Hot and Dense QCD at Strong Coupling & Neutron Star Phenomenology

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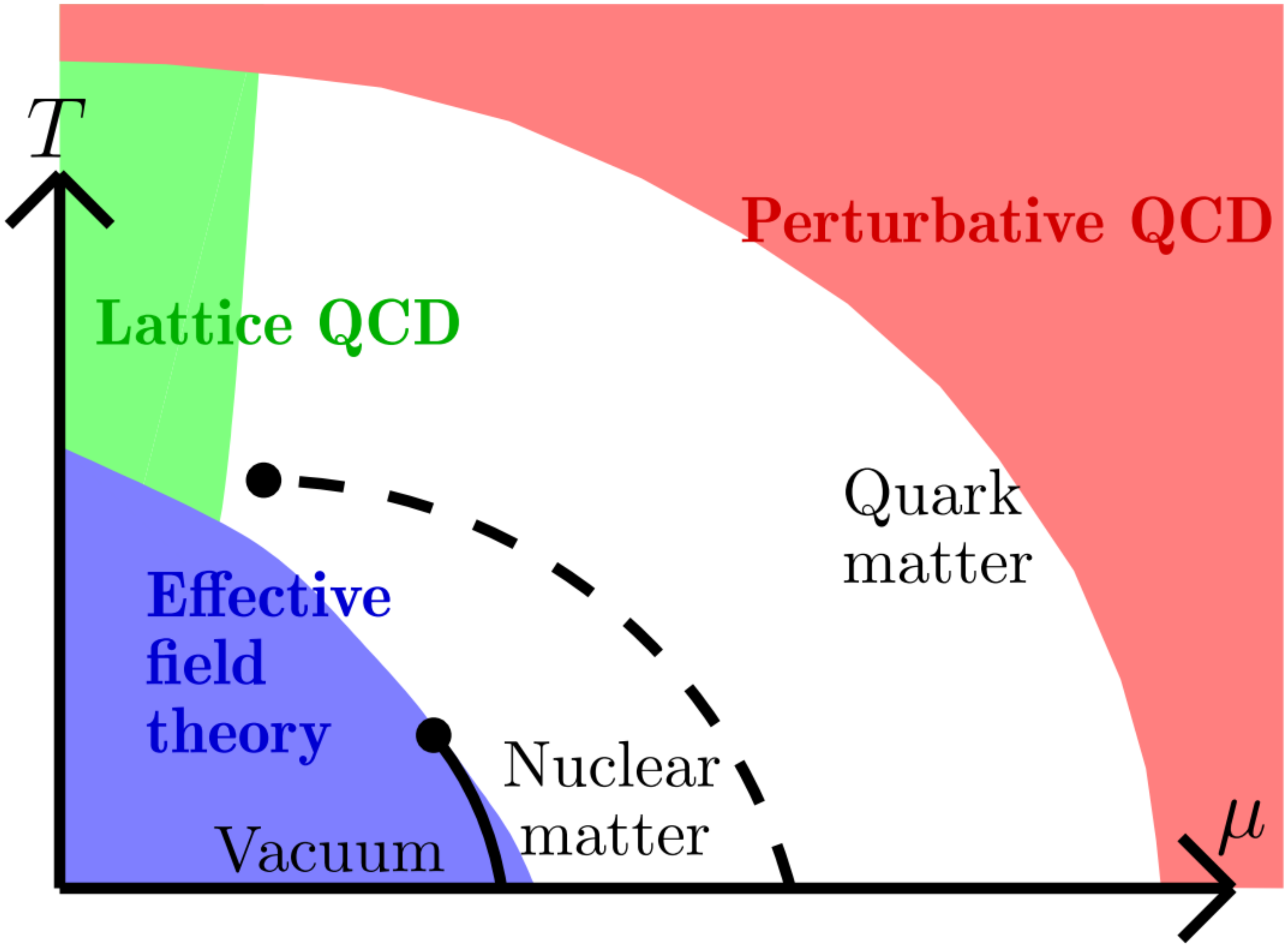
References : [1] 2009.10731, APJL; [2] 2112.12157, PRX;
[3] 2205.05691, SciPost; [4] 2211.10118, EPJ WC; [5] 2301.03173, Sym

The motivations to understand hot and dense QCD :

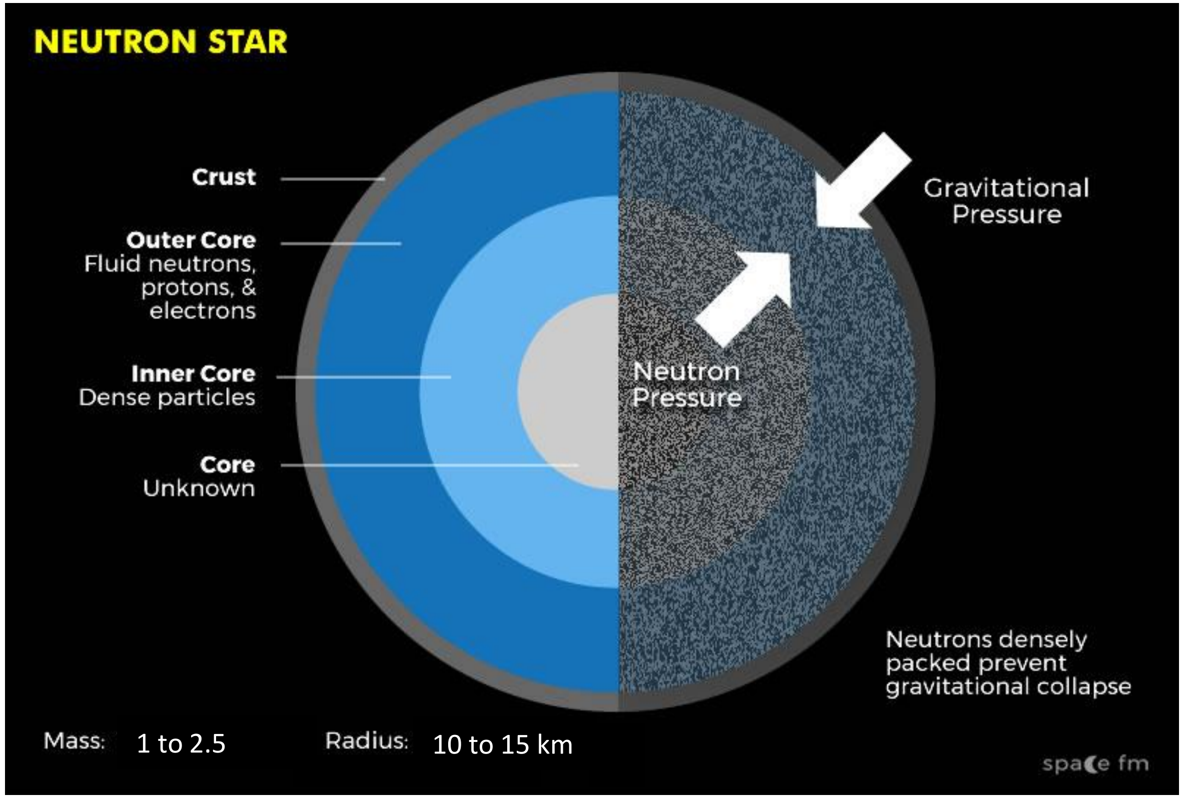
- A schematic sketch of the (possible) QCD phase diagram with applicability regimes of various theoretical and computational methods:



(M.Jarvinen 2021)

Neutron star as a blob of static dense QCD matter:

- With rough theoretical approximation, NS are compact objects of densely packed QCD (nuclear or quark) matter with fermi pressure and repulsive interactions which equilibrates it self-gravity (i.e. hydrostatic eq.).



- In GR, a static spherically symm. body is described by the Tolman-Oppenheimer-Volkoff (TOV) eqns:

$$p'(r) = -\frac{(\epsilon(p(r)) + p(r))(m(r) + 4\pi r^3 p(r))}{r^2\left(1 - \frac{2m(r)}{r}\right)}$$

$$m(r) = 4\pi \int_0^r d\hat{r}\, \hat{r}^2\, \epsilon(p(\hat{r}))$$

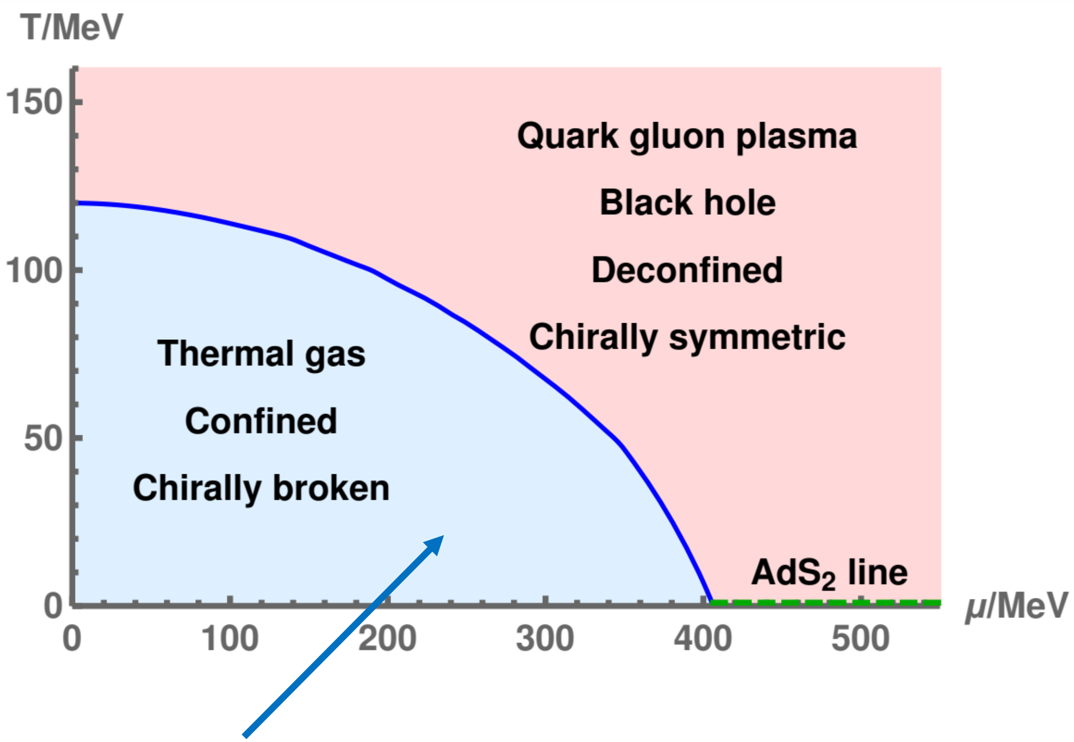
- The bottom line about TOV eqns.: They only depend the ulderlying theory via EoS, i.e. $\epsilon(p)$!!!

- There is one-to-one correspondence between $\epsilon(p)$ and $M(R)$.

- i) Mass measurements from pulsar binaries: $M_{\rm TOV} > 2M_{\odot}$ (lower bound)
  ii) Radii measurements from NICER: 10-15 km
  iii) Gravitational wave detection from Ligo/Virgo: $\Lambda_{1.4} < 580$ (upper bound)

- The bounds on max. mass and tidal deformability are complementary to each other. To achieve them, the EoS should be "stiff but not too stiff" (i.e. having high speed of sound $c_s^2 = dp/d\epsilon$).

The cold NM in V-QCD:



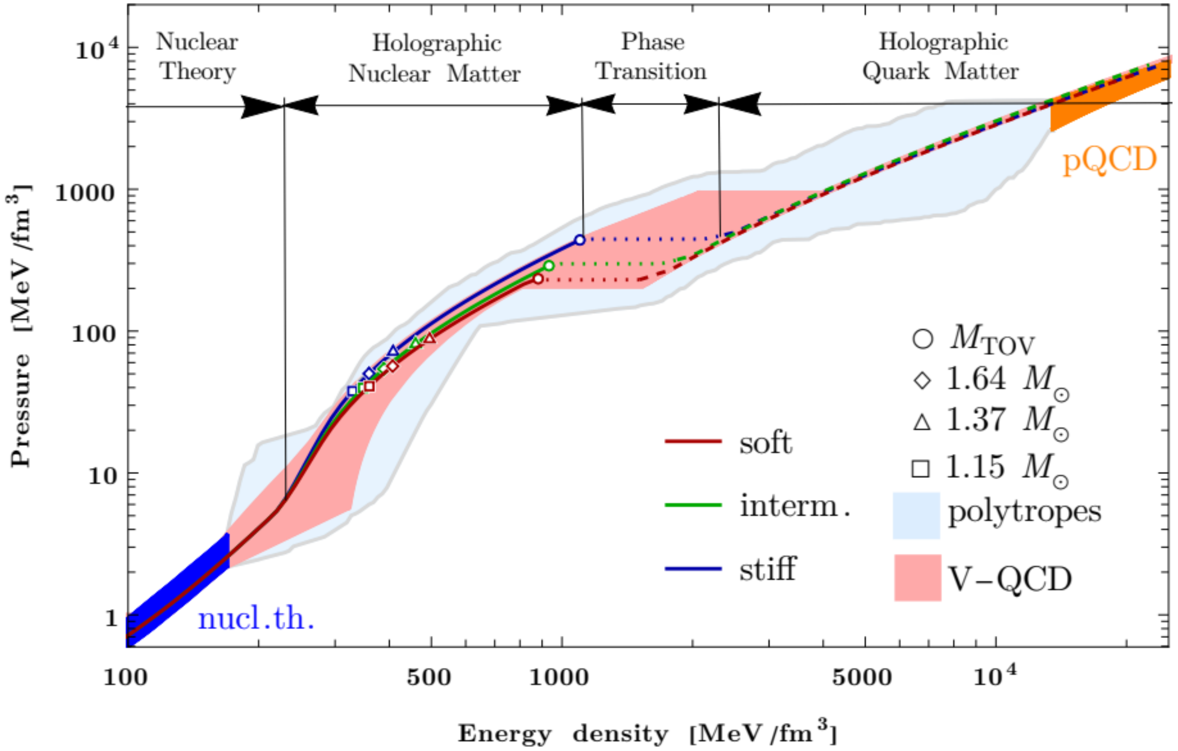
$$A_{L/R}(x_M) \mapsto \widehat{\Phi}(r)\mathbb{I}dt + A_{L/R}(x_M) \qquad A_L^i = -A_R^i = h(r)\sigma^i \qquad N_f = 2$$

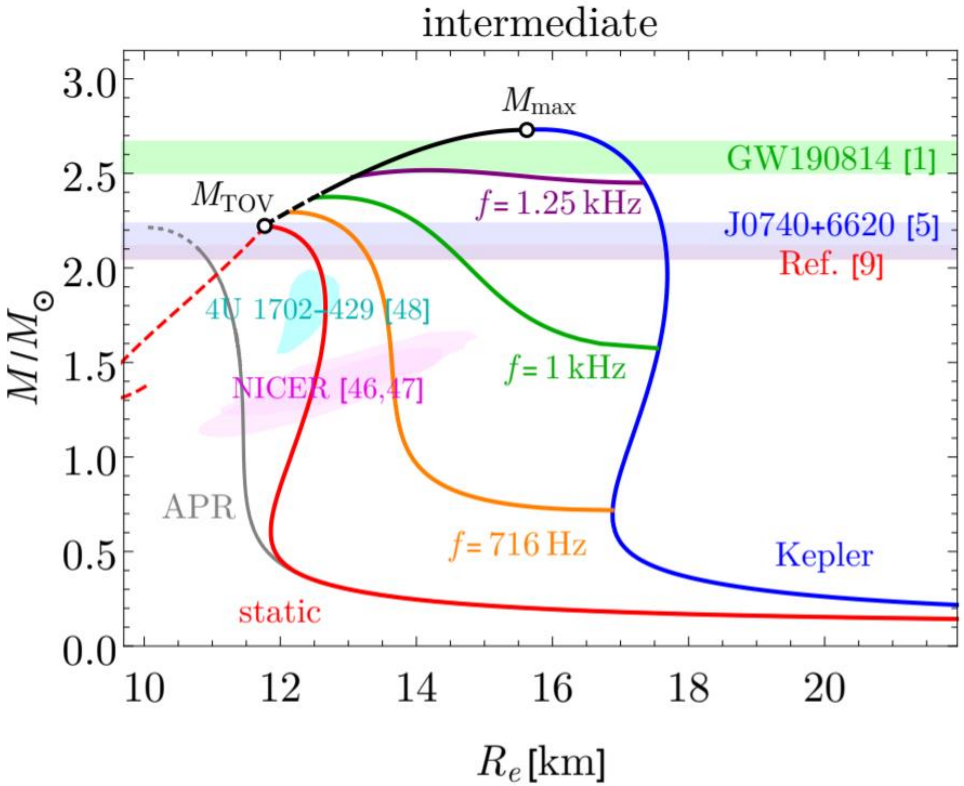
- Since, nucleons gets closer at high density, it is natural to treat them as homogeneous matter. The probe limit is employed.

(T.Alho, M.Jarvinen, K.Kajantie,E.Kiritsis, K.Tuominen 2013; M.Jarvinen 2015)

Hybrid EoSs:

- The hybrid EoSs are V-QCD EoSs with NM and QM, supplemented by other NM models below
- It has been shown that the hybrid EoSs are feasible are in good agreement with the available data (they are stiff enough).





(C.Ecker, M.Jarvinen, G.Nijs, W. van der schee, 2020, N.Jokela, M.Jarvinen, G.Nijs ,J.Remes, 2021; T.Demircik, C.Ecker, M.Jarvinen, 2020)

Finite-T generalization of the cold hybrid EoS:

- In the absence of other reliable ways to estimate the temperature dependence of the EoS for the dense NM, we use the simplest approach: a "van der Waals type model of NM", i.e., a free gas of nucleons with excluded volume corrections and an effective potential.

- Ideal gas of hadrons:

$$p_{id}(T,\{\mu_i,m_i,m_j\}) = \sum_i p_{id}^{(i)}(T,\mu_i,m_i) + \sum_j p_{id}^{(j)}(T,m_j) + p_{id}^{\gamma}(T)$$

$$i \in \{n,\bar{n},p,\bar{p},e,\bar{e}\}$$

$$m_j \leq 1 \text{ GeV}$$

$$p_{id}^{(\gamma)}(T) = \pi^2 T^4/45$$

- Hardcore repulsive interactions via excluded volume correction via thermodynamically consistent procedure: (D.Rischke, M.Gorenstein, H.Stoecker,W. Greiner, 1991)

$$p_{ex}(T,\{\mu_i\}) = p_{id}(T,\{\tilde{\mu}_i\}); \qquad v_0 = 0.56\text{fm}^3$$
$$\tilde{\mu}_i = \mu_i - v_i p_{ex}(T,\{\mu_i\}),$$

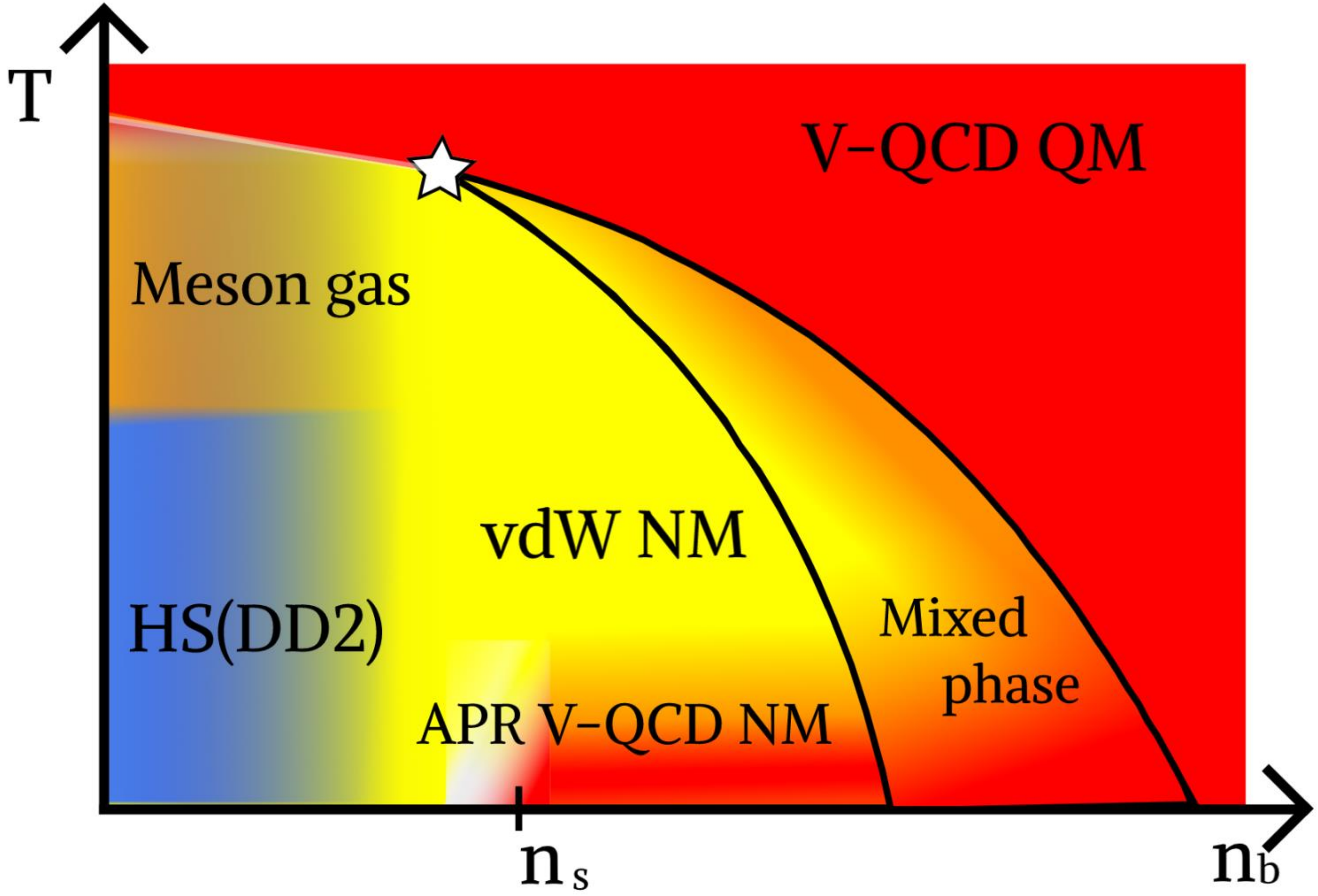
- Attractive interactions via matching cold V-QCD hybrid EoS (and DD2 for Y_q -dependence): (HSDD2: M. Hempel and J. Schaffner-Bielich, 2010 S. Typel, G. Ropke, T. Klahn, D. Blaschke, and H. H. Wolter,2010

$$f(T,n_b,Y_q) = f_{ex}(T,n_b,Y_q) + \Delta f(n_b,Y_q)$$

$$\Delta f(n_b,Y_q) = f_{VQCD}(n_b) + f_{e\bar{e}\gamma}(T_{min}) - f_{ex}(T_{min},n_b,Y_q) + f_{DD2}(T_{min},n_b,Y_q) - f_{DD2}(T_{min},n_b,Y_q^{eq})$$

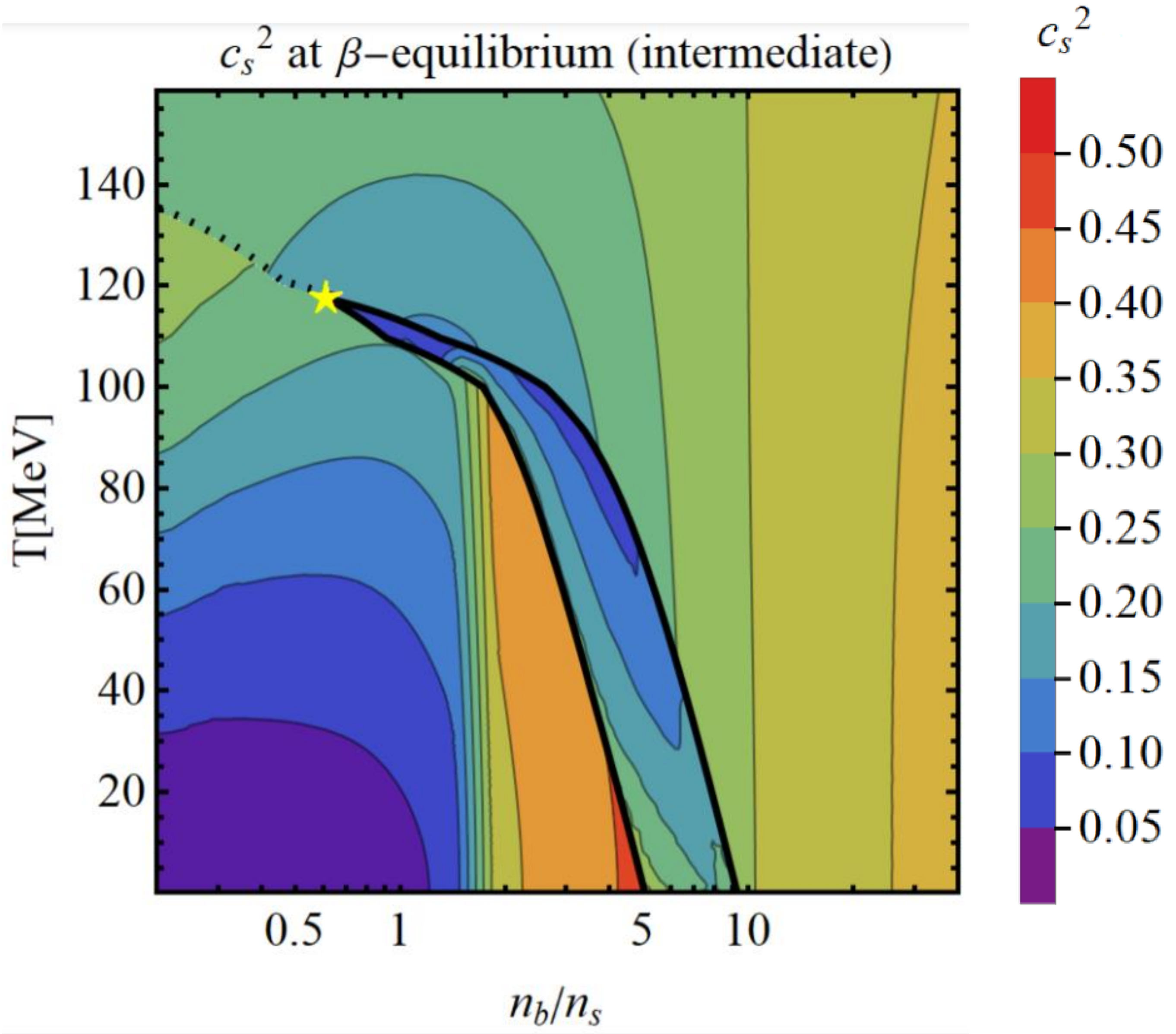
Finite-T generalization of the cold hybrid EoS:

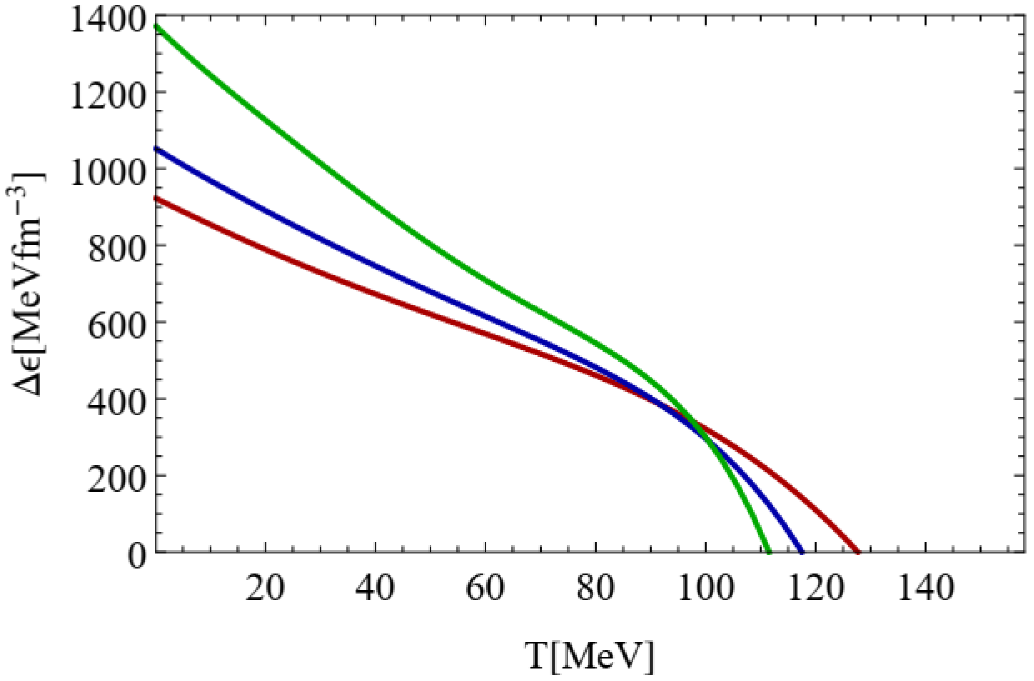
A schematic diagram that shows the construction of model:



(T.Demircik, C.Ecker, M.Jarvinen, 2021)

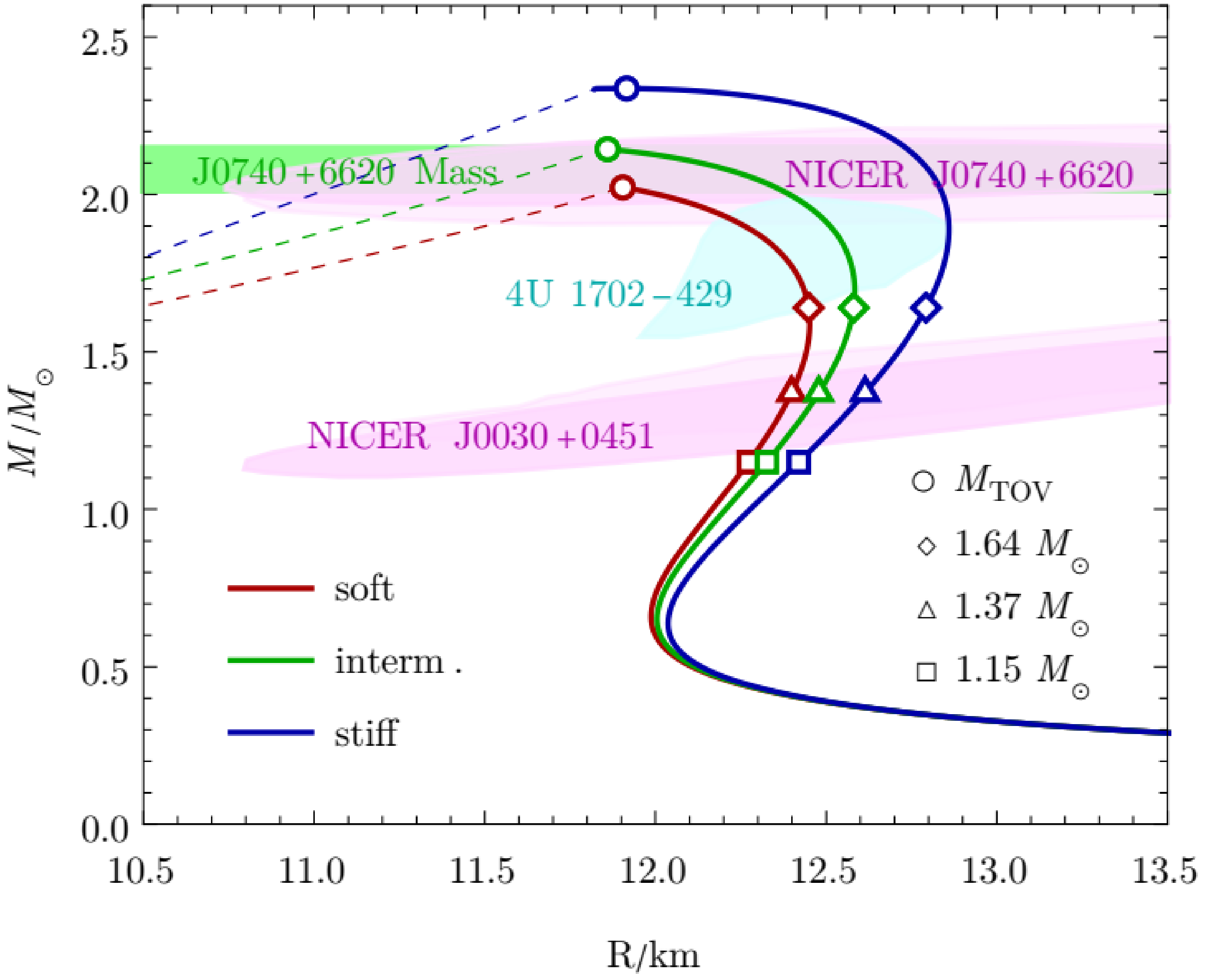
- A contour plots for speed of sound squared at β-equilibrium and latent heat.





(T.Demircik, C.Ecker, M.Jarvinen, 2021)

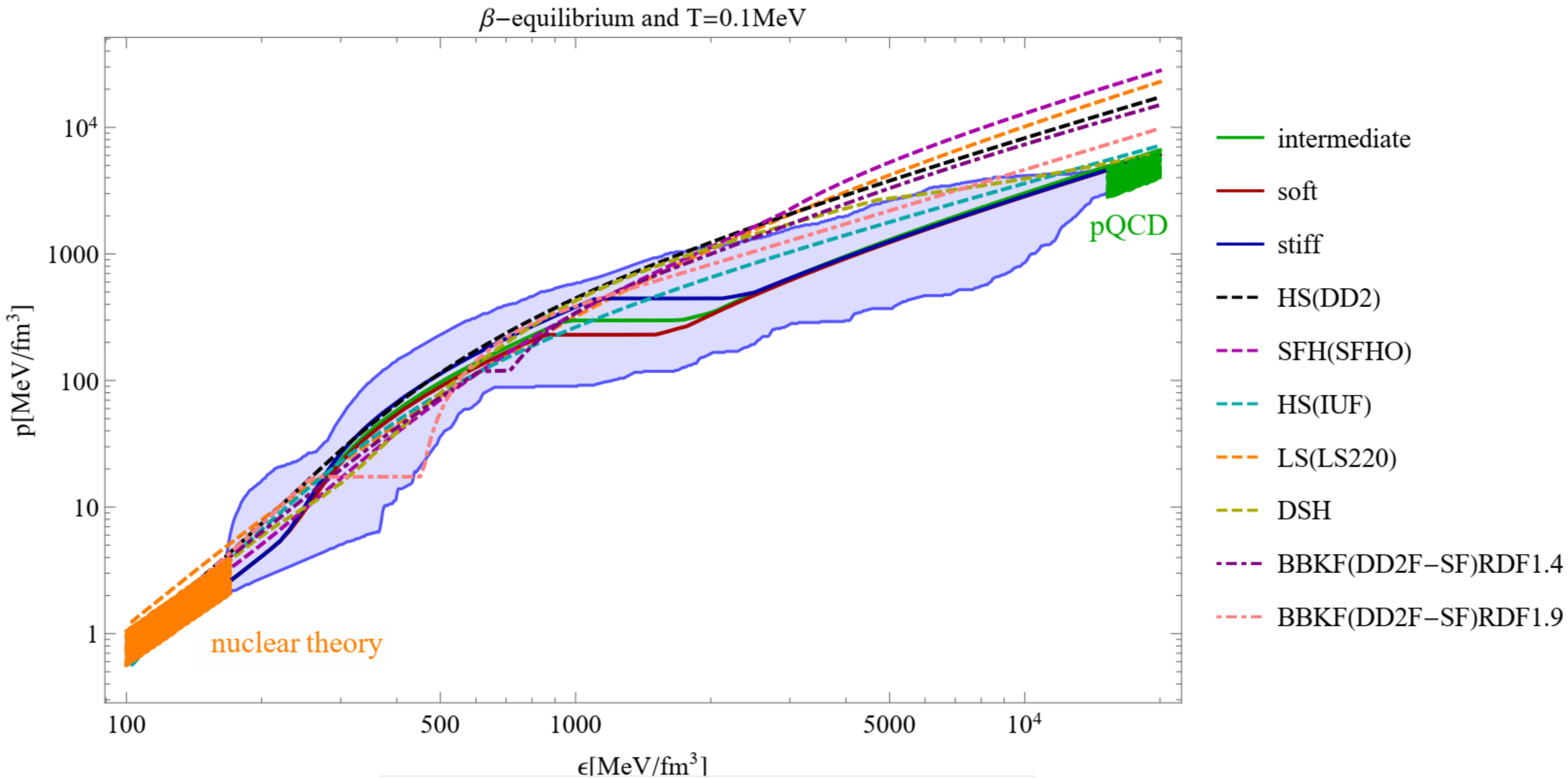
- Mass-radius relation of non-rotating stars and summary of the results for NS properties.



Model	$\frac{n_{bc}}{n_s}$	$\frac{\mu_{bc}}{\text{MeV}}$	$\frac{T_c}{\text{MeV}}$	$\frac{M_{TOV}}{M_\odot}$	$\frac{R_{e,1.4}}{\text{km}}$	$\Lambda_{1.4}$
soft	0.46	485	128	2.02	12.41	483
interm.	0.62	575	118	2.14	12.50	511
stiff	0.32	565	112	2.34	12.64	560
HS(DD2)	–	–	–	2.45	13.2	686

(T.Demircik, C.Ecker, M.Jarvinen, 2021)

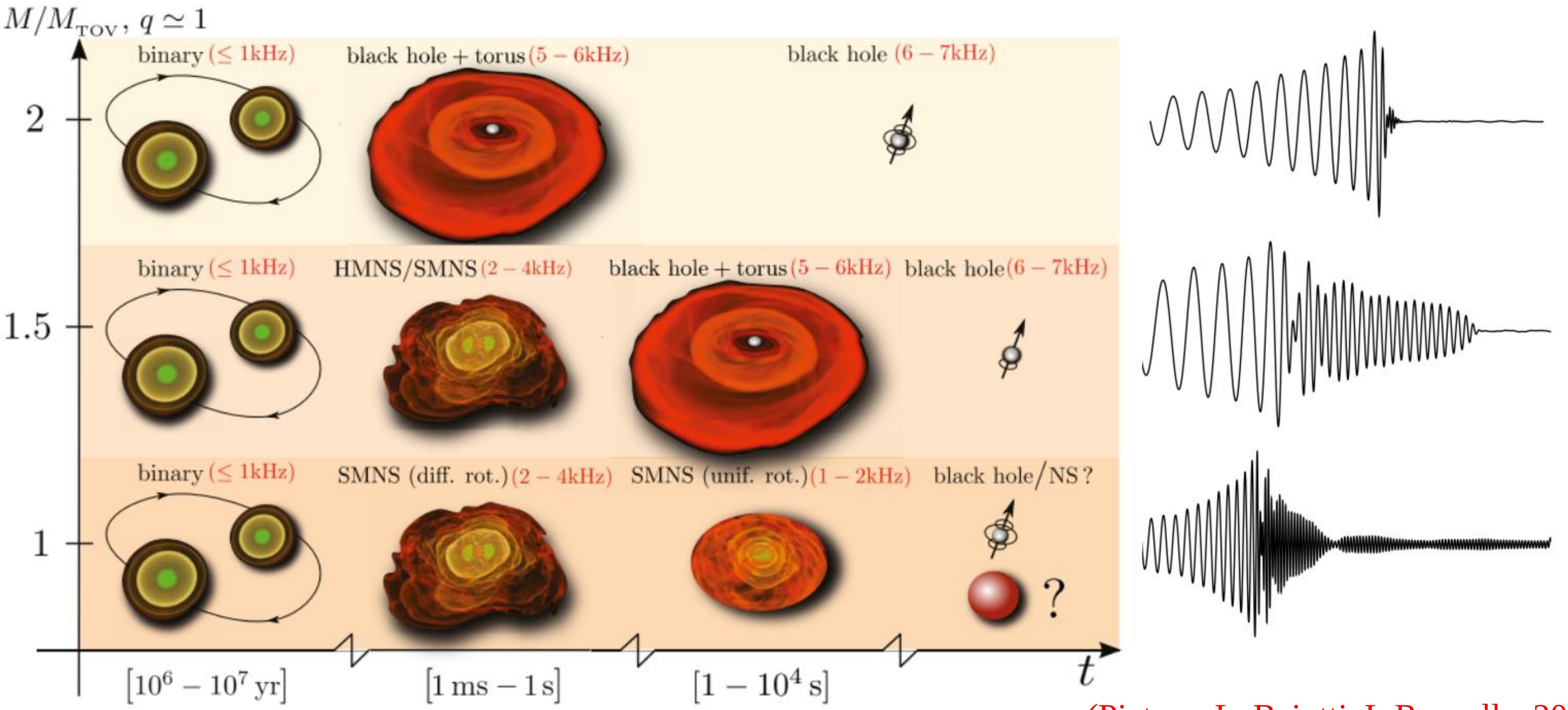
Comparison of EoSs with other EoSs



(T.Demircik, C.Ecker, M.Jarvinen, 2021)

Neutron star simulations of "the finite-T EoS":

- There are three scenarios for NS-merger event depending of initial total mass of the binary:



(Picture: L. Baiotti, L.Rezzolla, 2016)

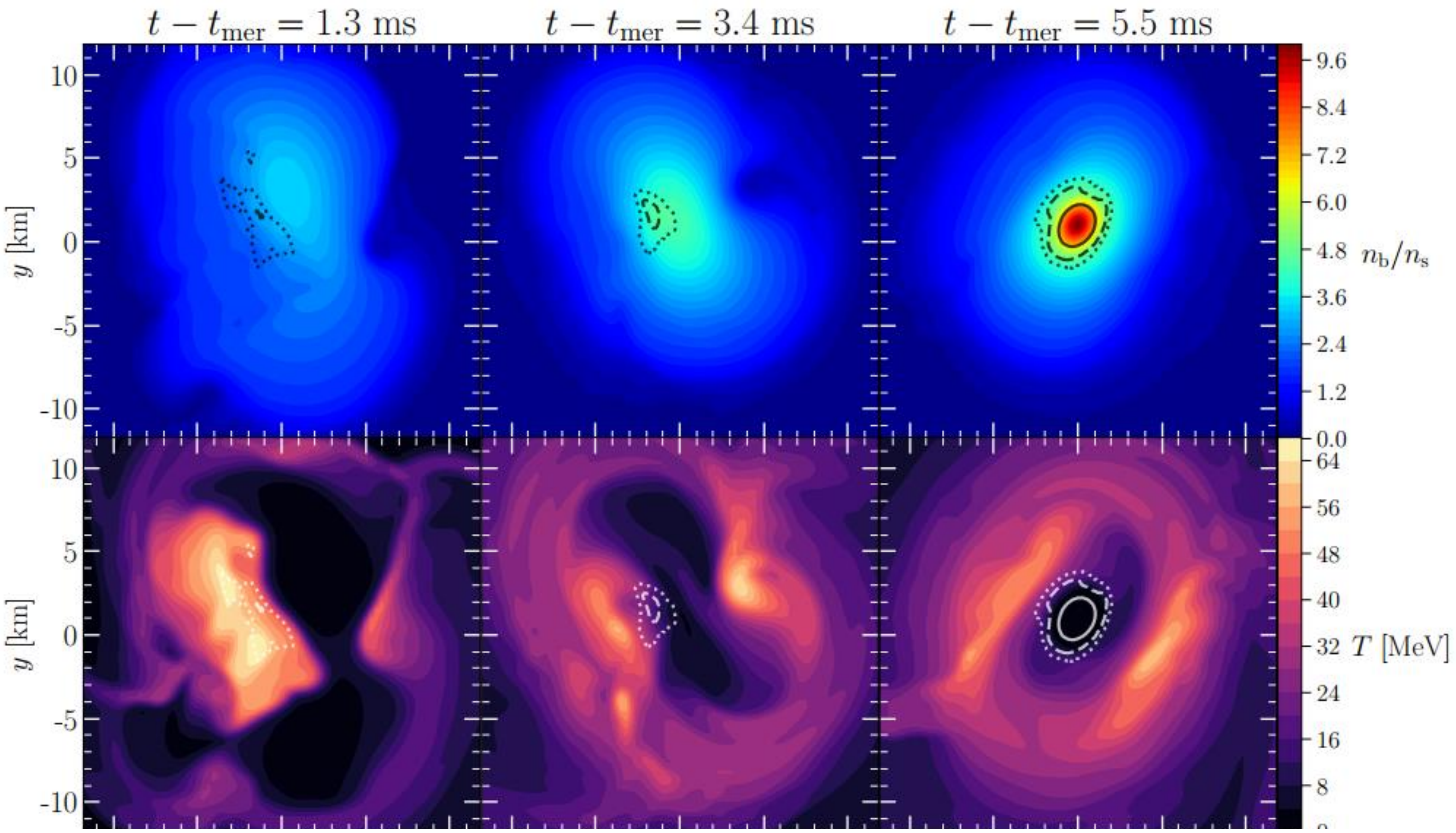
- It is not clear that transition from dense NM to QM can happen in NS merger event but if it can, the imprint of the transition may be visible in the GW and EMW signals of post-merger HMNS stage.

- We performed full 3+1 dimensional GR hydrodynamic simulations of equal/unequal mass binaries satisfying the properties of inspiral stage of GW170817.

- We identify three different stages of quark matter formation during the postmerger HMNS evolution:
  hot-quark (HQ), warm-quark (WQ) and cold-dense-quark (CDQ) stages.

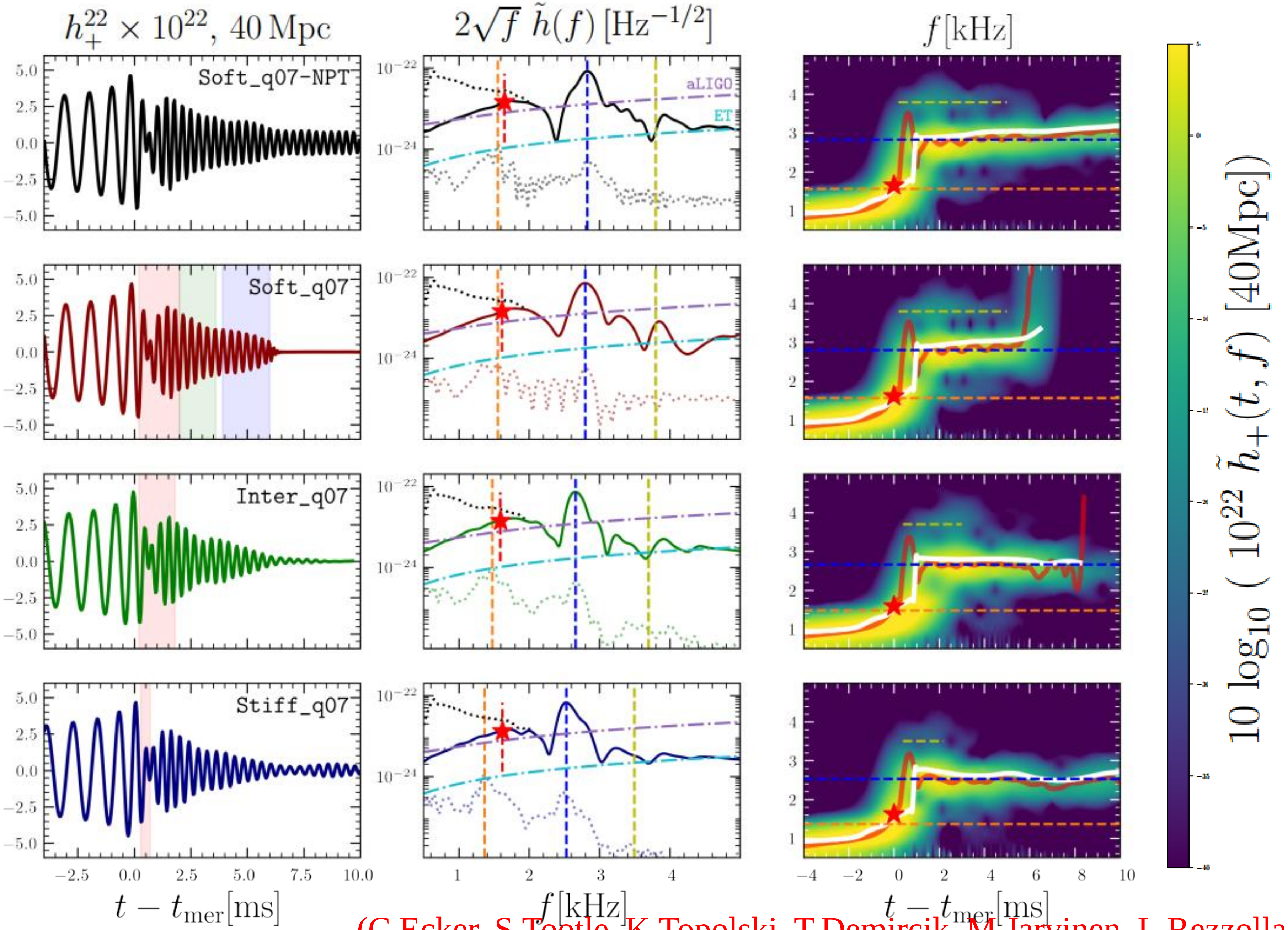
- The clearest imprint on the waveform is the early termination caused by the PT-triggered collapse to BH in CDQ stage.

- HQ (due to strong shocks and steep rise in T)
- WQ (in the significantly cooler and slightly denser region in highly nonaxisymmetric way)
- CDQ (in the dense and cold center of remnant after settle down of the violent post merger period)



(C.Ecker, S.Tootle, K.Topolski, T.Demircik, M.Jarvinen, L.Rezzolla, 2022)

- Futhermore, we analyzed waveform, power spectral density, spectrogram:



(C.Ecker, S.Tootle, K.Topolski, T.Demircik, M.Jarvinen, L.Rezzolla, 2022)

Summary:

- ✓Cold hybrid EoS was used to investigate GW190814 event.
- ✓The three variants of cold hybrid EoSs was generalized to 3D EoSs (via vdW-HG model with thermodynamically consistent procedure.
- ✓Various observables of NS were calculated and it was shown the EoS are feasible, i.e. they easily satisfy astrophysical constraints.
- ✓The phase diagram was computed with the location of QCD critical point.
- ✓Merger simulations for GW170817 were perform for different variants and with different scenarios.
- ✓Three different stages of the quark matter formation are catagorized
- ✓Early terminaiton of the signal is the most significant imprint for PT in NS
- ✓Detailed spectral analyses were performed, soft eos is ruled out due to its tension with expected life time.

Conclusion:

Gauge/gravity duality (combined with other methods) are useful to study QCD, i.e. since it accommodates strong coupling effects, phase transitions.

Extras

Outline:

- Motivations
- Brief review about the NSs
- Brief review about the holographic model and cold EoSs
- Finite-T EoSs
- NS phenomenology
- Summary and Conclusion

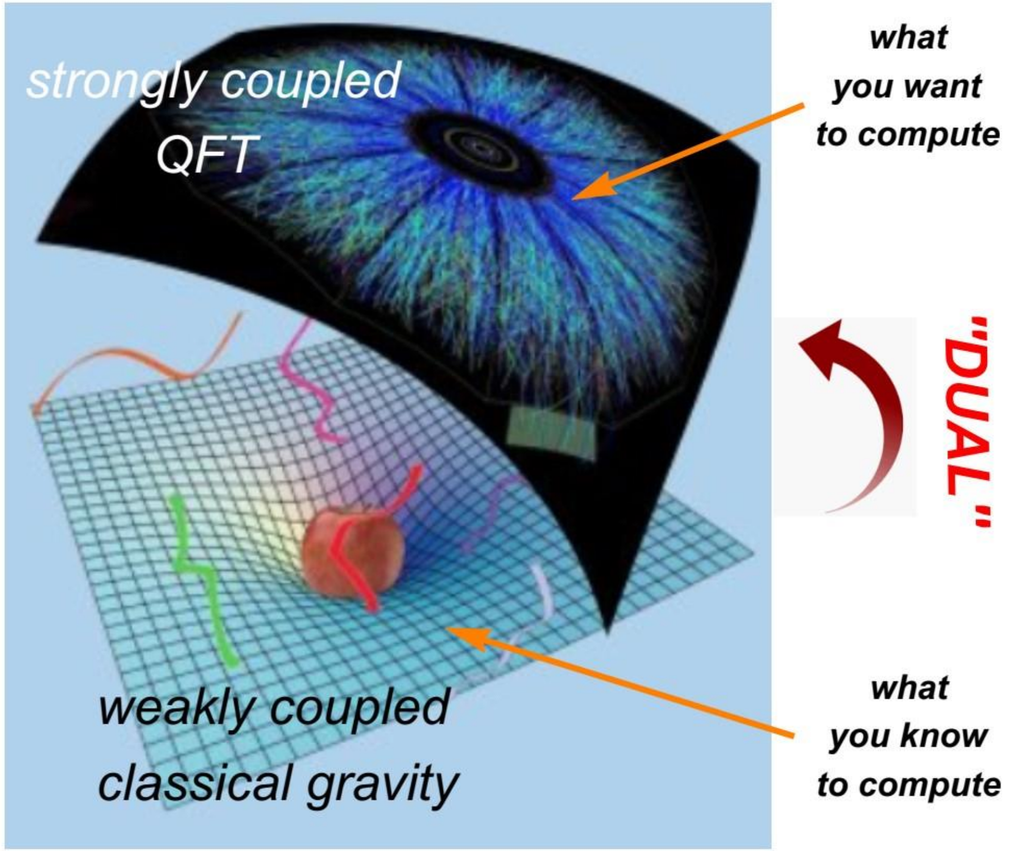
The motivations to understand hot and dense QCD :

- Determining the phase diagram of QCD is challenging task, particularly in low temperature and intermediate density regime (but not asymptotically hot and dense) due to the strongly interacting nature of QCD.

- However, It is timely and exciting to working on it due the boost created by ongoing and future experiments:
  i) Efforts in heavy-ion collisions to probe the QCD critical point regime (RHIC, LHC, FAIR, …)
  ii) Various experiments to determine neutron star properties (Ligo/Virgo, NICER, …)

Gauge/Gravity Duality:

- Holography (AdS/CFT) is a duality between strongly coupled gauge theory in 4D and weakly coupled gravity theory in 5D



(Picture: M. Baggioli, 2019)

- GKP-Witten Relation

$$Z_{gauge} = Z_{AdS}$$

- Holographic dictionary

$$\frac{\lambda}{N} = 2\pi g_s$$

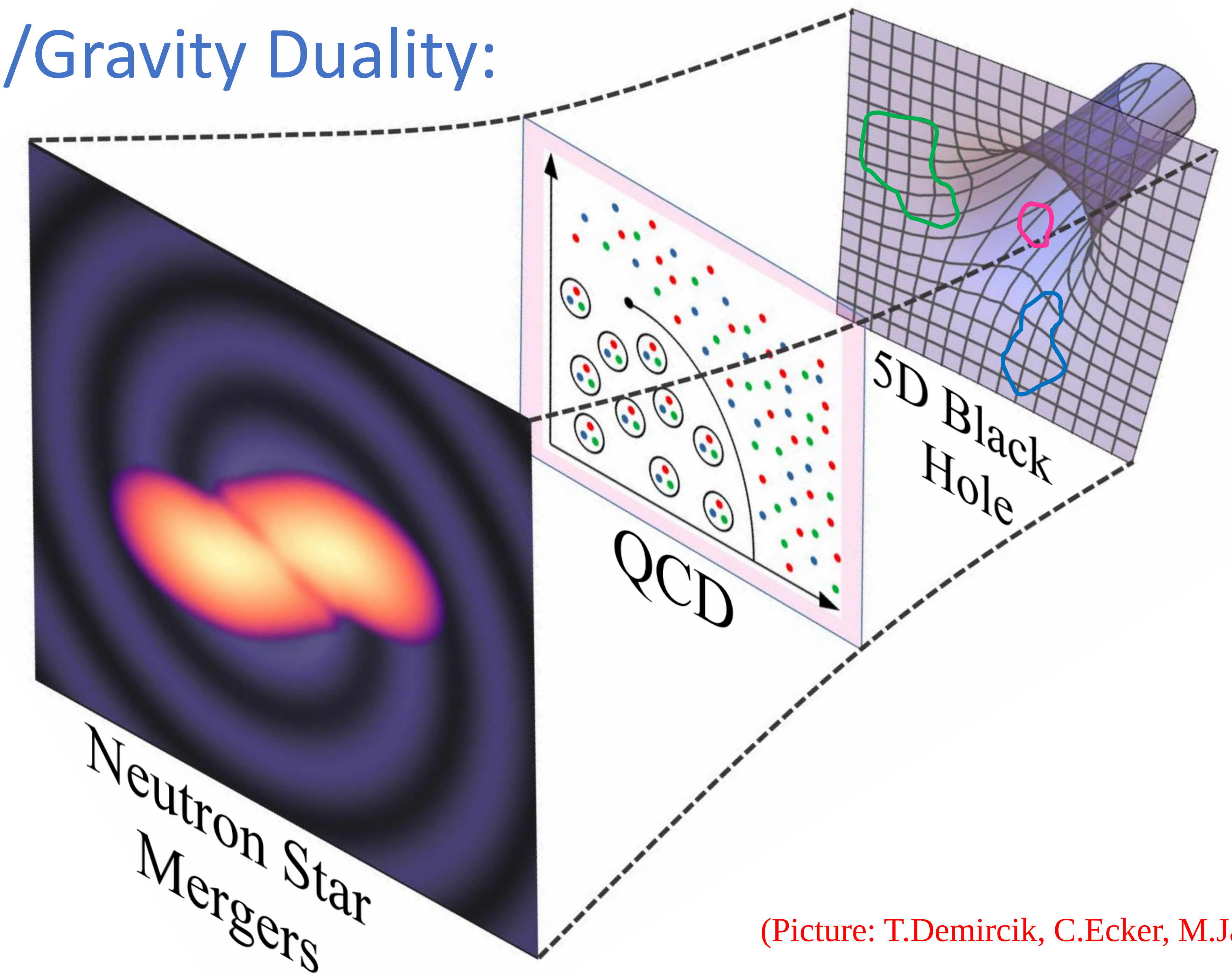
$$2\lambda = \frac{L^4}{l_s^4}$$

$$\lambda = g_{YM}^2 N$$

- N is Large ('t Hooft limit):
- Weak coupling limit of sting theory, one gets only tree level diagrams:
- It is reduced to classical string theory
- λ is kept constant
- g_{YM} is small (but the significance of higher order diagrams remains same)
- $L > l_s$ classical gravity theory of the point particles

note: $Z_{\mathrm{CFT}(N_c \gg 1)} = e^{-\underline{\mathrm{S_E}} + O(l_s^2)}$ $\quad Z_{\mathrm{CFT}\,(N_c \gg \lambda \gg 1)} = e^{-\underline{\mathrm{S_E}}}$

Gauge/Gravity Duality:

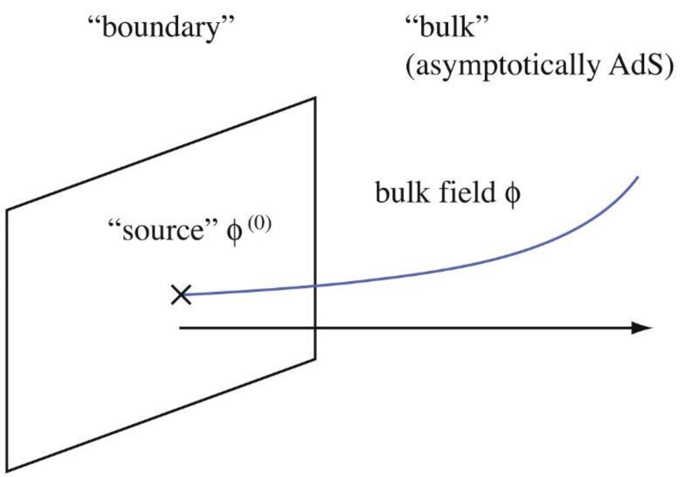


(Picture: T.Demircik, C.Ecker, M.Jarvinen, 2022)

Gubser-**K**lebanov-**P**olyakov-**W**itten-relation:

$$\mathcal{Z}_{gauge} = \mathcal{Z}_{AdS}$$

$$\left\langle exp\left(i\int \phi^{(0)} O\right)\right\rangle = exp\left(i\bar{S}[\phi|_{r=r_B} = \phi^{(0)}]\right)$$



(i.e. Bulk fields act as external sources of boundary operator!)

$$Z = e^{-S_E}, \quad S_E = \beta F$$

e.g. Boundary operator		External sources		Bulk fields
$T^{\mu\nu}$	$\leftrightarrow$	$g^{(0)}_{\mu\nu}$	$\rightarrow$	g_{MN}
J^{μ}	$\leftrightarrow$	$A^{(0)}_{\mu}$	$\rightarrow$	A_M

$$s = -\frac{1}{V_3}\partial_T F$$

$$P = -\partial_{V_3} F$$

$$\varepsilon = \frac{F}{V_3} + Ts$$

The boundary expansion of the bulk field is always in this form

$$\Phi(z, t, \mathbf{x}) = leading(t, \mathbf{x})\, z^{\Delta_L}\ (1 + \dots) + subleading(t, \mathbf{x})\, z^{\Delta_S}\ (1 + \dots) \qquad \Delta_L < \Delta_S$$

GKP-W:

Bulk	Boundary
leading	external source ϕ_0
subleading	expectation value of boundary $\langle O \rangle$
subleading/leading	the Green's function G_R^{OO}

V-QCD:

- V-QCD is a bottom-up holographic model which contains both gluon and flavor sectors.
- The backreaction is implemented via the "Veneziano limit" (G.Veneziano, 1979) :

$$N_c \to \infty \;, \quad N_f \to \infty \;, \quad \text{with } x_f \equiv \frac{N_f}{N_c} \text{ fixed}$$

- The main building blocks of V-QCD:

$$S_{\mathrm{V-QCD}} = S_{\mathrm{IHQCD}} + S_f$$

The gluon sector with the good description of pure YM via nontrival dilaton:
Asymptotic freedom,
Confinement,
Glueball spectrum,
Finite-T thermodynamics

(U.Gursoy, E.Kritsis, F.Nitti,2007)

Tachyonic DBI action (motivated by space filling) :
Breaking/Restoration of Chiral Symm,
Meson Spectrum,
Finite-T thermodynamics,

(M.Jarvinen, E.Kiritsis, 2012)

Holographic dictionary for V-QCD:

- Field/Operator correspondence:

field	operator	source
$\lambda = e^{\phi}$	$\mathbb{T}r\, G^2$	$\lambda_{'tH} = g^2 N_c$
g_{MN}	$T_{\mu\nu}$	$\eta_{\mu\nu}$
T^{ij}	$\bar{\psi}^i \psi^j$	M_q^{ij}
A_L^{ij}	$\bar{\psi}^i (1+\gamma_5)\gamma_\mu \psi^j$	$A_{L\,\mu}^{(\mathrm{ext})ij}$
A_R^{ij}	$\bar{\psi}^i (1-\gamma_5)\gamma_\mu \psi^j$	$A_{R\,\mu}^{(\mathrm{ext})ij}$

V-QCD at finite-T & finite-μ

- V-QCD action for finite-T and finite- n (with flavor indep. quark mass):

$$S^{\rm bg}_{\rm V-QCD} = M_{\rm p}^3 N_c^2 \int d^5x \sqrt{-\det g} \left[R - \frac{4\,(\partial\lambda)^2}{3\lambda^2} + V_g(\lambda) \right]$$

$$- x_f M_{\rm p}^3 N_c^2 \int d^5x\, V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)\hat{F}_{\mu\nu})}$$

The source of dilaton is identified as energy scale $\Lambda_{UV} \sim \Lambda_{QCD}$.

$$ds^2 = e^{2A(r)}\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2\right)$$

$A(r)$ is dual to log of field theory energy $A \sim Log\mu$.

Flow λ and A defines the mapping between holographic RG flow and field theory RG flow.

$$T = \tau(r)\mathbb{I}$$

$$A_t(r) = \Phi(r)\ , \quad F_{rt} = \Phi'(r)\ , \quad \mu = \Phi(r=0)$$

V-QCD potentials:

- V-QCD potentials are the “knobs” to tune with essential QCD features

$$S^{\rm bg}_{\rm V-QCD} = M_{\rm p}^3 N_c^2 \int d^5x \sqrt{-\det g} \left[R - \frac{4\,(\partial\lambda)^2}{3\lambda^2} + V_g(\lambda) \right]$$

$$- x_f M_{\rm p}^3 N_c^2 \int d^5x\, V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)\hat{F}_{\mu\nu})}$$

The main strategy is, 1st fixing asymptotics in weak coupling with perturbative results, 2nd fixing asymptotics at strong coupling with qualitative features of QCD

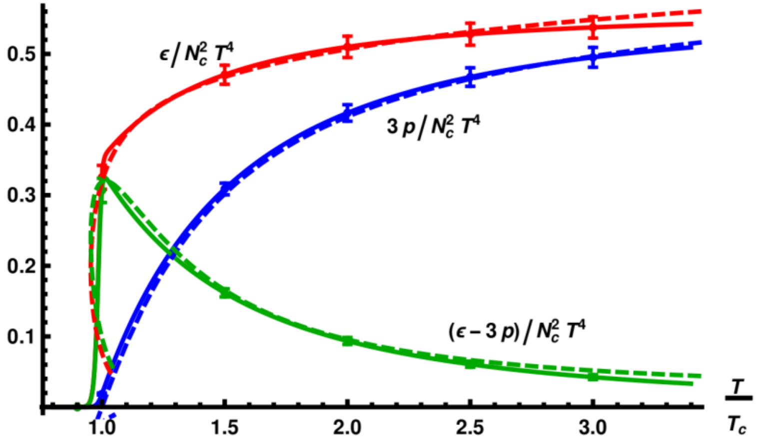
And the remaining task is, interpolating between both ends with the lattice data.

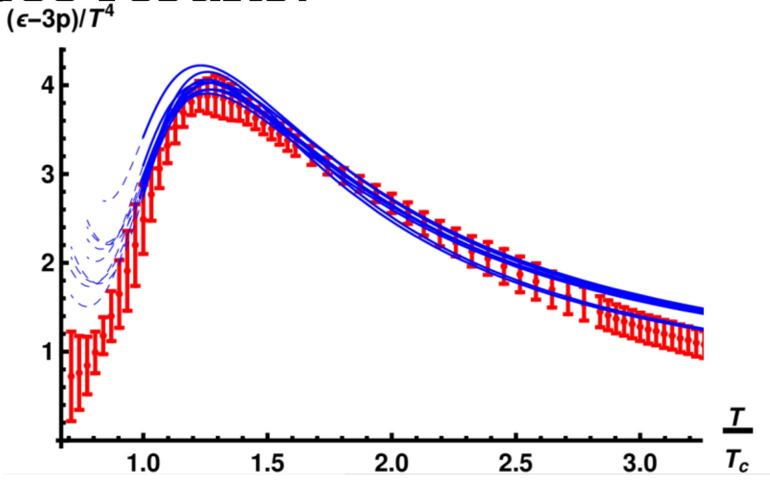
Parameters for V-QCD potentials:

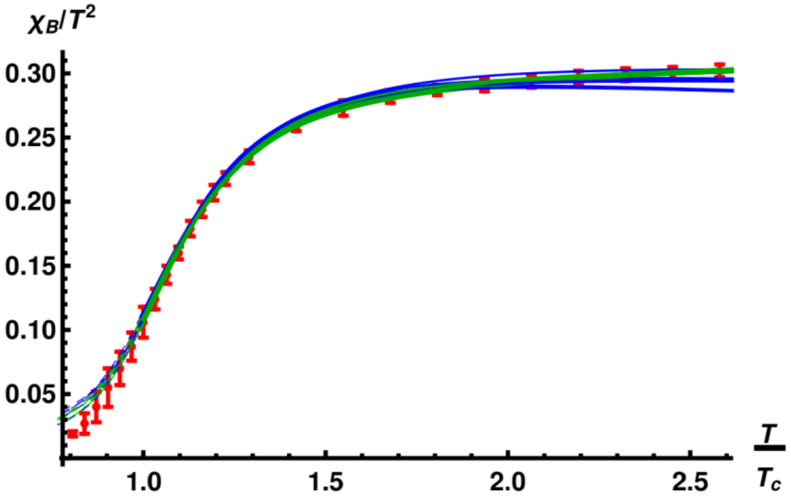
	5b	**7a**	**8b**
W_0	1.0	2.5	5.886
$W_{\rm IR}$	0.85	0.9	1.0
w_0	0.57	1.28	1.09
w_1	3.0	0	1.0
$\bar{w}_0$	65	18	22
$8\pi^2/\hat{\lambda}_0$	0.94	1.18	1.16
$\bar{\kappa}_0$	1.8	1.8	3.029
$\bar{\kappa}_1$	-0.857	-0.23	0
$\Lambda_{\rm UV}/{\rm MeV}$	226	211	157
$180\pi^2 M_{\rm p}^3\ell^3/11$	1.34	1.32	1.22

V-QCD potentials fixing via lattice comparison:

- Once asymptotics are fixed, a tuning in the middle makes the V-QCD agrees with the available lattice results:



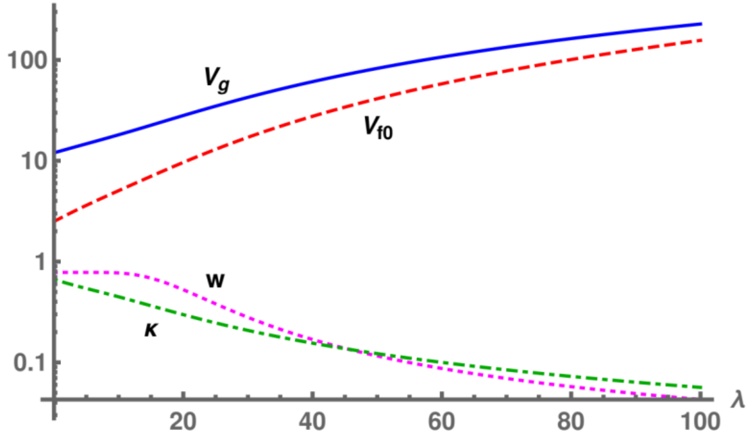




$V_g(\lambda)$: (Large-N_c data for pure YM)

$V_{f0}(\lambda)$, $\kappa(\lambda)$: ($N_c = N_f = 3$ data) ,

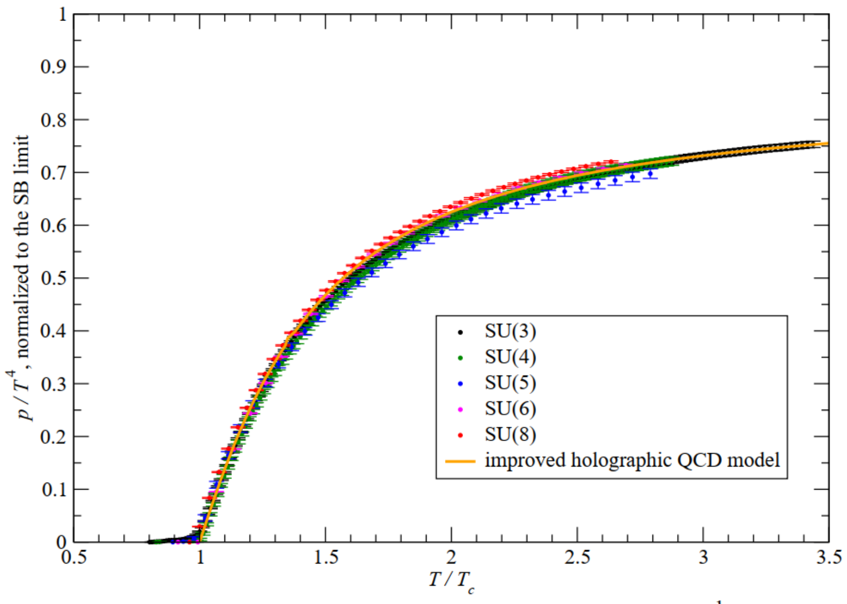
$w(\lambda)$: ($N_c = N_f = 3$ data at small density)

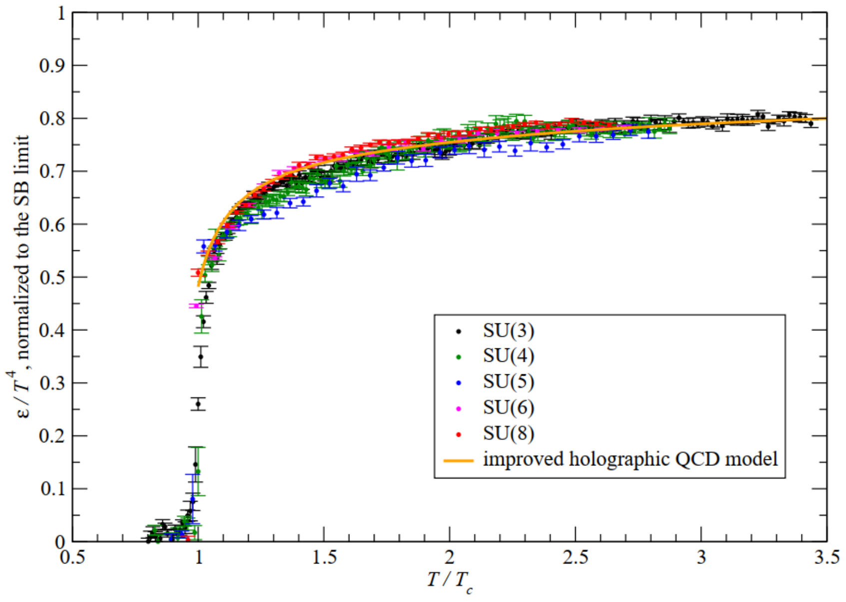


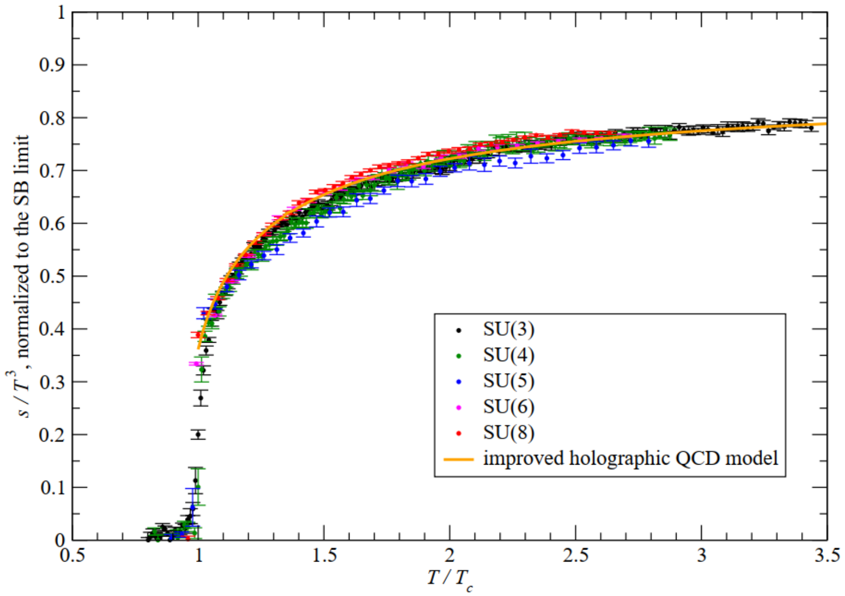
- The shapes of the curves are the first and foremost predictions of V-QCD, the precise matching is only small tuning. (The potentials are just simple monotonic functions, not tailored complicated functions.) !!!

(M.Panero, 2009; S.Borsanyi, Z.Fodor,C.Hoelbling,S.Katz,.S.Krieg,K.Szabo, 2014; S.Borsanyi, Z.Fodor,S.Katz,.S.Krieg,C.Ratti,K.Szabo, 2012; N.Jokela, M.Jarvinen, J.Remes, 2018; M.Jarvinen, 2021)

Gauge/Gravity Duality: comparison with the lattice







(Panero, 2019)

V-QCD at weak coupling

- Leading behavior of bulk field ⟷ Leading UV dimension of dual operator.

  RG-flow of bulk field ⟷ RG flow of dual operator.

$$S^{\rm bg}_{\rm V-QCD} = M_{\rm p}^3 N_c^2 \int d^5x \sqrt{-\det g}\left[R - \frac{4\,(\partial\lambda)^2}{3\lambda^2} + V_g(\lambda)\right]$$

$$- x_f M_{\rm p}^3 N_c^2 \int d^5x\, V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)\hat{F}_{\mu\nu})}$$

$$V_{\rm eff}(\lambda) = V_g(\lambda) - x_f V_{f0}(\lambda) \to \frac{12}{\ell^2} \qquad \text{as} \quad \lambda \to 0$$

The next terms are just given by Taylor exp. Whose coefficients are fixed by : $\frac{d\lambda}{dA} = \frac{\lambda'(r)}{A'(r)} \longleftrightarrow \beta(\lambda_{'\rm tH}) \equiv \frac{d\lambda_{'\rm tH}}{d\log\mu}$

$\kappa(\lambda)$ is given just given by Taylor exp. Whose coefficients are fixed by quark mass, quark mass operator and perturbative one loop anomalous dim of QCD in Veneziano limit.

(U.Gursoy, E.Kiritsis, 2012; M.Jarvinen, E.Kiritsis, 2012; D.Arean, I.Iatrakis, M.Jarvinen, E.Kiritsis, 2013)

V-QCD at strong coupling:

$$S_{\mathrm{V-QCD}}^{\mathrm{bg}} = M_{\mathrm{p}}^3 N_c^2 \int d^5x \sqrt{-\det g} \left[R - \frac{4\,(\partial\lambda)^2}{3\lambda^2} + V_g(\lambda) \right]$$

$$- x_f M_{\mathrm{p}}^3 N_c^2 \int d^5x\, V_{f0}(\lambda) e^{-\tau^2} \sqrt{-\det(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)\hat{F}_{\mu\nu})}$$

By requiring Confinement: $V_g \sim \lambda^{4/3}(\log\lambda)^{1/2} \quad (\lambda \to \infty)$

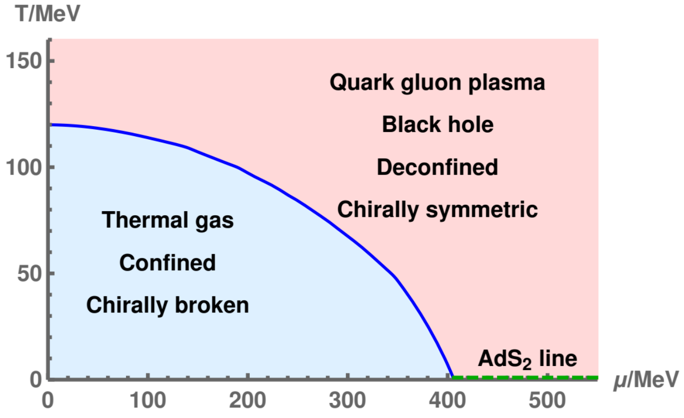
By requiring Chiral Symm. Breaking (Annihilation of brane due to divergence of tachyon): $V_{f0}(\lambda) \sim \lambda^{v_p} \quad \kappa(\lambda) \sim \lambda^{-4/3}(\log\lambda)^{1/2}$

Phase diag. at finite density: $w(\lambda) \sim \lambda^{-4/3}(\log\lambda)^{-w_p} \qquad v_p \approx 2\,, \quad w_p < -\frac{1}{2}$

(U.Gursoy, E.Kiritsis, 2012; M.Jarvinen, E.Kiritsis, 2012; D.Arean, I.Iatrakis, M.Jarvinen, E.Kiritsis, 2013)

The phase structure of V-QCD at finite-T & finite-μ:

- There two possible geometries at finite-T and at finite-μ (restricting to homo. and time indep. config and choices for potentials)



$$ds^2 = e^{2A(r)}\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2\right)$$

The chirally broken confined phase of hadron gas

Horizonless geometry ending at a "good" kind of IR singularity. That is This geometry is independent of T and μ (the only regular solution has and const.). It has trivial thermodynamics with vanishing entropy and pressure.

The chirally symmetric deconfined phase of dense quark matter

Charged black hole geometry ending with a "planar" horizon. That is $f(r_h) = 0$.

Temperature and entropy density equal:

$$T = \frac{1}{4\pi}|f'(r_h)|\ , \quad s = \frac{1}{4G_5}e^{3A(r_h)}$$

Nontrivial profile of n lead non-zero Φ and μ

(Phase Diagram: $\Omega = -p$ $, d\Omega = -sdT - nd\mu$)

(T.Alho, M.Jarvinen, K.Kajantie,E.Kiritsis, K.Tuominen 2013; M.Jarvinen 2015)

The cold NM in V-QCD:

- Since, nucleons gets closer at high density, it is natural to treat them as homogeneous matter.

- The probe limit is employed, i.e. DBI action will be expanded to first nontrivial (quadratic) order and the backreaction of baryons is neglected.

- The ansatz for the non-Abelian field that corresponds to baryons (in the case of two light flavors $N_f = 2$):

$$A_{L/R}(x_M) \mapsto \widehat{\Phi}(r)\mathbb{I}dt + A_{L/R}(x_M) \qquad A_L^i = -A_R^i = h(r)\sigma^i$$

- With the homogeneous ansatz, the geometry is “horizonless thermal gas with the probe h condensate in the bulk”. It is dual to “chirally broken confined phase with cold homogeneous NM”. Since the baryons are treated in probe limit on hadron gas which is T-independent, It is T-indep too!!

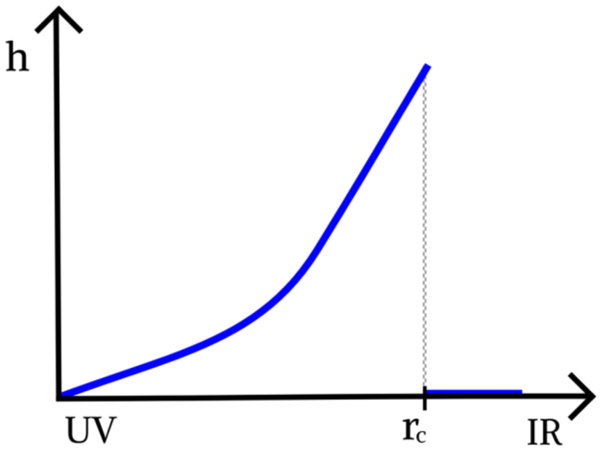
The cold NM in V-QCD

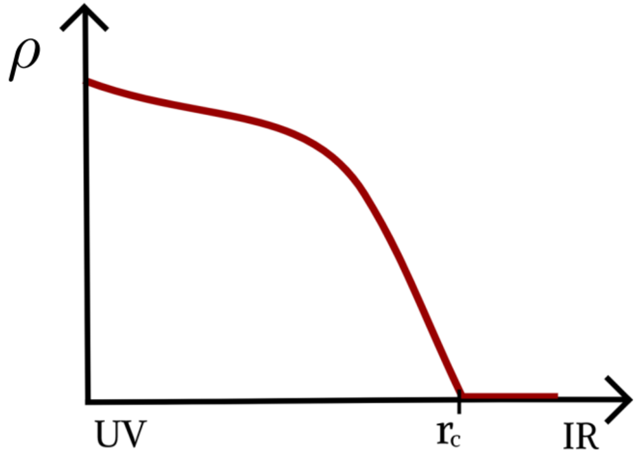
- The baryon density is obtained by varying action (with CS since the coupling of Φ and h only therein):

$$\rho = -\frac{\partial \mathcal{L}}{\partial \widehat{\Phi}'} = \begin{cases} \rho_0 + \frac{2}{\pi^2} e^{-b\,\tau^2} h^3 (1 - 2b\,\tau^2)\,, & (r < r_c) \\ \frac{2}{\pi^2} e^{-b\,\tau^2} h^3 (1 - 2b\,\tau^2)\,, & (r > r_c) \end{cases}$$

- The diverging τ in the IR and vanishing of baryon field h in UV enforce baryon density ρ to vanish. A nonzero ρ can however arise from abrupt discontinuity of h:

$$\rho_0 = \rho(r=0) = \frac{2}{\pi^2} e^{-b\,\tau(r_c)^2} (1 - 2b\,\tau(r_c)^2) \left[h(r_c+\epsilon)^3 - h(r_c-\epsilon)^3\right]$$





(T.Ishii, M.Jarvinen, G.Nijs, 2019)

Thermodynamically consistent EV

$$p_{ex}(T,\{\mu_i\}) = p_{id}(T,\{\tilde{\mu}_i\}); \qquad p_{id}(T,\mu,m) = \frac{g}{6\pi^2}\int_0^\infty \frac{p^4}{E_p}\frac{dp}{e^{(E_p-\mu)/T}\pm 1}$$

$$\tilde{\mu}_i = \mu_i - v_i p_{ex}(T,\{\mu_i\}),$$

$$n_{ex}^{(i)}(T,\{\tilde{\mu}_i\}) = \left(\frac{\partial p_{ex}}{\partial \mu_i}\right)_T = \frac{n_{id}^{(i)}(T,\{\tilde{\mu}_i\})}{1+v_0\sum_k n_{id}^{(k)}(T,\{\tilde{\mu}_k\})}$$

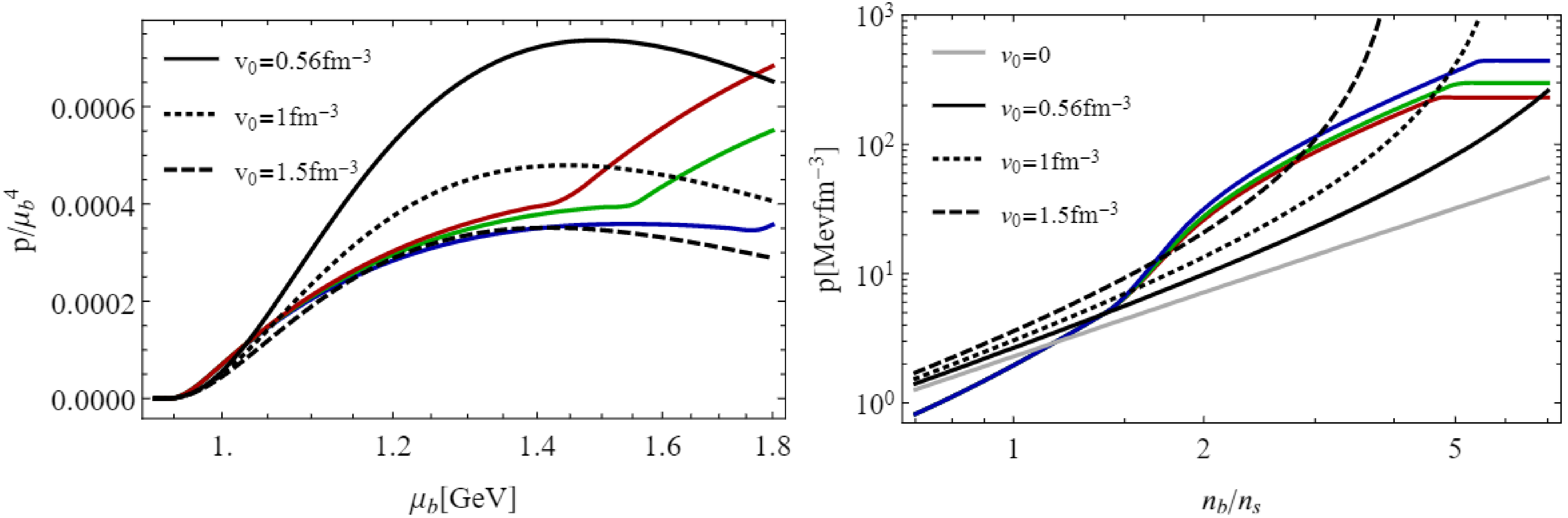
$$s_{ex}(T,\{\tilde{\mu}_i\}) = \left(\frac{\partial p_{ex}}{\partial T}\right)_{\mu_i} = \frac{s_{id}(T,\{\tilde{\mu}_i\})}{1+v_0\sum_k n_{id}^{(k)}(T,\{\tilde{\mu}_k\})}$$

$$\epsilon_{ex}(T,\{\tilde{\mu}_i\}) = Ts_{ex} - p_{ex} + \sum_i \mu_k n_{ex}^{(i)} = \frac{\epsilon_{id}(T,\{\tilde{\mu}_i\})}{1+v_0\sum_k n_{id}^{(k)}(T,\{\tilde{\mu}_k\})}$$

$$dp_{ex} = s_{ex}dT + n_b d\mu_b + n_b Y_q \mu_{le}, \qquad \mu_i = B_i\mu_b + Q_i\mu_q + L_i\mu_{le}$$

$$df_{ex} = s_{ex}dT + (\mu_{le}Y_q + \mu_b)dn_b + \mu_{le}n_b dY_q \qquad \mu_{le} = \mu_p + \mu_e - \mu_n$$

Different choices of the exc. volume



Details of the QM phase:

We assume that the QM is made of three quarks u, d, s, which are not distinguished by the strong interactions and the chemical potentials of the quarks are equal in the absence of electrons. We also impose charge neutrality in the presence of electrons.

$$n = (n_d + n_u + n_s)/3$$

$$n_d + n_s = 2n_u$$

$$f_{\rm QM}(T, n_b, Y_q) = f_{e\bar{e}\gamma}(T, Y_q n_b) + f_{\rm V-QCD}(T, n_b)$$

$$Y_q = \frac{n_e}{n} = \frac{2n_u - n_d - n_s}{3n}$$

Details of the Mixed phase:

In order to determine the phase transition between the NM and QM phases we should use the grand canonical ensemble.

$$p_{\rm NM}(\mu_b, \mu_{le}, T) = p_{\rm QM}(\mu_b, \mu_{le}, T)$$

This means that as functions of number densities there will be a mixed phase. The phase transition is of course simpler in the grand canonical, but in the end we need to compute the EOS tables in the canonical ensemble. Working directly in the canonical ensemble,

$$\begin{aligned}
p_{\rm NM}(n_b^{(1)}, Y_q^{(1)}, T) &= p_{\rm QM}(n_b^{(2)}, Y_q^{(2)}, T) \ , \\
\mu_b^{\rm (NM)}(n_b^{(1)}, Y_q^{(1)}, T) &= \mu_b^{\rm (QM)}(n_b^{(2)}, Y_q^{(2)}, T) \ , \\
\mu_{le}^{\rm (NM)}(n_b^{(1)}, Y_q^{(1)}, T) &= \mu_{le}^{\rm (QM)}(n_b^{(2)}, Y_q^{(2)}, T) \ .
\end{aligned}$$

The thermodynamics in the mixed phase involves a mixture of NM and QM matter in the equilibrium defined above. This is the Gibbs construction.

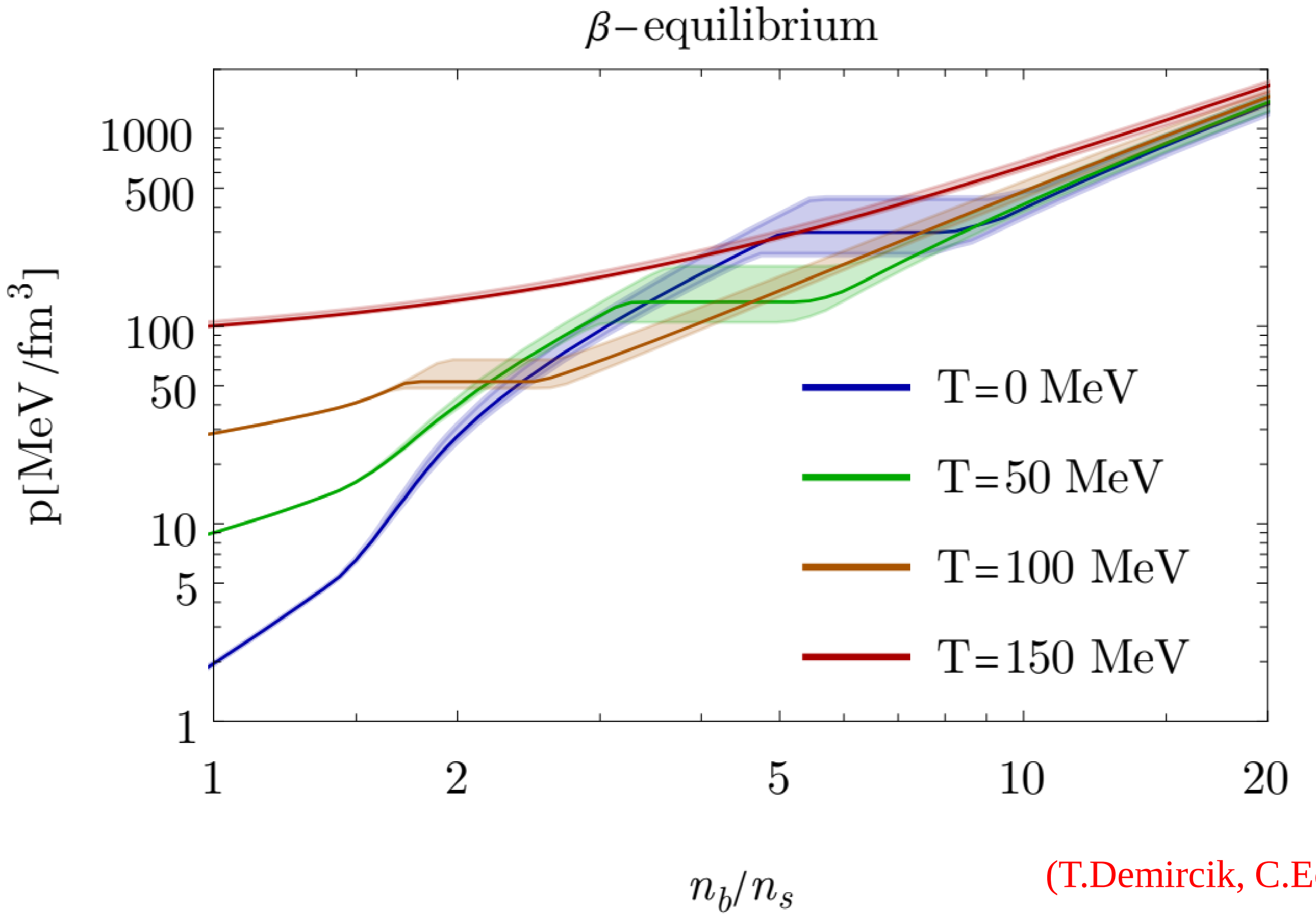
HS(DD2) and NM macthing:

$$\hat{f}_{\mathrm{HS(DD2)}}(T, n_b, Y_q) = f_{\mathrm{HS(DD2)}}(T, n_b, Y_q) + \sum_j p_{\mathrm{BE}}^{(j)}(T, m_j)$$

$$f_{\mathrm{NM}}(T, n_b, Y_q) = [1 - w(n_b)]\hat{f}_{\mathrm{HS(DD2)}}(T, n_b, Y_q) + w(n_b) f_{\mathrm{vdW}}(T, n_b, Y_q)$$

$$w(n_b) = \frac{1}{2}\left[1 + \tanh\left(\frac{\log(n_b/n_0)}{1.75}\right)\right] = \frac{(n_b/n_0)^{8/7}}{1 + (n_b/n_0)^{8/7}}$$

- Pressure as a function of the baryon number density in beta-equilibrium for different values of the temperatures.



(T.Demircik, C.Ecker, M.Jarvinen, 2021)

Binary NS Mergers

- 3+1D General Relativistic Hydrodynamics:

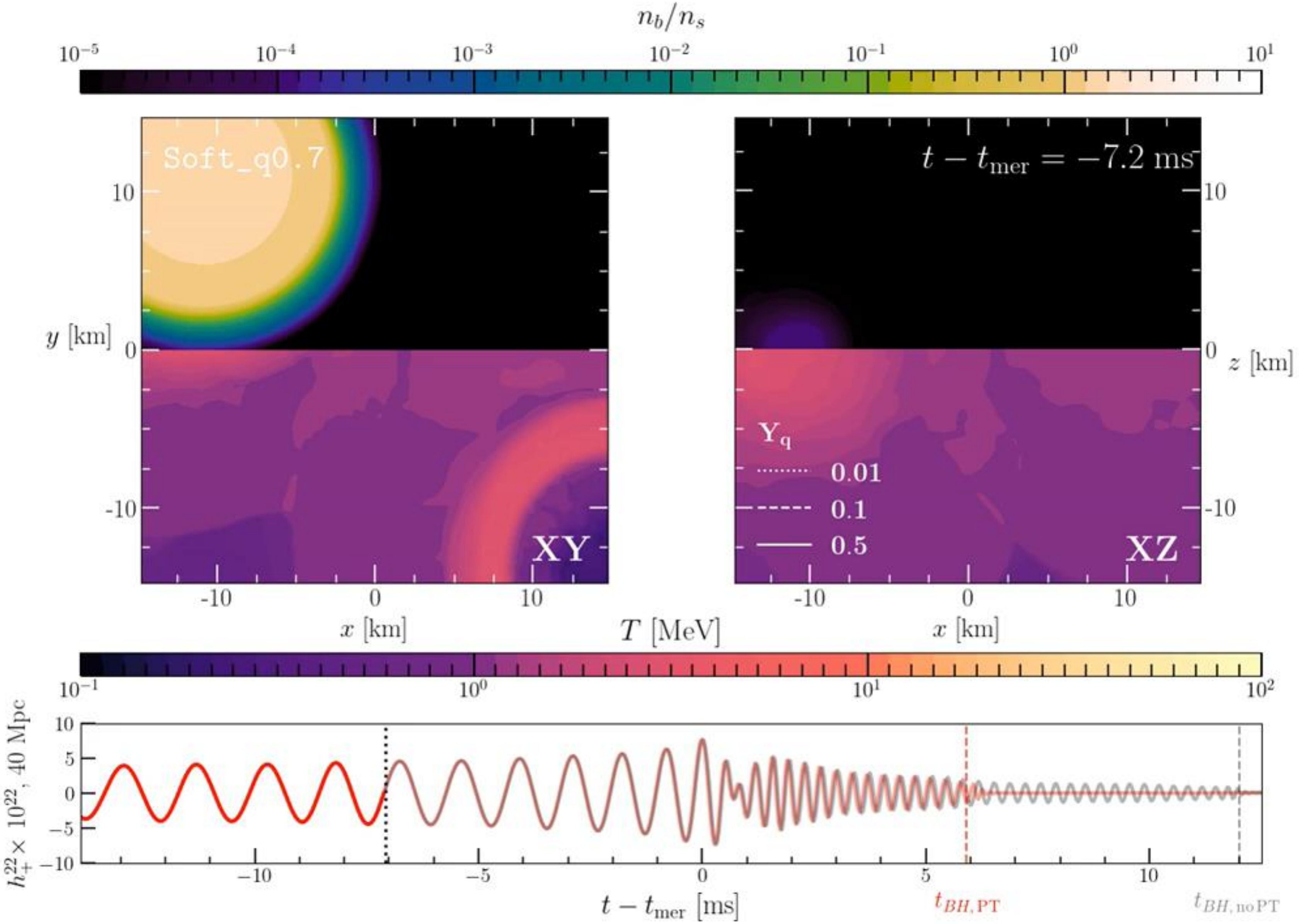
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \,, \quad \nabla_\mu T^{\mu\nu} = 0 \,, \quad \nabla_\mu J^\mu = 0 \,.$$

- Equation of State (EoS) $p = p(n_b, T, Y_e)$ as input required.
- Pseudo spectral code Frankfurt University/Kadath (FUKA) for initial data. [Papenfort, Tootle, Grandclément, Most, Rezzolla arXiv:2103.09911]
- Frankfurt/Illinois (FIL) code for binary evolution with tabulated EoS. [Most, Papenfort, Rezzolla arXiv:1907.10328]
- Implemented in the Einstein Toolkit framework. [https://einsteintoolkit.org/]

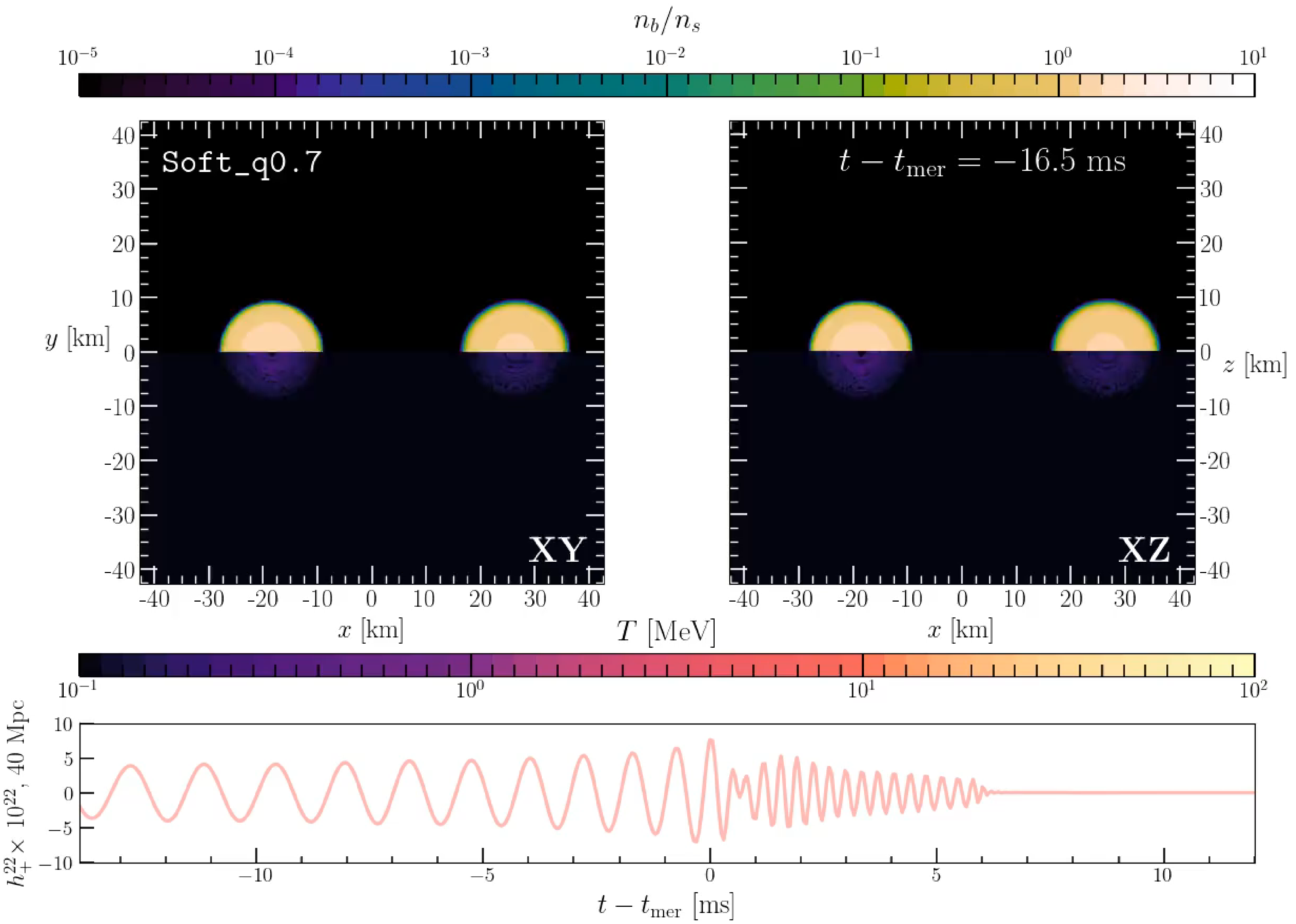
High-performance Computing Center

- Project BNSMIC with 100 million core-hours on HAWK.

	q	$M_{\rm TOV}$ $[M_\odot]$	M_1 $[M_\odot]$	M_2 $[M_\odot]$	R_1 [km]	R_2 [km]	Λ_1	Λ_2	$\tilde{\Lambda}$	$\Lambda_{1.4}$	$f_{\rm mer}$ kHz	f_3 kHz	$f^{2,1}$ kHz	$f^{2,2}$ kHz	$t_{\rm BH}$ [ms]
`Soft_q10`	1.0	2.02	1.37	1.37	12.37	12.37	537	537	537	475	1.77	4.00	1.44	2.92	9.5
`Soft_q07`	0.7	2.02	1.64	1.15	12.42	12.24	183	1393	517	475	1.63	3.80	1.55	2.80	5.8
`Soft_q10-NPT`	1.0	2.06	1.37	1.37	12.37	12.37	537	537	537	475	1.76	4.00	1.62	2.85	> 37
`Soft_q07-NPT`	1.0	2.06	1.64	1.15	12.42	12.24	183	1393	517	475	1.64	3.85	1.47	2.79	11
`Inter_q10`	1.0	2.14	1.37	1.37	12.45	12.45	565	565	565	511	1.74	4.00	1.51	2.73	> 35
`Inter_q07`	0.7	2.14	1.64	1.15	12.56	12.30	201	1437	543	511	1.63	3.70	1.42	2.67	> 37
`Stiff_q10`	1.0	2.34	1.37	1.37	12.58	12.58	617	617	617	560	1.74	3.90	1.39	2.49	> 37
`Stiff_q07`	0.7	2.34	1.64	1.15	12.76	12.38	231	1525	591	560	1.59	3.50	1.39	2.48	> 38

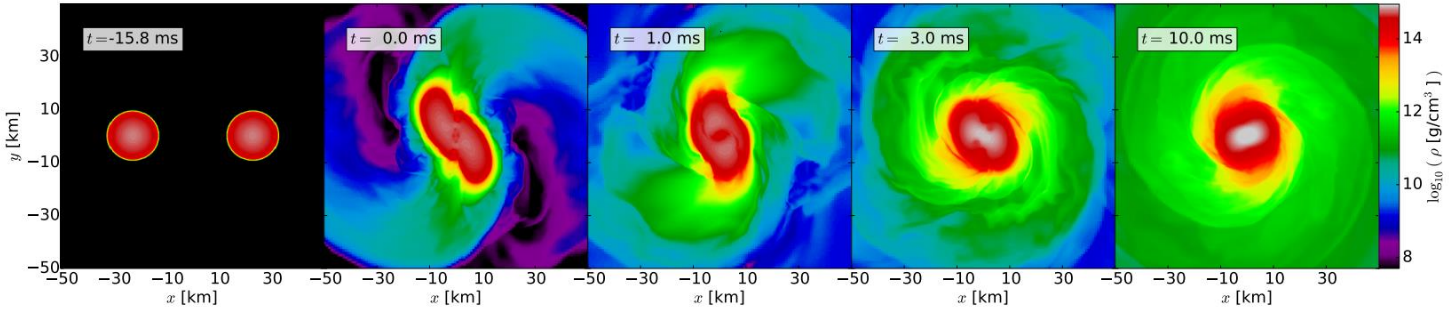


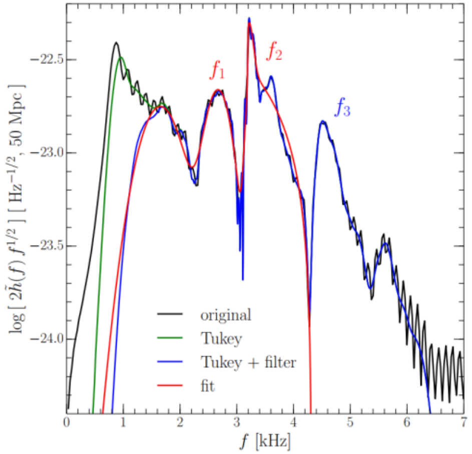
n_b/n_s
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
Soft_q0.7
$t - t_{\rm mer} = -7.2$ ms
y [km]
z [km]
Y_q
0.01
0.1
0.5
XY
XZ
x [km]
x [km]
T [MeV]
10^{-1} 10^{0} 10^{1} 10^{2}
$h_+^{22} \times 10^{22}$, 40 Mpc
$t - t_{\rm mer}$ [ms]
$t_{BH,\rm PT}$
$t_{BH,\rm noPT}$



n_b/n_s
Soft_q0.7
$t - t_{\rm mer} = -16.5$ ms
XY
XZ
y [km]
z [km]
x [km]
T [MeV]
$h_+^{22} \times 10^{22}$, 40 Mpc
$t - t_{\rm mer}$ [ms]

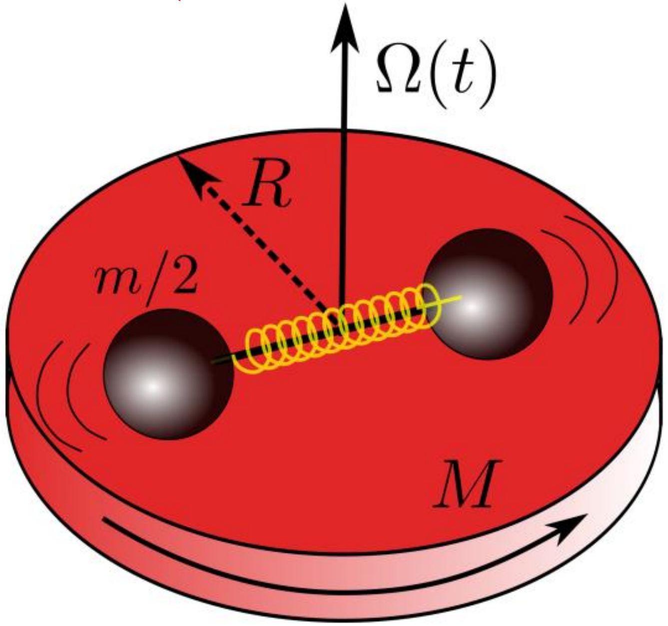
- f2 : related with oscillations of HMNS (αRmax)
  f1&f3: related with the merger process (collision and bounce-off of the stellar cores) (α C)





(Pictures: K.Takami, L.Rezzolla, L. Baiotti, 2015)

- A mechanical toy model:
  i) x ↑ , I ↑, ω ↓, Ω(t) → Ω1 ↓
  ii) x ↓ , I ↓, ω ↑, Ω(t) → Ω3 ↑
  iii) secular t: Ω(t) → (Ω1+ Ω3)/2



Gravitational Waves

- GW polarization amplitudes are related to the Weyl curvature scalars ψ_4 by:

$$\ddot{h}_+ - \mathrm{i}\ddot{h}_\times = \psi_4 = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \psi_4^{\ell m} \; {}_{-2}Y_{\ell m}(\theta, \varphi),$$

Which can be expanded as spin weighted spherical harmonics s=-2, the dominant mode l,m=2 in both in spiral and after the merger

$$h_{+,\times} = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{+,\times}^{\ell m} \; {}_{-2}Y_{\ell m}\left(\theta, \varphi\right) \approx h_{+,\times}^{22} \; {}_{-2}Y_{22}\left(\theta, \varphi\right)$$

PSD of the effective amplitude and inst. frequency of the GW:

$$\tilde{h}(f) \equiv \sqrt{\frac{|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2}{2}}$$

$$f_{\mathrm{GW}} \equiv \frac{1}{2\pi} \frac{d\phi}{dt}$$

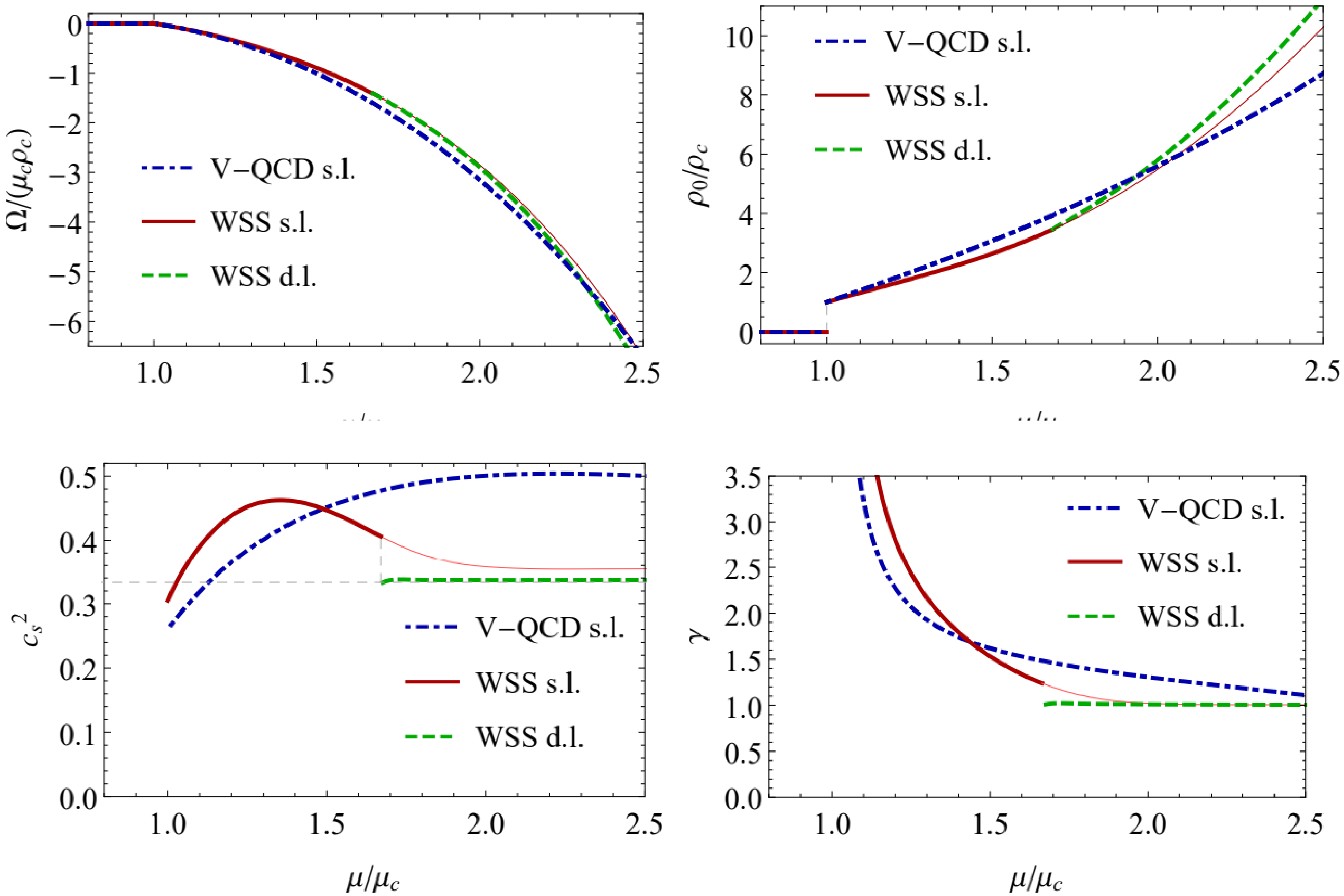
$$f_{\max} \equiv f_{\mathrm{GW}}(t = 0)$$

The cold NM in V-QCD- (Double layer Configuration)

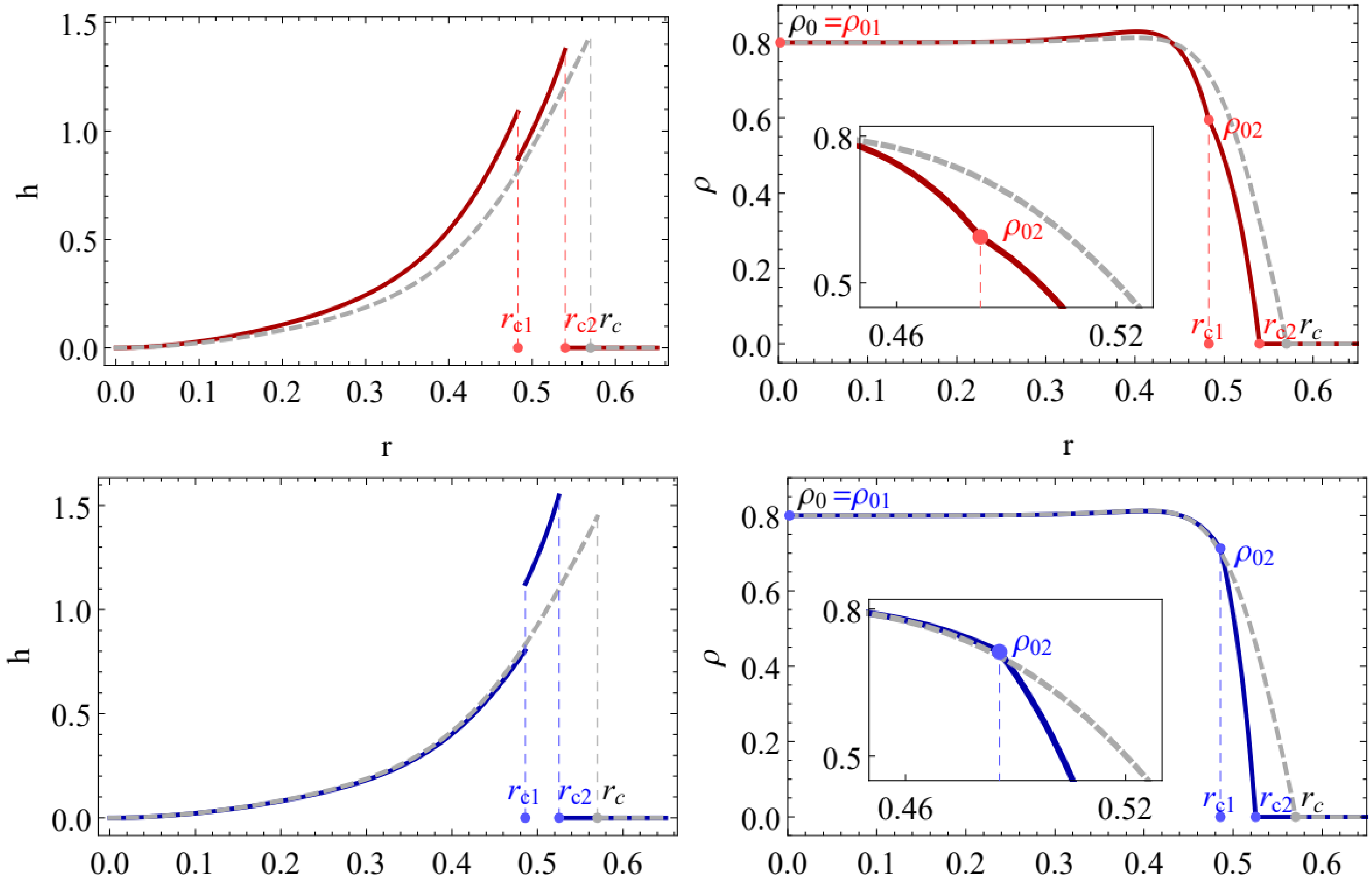
$$\rho = \begin{cases} \rho_{01} + \frac{2N_c}{\pi^2} e^{-b\tau^2} h^3 (1 - 2b\tau^2), & (r < r_{c1}) \\ \rho_{02} + \frac{2N_c}{\pi^2} e^{-b\tau^2} h^3 (1 - 2b\tau^2), & (r_{c1} < r < r_{c2}) \\ \frac{2N_c}{\pi^2} e^{-b\tau^2} h^3 (1 - 2b\tau^2), & (r > r_{c2}) \end{cases}$$

$$\rho_0 = \rho(r = 0) = \rho_{01} = -\frac{2N_c}{\pi^2} \sum_{i=1}^{2} (1 - 2b\tau^2) e^{-b\tau^2} \mathrm{Disc}\, h^3|_{r=r_{ci}}$$

$$\rho_{02} = -\frac{2N_c}{\pi^2}(1 - 2b\tau^2) e^{-b\tau^2} \mathrm{Disc}\, h^3|_{r=r_{c2}}\,, \qquad \rho_{01} - \rho_{02} = -\frac{2N_c}{\pi^2}(1 - 2b\tau^2) e^{-b\tau^2} \mathrm{Disc}\, h^3|_{r=r_{c1}}$$



$\Omega/(\mu_c\rho_c)$
V−QCD s.l.
WSS s.l.
WSS d.l.
ρ_0/ρ_c
V−QCD s.l.
WSS s.l.
WSS d.l.
c_s^2
V−QCD s.l.
WSS s.l.
WSS d.l.
μ/μ_c
γ
V−QCD s.l.
WSS s.l.
WSS d.l.
μ/μ_c



h
r
r_{c1}
r_{c2}
r_c
ρ
$\rho_0 = \rho_{01}$
ρ_{02}
0.46
0.52