Holographic CME with dynamical axial charge

Holographic Perspectives on chiral transport

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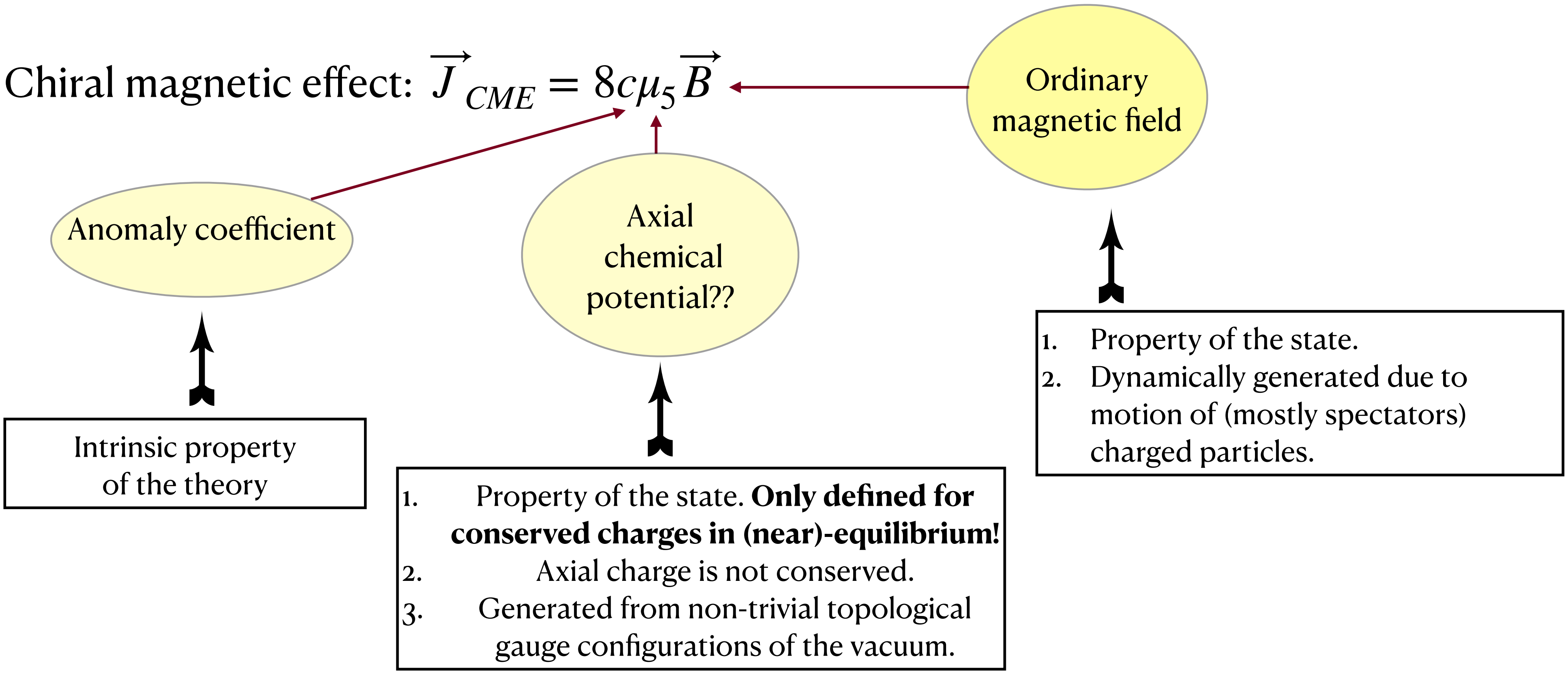
EXCELENCIA SEVERO OCHOA

Outline

- Introduction
- Holographic model: $U(1)_V \times U(1)_A$ + Stueckelberg ⟶ Real-time simulations
- Non-expanding plasma:
    - LHC/RHIC-like simulations.
- Expanding plasma.

Introduction

- Chiral magnetic effect: $\overrightarrow{J}_{CME} = 8c\mu_5\overrightarrow{B}$



Introduction

- Physical system and setup :

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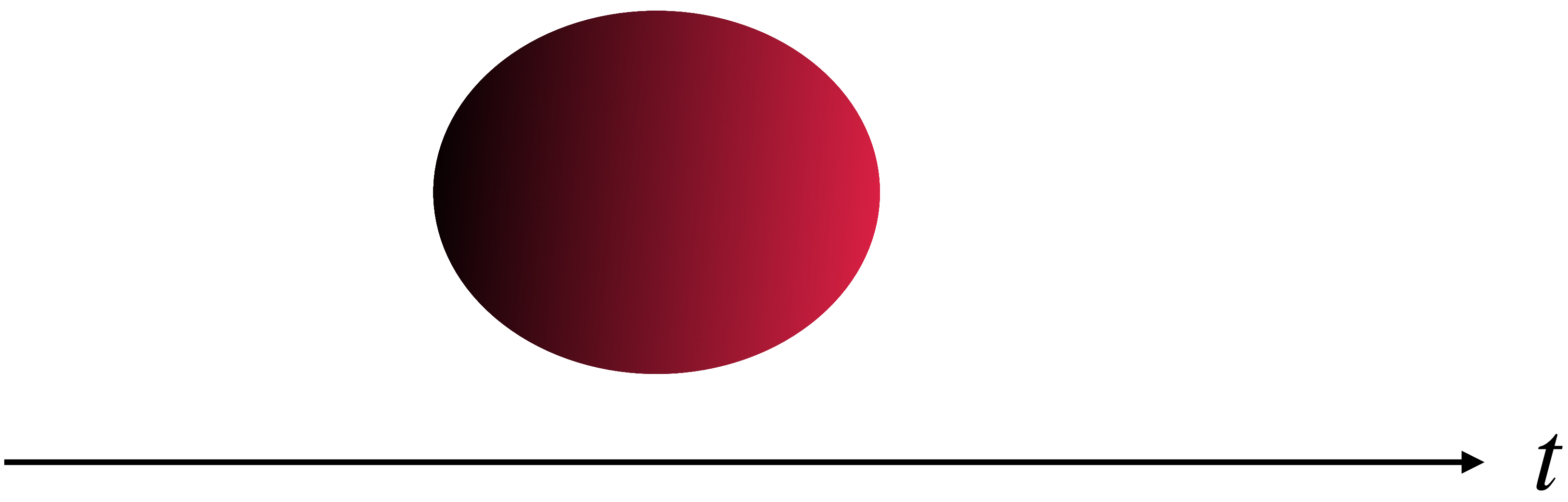

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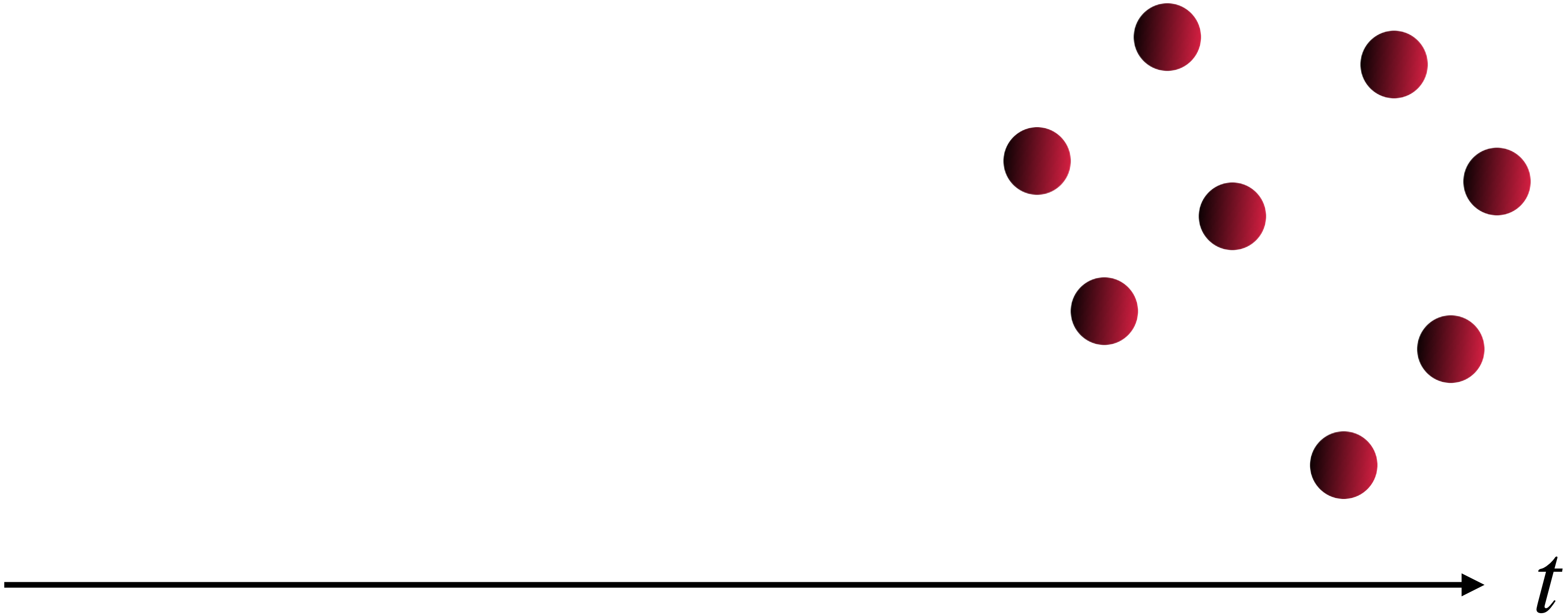
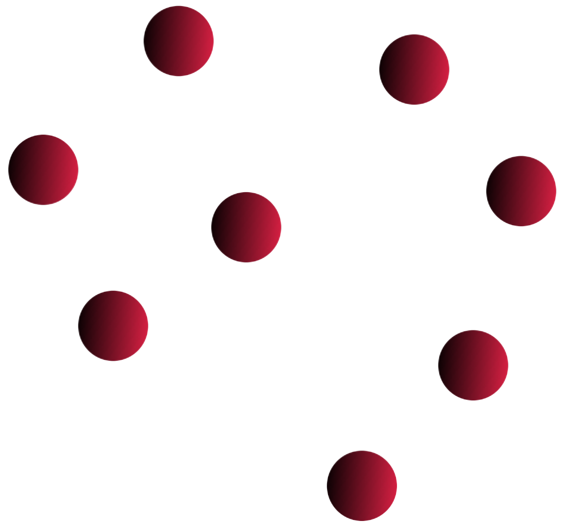


Introduction

- Physical system and setup :
  - Off-centered collision -> Almond shape
  - Strongly coupled plasma -> Good for holography
  - Short-lived magnetic field $\tau_B^{RHIC} \simeq 0.6\, fm/c$

    $\tau_B^{LHC} \simeq 0.02\, fm/c$



- Focus on non-conservation of axial charge

Introduction

- Holographic dictionary

QFT	AdS
Energy momentum tensor $T^{\mu\nu}$	Metric $g_{\mu\nu}$
Conserved current J^{μ}	Gauge field A_{μ}
Scalar operator $\mathcal{O}$	Scalar field ϕ
Temperature T	Black hole

Anomalies in currents in QFT due to coupling to non-dynamical gauge field. ($F\tilde{F}$ term in QCD axial anomaly) ⟷ Chern-Simons term in the bulk

Anomalies in currents in QFT due to dynamical gauge field. ($G\tilde{G}$ term in QCD axial anomaly) ⟷ Mass term for the gauge field in the bulk

Holographic model

$$S = \frac{1}{2\kappa^2}\int_{\mathscr{M}} d^5x\sqrt{-g}\,\Big[R+\frac{12}{L^2}-\frac{1}{4}F^2-\frac{1}{4}F^2_{(5)}-\frac{m^2}{2}(A_\mu-\partial_\mu\theta)(A^\mu-\partial^\mu\theta)$$
$$+\frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}(A_\mu-\partial_\mu\theta)\Big(3F_{\nu\rho}F_{\sigma\tau}+F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\Big)\Big] + S_{GHY}+S_{ct}$$

$F_{(5)} = dA$
$F = dV$
$[J_5] = 3+\Delta$
$\Delta = -1+\sqrt{1+m^2}$

- Gauge invariance $A_\mu \to A_\mu + \partial_\mu\lambda\,;\;\; \theta \to \theta + \lambda\,.$
- θ dual field to $Tr\{G\tilde{G}\}$
- The number of counterterms diverges as $\Delta \to 1$ and is minimum for $\Delta < \frac{1}{3}$ ➡ Focus on $\Delta \in (0, 1/3)$

”Anomalous Magneto Response and the Stüeckelberg Axion in Holography”
arXiv:1407.8162v1 [A. Jimenez-Alba, K. Landsteiner, L. Melgar]

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- We study CME and Axial Charge in:
  - Non-expanding plasma ⟶ Explore parameter space (α, Δ)
    Explore different states $(B, T, n_5(0), \dot{n}_5(0))$
    Simulations for RHIC and LHC
  - Expanding plasma ⟶ Simulations for RHIC

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Focus of the talk

Non-expanding plasma

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Asymptotically AdS

- Asymptotic solution:

$$\Sigma \simeq \frac{1}{u} + \dots$$

$$\theta \simeq \theta_0(v) + \dots$$

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Simplification!
Small charge

$$\Sigma \simeq \frac{1}{u} + \dots \qquad \theta \simeq \theta_0(v) + \dots$$

Justification!

$n_5/s \lessapprox 0.1$ in collisions

$$\xi \simeq u^4\left(\xi_4 - \frac{B^2}{12}\log u\right) + \dots \qquad V \simeq u^2 V_2(v) + \dots$$

[S. Shi, Y. Jiang, E. Lilleeskov, J. Liao](2018)

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Interest!
Numerical simplification
and stability

Non-expanding plasma

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QFT quantities

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Asymptotically AdS

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Initial state

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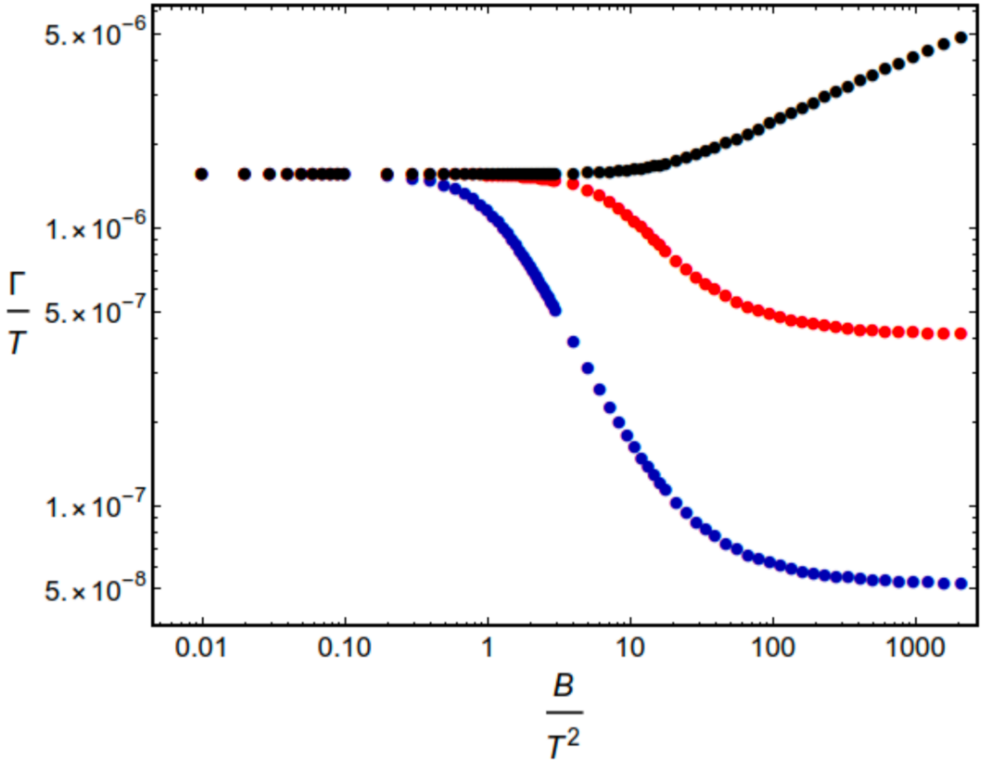
Static Background

Quasinormal modes

$\Delta << 1$



$\alpha = 0$

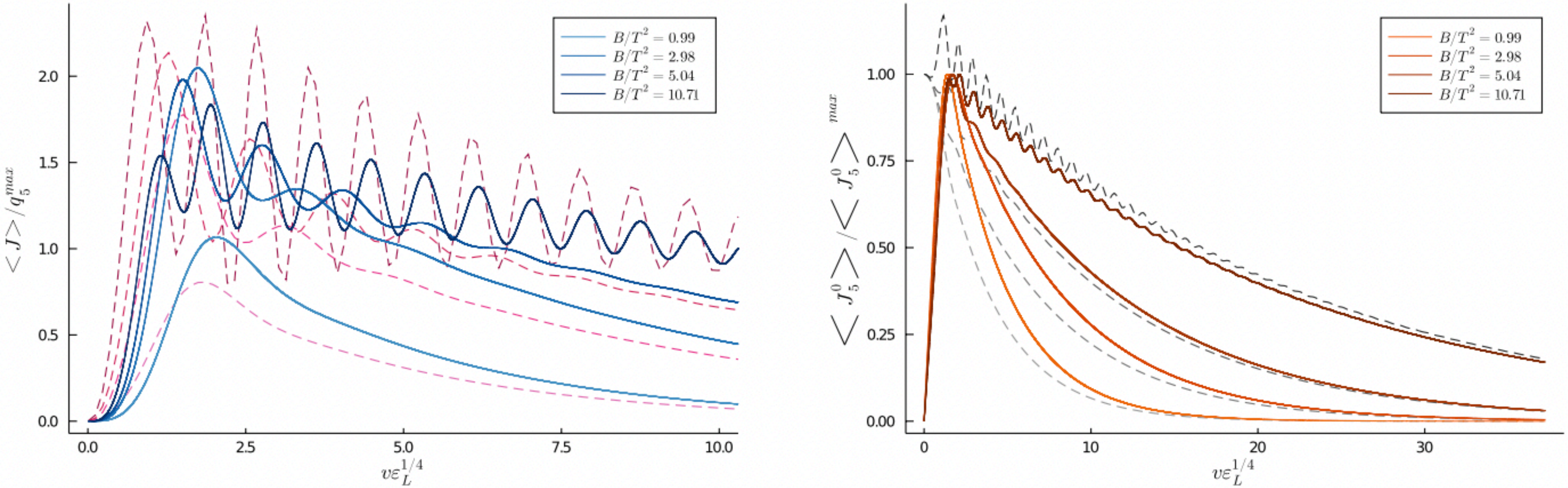
$\alpha = 0.316$

$\alpha = 2.5$

$$n_5 = n_5(0)e^{-\Gamma t}$$

$$\frac{1}{\tau} = \frac{\Gamma_{CS}}{2T\chi_A}$$

Quasinormal modes



RHIC and LHC simulations

Centrality bin	10 – 20%	20 – 30%	30 – 40%	40 – 50%
$(n_5/s)_0$	0.065	0.078	0.095	0.119
T_0 (GeV)	0.341	0.329	0.312	0.294
$eB_{max}(m_\pi^2)$	2.34	3.1	3.62	4.01
T_{sim} (GeV)	0.429	0.414	0.393	0.370
$eB_{sim}(m_\pi^2)$	1.87	2.48	2.90	3.20

Centrality bin	10 – 20%	20 – 30%	30 – 40%	40 – 50%
$(n_5/s)_0$	0.039	0.045	0.059	0.075
T_0 (GeV)	0.48	0.47	0.43	0.40
$eB_{max}(m_\pi^2)$	59.2	78.5	91.7	101.6
T_{sim} (GeV)	0.87	0.85	0.78	0.73
$eB_{sim}(m_\pi^2)$	2.28	3.02	3.53	3.91

$$B(\tau) = \frac{B_{max}}{1 + \tau^2/\tau_B^2} \qquad T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$$

[S. Shi, Y. Jiang, E. Lilleeskov, J. Liao](2018)

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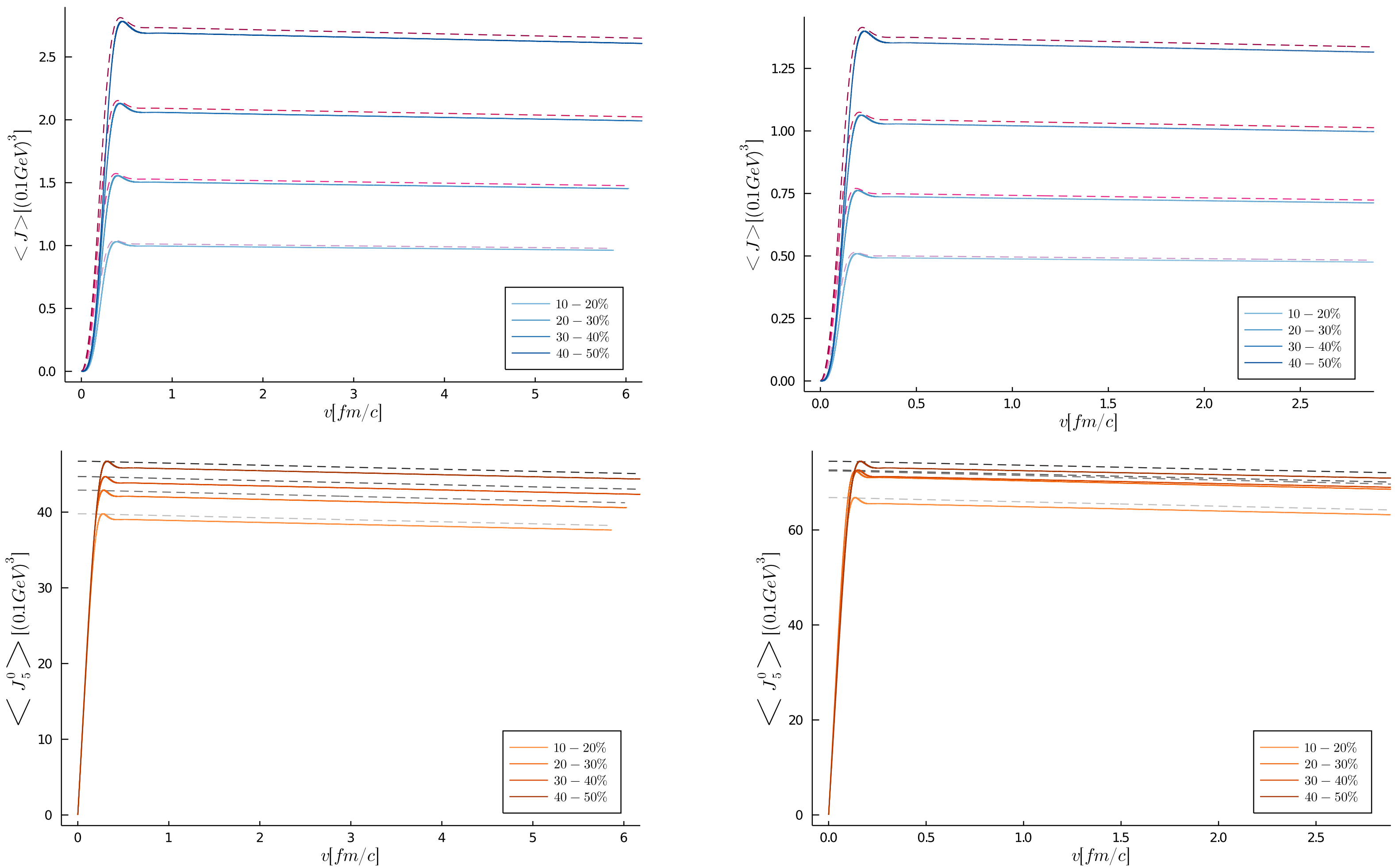

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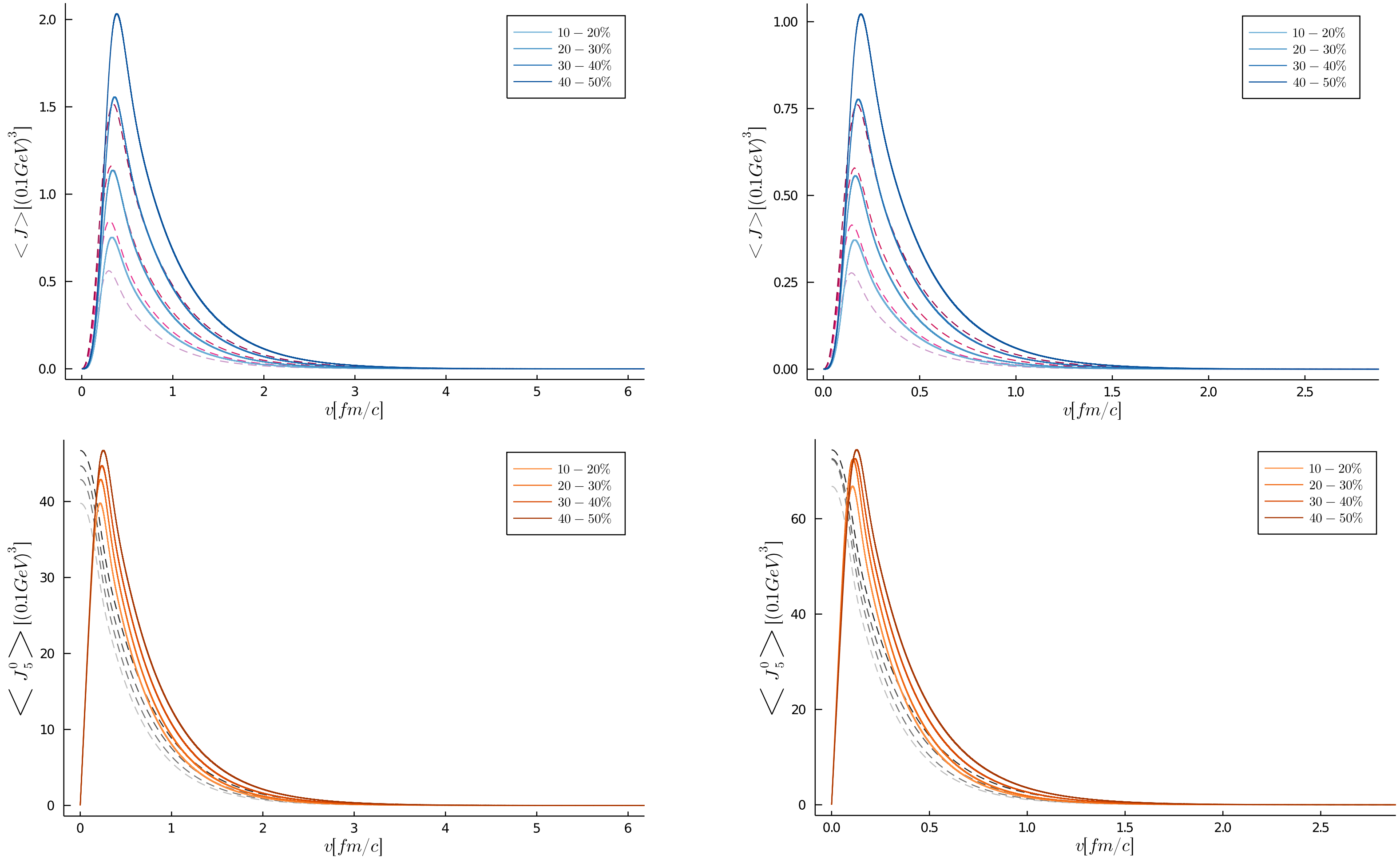

Expanding plasma

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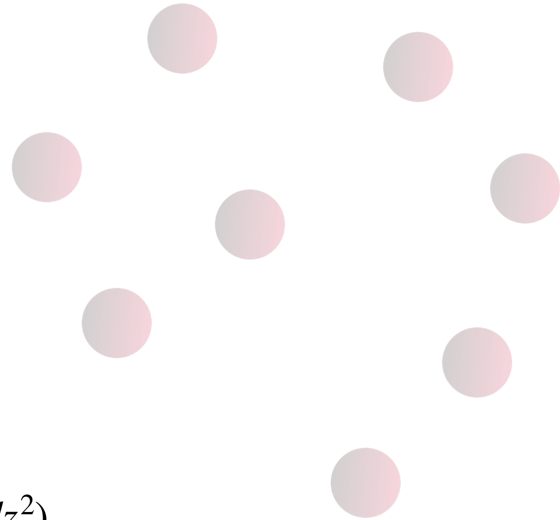

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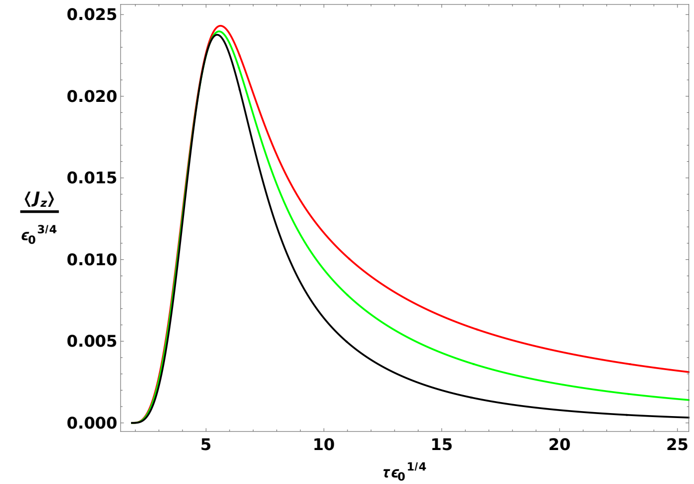
$$\lim_{u\to 0} ds^2 = \frac{1}{u^2L^2}(-dv^2+v^2dx^2+dy^2+dz^2)$$

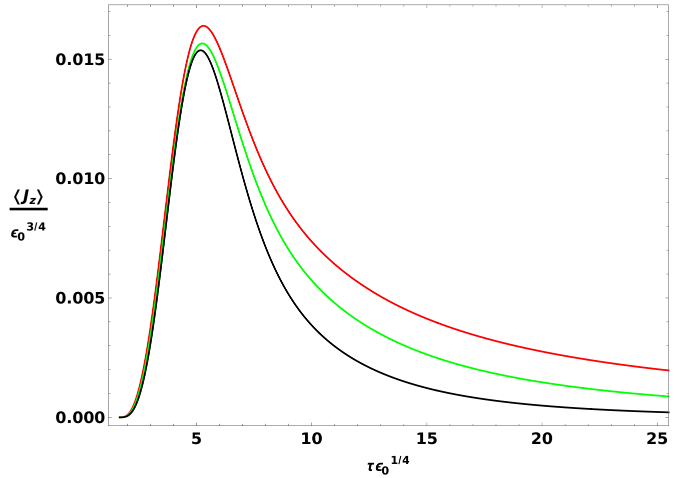
”Energy dependence of the CME in expanding holographic plasma” arXiv:2112.13857
[C.Cartwright, M. Kaminski, B. Schenke]

Expanding simulations

$T = 300 MeV$

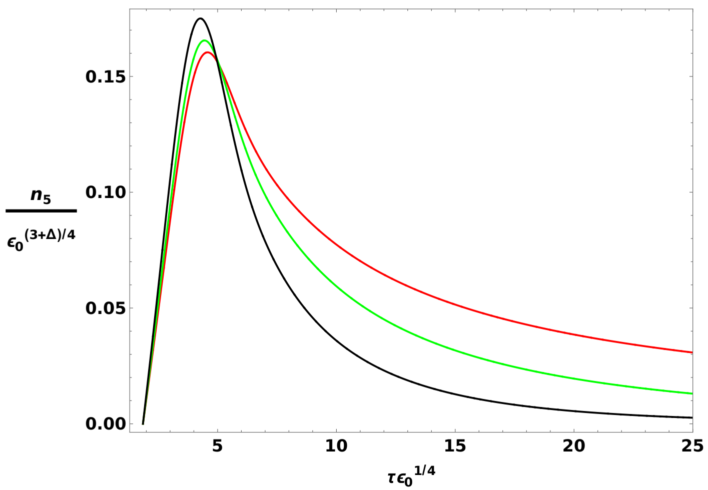
$B = m_\pi^2$

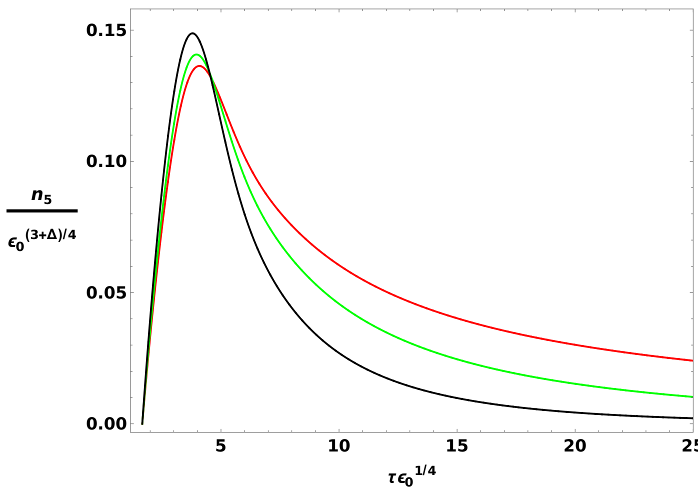




$T = 266 MeV$

$B = 0.65 m_\pi^2$





Summary & Outlook

- Axial charge dissipation weakens at $B \upuparrows$ in the presence of the anomaly.
- For small values of Δ both initial states considered result into the same CME signal. For bigger values of Δ the CME signal may be up to 25% smaller.
- The CME signal obtained for RHIC and LHC are of the same order. The threshold between higher signal at RHIC or LHC is at $n_5^{LHC}/n_5^{RHIC} \sim 3$.
- The different lifetimes of B in both simulations is key to arrive at these conclusions.
- Go beyond small charge limit, include time dependent B, include masses of quarks...