Non-invertible symmetries and Goldstone modes

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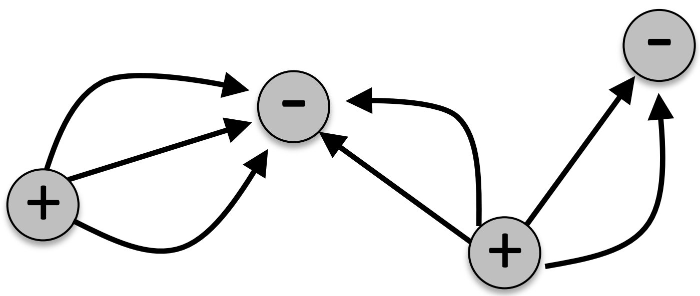
Symmetries of QED

In this talk, I'll discuss some new ways to think about a very familiar system: massless QED. (Applications to QCD as well).

$$S = \int d^4x \left[-\frac{1}{g^2} F^2 + \bar{\psi} \left(\partial\!\!\!/ + A\!\!\!/ \right) \psi \right]$$

Consists of massless fermions interacting via electric fields.



What are the symmetries, exactly?

Symmetries of QED

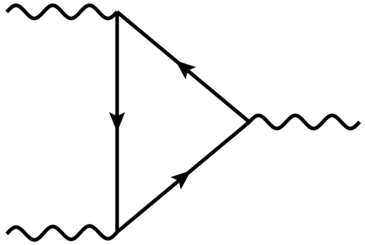
First, let us consider freezing EM; view A as an external gauge field.

$$S = \int d^4x \left[-\frac{1}{g^2} F^2 + \bar{\psi} \left(\partial\!\!\!/ + A\!\!\!/ \right) \psi \right]$$

Global symmetries are very well-understood:

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi \qquad \partial_\mu j_V^\mu = 0$$

$$j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi \qquad \partial_\mu j_A^\mu = \frac{1}{4\pi^2} F \wedge F$$



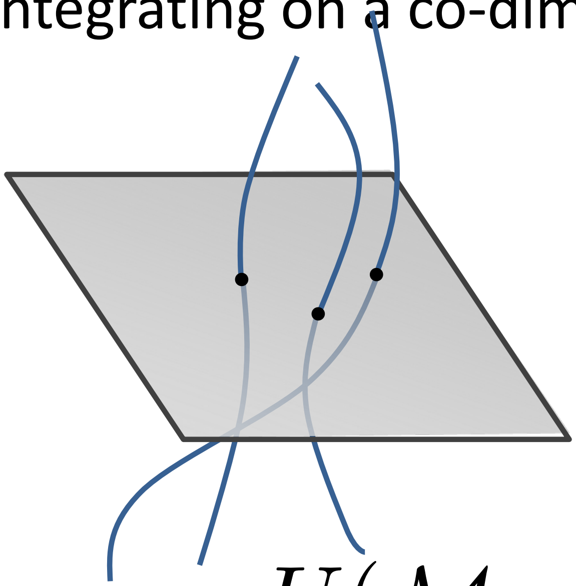
't Hooft anomaly: operator on RHS is a fixed external source. What about dynamical EM?

Generalizations of symmetry

To understand this, let's review ordinary 1-index currents first.

$$\nabla_\mu j^\mu = 0 \qquad d \star j = 0$$

An ordinary current counts particles; "catch them all" by integrating on a co-dimension 1 subspace. In Euclidean:



$$Q = \int_{\mathcal{M}_{d-1}} \star j$$

Defines a U(1)-valued topological codimension-1 surface operator.

$$U(\mathcal{M}_{d-1}) = \exp(i\alpha Q(\mathcal{M}_{d-1}))$$

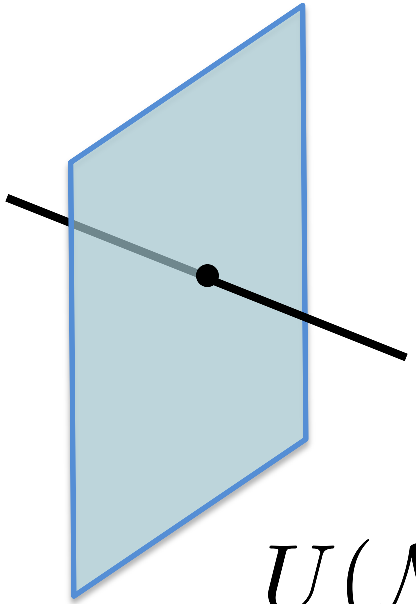
(Fancy way to talk about "conserved charge"; call this a 0-form symmetry)

Higher form symmetries

Now a 2-index current (Gaiotto, Kapustin, Seiberg, Willet):

$$\nabla_\mu J^{\mu\nu} = 0 \qquad d \star J = 0$$

A 2-index current counts strings; as they don't end in space or time, "catch them all" by integrating on a co-dimension 2 subspace:



$$Q = \int_{\mathcal{M}_{d-2}} \star J$$

Local current defines a U(1)-valued topological codimension-2 surface operator.

$$U(\mathcal{M}_{d-2}) = \exp(i\alpha Q(\mathcal{M}_{d-2}))$$

This is called a 1-form symmetry.

Symmetries of QED I

Now consider the situation where A is dynamical.

$$S = \int d^4x \left[-\frac{1}{g^2} F^2 + \bar{\psi} \left(\partial\!\!\!/ + A\!\!\!/ \right) \psi \right]$$

Vector current is gauged; no longer a global symmetry.

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi \qquad \partial_\mu j_V^\mu = 0$$

New 1-form global symmetry associated with conservation of magnetic flux.

$$J^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \qquad \nabla_\mu J^{\mu\nu} = 0$$

No magnetic monopoles means that magnetic field lines don't end. $J^{\mu\nu}$ counts magnetic flux density. (e.g. 4d photon is a Goldstone mode!)

Symmetries of QED II

$$S = \int d^4x \left[-\frac{1}{g^2} F^2 + \bar{\psi} \left(\partial\!\!\!/ + A\!\!\!/ \right) \psi \right]$$

How about the axial current?

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

$$\partial_\mu j_A^\mu = \frac{1}{4\pi^2} F \wedge F = -\frac{1}{4\pi^2} J \wedge J$$

Now the right hand side of the anomaly equation is an operator, not a fixed source; it appears that the current is simply not conserved?

True situation is somewhat more subtle…

Symmetries of QED II

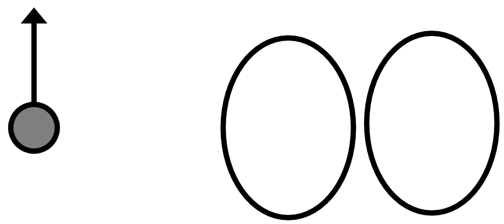
Summary of symmetries in massless QED:

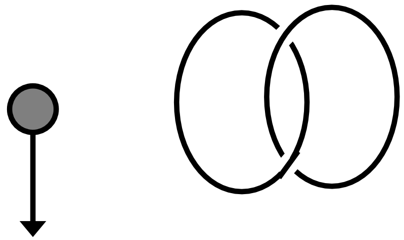
$$\partial_\mu J^{\mu\nu} = 0$$

$$\partial_\mu j^\mu_A = -\frac{1}{4\pi^2} J \wedge J$$

1-form symmetry for magnetic flux conservation.

Non-conservation of axial charge given in terms of 2-form current.



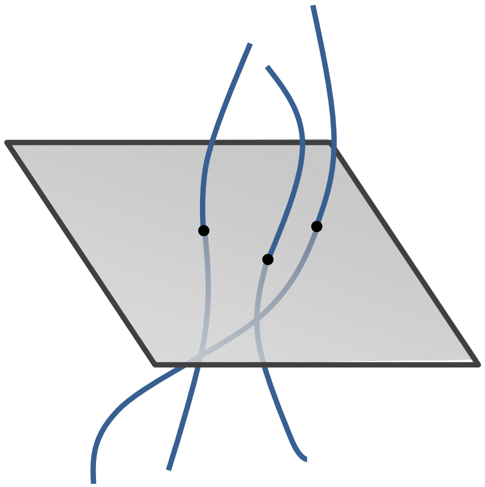


Right way to think about this was explained recently in terms of non-invertible symmetry (Choi, Lam, Shao; Cordova, Ohmori).

Non-invertible symmetry in QED

No conserved gauge-invariant current. Conserved charge?

$$Q(\mathcal{M}_3) = \int_{\mathcal{M}_3} \left(\star j_A - \frac{1}{4\pi^2} A \wedge dA \right)$$



Try and make a topological surface operator like before:

$$U(\mathcal{M}_3) = \exp\left(i\alpha Q(\mathcal{M}_3)\right)$$

This is gauge-invariant under small gauge transformations, but not large ones, unless α is 2*π* integer – but then the operator is always 1.

Feels like there is no useful conserved charge...

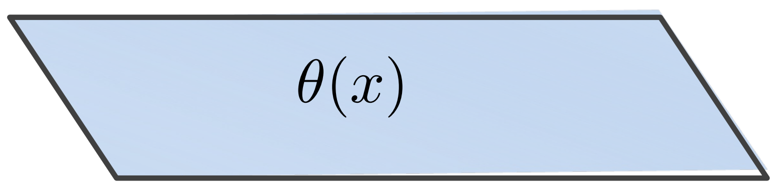
To fix this:

We can fix this by introducing a new degree of freedom. Original profound idea by Choi, Lam, Shao; Cordova, Ohmori. This reformulation by I. García-Etxebarria, NI; A. Karasik.

$$U_\alpha(\mathcal{M}_3) = \int [d\theta] \exp\left(i\alpha \int_{\mathcal{M}_3} \left(\star j_A - \frac{1}{4\pi^2}(A - d\theta) \wedge dA \right) \right)$$

$$\star \tilde{j}$$

Introduce a compact scalar field θ on the defect:



Transforms like a Higgs phase; this now defines a topological operator (i.e. a new conserved charge).

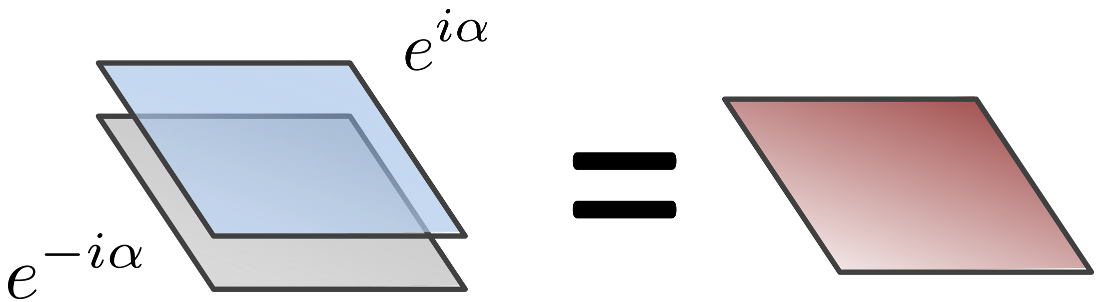
- There is now a gauge-invariant local current operator defined on the defect.
- Operator is topological! (can be interpreted as a conserved charge)

Some remarks

$$U_\alpha(\mathcal{M}_3) = \int [d\theta] \exp\left(i\alpha \int_{\mathcal{M}_3} \left(\star j_A - \frac{1}{4\pi^2}(A - d\theta) \wedge dA \right)\right)$$

$$\psi \to e^{i\alpha\gamma^5}\psi$$

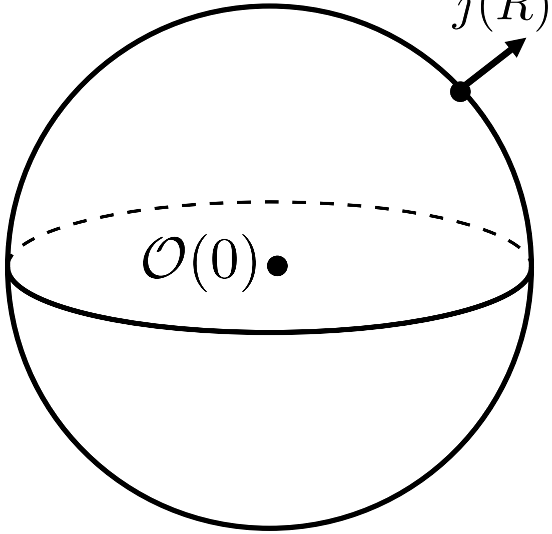
- Claim: this is the correct way to think about axial symmetry in QED.
- Called non-invertible symmetry (much recent work!), as it has no inverse; Due to new degree of freedom, acting with the opposite charge doesn't give a trivial operator.



- Opens up many doors: what is the phase structure of these symmetries? Is there a Goldstone theorem? (Yes.) Can we do hydrodynamics? (Wait ~5 mins). Etc. etc.

Goldstone's theorem I

Let's first formulate Goldstone's theorem for ordinary symmetries in Euclidean path-integral language (Hofman, NI).



Ward identity for U(1) current:

$$d\star j(x)\mathcal{O}(0) = iq\delta^{(4)}(x)\mathcal{O}(0)$$

Integrate both sides over interior of S^3.

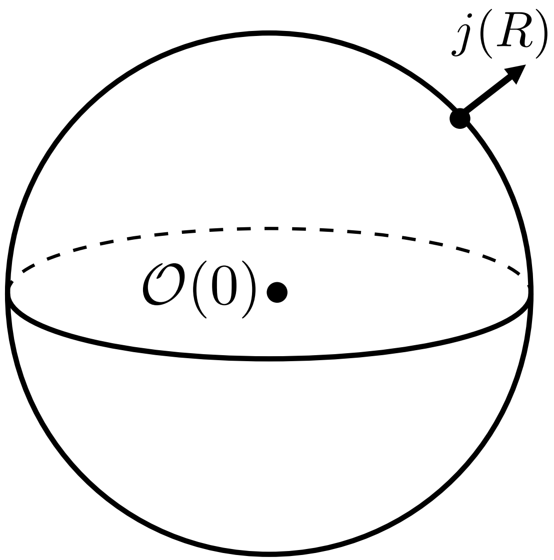
$$\int_{S^3} \star j(R)\mathcal{O}(0) = iq\mathcal{O}(0)$$

$$\langle j(R)\mathcal{O}(0)\rangle \sim \frac{iq\langle\mathcal{O}\rangle}{R^3}$$

If vev of charged operator is nonzero, must be a power-law correlation in theory; this is Goldstone mode.

Goldstone's theorem II: non-invertible

Let's now formulate Goldstone's theorem for these non-invertible symmetries.



Wrap the operator with the charge defect:

$$U_\alpha(S^3)O(0) = e^{iq\alpha}O(0)$$

Take a derivative with respect to symmetry parameter:

$$\int_{S^3} \star\tilde{j}(R)\mathcal{O}(0) = iq\mathcal{O}(0)$$

$$\langle \tilde{j}(R)\mathcal{O}(0)\rangle_c \sim \frac{iq\langle\mathcal{O}\rangle}{R^3}$$

Proof is essentially the same (except current operator is localized on defect). We have Goldstone mode.

Axion effective theory

What is the low-energy theory of the Goldstone mode? Axion theory:

$$S[\phi, A] = \int \left(\frac{1}{2\gamma^2} d\phi^2 + \frac{1}{4g^2} F^2 + i\phi F \wedge F + \cdots \right)$$

"Why" is this theory massless?

Often some words about "no U(1) instantons"; from our point of view, it is massless because it is the Goldstone mode of a spontaneously broken non-invertible symmetry.

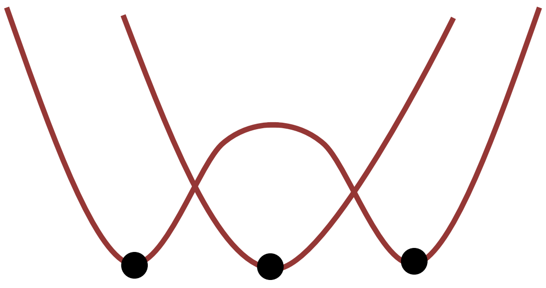
(Same words apply in application to chiral limit QCD: explanation of why π_0 is massless despite anomaly. Also protects masslessness of many fields in string theory.)

Future directions:

Physical applications of higher-form and non-invertible symmetries are still in their infancy (See Saso's talk yesterday). Basic questions are open:

Which systems have these symmetries? When is there a Goldstone theorem?

What is the phase structure? Phase transitions and critical phenomena?

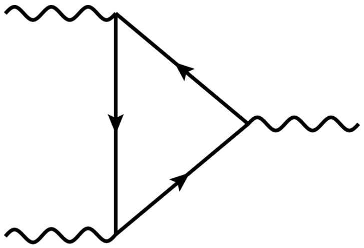


For QED: can we reformulate chiral magnetohydrodynamics with these non-invertible symmetries as a guiding principle? Arpit's talk next!

What is the new Landau paradigm built around these new symmetries?

Summary

- Non-invertible symmetry is a new way to think about systems with an ABJ anomaly.
- There is a Goldstone theorem for (some of) these non-invertible symmetries.
- A new non-invertible Landau paradigm?



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