The bosonic skin effect: boundary condensation in asymmetric transport

Louis Garbe

Trento, 05/05/2023

TU WIEN

FWF Der Wissenschaftsfonds.

VCQ

No boson is an island

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VCQ

No man is an island, entire of itself;
Every man is a piece of the continent,
A part of the main.

J. Donne

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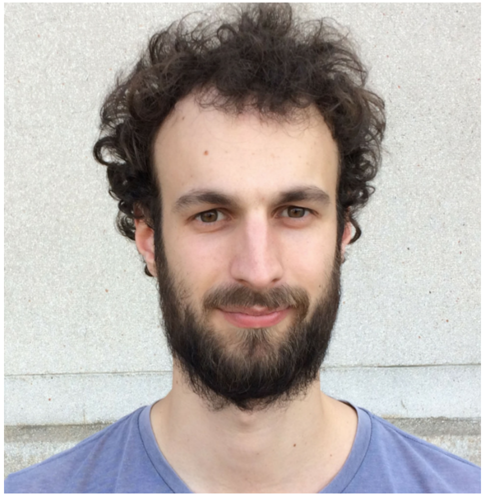
J. Donne

Bose statistics → intrinsically **collective** behavior

Effects in the context of **transport**

Yuri Minoguchi



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L. Garbe, Y. Minoguchi, J. Huber, P. Rabl, arXiv:2301.11339

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- Bosonic transport: the ASIP
- Non-Gaussian fluctuations
- Driven transport: the bosonic skin effect

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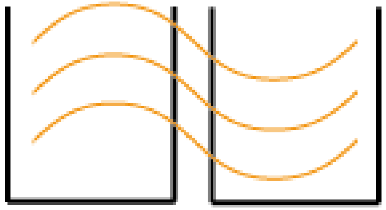
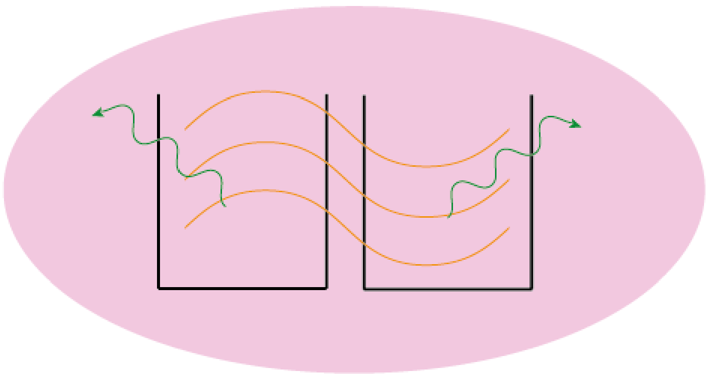
Incoherent bosonic transport



$$\hat{H} = -J(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2)$$

Tight-binding Hamiltonian
without interactions

Incoherent bosonic transport



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Tight-binding Hamiltonian
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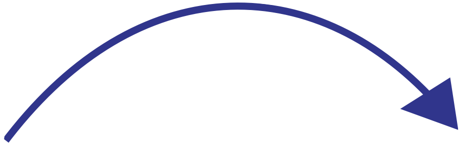
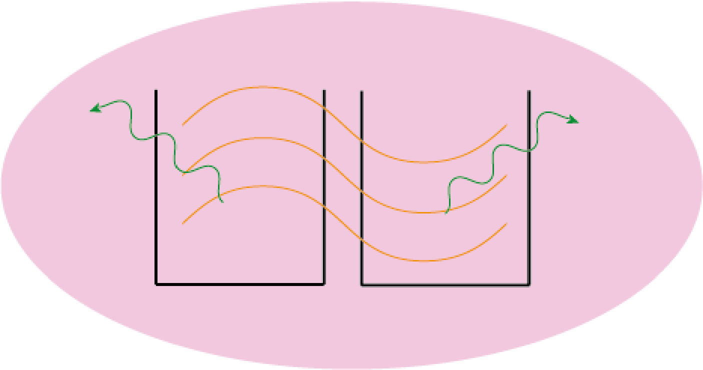
Decoherence



Coherence loss
No particle loss

?

Incoherent bosonic transport



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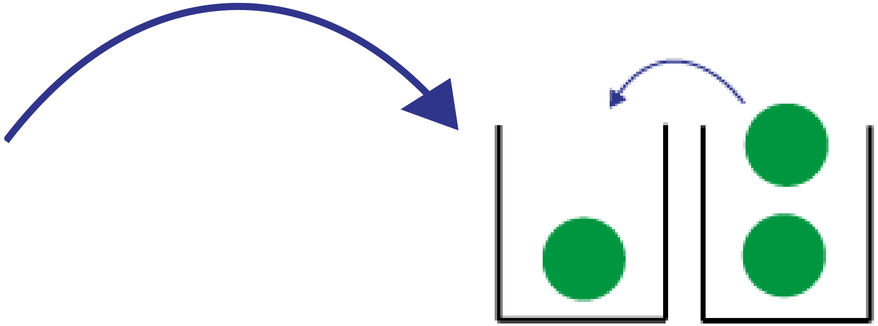


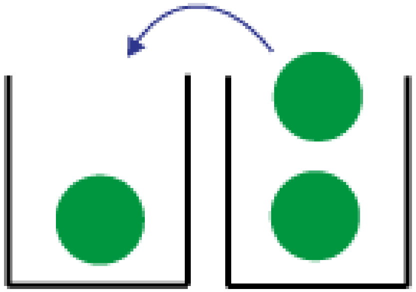
Classical picture:
independent particles?

$$P_{\text{ind}} = \Gamma$$

$$P[p \to p+1] \propto \Gamma n_p$$

Bose-enhance transport

$$\mathcal{L} \sim D[\hat{a}_1^\dagger \hat{a}_2] + D[\hat{a}_2^\dagger \hat{a}_1]$$



$$P[p \to p+1] \propto \Gamma n_p (1 + n_{p+1})$$

Bose-enhanced transport

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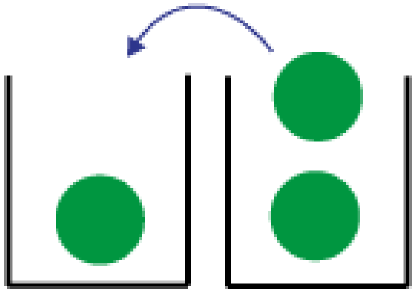

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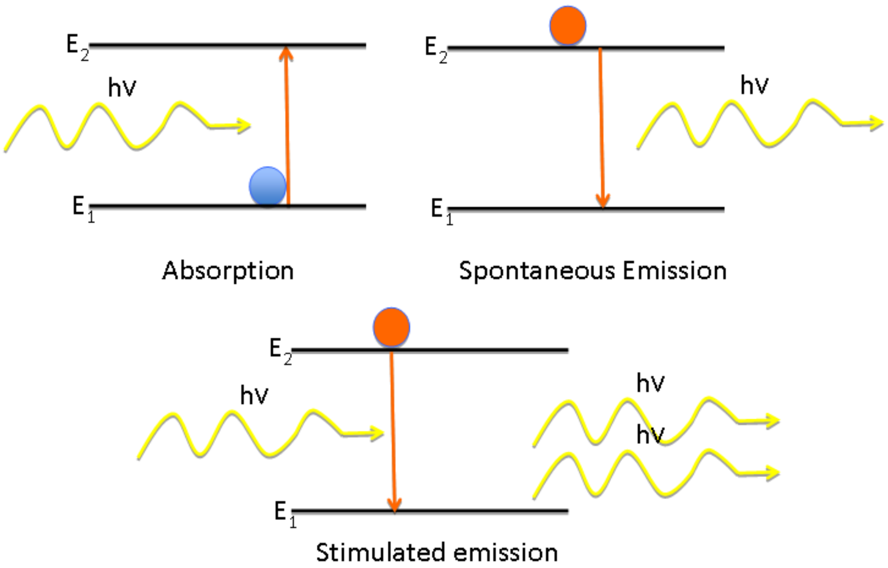
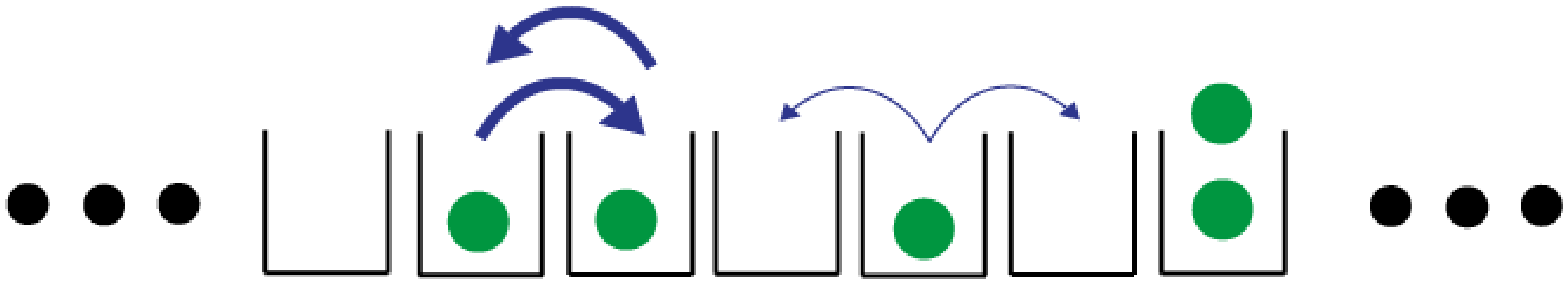


Even with neither coherence nor interaction, correlations induced by the bosonic statistics

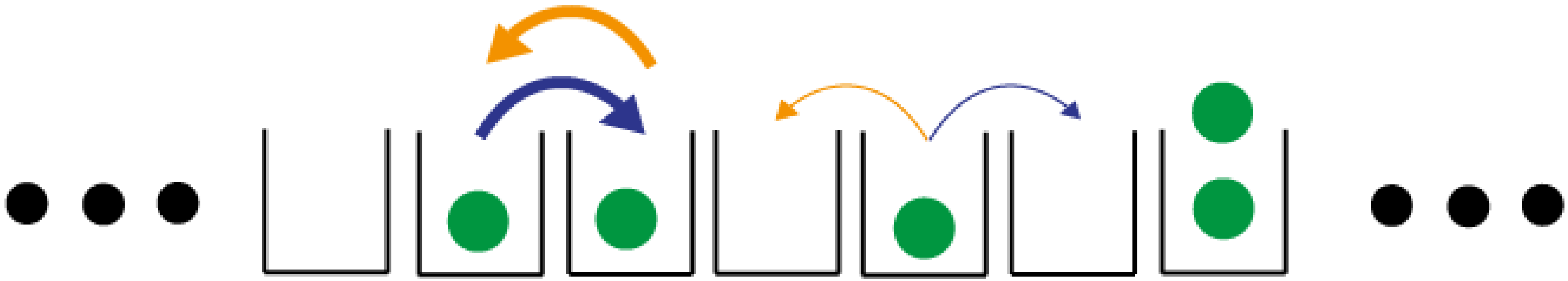
Bose-enhanced transport

Symmetric transport: opposite contributions cancel out



Bose-enhanced transport

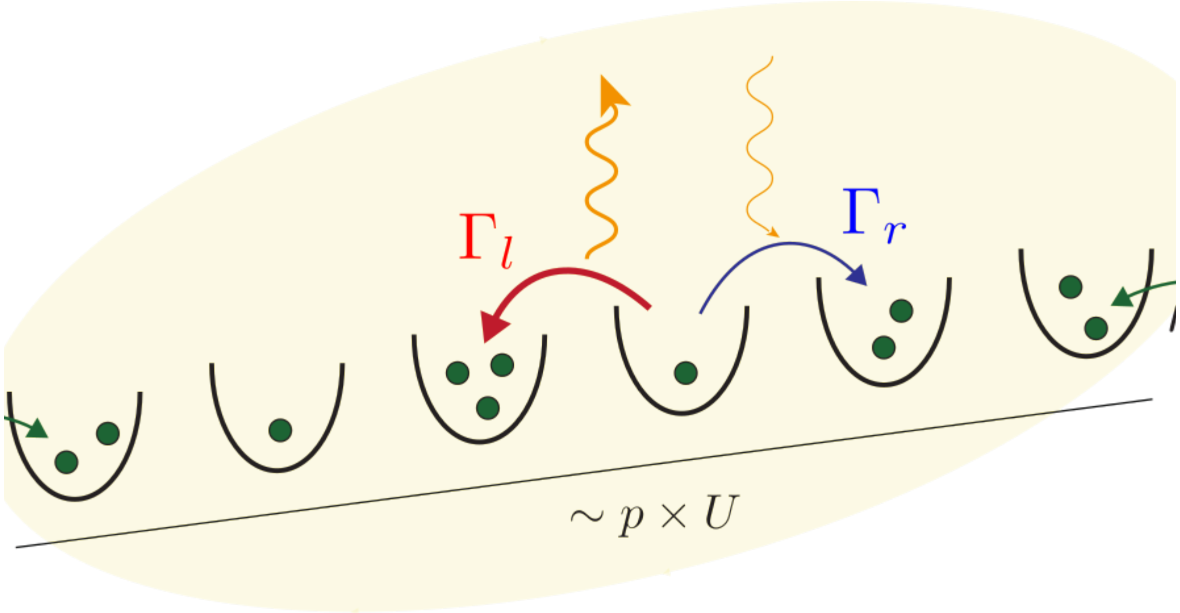
Need asymmetric transport



Our Model: hopping on a tilted lattice

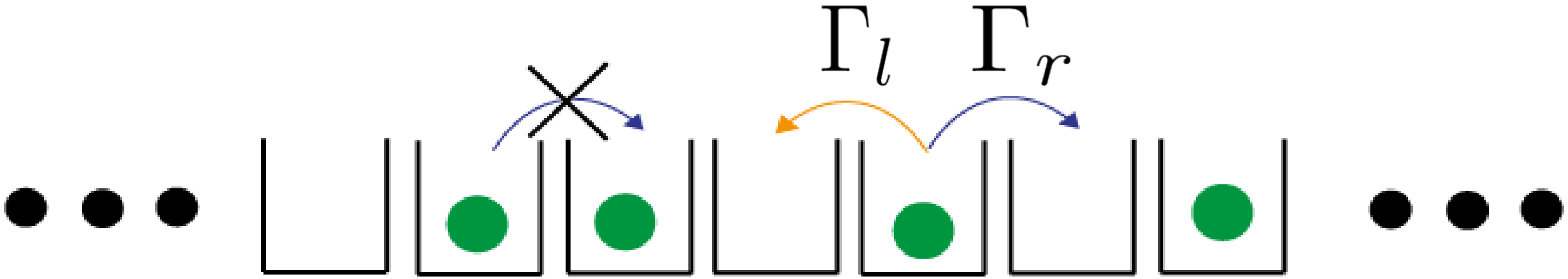
Asymmetry set by the **tilt** and the environment **temperature**

$$\Gamma_r \neq \Gamma_l$$



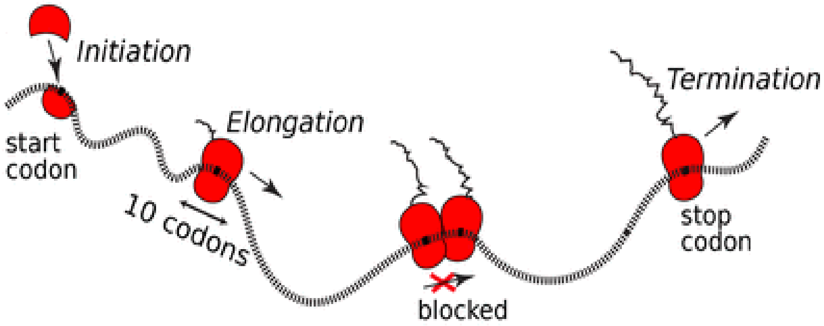
$$\mathcal{L}_{\text{hop}}\rho = \sum_{p=1}^{L-1} \Gamma_l \mathcal{D}[a_p^\dagger a_{p+1}]\rho + \Gamma_r \mathcal{D}[a_{p+1}^\dagger a_p]\rho$$

Asymmetric Simple Exclusion Process

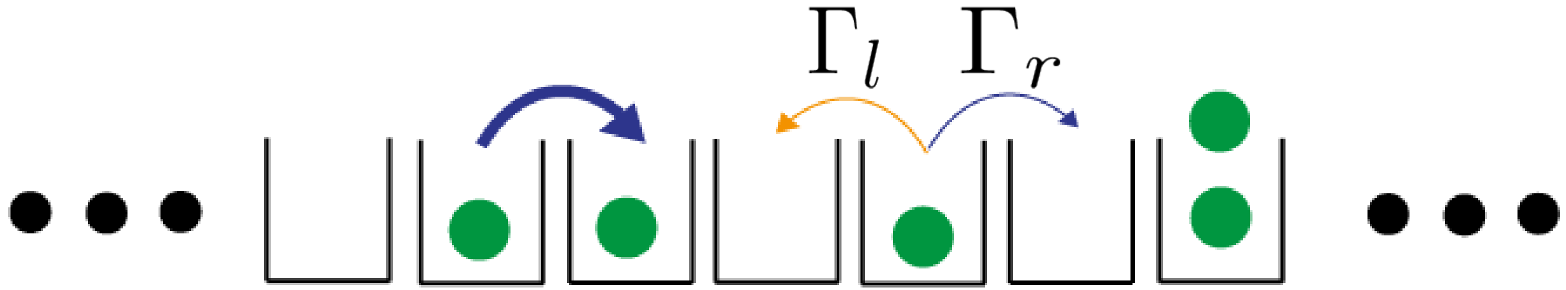


$$\Gamma_r \neq \Gamma_l$$

$$P[p \to p+1] \propto \Gamma_r(1 - n_{p+1})$$

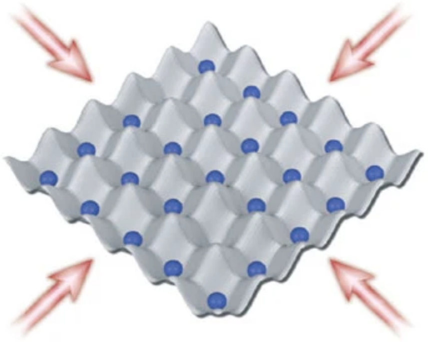


Asymmetric Simple Inclusion Process

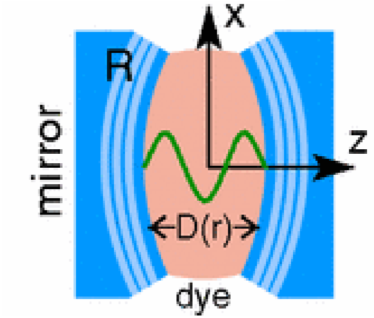


$$\Gamma_r \neq \Gamma_l$$

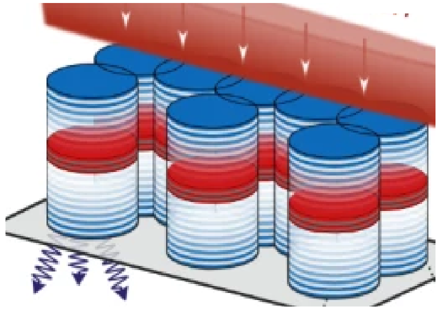
$$P[p \rightarrow p+1] \propto \Gamma_r(1+n_{p+1})$$



Cold atoms



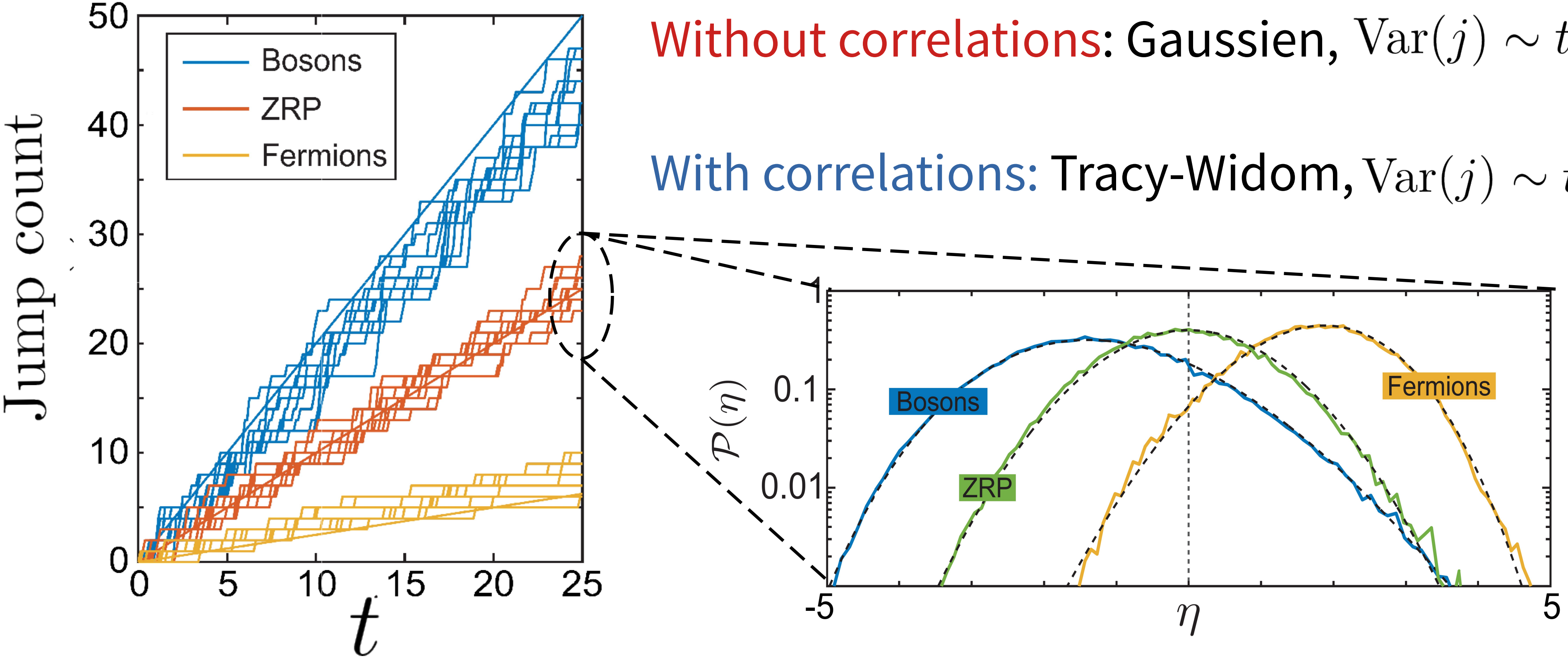
Photons condensates



Polaritons condensates

- Bosonic transport: the ASIP
- Non-Gaussian fluctuations
- Driven transport: the bosonic skin effect

Current fluctuations: KPZ statistics

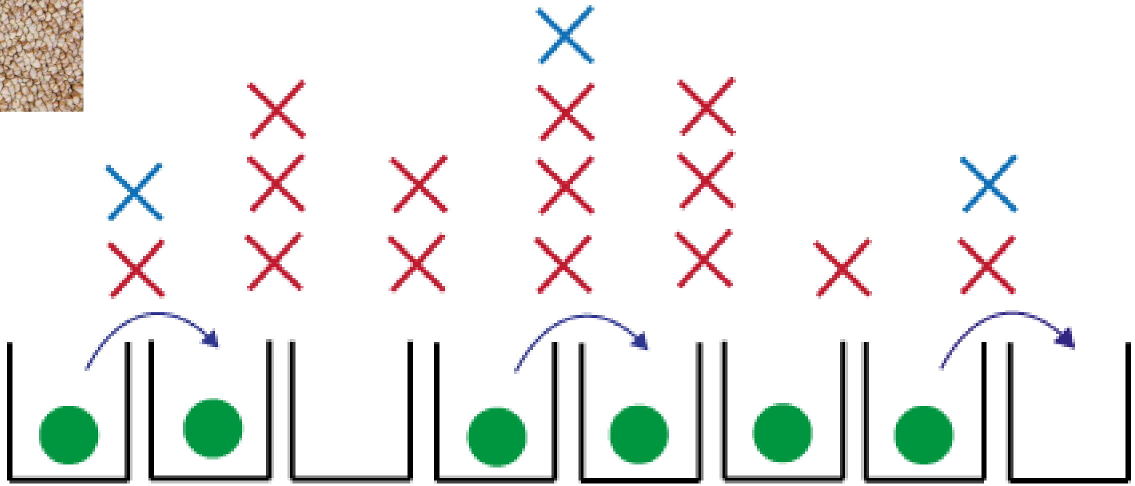


Without correlations: Gaussien, $\mathrm{Var}(j) \sim t^{1/2}$

With correlations: Tracy-Widom, $\mathrm{Var}(j) \sim t^{2/3}$

KPZ universality class

KPZ model: interface growth



- Bosonic transport: the ASIP
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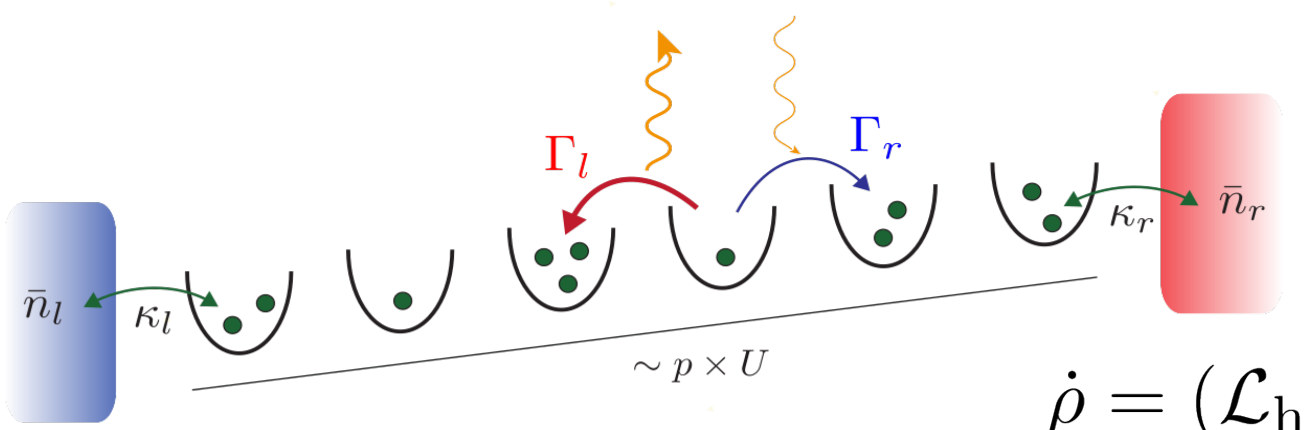
Driven transport

Out-of-equilibrium transport →
linear T gradient, Fourier law



$$\dot{\rho} = (\underbrace{\mathcal{L}_{\text{hop}}}_{\text{Chain}} + \underbrace{\mathcal{L}_{B,r} + \mathcal{L}_{B,l}}_{\text{Reservoirs}})\rho$$

Mean-field dynamics

We focus on mean values: $n_p = \langle a_p^\dagger a_p \rangle$

$$\frac{dn_p}{dt} = J_{p,p+1} - J_{p-1,p} = \nabla J$$

$$J_{p,p+1} \simeq \Gamma_l n_{p+1} - \Gamma_r n_p + (\Gamma_l - \Gamma_r) n_p n_{p+1}$$

Hydrodynamics equations

$$\Gamma_A = \Gamma_l - \Gamma_r$$

Long-wavelength limit: $n_i(t) \to n(x,t)$

$$\nu = \frac{\Gamma_l + \Gamma_r}{2}$$

$$\partial_t n = \Gamma_A(1-2n)\partial_x n + \nu \partial_x^2 n$$

Hydrodynamics equations

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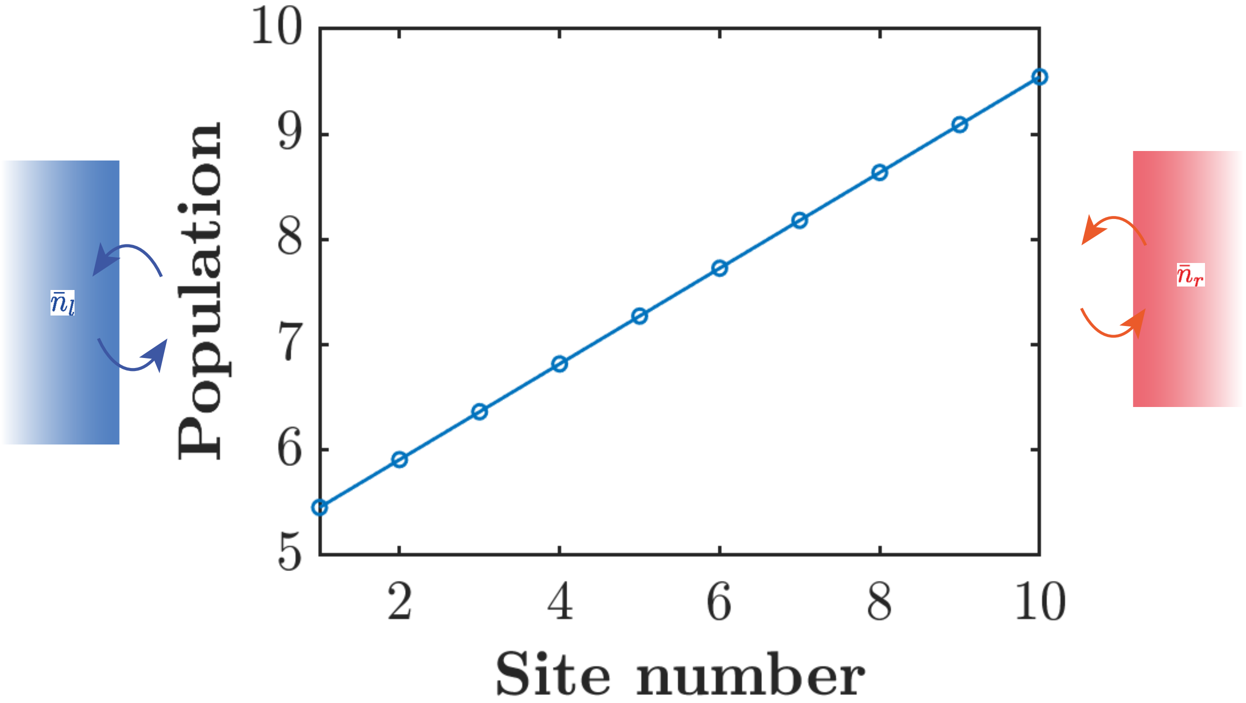
Burgers' equation

We recover diffusion behavior in the limit $\Gamma_r = \Gamma_l$

Density profile

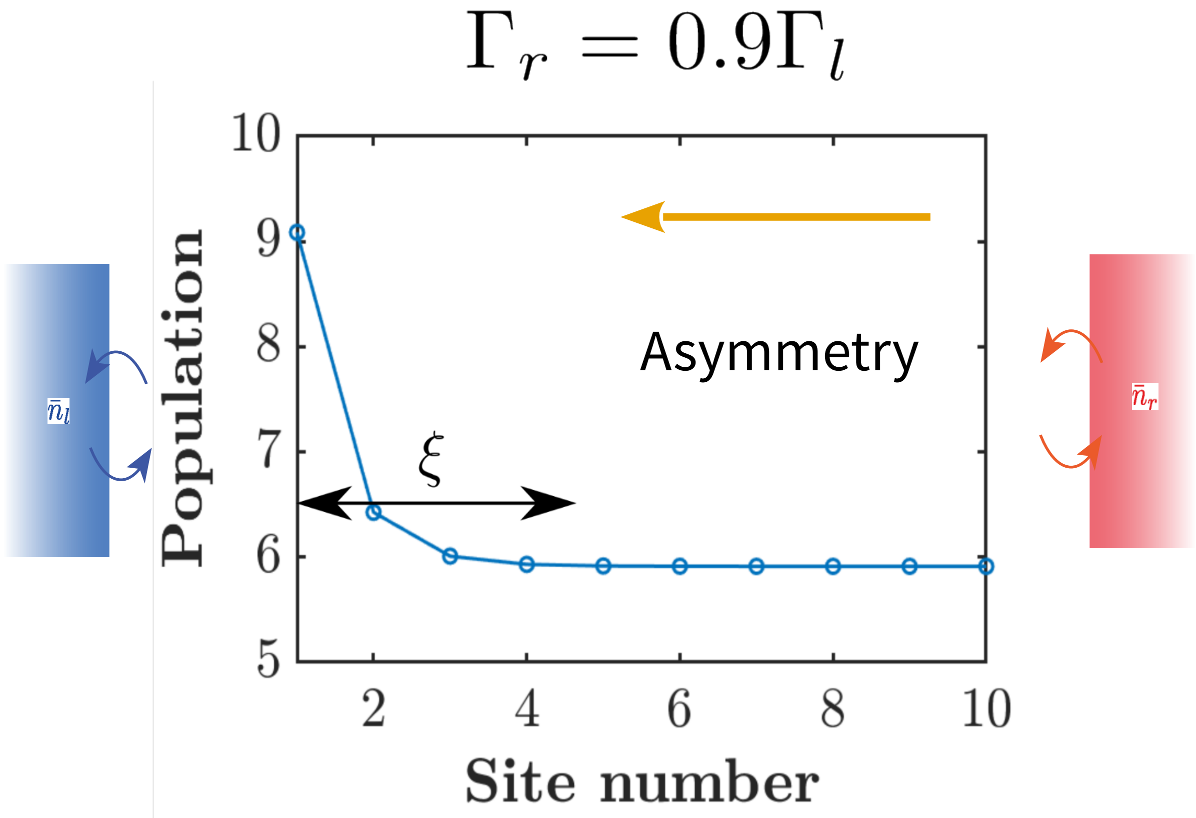
$$\Gamma_r = \Gamma_l$$



Diffusive transport

Linear population profile

$$J \propto \frac{\bar{n}_r - \bar{n}_l}{L}$$

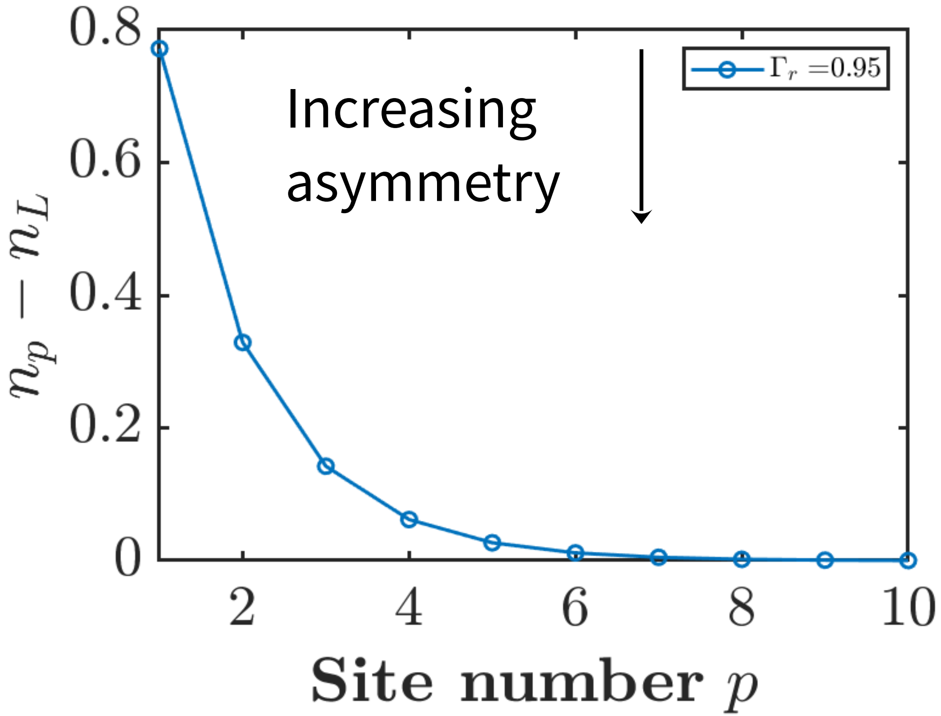


Accumulation on the edge
→ bosonic skin effect

Ballistic transport

$$J \propto \bar{n}_r$$

Asymmetry $\Gamma_l - \Gamma_r$ increases: profile more peaked?



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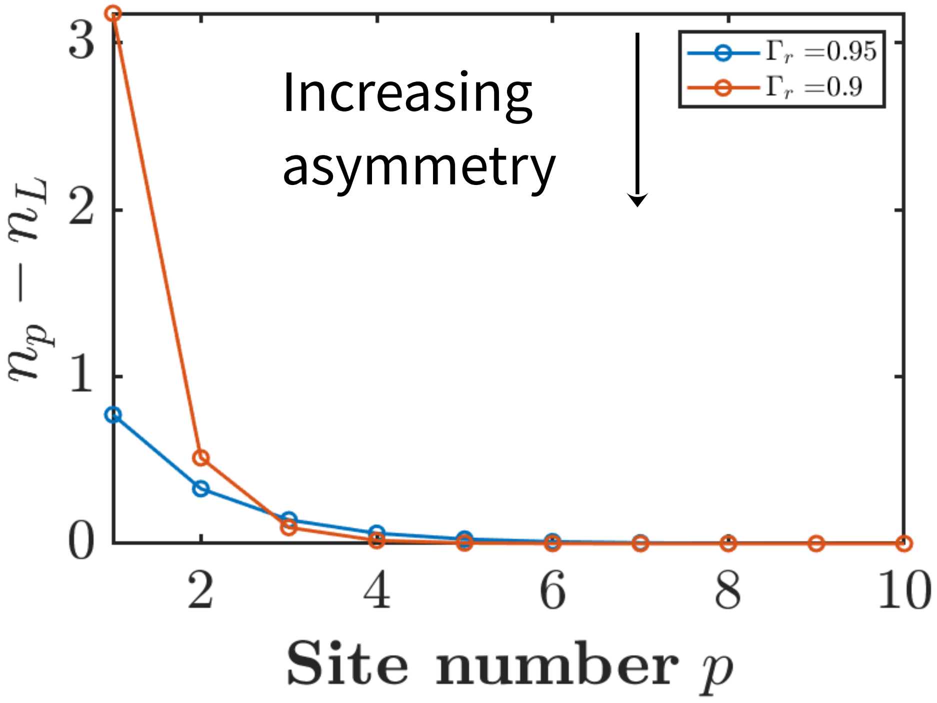

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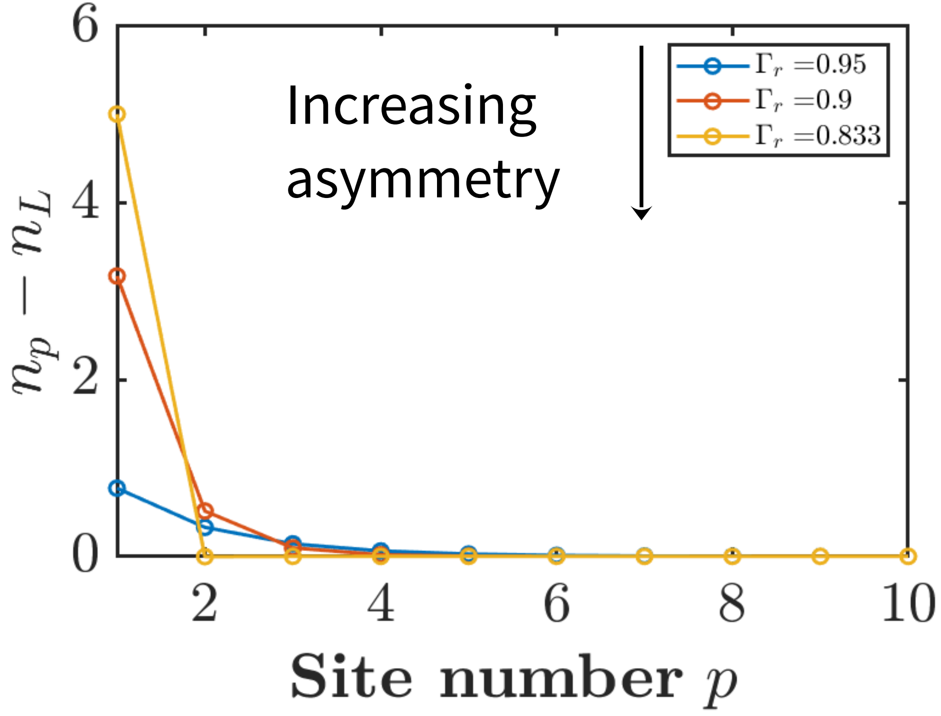

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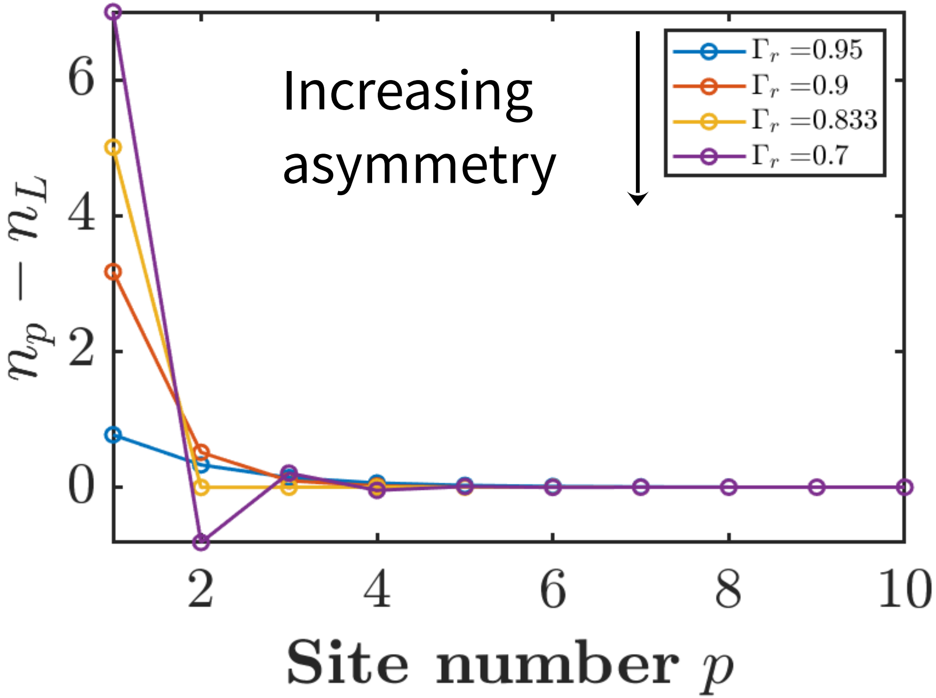

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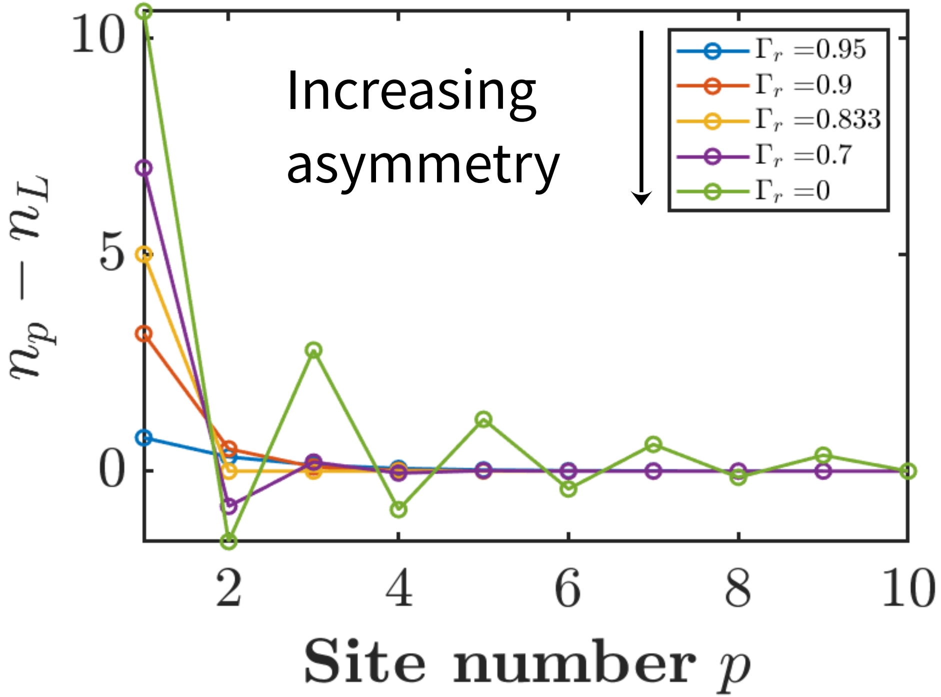

Zig-zag structure → failure of hydrodynamic treatment

Pile-up and clustering of particles → bosonic behavior

Staggered configuration

$$J_{p,p+1} \simeq \Gamma_l n_{p+1} - \Gamma_r n_p + (\Gamma_l - \Gamma_r) n_p n_{p+1}$$

$$\boxed{J_{p,p+1} \sim n_p n_{p+1}}$$

Extreme case: non-linear term only

Staggered configuration

$$J_{p,p+1} \simeq \Gamma_l n_{p+1} - \Gamma_r n_p + (\Gamma_l - \Gamma_r) n_p n_{p+1}$$

Extreme case: non-linear term only

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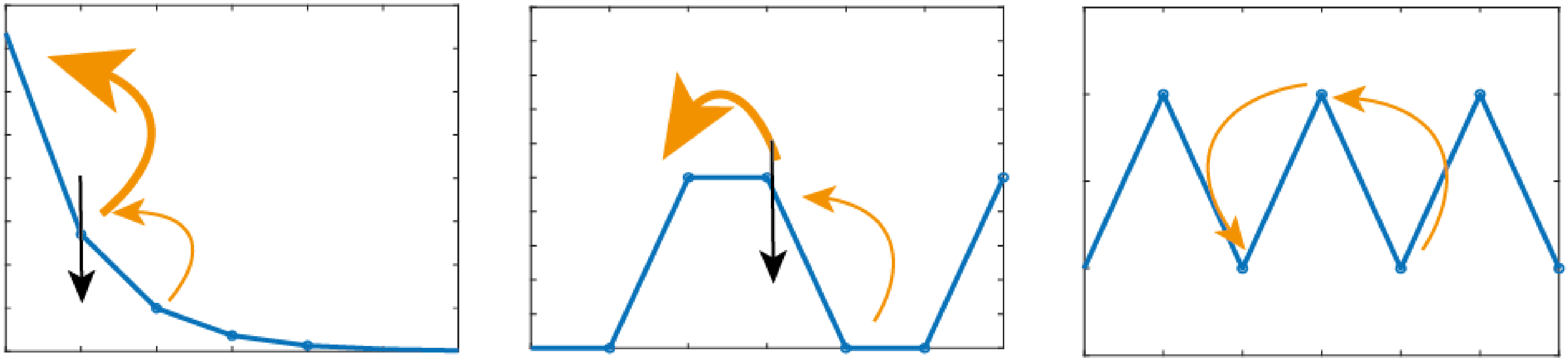


$n_1 > n_2 > n_3$	$n_1 = n_2 > n_3$	$n_1 = n_3$
$n_1 n_2 > n_2 n_3$	$n_1 n_2 > n_2 n_3$	$n_1 n_2 = n_2 n_3$
Unstable	Unstable	Stable

Population fluctuations

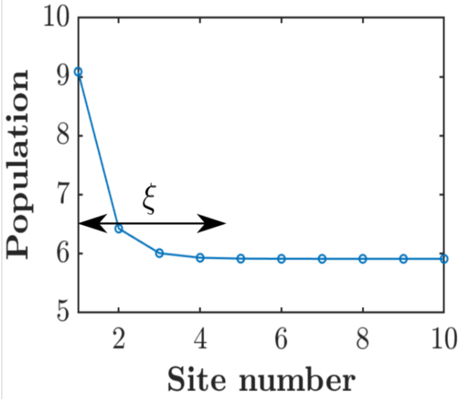
So far: we only considered the average population $\langle \hat{n}_p \rangle$. What about the fluctuations?

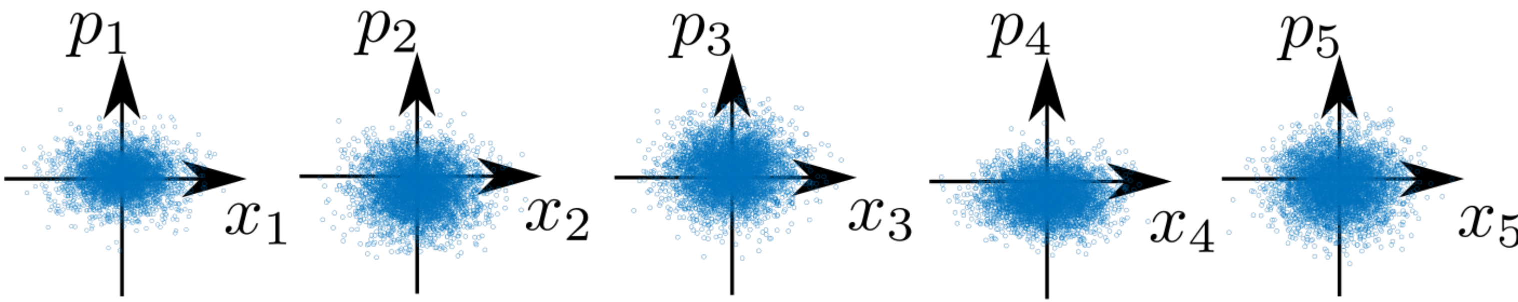
Wigner distribution in quadrature space:

$$x_q = a_q^\dagger + a_q$$

$$p_q = i(a_q^\dagger - a_q)$$

Weak asymmetry:





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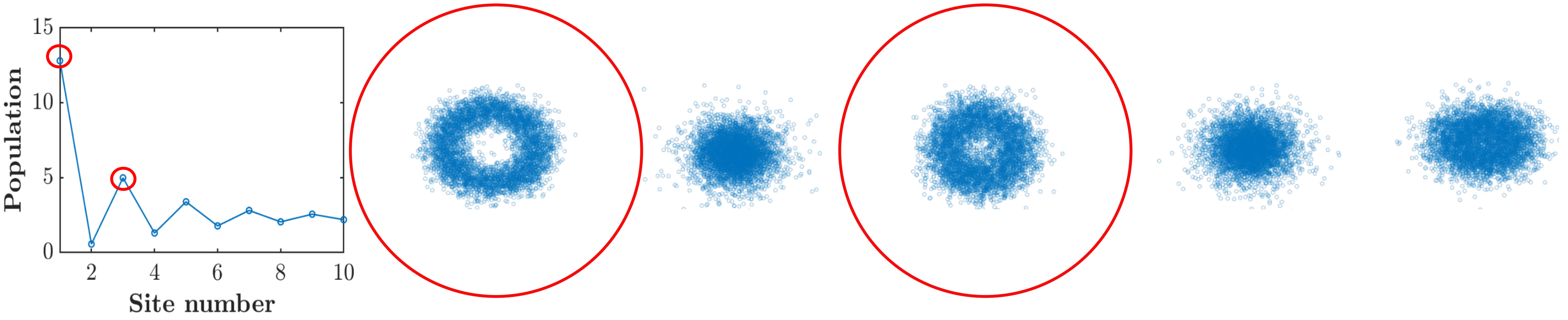
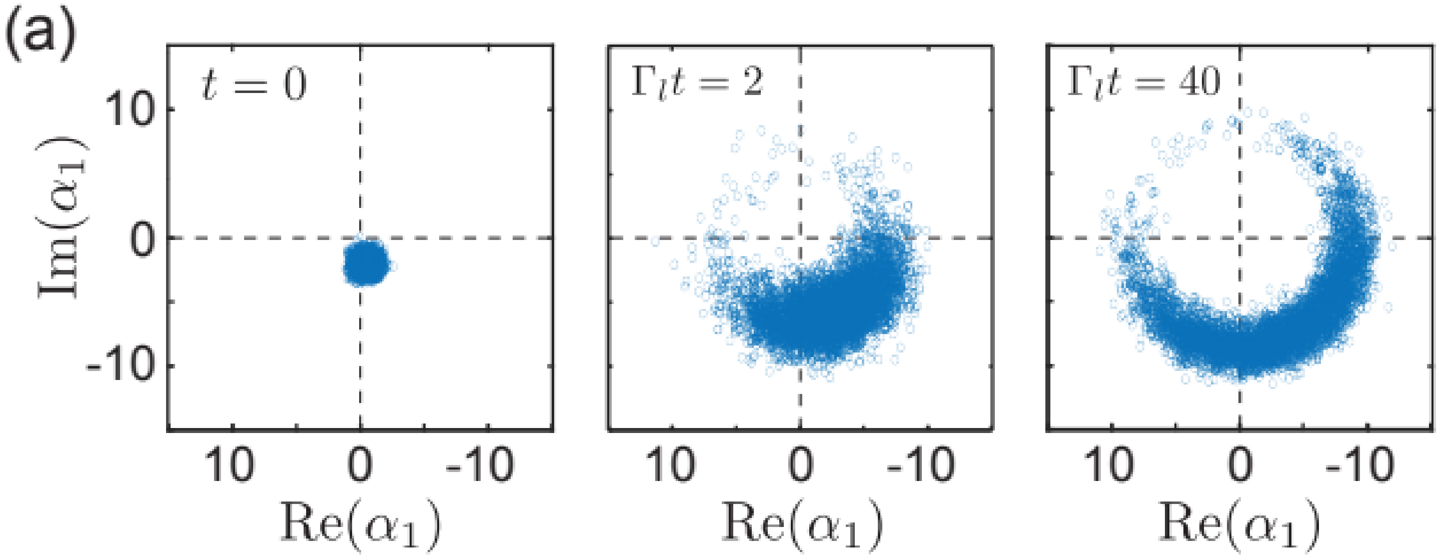
Strong asymmetry:



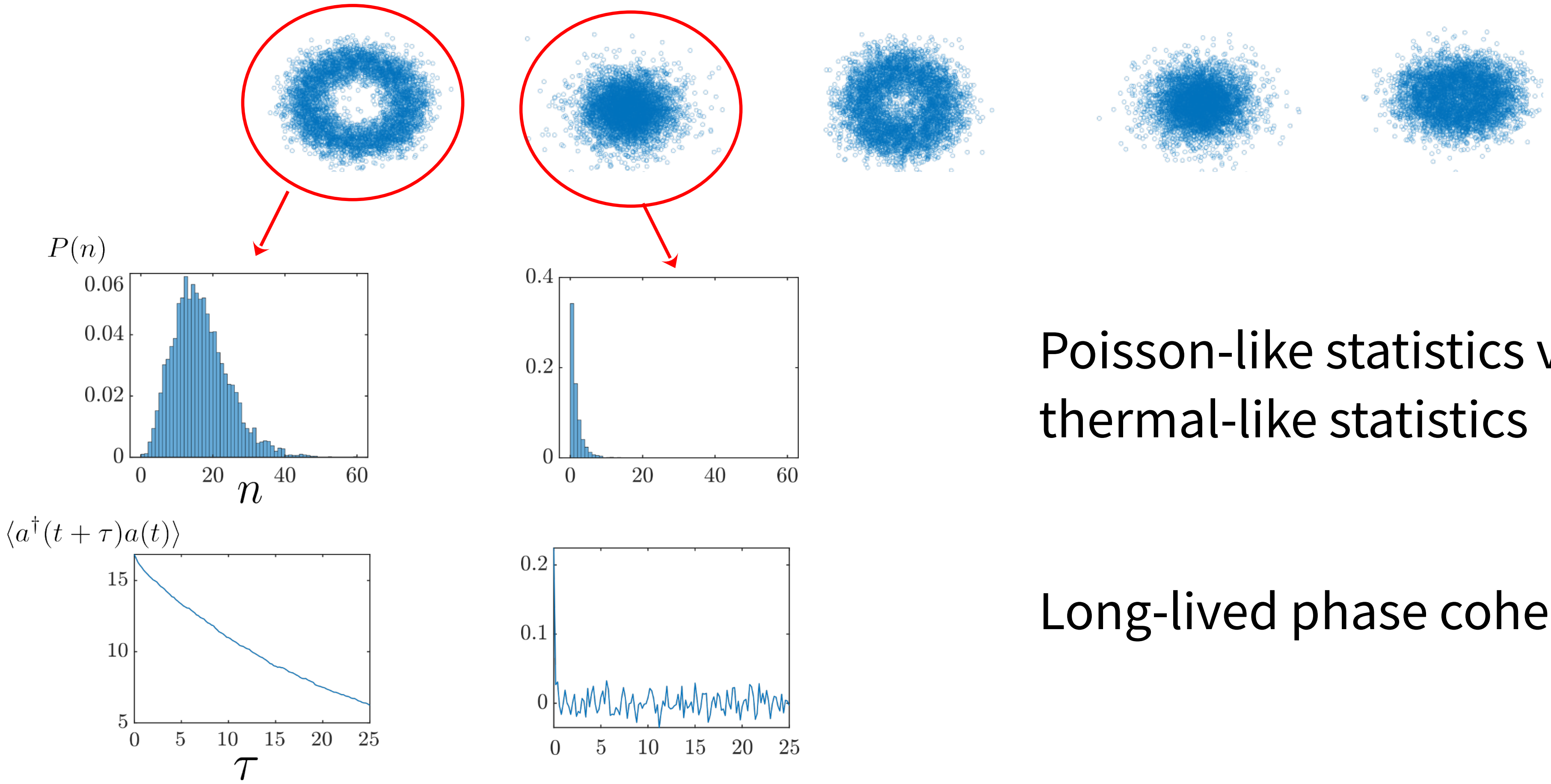
Out-of-equilibrium condensation



U(1) symmetry breaking→ lasing/condensation effect
on every other site

Signature of a properly bosonic behavior

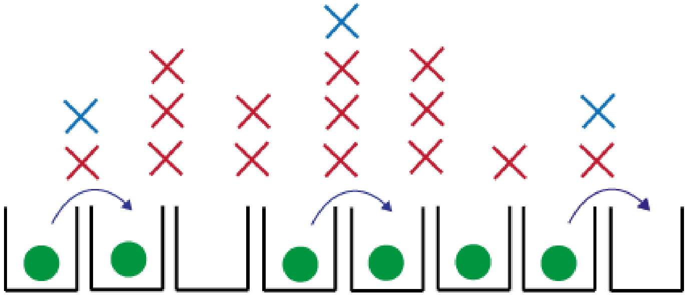
Out-of-equilibrium condensation

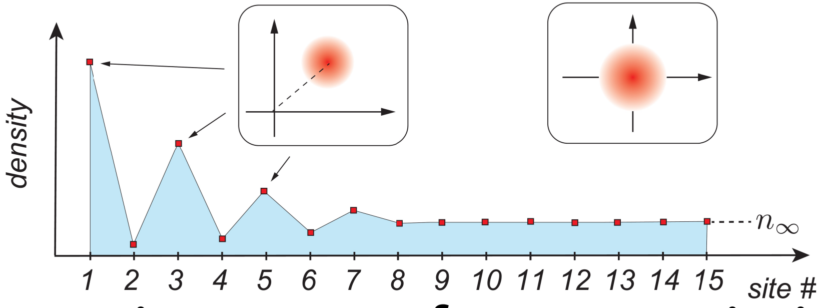


Poisson-like statistics vs thermal-like statistics

Long-lived phase coherence

Summary





- Even without coherence nor interaction, signature of Bose statistics
- Correlations induced in a transport scenario, hydrodynamic behavior
- Emergence of non-Gaussian statistics, in the KPZ class
- Bosonic skin effect: exotic condensation phase

No boson is an island, entire on its own;
Each is a piece of the condensate,
Of the collective wave.

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Each is a piece of the condensate,
Of the collective wave.

If a quanta be washed out, current is the less;
As well as if all of them where,
As well as if combinatorics itself
Was carried with it.

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Each is a piece of the condensate,
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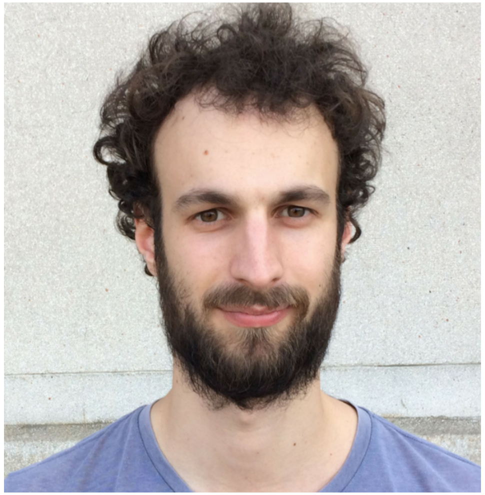
If a quanta be washed out, current is the less;
As well as if all of them where,
As well as if combinatorics itself
Was carried with it.

Every annihilator affects them all ,
For they are indistinguishable.
And therefore do not seek to know for which the bell tolls;
It tolls for them all.

Thanks for your attention!

Yuri Minoguchi



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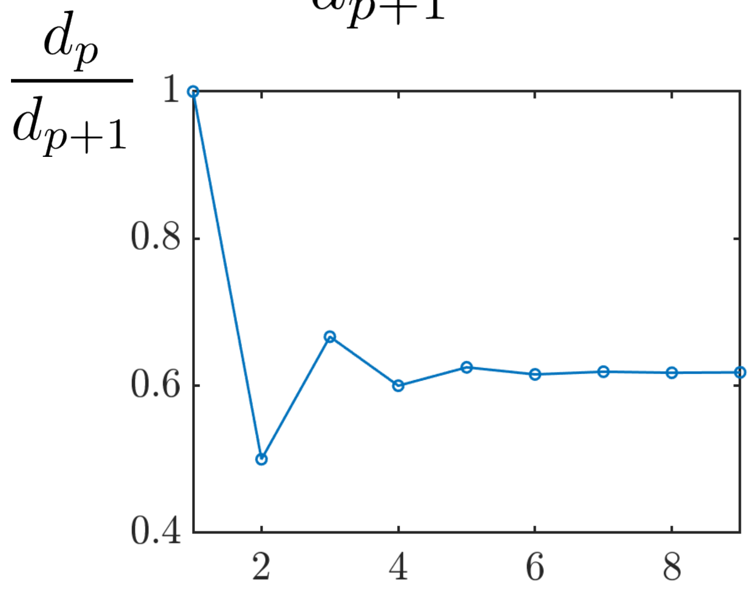
Outro: of rabbits and sunflower

Fibonacci sequence: $d_p = 1, 1, 2, 3, \ldots$

$$\frac{d_p}{d_{p+1}} \to \phi$$



Here: $n_p \propto \frac{d_p}{d_{p+1}}$

$$d_{p+2} = d_{p+1} + a d_p$$

Lucas sequence

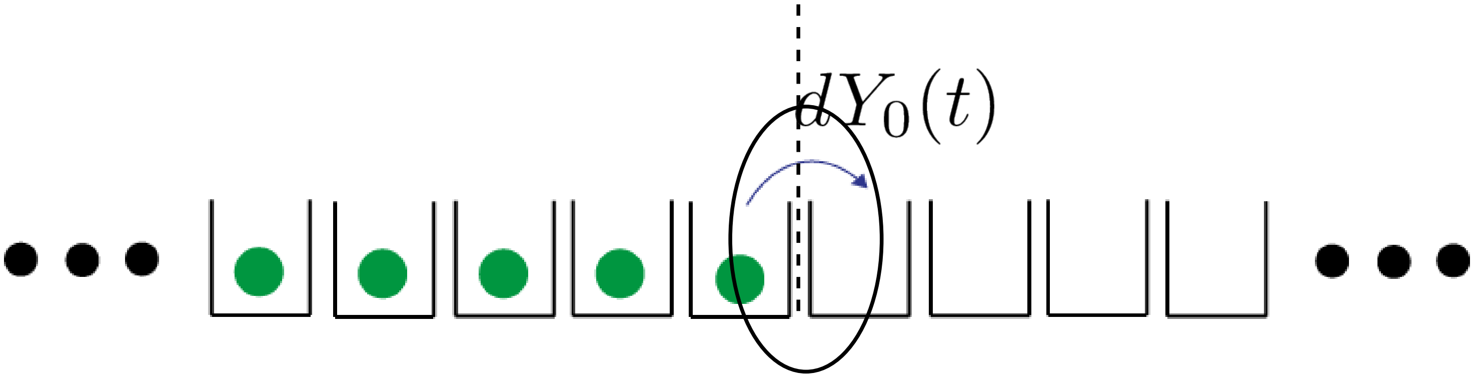
Integrated current

$$dn_i = dY_{i-1}(t) - dY_i(t)$$

Conservation relation

$$n_i \leftrightarrow h_i(t) = h_i(0) + \int_0^t dY_i(\tau)d\tau$$

Integrated current



Connection with non-Hermitian physics

Linearization: $$n_p(t) = n_\infty + \epsilon_p(t)$$ $$\frac{d\vec{\epsilon}}{dt} \sim -iH_{\text{eff}}\vec{\epsilon}$$

Dynamical matrix gives a *non-Hermitian* Hamiltonian

$$c = (\Gamma_r - \Gamma_l)n_\infty$$

$$\nu = \Gamma_r + \Gamma_l$$

$$H_{\text{eff}} = i\begin{bmatrix} c+\frac{\nu}{2} & \frac{\nu}{2}+c & 0 & 0 & 0 & \dots \\ \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & 0 & \dots \\ 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & \dots \\ 0 & 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix} - i\nu\mathbf{1}$$

Connection with non-Hermitian physics

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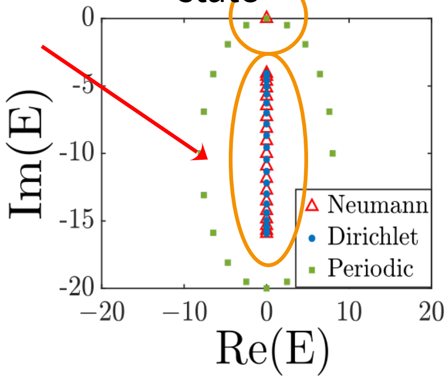
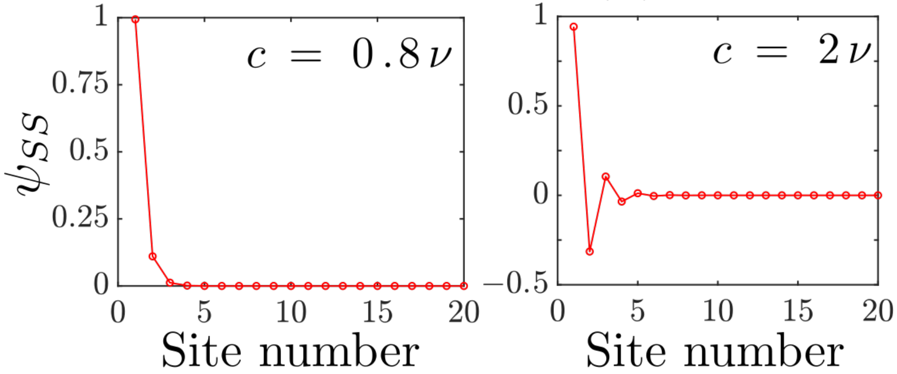
Stable steady-state

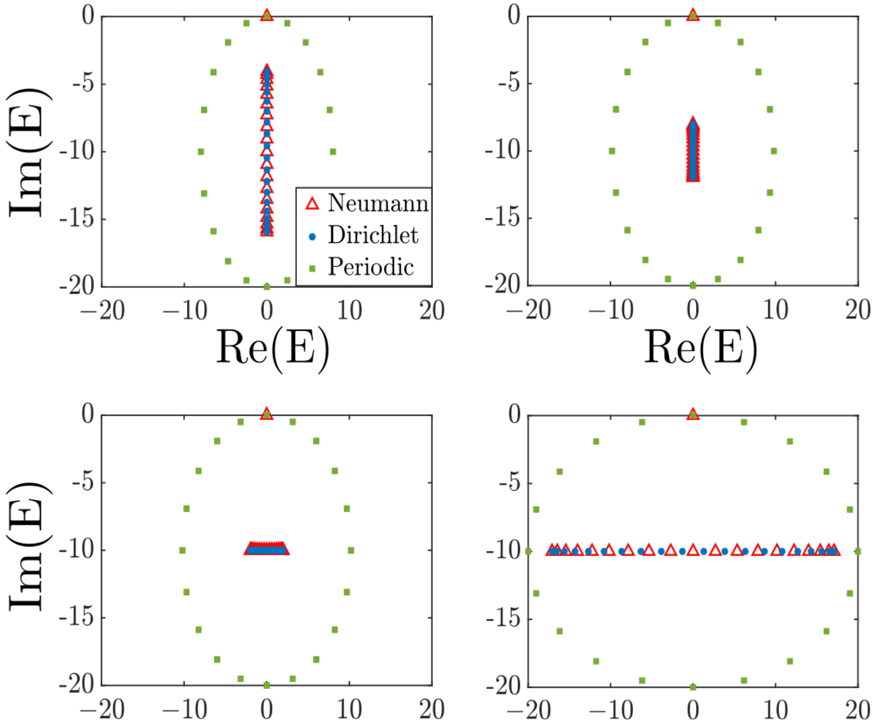
Decaying states



The steady-state shows a zig-zag transition for $c = \dfrac{\nu}{2}$



Connection with non-Hermitian physics



At the transition: excited states *coalesce*

$$H_{\text{eff}} = 2ci \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & -1 & 1 & 0 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & 0 & -1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix}$$

Non-diagonalizable Jordan form

→ *Exceptional point* in the excited states associated with the transition in the steady-state.