Baryons in V-QCD

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Holographic perspectives on chiral transport

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based on ArXiv:2209.05869, 2212.06747 with

E. Kiritsis, M. Jarvinen, E. Preau

Introduction

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- Essential ingredients: Chern-Simons terms.

Outline

- Review of the V-QCD model
- Chern-Simons Terms
- Constructing Baryons

States in holography

Finite energy configuration on the gravity side

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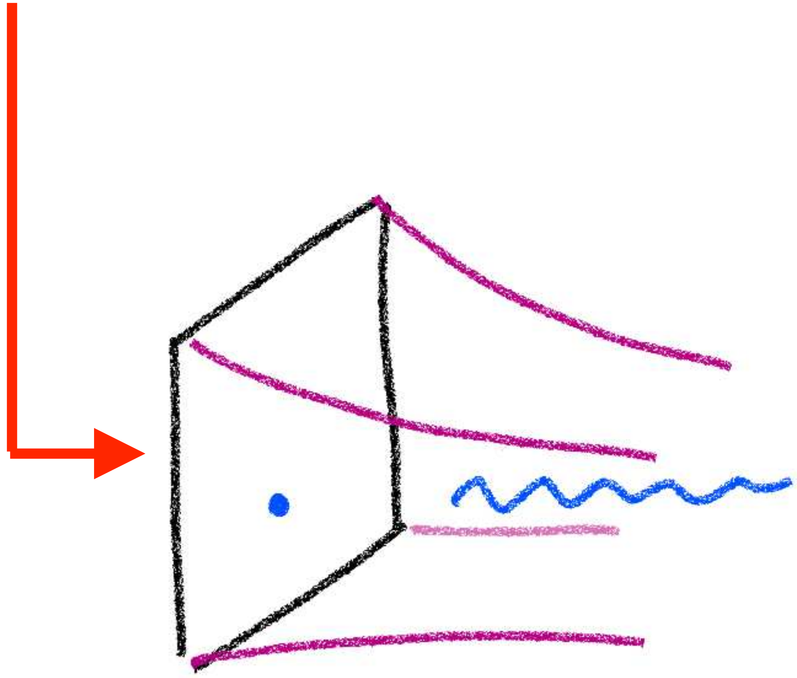

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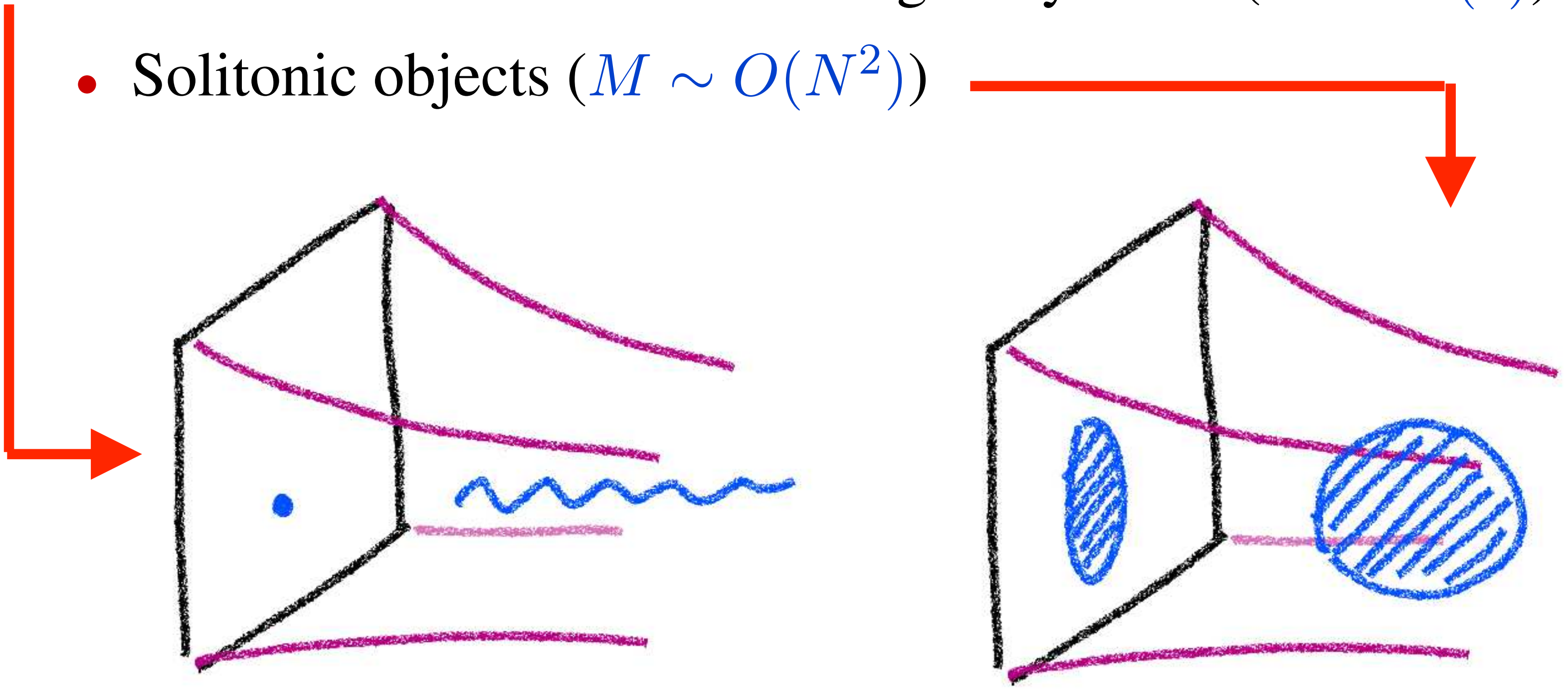
Gauge-singlet (composite) state in the Hilbert space of the QFT

- Perturbative excitations of the gravity fields ($M \sim O(1)$)
- Solitonic objects ($M \sim O(N^2)$)



Minimal holographic YM

- The bulk theory is five-dimensional (x^μ + RG coordinate r)
- Include only lowest dimension YM operators ($\Delta = 4$)

4D Operator		Bulk field	Coupling
TrF^2	$\Leftrightarrow$	Φ	$N \int e^{-\Phi}\, TrF^2$
$T_{\mu\nu}$	$\Leftrightarrow$	$g_{\mu\nu}$	$\int g_{\mu\nu} T^{\mu\nu}$

$\lambda = N g_{YM}^2 = e^\Phi$ (finite in the large N limit).

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- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).
- (Eventually: add axion field $a \Rightarrow TrF\tilde{F}$)

5-D Eistein-Dilaton Theory

Gursoy, Kiritsis, FN, 2007

Bulk dynamics described by a 2-derivative action:

$$S_c = -M_p^3 N_c^2 \int d^5x \sqrt{-g} \left[R + \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} - V(\lambda) \right]$$

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- Effective Planck scale $\sim N_c^2$ is large.

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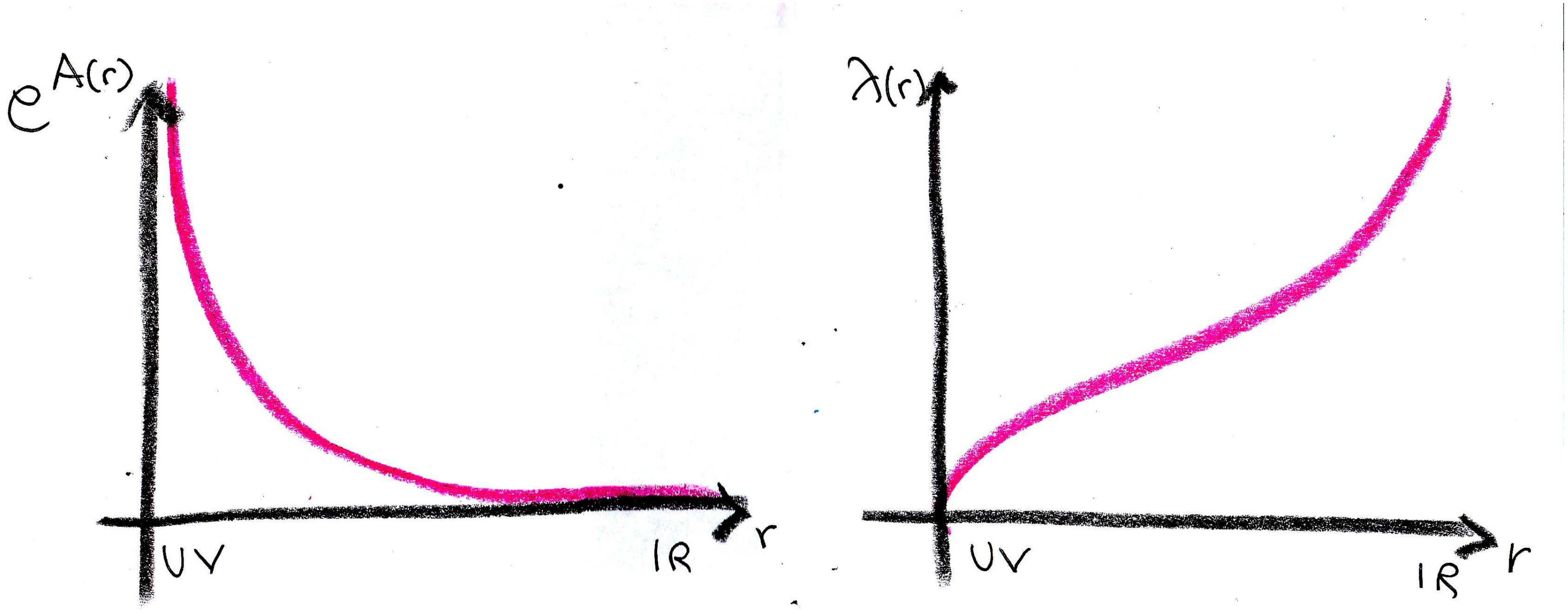
- Effective Planck scale $\sim N_c^2$ is large.
- Features: asymptotic freedom, confinement, discrete linear glueball spectrum, correct thermodynamics and phase diagram

Five dimensional setup: Yang-Mills

The Poincaré-invariant vacuum solution has the general form:

$$ds^2 = e^{2A(r)}(dr^2 + dx_\mu dx^\mu), \quad \lambda = \lambda(r), \quad 0 < r < +\infty$$

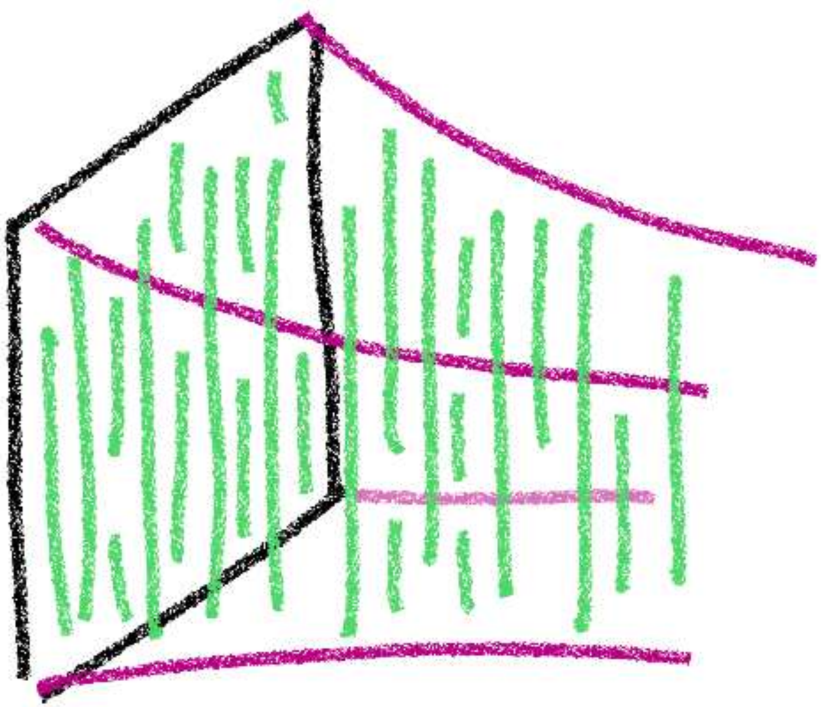
- $e^A(r) \propto$ 4D energy scale
- $\lambda(r) \propto$ running 't Hooft coupling



Adding Flavor: V-QCD

Jarvinen, Kiritsis 2011

N_f quark flavors $\Leftrightarrow$ N_f space-filling branes-antibranes.



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Flavor brane worldvolume fields:

- $U(N_f)_L \times U(N_f)_R$ gauge fields

$$A_B^{a;L}, A_B^{a;R} \quad \Leftrightarrow \quad J_\mu^{a;L,R} \equiv \bar{q}^i \gamma_\mu \left(\tau^a\right)_i^j (1 \pm \gamma_5) q_j$$

$a = 1 \ldots N_f^2,\ i, j = 1 \ldots N_f$

$U_B(1)$ current ⇔ abelian vector $A_\mu^{(L)} + A_\mu^{(R)}$

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- Bi-fundamental scalars Scalars

$$\mathcal{T}_j^i \Leftrightarrow \bar{q}^i q_j \qquad m^2 = -3 \Leftrightarrow \Delta = 3$$

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$$\mathcal{T}_j^i \Leftrightarrow \bar{q}^i q_j \qquad m^2 = -3 \Leftrightarrow \Delta = 3$$

$$S_{VQCD} = S_c + S_{DBI} + S_{CS}$$

Action: DBI term

$$S_{DBI} = -M_p^3 N_c Tr \int d^5x\, V_f(\lambda, \mathcal{T}^\dagger \mathcal{T}) \left[\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}}\right]$$

$$\mathbf{A}_{ab} = g_{ab} + w(\lambda, \mathcal{T}^\dagger \mathcal{T}) F_{ab} + \kappa(\lambda, \mathcal{T}^\dagger \mathcal{T}) (D_a \mathcal{T})^\dagger D_b \mathcal{T} \ + \ h.c.$$

Inspired by Sen’s brane-antibrane action

Vacuum and chiral symmetry breaking

- $V_f \sim -3\mathcal{T}^\dagger\mathcal{T}$ as $\mathcal{T} \to 0$
- $V_f \sim exp[-\mathcal{T}^\dagger\mathcal{T}]$ as $\mathcal{T} \to \infty$;

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- Vacuum state : $\mathcal{T}^i_j = \tau(r)\delta^i_j$ (equal quark masses)

$$\tau \sim m_q\, r + \sigma r^3 + \ldots \qquad r \to 0$$

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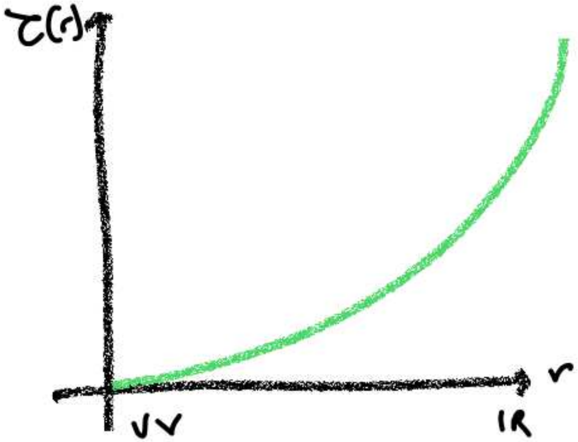

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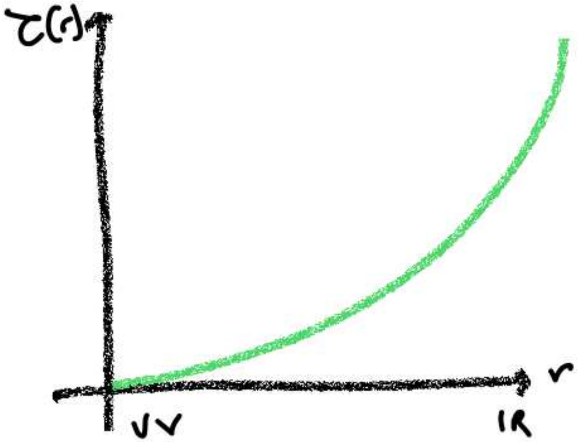
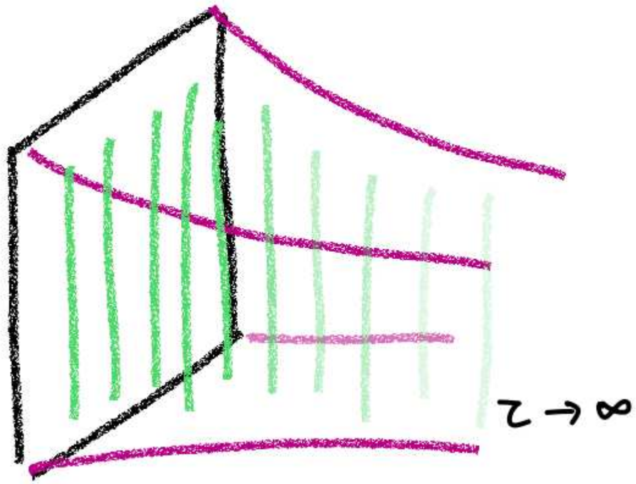




- IR: $S_{DBI} \propto V_f \to 0$, flavor disappear, color remains.

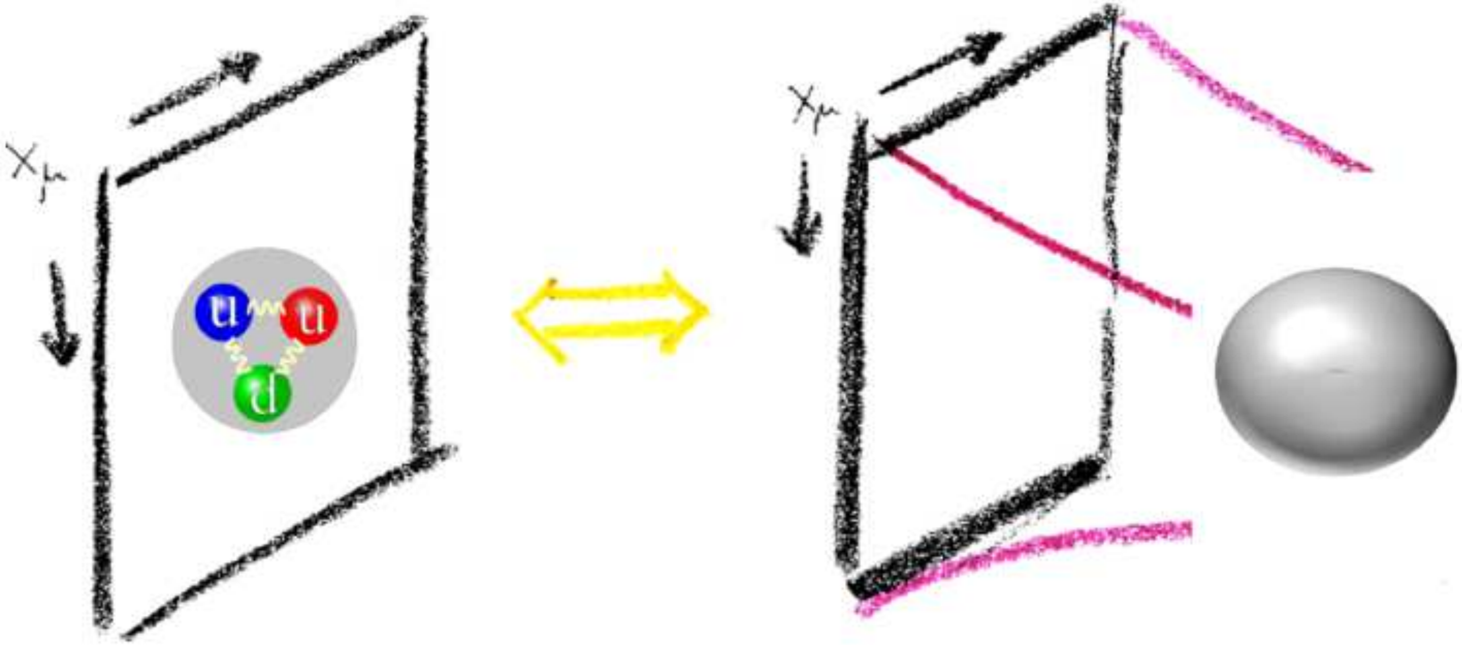
Action: Chern-Simons terms

$$S_{CS} = \frac{iN_c}{4\pi^2}\int \Omega_5(\mathcal{T}, \mathbf{A}^{(L,R)})$$

- EoM vanish on homogeneous configurations $\Rightarrow$ does not affect the bacgkground
- Starts at (at least) cubic order in the fields $\Rightarrow$ No contribution to the background, 2-point functions
- Possible contribution to 3- and higher-point function (possibly interesting)
- Crucial for constructing baryon states

Baryons in Holography

A single baryon is a solitonic object in the bulk, charged under the flavor gauge-fields.



Baryons as axial bulk instantons

Schematically:

$$S_{CS} \supset \int \omega_5(\mathbf{A}^{(L)}) - \int \omega_5(\mathbf{A}^{(R)}), \quad \omega_5(\mathbf{A}) = Tr\left(\mathbf{A} \wedge F + \frac{1}{2}\mathbf{A}^3 \wedge F - \frac{1}{10}\mathbf{A}^5\right)$$

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Constant baryon chemical potential: abelian $A_0^{(V)} = \mu$

$$S_{CS} = \int dt\, \mu \int d^3x dr \left[Tr\left(F^{(L)} \wedge F^{(L)}\right) - Tr\left(F^{(R)} \wedge F^{(R)}\right)\right]$$

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Baryon is a euclidean instanton of the $SU(N_f) \times SU(N_f)$ gauge fields, extended in the spatial + holographic directions

CS action in V-QCD

$$S_{CS} = \frac{iN_c}{4\pi^2} \int \Omega_5 \left(\mathcal{T}, \mathbf{A}^{(L,R)} \right)$$

Find most general Ω_5 such that:

- Parity invariant ($x^\mu \to -x^\mu$ and $L \leftrightarrow R$)
- Invariant under bulk flavor gauge transformations $U(N_f)_L \times U(N_F)_R$ up to a boundary term
- The boundary variation matches the QCD chiral anomalies

$$\int d^5 x \delta_\Lambda \Omega_5 = \int d^4 x \left(\Lambda^{(L)} D_\mu J^\mu_L + \Lambda^{(R)} D_\mu J^\mu_R \right)$$

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Work in the special case of zero quark masses (chiral limit) and set:

$$\mathcal{T} = \tau U$$

Real scalar

Unitary Nf x Nf matrix

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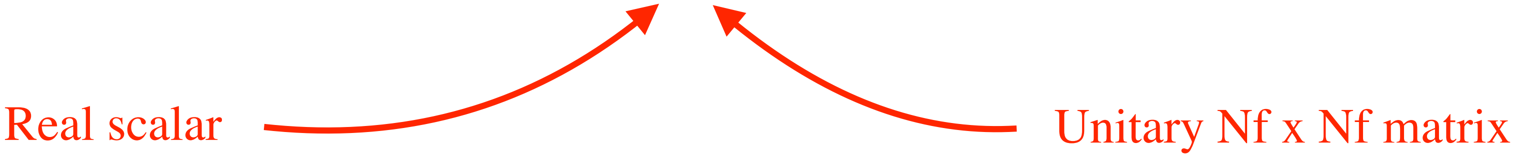

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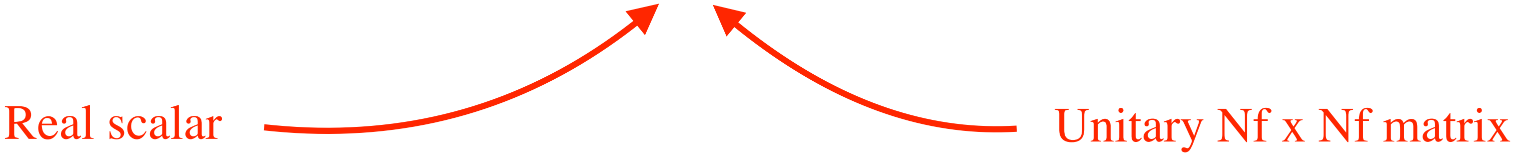

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- Build Ω_5 out of $\mathbf{A}^{(L,R)}$, U and τ

$$\Omega_5 = \Omega_5^0 + \Omega_5^c + dG_4$$

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Closed

Fixed by anomalies

Does not contribute to instanton EOM

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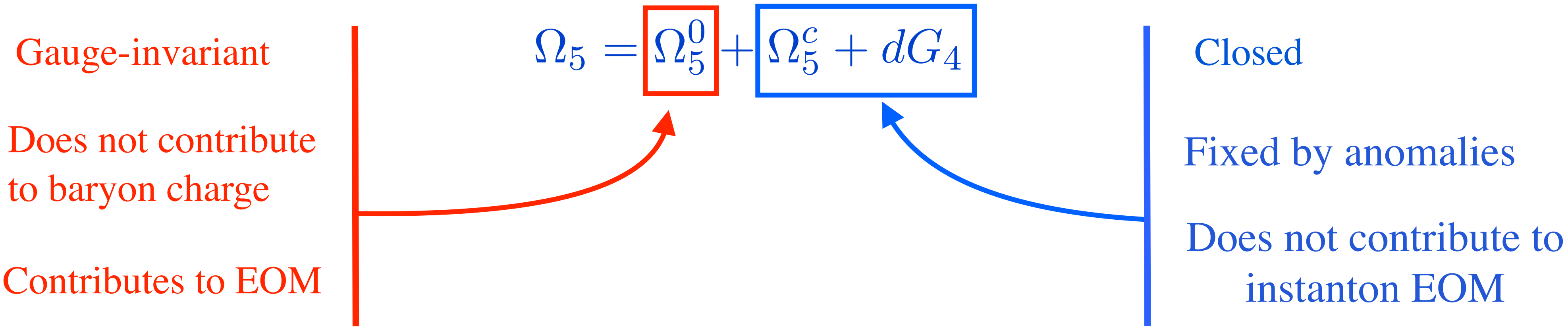

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Gauge-invariant

Does not contribute to baryon charge

Contributes to EOM

Closed

Fixed by anomalies

Does not contribute to instanton EOM

The V-QCD Baryon

Look for solutions to the EoM with following features:

- Finite energy (UV: normalzable boundary conditions, IR: regular)
- Static, axially symmetric (depend only on r, $|\vec{x}|$)
- Unit baryon charge

$$\frac{1}{8\pi^2}\int_{r,\vec{x}} Tr\left(F^L \wedge F^L - F^R \wedge F^R\right) = 1$$

The V-QCD Baryon

Ansatz: $SU(2) \subset SU(N_f)$ instanton, $U(1)_B$ turned on:

$$A^a_{L,i} = h_1(\xi,r)\epsilon_{iak}x^k + h_2(\xi,r)(\delta_{ia} - x_i x_a) + h_3(\xi,r)x_i x_a \qquad \xi \equiv |\vec{x}|$$

$$A^a_{L,z} = A(\xi,r)x^a, \quad A_{L,0} = \Phi(\xi,r), \quad U = exp\left(i\theta(\xi,r)\,\frac{\vec{x}\cdot\vec{\sigma}}{\xi}\right)$$

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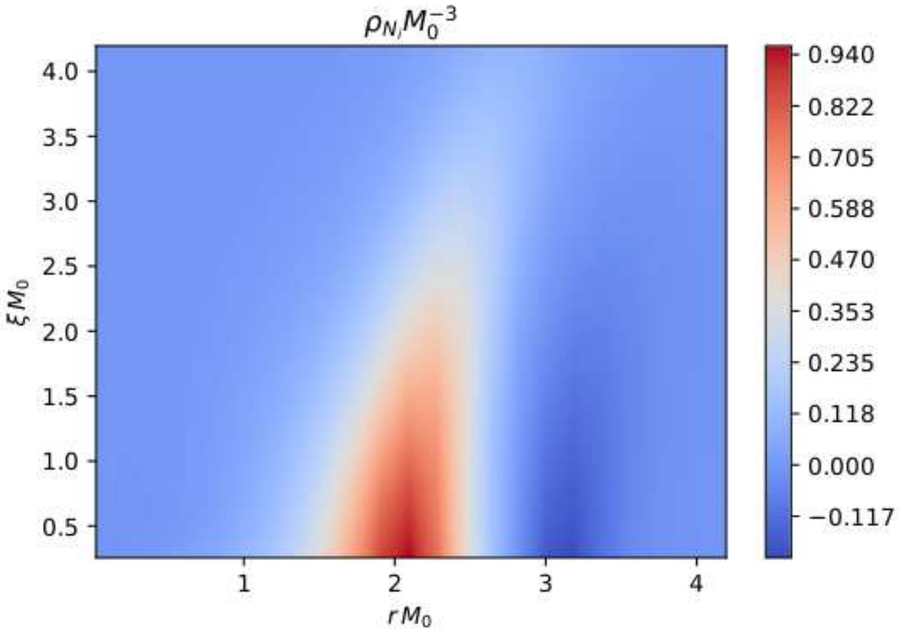
- Ignore backreaction on metric, dilaton and tachyon background profile $\mathcal{T} = \bar{\tau}(r)U(r, \xi)$ (corrections $\sim O(1/N_f, 1/N_c)$).
- 2d problem in (ξ, r)
- Instanton number = 2d winding number
- Impose normalizability in the UV, regularity in the IR.

Numerical Solution

Jarvinen, Kiritsis, FN, Préau, 2212.06747

Solution found using relaxation method.



Instanton number density

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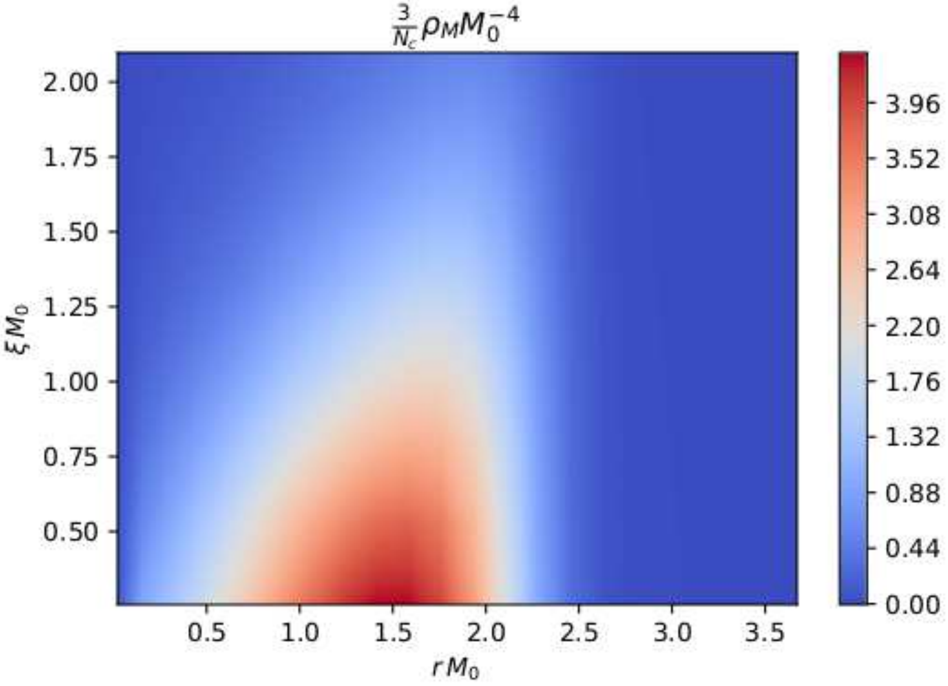

Baryon energy density

$$M_{class} = S_{on-shell} \approx 1 \text{ GeV}$$

Conclusion

More: (see 2212.06747)

- Excited state (rotations, isospin)

Next:

- More excited states (vibrations in r-direction);
- CS action and Baryon for non-zero quark masses;
- Use features of single baryon solution to construct more precise description of hardronic matter:
  - holographic fluid of baryons with bulk equation of state.
  - Understand particle/interface approximation

THANK YOU !

CS form: closed part

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- completely fixed by matching the QCD chiral anomalies. It reproduces the WZW action.
- Non-exact part is the ungauged WZW term:

$$\Omega_5^c = g_0 Tr(U^\dagger dU)^5$$

- The boundary value of the exact part is the gauging of the WZW term by left and right gauge fields $\mathbf{A}^{(L,R)}$ (it depends only on the boundary pion matrix $U(x, r = 0)$)

CS form: gauge-invariant part

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Most general gauge-invariant 5-form conistent with the discrete symmetries of QCD:

$$\Omega_5^0 = \sum_{i=1}^{4} f_i(\tau)\Omega_i^0$$

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- Ω_i^0: four gauge-invariant 5-forms built out of $F^{L,R}$ and U;
- $f_i(\tau)$ four new potentials.

$$f_i(\tau) \sim e^{-b\tau^2} \; \tau \to \infty \qquad f_i(\tau) \to const. \; \tau \to 0$$

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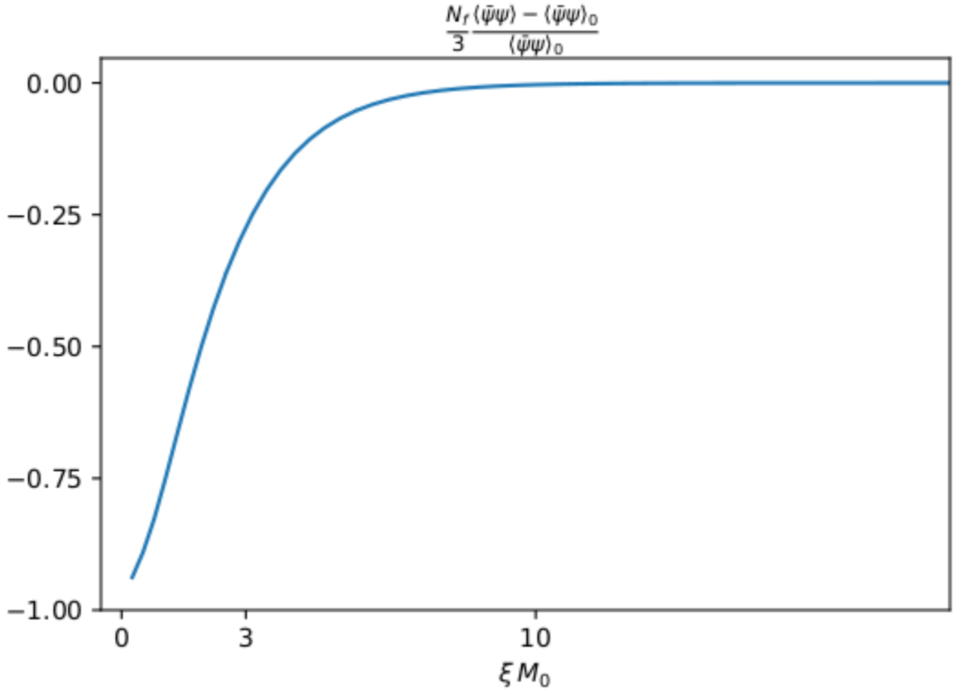
- It does not contribute to anomalies nor to baryon charge.

Backreaction

We computed the leading ($\sim O(1/N_f)$) backreaction on the tachyon profile.



Qualitatively: the chiral condensate decreases inside the baryon.

Excited states

Turning on rotational modes produces excited states.

- Slowly rotating solution:

$$E = M_0 + \frac{1}{2}\lambda\vec{\omega}^2$$

λ = moment of inertia (cab compute numerically)

- Quantize the rotational modes:

$$M_s = M_0 + \frac{1}{\lambda}s(s+1) \qquad s = 1/2, 3/2$$

Spin	V-QCD mass	Experimental mass
$s = \frac{1}{2}$	$M_N \simeq 1170$ MeV	$M_N = 940$ MeV
$s = \frac{3}{2}$	$M_\Delta \simeq 1260$ MeV	$M_\Delta = 1234$ MeV

(for $N_c = 3, N_f = 2$).

Baryon number is quantized

$$N_{inst}^{(A)} = \frac{1}{8\pi^2}\int_{r,\vec{x}} Tr\left(F^L \wedge F^L - F^R \wedge F^R\right) = N_L - N_R$$

Boundary baryon number:

$$N_B = \frac{1}{24\pi^2}\int d^3x \left[dUU^\dagger \wedge dUU^\dagger \wedge dUU^\dagger\right]_{UV} = N_{inst} + [\Delta N]_{IR}$$

$$U(r,\xi) = exp\left(i\theta(r,\xi)\frac{x^a\sigma^a}{\xi}\right)$$

- IR regularity: $[\Delta N]_{IR} = 0$
- Finite energy: $N_L = -N_R = n/2$ or equivalently $\theta(\xi = \infty) - \theta(\xi = 0) = 2n\pi$
- $N_B = \frac{1}{\pi}\left[\theta(r = 0, \xi = \infty) - \theta(r = 0, \xi = 0)\right]$

Baryons in the WSS model

Witten ’98, Sakai and Sugimoto ’05, Hata, Sakai, Sugimoto, Yamato ’07

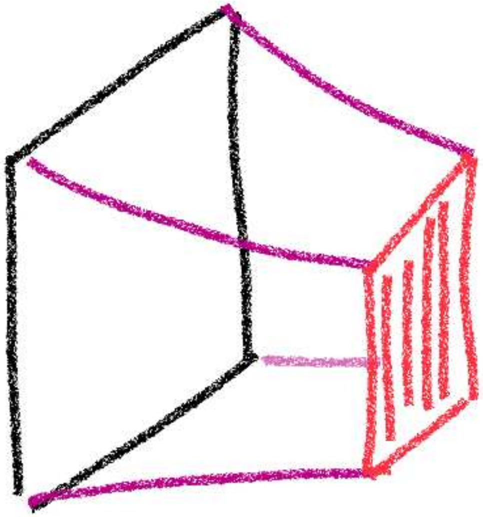
More realistic top-down model: 10d Witten-Sakai-Sugimoto

- N_c D4 branes (color) + N_f D8/anti-D8 branes (flavor)
- Gravity dual = near-horizon geometry created by the color branes.
- Geometry closes off in the IR: mass gap and confinement
- Flavor branes merge at the bottom: chiral symmetry breaking
- Baryons = instanton of worldvolume gauge field

Baryons in the Hard-Wall model

Erlich, Katz, Son, Stephanov ’05, Da Rold and Pomarol ’05

Hard-wall model: phenomenological 5d model implementing confinement



- 5d metric is AdS_5 down to the IR walll;

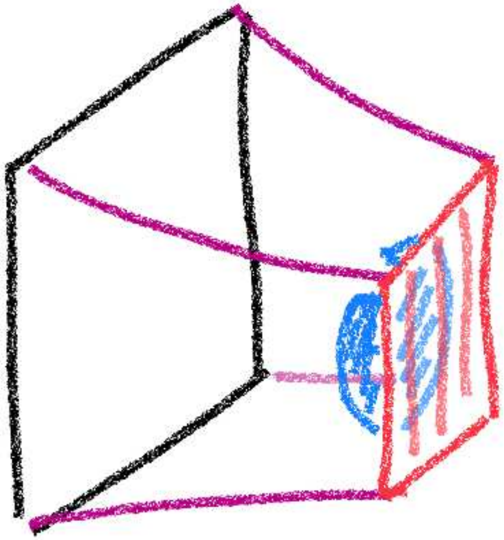
$$ds^2 = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \quad 0 < r < r_{IR}$$

- Dynamical 5d Fields: flavor gauge fields $\mathbf{A}^{(L,R)}$, tachyon $\mathcal{T}$

Baryons in the Hard-Wall model

Pomarol and Wulzer, ’07

- Look for an axiallly symmetric instanton configurations in AdS with a hard wall.
- The core of the bulk instanton is near the IR wall : solution sensitive to details of the IR physics.



- In a more realistic model, the interplay between geometry, CS and other interactions (dilaton and tachyon profiles) should stabilise the instanton.

Baryons in the WSS model

Witten '98, Sakai and Sugimoto '05, Hata, Sakai, Sugimoto, Yamato '07

More realistic top-down model: 10d Witten-Sakai-Sugimoto

- N_c D4 branes (color) + N_f D8/anti-D8 branes (flavor)
- Gravity dual = near-horizon geometry created by the color branes.
- Geometry closes off in the IR: mass gap and confinement
- Flavor branes merge at the bottom: chiral symmetry breaking
- Baryons = instanton of worldvolume gauge field
- Drawbacks
  - No tachyon in the chiral limit
  - Really a single worldvolume gauge field (D8 branes merge)