Chiral magnetohydrodynamics, holography & EFT

2205.03619 [hep-th]

with Ruth Gregory and Nabil Iqbal

(Holographic Model)

2212.09787 [hep-th]

with Nabil Iqbal and Napat Poovuttikul

(Towards an EFT)

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Symmetries of QED (Lessons from Nabil's and Saso's talks)

$$S_{EM}[A_\mu, \psi] = \int d^4x \left(-\frac{1}{g^2}F^2 \quad + \quad \overline{\psi}\gamma^\mu \left(\partial_\mu - iA_\mu\right)\psi \right)$$

- $U(1)_V$ Vector Symmetry: $\partial_\mu j^\mu_V = 0$, where $j^\mu_V = \overline{\psi}\gamma^\mu\psi$ → gauged away and no longer a global symmetry

- $U(1)^{(1)}$ 1 - form symmetry: New 1-form global symmetry associated to the conservation of magnetic flux (Bianchi Identity)

$$J^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \qquad \nabla_\mu J^{\mu\nu} = 0 \qquad \begin{cases} J^{0i} = \mathscr{B}^i \\ J^{ij} = \epsilon^{ijk}\mathscr{E}_k \end{cases}$$

- $U(1)_A$ Axial Symmetry: $\partial_\mu j^\mu_A = 0$, where $j^\mu_A = \overline{\psi}\gamma^\mu\gamma^5\psi$, is a (classical) symmetry which (now) at the quantum level leads to ABJ anomaly,

$$\partial_\mu j^\mu_A = -\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = \frac{1}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}J^{\mu\nu}J^{\rho\sigma} = k\epsilon_{\mu\nu\rho\sigma}J^{\mu\nu}J^{\rho\sigma}$$

(True situation is more subtle; recently explained in terms of non-invertible symmetry: Choi, Lam, Shao: Cordova, Ohmori; Karasik; Etxebarria, Iqbal)

Previous work

- Previous work: MHD with higher-form symmetries

(Ref: Grozdanov, Hofman, Iqbal; Grozdanov, Poovuttikul)

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^{\mu\nu} = 0$$

(b^μ: magnetic field as hydro variable)

↑

(MHD)

- Phenomenological model: Anomalous MHD Eqns = Maxwell Eqns + anomaly contribution

$$\nabla \times \mathscr{B} + \partial_t \mathscr{E} = j_V \qquad j_V = \sigma \mathscr{E} + 4\mu_A \mathscr{B}$$

(CME)

('t Hooft anomaly hydro)

(Refs: Boyarsky, Frohlich, Ruchayskiy; Joyce, Shaposhnikov; others)

(Refs: Son, Surowka; Neilsen, Oz; others)

Chiral Charge Relaxation

- Using anomalous MHD equations, it can be shown,

(Ref: Figueroa, Florio, Shaposhnikov)

$$\dot{\mu}_A \sim -\frac{k^2 b^2}{\chi_A \sigma}\mu_A = -\Gamma_A \mu_A \qquad \Rightarrow \quad \mu_A = e^{-\Gamma_A t} \qquad (\text{with } \rho_A = \chi_A \mu_A) \qquad \ldots(2)$$

(μ_A = chiral chem. pot., χ_A = chiral susceptibility, Γ_A = dissociation rate)

- We are interested in computing Γ_A using holography.

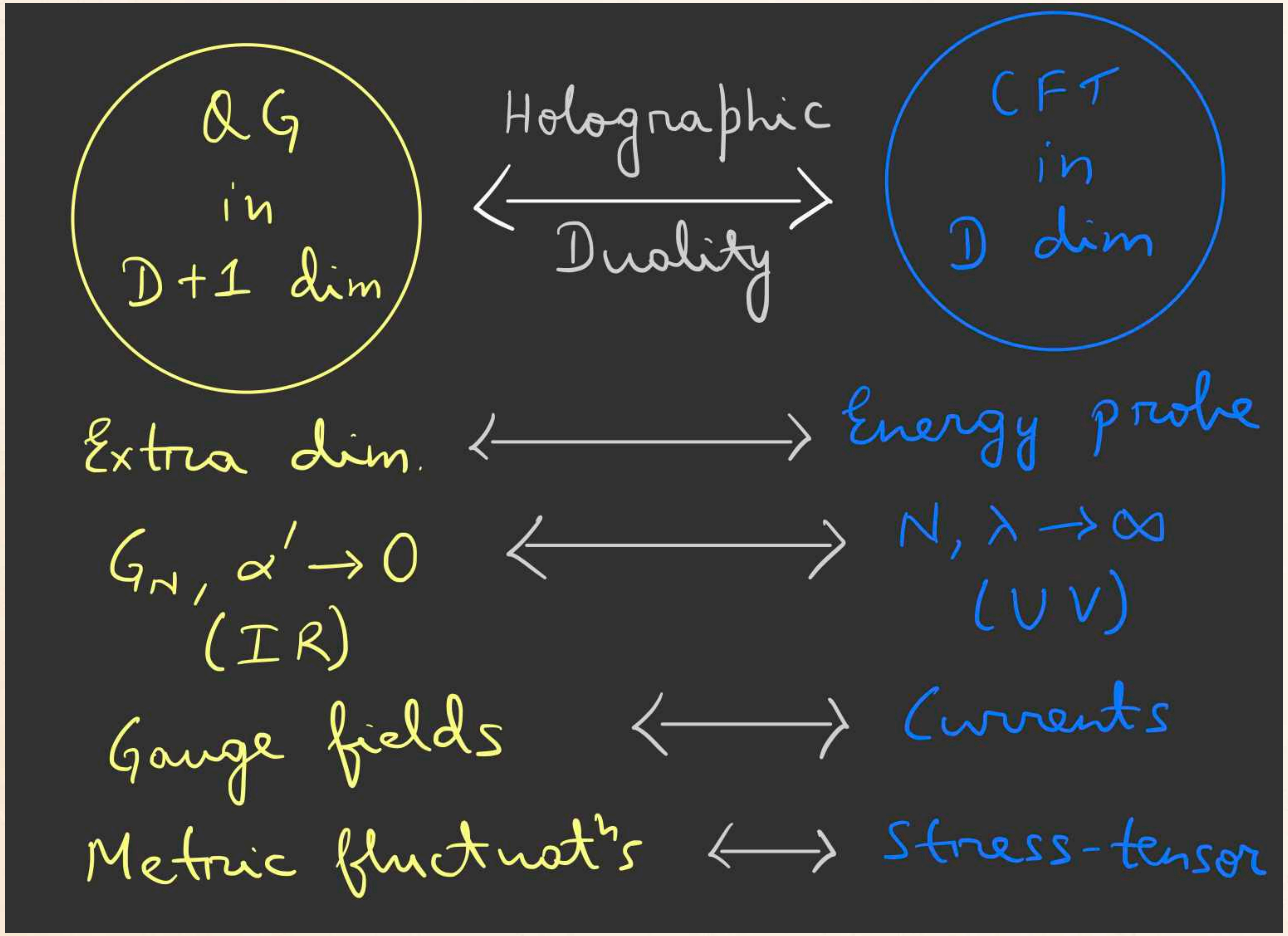
The set up

- There has been previous work in this direction where it has been found that $\Gamma_A = \zeta b^2$ (where $b =$ magnetic field). Few modern references are,
  - arXiv:1711.08450: Elementary physics arguments (of chiral MHD), self-consistent only if the anomaly coefficient k is treated perturbatively.
  - arXiv:1707.09967: Lattice simulations to obtain Γ_A and found a factor of 10 discrepancy in ζ, with above elementary physics arguments of chiral MHD.
- Weakly-coupled physics: Lagrangian describing a massless Dirac fermion ψ coupled to dynamical electromagnetism with photon A (with $F = dA$),

$$S[A,\psi] = \int d^4x \left(-\frac{1}{g^2}F^2 + \overline{\psi}\gamma^\mu \left(\partial_\mu - iA_\mu\right)\psi \right) \qquad ...(3)$$

- We shall construct and study a holographic model possessing the symmetries of the above action $S[A,\psi]$.

AdS/CFT



→

$$T^{\mu\nu} \sim \left.\frac{\delta S}{\delta g^{\mu\nu}}\right|_{r\to\infty}$$

$$j^{\mu} \sim \left.\frac{\delta S}{\delta A^{\mu}}\right|_{r\to\infty}$$

Holographic Construction

- Recall, we had two currents, j_A^μ and $J^{\mu\nu}$. So, $\longrightarrow$ $$\begin{aligned} E_1 &\leftrightarrow j_A^\mu \\ B_2 &\leftrightarrow J^{\mu\nu} \end{aligned}$$

- Gauge invariance: $B_2 \to B_2 + d\Lambda_1$ $\longrightarrow$ (leads to conservation of $J^{\mu\nu}$)

- However, E_1 is a 1-form with mutilated gauge invariance to allow for the non-conservation,

$$d \star j_A = k\, J \wedge J$$

Poincare Duality

- Start with the following well-understood action,

$$S\left[A_1, V_1\right] = \int_{\mathcal{M}^5} \left(-\frac{1}{2} F_2 \wedge \star F_2 - \frac{1}{2} G_2 \wedge \star G_2 - k A_1 \wedge F_2 \wedge F_2 - \frac{k}{3} A_1 \wedge G_2 \wedge G_2 \right)$$

$$(F_2 = dV_1,\ \ G_2 = dA_1)$$

('t Hooft anomaly)

↑

- Gauge-invariance (upto boundary terms):

$$A_1 \rightarrow A_1 + d\lambda_0 \qquad V_1 \rightarrow V_1 + d\tilde{\lambda}_0 \qquad \longrightarrow \qquad \partial_\mu j^\mu_A = k\epsilon^{\mu\nu\rho\sigma}\left(F_{\mu\nu}F_{\rho\sigma} + \frac{1}{3} G_{\mu\nu}G_{\rho\sigma} \right)$$

Now gauge away V_1 on the boundary $\leftrightarrow$ performing a bulk Poincare duality leading to the replacement: $V_1 \leftrightarrow B_2$. In this process $A_1 \rightarrow A_1 - d\phi_0 =: E_1$.

The bulk action

$$S_5 = \int_{\mathscr{M}^5} \left| dE_1 \right|^2 + \left| dB_2 \right|^2 + k\, E_1 \wedge \star dB_2 \wedge \star dB_2 + \cdots$$

(contains "mass" terms for E_1 like $(dB_2) \cdot E_1 \cdot (dB_2)$)

$\left(\mathscr{M}_5 = \mathrm{Sch} - AdS_5 \right)$

- As stated before, the boundary duals of the bulk 2-form field B_2 and the bulk 1-form field E_1 are the 2-form current $J^{\mu\nu}$ and the 1-form current j_A^μ respectively. Thus,

$$J^{\mu\nu} = \frac{\delta S_5}{\delta B^{\mu\nu}(\infty)} = \frac{\delta S_5}{\delta \left(\partial_r B_{\mu\nu} \right)} \qquad j_A^\mu = \frac{\delta S_5}{\delta E^\mu(\infty)}$$

- From above eqns. (and using EoMs), we can retrieve,

$$\partial_\mu J^{\mu\nu} = 0 \qquad \partial_\mu j_A^\mu = k \epsilon_{\mu\nu\rho\sigma} J^{\mu\nu} J^{\rho\sigma} \longrightarrow$$ "same symmetry structure"

Bulk action

$$S_{5p} = \int_{\mathcal{M}^5} \sqrt{-g}\, d^5x \left[\left\{ -\frac{H^2}{12} - \frac{k}{12}\epsilon_{PQRMN} H^{PQR} E_D H^{DMN} + k^2 \left((E \cdot H)^2 - \frac{2}{3} E^2 H^2 \right) \right. \right.$$
$$\left. -k^3 \epsilon_{PQRMN} E^P E_L H^{LQR} E_J H^{JMN} + 4k^4 \left(E^2 (E \cdot H)^2 - \frac{1}{3} E^4 H^2 \right) \right\} \tilde{c}_1^2$$
$$\left. + \left\{ \frac{k}{12} \epsilon^{PQRMN} E_P G_{QR} G_{MN} - \frac{1}{4} G^2 \right\} \right] \qquad (3.26)$$

$(\forall\ k)$

$$S[E, B] = \int d^5x \sqrt{-g} \left[-\frac{1}{4} G^2 - \frac{1}{12} H^2 + k^2 (E \cdot H)^2 - \frac{k}{12} \epsilon_{PQRMN} H^{PQR} E_L H^{LMN} \right]$$

$\left(\text{upto } \mathcal{O}(k^2)\right)$

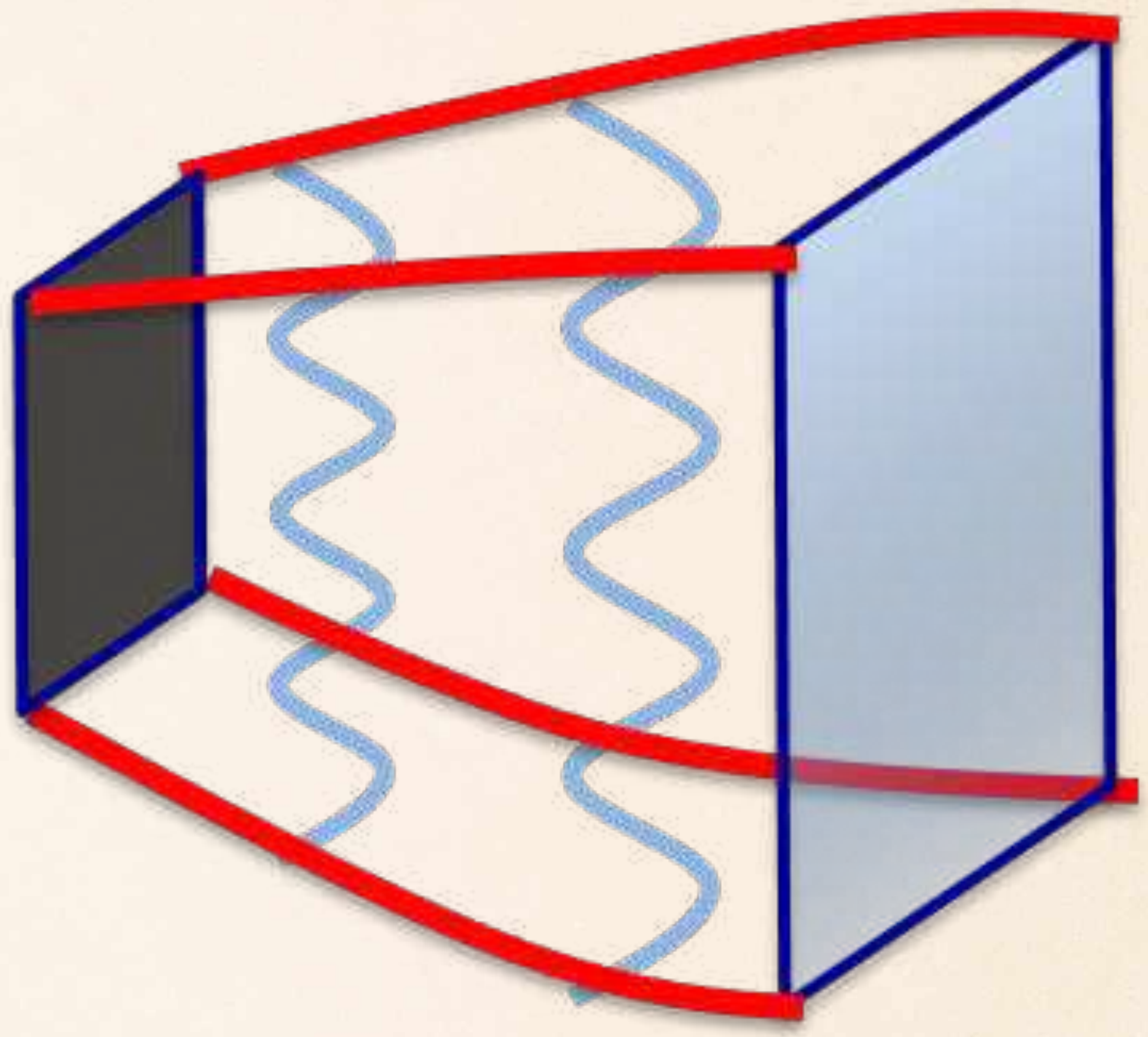
$$\left(G = dE,\ H = dB,\ (E \cdot H)^2 = E^L H_{LMN} H^{PMN} E_p,\ \tilde{c}_1 = \frac{1}{1 + 4k^2 E^2} \right)$$

Compute QNMs

- We are interested in the strong coupling calculation of the dissociation rate Γ_A on the field theory side.
- This rate is dual to the ω_{qnm} on the gravity side which is now in the weak coupling regime.
- So we need to compute the QNM of the Sch $- AdS_5$ black hole to get an estimate on the dissociation rate.

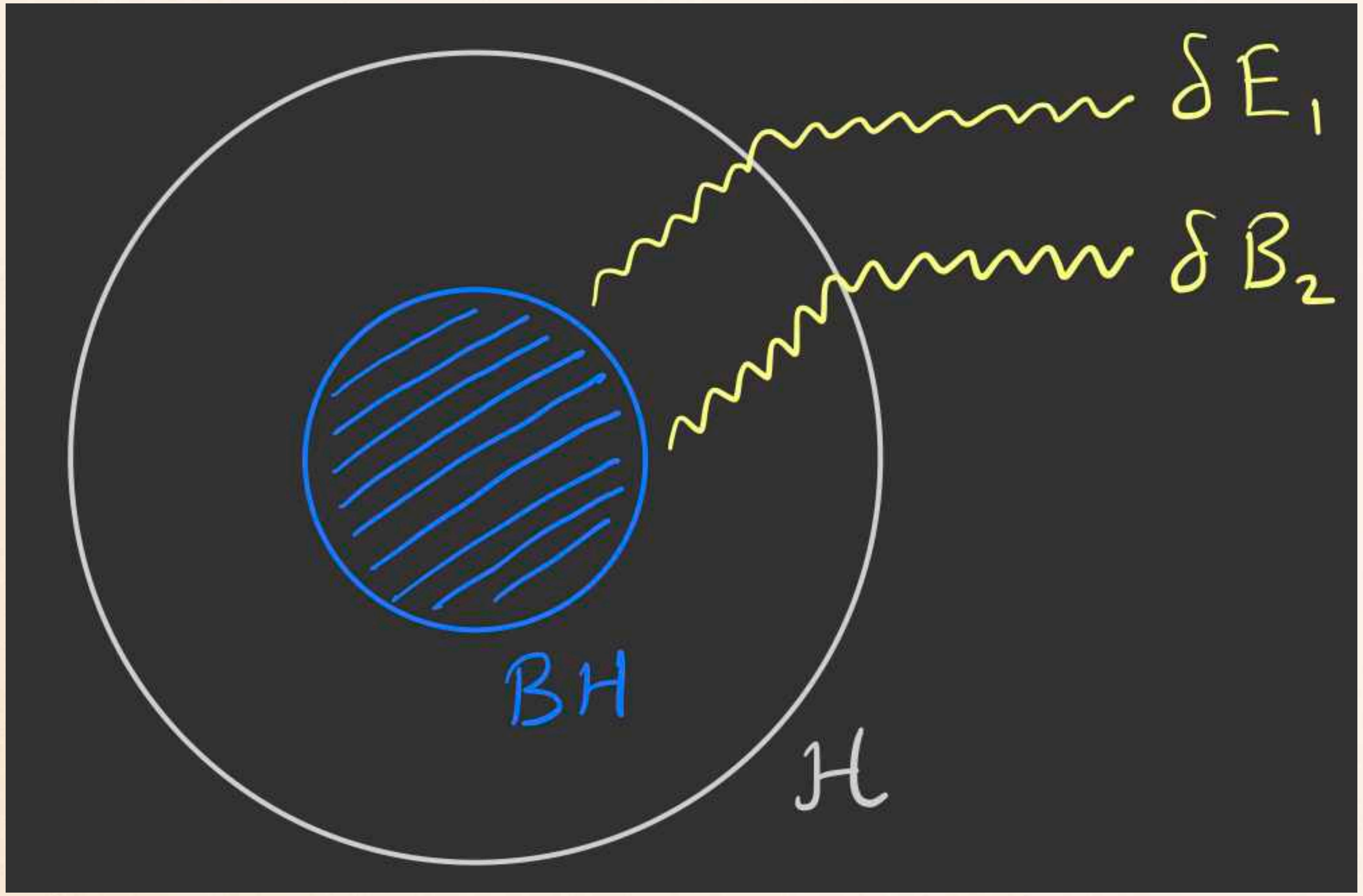
(work in probe limit & ignore gravitational back-reaction)

(background - planar black brane)



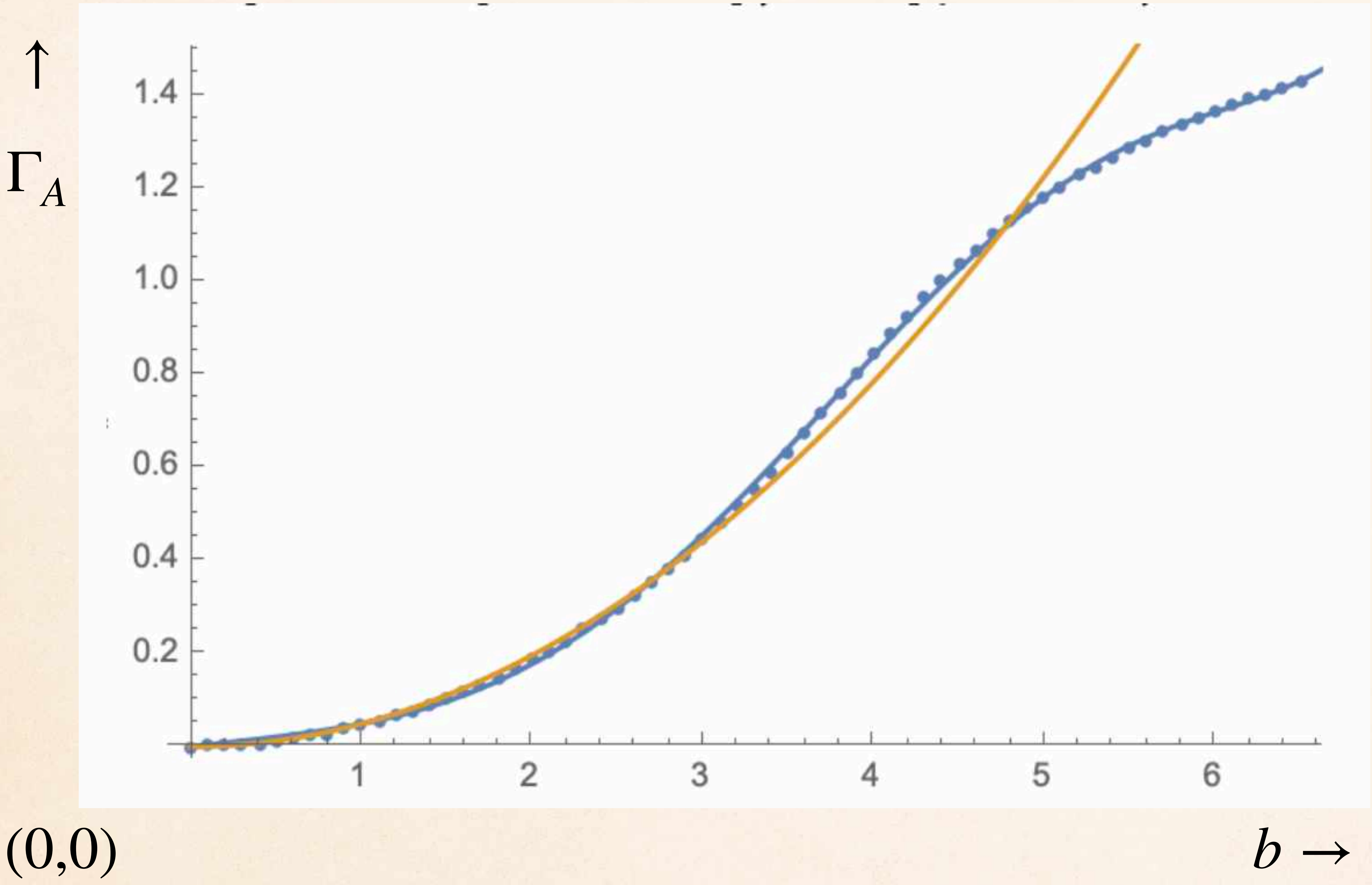
- Heat up S_5, put in Sch-AdS interior. Look at fluctuations in usual manner and compute QNMs

Further steps



- We study the bulk model in a Schwarzschild $- AdS_5$ background.
- Next, we move on to find the background magnetic field solution of EoMs of the action S_5 and study fluctuations of δE_1 and δB_2 in this background.
- We numerically solve for these fluctuations (by solving their EoMs) and then compute the lowest QNM from these solutions by imposing infalling boundary conditions at the horizon.

Results



"universal hydro description at small b?"

- This QNM $\Gamma_A(b)$ goes as (for a small neighbourhood around $b = 0$),

$$\Gamma_A(b) \; = \zeta \, b^2 \qquad \ldots(4)$$

- ζ (above) is in agreement with previous chiral MHD results but differs from lattice simulations.

- However, the nice quadratic behaviour of $\Gamma_A(b)$ stops as $b >> 0$.

Towards an EFT I

- Need to construct real-time (fields are doubled) effective theory in which the axial current is non-conserved in the right way: rough structure of the action (very-schematic),

$$\mathcal{Z}[a_1, a_2] = \int [d\phi_1, d\phi_2] \exp\left(S\left(a_1 - d\phi_1, a_2 - d\phi_2\right)\right) \qquad \mathcal{Z}[b_1, b_2] = \int [d\gamma_1, d\gamma_2] \exp\left(S\left(b_1 - d\gamma_1, b_2 - d\gamma_2\right)\right)$$

$$\mathcal{Z}[b, a] = \int [d\gamma d\phi] \exp\left(S_{inv} + k\phi\, J \wedge J\right) \qquad j^\mu = \frac{\delta \mathcal{Z}}{\delta a_\mu} \qquad J^{\mu\nu} = \frac{\delta \mathcal{Z}}{\delta b_{\mu\nu}}$$

- Verified that we can construct (two of) non-invertible defect operators → microscopic symmetries are realised in the dissipative action.

Full disclosure: difficult to realise phase correctly: “chemical shift” is violated. (Work in progress)

(Ref: Crossley, Glorioso, Liu; Grozdanov, Leutheusser, Liu, Vardhan)

Towards an EFT II

- If we ignore this, we find anomaly-induced term

$$E = \rho \left(\chi_b^{-1} \nabla \times B - 2k\, \mu_5 B \right) + \ldots$$

(Same as "pheno", but now coefficients have universal meaning...)

- Universality: $\zeta \sim \dfrac{k^2 \rho}{\chi_A} \sim \dfrac{Im\left(E^i E^i\right)}{\chi_A}$

$(\Gamma_A = \zeta b^2)$

(Ref: Grozdanov, Hofman, Iqbal)

Future Directions

- Fix the chemical shift issue

- Hydro-loops? What happens to Γ_A (the chiral charge relaxation rate) in the limit of vanishing magnetic field - preliminary calculations suggest it is "vanishing". Understand from EFT why?

Thank You for listening…

Anomalies -> CME (Application)

- Eq.(1) implies (as $\tilde{F}^{\mu\nu}F_{\mu\nu} \sim \mathscr{E} \cdot \mathscr{B}$),

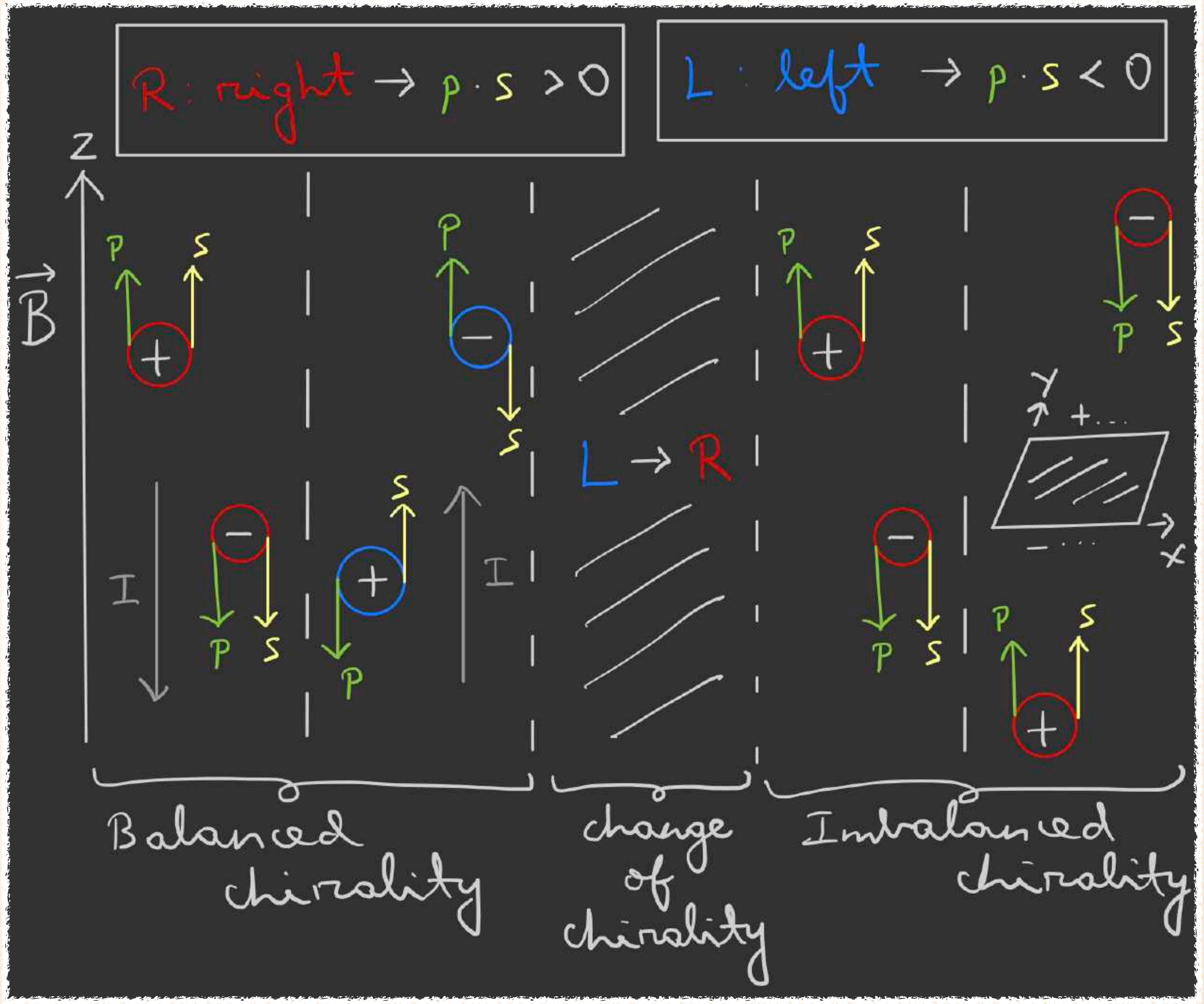
$$\dot{\rho}_A \sim \mathscr{E} \cdot \mathscr{B}$$

which means that in the presence of (external) parallel electric and magnetic fields, the axial charge density (ρ_A) can change.

- Such changes in axial charge density results in chirality imbalance and thereby leads to the generation of an electric current parallel to the external magnetic field:

Chiral Magnetic Effect (CME)

(Ref: Kharzeev, Landsteiner)



Towards an EFT - Equilibrium

- Now let us write down the equilibrium *effective* action:

$$\mathcal{S}_{\text{eqm}}\left[\psi\right] \sim \int_{\mathbb{R}^3} d^3x \left[\frac{\chi_A}{2}\left(a_\tau\right)^2 + \frac{\chi_B}{2}\left(B_i B^i\right) + \frac{\chi_O}{12}(db)_{ijk}(db)^{ijk} + \underbrace{k\, a_i V^i}\right]$$

(Gauge non-invariant term - to saturate anomaly)

- We can solve for V^i, in an order-by-order in k expression, using EoMs and the anomaly equation. We get,

$$V^i = \chi_B \chi_O \,|\, h \,|\, B^i + \frac{k}{2}\left(\chi_O \chi_B\right)\left[\chi_O \,|\, h\,|^2 a^i + \chi_B\,(a\cdot B)\,B^i\right] + \mathcal{O}(k^2)$$

- Now using expressions for currents and EoMs we get: $j^i \sim \underbrace{k\mu_{\text{el}}\mathcal{B}^i}_{\text{(CSE)}}$ (where, $\mathcal{E}_m \sim \partial_m \mu_{\text{el}}$).

Towards an EFT - Symmetries

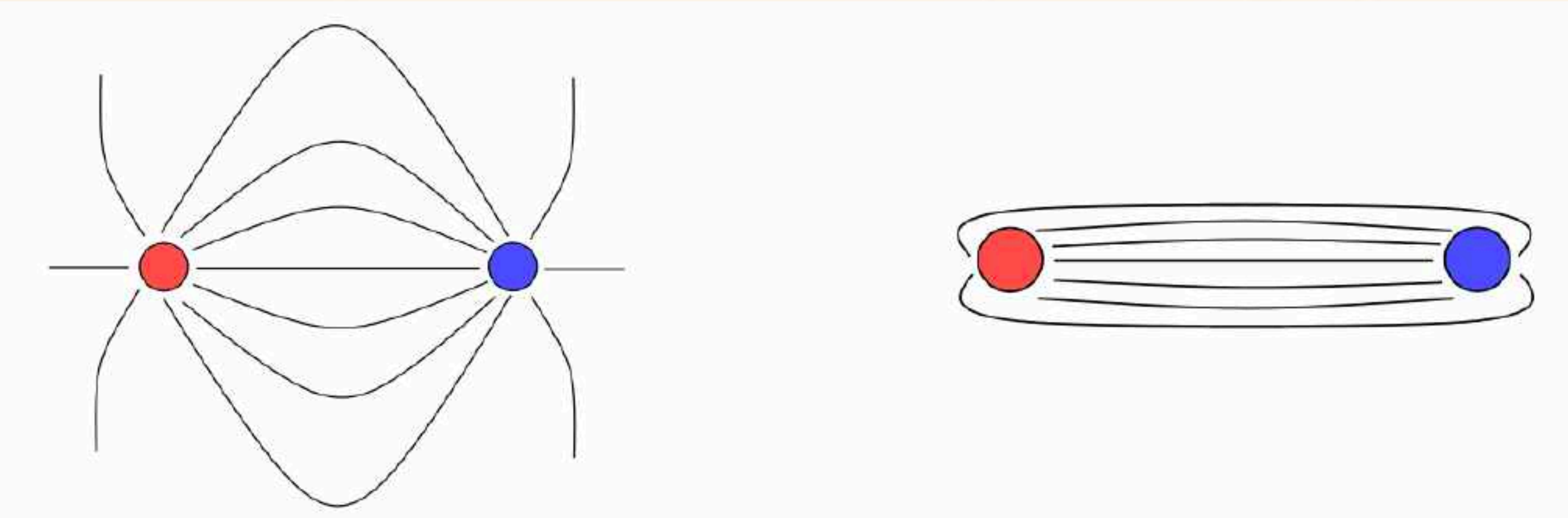
- First let us consider the theory on $S^1 \times \mathbb{R}^3$ (τ, x, y, z). $J^{\mu\nu}$ decomposes on $\mathbb{R}^3$ as:

  $U(1)^{(0)}$ 0-form symmetry $\to J^{i\tau} \to \mathcal{B}^i = J^{i\tau}$ is magnetic 3-vector   source: $b_{i\tau} \to b_{i\tau} + \partial_i \Lambda_\tau$

  $U(1)^{(1)}$ 1-form symmetry $\to J^{ij} \to \mathcal{E}^i = \frac{1}{2}\epsilon^{ijk} J_{jk}$ is electric 3-vector   source: $b_{ij} \to b_{ij} + \partial_i \Lambda_j - \partial_j \Lambda_i$

- $U(1)^{(0)}$ is *spontaneously broken* in the "normal phase" $\equiv$ magnetic field is unscreened in the deconfined plasma

(Ref: Iqbal, Poovuttikul; Tong)



$\psi \to \psi + \Lambda_\tau(x^i)$ (Goldstone mode)

invariant combo: $B_i \equiv \psi_i - b_{i\tau}$

- j^μ decomposes on $S^1 \times \mathbb{R}^3$ into: j^τ and j^i. The sources are: $a_\tau \to a_\tau$ and $a_i \to a_i + \partial_i \lambda$.

(anomaly equation)

$$\left\{ \partial_i j^i = k\epsilon_{ijk} J^{ij} J^{k\tau} \right\}$$

Towards an EFT - Equilibrium (derivation)

- Now let us write down the equilibrium *effective* action:

$$\mathcal{S}_{\text{eqm}}[\psi] \sim \int_{\mathbb{R}^3} d^3x \left[\chi_A\left(a_\tau\right)^2 + \chi_B\left(B_i B^i\right) + \chi_O\,(db)_{ijk}(db)^{ijk} + \underbrace{k\, a_i V^i}\right]$$

(Gauge non-invariant term - to saturate anomaly)

- Currents: $j^\tau = \dfrac{\delta \mathcal{S}}{\delta a_\tau} = \chi_A\, a_\tau\,, \qquad j^i = \dfrac{\delta \mathcal{S}}{\delta a_i} = k\,V^i + k\,\dfrac{\delta V^j}{\delta a_i} a_j \quad \rightarrow \quad \partial_i j^i = k\,\partial_i\left[V^i + \dfrac{\delta V^j}{\delta a_i} a_j\right]$

$$J^{i\tau} = \frac{\delta \mathcal{S}}{\delta b_{i\tau}} = -\chi_B B^i + k\,\frac{\delta V^j}{\delta b_{i\tau}} a_j\,, \qquad J^{ij} = \frac{\delta \mathcal{S}}{\delta b_{ij}} = -\,\epsilon^{ijk}\partial_k f \qquad \text{where, } f = \epsilon^{ijk}\left[\frac{\chi_O}{12} h_{ijk} + \frac{k}{6} a_m \frac{\partial\left(V^m\right)}{\partial(\partial_k b_{ij})}\right]$$

$$\Rightarrow \partial_i\left[V^i + \frac{\delta V^j}{\delta a_i} a_j\right] = \epsilon_{ijk} J^{ij} J^{k\tau} = -\,2\left[\partial_i\left(J^{i\tau} f\right) - f\partial_i J^{i\tau}\right] = -\,2\partial_i(J^{i\tau} f) \quad \text{and, } \; V^i = V^i_{(0)} + k\,V^i_{(1)} + \mathcal{O}(k^2)$$

References

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