The butterfly effect
in a
holographic chiral system

Navid Abbasi
Lanzhou University

Holographic perspectives on chiral transport

ECT, March 13

Motivation

- Quantum chaos is associated with **energy** dynamics (holographic system)

- In chiral systems, **energy** transport through the CME

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→ Any connection between "Quantum chaos" and "CME" ?

Quantum chaos

- One important aspect of thermal systems:

Atypicality $\rightarrow$ evolution $\rightarrow$ typicality

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Scrambling time is conjectured as $t \sim \beta \log S$

[Sekino, Susskind 0808.2096]

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Scrambling time is conjectured as $t \sim \beta \log S \overset{\boxed{N \gg 1}}{\gg} \beta \sim t_{\text{diffusion}}$

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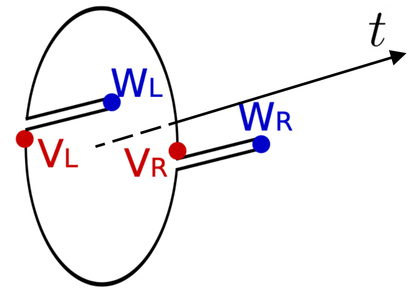
Quantifying the butterfly effect

Exponential decrease of "Out-of-time-ordered correlators" (OTOC)

OTOC = $F(t) = \langle \Psi | V_L W_R(t) V_R W_L(t) | \Psi \rangle$



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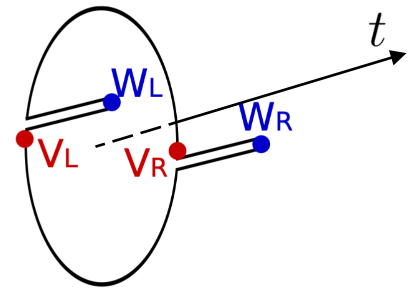

- For large N holographic CFTs

$$F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$$

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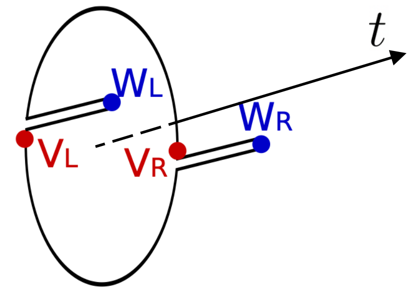

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Lyapunov exponent

Butterfly velocity

- The squared commutator

$$C(t_w, |x-y|) = \mathrm{tr}\left\{\rho(\beta)[W_x(t_w), W_y]^\dagger [W_x(t_w), W_y]\right\}$$
$$= \ 2 - 2\,\mathrm{Re}\,\langle TFD|\, W_y W_x\,(t_w) W_y W_x(t_w)\,|TFD\rangle$$

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\begin{aligned}
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[Roberts, Shenker, Stanford, 1409.8180]

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$$t^* \sim \frac{\beta}{2\pi} \log N^2$$

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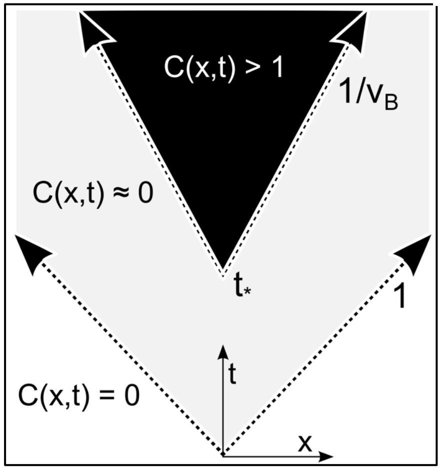
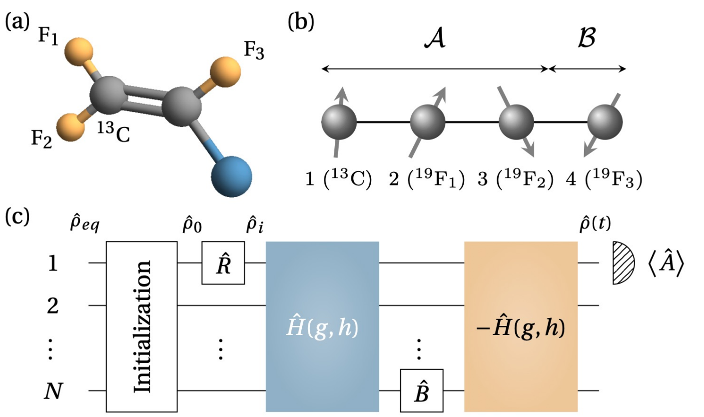


$$x-y=\mathrm{x}$$

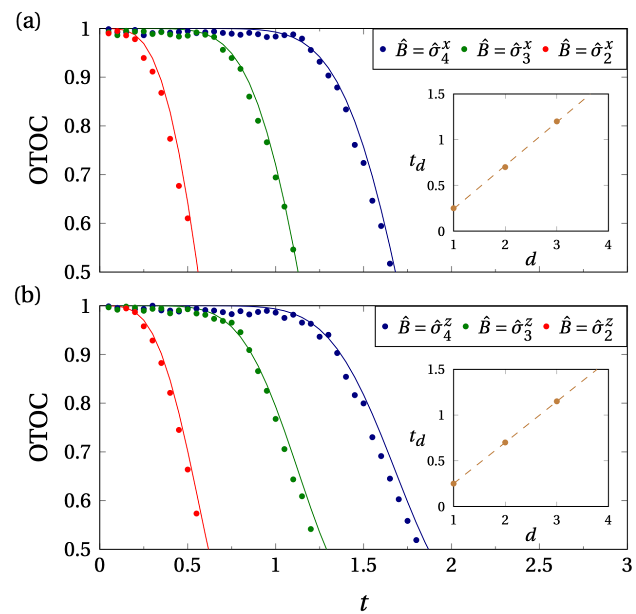
OTOC and v_B from experiment

Ising spin chain on a nuclear magnetic resonance (NMR) quantum simulator

[Li, Fan Wang, Ye, Zeng, Zhai, Peng, Du 1609.01246]



$$\hat{H}_{\rm NMR} = -\sum_{i=1}^{4} \frac{\omega_{0i}}{2} \hat{\sigma}_i^z + \sum_{i<j,=1}^{4} \frac{\pi J_{ij}}{2} \hat{\sigma}_i^z \hat{\sigma}_j^z$$



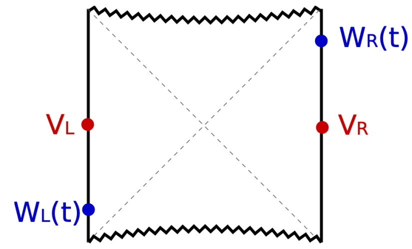
Several other experiments:

[Garttner, Bohnet, Safavi, Wall, Bollinger, Rey 1608.08938]
[Cao, Zhu, Del Campo 2111.12475]
[Swingle, Bentsen, Schleier-Smith, Hayden 1602.06271]
[...]

OTOC from eternal black hole

[Maldacena 0106112]
[Van Raamsdonk 1005.3035]
[Shenker, Stanford 1412.6987]

$$\langle V_L W_R(t) V_R W_L(t) \rangle$$



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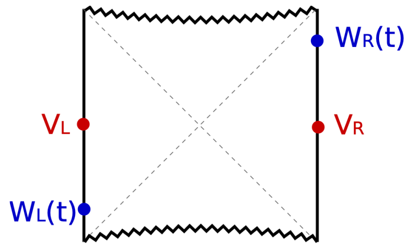

- More generally, we consider: $\langle V_{x_1}(t_1) W_{x_2}(t_2) V_{x_3}(t_3) W_{x_4}(t_4) \rangle$

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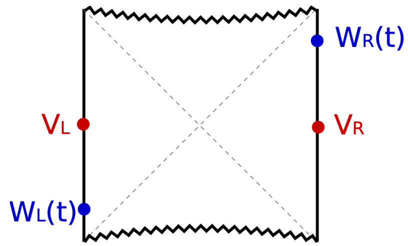

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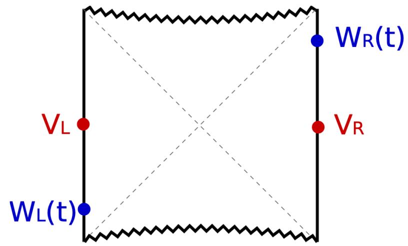

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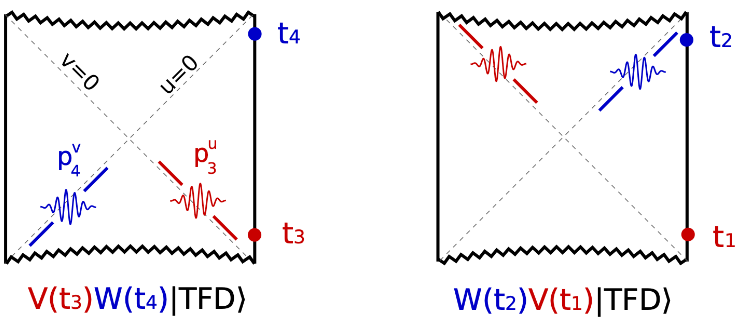

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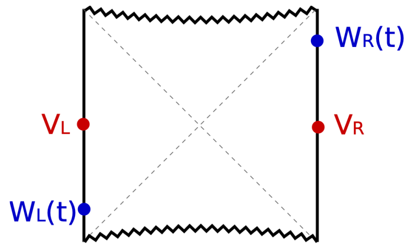

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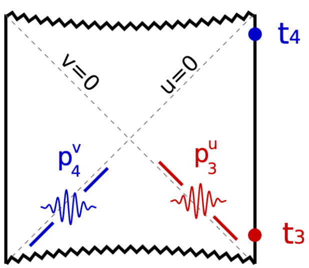

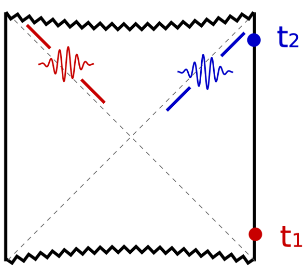

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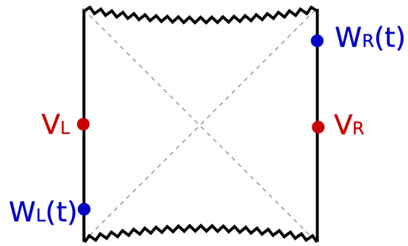

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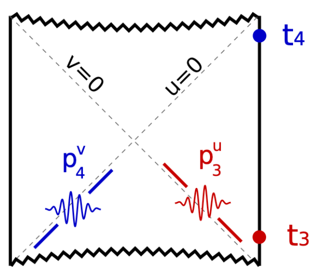

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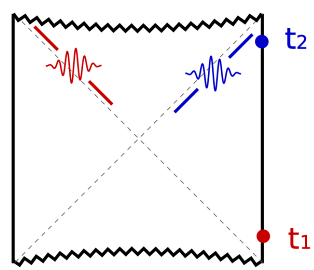

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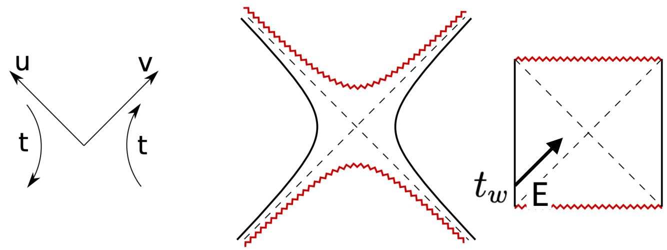
- In the Eikonal approximation, one should calculate "the phase shift".

$$|p_1^u, x_1; p_2^v, x_2\rangle_{out} \approx e^{i\delta(s,b)} |p_1^u, x_1; p_2^v, x_2\rangle_{in} + |\chi\rangle$$

Shock wave geometry in AdS

[Shenker, Stanford 1306.0622]
[Sfetsos 9408169]
[Aichelburg, Sexl 1971]
[Dray, 't Hooft 1985]

- If a "small" quanta of energy E is thrown into the BH



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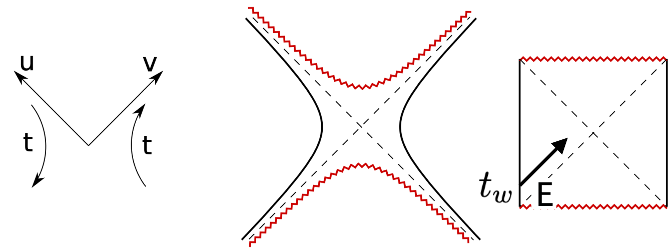

- The geometry eventually will be affected, because of blueshift

$$T_{uu} = \frac{E}{\ell_{AdS}^{d+1}} e^{2\pi t_w/\beta} \delta(u) a_0(x)$$

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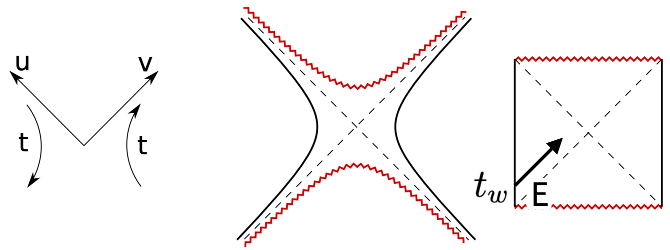

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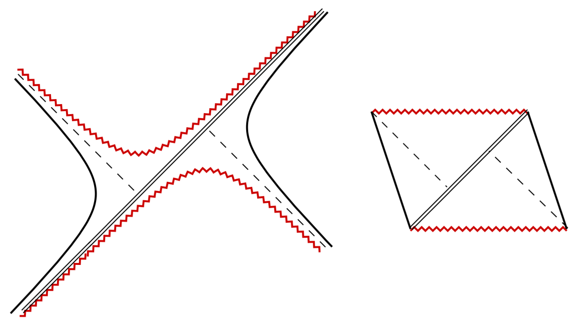

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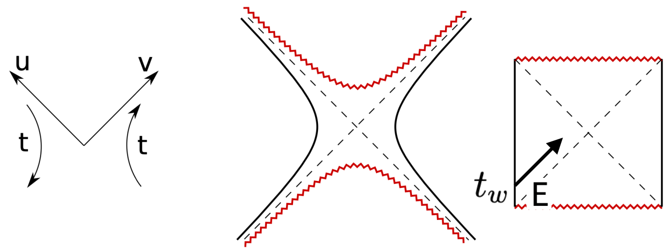

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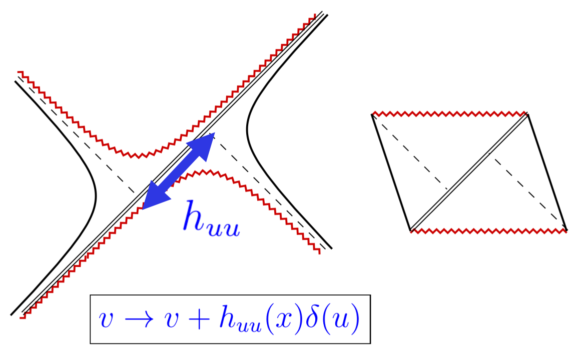

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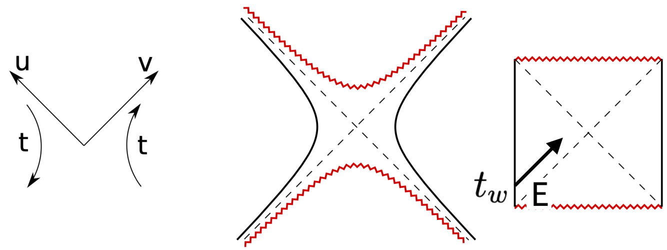

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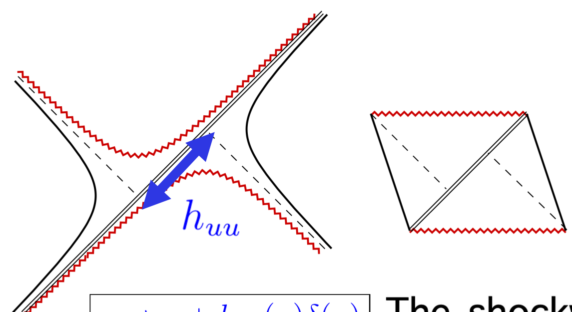

$v \to v + h_{uu}(x)\delta(u)$ The shockwave geometry

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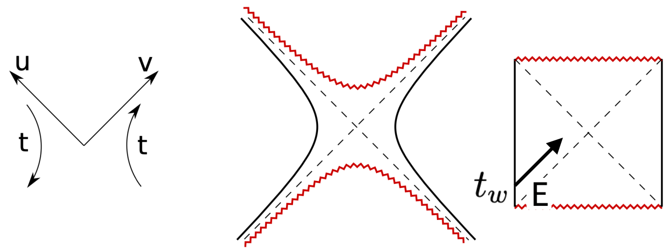

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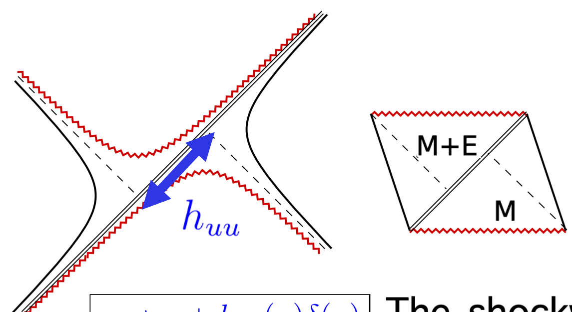

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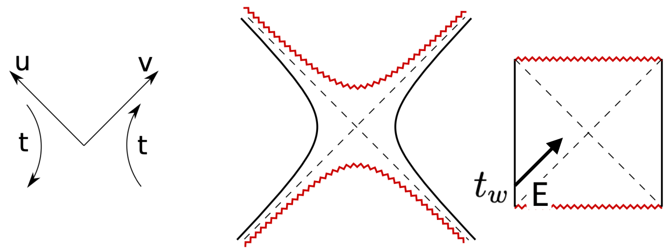

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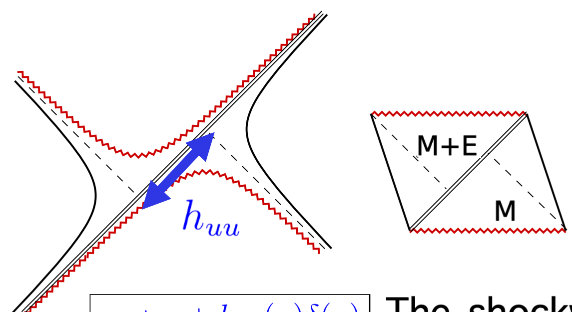

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$$\delta(s,b) = S_{cl}$$

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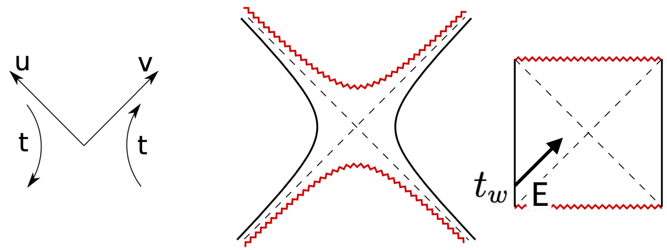

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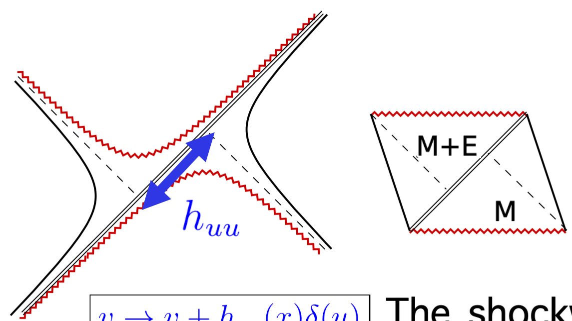

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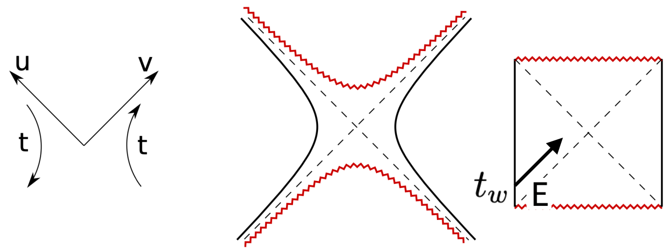

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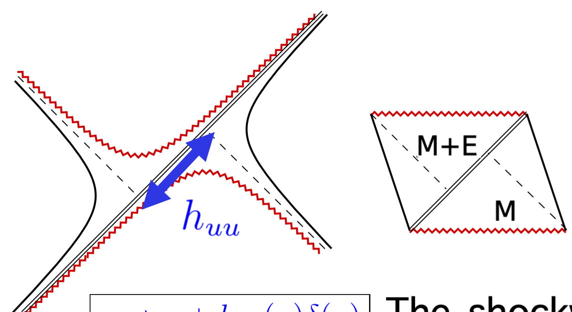

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[Shenker, Stanford 1412.6987]

Butterfly velocity from BH

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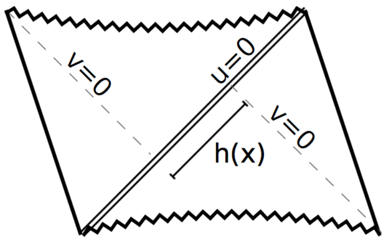

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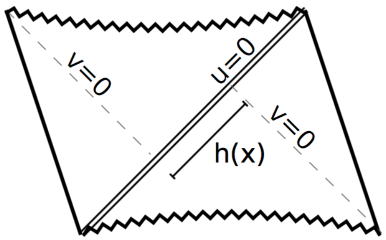

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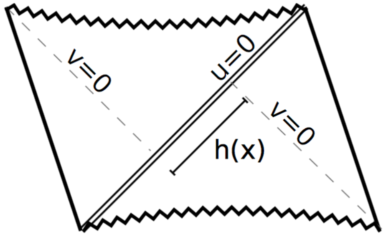

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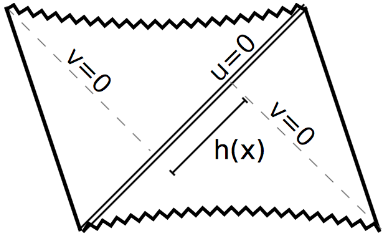

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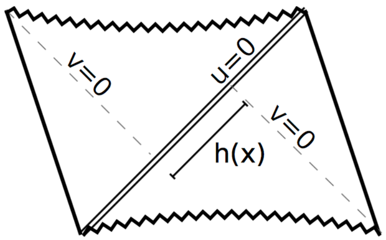

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By comparison:

➢ One finds $\boxed{\lambda = 2\pi T}$ $\boxed{v_B = \sqrt{\frac{D-1}{2(D-2)}}}$

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in a holographic chiral system in the presence of a B

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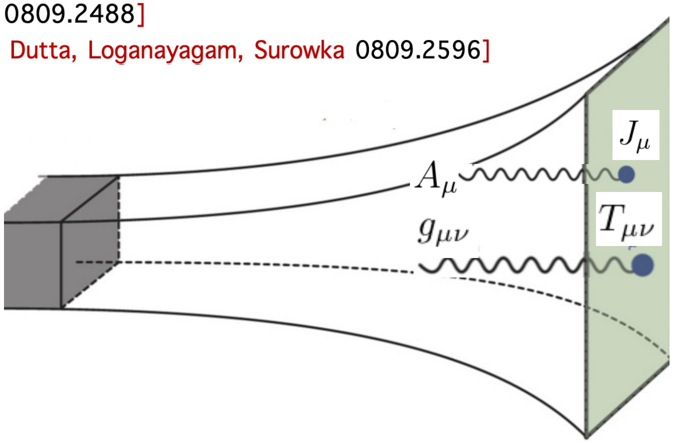

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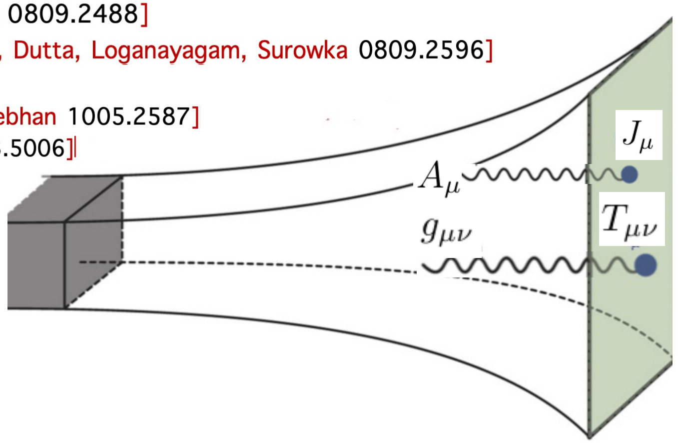

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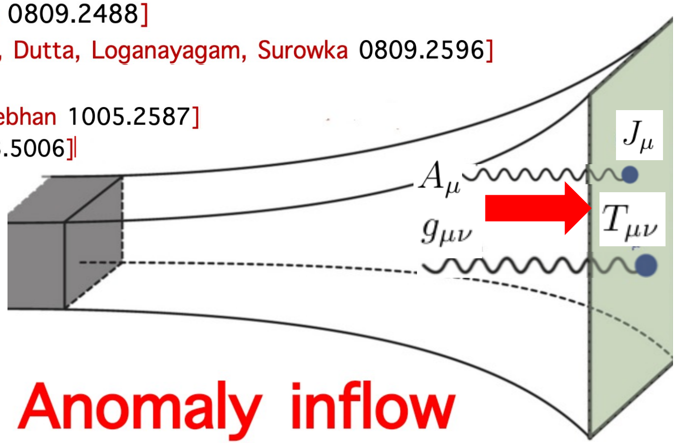

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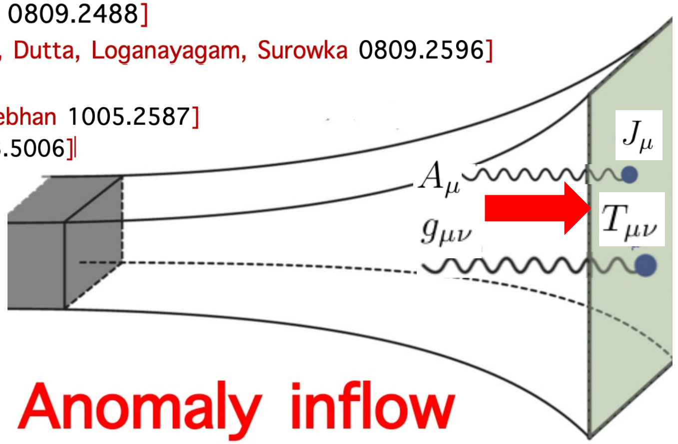

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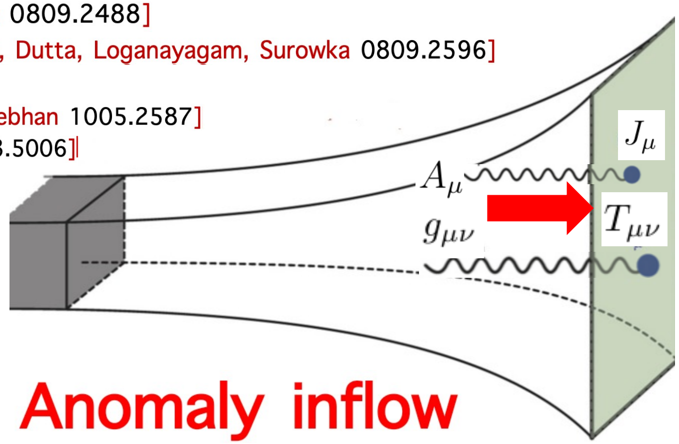

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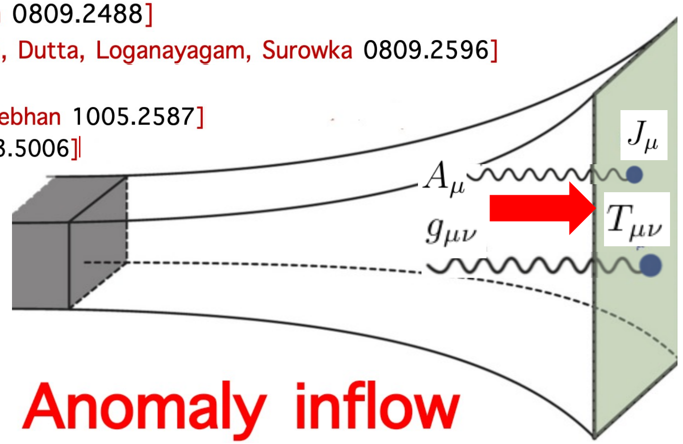

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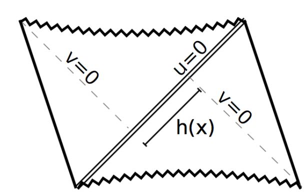
- What we need to find: the function h(x) on this background



BH solution

EOM:

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$$V_2(r) = \frac{S_2(r) + T_2(r)}{3}, \qquad W_2(r) = \frac{S_2(r) - 2T_2(r)}{3}$$

What we are going to do:

- In [NA, Ghazi, Taghinavaz, Tavakol, 1812.11310],

in addition to perturbation over $b = \frac{B}{(\pi T)^2}$

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$$\begin{gathered} 4\pi T\,(r - r_h) \\ +\tfrac{1}{2}\left(-4 + \tfrac{56\nu^2}{3} + b^2\left(\tfrac{8}{9}\nu^2\left(42\kappa^2(\log(2)-1)-17\right) + \tfrac{10}{3}\right)\right)(r-r_h)^2 \\ +\tfrac{1}{6}\left(\tfrac{24}{\pi T} - \tfrac{144\nu^2}{\pi T} + b^2\left(\tfrac{8\nu^2\left(\kappa^2(363-324\log(2))+155\right)}{9\pi T} - \tfrac{64}{3\pi T}\right)\right)(r-r_h)^3 \end{gathered}$$

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We perform calculations fully analytically

in a double expansion over ν and b.

Shockwave geometry

Applying $V \to V + h(x)\delta(U)$

$$\begin{aligned} ds^2_{\text{future}} &= ds^2_{\text{past}} - A(UV)h(x)\delta(U)\,dU^2 - \frac{D(UV)}{V}h(x)\delta(U)\,dU\,dx_3 \\ F_{\text{future}} &= F_{\text{past}} - \frac{H(UV)}{V}h(x)\delta(U)\,dx_3 \wedge dU. \end{aligned}$$

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- Einstein equations then give

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[NA, Tabatabaei 1910.13696]

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\mathfrak{p} = (\pi T)\,\kappa\,(\log(4) - 1)\nu^2
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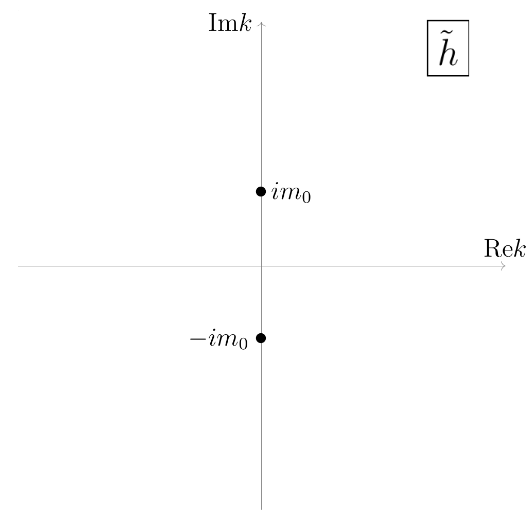

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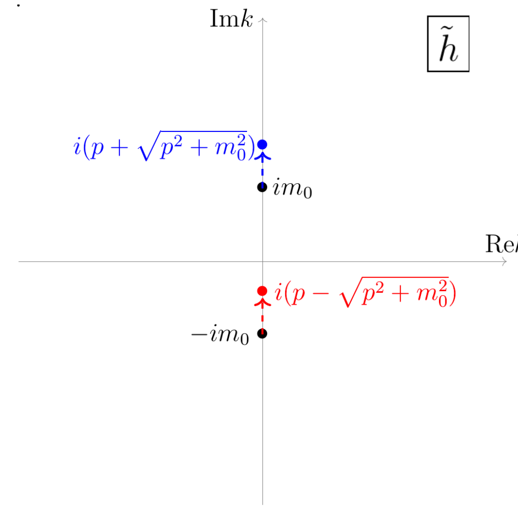

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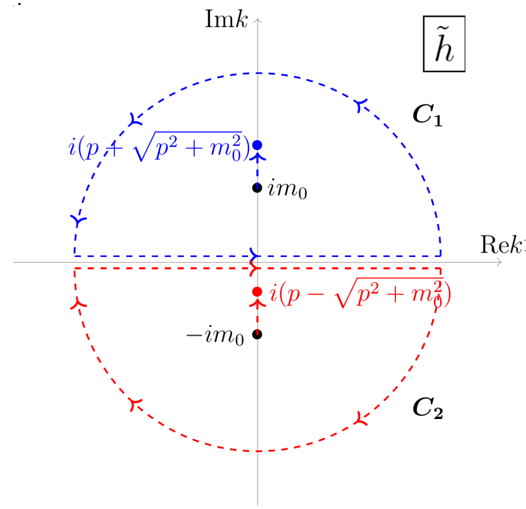

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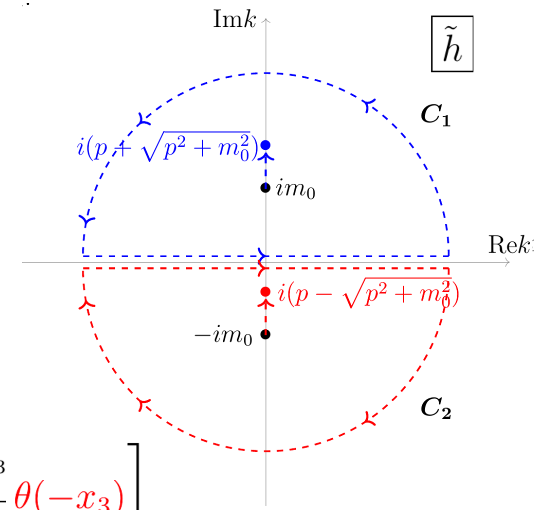

$$h(x_3) \sim (2\pi i)(-\tilde{E})\left[\frac{e^{-(\mathfrak{p}+\sqrt{\mathfrak{p}^2+m_0^2})x_3}}{2i\sqrt{\mathfrak{p}^2+m^2}}\theta(x_3) + \frac{e^{-(\mathfrak{p}-\sqrt{\mathfrak{p}^2+m_0^2})x_3}}{2i\sqrt{\mathfrak{p}^2+m^2}}\theta(-x_3)\right]$$

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$$C(t,x) = 2 - 2\langle W(t,\vec{x})W(0)W(t,\vec{x})W(0)\rangle_\beta \sim \frac{1}{N^2}\,e^{\lambda\left(t-\frac{x}{v_B}\right)}$$

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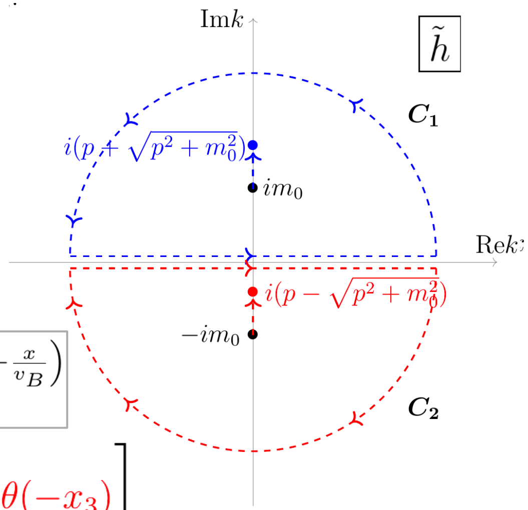

Asymmetry

Two different velocities

$$x_3 > 0 : \qquad v_B^{L_1} = \frac{2\pi T}{m_0^2}\left(\sqrt{\mathfrak{p}^2 + m_0^2} - \mathfrak{p}\right)$$

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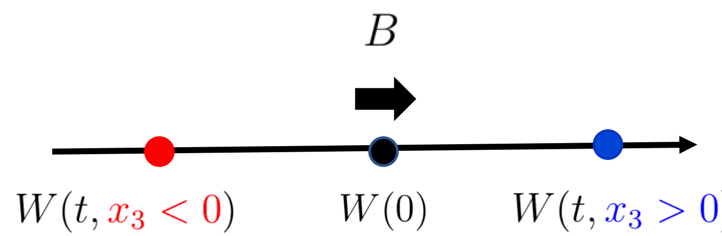

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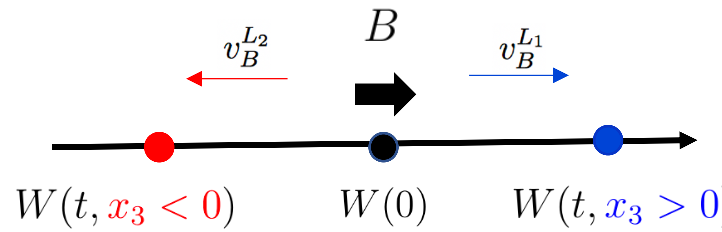

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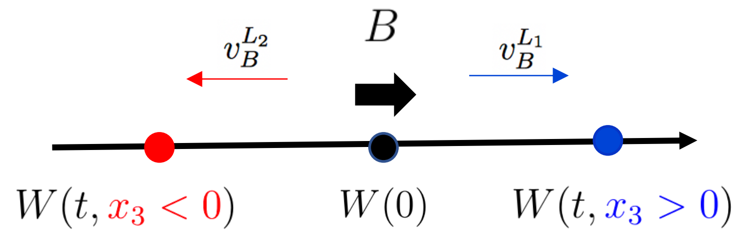

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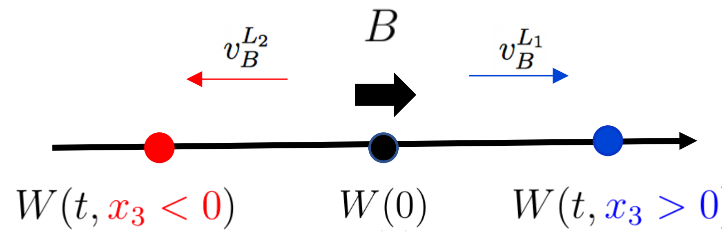

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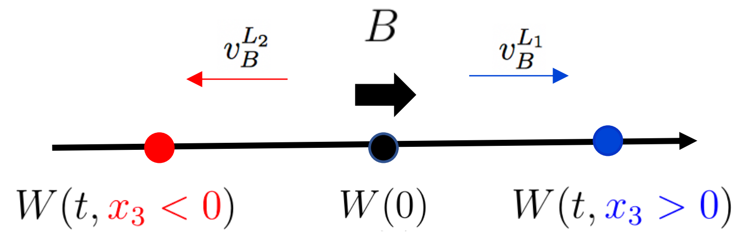

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The difference is:

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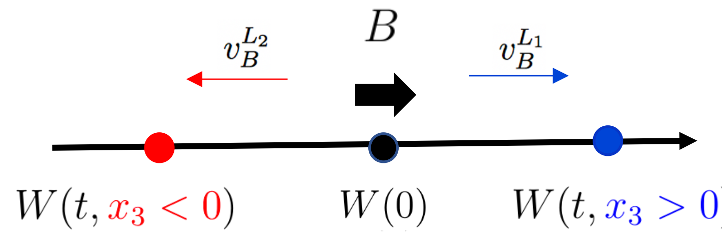

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This is actually a diagnostic of the chiral anomaly. [NA, Tabatabaei 1910.13696]

Connection to Chiral Transport

Hydrodynamic spectrum in a chiral system

Constitutive relations:

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\begin{aligned}
T^{\mu\nu} &= wu^\mu u^\nu + pg^{\mu\nu} + \sigma_\epsilon^{\mathcal{B}}(u^\mu B^\nu + u^\nu B^\mu) + \sigma_\epsilon^{\mathcal{V}}(u^\mu \omega^\nu + u^\nu \omega^\mu), \\
J^\mu &= nu^\mu + \sigma^{\mathcal{B}} B^\mu + \sigma^{\mathcal{V}} \omega^\mu,
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[Neiman, Oz 1011.5107]
[Kharzeev, Yee 1105.6360]
[Banerjee , Bhattacharya, Bhattacharya, Jain, Minwalla, Sharma 1203.3544]
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[Kharzeev, Yee, Yin 1501.002222]
[Yamamoto 1505.05444]
[Chernodub 1509.01245]
[NA, Davody, Hejazi, Rezaei 1509.08878]
[Ammon, Kaminski, Koirala, Leiber, Wu 1701.0565]
[NA, Naderi, Taghinavaz 1712.06175]
[Ammon, Greiner, Hernandez, Kaminski, Koirala, Leiber, Wu 2102.09183]

Hydro modes:

$$
\begin{aligned}
\omega &= \pm \frac{nB}{w} + \frac{n\,B}{w^2}\,\sigma_\epsilon^{\mathcal{V}}\,k \\
\omega &= \frac{B}{w}\frac{1}{[\beta,\alpha]}\Big(w(\alpha_1\partial_\mu + \alpha_2\partial_T)\sigma^{\mathcal{B}} - n(\alpha_1\partial_\mu + \alpha_2\partial_T)\sigma_\epsilon^{\mathcal{B}}\Big)\,k \\
\omega &= \pm c_s k + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(-1 + \frac{(n\alpha_2 - w\beta_2)\partial_T - (n\alpha_1 - w\beta_1)\partial_\mu}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma^{\mathcal{B}}\,k \\
&\quad + \frac{B}{w}\left(1 - \frac{[\gamma,\beta]}{[\alpha,\beta]}\right)\sigma_\epsilon^{\mathcal{B}}\,k + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(\frac{(n\alpha_1 - w\beta_1)\partial_\mu - (n\alpha_2 - w\beta_2)\partial_T}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma_\epsilon^{\mathcal{B}}\,k
\end{aligned}
$$

Hydrodynamic spectrum in a chiral system

Constitutive relations:

$$
\begin{aligned}
T^{\mu\nu} &= w u^\mu u^\nu + p g^{\mu\nu} + \sigma_\epsilon^{\mathcal{B}}(u^\mu B^\nu + u^\nu B^\mu) + \sigma_\epsilon^{\mathcal{V}}(u^\mu \omega^\nu + u^\nu \omega^\mu),\\
J^\mu &= n u^\mu + \sigma^{\mathcal{B}} B^\mu + \sigma^{\mathcal{V}} \omega^\mu,
\end{aligned}
$$

[Neiman, Oz 1011.5107]
[Kharzeev, Yee 1105.6360]
[Banerjee , Bhattacharya, Bhattacharya, Jain, Minwalla, Sharma 1203.3544]
[Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom 1203.3556]
[Landsteiner 1610.04413]

Hydro equations

$$
\begin{aligned}
\partial_\mu T^{\mu\nu} &= 0\\
\partial_\mu J^\mu &= C E^\mu B_\mu
\end{aligned}
$$

[Kharzeev, Yee, Yin 1501.002222]
[Yamamoto 1505.05444]
[Chernodub 1509.01245]
[NA, Davody, Hejazi, Rezaei 1509.08878]
[Ammon, Kaminski, Koirala, Leiber, Wu 1701.0565]
[NA, Naderi, Taghinavaz 1712.06175]
[Ammon, Greiner, Hernandez, Kaminski, Koirala, Leiber, Wu 2102.09183]

Hydro modes:

$$
\begin{aligned}
\omega &= \pm\frac{nB}{w} + \frac{n\,B}{w^2}\,\sigma_\epsilon^{\mathcal{V}}\,k\\
\omega &= \frac{B}{w}\frac{1}{[\beta,\alpha]}\Big(w\big(\alpha_1\partial_\mu + \alpha_2\partial_T\big)\sigma^{\mathcal{B}} - n\big(\alpha_1\partial_\mu + \alpha_2\partial_T\big)\sigma_\epsilon^{\mathcal{B}}\Big)\,k\\
\omega &= \pm c_s k + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(-1 + \frac{(n\alpha_2 - w\beta_2)\partial_T - (n\alpha_1 - w\beta_1)\partial_\mu}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma^{\mathcal{B}}\,k\\
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\end{aligned}
$$

$$
\begin{aligned}
\alpha_1 &= \left(\frac{\partial\epsilon}{\partial T}\right)_\mu\\
\alpha_2 &= \left(\frac{\partial\epsilon}{\partial \mu}\right)_T\\
\beta_1 &= \left(\frac{\partial n}{\partial T}\right)_\mu\\
\beta_2 &= \left(\frac{\partial n}{\partial \mu}\right)_T\\
\gamma_1 &= \left(\frac{\partial p}{\partial T}\right)_\mu\\
\gamma_2 &= \left(\frac{\partial p}{\partial \mu}\right)_T
\end{aligned}
$$

Special case: holographic chiral system

Thermodynamics: [NA, Ghazi, Taghinavaz, Tavakol, 1812.11310]

$$\epsilon = \frac{N_c^2}{8\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N_c^2 B^2}{4\pi^2}\left((1 - \log(\frac{\pi T}{\Delta})) - \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\log(2) - 3)\right)$$

$$p = \frac{N_c^2}{24\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N_c^2 B^2}{4\pi^2}\left(\log(\frac{\pi T}{\Delta}) + \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\log(2) - 3)\right)$$

$$n = \frac{N_c^2}{3\pi^2}\left(3(\pi T)^2\mu + 4\mu^3\right) + \frac{N_c^2 B^2}{3\pi^2}\frac{\mu}{(\pi T)^2}(8\log(2) - 3)$$

Transport coefficients: [Landsteiner, Megias, Pena-Benitez 1207.5808]

$$\begin{aligned}
\sigma^{\mathcal{B}} &= -\frac{\kappa}{2\pi G_5}\mu = \frac{2N_c^2}{\pi^2\sqrt{3}}\mu \\
\sigma^{\mathcal{V}} = \sigma^{\mathcal{B}}_{\epsilon} &= -\frac{\kappa}{2\pi G_5}\mu^2 = \frac{N_c^2}{\pi^2\sqrt{3}}\mu^2 \\
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Hydro modes: [NA, Tabatabaei 1910.13696]

$$v_{\text{CAW}} = 0$$

$$v_{\text{CMW}} = \frac{2}{\sqrt{3}} b\left(1 - 2\nu^2\right)$$

$$v_{sound} = \frac{1}{\sqrt{3}}\left(\pm 1 + \frac{4}{3}\nu^2 b\right)$$

Sound vs butterfly velocity

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1. Splitting of butterfly velocities might be originated just from chiral magnetic effects.

$$v_{sound} = \pm c_s + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(-1 + \frac{(n\alpha_2 - w\beta_2)\partial_T - (n\alpha_1 - w\beta_1)\partial_\mu}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma^{\mathcal{B}} \tag{2.41}$$
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Confirmed by Pole-skipping

[Grozdanov, Schalm, Scopelliti 1710.00921]
[Blake, Lee, Liu 1801.00010]
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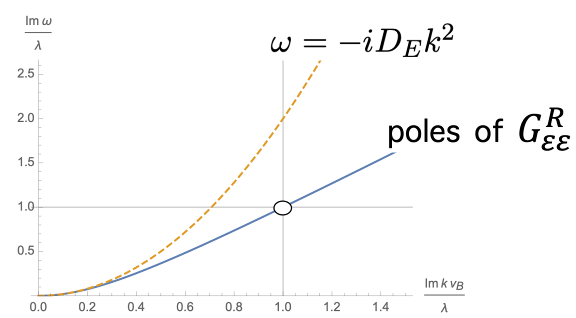

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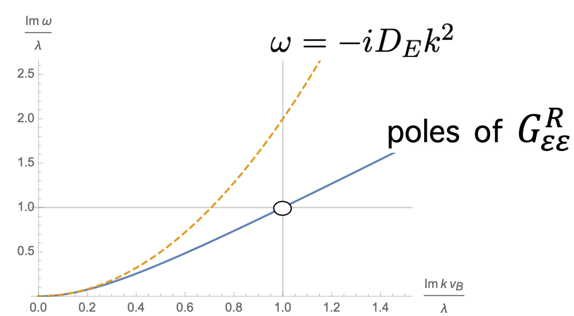

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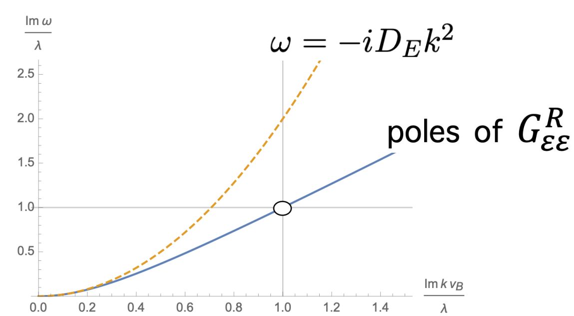

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Discussion

- Butterfly velocity is a measurable quantity in experiment.
- Observing the advertised splitting is an implicit sign of chiral anomaly
- Can it be regraded as a sign of CME?

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needs more exploration!

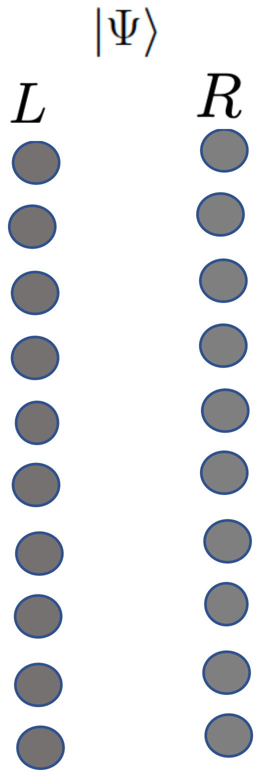
Thank you for your attention

Scrambling measures

In a qubit system with $H_L = \sum_{i=1}^{10} \left\{ \sigma_z^{(i)} \sigma_z^{(i+1)} - 1.05\ \sigma_x^{(i)} + 0.5\ \sigma_z^{(i)} \right\}$

Prepare the system in the thermofield state $|\Psi\rangle = \frac{1}{Z^{1/2}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R$

[Shenker, Stanford 1306.0622]



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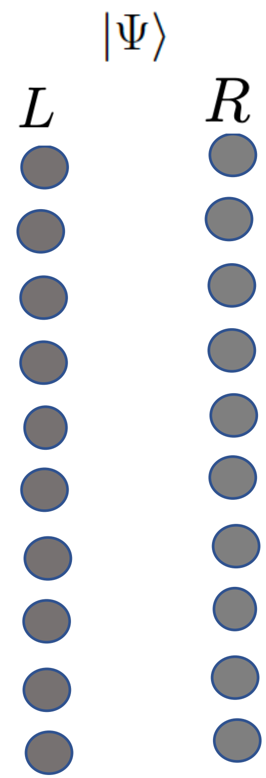

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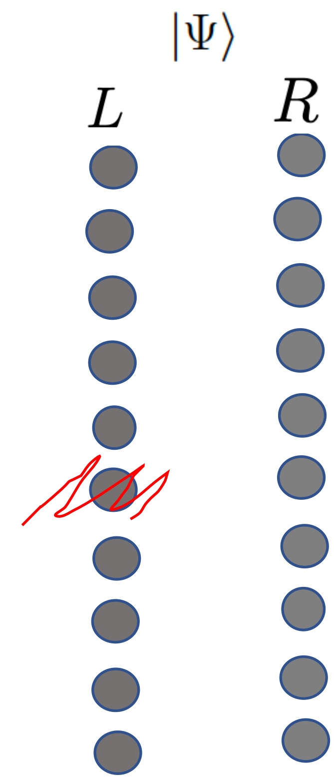

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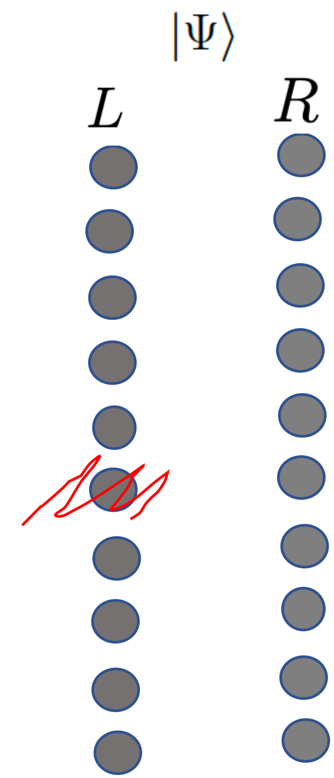

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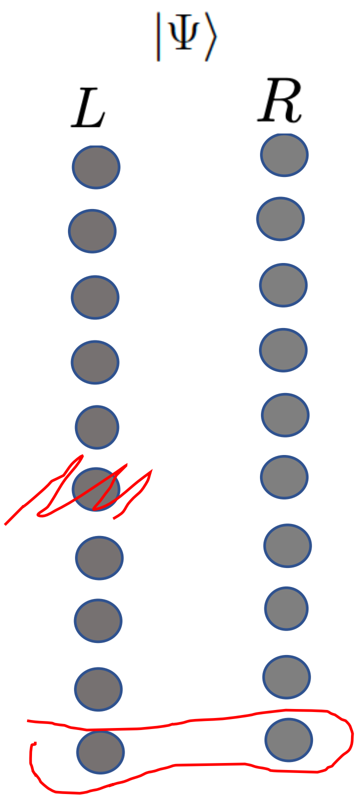

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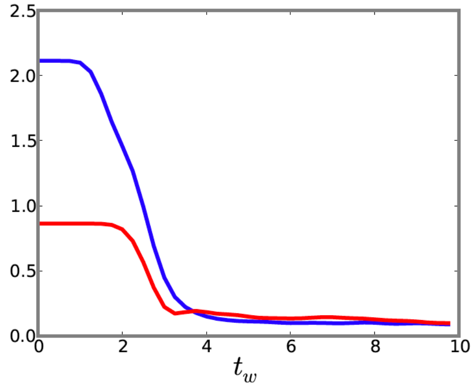
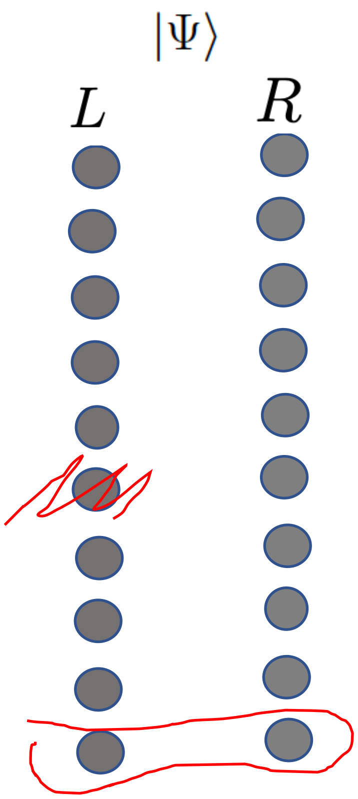


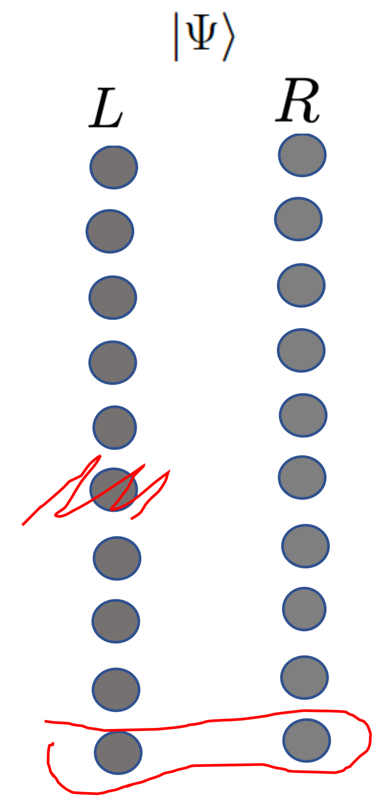


Scrambling measures

In a qubit system with $H_L = \sum_{i=1}^{10} \left\{ \sigma_z^{(i)} \sigma_z^{(i+1)} - 1.05\ \sigma_x^{(i)} + 0.5\ \sigma_z^{(i)} \right\}$

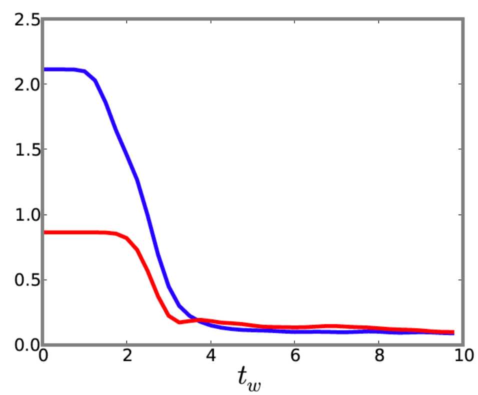
Prepare the system in the thermofield state $|\Psi\rangle = \frac{1}{Z^{1/2}} \sum_n e^{-\beta E_n/2}\ |n\rangle_L |n\rangle_R$

[Shenker, Stanford 1306.0622]



- perturb the system: apply $\sigma_z^{(5,L)}$ at t_w
- The state becomes: $|\Psi'\rangle = e^{-iH_L t_w} \sigma_z^{(5,L)} e^{iH_L t_w} |\Psi\rangle$

$\langle \Psi' | \sigma_z^{(1,L)} \sigma_z^{(1,R)} | \Psi' \rangle$



- Scrambling destroys spin correlation

Out-of-time-order correlator: OTOC

The correlator $\langle\Psi'|\sigma_z^{(1,L)}\sigma_z^{(1,R)}|\Psi'\rangle$ is in fact: $|\Psi'\rangle = e^{-iH_L t_w}\sigma_z^{(5,L)}e^{iH_L t_w}|\Psi\rangle$

$$\langle\Psi|\sigma_z^{(5,L)}(t_w)\sigma_z^{(1,L)}\sigma_z^{(1,R)}\sigma_z^{(5,L)}(t_w)|\Psi\rangle$$

- Or more generally: $\langle\Psi|W_L(t)V_L V_R W_L(t)|\Psi\rangle$

- Similarly: OTOC = $F(t) = \langle\Psi|V_L W_R(t) V_R W_L(t)|\Psi\rangle$

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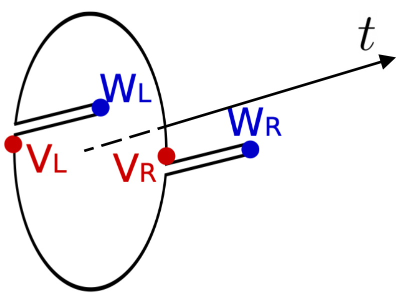

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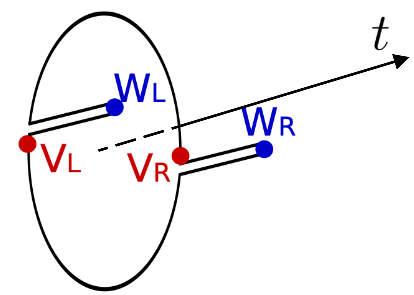
- Or more generally: $\langle\Psi|W_L(t)V_LV_RW_L(t)|\Psi\rangle$
- Similarly: OTOC = $F(t) = \langle\Psi|V_LW_R(t)V_RW_L(t)|\Psi\rangle$



- For large N holographic CFTs

$$F(t) = f_0 - \frac{f_1}{N^2}\exp\frac{2\pi}{\beta}t + \mathcal{O}(N^{-4})$$

[Maldacena, Shenker, Stanford 1503.01406]

Out-of-time-order correlator: OTOC

The correlator $\langle\Psi'|\sigma_z^{(1,L)}\sigma_z^{(1,R)}|\Psi'\rangle$ is in fact: $|\Psi'\rangle = e^{-iH_L t_w}\sigma_z^{(5,L)}e^{iH_L t_w}|\Psi\rangle$

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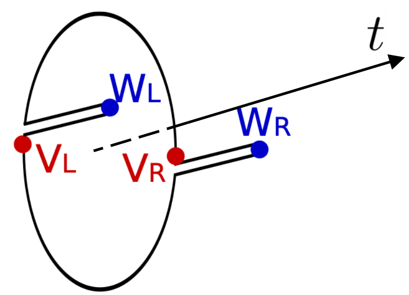

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[Maldacena, Shenker, Stanford 1503.01406]

Lyapunov exponent

Size of precursor growth

- The squared commutator

$$C(t_w, |x-y|) = \mathrm{tr}\left\{\rho(\beta)[W_x(t_w), W_y]^\dagger [W_x(t_w), W_y]\right\}$$
$$= 2 - 2\,\mathrm{Re}\,\langle TFD|\, W_y W_x\,(t_w) W_y W_x(t_w)\,|TFD\rangle$$

- Size of precursor $s[W_x(t_w)]$ is the volume of region in y such that $C \geq 1$

= a ball centered at x of the radius

$$r[W_x(t_w)] \approx v_B(t_w - t_*)$$

$$C(t) = 2 - 2\langle W(t,\vec{x})V(0)W(t,\vec{x})V(0)\rangle_\beta \sim \frac{1}{N} e^{\lambda\left(t - \frac{x}{v_B}\right)}$$

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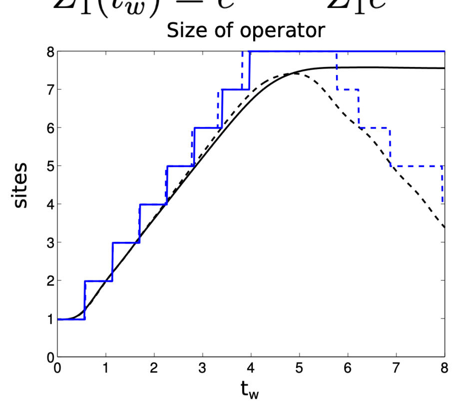
- Linear growth in time is checked numerically

in the spin chain $H = -\sum_i Z_i Z_{i+1} + g X_i + h Z_i$

$$Z_1(t_w) = e^{-iHt_w} Z_1 e^{iHt_w}$$



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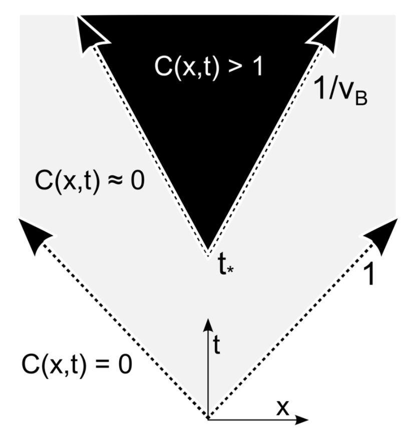
Butterfly speed

[Roberts, Shenker, Stanford, 1409.8180]



$y = 0$