Electric Conductivity and Chiral Anomaly — perturbative and holographic perspectives

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— Holographic perspectives on chiral transport —

Chiral Anomaly

From a review (Aspects of Chiral Sym) by Smilga (2000)

$$K_{\mu\nu}^{AB,\ \mathcal{H}}(q) \;=\; i\int \langle T\{j_{\mu 5}^{A}(x) j_{\nu}^{B}(0)\}\rangle_{\mathcal{H}}\ e^{iqx} d^4x$$

('t Hooft)

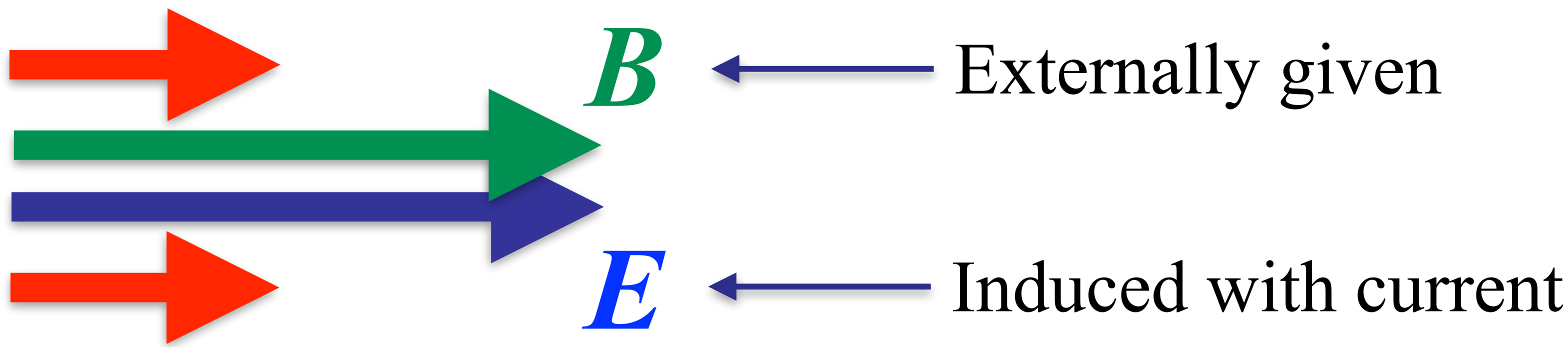
One loop calculation leads to

$$K_{\mu\nu}^{AB,\ \mathcal{H}}(q) \;=\; -\frac{\mathcal{H}}{2\pi^2}\frac{q_\mu \tilde{\epsilon}_{\nu\alpha} q^\alpha}{q^2} \cdot N_c \cdot \frac{1}{2}\delta^{AB}$$

This formula tells us a lot: CME is included, the difference in the order of limits is included, etc.

Chiral Magnetic Effect

$$n_5 \propto \boldsymbol{E}\cdot\boldsymbol{B}\,t$$



$$\boldsymbol{j}_{\mathrm{Ohm}} = \sigma\boldsymbol{E}$$

$$\boldsymbol{j}_{\mathrm{CME}} \propto n_5\,\boldsymbol{B} \propto (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B}\,t \quad (\sigma=\infty!)$$

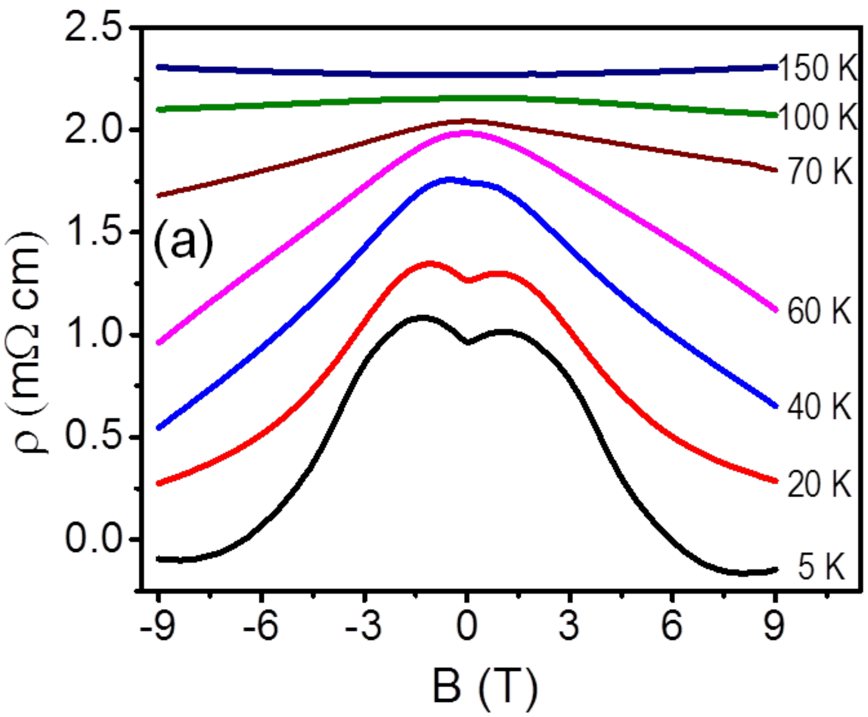
If this linear rise is balanced by "relaxation" $\sigma_{\mathrm{CME}} \propto B^2$

Chiral Magnetic Effect

$$j = (\sigma_{\rm Ohm} + \sigma_{\rm CME})E \qquad \sigma_{\rm CME} \propto B^2$$

Son-Spivak (2012)

Li et al. Nature Phys. 12, 550 (2016)



Two Questions:

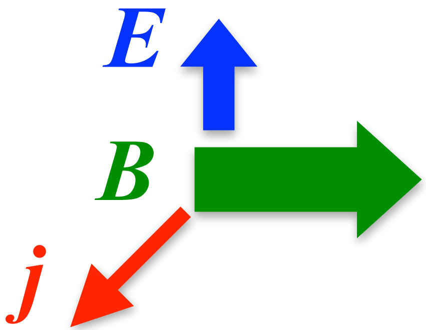
* **Microscopic calculation of $\sigma_{\rm CME}$ beyond the relaxation time approx. ?**

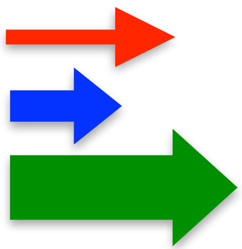
* ***B* dependece in the Ohmic part: $\sigma_{\rm Ohm}$?**

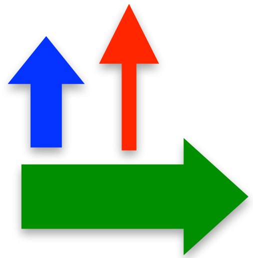
Electric Conductivity

Tensor Decomposition

$$\sigma^{ij} = \sigma_H \,\epsilon^{ijk} \hat{B}^k + \sigma_\parallel \hat{B}^i \hat{B}^j + \sigma_\perp (\delta^{ij} - \hat{B}^i \hat{B}^j)$$







$$\sigma_H = \frac{n_0}{B}$$

Hall conductivity

CME

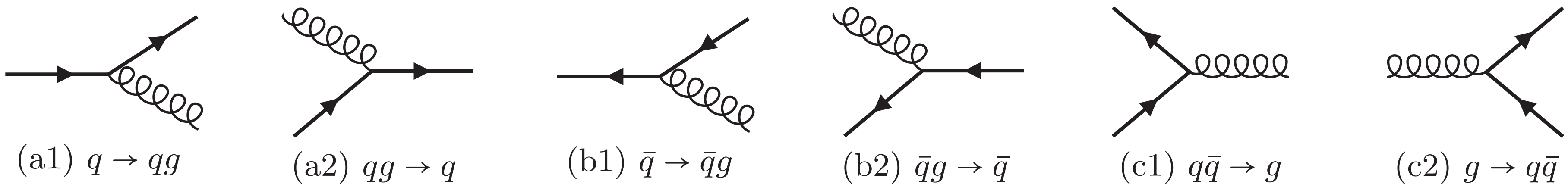
$$\frac{\sigma_\perp}{T} \sim \frac{g^2 T^2}{|qB|}$$

Suppressed

$$T \gtrsim \sqrt{qB} \gg gT$$

Perturbative Approach

Magnetic field opens 1→2 (2→1) scattering channels.
(Compton scattering with one photon replaced by B
+ Pair creation/annihilation processes)



Fukushima-Hidaka, PRL (2018) / JHEP (2020)

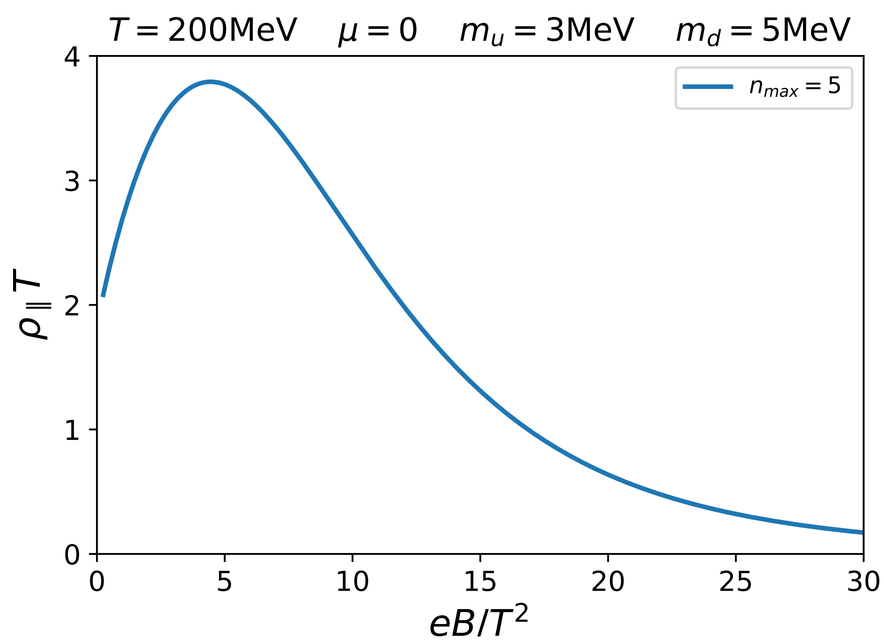
Calculated the conductivity for $T \sim \sqrt{eB} \gg gT$
(dropping the thermal screening effects)

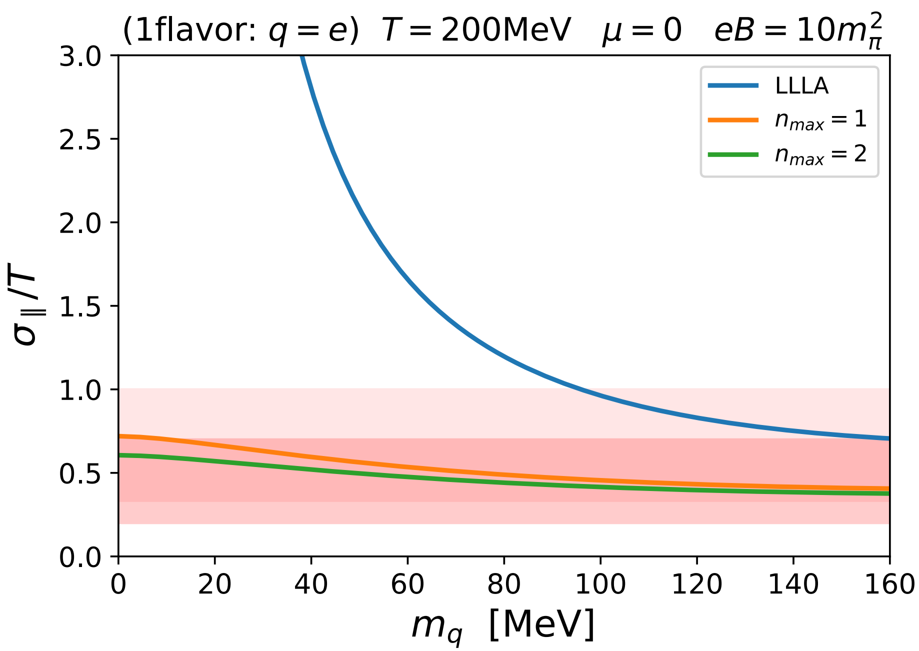
The question: is chiral anomaly included?

Perturbative Approach

Kubo formula → Pinch singularities → Kinetic equation

$f + \delta f : \delta f \propto E$ **Linearlized kinetic eq. solved →** σ





Fukushima-Hidaka (2018 / 2020)

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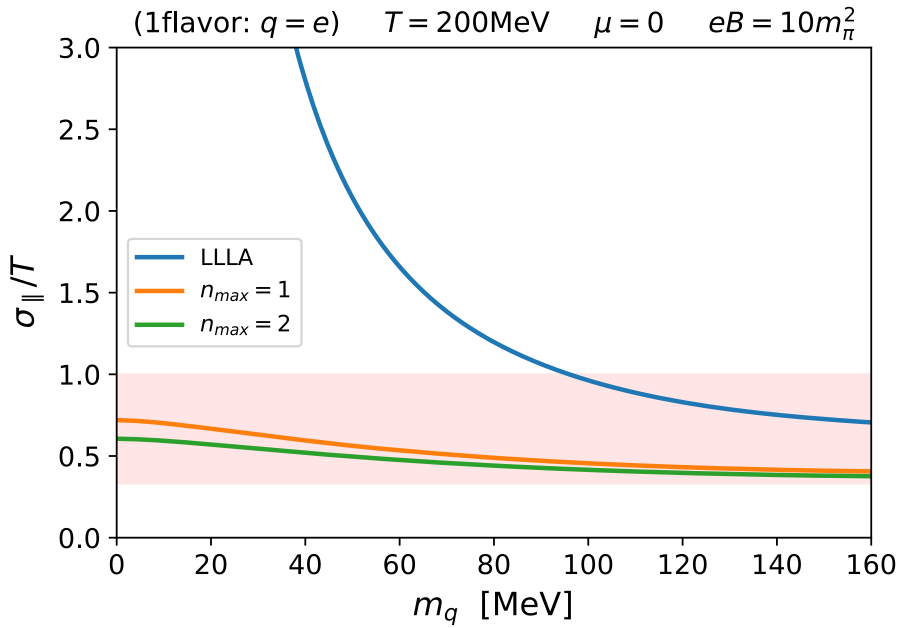
Perturbative Approach



At first, we thought that $\sigma \to \infty$ at $m \to 0$ is an artifact from the LLL

No scattering in (1+1)D

Referee pointed out : $\sigma \to \infty$ must be the answer!

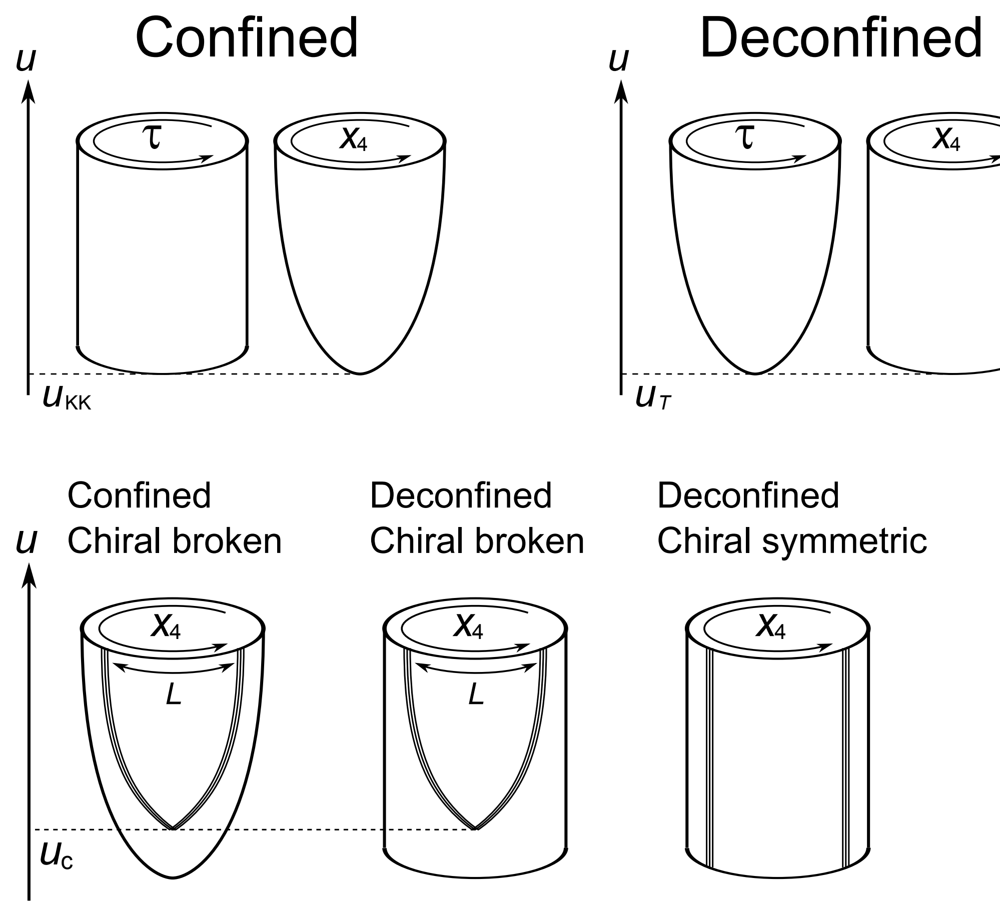
The problem was: our δf did not include chiral charge.

n_5 should be treated as a hydro mode…

Holographic Approach

Sakai-Sugimoto Model

Fukushima-Okutsu (2022)



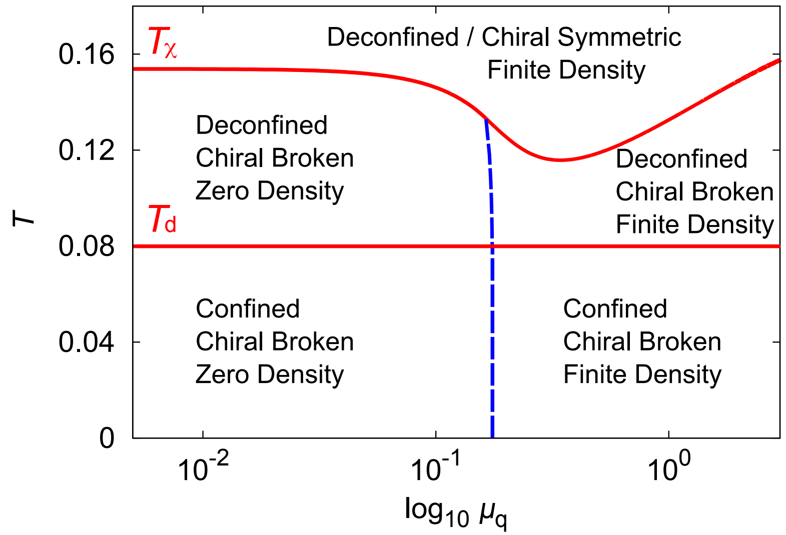
Chiral symmetry is realized in the same way as QCD.

Chiral anomaly is easily seen from the equation of motion.

CME has been well investigated: Yee, Rebhan, Schmitt, Stricker, etc.

Holographic Approach

Suggestive Phase Structure in Sakai-Sugimoto Model



Hawking-Page Transition

$$T_c = M_{\rm KK}/(2\pi)$$

$$M_{KK} = 0.95\,\mathrm{GeV}$$

$$\lambda = 16.63$$

Compute the current with $E \to \sigma = j/E$

[*B*=0]

$$\frac{\sigma}{C_e T} = \frac{2\lambda N_c T}{27\pi M_{\rm KK}} = \frac{\lambda}{9\pi^2}\left(\frac{T}{T_c}\right)$$

(per one flavor)

$\sigma/(C_e T)$	$1.1T_c$	$1.3T_c$	$1.5T_c$
This work	0.206	0.243	0.281
Lattice-QCD [52]	0.201-0.703	0.203-0.388	0.218-0.413

Fukushima-Okutsu (2022)

Holographic Approach

With nonzero *B*, transverse σ is easy to compute.

$$\sigma_\perp = \frac{\sigma(B=0)}{\sqrt{1+B^2 u_T^{-3}}}$$

Conductivity suppressed for large *B*, consistent with the Landau orbit picture.

With nonzero *B*, a time-independent term appear from the Chern-Simons term, leading to $\sigma_{\|} \to \infty$!

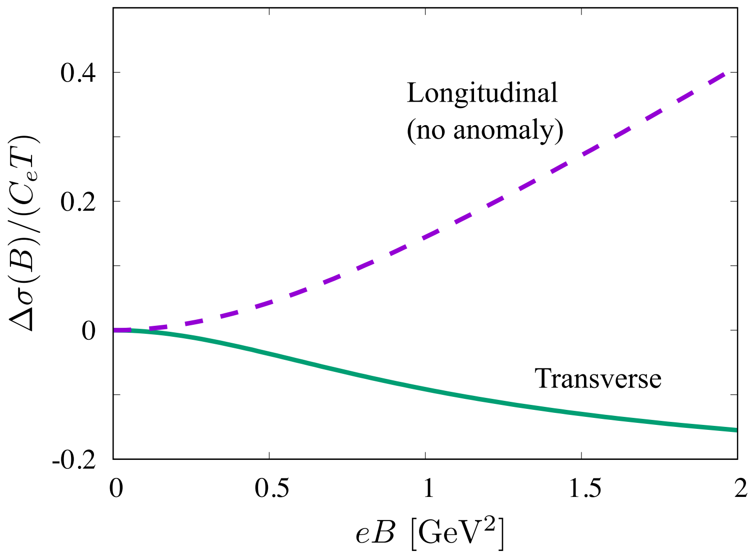
*** We can adjust the anomaly to be zero…**

Coefficient α in the Chern-Simons term changed.

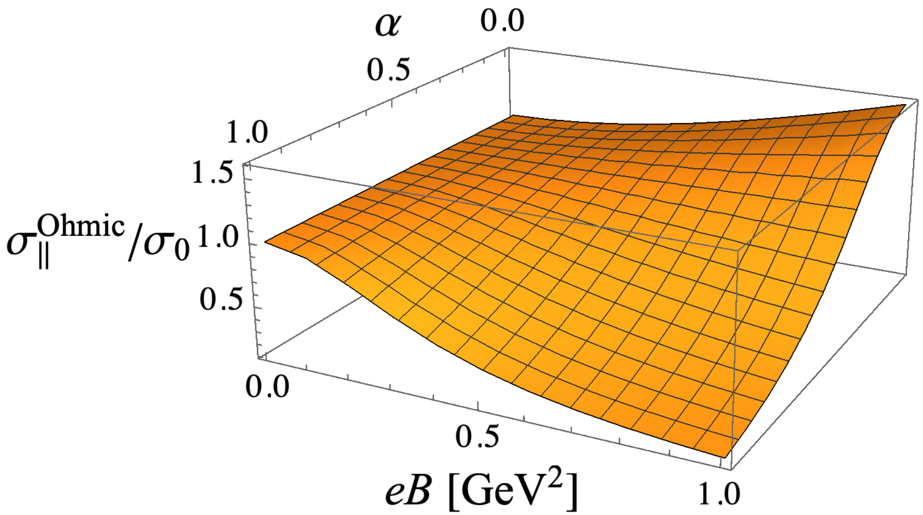
*** We can drop the divergent term to be zero…**

Extracting the Ohmic part of the conductivity.

Holographic Approach



Bad news — conductivity is enhanced even without the Chern-Simons term (maybe without the chiral anomaly).



Good news — Ohmic part with the chiral anomaly shows the *positive* magnetoresistance, not contaminating the CME signature (negative mag.).

Conclusions / Outlooks

- **Still, the interpretation of the conductivity in terms of chiral anomaly is a subtle issue…**
- **Full perturbative calculation with the chiral charge taken into account is needed (maybe somebody already did it?)**
- **How to reliably calculate the magnetic dependence in the Ohmic part? Is this a well-defined question?**
- **Personally, I am interested in a question of how to formulate the relaxation time in a way calculable in the lattice or in the holographic model.**