Collective axions in chiral media: an effective field theory approach

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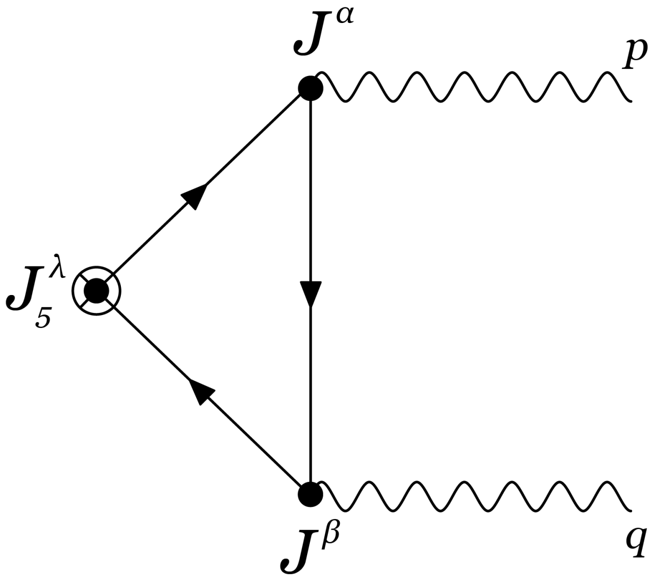
based on work in progress in collaboration with E. Mottola and A. Stergiou

Holographic Perspectives on Chiral Transport, ECT*, Trento, Italy

A.D. Dolgov, V.I. Zakharov, NPB, 1971



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$$\partial_\alpha J_5^\alpha = \frac{2\alpha}{\pi} E \cdot B$$

$$J_5^\lambda = \frac{2\alpha}{\pi} \frac{\partial^\lambda}{\Box} E \cdot B + J_{5,\perp}^\alpha$$

- Does this pole correspond to some collective mode?

- Are there other poles in the transverse part?

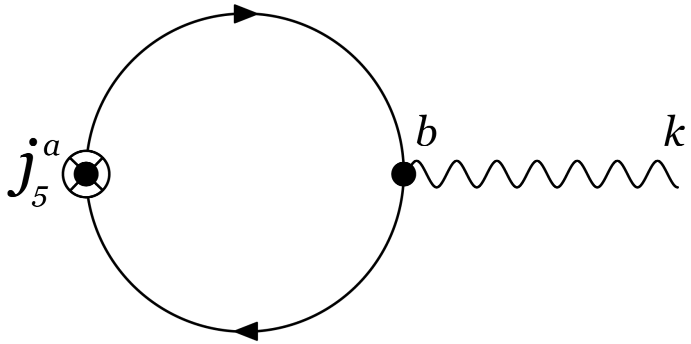
- What happens with this mode in the low-energy limit?

Bosonization

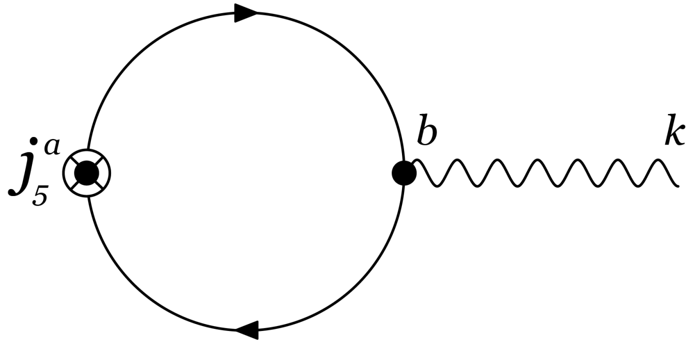
$$S = \int d^2x \; \{ i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi \}$$

$$j_5^a = -\epsilon^{ab} j_b$$

$$\Pi_2^{ab}(k) = \frac{1}{\pi k^2}\left(k^a k^b - g^{ab} k^2\right)$$



$\sim \varepsilon *$



$$\Pi_2^{ab}(k) = \frac{1}{\pi k^2}\left(k^a k^b - g^{ab} k^2\right)$$

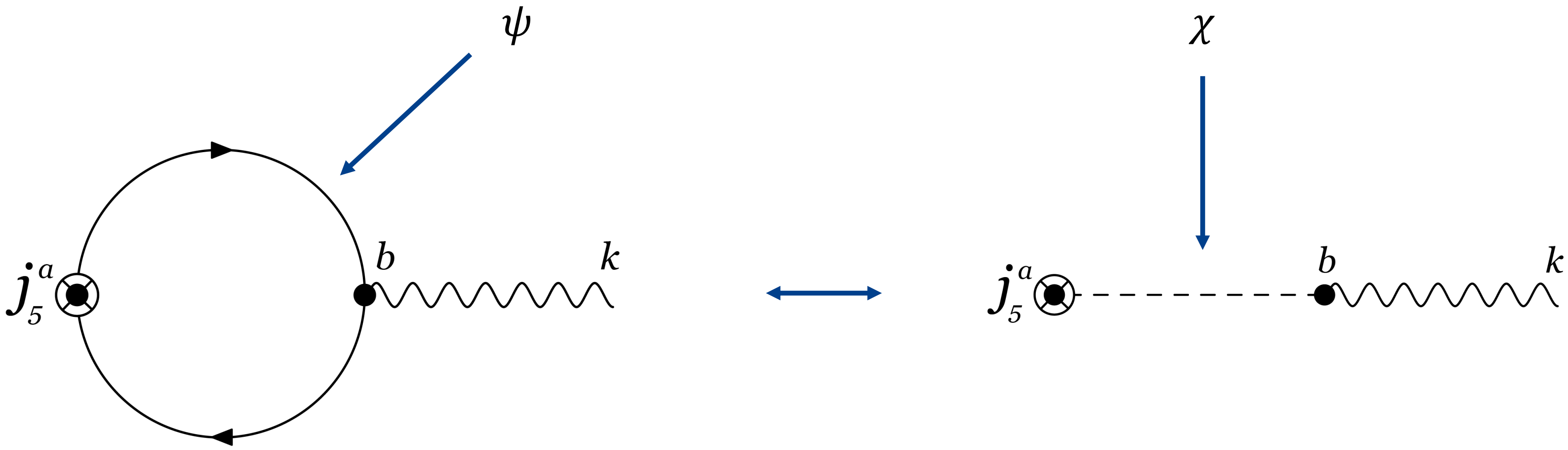
$$j_5^a = \frac{1}{\pi}\frac{\partial^a}{\Box}E$$

$$S = -\frac{1}{2\pi}\int d^2x\, E\frac{1}{\Box}E$$

↓

$$S = \int d^2x\, \left\{-\frac{\pi}{2}\partial_a\chi\partial^a\chi - E\chi\right\}$$

Schwinger boson



ψ
χ
j_5^a
b
k
j_5^a
b
k

G.M. Monteiro, A.G. Abanov, V.P. Nair, PRD, 2014

Anomalous hydrodynamics in D=2

$$S = \int d^2x \left\{ (\partial_a \eta + A_{5a}) j_5^a + \frac{1}{\pi} \eta E - \epsilon(n_5) \right\}$$

$$\frac{\delta S}{\eta} = 0 \qquad \frac{\delta S}{j_5^a} = 0$$

$$\longrightarrow$$

$$\partial_a T^{ab} = F^{bc} j_c$$

$$\partial_a j^a = 0$$

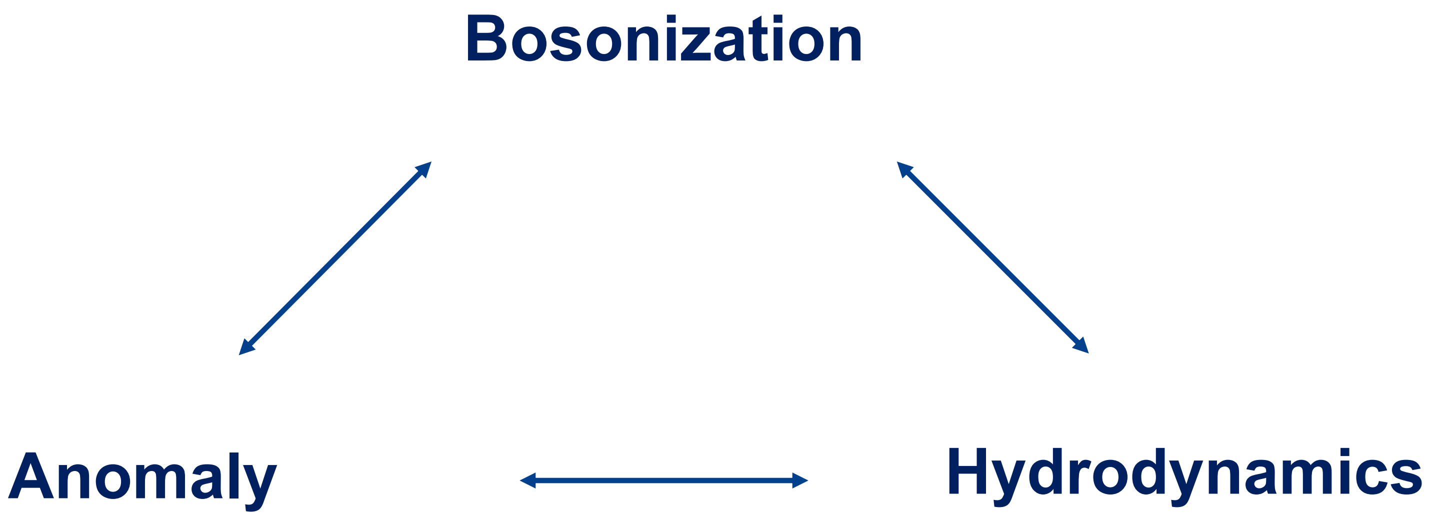
$$\partial_a j_5^a = \frac{1}{\pi} E$$

$$\epsilon = \frac{\pi}{2} n_5^2 = p = \frac{1}{2\pi} \mu_5^2$$

$$S = \int d^2x \left\{ (\partial_a \eta + A_{5a}) j_5^a + \frac{1}{\pi} \eta E - \epsilon(n_5) \right\}$$

$$\partial_a \eta = -\pi j_{5a} = -\pi \partial_a \chi$$

$$S = \int d^2x \left\{ -\frac{\pi}{2} \partial_a \chi \partial^a \chi - E\chi \right\}$$


Bosonization
Anomaly
Hydrodynamics

- In 2d the pole is a propagating collective mode

- It is also the sound of the 2d anomalous hydrodynamics

- No analogy for the transverse part

M. Giannotti, E. Mottola, PRD, 2009
E. Mottola, AS, NPB, 2021

Effective action in D=4

$$S = \frac{2\alpha}{\pi} \int d^4x \, \partial_\mu A_5^\mu \frac{1}{\Box} E \cdot B$$

$$\updownarrow$$

$$S = \int d^4x \, \left\{ (\partial_\lambda \eta + A_{5\lambda}) J_5^\lambda + \frac{2\alpha}{\pi} \eta \, E \cdot B \right\}$$

E. Mottola, AS, NPB, 2021

$$S = \int d^4x \left\{ (\partial_\lambda \eta + A_{5\lambda}) J_5^\lambda + \frac{2\alpha}{\pi} \eta \, E \cdot B - \epsilon(n_5) \right\}$$

↓

$$\frac{dn_5}{d\mu_5} \left(\partial_t^2 - v_s^2 \nabla^2 \right) \eta = \frac{2\alpha}{\pi} E \cdot B$$

↓

$$\mathbf{J}_5 = -\frac{2\alpha}{\pi} \frac{v_s^2}{\partial_t^2 - v_s^2 \nabla^2} \nabla E \cdot B$$

E. Mottola, AS, NPB, 2021

$$[\eta(t,\mathbf{x}),\Pi_\eta(t,\mathbf{y})] = i\delta^{(3)}(\mathbf{x}-\mathbf{y})$$

$$J^0 = -\frac{2\alpha}{\pi}B\cdot\nabla\eta$$

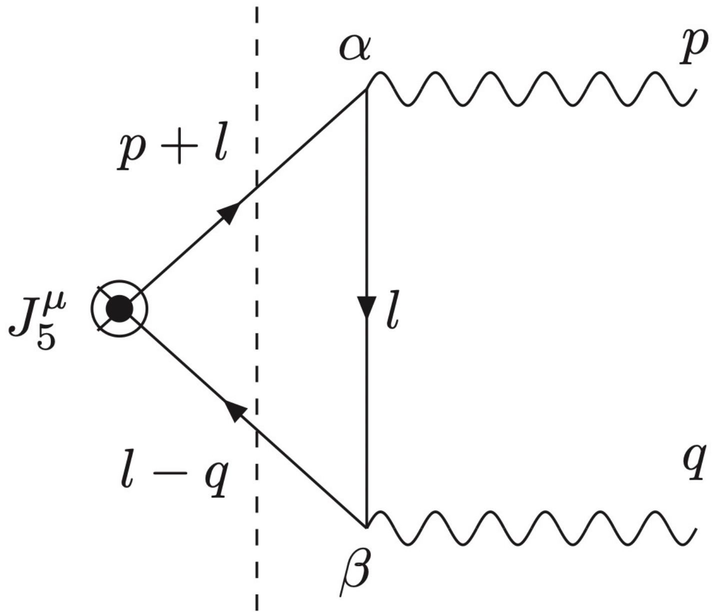
$$\mathbf{J} = \frac{2\alpha}{\pi}(\mathbf{B}\dot{\eta}-\mathbf{E}\times\nabla\eta)$$

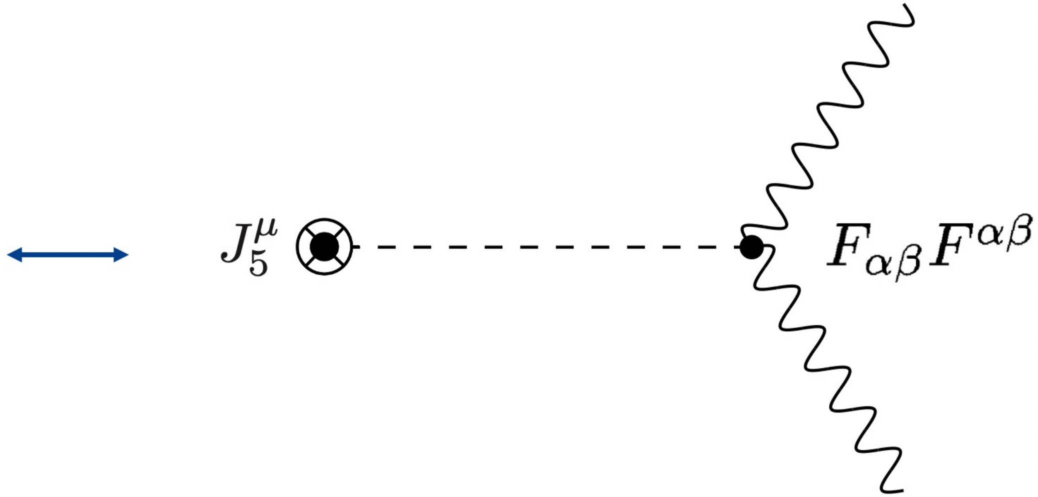
$$\Downarrow$$

$$[J^0(t,\mathbf{x}),J_5^0(t,\mathbf{y})] = -\frac{2\alpha}{\pi}\,i\,\mathbf{B}\cdot\nabla_x\delta^{(3)}(\mathbf{x}-\mathbf{y})$$

$$[\mathbf{J}(t,\mathbf{x}),J_5^0(t,\mathbf{y})] = -\frac{2\alpha}{\pi}\,i\,\mathbf{E}\times\nabla_x\delta^{(3)}(\mathbf{x}-\mathbf{y})$$

E. Mottola, AS, NPB, 2021





Interacting fermions

$$L_{\rm cl} = \bar{\psi}\gamma^\mu (i\overleftrightarrow{\partial}_\mu + A_\mu + A_{5\mu}\gamma^5)\psi - g\bar{\psi}\Phi\psi + L_\Phi \,,$$

$$\Phi = \sigma \exp(2i\zeta\gamma^5)$$

$$V_{\rm eff}(\sigma) = -2\mu_5^2\sigma^2 + V(\sigma)$$

$$\mu_5^2 = -(\partial\zeta + A_5)^2$$

$$\sigma = 0 \qquad\qquad 4\mu_5^2 < \kappa^2$$

$$\sigma = \sqrt{\frac{4\mu_5^2 - \kappa}{\lambda}} \neq 0 \qquad\qquad 4\mu_5^2 > \kappa^2$$

integrating out the fermions

$$\frac{dn_5}{d\mu_5}\left(\partial_t^2 - v_s^2\nabla^2\right)\zeta = \frac{2\alpha}{\pi} E\cdot B$$

$$v_s^2 = \frac{\sigma\, V''(\sigma) - V'(\sigma)}{\sigma\, V''(\sigma) + 3V'(\sigma)} = \frac{dP}{d\epsilon}$$

$$\frac{dn_5}{d\mu_5} = 4\sigma^2$$

$$\sigma = 0$$

$$\frac{dn_5}{d\mu_5}\left(\partial_t^2 - v_s^2\nabla^2\right)\eta = \frac{2\alpha}{\pi}E\cdot B$$

$$\sigma = \sqrt{\frac{4\mu_5^2 - \kappa}{\lambda}}$$

$$\frac{dn_5}{d\mu_5}\left(\partial_t^2 - v_s^2\nabla^2\right)\zeta = \frac{2\alpha}{\pi}E\cdot B$$

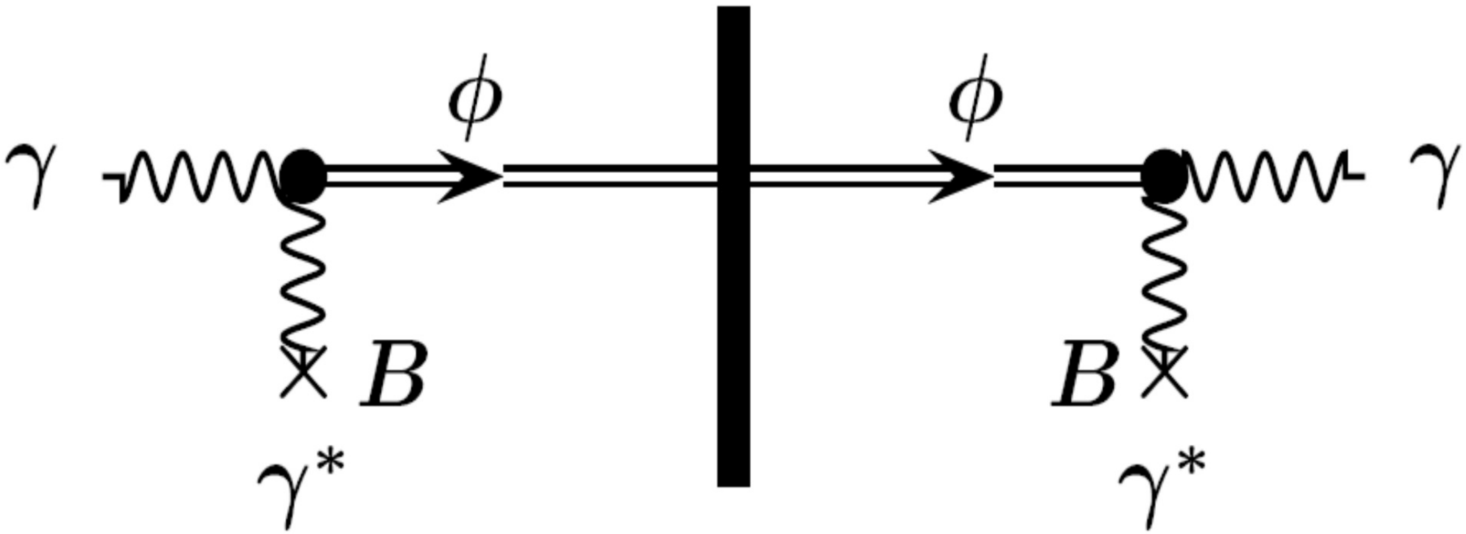
- In 4d the pole is a collective mode propagating over a background medium

- It is the sound of the anomalous hydrodynamics

- In the non-interacting limit one has to carefully study the transverse part of the anomalous diagram in the presence of (general) background

- For interacting fermions, one finds that the axionic mode is unmodified, while the EFT is under control (e.g. no transverse terms)

Weyl semi-metals



Okun 1982, Skivie 1983, Ansel'm 1985, Van Bibber et al. 1987

Weyl semi-metals

$$S_h = \frac{1}{2}\int d^4x\, T^{\mu\nu} h_{\mu\nu} = -\frac{n_5}{2}\int d^4x\, \left(\dot{h}_{ii} - 2\partial_i h_{0i}\right)$$

$$T^{\mu\nu}\Big|_{\text{fluid}} = \frac{\mu_5}{n_5} J_5^\mu J_5^\nu + g^{\mu\nu}\left\{\mu_5 n_5 - \epsilon(n_5)\right\}$$

Fermi velocity decreases the mass

$$\left(-\partial_t^2 + v_s^2\,\nabla^2\right)\zeta - \left(\frac{2\alpha}{\pi}\right)^2 \frac{d\mu_5}{dn_5}\,\frac{B^2\,\partial_t^2 - \left(B\cdot\nabla\right)^2}{\partial_t^2 - \nabla^2}\zeta$$

$$= \frac{\mu_5}{2} v_s^2\left(h_{ii} - 2\partial_i h_{0i}\right)$$

$$\mathbf{J} = \frac{2\alpha}{\pi}\left(\mathbf{B}\dot\eta - \mathbf{E}\times\nabla\eta\right)$$

Search for the particular pattern in the current

source the mode, e.g. with strain

Summary

- The anomalous pole leads to a propagating collective mode, at least in the presence of matter

- This mode is protected by a version of the anomaly matching condition

- The presence of the mode in the non-interacting limit highlights a non-trivial analogy between SSB and anomalies

- This mode is a collective axion, it fits into the ongoing search for axionic modes in condensed matter systems, and may lead to phenomenological implications in Weyl/Dirac semi-metals