\mathrm{d}y^2 + w(u)^2\mathrm{d}z^2\right)$$

$$V_\mu = (0, 0, -B/2\,y, B/2\,x, 0), \qquad A_\mu = (0, 0, 0, 0, 0)$$

- Consider time and space dependent fluctuations about this background (in Fourier space)
- Without axion ($m_s = 0, \theta \equiv 0$): axial charge and electric current can oscillate into each other $\rightarrow$ Chiral magnetic wave
- With axion: Axion couples via derivatives to axial gauge field and pulls axial charge into black hole $\rightarrow$ Chiral magnetic wave overdamped, finite lifetime of axial charge
- Special cases $\boldsymbol{k} = k_{||} : \{a_t, a_z, v_t, v_z, \theta\}$, $\boldsymbol{k} = k_x : \{h_{yz}, a_t, a_x, v_z, \theta\}$
- Also important: anisotropy of the background

Procedure

- Prepare initial state: magnetic brane at finite background magnetic field, no charges
- Compute electric current two-point function at fixed $B = B\, e_z$ and ω

$$\Delta G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}) \equiv G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}, m_s) - G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}, m_s = 0)$$

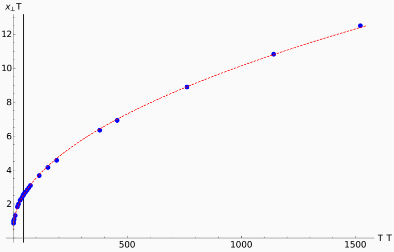
- Perform inverse (discrete) Fourier transform to real space
- Size is given by root mean square

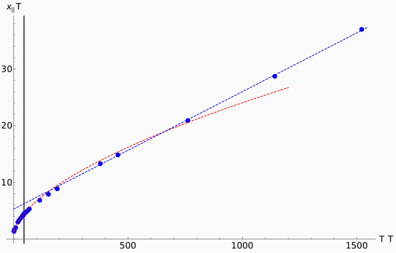
$$x_{\text{rms}} = \sqrt{\frac{\int \mathrm{d}x\, x^2\, \Delta G^{\text{ret}}_{J^z J^z}(x)}{\int \mathrm{d}x\, \Delta G^{\text{ret}}_{J^z J^z}(x)}}$$

- Encodes the information about spatial profile of induced axial charge by topological fluctuations for a given magnetic field and time interval

Sphaleron size

Fix $B/T^2 = 0.22$, $\alpha = 6/19$, $m_s = 0.3$.



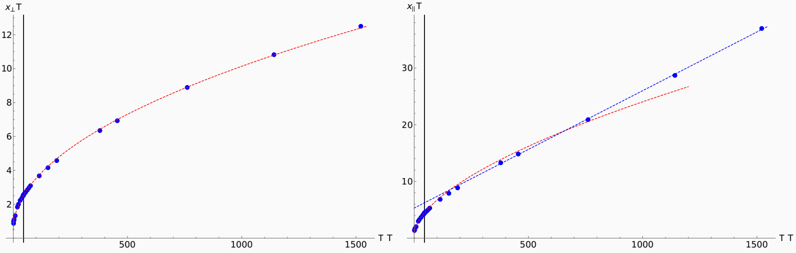


$\perp$ and $\parallel$ with respect to B; black line is relaxation time of axial charge

$$x_\perp T \sim a_1 + a_2 \sqrt{\mathcal{T} T}$$

$$x_\parallel T \sim a_3 + a_4 \sqrt{\mathcal{T} T} \text{ (for } \mathcal{T} T \text{ small)}, \quad x_\parallel T \sim a_5 + a_6 \mathcal{T} T \text{ (for } \mathcal{T} T \text{ large)}$$

Observations



- Only diffusion in transverse direction (exponent 1/2)
- For $k \| B$ ballistic behavior for sufficiently large time intervals
- Size enhanced along magnetic field
- Velocity: $\Delta x_\parallel / \Delta \mathcal{T} = 0.021 \ll 1$

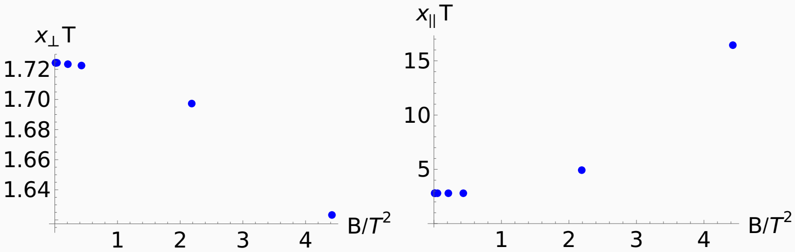
Dimensionful units

Let's put $T = 300\text{MeV}$, $B = 1 m_\pi^2$, $\mathcal{T} = 10\,\text{fm}$

$$x_\perp = 1.11\,\text{fm} \qquad \text{and} \qquad x_\parallel = 1.80\,\text{fm}$$

Dependence on magnetic field

Fix $\mathcal{T}T \approx 15$, $\alpha = 6/19$, $m_s = 0.3$.

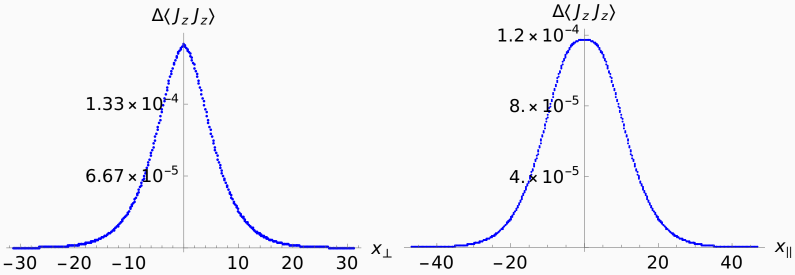


Observation

Significant enhancement in $x_{\|}$ $\Rightarrow$ become more elongated

Spatial distribution

Fix $\mathcal{T}T \approx 22.80$, $\alpha = 6/19$, $m_s = 0.3$.

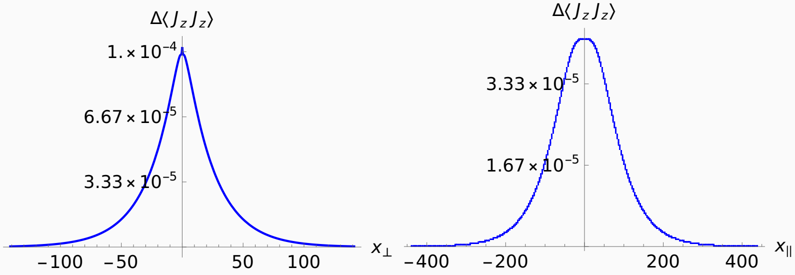


Observation

Longitudinal distribution flatter at top

Spatial distribution large $\mathcal{T}$

Fix $\mathcal{T}T \approx 1140$, $\alpha = 6/19$, $m_s = 0.3$.



Observation

Size increases, absolute value decreases, peak at zero looks more pronounced

Conclusions and Outlook

Conclusions

- Dynamical axial charge generation and topological dynamics at strong coupling
- Estimate of size of sphalerons at strong coupling
- Fluctuations of electric current are experimental observable

Outlook

- Improved holographic models closer to phenomenology
- Full non-linear, 3+1 dimensional dynamics with time-dependent magnetic fields

Thank you for your attention!