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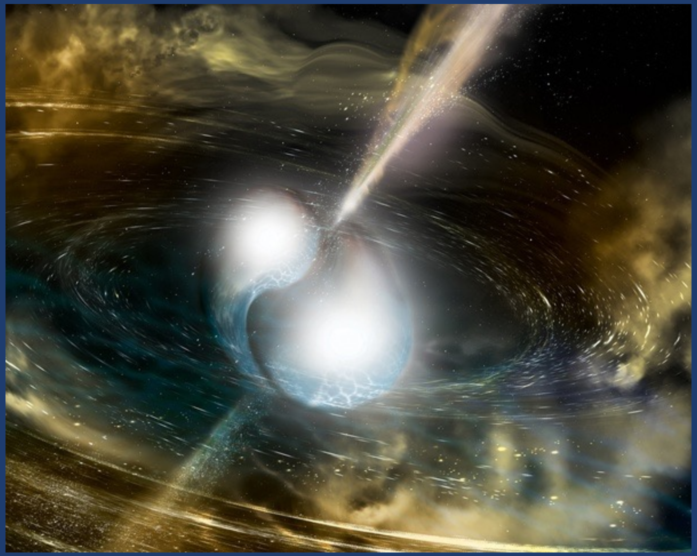
A NEW FORMULATION OF STRONG-FIELD MAGNETOHYDRODYNAMICS FOR NEUTRON STARS

TRENTO, 14.3.2023



motivation: dynamics of strong magnetic fields in neutron stars and applications of magnetohydrodynamics

old: microscopic model of Goldreich and Reisenegger (1992)

new: higher-form symmetries and effective field theory

MAGNETOHYDRODYNAMICS

MHD = Navier-Stokes + Maxwell (EM)

generalise

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\epsilon \vec{v}) = 0$$

continuity

$$\epsilon \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{v} = -\nabla p + \vec{J} \times \vec{B}$$

Euler (or Navier-Stokes)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\xcancel{\nabla \cdot \vec{E} = 0}$$

Maxwell

$$\vec{E} + \vec{v} \times \vec{B} = 0\,, \quad (\sigma \to \infty)$$

Ohm's law (ideal)

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \left(\frac{p}{\epsilon^\gamma} \right) = 0$$

equation of state

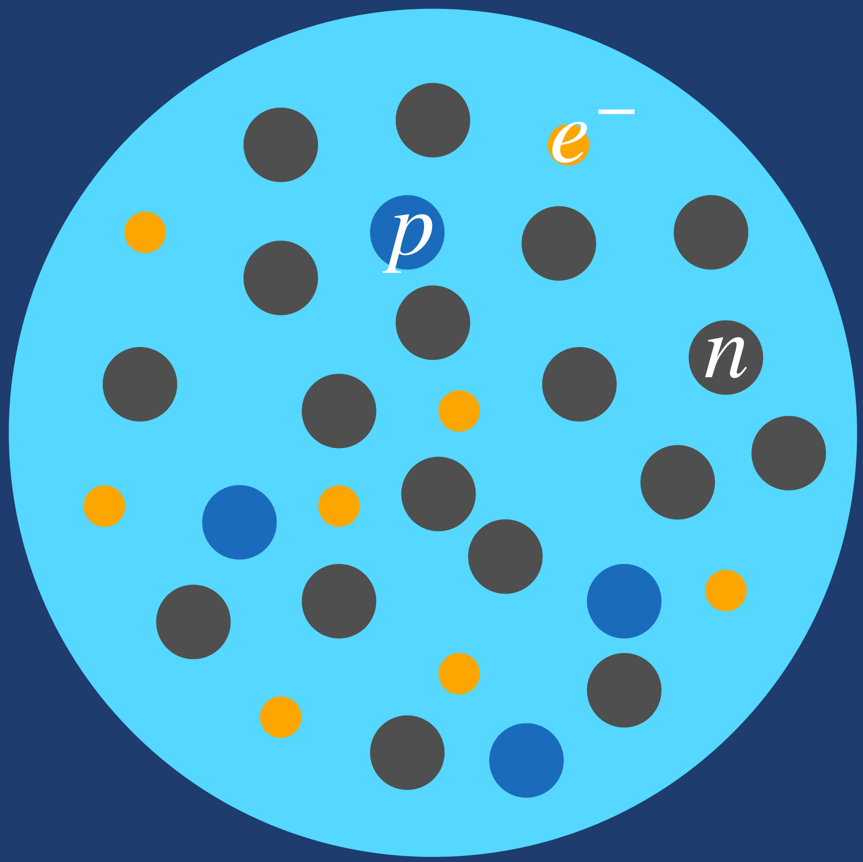
- avoid mixing macroscopic and microscopic concepts
- what are the symmetries?
- general equation of state

  $$B/T^2 \ll 1, \quad g \ll 1$$

- correct number of transport coefficients?
- quantum effects:
  pair-creation, Landau levels?

MAGNETIC DIFFUSION IN NEUTRON STARS

- theory of magnetic field evolution [Goldreich and Reisenegger (1992)]
- neutron star is a fluid made of neutrons, electrons and protons



$$\mu_n + m_n\psi = \text{constant}$$

$$m_p \frac{\partial \boldsymbol{v}_p}{\partial t} + m_p(\boldsymbol{v}_p \cdot \nabla)\boldsymbol{v}_p = -\nabla\mu_p - m_p\nabla\psi + e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_p}{c} \times \boldsymbol{B}\right) - \frac{m_p \boldsymbol{v}_p}{\tau_{pn}} - \frac{m_p(\boldsymbol{v}_p - \boldsymbol{v}_e)}{\tau_{pe}},$$

$$m_e^* \frac{\partial \boldsymbol{v}_e}{\partial t} + m_e^*(\boldsymbol{v}_e \cdot \nabla)\boldsymbol{v}_e = -\nabla\mu_e - e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_e}{c} \times \boldsymbol{B}\right) - \frac{m_e^* \boldsymbol{v}_e}{\tau_{en}} - \frac{m_e^*(\boldsymbol{v}_e - \boldsymbol{v}_p)}{\tau_{ep}}.$$

$$\Delta\Gamma \equiv \Gamma(p + e^- \rightarrow n + \nu_e) - \Gamma(n \rightarrow p + e^- + \bar{\nu}_e) = \lambda\Delta\mu$$

weak interactions influence continuity equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \left(\frac{\boldsymbol{j}}{\sigma_0}\right) + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right)\nabla \times \left(\frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c e}\right)$$

$$\sigma_0 = n_c e^2 \left(\frac{1}{\tau_{ep}/m_e^*} + \frac{1}{\tau_{pn}/m_p + \tau_{en}/m_e^*}\right)^{-1}$$

$$\frac{\boldsymbol{v}_p + \boldsymbol{v}_e}{2} = \boldsymbol{v} - \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right)\frac{\boldsymbol{j}}{2n_c e}$$

$$\boldsymbol{j} = \frac{c\nabla \times \boldsymbol{B}}{4\pi}$$

MAGNETIC DIFFUSION IN NEUTRON STARS

- theory of magnetic field evolution [Goldreich and Reisenegger (1992)]
- neutron star is a fluid made of neutrons, electrons and protons
- phenomenological considerations give MHD evolution equations for magnetic diffusion
- coefficients are determined in terms of microscopic quantites

$$\nabla \cdot \mathbf{B} = 0, \quad \partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = \eta\, \mathbf{j} + c_a\, (\mathbf{j} \times \mathbf{B}) \times \mathbf{B} + c_H\, \mathbf{j} \times \mathbf{B}$$

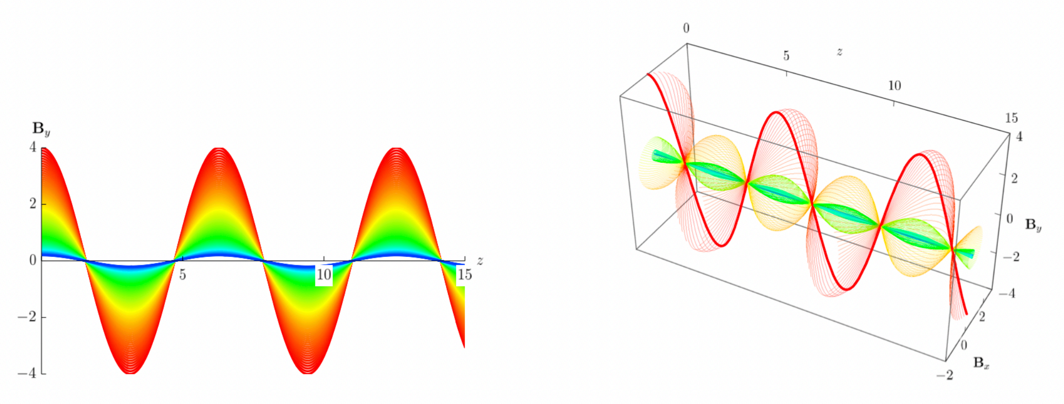
$$\mathbf{j} \equiv \nabla \times \mathbf{B}, \quad \eta,\, c_a,\, c_H = \text{const.}$$

Hall drift

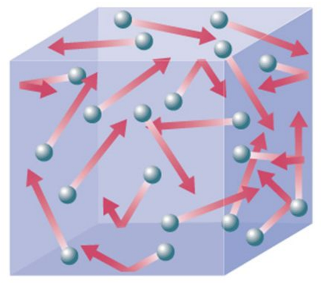
standard diffusion

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B}$$

ambipolar diffusion

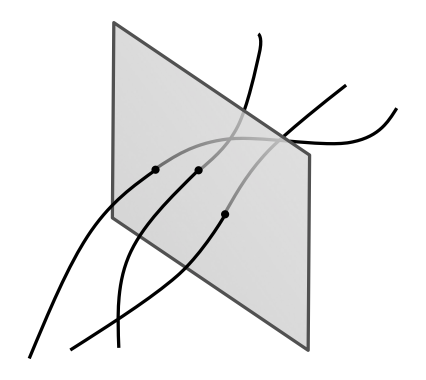


HIGHER-FORM SYMMETRIES



- zero-form symmetry (one-form conserved current)

$$\mathcal{O}(x) \to e^{iq\Lambda}\mathcal{O}(x) \qquad \partial_\mu J^\mu = 0$$



- one-form symmetry (two-form conserved current)
[Gaiotto, Kapustin, Seiberg, Willet (2014)]

$$\boxed{W(C) \to \exp\left(iq\int_C \Lambda\right) W(C) \qquad \partial_\mu J^{\mu\nu} = 0}$$

- *p*-form symmetries count higher-dimensional objects

$$\boxed{Q = \int_{S_{d-p}} \star J_{p+1}}$$

- EM coupled to matter (e.g., QED)

$$\boxed{J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}, \quad \partial_\mu J^{\mu\nu} = 0}$$

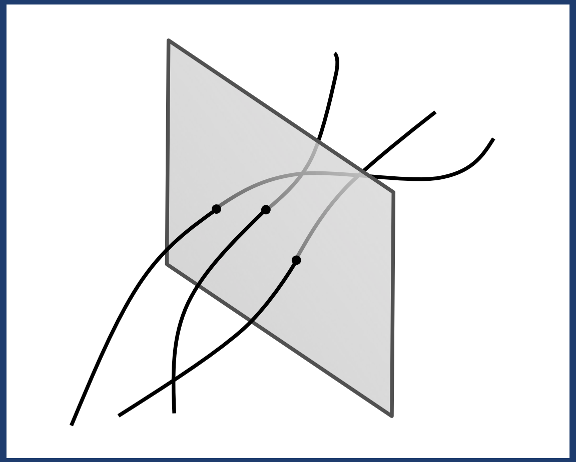
- one-form global symmetry is tautological in vacuum

$$\boxed{\partial_\mu J^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\partial_\mu\partial_\rho A_\sigma = 0}$$

- Goldstone boson of a broken symmetry is the photon

PLASMA AND MAGNETOHYDRODYNAMICS

- plasma is a phase with an unbroken one-form symmetry and no IR massless photons (Debye screening)
- $\partial_\mu J^{\mu\nu} = 0$ – Ward identity becomes powerful
- magnetohydrodynamics (MHD) is the IR EFT
- IR dynamics encoded in conservation laws



counting # of magnetic flux lines crossing a *2d* surface

$$\boxed{\begin{aligned}\partial_\mu T^{\mu\nu} &= 0\\ \partial_\mu J^{\mu\nu} &= 0\end{aligned}}$$

- general formulation for any plasma (EoM level) [Grozdanov, Hofman, Iqbal, PRD (2017)]
- any equation of state, transport coefficients (2+3 viscosities and 2 resistivities)
- new predictions for strong-***B*** dispersion relations
- symmetry-enhanced (non-dissipative) *T = 0* limit

$$\boxed{\begin{array}{ll}\eta_\perp \geq 0 & \eta_{||} \geq 0\\ r_\perp \geq 0 & r_{||} \geq 0\\ \zeta_\perp \geq 0 & \zeta_\perp \zeta_{||} \geq \zeta_\times^2\end{array}}$$

MAGNETOHYDRODYNAMICS

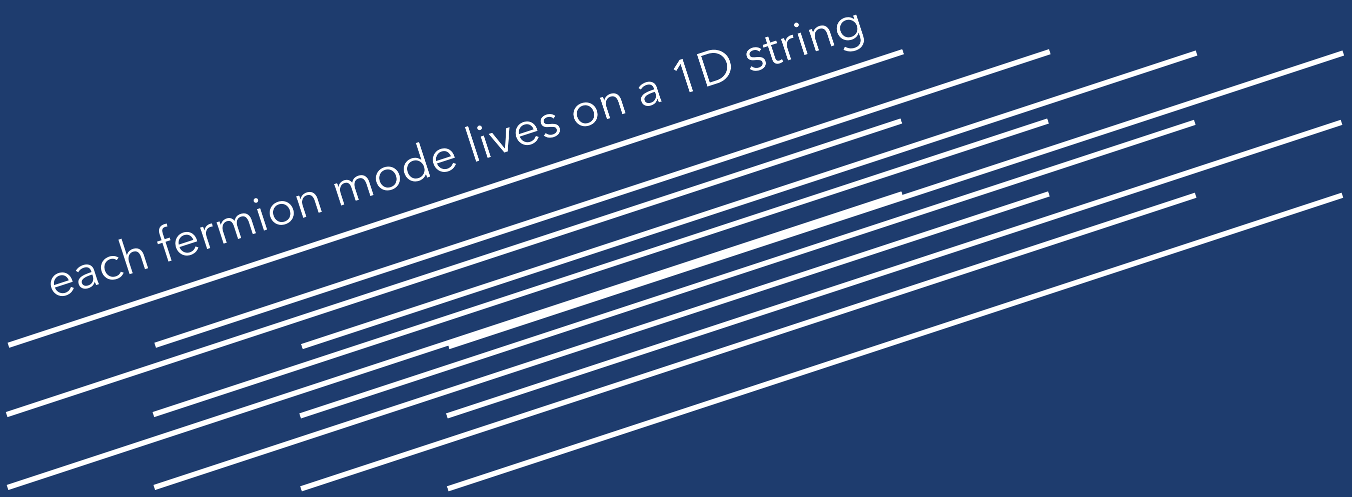
- old theories are special limits of new generalised MHD
- can be easily systematically extended using our formalism

first-order expansion in $B/T^2 \ll 1$ → standard, textbook ideal MHD with new dissipative terms

$$\boxed{P(B,T) = \frac{a}{4}T^4 + \frac{g^2}{2}B^2} + \frac{\beta}{4}\frac{B^4}{T^4} + \cdots$$

zero-temperature limit: *T* = *0*

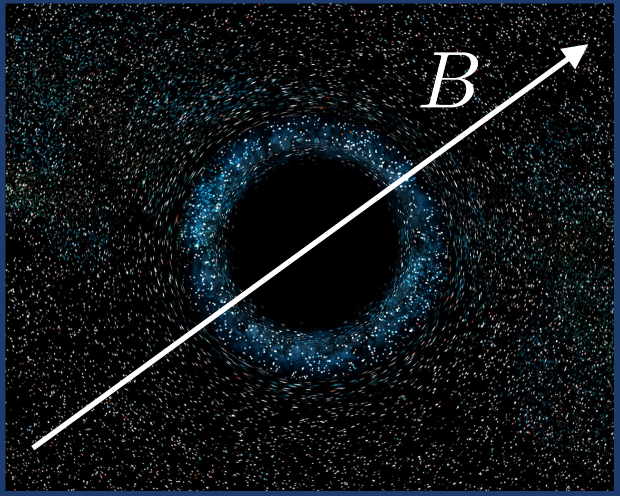
- massless lowest Landau level fermion in a strong, dynamical magnetic field
  + 2*d* bosonisation



- force-free electrodynamics used in astrophysics:
  Maxwell's equations
  + constraint

$$\boxed{\mathbf{E}\cdot\mathbf{B} = 0}$$

HOLOGRAPHIC MAGNETOHYDRODYNAMICS



- holographic dual of a magnetised plasma

  [Grozdanov, Poovuttikul, JHEP (2017); Hofman, Iqbal, SciPost (2017)]

- generating functional

$$\boxed{W\left[g_{\mu\nu}, b_{\mu\nu}\right] = \left\langle \exp\left[i\int d^4x\sqrt{-g}\left(\frac{1}{2}T^{\mu\nu}g_{\mu\nu} + J^{\mu\nu}b_{\mu\nu}\right)\right]\right\rangle}$$

- theory of gravity and two-form gauge field ($H = db$) in 5*d*

$$\boxed{S = \frac{N_c^2}{8\pi^2}\left[\int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{3e^2}H_{\mu\nu\rho}H^{\mu\nu\rho}\right) + \int_{\partial M} d^4x\sqrt{-\gamma}\left(2\,\mathrm{tr}K - 6 + \frac{1}{e^2}\mathcal{H}_{ab}\mathcal{H}^{ab}\ln\mathcal{C}\right)\right]}$$

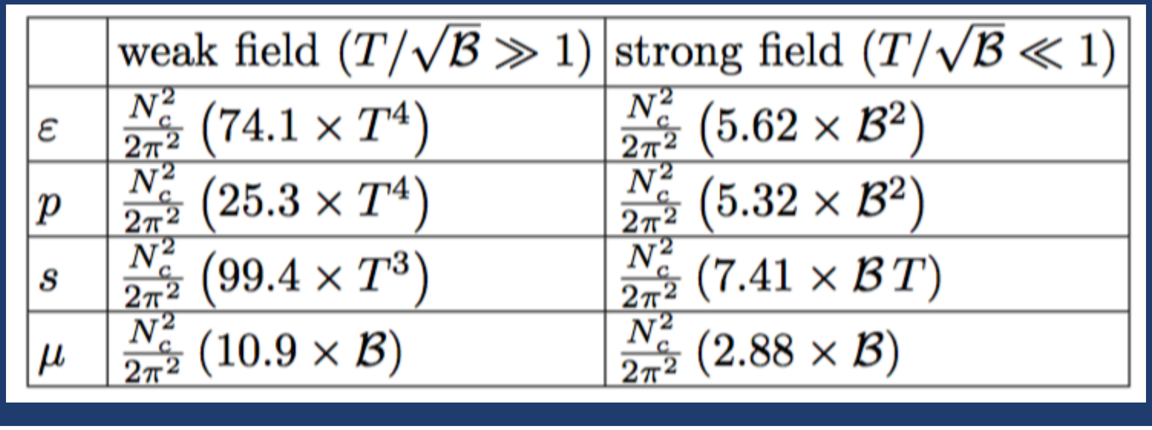
+ mixed boundary conditions

- holographic model with dynamical electromagnetism

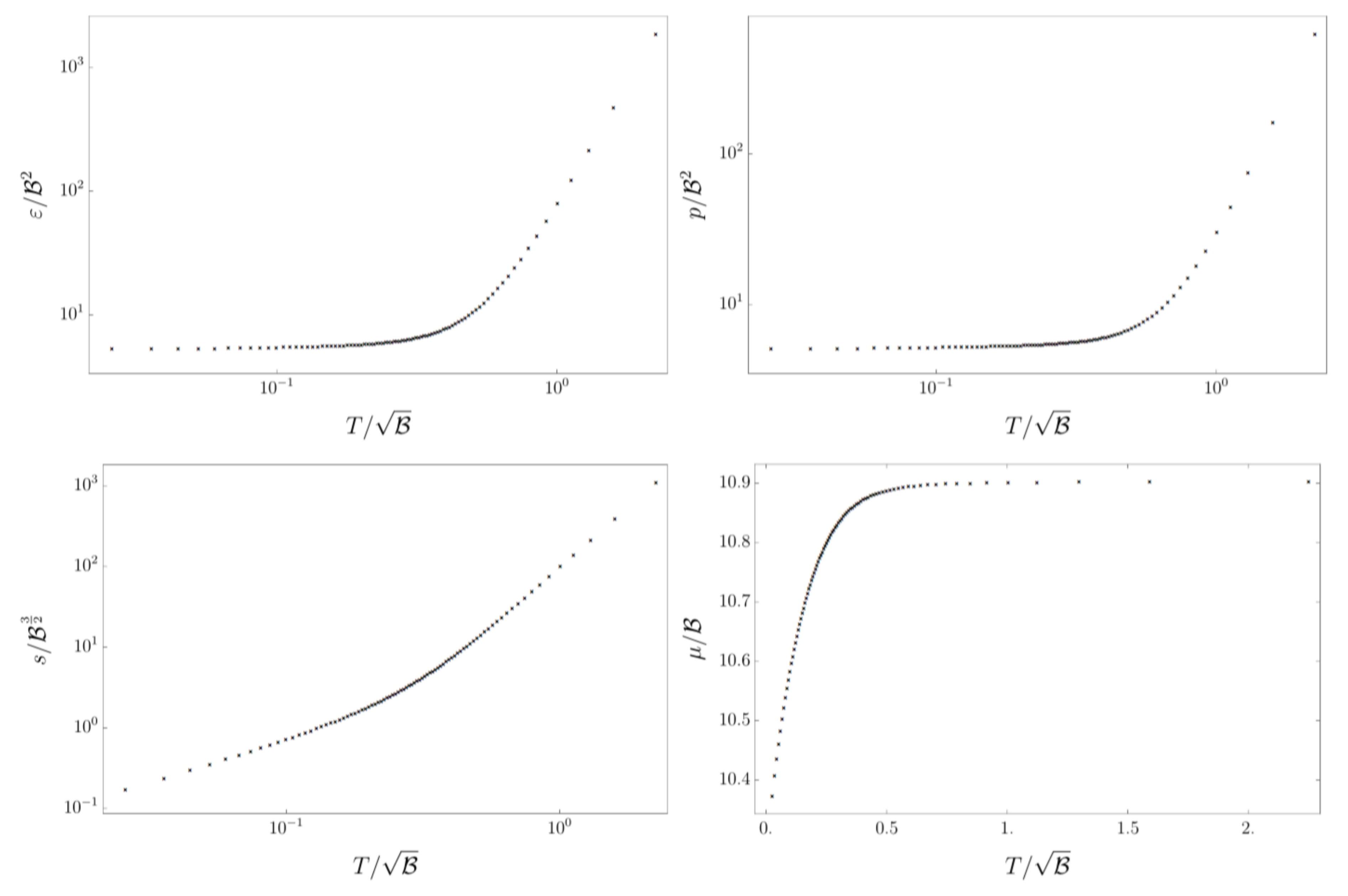
- EFTs beyond MHD (quasihydrodynamics): screened photons, Ampere's law, …

  [Grozdanov, Lucas, Poovuttikul, PRD (2018)]

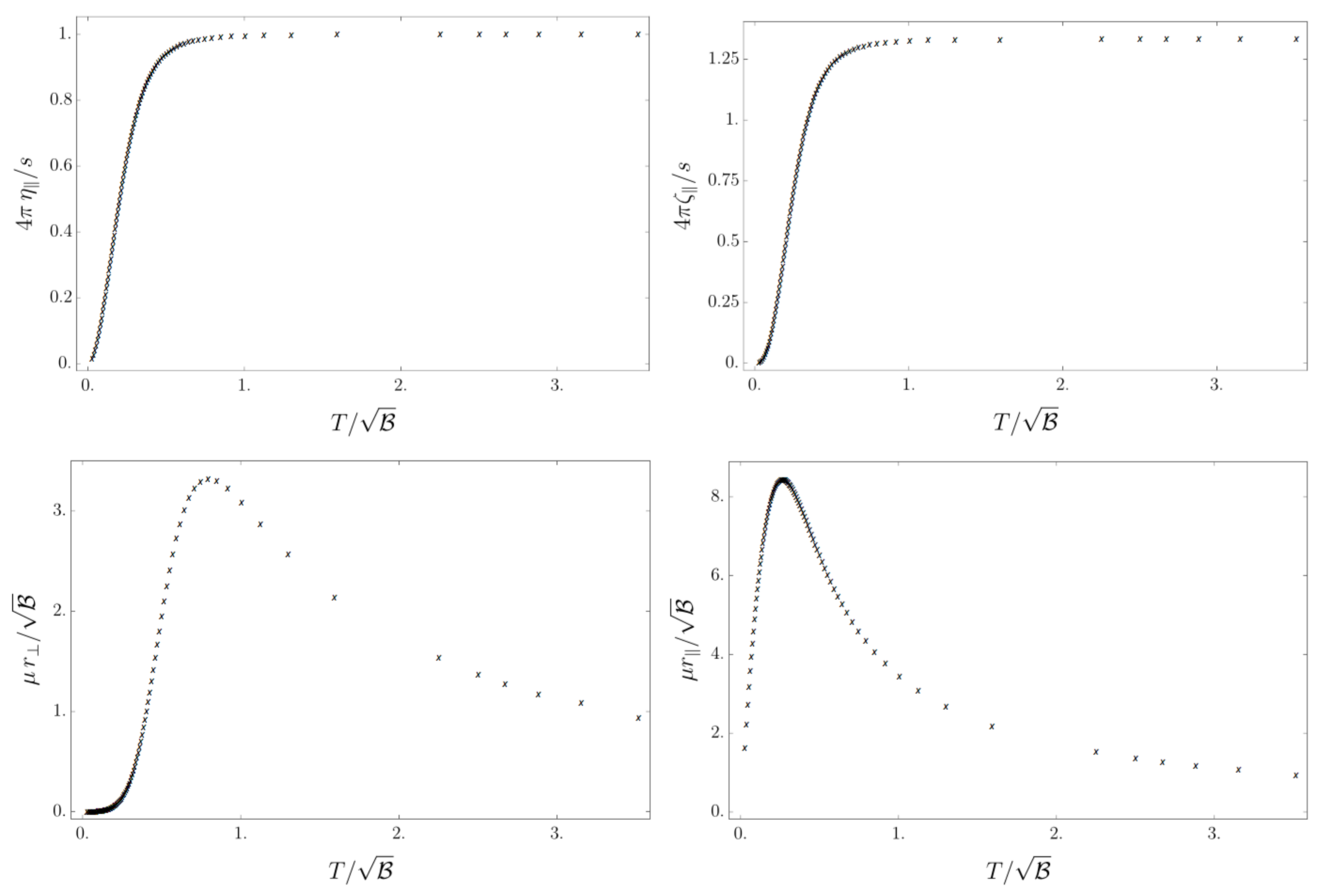
HOLOGRAPHIC MAGNETOHYDRODYNAMICS

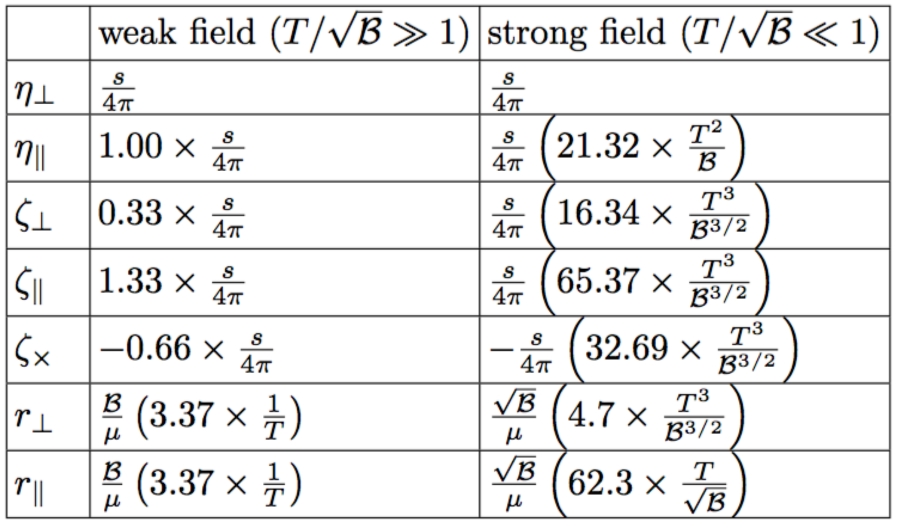


	weak field ($T/\sqrt{\mathcal{B}} \gg 1$)	strong field ($T/\sqrt{\mathcal{B}} \ll 1$)
ε	$\frac{N_c^2}{2\pi^2}\left(74.1 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.62 \times \mathcal{B}^2\right)$
p	$\frac{N_c^2}{2\pi^2}\left(25.3 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.32 \times \mathcal{B}^2\right)$
s	$\frac{N_c^2}{2\pi^2}\left(99.4 \times T^3\right)$	$\frac{N_c^2}{2\pi^2}\left(7.41 \times \mathcal{B}\,T\right)$
μ	$\frac{N_c^2}{2\pi^2}\left(10.9 \times \mathcal{B}\right)$	$\frac{N_c^2}{2\pi^2}\left(2.88 \times \mathcal{B}\right)$



HOLOGRAPHIC MAGNETOHYDRODYNAMICS





	weak field $(T/\sqrt{\mathcal{B}} \gg 1)$	strong field $(T/\sqrt{\mathcal{B}} \ll 1)$
$\eta_\perp$	$\frac{s}{4\pi}$	$\frac{s}{4\pi}$
$\eta_\parallel$	$1.00 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(21.32 \times \frac{T^2}{\mathcal{B}}\right)$
$\zeta_\perp$	$0.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(16.34 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\parallel$	$1.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(65.37 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\times$	$-0.66 \times \frac{s}{4\pi}$	$-\frac{s}{4\pi}\left(32.69 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\perp$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(4.7 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\parallel$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(62.3 \times \frac{T}{\sqrt{\mathcal{B}}}\right)$

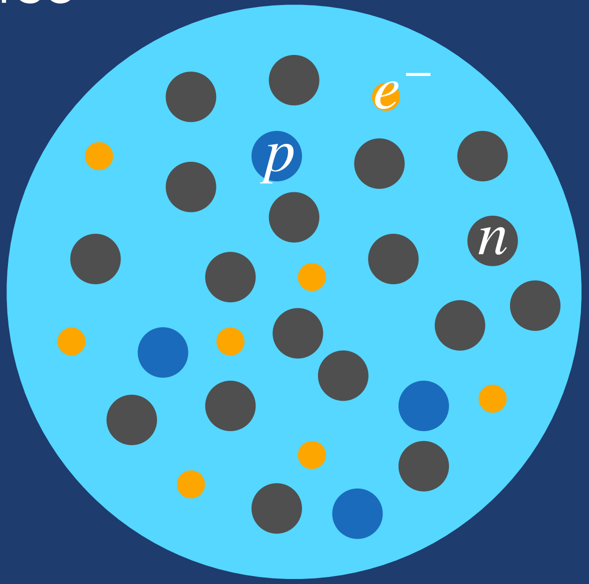
- maximal resistivity
- bulk viscosities

$$\boxed{\zeta_\perp \zeta_\parallel = \zeta_\times^2}$$

- all transport coefficients must vanish at *T = 0*
  [Grozdanov, Hofman, Iqbal, PRD (2017)]
- holography agrees
  [Grozdanov, Poovuttikul, JHEP (2017)]

MAGNETIC DIFFUSION IN NEUTRON STARS

- EFT (CTP formalism) for the one-form symmetry with broken *C*-invariance
  [Grozdanov, Leutheusser, Liu, Vardhan, 2207.01636 & *to appear*]



$$\boxed{\begin{aligned} J^{0i} &= a\,G_{0i} - \beta_0(c\,\delta_{ij} + \tilde{c}\,G_{0i}G_{0j})\partial_0 G_{0j} + h\epsilon_{ijn}G_{0n}\partial_0 G_{0j} + m\delta_{ij}\epsilon_{kln}G_{0n}H_{jkl} \\ J^{ij} &= -2\beta_0(d\,\delta_{ik}\delta_{jl} + \tilde{d}\,\epsilon_{ijm}\epsilon_{kln}G_{0m}G_{0n})\partial_0 G_{kl} - m\epsilon_{lij}\partial_l G_{0k}^2 + 2p\delta_{k[i}\epsilon_{j]ln}G_{0n}\partial_0 G_{kl} \end{aligned}}$$

- new magnetic diffusion equation

$$\nabla \cdot \mathbf{B} = 0, \quad \partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\begin{aligned} \mathbf{E} = \eta\,\mathbf{j} + c_a\,(\mathbf{j} \times \mathbf{B}) \times \mathbf{B} + c_H\,\mathbf{j} \times \mathbf{B} \\ - c_H(\nabla \ln a(\mathbf{B}^2) \times \mathbf{B}) \times \mathbf{B} - c_\eta \nabla \ln a(\mathbf{B}^2) \times \mathbf{B} + \nabla f \end{aligned}$$

$$\eta = c_\eta + c_a \mathbf{B}^2, \; c_\eta = \frac{2\beta_0 d}{a}, \; c_a = \frac{4\beta_0 \tilde{d}}{a^3}, \; c_H = -\frac{p}{a^2}$$

all functions of ***B***

- one can compute all correlation functions of ***B*** and ***E*** from auxiliary higher-form variables – a much more 'natural' description

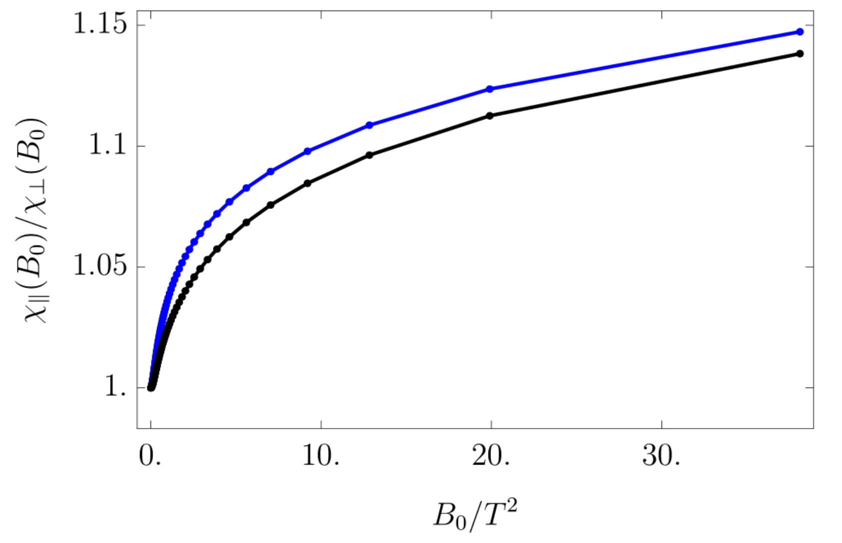
MAGNETIC DIFFUSION IN NEUTRON STARS

- magnetic field dependence of susceptibilities

$$\chi_\parallel = a + \tilde{a} B_0^2, \quad \chi_\perp = a, \quad \tilde{a} = 2a'(B_0^2)$$

- Kubo formulae:

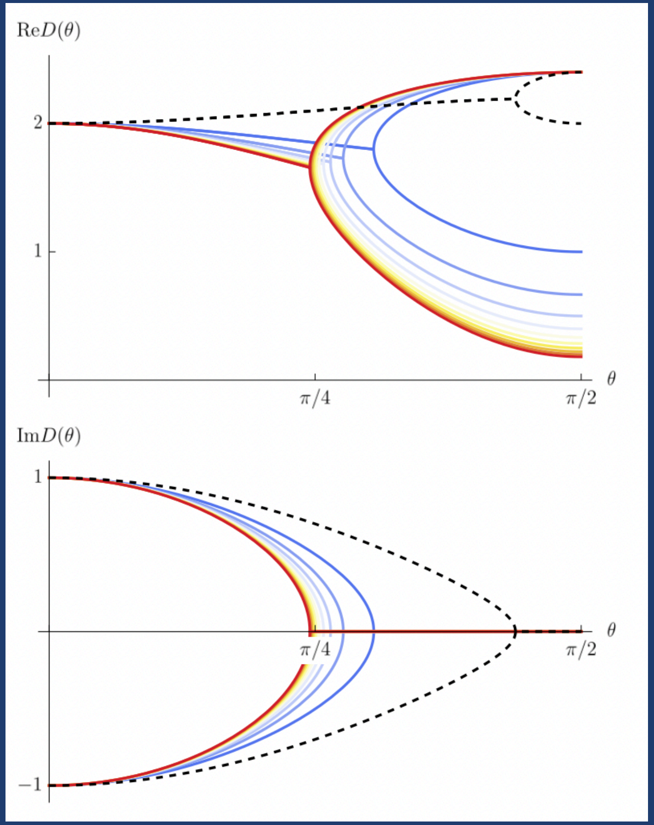
$$\chi_\parallel = \lim_{\mathbf{k}\to 0} \lim_{\omega\to 0} G^R(B_z, B_z), \quad \chi_\perp = \lim_{\mathbf{k}\to 0} \lim_{\omega\to 0} G^R(B_x, B_x)$$



$$\sigma^\parallel = \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{1}{i\omega} G^R(E_z, E_z), \quad \sigma_1^\perp = \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{1}{i\omega} G^R(E_x, E_x), \quad \sigma_2^\perp = \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{1}{i\omega} G^R(E_x, E_y)$$

- 'corrected' diffusive dispersion relation

$$\omega = -i\frac{\sigma_1^\perp}{\chi_\perp} k_z^2 - \frac{i}{2}\left(\frac{\sigma^\parallel}{\chi_\perp} + \frac{\sigma_1^\perp}{\chi_\parallel}\right) k_\perp^2$$
$$\mp \frac{i}{2}\sqrt{\left(\frac{\sigma^\parallel}{\chi_\perp} - \frac{\sigma_1^\perp}{\chi_\parallel}\right)^2 k_\perp^4 - 4\left(\frac{\sigma_2^\perp}{\chi_\perp}\right)^2 \left(k_z^2 + \frac{\chi_\perp}{\chi_\parallel} k_\perp^2\right) k_z^2}$$



$$\omega = -iD(\theta)k^2$$

SUMMARY AND FUTURE DIRECTIONS

- formal approach to QFTs enables new physical insights
- plasma can be understood as a string fluid
- higher-form symmetries and new developments in EFTs allow for a new, general formulation of magnetohydrodynamics
- new predictions, including a new theory of magnetic diffusion in neutron stars
- next step: non-linear evolution of the diffusion equation (best variables?)
- include: chiral effects, superfluidity, …
- how large are new effects in realistic neutron stars?

THANK YOU!