Thermodynamics and hydrodynamics in the holographic Stückelberg model

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Other references: K.Landsteiner, E.M., F.Peña-Benítez, PRL107(2011); Lect. Notes Phys. 81 (2013); E.M., M.Valle, JHEP1411 (2014).

Issues

1. Introduction: anomalous transport
   - Example: chiral magnetic effect
   - Hydrodynamics of relativistic fluids
   - Kubo Formulae
2. Holographic Stückelberg mechanism
   - Holographic Stückelberg mechanism with a U(1) gauge field
   - Thermodynamics of the Stückelberg model
3. Transport properties in the holographic Stückelberg model
   - RN-AdS_5 background
   - Perturbations: One-point and two-point functions
   - Results: correlators and conductivities

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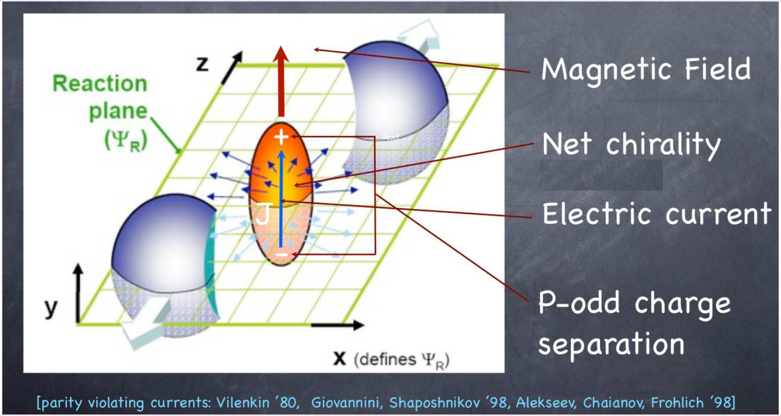
Issues

The Chiral Magnetic Effect (CME)

[Kharzeev, McLerran, Warringa '07]



Strong Magnetic field induces a $\mathcal{P}$-odd charge separation $\Longrightarrow$

$\Longrightarrow$ Electric current: $\vec{J} = \sigma_B \vec{B}$.

Issues

Hydrodynamics of relativistic fluids

Hydrodynamics approach: $\omega << 1/\tau_{\text{mft}}, \quad k << 1/\ell_{\text{mfp}}$

[Son,Surowka], [Eling,Neiman,Oz], [Erdmenger et al.], [Banerjee et al.], [Loganayagam], [Kharzeev, Yee], [Sadovyev et al.], [Landsteiner, EM, Pena-Benitez], ...

$$\langle T^{\mu\nu}\rangle = \underbrace{(\varepsilon+\mathcal{P})u^\mu u^\nu + \mathcal{P}g^{\mu\nu}}_{\text{Ideal Hydro}} + \underbrace{\langle T^{\mu\nu}\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}},$$

$$\langle J^{\mu}\rangle = \underbrace{nu^\mu}_{\text{Ideal Hydro}} + \underbrace{\langle J^{\mu}\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}}.$$

- Landau frame: $\langle T^{0i}\rangle \sim u^i$

$$\langle T^{\mu\nu}\rangle_{\text{diss \& anom}} = -\zeta P^{\mu\nu}D^\alpha u_\alpha + \underbrace{\sigma_B^\varepsilon (B^\mu u^\nu + u^\nu B^\mu)}_{\text{CME of energy current}} + \underbrace{\sigma_V^\varepsilon (\omega^\mu u^\nu + u^\nu \omega^\mu)}_{\text{CVE of energy current}} + \cdots$$

$$\langle J^{\mu}\rangle_{\text{diss \& anom}} = -\sigma T P^{\mu\nu} D_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu + \underbrace{\sigma_B B^\mu}_{\text{CME}} + \underbrace{\sigma_V \omega^\mu}_{\text{CVE}} + \cdots$$

where $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, and
vorticity: $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}u_\nu D_\rho u_\lambda$ → Chiral Vortical Effect (CVE).

Issues

Kubo Formulae: weak coupling

- Constitutive relation for the current:

$$\langle J^\mu \rangle = \underbrace{n u^\mu}_{\text{Ideal Hydro}} + \underbrace{\sigma_B B^\mu + \sigma_V \omega^\mu}_{\text{Anomalous}} + \cdots .$$

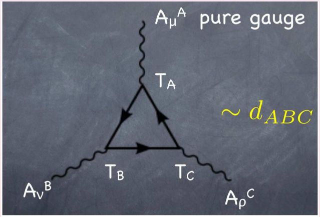
- Chiral Magnetic and Vortical Conductivities **[Kharzeev'09, Amado'11]**

$$\sigma_B = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a J^b \rangle|_{\omega=0} \,, \qquad \sigma_V = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a T^{0b} \rangle|_{\omega=0} \,.$$

- 1-loop calculation **[Kharzeev, Warringa '09], [EM, Landsteiner, PB '11]**

$$(\sigma_B)_{AB} = \frac{1}{4\pi^2} \underbrace{d_{ABC}}_{\text{Chiral Anom Coef}} \mu^C$$

$$d_{ABC} = \frac{1}{2} \mathrm{tr}(T_A \{T_B, T_C\})$$



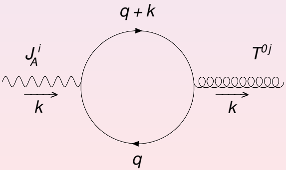
Chiral Magnetic Conductivity induced by the Chiral Anomaly.

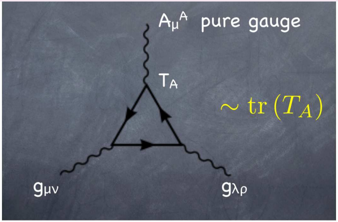
Chiral Vortical Effect

- Theory of N Free Chiral fermions $\Longrightarrow$ 1 loop calculation **[Landsteiner, Megías, Pena-Benitez, PRL107 '11]**:

$$(\sigma_V)_A = \lim_{k_n \to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle J_A^i T^{0j} \rangle|_{\omega=0}$$

$$= \underbrace{\frac{1}{8\pi^2} \sum_{B,C} d_{ABC}\, \mu^B \mu^C}_{\text{Chiral Anomaly}} + \underbrace{\frac{T^2}{24} \mathrm{tr}\,(T_A)}_{\text{Mixed Gauge-Gravitational Anomaly}}$$





Chiral Vortical Conductivity induced by the Chiral and Gauge-Gravitational Anomalies.

Anomalies

- Anomaly:

$$D_\mu J^\mu_a = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{abc}}{32\pi^2}F^b_{\mu\nu}F^c_{\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right),$$

  - Chiral anomaly coefficient:

$$d_{abc} = \frac{1}{2}[\mathrm{tr}(T_a\{T_b, T_c\})_R - \mathrm{tr}(T_a\{T_b, T_c\})_L],$$

  - Mixed gauge-gravitational anomaly coefficient:

$$b_a = \mathrm{tr}(T_a)_R - \mathrm{tr}(T_a)_L .$$

- Axial vortical conductivity:

$$\sigma_{V\,\mathrm{axial}} = \frac{\mu^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12} \qquad (\text{Chiral Anomaly}) + (\text{Mixed Anomaly})$$

See also **[Vilenkin '80]**.

Issues

Stückelberg mechanism with a U(1) gauge field

- When including dynamical gauge fields ⟶ anomalous conductivities do get renormalized **[Golkar,Son'12], [Hou,Liu,Ren'12].**
- Anomaly of the singlet $U_A(1)$ current in QCD:

$$\partial_\mu J^\mu_A = \epsilon^{\alpha\beta\gamma\delta}\left(\frac{N_c\sum_f q_f^2}{32\pi^2}\underbrace{F_{\alpha\beta}F_{\gamma\delta}}_{\text{electromagnetic field strengths}} + \overbrace{\frac{N_f}{16\pi^2}\mathrm{tr}\big(\underbrace{G_{\alpha\beta}G_{\gamma\delta}}_{\text{gluon field strengths}}\big) + \frac{N_c N_f}{96\pi^2}F^5_{\alpha\beta}F^5_{\gamma\delta}}^{\mathcal{O}_A}\right)$$

- Axial vortical conductivity receives two loop corrections:

$$\sigma_{V\,\mathrm{axial}} \sim \frac{T^2}{12}\left(1 + \frac{N_c^2-1}{2N_c}\frac{g^2}{4\pi^2}\right)$$

- In quantum field theory

$$Z = \int D\Psi D\bar{\Psi}\underbrace{D\mathcal{A}}_{\text{dynamical}}\exp\left[i\int d^4x\left(-\frac{1}{2}\mathrm{tr}(G.G) + \bar{\Psi}\underbrace{\not{D}}_{\text{dynamical}}\Psi + \Theta\mathcal{O}_A + \underbrace{J.A}_{\text{external}}\right)\right]$$

  is gauge invariant under $\delta A_\mu = \partial_\mu\lambda$ and $\delta\Theta = -\lambda$.
- Holographic dual ⟶ Theory with a massive gauge field via the Stückelberg mechanism.

Stückelberg mechanism with a U(1) gauge field

[A.Jimenez-Alba, K.Landsteiner, L.Melgar, PRD90, 126004 (2014)].

- Holographic model with a massive U(1) gauge boson including Chern-Simons terms:

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\Big[R+2\Lambda-\frac{1}{4g_5^2}F_{MN}F^{MN}-\frac{m^2}{2g_5^2}(A_M-\partial_M\Theta)(A^M-\partial^M\Theta)$$

$$+\epsilon^{MNPQR}(A_M-\partial_M\theta)\Big(\underbrace{\frac{\kappa}{3}F_{NP}F_{QR}}_{\text{Chiral CS}}+\underbrace{\lambda R^A{}_{BNP}R^B{}_{AQR}}_{\text{Mixed Gauge-grav. CS}}\Big)\Big]+S_{GH}+S_{CSK}\,.$$

- Θ field needed to respect gauge invariance.
- Gibbons-Hawking boundary term

$$S_{GH} = \frac{1}{8\pi G}\int_\partial d^4x\sqrt{-h}K\,.$$

- Extra boundary contribution $\sim \lambda$:

$$S_{CSK} = -\frac{\lambda}{2\pi G}\int_\partial d^4x\sqrt{-h}n_M\epsilon^{MNPQR}(A_N-\partial_N\Theta)K_{PL}D_QK^L_R\,.$$

Stückelberg mechanism with a U(1) gauge field

- Equations of motion

$$\begin{aligned} G_{MN} - \Lambda g_{MN} &= \frac{1}{2g_5^2} F_{ML} F_N{}^L - \frac{1}{8g_5^2} F^2 g_{MN} + \frac{m^2}{2g_5^2}(A_M - \partial_M\Theta)(A_N - \partial_N\Theta \\ &\quad - \frac{m^2}{4g_5^2}(A_P - \partial_P\Theta)^2 g_{MN} + 2\lambda\epsilon_{LPQR(M}\nabla_B\left(F^{PL}R^B{}_{N)}{}^{QR}\right), \\ \nabla_N F^{NM} &= -\epsilon^{MNPQR}\left(\kappa F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right) + m^2(A^M - \partial^M\Theta \\ \nabla_M(A^M - \partial^M\Theta) &= 0\,. \end{aligned}$$

- We can define $B_M := A_M - \partial_M\Theta$.
- Metric in Fefferman-Graham coordinates

$$ds^2 = -\frac{\ell^2}{\rho}g_{\tau\tau}(\rho)d\tau^2 + \frac{\ell^2}{\rho}g_{xx}(\rho)d\vec{x}^2 + \frac{\ell^2}{4\rho^2}d\rho^2\,.$$

Issues

Thermodynamics of the Stückelberg model

[E.M., J. Phys. Conf. Ser. 1416 (2019) 012022].

- In conformal coordinates: $ds^2 = \frac{\ell^2}{z^2}\left[g_{\mu\nu}dx^\mu dx^\nu + dz^2\right]$; the behavior of the time component of the gauge field is

$$A_\tau(z) = A_{(0)\tau}\left(\frac{z}{\ell}\right)^{-\Delta} + \mathcal{O}_A^\tau\left(\frac{z}{\ell}\right)^{\Delta+2} + \cdots .$$

  with $A_{(0)\tau}$ the deformation of the CFT

$$\int d^4x\, A_{(0)\tau}\mathcal{J} = \int d^4x\, A_{(0)\tau}(\Delta^{3-\tilde{\Delta}}\mathcal{O}_A^\tau)\,,$$

  and $m^2\ell^2 = \Delta(\Delta+2)$ $\quad(0 \le \Delta < 1)$. $-\Delta$ is the conformal dimension of the U(1) current dual to the gauge field.
- Black hole solution (RN-AdS$_5$ background):

$$ds^2 = \frac{r^2 f(r)}{\ell^2}dt^2 + \frac{\ell^2}{r^2 g(r)}dr^2 + \frac{r^2}{\ell^2}d\vec{x}^2\,, \quad \Theta(r) = \mathrm{cte}\,, \qquad r = \frac{\ell^2}{z}\,.$$

- Seek for solutions in an expansion in Δ.

Thermodynamics of the Stückelberg model

- Leading order solution: $(\Delta = 0) \implies (m = 0)$

$$f_0(r) = g_0(r) = \left(1 - \frac{r_h^2}{r^2}\right)\left(1 + \frac{r_h^2}{r^2} - \frac{q^2}{r_h^2 r^4}\right),$$

$$A_{\tau,0}(r) = A_{(0)\tau}\left(1 - \frac{r_h^2}{r^2}\right), \qquad T_0 = \frac{r_h}{\pi \ell^2}\left(1 - \frac{q^2}{2r_h^6}\right).$$

- Corrections of $\mathcal{O}(\Delta)$: $(\Delta \neq 0) \implies (m \neq 0)$

$$f(r) = f_0(r)\left(1 + F(r)\right), \quad g(r) = f_0(r)\left(1 + F(r) + G(r)\right),$$
$$A_\tau(r) = A_{\tau,0}(r) + J(r),$$
$$T = T_0\left(1 + \frac{\delta T}{T_0}\right), \qquad \frac{\delta T}{T_0} = F(r_h) + \frac{1}{2}G(r_h).$$

- Renormalization:

$$r = \frac{\ell^2}{z} + a_1 z + a_3 z^3 + a_5 z^5 + b_5 z^5 \log z + \cdots, \qquad z \to 0.$$

Thermodynamics of the Stückelberg model

- Renormalized action $S_{\rm ren} = S_{\rm reg} + S_{\rm ct} \equiv \frac{V_3}{T}\Omega$, where the counterterms needed to cancel the divergences are

$$S_{\rm ct} = \frac{1}{16\pi G}\int d^4x\sqrt{h}\frac{6}{\ell} + \frac{\ell}{4}m^2\int d^4x\sqrt{h}h^{\mu\nu}\left(A_\mu - \partial_\mu\Theta\right)\left(A_\nu - \partial_\nu\Theta\right) .$$

- Thermodynamical variables:

$$p = -\Omega\,, \qquad \varepsilon = -T^2\frac{\partial}{\partial T}\left(\frac{\Omega}{T}\right)\Big|_\mu , \qquad s = -\frac{\partial\Omega}{\partial T}\Big|_\mu , \qquad n = -\frac{\partial\Omega}{\partial\mu}\Big|_T .$$

At high temperature $|A_{(0)\tau}| \ll T$:

$$\varepsilon - 3p = \Delta\frac{2\pi^2\ell}{g_5^2}A_{(0)\tau}^2 T^2 + \cdots , \qquad c_s^2 = \left(\frac{\partial p}{\partial\varepsilon}\right)_{s/n} = \frac{1}{3} - \Delta\frac{A_{(0)\tau}^2}{3g_5^2\xi^2 T^2} + \cdots$$

- Breaking of conformal invariance by the mass $\propto \Delta$.
- Bekenstein-Hawking entropy formula is still valid, at least up to $\mathcal{O}(\Delta)$:

$$S = \frac{\mathcal{A}}{4G}$$

Issues

RN-AdS$_5$ background

[E.M., N.Rai, 2301.00361].

- To account for chiral vortical effects ⟶ Computation beyond the probe limit ⟶ Full backreaction of the gauge field on the metric.
- Metric in Fefferman-Graham coordinates

$$ds^2 = -\frac{\ell^2}{\rho} g_{\tau\tau}(\rho) d\tau^2 + \frac{\ell^2}{\rho} g_{xx}(\rho) d\vec{x}^2 + \frac{\ell^2}{4\rho^2} d\rho^2 \,.$$

- Temperature of the black hole: $T = \frac{1}{2\pi}\sqrt{2\rho_h g''_{\tau\tau}(\rho_h)}$.
- Schwarzschild black hole ($B_M = 0$) in Fefferman-Graham coordinates:

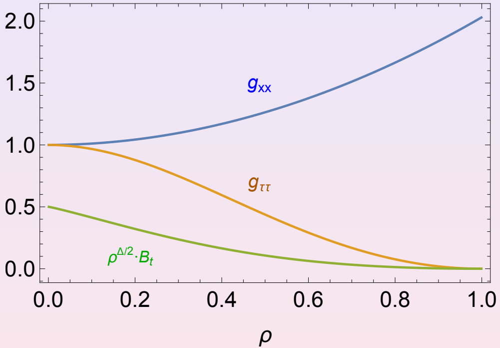
$$g_{\tau\tau}(\rho) = \frac{1}{\rho_h^2}\frac{(\rho_h^2 - \rho^2)^2}{\rho_h^2 + \rho^2} \,, \qquad g_{xx}(\rho) = 1 + \frac{\rho^2}{\rho_h^2} \,, \qquad T = \frac{1}{\pi}\sqrt{\frac{2}{\rho_h}} \,.$$

- The inclusion of the chemical potential demands a numerical solution of the EoM:

$$B_\tau(\rho_h) = 0 \,, \quad \lim_{\rho\to 0}\left(\rho^{\Delta/2} B_\tau(\rho)\right) = \mu_5 \,.$$

RN-AdS$_5$ background and perturbations

Background $(\rho_h = 1)$



- Consider fluctuations of the fields around this background:

$$g_{MN} = g^{(0)}_{MN} + \epsilon\, h_{MN}\,, \qquad B_M = B^{(0)}_M + \epsilon\, b_M\,.$$

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Perturbations

- Let us consider the perturbations

$$g_{MN} = g^{(0)}_{MN} + \epsilon\, h_{MN}\,, \qquad B_M = B^{(0)}_M + \epsilon\, b_M\,.$$

- Momentum in z-direction ($\vec{k} = k\hat{z}$), and use Fourier expansion:

$$h_{MN}(\rho, x^\mu) = \int \frac{d^d k}{(2\pi)^d} e^{i(\vec{k}\vec{x} - \omega t)} h_{MN}(\rho)\,, \quad B_M(\rho, x^\mu) = \int \frac{d^d k}{(2\pi)^d} e^{i(\vec{k}\vec{x} - \omega t)} B_M(\rho)\,.$$

- The perturbations can be of the following three types under O(2) transformations:
  - vector type: h^x_t, h^y_t, h^x_z, h^y_z and b_x, b_y.
  - scalar type: h_{tt}, h^z_t, $h_{xx} = h_{yy}$, h_{zz} and b_t, b_z.
  - tensor type: h_{xy}, $h_{xx} = -h_{yy}$.
- $\kappa\epsilon^{MNPQR} A_M F_{NP} F_{QR}$ contributes only to *vector* type.
- $\lambda\epsilon^{MNPQR} A_M R^A{}_{BNP} R^B{}_{AQR}$ contributes to *vector* and *tensor* type.
- We will concentrate on the vector type perturbations:

$$\{h^i_t(\rho)\,,\ b_i(\rho)\}\,, \qquad i = x, y\,.$$

One-point and two-point functions

➙ Procedure to compute the transport coefficients:

- Consider fluctuations of the fields: b_M, h_{MN}.
- Renormalized on-shell action up to quadratic order in fluctuations

$$S^{(2)}_{\rm ren}(b_M, h_{MN}) = S^{(2)}_{\rm on\text{-}shell} + S^{(2)}_{\rm ct} \,.$$

- Variations of the on-shell action ➙ one-point functions

$$\langle J^\mu \rangle = \frac{\partial S^{(2)}_{\rm ren}}{\partial b_\mu} \,, \qquad \langle T^{\mu\nu} \rangle = \frac{\partial S^{(2)}_{\rm ren}}{\partial h_{\mu\nu}} \,.$$

- Two-point functions (Kubo formulae)

$$\langle J^\mu J^\nu \rangle = \frac{\partial^2 S^{(2)}_{\rm ren}}{\partial b_\mu \partial b_\nu} \,, \qquad \langle J^\mu T^{\nu\sigma} \rangle = \frac{\partial^2 S^{(2)}_{\rm ren}}{\partial b_\mu \partial h_{\nu\sigma}} \,, \quad \cdots$$

Perturbations

- Asymptotic analysis near the boundary ($\rho \to 0$) shows

$$b_i(\rho) = \underbrace{b_i^{(0)}}_{\text{Sources}} \cdot \rho^{-\Delta/2} + b_i^{(1)} \cdot \rho^{(\Delta+2)/2} + \cdots ,$$

$$h_t^i(\rho) = \overbrace{h_t^{i\,(0)}} + h_t^{i\,(1)} \cdot \rho^2 + \cdots , \quad \text{with} \quad m^2 = \Delta(\Delta+2)/\ell^2 .$$

- Remember: $\Delta = 0 \quad \rightarrow \quad m^2 = 0.$
- Renormalization of the theory ($ds^2 = G_{\mu\nu}dx^\mu dx^\nu + \frac{\ell^2}{4\rho^2}d\rho^2$):

$$\delta S_{\rm ct} = -\frac{1}{2\kappa^2}\int_{\rho=\varepsilon} d^4x\sqrt{-\gamma}(K^{\mu\nu} - K\gamma^{\mu\nu})\delta G_{\mu\nu} - \frac{\Delta}{\ell}\int d^4x\sqrt{-\gamma}\gamma^{\mu\nu}B_\mu \delta B_\nu$$

The Chern-Simons terms do not introduce new divergences.

One-point and two-point functions

- One-point functions:

$$\langle J_a \rangle = \frac{\delta S_{\rm ren}}{\delta b_a^{(0)}} = -\frac{2}{16\pi G}(\Delta+1) b_a^{(1)} \,, \qquad (a = x, y)$$

$$\langle T_{0a} \rangle = \frac{\delta S_{\rm ren}}{\delta h_t^{a(0)}} = \frac{1}{16\pi G}(2 h_t^{a(0)} + h_t^{a(1)}) \,, \qquad (a = x, y)$$

- Two-point functions:

$$\langle J_a J_b \rangle = \frac{\delta \langle J_a \rangle}{\delta b_b^{(0)}} \,, \qquad \langle J_a T_{0b} \rangle = \frac{\delta \langle J_a \rangle}{\delta h_t^{b(0)}} \,,$$

$$\langle T_{0a} J_b \rangle = \frac{\delta \langle T_{0a} \rangle}{\delta b_b^{(0)}} \,, \qquad \langle T_{0a} T_{0b} \rangle = \frac{\delta \langle T_{0a} \rangle}{\delta h_t^{b(0)}} \,, \qquad (a, b = x, y)$$

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Massless case

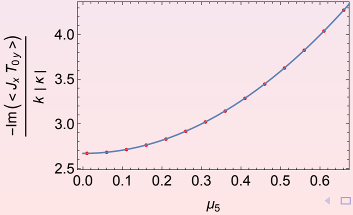
[Policastro, Son, Starinets '02], [Kovtun '05], [Matsuo et al. '08, '10], [Gynther et al. '11], [Landsteiner '11].

$$\langle J_x J_y \rangle = -ik\frac{(3\mu_5 - \alpha)}{12\pi^2}\,, \qquad \langle J_x J_x \rangle = 0\,, \qquad [A_\tau(\rho=0)=\alpha]\,,$$

$$\langle J_x T_{0x} \rangle = \frac{\sqrt{3}Q}{4\pi G\ell^3}\,, \qquad \langle T_{0x} J_x \rangle = 0\,,$$

$$\langle J_x T_{0y} \rangle = -ik\left(\frac{\mu_5^2}{8\pi^2} + \frac{T^2}{24}\right)\,,$$

$$\langle T_{0x} T_{0x} \rangle = \frac{M}{16\pi G\ell^3}\,, \qquad \langle T_{0x} T_{0y} \rangle = -ik\left(\frac{\mu_5^3}{12\pi^2} + \frac{\mu_5 T^2}{12}\right)\,.$$



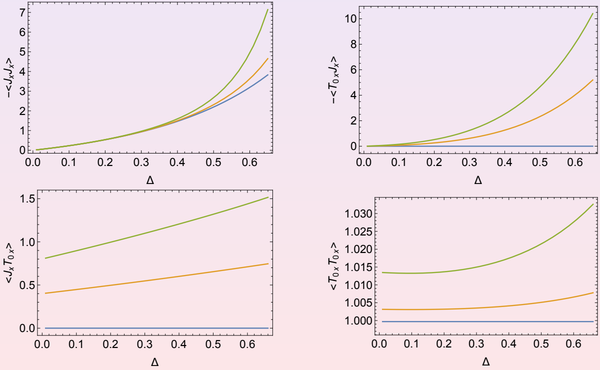
Massive case

[Jimenez-Alba et al. '14], [Rai, E.M. '23].

→ Numerical solution of the EoM.

$$\langle J_x J_y \rangle\,, \quad \langle J_x T_{0x} \rangle\,, \quad \langle J_x T_{0y} \rangle\,, \quad \langle T_{0x} T_{0x} \rangle\,, \quad \langle T_{0x} T_{0y} \rangle\,,$$

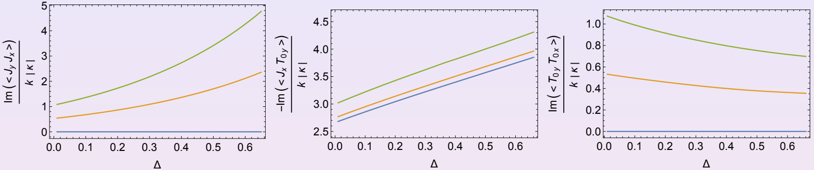
receive contributions from Δ. In addition $\langle J_x J_x \rangle \neq 0\,, \quad \langle T_{0x} J_x \rangle \neq 0$.



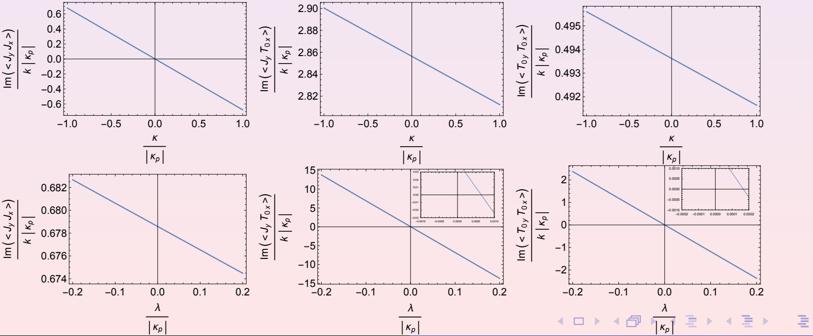
$\mu_5 = 0, 0.15, 0.3$ (blue, orange, green).

Massive case: anomalous correlators $\propto \epsilon^{\mu\nu\lambda\sigma}$

- Dependence with Δ: $[\langle J_x T_{0y}\rangle = \langle T_{0x} J_y\rangle]$ $\mu_5 = 0, 0.15, 0.3$ (blue,orange,green).



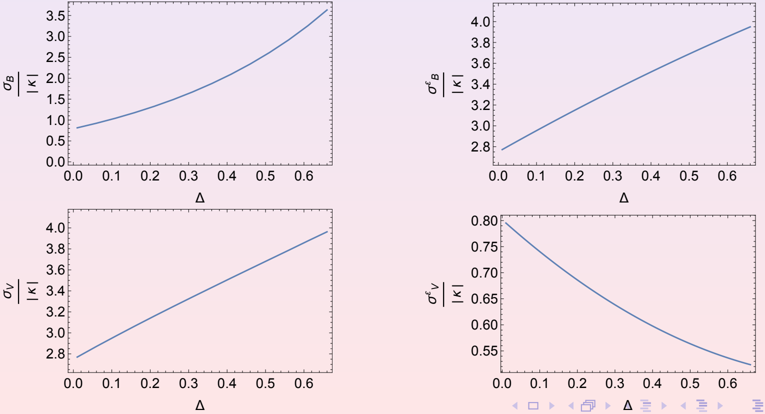
- Dependence with κ and λ: $(\mu_5 = 0.1, \Delta = 0.1)$



Massive case: anomalous conductivities $\propto \epsilon^{\mu\nu\lambda\sigma}$

$$\sigma_B = -\lim_{k\to 0}\frac{1}{k}\mathrm{Im}\langle J_x J_y\rangle\,, \qquad \sigma_B^{\varepsilon} = -\lim_{k\to 0}\frac{1}{k}\mathrm{Im}\langle T_{0x} J_y\rangle\,.$$

$$\sigma_V = -\lim_{k\to 0}\frac{1}{k}\mathrm{Im}\langle J_x T_{0y}\rangle\,, \qquad \sigma_V^{\varepsilon} = -\lim_{k\to 0}\frac{1}{k}\mathrm{Im}\langle T_{0x} T_{0y}\rangle\,,$$

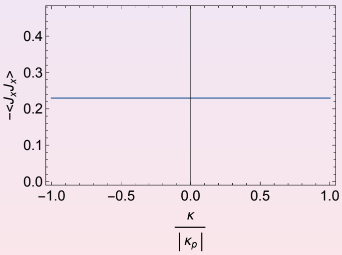


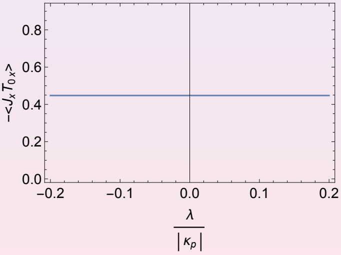
Massive case: non-anomalous correlators $\not\propto \epsilon^{\mu\nu\lambda\sigma}$

The non-anomalous correlators

$$\langle J_x J_x \rangle \,, \qquad \langle J_x T_{0x} \rangle \,, \qquad \langle T_{0x} J_x \rangle \,, \qquad \langle T_{0x} T_{0x} \rangle \,,$$

are independent of κ and λ.





$(\mu_5 = 0.1, \Delta = 0.1)$

Conclusions

- We have studied the effects of *external magnetic fields* and *vortices* in relativistic hydrodynamics $\Longrightarrow$ Anomalous Transport.
- Transport properties in holography including pure gauge and mixed gauge-gravitational Chern Simons terms $\Longrightarrow$ Kubo Formulae.
- Holographic Stückelberg model with a U(1) gauge field ⟶ thermodynamics and anomalous transport properties:
  - Breaking of conformal invariance by the mass.
  - Thermodynamics: analytically as an expansion in Δ.
  - Conductivities: numerically by solving the EoM of the fluctuations.
  - Good agreement with previous results on the CME in the *'probe limit'* **[A.Jimenez-Alba et al '14]**.
  - Computation including full backreaction $\Longrightarrow$ account for CVE and energy transport effects.
- Future directions:
  - Extension to the $U(1)_V \times U(1)_A$ gauge group.
  - Application to condensed matter systems ⟶ Superfluids, Weyl semi-metals **[Basar, Kharzeev, Yee '14], [Landsteiner '14]**.

Thank You!